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FOREWORD

This report was prepared by Mr. J. C. Halpin of the Elastomers and Coatings Branch, Nonmetallic Materials Division, Air Force Materials Laboratory, and Mr. E. A. Meinecke, The University of Akron, Akron, Ohio. The work was conducted under Project 7342, "Fundamental Research on Macromolecular Materials and Lubrication Phenomena," Task 734202, "Studies on the Structure-Property Relationships of Polymeric Materials," and was administered by the Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio with Mr. J. C. Halpin serving as Project Engineer.

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ABSTRACT

Delayed buckling of a viscoelastic material was investigated experimentally as a function of buckling load, time, and temperature. The concept of "Reduced Variables" was shown, for the first time, to be applicable to this physical phenomenon. Linear viscoelastic analysis was employed to give a qualitative and a semiquantitative description of the buckling process and the prediction of the critical time to buckle.

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INTRODUCTION

It is commonly observed that when a sufficient compressive load is administered to a slender bar it collapses. If the bar consists of a viscoelastic material, there will be a delay in time between the application of the steady compressive load and the catastrophic collapse of the bar. This phenomenon, of a delayed instability, is the simplest of a family of such phenomenon which underlie such physical processes as time-temperature dependent fracture, delayed yielding, etc. Because such processes are poorly understood, if at all, and are of paramount importance in modern technology, a critical examination of this class of phenomena is most timely. The conventional approaches to this problem are reviewed by Hoff (Reference 1), Boley and Weiner (Reference 2), and Hilton (Reference 3). These reviews can be summarized by the following statements (Reference 2):

1. The phenomenon of creep buckling is both physically and mathematically quite different from the classical type of buckling phenomenon. The usual buckling load represents a "point of bifurcations" on a load vs. deflection plot, a point beyond which more than one configuration is possible; creep buckling is characterized by deflections or, in some cases, velocities increasing beyond all bounds.

2. Creep buckling can occur at a finite time only if the material follows a nonlinear creep law.

3. The column will undergo creep buckling at any value of the compressive axial load, no matter how small.

4. Creep buckling will occur whenever the column has initial imperfections, and only then; in fact, for a perfect column the critical time becomes infinite.

5. The value of t_B, the critical time required to buckle the column, depends on the initial deflection or curvature of the column and on the magnitude of the load; it has been found to be not too strongly affected by changes in the former but very sensitive to changes in the latter.

6. The elasticity, small-deflections analysis is not valid in the immediate neighborhood of the critical time because the deflections are then too large. The magnitude of the deflections, however, varies very rapidly near the critical time and, in fact, increases from small values to very large ones so quickly that it is often said that the column "snaps through" at that time although, of course, no instantaneous buckling process occurs. This implies that calculations based on a small-deflection theory are valid up to times very close to the critical, and thus cover in effect the entire range of practical interest.

It is the intention of the authors to demonstrate, unequivocally, that statements 2 and 3 are not generally correct but are the results of an analysis which does not correctly portray physical reality.

OBSERVATIONS OF THE PHENOMENA

A satisfactory analysis of the kinetic processes leading to a delayed instability can be achieved through a study of the creep buckling of elastomeric rods. Rubberlike materials are particularly attractive because they allow for the detailed observation of buckling deflection modes and rates of deflection coupled with a known nonlinear viscoelastic response (Reference 4) which is qualitatively different from that observed with conventional structural materials (Reference 1). For example, consider the time lapse photographic sequence, Figure 1, of an elastomeric rod deflecting under the action of an axially applied compressive stress. In this figure one should note that there is an initial bending of the column with the application of the load and that this deflection grows in time until the column collapses. Measurements of the midspan deflection, Figure 2, clearly indicate a slow "creeping" like beam deflection followed by a rapid increase in deflection rate as the critical time is approached as suggested by statements 1 and 6 above.

If one now wishes to experimentally examine the hypothesis that the deflection process and consequently the time required to buckle are controlled by the viscoelastic character of the column, a series of buckling experiments can be performed at various loads and temperatures. Consider, for example, the relationship between the buckling lifetime, t_B, and compressive load, P. For each value of load there exists a distinctive buckling lifetime, Figure 3. The repetition of an experiment with the same value of P at different temperatures yields a variation in t_B. This dependence of time upon the ambient temperature is a normal manifestation of viscoelastic processes and can be handled within the concept of reduced variables (References 5-8). By employing the well known WLF equations, a time-temperature reduction procedure was used to construct a temperature-reduced buckling lifetime, t_B/A_T, load master curve, Figure 4, extending over eight decades in t_B. This successful reduction procedure parallels similar experiences with large deformative (Reference 4) and ultimate properties (Reference 9) of these systems. Clearly, the rate processes controlling the buckling phenomena are dominated by the viscoelastic rate processes.

This point may be amplified by noting that the linear theory of elastic stability (Reference 10) defines the critical buckling load as

$$P_{cr} = 4I \left(\frac{\pi}{L}\right)^2 E \tag{1}$$

where P_{cr} is the buckling load, I is the minimum moment of inertia of the crossection, E is the modulus of elasticity, and L is the length of the column. Since the deflections in Figure 1 are slow until the critical condition is reached, one may assume creep buckling to be a quasi-static process. By making use of the results of References 8 and 11, time dependence can be incorporated in Equation 1 by simply allowing E to be redefined as the relaxation modulus so that

$$\log P_{cr}(t) = \log\left(4\left(\frac{\pi}{L}\right)^2\right) + \log E(t) \tag{2}$$

Thus, the results of Figure 4 replotted on a log-log basis should yield a curve parallel to a plot of log E(t) versus log (t/A_T). Such agreement is illustrated in Figure 5, where the distance between the curves determines the constants of Equation 2. Note the superposition of the E(t) data points (filled boxes) and the buckling loads P(t) (the open circles). This correlation of P(t) with E(t) can also be illustrated by performing buckling experiments in a testing machine which compresses the column at a constant rate, as illustrated in Figure 6. Note that the parallelism between the rate-dependent modulus and the buckling load when plotted against the logarithmic temperature will reduce strain rate, V/A_T. Thus, Equation 2 is in accord with experimental reality. Accordingly, it is clear that a column buckles when the modulus decays to a value small enough to satisfy the criteria of Equation 1 for a given value of P. Accordingly, the lifetime of the column t_B is determined to be the time, t/A_T, required for the modulus to decay from its initial value to that required by Equation 1.

Further confidence in this conclusion can be achieved by the predictions of the time-dependence of midspan deflection. Our initial statements 1, 2, and 6 indicate that we must show that the theory assumed in Equation 2 allows for an unbounded deflection at a finite time, t_B. From Reference 10 we note that the approximate elasticity treatment yields an expression for the midspan deflection,

$$y_m = \frac{y_0}{P_{cr}/P - 1} \tag{3}$$

where y_m is the central deflection illustrated in Figure 1 and y_0 is the initial imperfection or curvature of the column. If Equation 2 is a proper statement, then the time dependent deflection should be approximated by

$$y_m(t) = \frac{y_0}{\dfrac{P_{cr}(t)}{P} - 1} \tag{4}$$

where

$$P_{cr}(t) = \frac{4\pi^2 I}{L^2} E(t).$$

Performing the indicated operations, we may compute, from linear viscoelastic theory, the solid curves of Figure 1. The agreement is rather good in view of the assumptions implicit in Equations 1 and 3, as derived from elasticity theory.

CONCLUSIONS

The initial statements 2 and 3 are incorrect because, as shown, linear viscoelastic analysis does yield a qualitative and semi-quantitative description of the delayed instability phenomena in time and temperature dependent materials. Corrections for nonlinear viscoelasticity will not alter the qualitative form of the elementary theory as outlined here, but it will lead, in most practical cases, to lifetimes which will be shorter than those predicted from linear theory. Similar results can be expected for a more rigorous deriviation of Equations 1 and 3 in elasticity theory. Finally, these results are in accord with the successful approach of Bueche and Halpin (Reference 9) to the fracture of viscoelastic solids, another form of an elastic stability process similar to that studied here.

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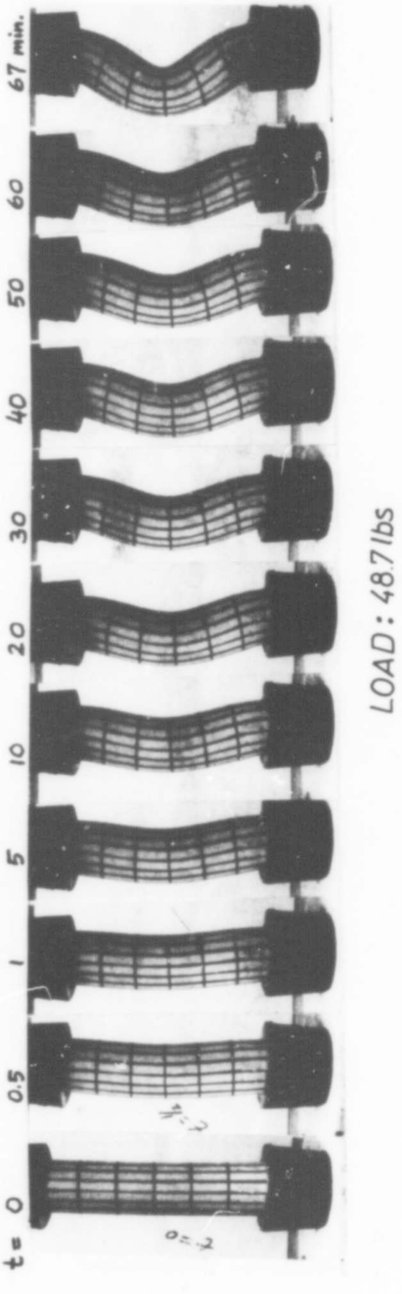

Figure 1. Time Dependent Buckling of Viscoelastic Column; SBR rubber

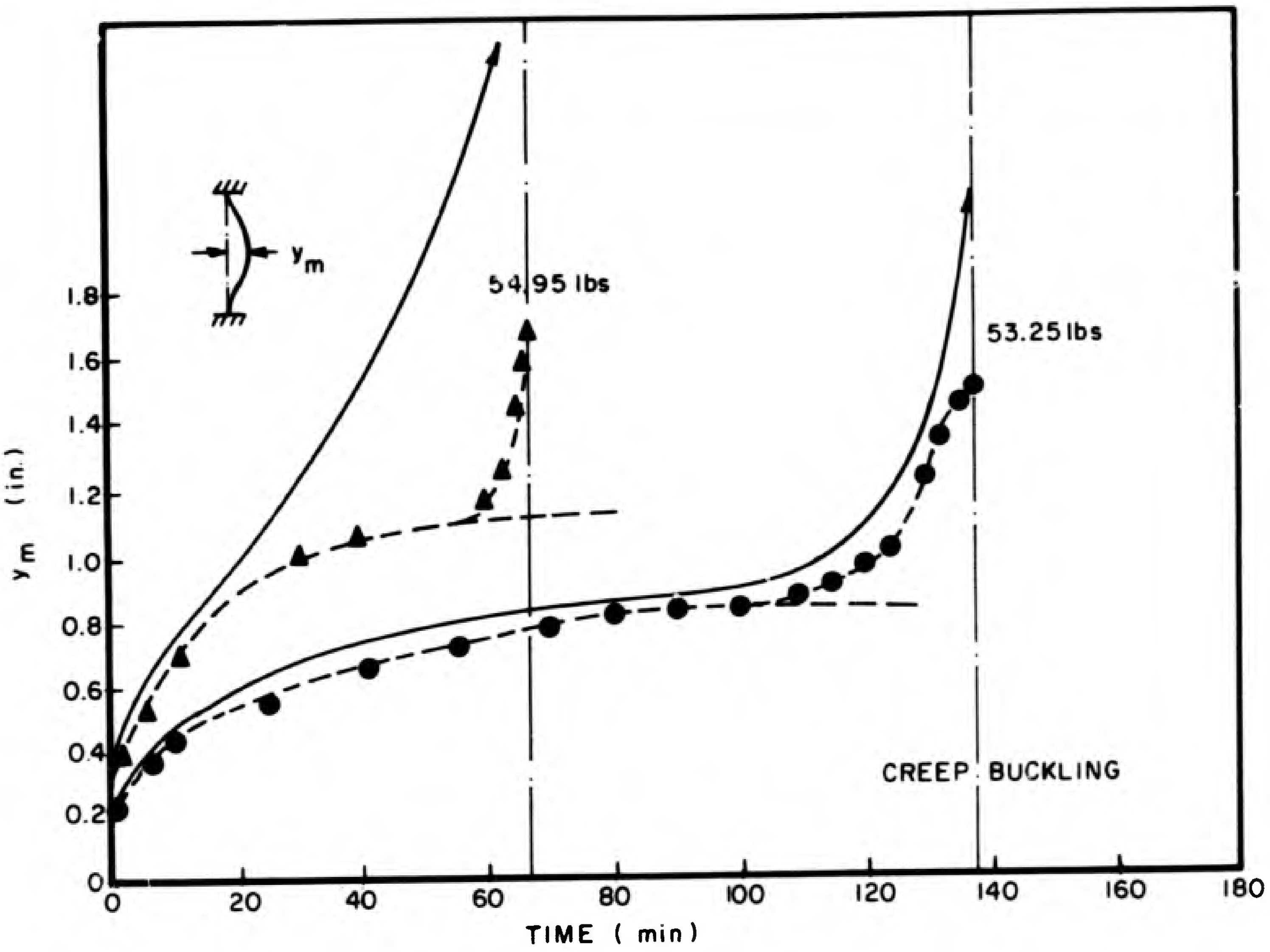

Figure 2. Time-Temperature Dependent Midspan Deflection in a Buckling Column

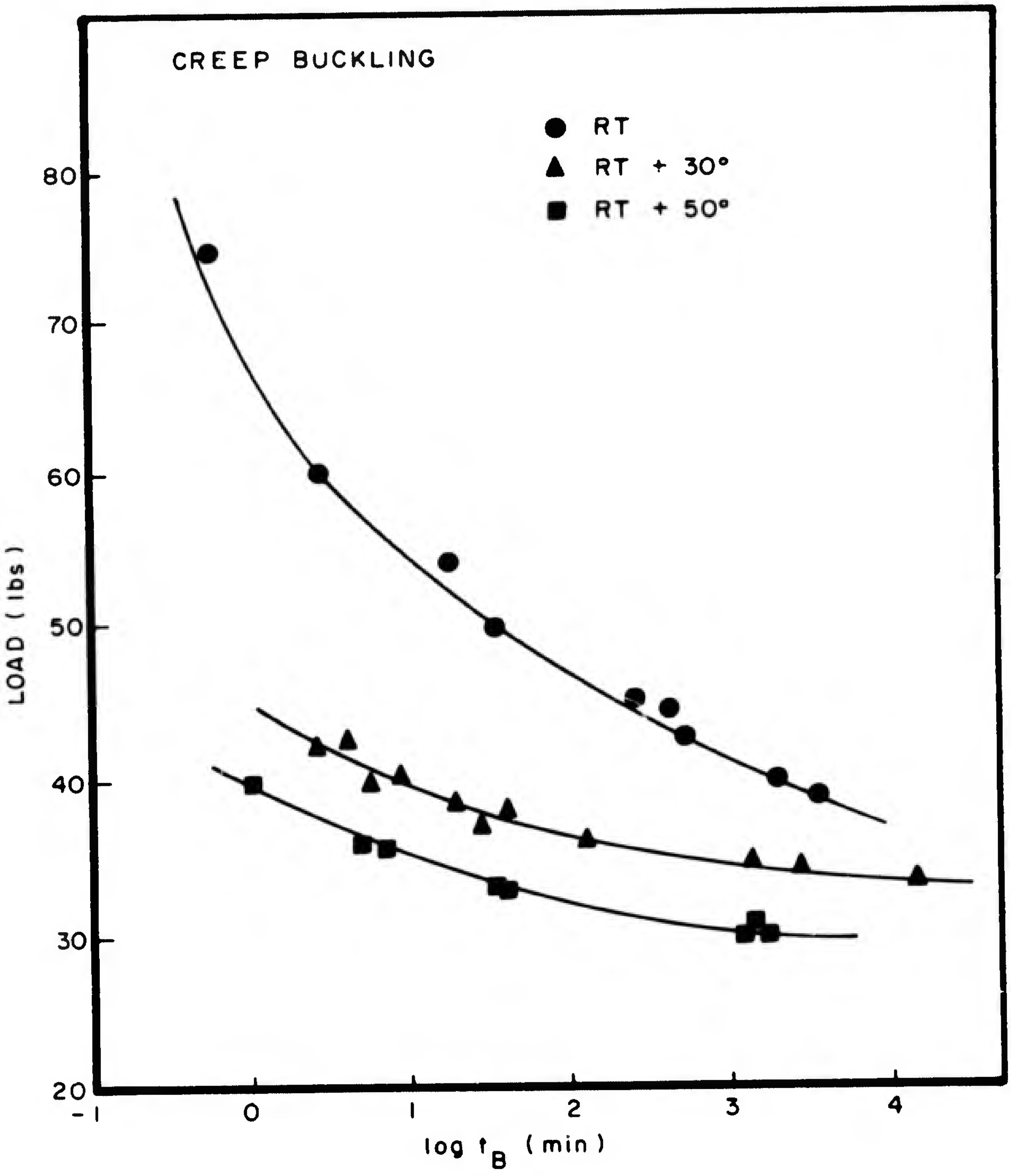

Figure 3. Dependence of Buckling Load on Time and Temperature

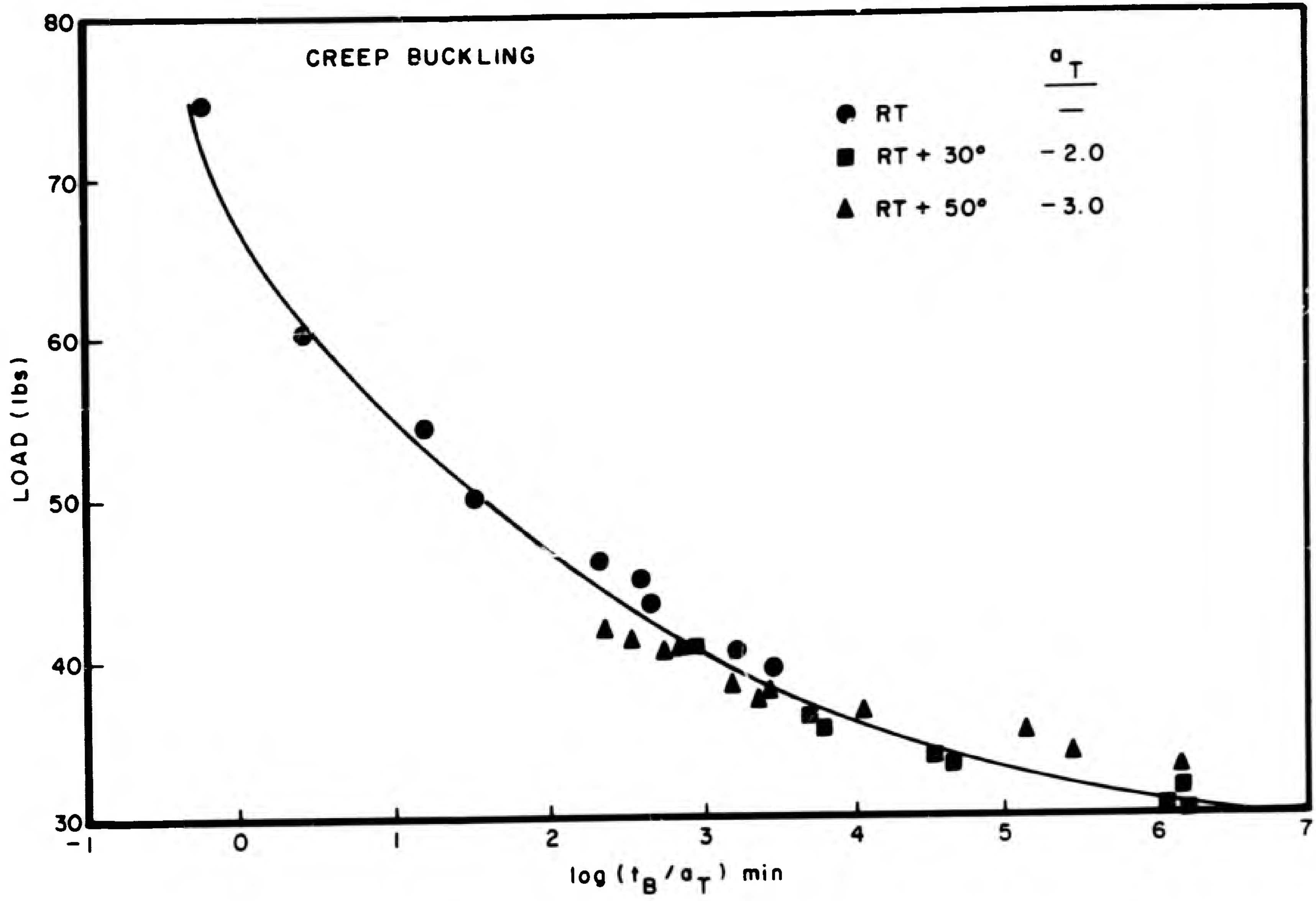

Figure 4. Buckling Lifetime Master Curve

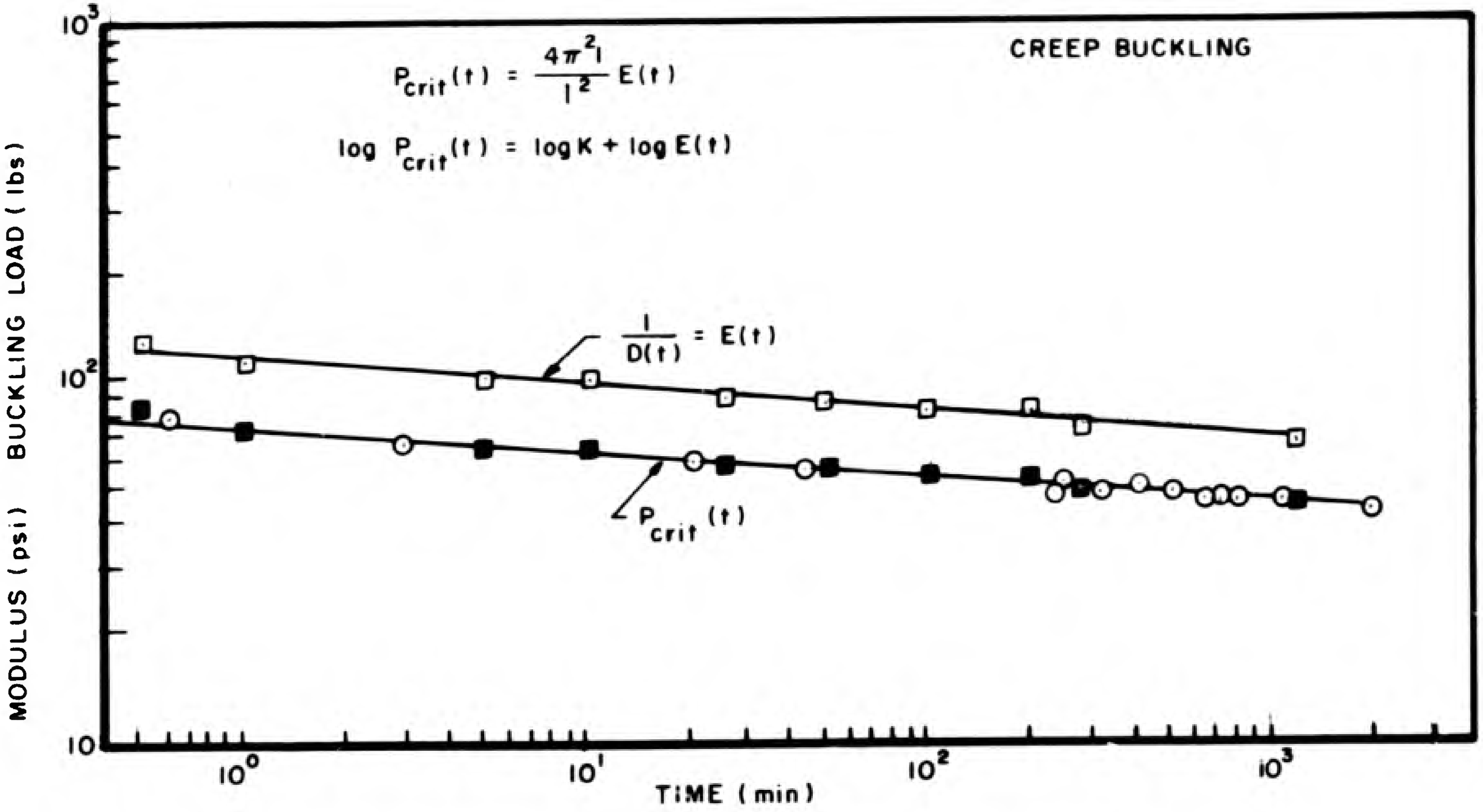

Figure 5. Correlation of Time Dependent Buckling with Relaxation Modulus

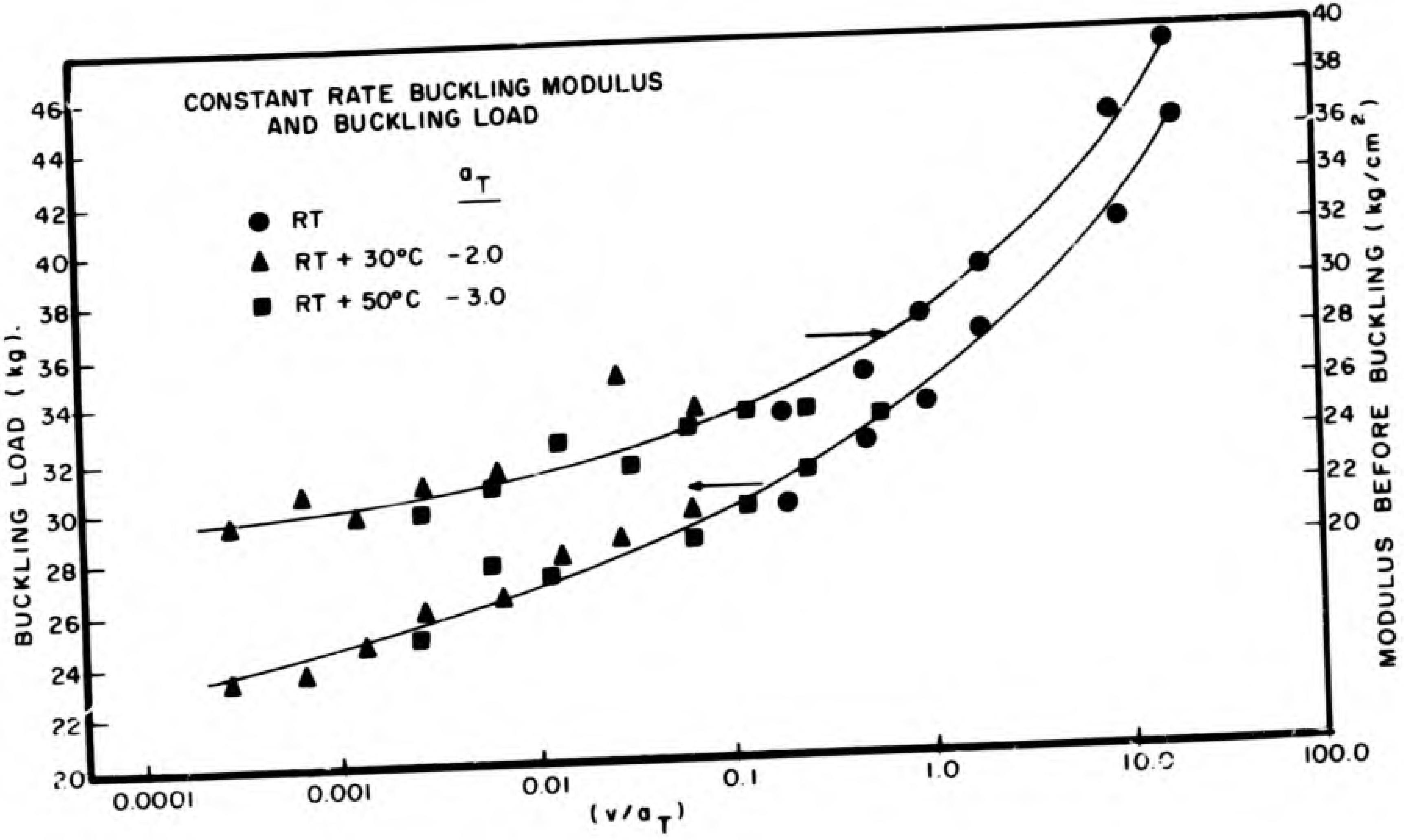

Figure 6. Time-Temperature Dependent Buckling in Constant Rate of Compression Test

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