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Negative Conductivity in Solid State Avalanche Diodes

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ABSTRACT

A new and useful parameter (σ) for avalanche diodes is obtained which possesses the properties associated with negative conductivity. It is shown how σ unifies the description of various aspects of device behavior such as diode impedance Z, total current, and the effect of device radius on performance. A greatly simplified, approximate formula for Z is obtained, in terms of σ, which predicts reasonably well the significant trends, zero-crossings, and peaks.

Accepted for the Air Force
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NEGATIVE CONDUCTIVITY IN SOLID STATE AVALANCHE DIODES

I. INTRODUCTION

The resistance R of a simple, classical resistor is calculated in an elementary fashion from

$$R = \frac{L}{\sigma_o A} \tag{1}$$

which relates the geometric factors (L = length, A = cross-sectional area) and material parameter (σ_o = conductivity) to R. The impedance Z of a simple parallel-plate capacitor of separation L, and cross-sectional area A, completely filled with a lossy dielectric of permittivity ϵ and conductivity σ_o, is

$$Z = \frac{L}{(j\omega\epsilon + \sigma_o)\, A} \quad . \tag{2}$$

This report considers the description of an avalanche diode in a new simplified manner using an effective conductivity σ. The impedance Z_a of the avalanche region will be shown to be given approximately by

$$Z_a \simeq \frac{L}{(j\omega\epsilon + \sigma_1)\, A} \quad . \tag{3}$$

In addition, the total current density (particle plus displacement) will be shown to be specified without approximation by

$$J_T = (\sigma_1 + j\omega\epsilon)\, B_1 \tag{4}$$

(where B_1 is an amplitude constant) and the radial variation of the electric field and currents will be shown to be given for the normal IMPATT mode by $J_o(Tr)$, where

$$T^2 = -j\omega\mu_o(\sigma_1 + j\omega\epsilon) \quad , \tag{5}$$

μ_o = material permeability, and $J_o(x)$ is the Bessel function of the 1st kind and of zeroth order.

Previous "exact" expressions for Z_a in the literature are algebraically complex expressions which require lengthy numerical calculations before their content can be made explicit. The dependence of the total current, J_T, on radian frequency (ω), DC current density bias (J_o), and the material parameters has not been previously developed in the literature. Finally, the radial variation of E_x, $J_n = -qV_s n$, and $J_p = -qV_s p$ (the electron and hole current densities with V_s = saturated drift speed) has not been previously evaluated, although they may begin to be noticeable in sufficiently large-diameter ring diodes of the type described by Gibbons and Misawa.[1]

II. TOTAL CURRENT DENSITY

The total current density is defined by

$$J_T = J_n + J_p + j\omega\epsilon\, E_x \quad . \tag{6}$$

It is shown in the Appendix that

$$E_x = \sum_{i=1}^{3} E_i \qquad \left(E_i \equiv B_i\, e^{-jK_i x}\right) \quad , \tag{7}$$

$$J_n + J_p = \sum_{i=1}^{3} \sigma_i E_i \quad , \tag{8}$$

where B_1, $B_2 = B_3$ are amplitude coefficients, and

$$\sigma_i = \frac{-2j\omega\alpha_o' J_o V_s}{\omega^2 - (K_i V_s)^2 + 2j\omega\alpha_o V_s} \qquad (i = 1, 2, 3) \tag{9}$$

are temporally and spatially dependent conductivities, $K_1 = 0$, and

$$K_2 = -K_3 = \sqrt{(\omega/V_s)^2 + 2j(\omega/V_s)\,\alpha_o - 2\alpha_o' J_o/\epsilon V_s}$$

are wavenumbers. Here α_o and α_o' are the ionization coefficient and $d\alpha/dE$ evaluated at the DC electric field value E_o. By direct addition of (8) and $j\omega\epsilon$ times (7),

$$J_T = (\sigma_1 + j\omega\epsilon)\, B_1 + 2B_2(\sigma_2 + j\omega\epsilon)\cos(K_2 x) \quad . \tag{10}$$

Direct calculation yields $\sigma_2 = -j\omega\epsilon$, which leaves the final result

$$J_T = (\sigma_1 + j\omega\epsilon)\, B_1 \quad , \tag{11}$$

where

$$\sigma_1 = -\frac{2j\alpha_o' J_o V_s}{\omega + 2j\alpha_o V_s}$$

or

$$J_T = \left(\frac{j\omega\epsilon + 2\alpha_o V_s \epsilon - 2j\alpha_o' J_o V_s}{\omega + 2j\alpha_o V_s}\right) B_1 \tag{12}$$

for which normalized values are plotted in Fig. 1.

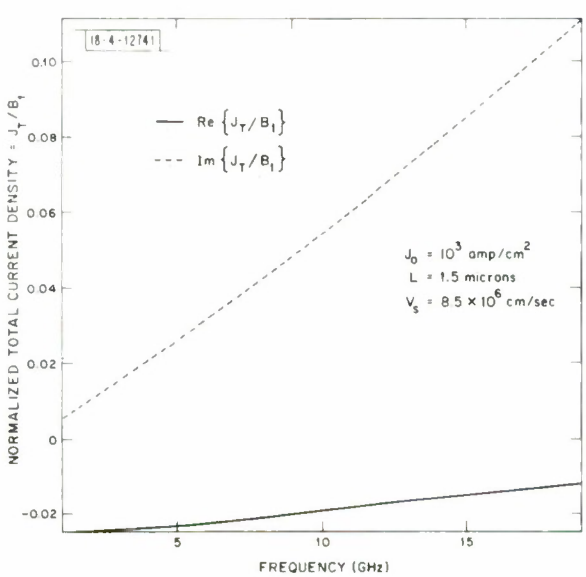

Fig. 1. The real and imaginary components of the total current density (J_T) as a function of frequency (f) for an X-band silicon diode in which the avalanche zone is 1.5 microns in length, $J_o = 10^3$ amp/cm^2, $\alpha_o = 6.6 \times 10^3$, $\alpha_o' = 0.164$, and $\epsilon = 10^{-12}$.

III. DEVICE IMPEDANCE

The impedance is defined by

$$Z_a = V/J_T A \quad , \tag{13}$$

where A = cross-sectional area, and

$$V = -\int_{-L/2}^{+L/2} E_x \, dx \tag{14}$$

is the voltage across the diode. Neglecting space-charge, integration of Eq. (6) (see Appendix) gives

$$E_x = \text{constant} = B_1$$

so that

$$V = -E_x L = -B_1 L \quad . \tag{15}$$

From Eqs. (11), (13), and (15), Eq. (3) is obtained for Z_a. The exact result (see Appendix) is

$$Z_a = -\frac{\left(1 + \dfrac{M \sin\theta_2}{\theta_2}\right) L}{(\sigma_1 + j\omega\epsilon)} \quad , \tag{16}$$

where

$$\theta_2 = K_2 L/2 = (L/2)\sqrt{(\omega/V_s)^2 + 2j(\omega/V_s)\,\alpha_o - 2\alpha_o' J_o/\epsilon V_s} \quad ,$$

and

$$M = \frac{-2j\alpha_o' J_o V_s}{\epsilon\,(\omega + 2j\alpha_o V_s)\,(j\omega \cos\theta_2 - V_s K_2 \sin\theta_2)} \quad . \tag{17}$$

Figures 2 through 5 show that the activity threshold and peak impedance frequencies are reasonably well predicted by the approximate result of Eq. (3) (in conjunction with the drift zone impedance), although the values of Z_a in the vicinity of these points are not precise. Thus, the significant trends, peaks, and zero-crossings are revealed by Eq. (3), although the magnitudes show only semi-quantitative agreement in certain ranges. However, this is a far better approximation for Z_a than provided by previous simplified analyses, such as that of Gilden and Hines,[2] which, for example, shows no negative resistance effects associated with the avalanche region.

IV. RADIAL VARIATIONS

A lengthier analysis[3] reveals that the usual quasi-static, one-dimensional picture may, in a more accurate field theory, be described as a quasi-TEM radial wave mode. One may attempt to approximate this by a TEM radial wave mode (which will only satisfy boundary conditions approximately). The equations for a radial mode, with the only non-zero components being E_x and H_φ, are

$$\frac{1}{r}\frac{\partial}{\partial r}(rH_{\varphi i}) = J_{Ti} = (\sigma_i + j\omega\epsilon)\, E_i \quad , \tag{18}$$

$$\frac{\partial E_i}{\partial r} = j\omega\mu_o H_{\varphi i} \quad , \tag{19}$$

which combine to yield

$$\frac{1}{r}\frac{\partial}{\partial r}\, r\left(\frac{\partial E_i}{\partial r}\right) = j\omega\mu_o(\sigma_i + j\omega\epsilon)\, E_i \equiv T_i^2 E_i \quad . \tag{20}$$

The non-singular solution to Eq. (20) is

$$E_i = J_o(T_i r)\, B_i\, e^{-jK_i x} \qquad (i = 1, 2, 3) \quad . \tag{21}$$

By direct calculation $T_1^2 = -j\omega\mu_o(\sigma_1 + j\omega\epsilon)$, shown in Fig. 6, while $\sigma_2 = \sigma_3 = -j\omega\epsilon$ implies that $T_2^2 = T_3^2 = 0$. This behavior is peculiar in that the radial TM wave mode, with non-zero E_x, H_φ, and E_r field components (each of which varies as $J_o(Tr)$ with the same T), does not smoothly reduce to the radial TEM mode wave. When $|T|\, r << 1$, then $J_o(Tr) \simeq 1$ and the usual one-dimensional results are retrieved. The current density is similarly described as

$$J_n + J_p = \sum_{i=1}^{3} J_o(T_i r)\, \sigma_i B_i\, e^{-jK_i x} \quad . \tag{22}$$

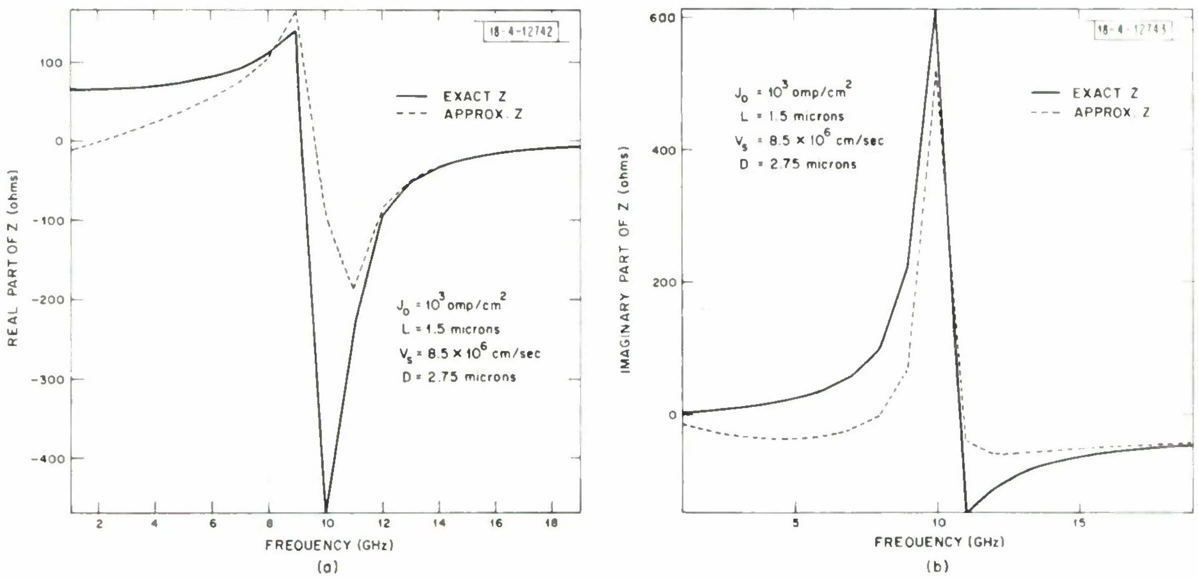

Fig. 2. The approximate and exact silicon IMPATT diode impedance vs frequency (f) for a diode in which the avalanche zone (L) is 1.5 microns in length, the drift zone (D) is 2.75 microns, $J_o = 10^3\,\mathrm{amp/cm^2}$, $\alpha_o = 6.6 \times 10^3$, $\alpha_o' = 0.164$. (a) Real part; (b) imaginary part.

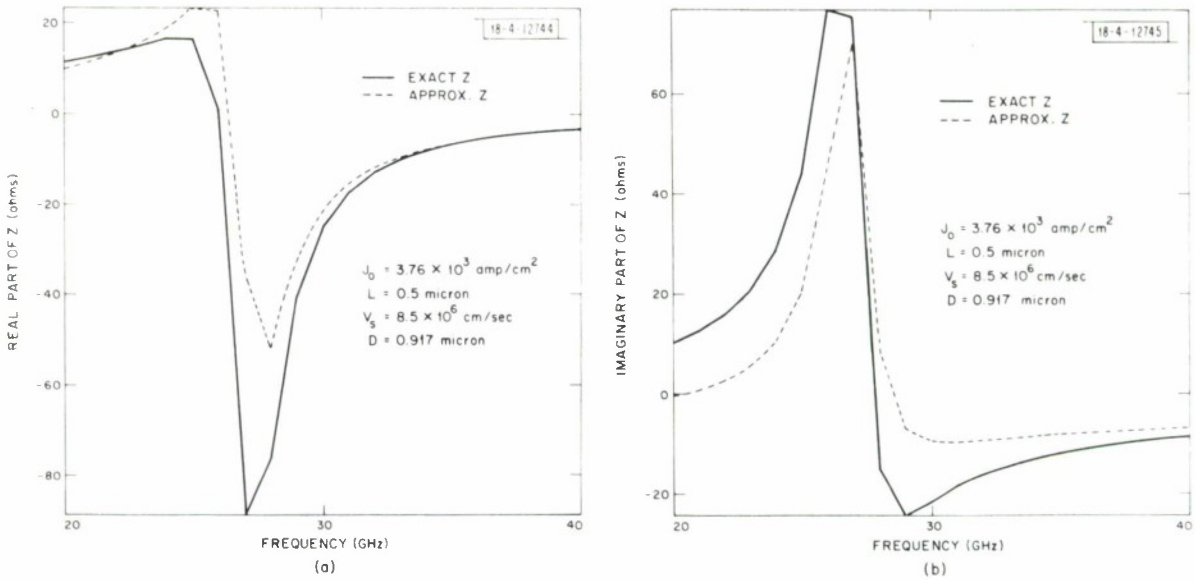

Fig. 3. The approximate and exact silicon IMPATT diode impedance vs frequency (f) for a diode in which the avalanche zone (L) is 0.5 micron in length, the drift zone (D) is 0.92 micron, $J_O = 3.76 \times 10^3$ amp/cm^2, $\alpha_O = 2 \times 10^4$, $\alpha'_O = 0.315$. (a) Real part; (b) imaginary part.

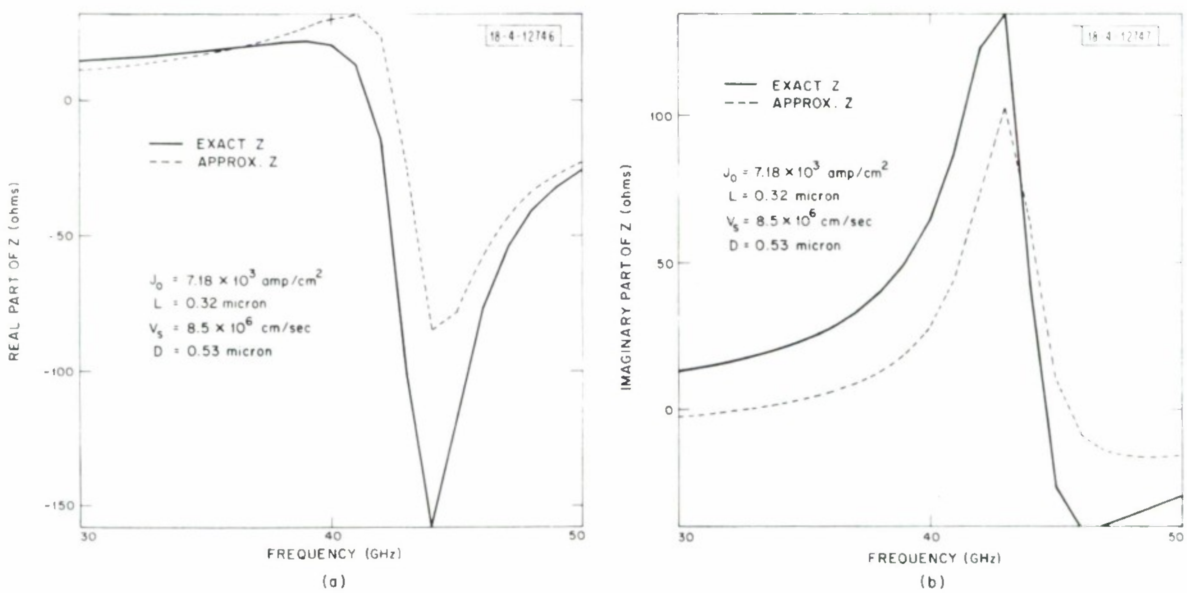

Fig. 4. The approximate and exact silicon IMPATT diode impedance vs frequency (f) for a diode in which the avalanche zone (L) is 0.32 micron in length, the drift zone (D) is 0.53 micron, $J_O = 7.18 \times 10^3$ amp/cm^2, $\alpha_O = 3.125 \times 10^4$, $\alpha'_O = 0.427$. (a) Real part; (b) imaginary part.

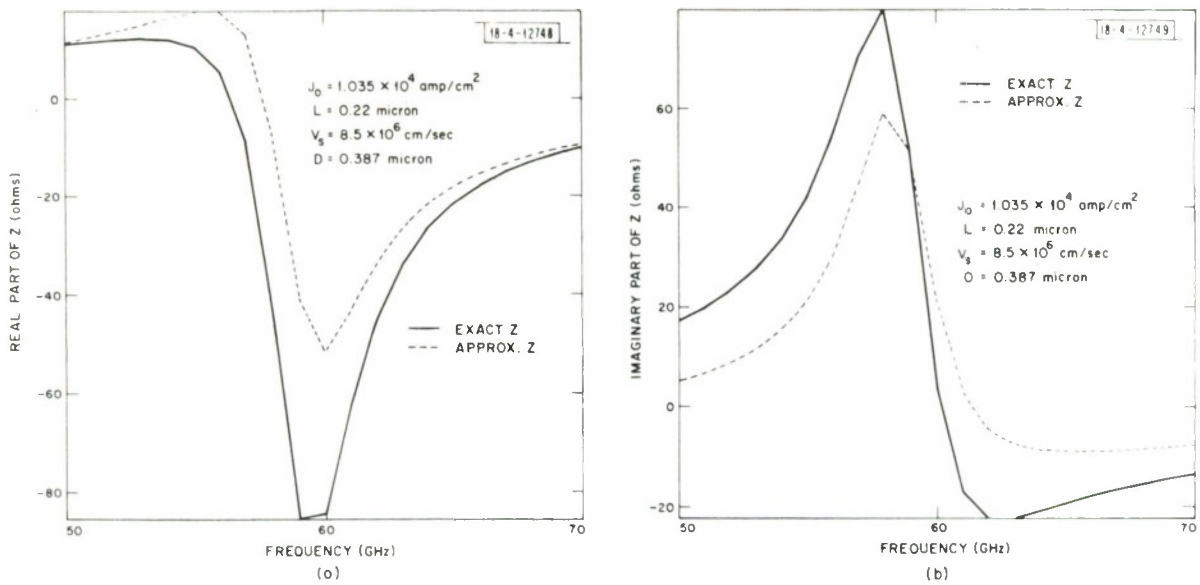

Fig. 5. The approximate and exact silicon IMPATT diode impedance vs frequency (f) for a diode in which the avalanche zone (L) is 0.22 micron in length, the drift zone (D) is 0.387 micron, $J_o = 1.035 \times 10^4$ amp/cm^2, $\alpha_o = 4.545 \times 10^4$, $\alpha'_o = 0.542$. (a) Real part; (b) imaginary part.

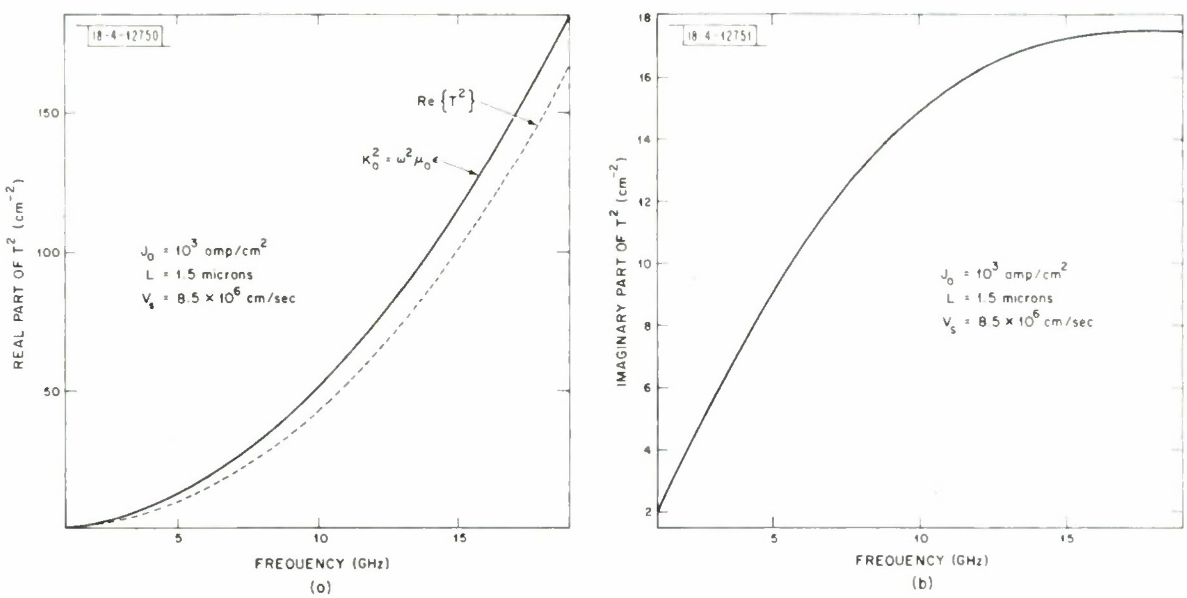

Fig. 6. The real and imaginary parts of T^2 vs frequency (f) for an X-band diode with L = 1.5 microns, $J_o = 10^3$ amp/cm^2, $\alpha_o = 6.6 \times 10^3$, $\alpha'_o = 0.164$, and $\epsilon = 10^{-12}$ farads/cm. The square of the radial wavenumber (for a TEM cylindrical wave) K_o^2 (which is purely real) is indicated for comparison. (a) Re $\{T^2\}$; (b) Im $\{T^2\}$.

APPENDIX
IMPEDANCE DERIVATION

The continuity equations for electron and hole currents are[4]

$$\partial n/\partial t = (1/q)\, \partial J_n/\partial x + g \quad , \tag{23}$$

$$\partial p/\partial t = -(1/q)\, \partial J_p/\partial x + g \quad , \tag{24}$$

where $g = \alpha_o V_s(n + p) + \alpha_o' V_s E_x(n_o + p_o)$ is the small-signal AC component of the electron-hole generation rate, V is the saturated drift speed of the carriers, n and p are the AC electron and hole current densities, and $n_o + p_o$ are the DC electron and hole current densities. After some algebraic manipulation, along with the assumption that all quantities vary as $e^{j\omega t}$, Eqs. (23) and (24) may be written

$$D_n n - \alpha_o V_s p - C E_x = 0 \quad , \tag{25}$$

$$D_p p - \alpha_o V_s n - C E_x = 0 \quad , \tag{26}$$

where

$$D_n \equiv (\partial/\partial t) - V_s \partial/\partial x - \alpha_o V_s) \quad ,$$

$$D_p \equiv (\partial/\partial t + V_s \partial/\partial x - \alpha_o V_s) \quad ,$$

$$C \equiv \alpha_o' V_s^2 (n_o + p_o) \quad .$$

From these equations it can be readily shown that

$$\begin{aligned} [D_n D_p - (\alpha_o V_s)^2]\, J_n &= -qV_s C\, [D_p + \alpha_o V_s]\, E_x \\ [D_n D_p - (\alpha_o V_s)^2]\, J_p &= -qV_s C\, [D_n + \alpha_o V_s]\, E_x \quad . \end{aligned} \tag{27}$$

For $e^{j(\omega t - K_i x)}$ dependence it is recognized that Eqs. (6), (23), and (24) yield a third-order system of equations for which the cubic dispersion relation reveals the three roots for K_i given in Sec. II. Then E_x, J_n, and J_p must have the form

$$E_x = \sum_{i=1}^{3} E_i \qquad \left(E_i \equiv B_i\, e^{-jK_i x}\right) \quad , \tag{7}$$

$$J_n = \sum_{i=1}^{3} \sigma_{ni} E_i \quad , \qquad J_p = \sum_{i=1}^{3} \sigma_{pi} E_i \quad , \tag{28}$$

where σ_{ni} and σ_{pi} can be determined by substituting Eq. (7) into Eqs. (27) and turn out to be given by

$$\sigma_{ni} = \frac{\sigma_i}{2}\left[1 + \frac{VK_i}{\omega}\right] \quad ,$$

$$\sigma_{pi} = \frac{\sigma_i}{2}\left[1 - \frac{VK_i}{\omega}\right] \quad , \tag{29}$$

where σ_i is given in Eq. (9). Applying the usual boundary conditions

$$J_n = 0 \quad \text{at} \quad x = -L/2$$

$$J_p = 0 \quad \text{at} \quad x = L/2 \tag{30}$$

it follows that $B_2 = B_3$ and

$$B_2/B_1 = B_3/B_1 = M \tag{31}$$

where M is defined in Eq. (17). Thus, Eq. (7) can be written

$$E_x = B_1 \left[1 + M \cos(K_2 X)\right] \quad .$$

Eq. (18) is obtained by summing Eqs. (28).

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13. ABSTRACT

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14. KEY WORDS

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