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SUMMARY

This Report gives an analytical solution for the axial flow of gas in gas-film lubricated journal bearings with film thickness varying around the journal, when subjected to different constant end pressures. The mass flow formulae are shown to be the same as for stationary concentric circular cylinders, except that the clearance cubed is replaced by the mean of the clearance cubed. An extended form of Elrod's mass content rule is also obtained.

Departmental Reference: Space 299

CONTENTS

1 INTRODUCTION

Gas lubricated journal and thrust bearing assemblies are often arranged so that the journal bearing connects a pair of opposed thrust bearings. When an axial load is applied to such a bearing system, there is a flow of gas along the journal if the centres of the thrust plates are not vented to the atmosphere. This flow must be balanced by a coupled flow of gas across the faces of the thrust plates, which causes a reduction in the axial stiffness of the bearing system.

Until recently[1,2] in gyro design, axial flow has been ignored largely because of the complexity of computation. One reason for this is that the clearance usually varies around the bearing, either because of deliberate mutilations of the surface intended to suppress the half-speed instability or because the bearing is deflected by a load. However under certain conditions, the axial flow through journal bearings can be found analytically. This simplifies the computations required for thrust plate performance.

The axial flow analysis given in this Report applies to rotating seal bearings as well as gyro-type bearings.

The conditions under which the axial flow can be found analytically are introduced in sections 2 and 3. Basically they are all introduced to make the Reynolds equation integrable. These restricting conditions are then listed in section 6.

Section 4 gives an extended form of Elrod's[3] mass content rule and in section 5 the integral around the bearing of the clearance cubed is found for typical clearance profiles.

2 ELIMINATION OF TIME DEPENDENCE

Consider co-ordinates θ, z fixed in space where θ is the angle around the journal and z is the axial distance from the geometric centre of the journal (Fig.1). Let p and h be the pressure and clearance respectively at the point (θ, z). Let t be the time, μ the viscosity of gas, R the journal radius and ω the angular velocity (μ, R and ω are assumed constant).

The steady-state Reynolds equation for isothermal flow with cylindrical co-ordinates fixed in space is[4]:

$$\frac{1}{R^2}\frac{\partial}{\partial\theta}\left(p\,h^3\frac{\partial p}{\partial\theta}\right)+\frac{\partial}{\partial z}\left(p\,h^3\frac{\partial p}{\partial z}\right) = 6\mu\,\omega\frac{\partial}{\partial\theta}(ph)\ . \qquad (1)$$

The steady-state condition implies that the moving surface must be circular in cross-section and rotating with its centre fixed. In practice this restriction is unnecessarily severe as can be seen from consideration of the time-dependent Reynolds equation.

The time-dependent Reynolds equation for isothermal flow with co-ordinates fixed in space is[4]:

$$\frac{1}{R^2}\frac{\partial}{\partial\theta}\left(p\,h^3\frac{\partial p}{\partial\theta}\right)+\frac{\partial}{\partial z}\left(p\,h^3\frac{\partial p}{\partial z}\right) = 6\mu\,\omega\frac{\partial}{\partial\theta}(p\,h)+12\mu\frac{\partial}{\partial t}(p\,h). \qquad (2)$$

Introduce a co-ordinate ξ measured from an axis rotating with angular velocity ω_1, so that:

$$\theta = \xi + \omega_1\,t\ . \qquad (3)$$

Then:

$$\left(\frac{\partial}{\partial\theta}\right)_t = \left(\frac{\partial}{\partial\xi}\right)_t \qquad (4)$$

and:

$$\left(\frac{\partial}{\partial t}\right)_\theta = \left(\frac{\partial}{\partial t}\right)_\xi - \omega_1\left(\frac{\partial}{\partial\xi}\right)_t\ . \qquad (5)$$

Reynolds equation in rotating co-ordinates then becomes:

$$\frac{1}{R^2}\frac{\partial}{\partial\xi}\left(p\,h^3\frac{\partial p}{\partial\xi}\right)+\frac{\partial}{\partial z}\left(p\,h^3\frac{\partial p}{\partial z}\right) = 6\mu(\omega-2\omega_1)\frac{\partial}{\partial\xi}(p\,h)+12\mu\frac{\partial}{\partial t}(p\,h)\ . \qquad \ldots(6)$$

Now if ω_1 can be so chosen that:

$$\left(\frac{\partial}{\partial t}(p\,h)\right)_\xi = 0\ , \qquad (7)$$

the time-dependent terms are absorbed entirely into the variable ξ and the axial mass flow may then be treated exactly as in steady-state case. For practical cases in aerodynamic bearings (7) is only true when:

$$\frac{\partial p}{\partial t} = \frac{\partial h}{\partial t} = 0 \; . \tag{8}$$

These conditions apply to a limited class of time-dependent cases, namely those where the pressure and clearance fields rotate with constant angular velocity ω_1. In general at least one of the bearing members is circular in cross-section. One however may be non-circular in cross-section but possessing symmetry with a well-defined geometric centre. Thus the time dependent effects do not appear in the following three practical cases:

(i) both bearing surfaces circular with their centres at a constant separation, the centre of the rotating surface orbiting the centre of the stationary surface with any constant angular velocity ω_1;

(ii) the rotating surface non-circular but concentric with a stationary circular surface (in this case $\omega_1 = \omega$);

(iii) the rotating surface non-circular with its centre at a constant distance from the centre of a stationary circular surface, the centre of the rotating surface orbiting the centre of the stationary surface with constant angular velocity $\omega_1 = \omega$.

It should be noted that the important case of a non-circular rotating surface and a stationay circular surface with their centres fixed but not coincident, is not included.

With the sufficient conditions applied (1) and (6) reduce to the form:

$$\frac{1}{R^2}\frac{\partial}{\partial \xi}\left(p\, h^3 \frac{\partial p}{\partial \xi}\right) + \frac{\partial}{\partial z}\left(p\, h^3 \frac{\partial p}{\partial z}\right) = 6\mu\,(\omega - 2\omega_1)\frac{\partial}{\partial \xi}(p\, h), \tag{9}$$

the steady-state case being a particular case, $\omega_1 = 0$.

Now p, h and their derivatives with respect to z are all continuous and cyclic in ξ, so integrating (9) around the bearing gives:

$$\oint \frac{\partial}{\partial z}\left(p\, h^3 \frac{\partial p}{\partial z}\right) d\xi = 0. \tag{10}$$

Again since p, h are their derivatives with respect to z are continuous and cyclic in ξ, the order of integration and differentiation may be interchanged, so that:

$$\frac{d}{dz}\oint p\, h^3 \frac{\partial p}{\partial z}\, d\xi = 0 \; ; \qquad (11)$$

or:

$$\oint p\, h^3 \frac{dp}{dz}\, d\xi = \frac{k}{2} \, , \qquad (12)$$

where k is a constant since the integral is independent of ξ and therefore of θ and t.

It will be shown in the next section that the axial mass flow is proportional to k.

3 AXIAL MASS FLOW

With the imposed restrictions the pressure-field is given by the solution of equation (9), subject to boundary conditions, as a function of ξ and z :

$$p = p(\xi, z) \, . \qquad (13)$$

The functional dependence of p will be taken as understood.

Consider a bearing whose clearance is a function of ξ only:

$$h = h(\xi) \, , \qquad (14)$$

which is subjected to end pressures:

$$\left.\begin{aligned} p &= p_1 \text{ at } z = -\ell/2 \, , \\ p &= p_2 \text{ at } z = +\ell/2 \, , \end{aligned}\right\} \qquad (15)$$

where p_1 and p_2 are independent of ξ, and ℓ is the length of the journal (Fig.1).

If M is the axial mass flow of gas then:

$$M = \oint \left\{ \int_0^{h(\xi)} \rho\, w\, dy \right\} R\, d\xi \, , \qquad (16)$$

where ρ is the density of gas, y is the radial direction, and w is obtained from the Stokes flow equation[4,5] :

$$w = \frac{1}{2\mu} y(y - h) \frac{\partial p}{\partial z} . \tag{17}$$

For isothermal flow:

$$\frac{\rho}{\rho_a} = \frac{p}{p_a} , \tag{18}$$

(subscript 'a' denoting ambient conditions), so that on integration with respect to y equation (16) yields:

$$M = 2A \oint h^3(\xi)\, p \frac{\partial p}{\partial z}\, d\xi \; (= A\, k) , \tag{19}$$

where,

$$A = -\frac{\rho_a R}{24\mu\, p_a} = -\frac{\rho_a d}{48\mu\, p_a} . \tag{20}$$

Since h has been assumed a function of ξ only, (12) gives:

$$k = \oint \frac{\partial}{\partial z} \{p^2 h^3(\xi)\}\, d\xi . \tag{21}$$

Again interchanging the order of integration and differentiation (21) yields:

$$k = \frac{\partial}{\partial z} \oint p^2 h^3(\xi)\, d\xi . \tag{22}$$

Integrating (22) with respect to z gives:

$$k\, z + C = \oint p^2 h^3(\xi)\, d\xi , \tag{23}$$

where C is another constant since the integral is independent of ξ and therefore of θ and t.

Using boundary conditions (15) in (23) gives:

$$k = -\frac{(p_1^2 - p_2^2)}{\ell} I$$

$$C = \frac{(p_1^2 + p_2^2)}{2} I$$

where, $$I = \oint h^3(\xi)\, d\xi \; . \tag{26}$$

For convenience the angular variable ξ may be replaced by an angular variable φ where φ is the angle measured from an arbitrary position on the non-circular member (Fig.2). Hence:

$$I = \oint h^3(\varphi)\, d\varphi \; . \tag{27}$$

By eliminating k from (19) the axial mass flow of gas (which as can be seen is independent of angular velocity) is given by:

$$\boxed{M = \frac{\rho_a}{48\mu\, p_a}\left(\frac{d}{\ell}\right)(p_1^2 - p_2^2) \oint h^3(\varphi)\, d\varphi} \tag{28}$$

4 MASS CONTENT RULE

The extended form of Elrod's[3] mass content rule is obtained by eliminating k and C from (23); it is:

$$\oint p^2 h^3\, d\xi = \left[\frac{p_1^2 + p_2^2}{2} - (p_1^2 - p_2^2)\frac{z}{\ell}\right] \oint h^3(\xi)\, d\xi \; . \tag{29}$$

The angular co-ordinate ξ may be replaced by the angular co-ordinate θ which is measured from a space-fixed frame of reference, thus giving the required form:

$$\boxed{\oint p^2 h^3\, d\theta = \left[\frac{p_1^2 + p_2^2}{2} - (p_1^2 - p_2^2)\;\frac{z}{\ell}\right] \oint h^3(\theta)\, d\theta} \tag{30}$$

This equation is valid for all the cases for which the axial mass flow formula is valid, listed in section 6.

5 EVALUATION OF THE PARAMETER I FOR TYPICAL CASES

Consider one bearing member circular and the other member having n ($\geqslant 2$) equally spaced cosine-shaped lobes[6] as in Fig.2. The illustration shows three lobes for simplicity. Let c be the minimum radial clearance between the two bearing members when they are concentric. Let the lobe-depth be b (with $\delta = b/2c$) and the eccentricity ε (i.e. the distance between centres is $c\varepsilon$) with eccentric angle α. Since h is a function of φ only, the lobes extend the full length of the journal but they need not cover the full sector $2\pi/n$. Let their sector angle be $2\pi\nu/n$ where $0 < \nu \leq 1$; the case of no lobing is covered by $b = 0$.

The clearance h is then given by:

$$h = c(1 + \varepsilon \cos(\varphi - \alpha) + \eta) , \tag{31}$$

where:

$$\eta = \delta + \delta \cos \left\{ \frac{n}{\nu} \left(\varphi - \frac{2\pi r}{n} \right) \right\} \tag{32}$$

for:

$$\frac{2\pi r}{n} - \frac{\pi\nu}{n} \leq \varphi \leq \frac{2\pi r}{n} + \frac{\pi\nu}{n} , \qquad r = 1, 2, \ldots, n$$

and:

$$\eta = 0, \quad \text{otherwise.} \tag{33}$$

In the case of the bearing member being fully lobed (31), (32), (33) reduce to:-

$$h = c(1 + \varepsilon \cos(\varphi - \alpha) + \delta + \delta \cos n\varphi) . \tag{34}$$

Integration of (37) with h given by (31), (32), (33) yields:

$$\boxed{I = 2\pi c^3 \left[\left(1 + \frac{3}{2}\varepsilon^2\right) + \frac{\nu\delta}{2}\left(6 + 3\varepsilon^2 + 9\delta + 5\delta^2\right) + \frac{3}{4}\delta\varepsilon^2 C_n(\nu,\alpha) \right]} \quad \ldots (35)$$

where:

$$C_n(\nu, \alpha) = \frac{2 \sin \pi\nu}{\pi(1 - \nu^2)} \cos 2\alpha, \qquad \text{for } n = 2; \tag{36}$$

and:

$$C_n(\nu,\alpha) = 0, \qquad \text{for } n > 2; \tag{37}$$

where $\dfrac{2\sin\pi\nu}{\pi(1-\nu^2)} = 1$, for $\nu = 1$.

Note that the value of I depends on the phase relationship of the eccentricity and the lobing for n = 2.

Particular cases of I are;

no lobing : $$I = 2\pi c^3\left(1 + \frac{3}{2}\varepsilon^2\right),$$

no eccentricity fully lobed : $$I = 2\pi[c(1+\delta)]^3\left\{1 + \frac{3}{2}\left(\frac{\delta}{1+\delta}\right)^2\right\},$$

no eccentricity : $$I = 2\pi\left[(1-\nu)\,c^3 + \nu[c(1+\delta)]^3\left\{1 + \frac{3}{2}\left(\frac{\delta}{1+\delta}\right)^2\right\}\right],$$

more than two lobes: $$I = 2\pi c^3\left[\left(1 + \frac{3}{2}\varepsilon^2\right) + \frac{\nu\delta}{2}\left(6 + 3\varepsilon^2 + 9\delta + 5\delta^2\right)\right].$$

6 CONCLUSIONS

In sections 2 and 3 restrictive conditions were applied in order to obtain the axial mass flow formula (28) and the mass content rule (30); they are listed below:

(i) constant pressures at both ends of the journal;

(ii) clearance varying only around the journal;

(iii) $\frac{\partial}{\partial t}(p\,h) = 0$ in space-fixed or rotating co-ordinates. In practice the last of these conditions is further restricted to one of the following alternative 'steady state' conditions:

(a) the rotating surface circular with its centre fixed, the other surface being stationary;

(b) the rotating surface non-circular but concentric with a stationary circular surface;

(c) the rotating surface non-circular with its centre at a constant distance from the centre of a stationary circular surface, the centre of the rotating surface orbiting the centre of the stationary surface with constant angular velocity equal to the rotational angular velocity (e.g. synchronous whirl caused by simple mass unbalance);

(d) both bearing surfaces circular with their centres at a constant separation, the centre of the rotating surface orbiting the centre of the stationary surface with any constant angular velocity.

These conditions are sufficient (though not necessary) for the analytic solutions (28) and (30).

In all cases it has been assumed that one bearing surface rotates with constant angular velocity while the other surface remains stationary, because in practice this is usual. It is however easy to see that when both surfaces are circular and constant magnitude circular whirl is present, equations (28) and (30) will still apply even though both surfaces are moving.

It may also be pointed out for example that, if p_1 and p_2 and known functions of ξ it might still be possible to evaluate k and C from (23) with the new relationships at the boundaries.

In conclusion it should be stated that all bearings which satisfy (ii) and are not included in the restricting conditions should be examined in their own right should there be a need to formulate equations of the form of (28) and (30).

SYMBOLS

A	constant $(= - \rho_a R/24\mu p_a)$
b	lobe depth
C	constant of integration
$C_n(\nu,\alpha)$	integration function
c	minimum radial clearance with bearing members concentric
d	diameter of journal
h	clearance
I	integral of clearance cubed around the journal
k	constant
ℓ	length of journal
M	axial mass flow
n	number of lobes
p	pressure
p_1	end pressure at $z = - \ell/2$
p_2	end pressure at $z = + \ell/2$
p_a	ambient pressure
R	journal radius
r	dummy integer
t	time
w	velocity of gas in z-direction
y	radial co-ordinate
z	axial distance from centre of bearing
α	eccentric angle
δ	$b/2c$
ε	eccentricity
η	expression describing lobe-shape
θ	angle around journal from fixed line in space
μ	viscosity of gas
ν	proportion of surface that is lobed
ξ	angle around journal from a line rotating with angular velocity ω_1
ρ	density of gas
ρ_a	ambient density
φ	angle measured from line fixed in non-circular member
ω	angular velocity
ω_1	angular velocity of rotating co-ordinate

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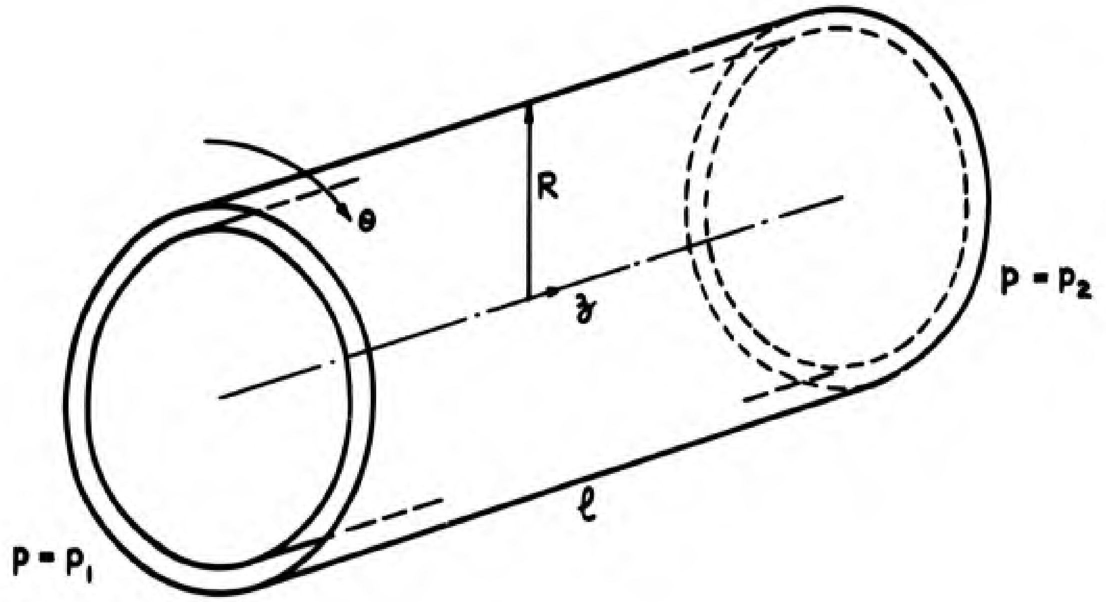

Fig. 1

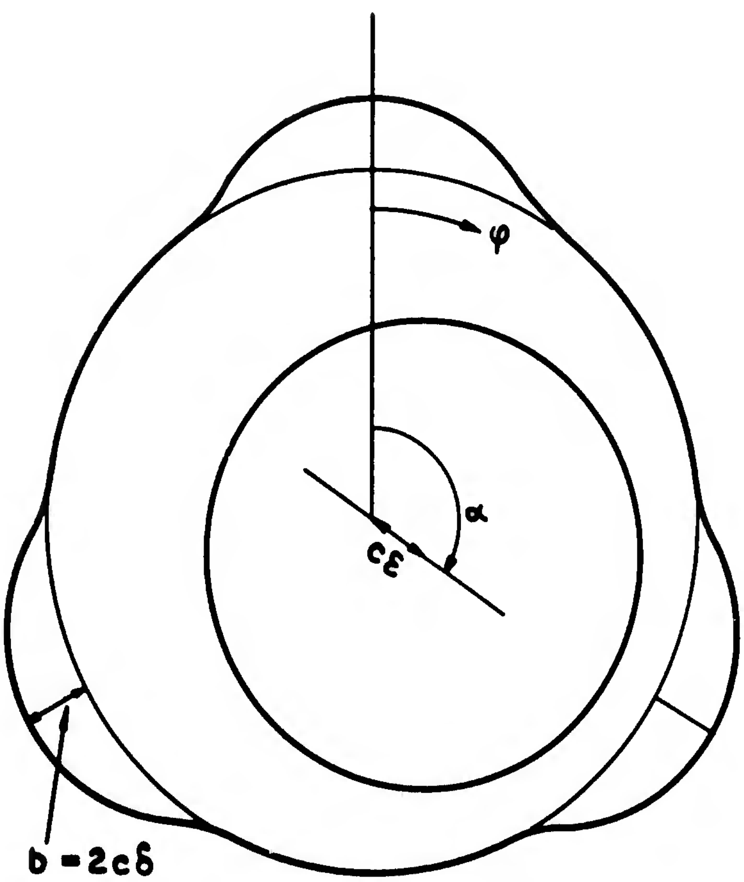

Fig.2