

University of Washington
College of Engineering
Department of Electrical Engineering
Seattle, Washington 98105

SCATTERING OF ELECTRIC DIPOLE RADIATION BY A MOVING, DISPERSIVE DIELECTRIC HALF-SPACE

by

J. Fred Holmes and Akira Ishimaru

Contract No. F 19(628)-68-C-0126
Project No. 4600
Task No. 460010
Work Unit No. 46001001

Scientific Report No. 3
TECHNICAL REPORT NO. 133

April 1969

Contract Monitor
Otho E. Kerr, Jr.
Microwave Physics Laboratory

Distribution of this document is unlimited. It may be released to the Clearinghouse,
Department of Commerce, for sale to the general public.

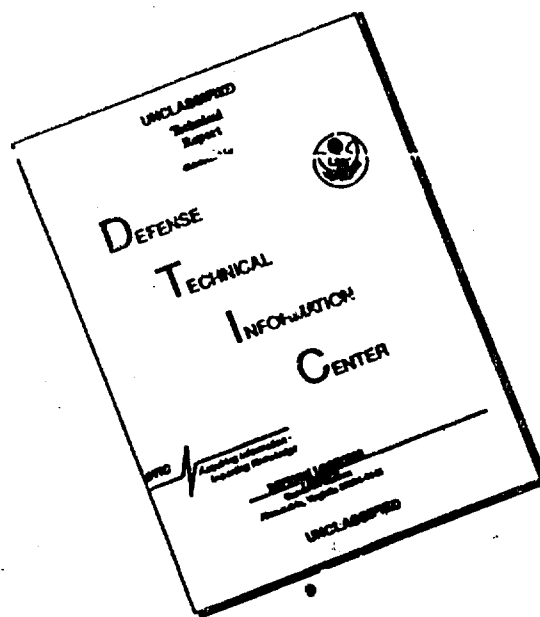


Prepared for
Air Force Cambridge Research Laboratories
Office of Aerospace Research
United States Air Force
Bedford, Massachusetts 01730

Reproduced by the
CLEARINGHOUSE
for Federal Scientific & Technical
Information Springfield Va. 22151

36

DISCLAIMER NOTICE



**THIS DOCUMENT IS BEST
QUALITY AVAILABLE. THE COPY
FURNISHED TO DTIC CONTAINED
A SIGNIFICANT NUMBER OF
PAGES WHICH DO NOT
REPRODUCE LEGIBLY.**

AFCRL-69-0200

University of Washington
College of Engineering
Department of Electrical Engineering
Seattle, Washington 98105

SCATTERING OF ELECTRIC DIPOLE RADIATION BY A
MOVING, DISPERSIVE DIELECTRIC HALF-SPACE

by

J. Fred Holmes and Akira Ishimaru

Contract No. F 19(628)-68-C-0126
Project No. 4600
Task No. 460010
Work Unit No. 46001001

Scientific Report No. 3

TECHNICAL REPORT NO. 133

April 1969

Contract Monitor
Otho E. Kerr, Jr.
Microwave Physics Laboratory

Distribution of this document is unlimited. It may be released to the
Clearinghouse, Department of Commerce, for sale to the general public.

Prepared for

Air Force Cambridge Research Laboratories
Office of Aerospace Research
United States Air Force
Bedford, Massachusetts 01730

Abstract

The problem of a point electric dipole moving over a dispersive dielectric half-space is studied. The dipole is located in the free space above the dielectric and is assumed to be time harmonic in its rest frame, oriented perpendicular to the interface and moving parallel to it. Previous work in this area has been mostly confined to the use of a plane wave as a source or a non-dispersive dielectric.

Solutions for this problem are obtained using integral transform techniques. The integration is performed for the free space region by making an asymptotic expansion for one integration and then using the saddle point approximation for the remaining integration. The solution obtained in this manner for the vector potential is then used to generate the electric and magnetic fields in both the rest frame of the source and the rest frame of the dielectric. This yields the reflected radiation fields for the case of an arbitrary, dispersive dielectric and the lateral wave fields and surface wave fields for the case of a lossless plasma.

The field patterns are distorted by the relative motion of the source and dielectric. In the rest frame of the source, all three waves exhibit the frequency of the source and in the rest frame of the dielectric, they all exhibit some form of doppler shift (different from the primary wave). The criteria for existence of the lateral wave is not modified by the relative motion of the source and dielectric; but the criteria for existence of the surface waves is greatly modified by the relative motion. For velocities greater than some critical velocity, a new type of surface wave comes into existence.

TABLE OF CONTENTS

1. INTRODUCTION	1
2. INTEGRAL FORMULATION	2
3. SCATTERED FIELDS	6
4. REFLECTED RADIATION FIELDS FOR A LOSSLESS PLASMA DIELECTRIC	12
5. LATERAL WAVE FIELDS FOR A LOSSLESS PLASMA DIELECTRIC	16
6. SURFACE WAVE FIELDS FOR A LOSSLESS PLASMA DIELECTRIC	
7. SUMMARY	24
ACKNOWLEDGEMENT	24
REFERENCES	25
LIST OF FIGURES	26

1. INTRODUCTION

A moving, temporally dispersive material is unusual in that it is both temporally and spatially dispersive as observed in any inertial reference frame other than its rest frame. In order to investigate the interaction of electromagnetic sources with such a material, the problem of a point electric dipole moving over a dispersive dielectric half-space has been studied. The dipole is located in the free space above the dielectric and is assumed to be time harmonic in its rest frame, oriented perpendicular to the interface and moving parallel to it.

Previous work in the area of scattering from a moving half-space has been mostly confined to the use of a plane wave as a source or a non-dispersive dielectric. Particular problems involving the reflection of plane waves from a moving half-space have been investigated in detail by Tai (1965), Yeh (1965, 1966, 1967) Pyati (1966) and others; and just recently Shiozawa and Hazama (1968) have solved the general problem of reflection and transmission of a plane electromagnetic wave at the interface between a stationary dielectric medium and a dielectric medium moving in an arbitrary direction parallel to the interface. It appears that the only work that has been done on the scattering of dipole radiation by a moving half-space was done by Pyati in 1966. He investigated the problem of a vertical or horizontal dipole positioned over a moving, dielectric (non-dispersive) half-space and solved for the reflected radiation fields in the rest frame of the source. No effort was made to investigate the effect of the motion on the lateral wave or surface waves that might be present.

In the work that follows, we utilize two inertial reference frames designated S and S' where it is assumed that reference frame S' is moving with velocity v in the \hat{z} direction with respect to reference frame S and that they coincide at $t = 0$. The dielectric half-space is stationary in reference frame S and occupies the region $x < 0$. The dipole is stationary in reference frame S' and is positioned at the point $(H, 0, 0)$ in that frame. The region $x > 0$, in which the dipole is located, is assumed to be free space.

Solutions for this problem are obtained using integral transform techniques. Section 2 develops integral formulations, involving three integrations, for the vector potential in reference frame S for both

regions of space and carries out one of the integrations. It appears that this is as far as the work can be carried out in general. Beyond this point some sort of approximation must be made. This is done in sections 3 through 6 where solutions are found for the fields at observation points far from the image source.

In section 3 the original contour for the integral is converted to contours around each singularity in the integrand, and a change of variable is made to gain control over the asymptotic expansion variable that is subsequently used. These singularities consist of two branch points and one or more poles. The integrations along the branch cuts are carried out in sections 4 and 5 and yield the reflected radiation fields for a generalized dispersive dielectric and the lateral wave fields for a lossless plasma dielectric. The integration around the poles is carried out in section 6 and yields surface waves for the case of a lossless plasma dielectric. In each case, the saddle point approximation is used for the final integration. The vector potential found in this manner is then used to generate the electric and magnetic fields (\vec{E} and \vec{B}) in reference frame S . These fields are then transformed to reference frame S' to obtain the electric and magnetic fields (\vec{E}' and \vec{B}') in the rest frame of the dipole.

2. INTEGRAL FORMULATION

In this section, an integral formulation for the vector potential in the rest frame of the dielectric is developed. Since the problem that is being considered is homogeneous with respect to the extension space coordinates y , z and t , it lends itself nicely to the use of Fourier transform techniques for the solution with respect to these coordinates. This leaves us with a one-dimensional boundary value problem to solve similar to the familiar Sommerfeld Half-Space problem. However, for the problem we are considering the source is more interesting since it is both temporally and spatially dispersive due to the motion of the source.

Before we can solve the boundary value problem in reference frame S , we must first obtain a formulation for the source as it would be observed in that reference frame. In reference frame S' , the source is described by

$$\vec{J}'(\vec{r}', t') = \hat{x} I_0 \delta(x' - H) \delta(y') \delta(z') e^{j\omega' t'} \quad (1)$$

We would like to convert $\vec{J}'(\vec{r}', t')$ to frequency space so that we may conveniently transform it to the rest frame of the dielectric. The four-dimensional Fourier transform of (1) is given by

$$\vec{J}'(\vec{k}', \omega') = \frac{\hat{x} I_0 e^{jk_1' H} \delta(\omega' - \omega_0)}{2\pi} \quad (2)$$

where the kernel

$$\frac{e^{-j\omega' t' + j\vec{k}' \cdot \vec{r}'}}{4\pi^2}$$

has been used in taking the transform. We will now use the relativistic spectral transformations (Holmes, 1968) to transform the spectral form of the current density given by (2) to reference frame S. The transformations are given by

$$J_x(\vec{k}, \omega) = J_x'(\vec{k}', \omega') \quad (3)$$

$$J_y(\vec{k}, \omega) = J_y'(\vec{k}', \omega') \quad (4)$$

$$J_z(\vec{k}, \omega) = \Gamma[J_z'(\vec{k}', \omega') + v\rho'(\vec{k}', \omega')] \quad (5)$$

The charge density $\rho'(\vec{k}', \omega')$ can be obtained from $\vec{J}'(\vec{k}', \omega')$ by using the continuity equation converted to frequency space. The spectral form for the current density in frame S then is given by

$$J(\vec{k}, \omega) = \frac{I_0}{2\pi\Gamma} e^{jk_1 H} \delta(\omega - (\frac{\omega_0}{\Gamma} + vk_3)) [\hat{z} \frac{vk_1}{(\omega - vk_3)} + \hat{x}] \quad (6)$$

where the transformations for the components of the wave four-vector (Jackson, 1962) have been used. Now inverting (6) with respect to k_1 we obtain

$$J(x, k_2, k_3, \omega) = \frac{I_0}{\Gamma\sqrt{2\pi}} \delta(\omega - (\frac{\omega}{T} + vk_3)) [R \delta(x-H) + \frac{2jv}{(\omega - vk_3)} \frac{d}{dx} \delta(x-H)] \quad (7)$$

which is the desired source term.

The one-dimensional boundary value problem we must solve is illustrated in figure 1 where use has been made of (7) and the fact that in free space the form of the differential equation for the vector potential is invariant. Since the solution of this type of problem in the manner we are proposing is well known, the details have not been included¹. However, we would like to make a few specific comments. At first glance it appears that difficulty might be encountered in finding the particular solution to the differential equation in region 1. However, due to the delta functions in the source terms it can be readily generated using the free space Green's function for the differential equation. Also, it turns out that the vector potential obtained from the solution of this boundary value problem contains the factor

$$\delta(\omega - (\frac{\omega}{T} + vk_3))$$

so that the inversion with respect to ω can be performed exactly. The vector potentials corresponding to the scattered and transmitted field are

$$A_{1zS}(\vec{r}, t) = \frac{jv\Gamma\mu_0 I_0}{2\omega_0 (2\pi)^2} e^{j\Gamma\omega_0 (t - \frac{v}{c} z)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [\frac{k-h}{k+h}] e^{-jf_1} dk_{33} dk_2 \quad (8)$$

$$A_{1xS}(\vec{r}, t) = \frac{-j\mu_0 I_0}{2(2\pi)^2} e^{j\Gamma\omega_0 (t - \frac{v}{c} z)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [\frac{2\Gamma^2 v}{\omega_0} \frac{K(k_{33} + \frac{v\omega}{c^2})(\epsilon_r - 1)}{(K\epsilon_r + h)(K+h)} + \frac{(K\epsilon_r - h)}{K(K\epsilon_r + h)}] e^{-jf_1} dk_{33} dk_2 \quad (9)$$

1. Considerable detail is included in the original work (Holmes, 1968).

$$A_{2zT} = \frac{j\mu_0 I v}{\omega_0 (2\pi)^2} e^{j\Gamma\omega_0(t - \frac{v}{c^2}z)} \iint_{-\infty}^{\infty} \frac{K}{(K+h)} e^{-jf_2} dk_{33} dk_2 \quad (10)$$

and

$$A_{2xT} = \frac{-j\mu_0 I v}{(2\pi)^2} e^{j\Gamma\omega_0(t - \frac{v}{c^2}z)} \iint_{-\infty}^{\infty} \left[\frac{\Gamma^2 v}{\omega_0} \frac{K(\epsilon_r - 1)(k_{33} + \frac{vw}{c^2})}{(K\epsilon_r + h)(K+h)} + \frac{\epsilon_r}{K\epsilon_r + h} \right] e^{-jf_2} dk_{33} dk_2 \quad (11)$$

where

$$K = \left(\frac{\omega^2}{c^2} - k_2^2 - k_{33}^2 \right)^{1/2} \quad (12)$$

$$h = \left[\epsilon_r \frac{\Gamma^2}{c^2} (\omega_0 + vk_{33})^2 - \Gamma^2 (k_{33} + \frac{vw}{c^2})^2 - k_2^2 \right]^{1/2} \quad (13)$$

$$\epsilon_r = \epsilon_r | \omega = \Gamma(\omega_0 + vk_{33}) \quad (14)$$

$$f_1 = K(H+x) + k_{33} \Gamma(z-vt) + k_2 y \quad (15)$$

$$f_2 = KH - hx + k_{33} \Gamma(z-vt) + k_2 y \quad (16)$$

and the change of variable

$$k_3 = \Gamma(k_{33} + \frac{vw}{c^2})$$

has been made to simplify the expressions. It appears that this is as far as the work can be carried out in general and that some sort of approximation must be made in order to perform the remaining two integrations. This is done for region 1 in the following sections.

3. SCATTERED FIELDS

We would like to evaluate the integrals for the scattered fields by obtaining the first term of an asymptotic expansion for the integration with respect to k_{33} and then using the saddle point approximation for the integration with respect to k_2 . We can gain control of the asymptotic expansion variable by putting the integral into the form (Van Der Waerden, 1950)

$$I = \int P(u) e^{-\lambda u} du \quad (17)$$

A convenient choice for λ is

$$\lambda = \underline{r}' \frac{\omega}{c} \quad (18)$$

where

$$\underline{r}' = [(H+x)^2 + (z-vt)^2 + y^2]^{1/2} \quad (19)$$

and is the distance in wave lengths from the image source to the field point in reference frame S' . Then for some suitably large λ , the first term in the asymptotic expansion should in most cases represent the integral adequately.

In the u plane, the integrands under consideration have singularities in the form of branch points and poles. The original contour can of course by the Cauchy-Goursat Theorem be replaced by paths around these singularities plus a suitable closure path. The closure path will be chosen as the right-half plane, to ensure that there will be no contribution to the integral from the closure. The branch cuts will be taken from the branch point along a path parallel to the real axis. This forces the integrand to decrease most rapidly along the branch cut paths as you move away from the branch point and will aid in our evaluation of the branch cut integrations. Our original integral then becomes

$$I = \left| \begin{array}{c} -\int P(u) e^{-\lambda u} du + 2\pi j \sum_i R_i \\ \text{Branch cut paths} \end{array} \right| \text{SIGN}(z-vt) \quad (20)$$

where R_i is the residue at the i th pole and $\text{SIGN}(z-vt)$ is ± 1 depending on the sign of the quantity $(z-vt)$.

Let us now consider the procedure that will be followed in evaluating the branch cut integrations. The first step will be to transform the branch point to the origin and then to utilize the second branch, $P(u^-)$, for the path below the cut and the first branch, $P(u^+)$, for the path above the cut. The integration is completed, by expanding the integrand in a series and then keeping only the first term. The resultant expression for each branch cut integration becomes

$$I_B = -\text{SIGN}(z-vt) e^{-\lambda d} \underline{B}(k_2) \int_0^\infty \eta^N e^{-\lambda \eta} d\eta \quad (21)$$

where N and $\underline{B}(k_2)$ depend on the integrand under consideration and d is the location of the branch point. Evaluating the integral, this becomes

$$I_B = -\text{SIGN}(z-vt) e^{-\lambda d} \underline{B}(k_2) \lambda^{-(N+1)} \Gamma(N+1) \quad (22)$$

The integration with respect to k_2 will be done by a saddle point approximation. For the branch cut contributions the integral will be of the form

$$A_B = -\text{SIGN}(z-vt) \frac{\lambda^{-(N+1)}}{\Gamma(N+1)} \int_{-\infty}^{\infty} \underline{B}(k_2) e^{f(k_2)} dk_2 \quad (23)$$

where

$$f(k_2) = j \left[\Gamma \omega_0 \left(t - \frac{v}{2} z \right) - k_2 y \right] - \lambda d \quad (24)$$

The saddle point approximation gives us

$$A_B = -j \text{ SIGN}(z-vt) \sqrt{2\pi} \frac{\Gamma(N+1) BB(k_{20}) e^{-f(k_{20})}}{\lambda^{(N+1)} \sqrt{\frac{\partial^2 f(k_{20})}{\partial k_2^2}}} \quad (25)$$

where the saddle point k_{20} is found from

$$\frac{\partial f}{\partial k_2} = 0 \quad (26)$$

The integration with respect to k_2 for the pole contributions to the integral will also be done by the saddle point approximation. The details are discussed in section 6.

The above work describes in general the technique that will be used to evaluate the integrals. The actual integrals will be evaluated in sections 4 through 6 using the above results. We would now like to investigate the nature and location of the singularities that we will have to deal with in those sections.

Before examining the integrands for singularities we must transform them to the u plane. They will be given by

$$P(u) = W(u) \frac{dk_{33}}{du} \quad (27)$$

where $W(u)$ is the integrand in the k_{33} plane expressed as a function of u .

We therefore need to find k_{33} , $\frac{dk_{33}}{du}$ and h as a function of u . Using (9), (15) and (17) we find that the first three are given by

$$k_{33} = -\underline{r}' \frac{\omega_0}{c} \frac{1}{(\underline{r}'^2 - y^2)} [ju\Gamma(z-vt) + (H+x)\text{SIGN}(z-vt) \sqrt{u^2 + (1 - \frac{k_{2c}^2}{\omega_0^2})(1 - \frac{y^2}{\underline{r}'^2})}] \quad (28)$$

$$\frac{dk_{33}}{du} = \frac{-\frac{r'\omega}{c(r'^2 - y^2)}}{u(H+x)\text{SIGN}(z-vt) + j\Gamma(z-vt) \sqrt{u^2 + (1 - \frac{k_2^2 c^2}{\omega^2})(1 - \frac{y^2}{r'^2})}} \sqrt{u^2 + (1 - \frac{k_2^2 c^2}{\omega^2})(1 - \frac{y^2}{r'^2})} \quad (29)$$

and

$$\underline{k} = \frac{\frac{r'\omega}{c(r'^2 - y^2)}}{[-ju(H+x) + |\Gamma(z-vt)| \sqrt{u^2 + (1 - \frac{k_2^2 c^2}{\omega^2})(1 - \frac{y^2}{r'^2})}]} \quad (30)$$

The function $\underline{h}(u)$ cannot be found unless the dielectric has been specified. For the case of a lossless plasma

$$\epsilon_r = 1 - \frac{\omega_p^2}{\omega^2} \quad (31)$$

and \underline{h} takes on the particularly simple form

$$\underline{h} = \sqrt{\underline{k}^2 - \frac{\omega_p^2}{c^2}} \quad (32)$$

We are now ready to examine the integrands for the pertinent singular points. Each singular point of $P(u)$ that is enclosed by the contour will make a contribution to the integral. In lieu of directly finding which points are enclosed by the contour, and then evaluating their contribution to the integral, we have examined the contribution associated with each singular point and then retained only those contributions which correspond to waves of the outgoing type. The two approaches of course are equivalent; however, the latter technique is more tenable when a multiple integral is involved, since the location of the singular points will generally be a function of the subsequent integration variable.

From examining (8) and (9) we see that the integrands under consideration have branch points where

$$\underline{k} = 0 \quad (33)$$

and

$$\underline{h} = 0 \quad (34)$$

and that the integrand in (9) may have one or more poles where

$$\underline{k} \cdot \underline{e}_r + \underline{h} = 0 \quad (35)$$

In the u plane, the branch points corresponding to (33) and (34) are located at

$$d = j \sqrt{\left(1 - \frac{k_2^2 c^2}{\omega_0^2}\right) \left(1 - \frac{v^2}{r'^2}\right)} \quad (36)$$

and

$$d = \frac{jc}{\omega_0 r'} \left[\frac{\omega}{c} (H+x) + |\Gamma(z-vt)| \sqrt{\frac{\omega_0^2 - \omega^2}{c^2} - k_2^2} \right] \quad (37)$$

The poles associated with equation (35) will be discussed in section 6.

Our ultimate goal is to find the scattered electric and magnetic fields in reference frame S (\vec{E} and \vec{B}) and reference frame S' (\vec{E}' and \vec{B}'). Due to the nature of the integration techniques used in the above work and in the work that follows, these fields can be found from the vector potential in a simple manner. In x, k_2, k_3, ω space, the vector potential in reference frame S has the form

$$\vec{A} = \vec{A}(k_2, k_3, \omega) e^{-jK(H+x)} = \vec{A}(x, k_2, k_3, \omega) \quad (38)$$

In order to find the electric fields, we need to find an expression for the scalar potential. The scalar potential can be found using the Lorentz Gage condition (which is invariant in free space) transformed to the same coordinate space as \vec{A} . The scalar potential then is given by

$$\phi(x, k_2, k_3, \omega) = \frac{c^2}{\omega} [K A_x + k_3 A_z] \quad (39)$$

Using the vector potential and (39), the electric and magnetic fields can be generated. This form for the fields would in general have to be inverted with respect to k_2 , k_3 and ω . However, in the above work and in the work that follows, the integration process is such that the amplitude portions of the integrand are evaluated at some particular ω , k_2 and k_3 . It is therefore not necessary to go through this integration process again. We can use the expressions already obtained for A_z and A_x and replace ω , k_1 , and k_2 in the expressions for the fields by the values used in the particular integration process. Proceeding in this manner, the fields in reference frame S are given by

$$B_x = -jk_2 A_z \quad (40)$$

$$B_y = j[K A_z - \Gamma(k_{33} + v \frac{\omega}{c^2}) A_x] \quad (41)$$

$$B_z = jk_2 A_x \quad (42)$$

$$E_x = -j \frac{1}{\Gamma(\omega_0 + vk_{33})} [A_x (\Gamma^2(\omega_0 + vk_{33})^2 - c^2 k_2^2) - A_z c^2 K \Gamma(k_{33} + v \frac{\omega}{c^2})] \quad (43)$$

$$E_y = \frac{jk_2 c^2}{\Gamma(\omega_0 + vk_{33})} [K A_x + A_z \Gamma(k_{33} + v \frac{\omega}{c^2})] \quad (44)$$

and

$$E_z = j \frac{1}{\Gamma(\omega_0 + vk_{33})} [A_x c^2 K \Gamma(k_{33} + v \frac{\omega}{c^2}) - A_z (\omega_0^2 - c^2 k_{33}^2)] \quad (45)$$

where k_2 and k_{33} depend on the particular singularity under consideration and are yet to be determined.

4. REFLECTED RADIATION FIELDS FOR A LOSSLESS PLASMA DIELECTRIC

The generalized technique formulated in section 3 for evaluating the branch point contributions to A_{1xs} and A_{1zs} will be applied in this section to the branch point given by (36). It will be found that the contribution from this branch point corresponds to the reflected radiation fields.

In order to use (25) which gives a generalized solution for the branch cut integration we need to find the saddle point and to calculate

$\frac{\partial^2 f}{\partial k_2^2}(k_{20})$, $f(k_{20})$, N and $BB(k_{20})$. Performing these calculations and using the results in (25) we find that the contributions to the vector potential from the branch cut integration corresponding to equation (36) are given by

$$A_{1zs}(\vec{r}, t) = \frac{-\mu_0 I}{4\pi} \Gamma \frac{v}{c} \frac{(H+x)}{r'^2} \left[\frac{K-h}{K+h} \right] e^{j\omega_0 \left[\Gamma(t - \frac{v}{c} z) - \frac{r'}{c} \right]} \quad (46)$$

and

$$A_{1xs}(\vec{r}, t) = \frac{\mu_0 I \omega_0}{4\pi c} \frac{(H+x)}{r'^2} \left[2 \Gamma^2 \frac{v}{c} K_0 \frac{\left(\frac{\Gamma(z-vt)}{r'} + \frac{v}{c} \right) (\epsilon_{r_0} - 1)}{(K_0 \epsilon_{r_0} + h)(K_0 + h)} + \frac{(K_0 \epsilon_{r_0} - h)}{K_0 (K_0 \epsilon_{r_0} + h)} \right] e^{j\omega_0 \left[\Gamma(t - \frac{v}{c} z) - \frac{r'}{c} \right]} \quad (47)$$

where

$$K_0 = \frac{\omega_0}{c} \frac{(H+x)}{r'} \quad (48)$$

$$h_0 = \frac{\omega_0}{c} [\epsilon_{r_0} \Gamma^2 A^2 - \Gamma^2 D^2 - \frac{v^2}{r'^2}]^{1/2} \quad (49)$$

$$\epsilon_{r_0} = \epsilon_r|_{\omega = \Gamma \omega_0 A} \quad (50)$$

$$A = 1 + \frac{\beta \Gamma(z-vt)}{\underline{r}'} \quad (51)$$

and

$$D = \frac{\Gamma(z-vt)}{\underline{r}'} + \beta \quad (52)$$

Equations (46) and (47) correspond to the reflected radiation fields. We observe that the fields go down as $1/\underline{r}'$ where \underline{r}' is the distance from the image source to the field point and that although the frequency distribution is the same as that of the primary source, the distribution is centered on the image source. This means that whenever the velocity is non-zero, the frequency of the reflected radiation field as observed in reference frame S will differ from that of the primary field everywhere except on the interface and a vertical axis through the dipole. The frequency distribution is given by

$$\frac{\omega_R}{\omega_0} = \Gamma \left[1 + \frac{v}{c} \frac{\Gamma(z-vt)}{\underline{r}'} \right] \quad (53)$$

We also observe that in the rest frame of the source, the reflected radiation field will not have a doppler shift. It should be noted that (46) and (47) are valid for an arbitrary dispersive dielectric. When $v = 0$, (46) and (47) reduce to the well known result for the zero velocity case (Baños, 1966, pg. 35 & 179).

Next we would like to find the electric and magnetic fields in reference frame S and reference frame S'. Equations (40) through (45) give these fields in reference frame S in terms of the values of k_{33} and k_2 used in the above integration process and given here.

$$k_2 = \frac{\omega_0}{c} \frac{y}{\underline{r}'} \quad (54)$$

and

$$k_{33} = \frac{\omega_0}{c} \frac{\Gamma(z-vt)}{\underline{r}'} \quad (55)$$

Using these in (40) through (45), the electric and magnetic fields in reference frame S are given by

$$E_{xR} = - \frac{j\omega}{\Gamma(1 + \frac{v}{c} \frac{\Gamma(z-vt)}{\Gamma'})} [\Gamma^2(1 + \frac{v}{c} \frac{\Gamma(z-vt)}{\Gamma'})^2 - \frac{(H+x)^2}{\Gamma'^2}] A_{xR} - \frac{(H+x)}{\Gamma'} \Gamma(\frac{\Gamma(z-vt)}{\Gamma'} + \frac{v}{c}) A_{zR} \quad (56)$$

$$E_{yR} = \frac{j\omega y}{\Gamma' \Gamma(1 + \frac{v}{c} \frac{\Gamma(z-vt)}{\Gamma'})} [\frac{(H+x)}{\Gamma'} A_{xR} + \Gamma(\frac{\Gamma(z-vt)}{\Gamma'} + \frac{v}{c}) A_{zR}] \quad (57)$$

$$E_{zR} = \frac{j\omega}{\Gamma(1 + \frac{v}{c} \frac{\Gamma(z-vt)}{\Gamma'})} [\frac{\Gamma(H+x)}{\Gamma'} (\frac{\Gamma(z-vt)}{\Gamma'} + \frac{v}{c}) A_{xR} - (1 - \frac{\Gamma^2(z-vt)^2}{\Gamma'^2}) A_{zR}] \quad (58)$$

$$B_{xR} = -j \frac{\omega}{c} \frac{y}{\Gamma'} A_{zR} \quad (59)$$

$$B_{yR} = j \frac{\omega}{c} [\frac{(H+x)}{\Gamma'} A_{zR} - \Gamma(\frac{\Gamma(z-vt)}{\Gamma'} + \frac{v}{c}) A_{xR}] \quad (60)$$

and

$$B_{zR} = j \frac{\omega}{c} \frac{y}{\Gamma'} A_{xR} \quad (61)$$

where A_{xR} and A_{zR} are the x and z components of the vector potential for the reflected radiation field given by (46) and (47). We would now like to transform the electric and magnetic fields for the reflected radiation field to reference frame S' . The transformations are well known (Jackson, 1966) and yield

$$E'_{xR} = -j \frac{\omega}{\Gamma A} [\Gamma(A - \frac{(H+x')^2}{\Gamma'^2}) A_{xR} - \frac{(H+x')}{\Gamma'^2} z' A_{zR}] \quad (62)$$

$$E'_{yR} = j \frac{\omega_0 y'}{\Gamma \underline{r}^2 A} \left[\frac{\Gamma(H+x')}{\underline{r}'} A_{xR} + \frac{z'}{\underline{r}'} A_{zR} \right] \quad (63)$$

$$E'_{zR} = j \frac{\omega_0}{\Gamma A} \left[\frac{\Gamma(H+x')}{\underline{r}'} \left(\frac{z'}{\underline{r}'} + \frac{v}{c} \right) A_{xR} - \left(1 - \frac{z'^2}{\underline{r}'^2} \right) A_{zR} \right] \quad (64)$$

$$B'_{xR} = \frac{-j\omega_0 y'}{\Gamma c A \underline{r}'} \left[-\frac{v}{c} \frac{(H+x')}{\underline{r}'} A_{xR} + A_{zR} \right] \quad (65)$$

$$B'_{yR} = \frac{j\omega_0}{\Gamma c A} \left[-\Gamma \left(\frac{z'}{\underline{r}'} A + \frac{v}{c} \frac{(H+x')^2}{\underline{r}'^2} \right) A_{xR} + \frac{(H+x')}{\underline{r}'} A_{zR} \right] \quad (66)$$

and

$$B'_{zR} = j \frac{\omega_0}{c} \frac{y'}{\underline{r}'} A_{xR} \quad (67)$$

where $A = 1 + \frac{v}{c} \frac{z'}{\underline{r}'}$.

The reflected radiation fields for an arbitrary, dispersive dielectric are given by (56) through (61) in the rest frame of the dielectric and by (62) through (67) in the rest frame of the source. Although these equations are not terribly complex, it is nevertheless difficult to determine the effect of the motion on the fields by merely inspecting the equations. Therefore, the quantities $^2 (\vec{E}_R \cdot \vec{E}_R^*) \left(\frac{4\pi R^*}{\omega \mu_0 I} \right)$ and $(\vec{E}_R' \cdot \vec{E}_R'^*) \left(\frac{4\pi \underline{r}'}{\omega \mu_0 I} \right)^2$ were plotted for a lossless plasma with $\chi = \frac{\omega_p^2}{\omega^2} = 5$ in order to observe some of the effects caused by the relative motion of the source and dielectric. Figure 2 shows the field patterns in the x-z plane for $x > 0$ in a polar plot. For $\beta = 0.0$, the pattern is a symmetrical two lobe pattern. In the rest frame of the dielectric, as the velocity is increased, the lobe behind the source shrinks and the lobe in front on the source "peaks up" and bends down toward the direction of motion. This is probably what should be expected

2. R^* is an arbitrary fixed distance from the image source as measured in reference frame S.

since the reflected wave is highly dependent on the primary wave, and the primary wave has this same behavior (Fujioka, 1966). In the rest frame of the source, as the velocity of the dielectric increases the lobe in the direction of motion of the dielectric grows larger and bends down toward the interface and the other lobe shrinks.

5. LATERAL WAVE FIELDS FOR A LOSSLESS PLASMA DIELECTRIC

The generalized technique formulated in section 3 for evaluating the branch point contributions to A_{1xs} and A_{1zs} will be applied in this section to the branch point given by (37). It will be found that the contribution from this branch point corresponds to the lateral wave fields.

Proceeding as in section 4 we find that the contributions to the vector potential from this branch point are given by

$$A_{1zL} = \frac{jv\Gamma\mu_0 I_0 (1-X)^{1/2} X^{1/4} e^{f(k_{20})}}{2\pi\omega_0 R'^2 [\sqrt{X} - \frac{(H+X)}{R'} \sqrt{1-X}]^{3/2}} \quad (68)$$

and

$$A_{1xL} = \frac{-j\mu_0 I_0 c(1-X)^{1/2} e^{f(k_{20})}}{2\pi\omega_0 X^{1/4} R'^2 \epsilon_{r0}^2} \frac{\Gamma^2 \frac{v}{c} \left(\frac{\Gamma(z-vt)}{R'} \sqrt{1-X} + \frac{v}{c} \right) (\epsilon_{r0}^2 - 1) + \epsilon_{r0}}{[\sqrt{X} - \frac{(H+X)}{R'} \sqrt{1-X}]^{3/2}} \quad (69)$$

where

$$f(k_{20}) = j\omega_0 \left[\Gamma \left(t - \frac{v}{c} z \right) - \sqrt{X} \frac{(H+X)}{c} - \frac{R'}{c} \sqrt{1-X} \right] \quad (70)$$

and

$$\epsilon_{r0} = 1 - \frac{X}{\Gamma^2 \left(1 + \frac{v}{c} \frac{\Gamma(z-vt)}{R'} \sqrt{1-X} \right)^2} \quad (71)$$

The above is for the case when the dielectric is a lossless plasma. We should note that, since the location of the branch point that corresponds to the lateral wave is independent of the velocity, the criteria for existence of the lateral wave is the same as for the zero velocity case (Brekhovskikh, 1960). When $v = 0$, (68) and (69) reduce to the well known result for the zero velocity case (Brekhovskikh pg. 275, Baños pg. 184).

The frequency distribution in reference frame S for this lateral wave is given by

$$\frac{\omega_L}{\omega_0} = \left(1 + \frac{v}{c} \sqrt{1-X} \frac{\Gamma(z-vt)}{R'}\right) \quad (72)$$

where

$$R' = [y^2 + \Gamma^2(z-vt)^2]^{1/2} \quad (73)$$

We also observe that in the rest frame of the source, the lateral wave fields will not have a Doppler shift. The frequency distributions for the lateral wave fields ($X = 0.5$) and the reflected wave fields are shown in a polar plot in figure 3. The plots correspond to what would be observed on a y-z plane intersecting the image source. It should be noted that in making the lateral wave plot, the fact that the group velocity for the wave is not the speed of light, was properly accounted for in making the ray angle transformations.

We would now like to find the electric and magnetic fields in reference frame S (\vec{E} and \vec{H}) and reference frame S' (\vec{E}' and \vec{H}') corresponding to the lateral wave. A good starting point is (40) through (45) which give these fields in terms of the values of k_{33} and k_2 used in the above integration process and given here.

$$k_{33} = \frac{\omega}{c} \frac{\Gamma(z-vt)}{R'} \sqrt{1-X} \quad (74)$$

and

$$k_2 = \frac{\omega}{c} \frac{y}{R'} \sqrt{1-X} \quad (75)$$

Using (74) and (75) in (40) through (45) we obtain

$$B_{xL} = -j \frac{\omega}{c} \frac{y}{R'} \sqrt{1-X} A_{1zL} \quad (76)$$

$$B_{yL} = j \frac{\omega}{c} [\sqrt{X} A_{1zL} - \Gamma F A_{1xL}] \quad (77)$$

$$B_{zL} = j \frac{\omega}{c} \frac{y}{R'} \sqrt{1-X} A_{1xL} \quad (78)$$

$$E_{xL} = -j \frac{\omega}{\Gamma E} [A_{1xL} [\Gamma^2 E^2 - X] - \Gamma \sqrt{X} F A_{1zL}] \quad (79)$$

$$E_{yL} = j \frac{\omega}{\Gamma E} \frac{\sqrt{1-X}}{\Gamma E} [\sqrt{X} A_{1xL} + \Gamma F A_{1zL}] \quad (80)$$

and

$$E_{zL} = j \frac{\omega}{\Gamma E} [\sqrt{X} \Gamma F A_{1xL} - (1 - \frac{\Gamma^2 (z-vt)^2}{R'^2} (1-X)) A_{1zL}] \quad (81)$$

where

$$E = 1 + \frac{v}{c} \frac{\Gamma (z-vt)}{R'} \sqrt{1-X} \quad (82)$$

$$F = \frac{\Gamma (z-vt)}{R'} \sqrt{1-X} + \frac{v}{c} \quad (83)$$

and A_{1zL} and A_{1xL} are given by (68) and (69). We would now like to transform the electric and magnetic fields for the lateral wave to reference frame S' . The transformations yield

$$B'_{xL} = -j \frac{\omega y'}{c R'} \frac{\sqrt{1-X}}{\Gamma E} [A_{1zL} - \Gamma \frac{v}{c} \sqrt{X} A_{1xL}] \quad (84)$$

$$B'_{yL} = j \frac{\omega}{c \Gamma E} [\sqrt{X} A_{1zL} - (E \frac{z'}{R'} \sqrt{1-X} + \frac{v}{c} X) A_{1xL}] \quad (85)$$

$$B'_{zL} = j \frac{\omega y'}{c R'} \sqrt{1-X} A_{1xL} \quad (86)$$

$$E'_{xL} = \frac{-j\omega}{\Gamma E} [\Gamma (E-X) A_{1xL} - \sqrt{X} \sqrt{1-X} \frac{z'}{R'} A_{1zL}] \quad (87)$$

$$E'_{yL} = j\omega \frac{y}{R'} \frac{\sqrt{1-X}}{\Gamma E} [\Gamma \sqrt{X} A_{1xL} + \frac{z'}{R'} \sqrt{1-X} A_{1zL}] \quad (88)$$

and

$$E'_{zL} = \frac{j\omega}{\Gamma E} [\Gamma F \sqrt{X} A_{1xL} - (1 - \frac{z'^2}{R'^2} (1-X)) A_{1zL}] \quad (89)$$

The lateral wave fields for a lossless plasma are given by (76) through (81) in the rest frame of the dielectric and by (94) through (89) in the rest frame of the source. In analogy with the discussion on the reflected radiation fields, we would like to plot quantities

like $(\vec{E}'_L \cdot \vec{E}'_L) (\frac{2\pi R'^2}{\mu_0 \epsilon_0 c})^2$ and $(\vec{E}'_L \cdot \vec{E}'_L) (\frac{2\pi R'^2}{\mu_0 \epsilon_0 c})^2$. These are shown in figure

4, where again due account has been made in the reference frame S plots for the fact that the group velocity for the lateral wave is not the speed of light³. The above quantities are plotted in a polar plot for $X = 0.5$, the y-z plane and $(H+x) = 0$. For $\beta = 0.0$, the patterns are isotropic. In the rest frame of the source, as the velocity is increased, the pattern becomes elongated in the direction the dielectric is moving. It almost looks as if the energy being radiated by the source is being dragged along by the dielectric. In the rest frame of the dielectric, as the velocity increases the pattern is elongated in the direction of motion of the source.

6. SURFACE WAVE FIELDS FOR A LOSSLESS PLASMA DIELECTRIC

We will now examine the contribution to the integral for A_{1xs} for a lossless plasma dielectric due to the pole associated with the function $K \epsilon_p + h$. In order to find the location of the pole, we must solve the equation

$$(1 - k_e^2 - k_2^2)^{1/2} (1 - \frac{\omega_p^2}{\Gamma^2 \omega_0^2 (1 + \beta k_3)^2}) + (1 - k_3^2 - k_2^2 - \frac{\omega_p^2}{\omega_0^2})^{1/2} = 0 \quad (90)$$

where

$$k_3 = \frac{c}{\omega_0} k_{33} \quad (91)$$

and

$$k_2 = \frac{c}{\omega_0} k_2 \quad (92)$$

3. See Holmes (1968) for a more complete discussion of this aspect of the problem.

for the value of k_3 which satisfies it. Since this is virtually impossible to do analytically, we will assume that (90) has been solved and continue with the evaluation of the integral. Then later, after the solution has been formulated, (90) can be solved numerically.

The integration with respect to k_{3p} is then just $2 j\pi R_p \text{SIGN}(z-vt)$ where R_p is the residue at the pole. The contribution by the pole to the integral for A_{1xs} then is given by

$$A_{1xs} = \frac{\mu I \omega}{4\pi c} \text{SIGN}(z-vt) e^{j\Gamma \omega_0 (t - \frac{v}{c} z)} \int_{-\infty}^{\infty} R_p e^{j\Gamma' \frac{\omega}{c} f_2} dk_2 \quad (93)$$

where

$$R_p = \frac{-2[\Gamma^2 \beta(k_{3p} + \beta) - \epsilon_{rp} h_p]}{k_{3p} (\epsilon_{rp}^2 - 1) + 2K_{-p} \frac{\omega_p^2}{\omega_0^2} \frac{\beta}{\Gamma^2 (1 + \beta k_{-e})^3}} \quad (94)$$

$$f_2 = -\frac{(H+x)}{\Gamma'} K_p - k_{3p} \frac{\Gamma(z-vt)}{\Gamma'} - k_2 \frac{y}{\Gamma'} \quad (95)$$

$$K_p = (1 - k_{3p}^2 - k_2^2)^{1/2} \quad (96)$$

$$h_p = (1 - k_{3p}^2 - k_2^2 - \frac{\omega_p^2}{\omega_0^2})^{1/2} \quad (97)$$

$$\epsilon_{rp} = (1 - \frac{\omega_p^2}{\omega_0^2 \Gamma^2 (1 + \beta k_{3p})^2}) \quad (98)$$

k_{3p} equals the value of k_3 which satisfies (90) and the remaining variable of integration has been changed to k_2 . Using the saddle point technique to perform the integration with respect to k_2 , A_{1xs} becomes

$$A_{lxss} = \frac{\mu_0 I_0 R_{p0} e^{j \frac{\pi}{4} + j \Gamma \omega_0 (t - \frac{v}{2} z) + j r' \frac{\omega_0}{c} f_2(k_{3p}, k_{20})}}{\sqrt{\frac{c}{\omega_0}} 2\sqrt{2\pi} \sqrt{r'} \left[\frac{\partial^2 f_2}{\partial k_2^2}(k_{3p}, k_{20}) \right]^{1/2}} \quad (99)$$

where

$$R_{p0} = R_p|_{k_2 = k_{20}} \quad (100)$$

and k_{20} is obtained from the solution of

$$-\frac{(H+x)}{r'} \frac{dk_{-p}}{dk_2} - \frac{\Gamma(z-vt)}{r'} \frac{dk_{-3p}}{dk_2} - \frac{y}{r'} = 0 \quad (101)$$

Equation (101) contains the functions $\frac{dk_{-3p}}{dk_2}$ and $\frac{dk_{-p}}{dk_2}$. These can be obtained from (96) and (90). Equation (101), from which the saddle point k_{20} is found, then becomes

$$\frac{-k_2(1 - \epsilon_{rp}^2) \left[\frac{\Gamma(z-vt)}{r'} - \frac{k_{-3p}}{k_{-p}} \frac{(H+x)}{r'} \right]}{-k_{3p}(1 - \epsilon_{rp}^2) + \frac{2K_h B}{\Gamma^2(1 + \beta k_{-3p})^3} \chi} - \frac{y}{r'} + \frac{k_2}{k_{-p}} \frac{(H+x)}{r'} = 0 \quad (102)$$

The solution for A_{lxss} given by (99) "goes down as" $(r')^{-1/2}$ which implies that A_{lxsp} corresponds to a surface wave. In order to find out if a surface wave is possible, the phase function for A_{lxss} must be examined. When $v = 0$, (99) reduces to the well known result for the zero velocity case (Baños pg. 124).

A parameter associated with the surface wave which is of interest is the normalized frequency given by

$$\frac{\omega}{\omega_0} = [1 - \beta \operatorname{Re}(\frac{\partial f_2}{\partial (\cos \theta')})] \quad (103)$$

where use has been made of

$$\frac{\partial}{\partial t} = - \frac{\Gamma v}{\Gamma} \frac{\partial}{\partial \cos \theta'} \quad (104)$$

We note that in the rest frame of the source, the surface wave does not exhibit a Doppler shift.

We would now like to find the electric and magnetic fields in reference frame S (\vec{E} and \vec{H}) and reference frame S' (\vec{E}' and \vec{H}') corresponding to the surface wave(s). Proceeding as before and using (91) and (92) yields

$$E_{xs} = -j \frac{\omega_0 [\Gamma^2 (1 + \frac{v}{c} k_{3p})^2 - K_p^2]}{\Gamma (1 + \frac{v}{c} k_{3p})} A_{1xss} \quad (105)$$

$$E_{ys} = j \frac{\omega_0 k_{20} K_p}{\Gamma (1 + \frac{v}{c} k_{3p})} A_{1xss} \quad (106)$$

$$E_{zs} = j \frac{\omega_0 K_p (k_{3p} + \frac{v}{c})}{(1 + \frac{v}{c} k_{3p})} A_{1xss} \quad (107)$$

$$B_{xs} = 0 \quad (108)$$

$$B_{ys} = -j \frac{\Gamma \omega_0}{c} (k_{3p} + \frac{v}{c}) A_{1xss} \quad (109)$$

$$B_{zs} = j \frac{\omega_0}{c} k_{20} A_{1xss} \quad (110)$$

$$E'_{xs} = j \frac{\omega_0}{(1 + \frac{v}{c} k_{3p})} [K_p^2 - (1 + \frac{v}{c} k_{3p})] A_{1xss} \quad (111)$$

$$E'_{ys} = j \frac{\omega_0 k_{-20} K_{-p}}{(1 + \frac{v}{c} k_{-3p})} A_{1xss} \quad (112)$$

$$E'_{zs} = j \frac{\omega_0 K_{-p} (k_{-3p} + \frac{v}{c})}{(1 + \frac{v}{c} k_{-3p})} A_{1xss} \quad (113)$$

$$B'_{xs} = j \frac{\omega_0 v}{c} \frac{k_{-20} K_{-p}}{(1 + \frac{v}{c} k_{-3p})} A_{1xss} \quad (114)$$

$$B'_{ys} = -j \frac{\omega_0}{c(1 + \frac{v}{c} k_{-3p})} [k_{-3p}(1 + \frac{v}{c} k_{-3p}) + \frac{v}{c} K_{-p}^2] A_{1xss} \quad (115)$$

and

$$B'_{zs} = j \frac{\omega_0}{c} k_{-20} A_{1xss} \quad (116)$$

The surface wave fields for a lossless plasma are given by (105) through (110) for the rest frame of the dielectric and by (111) through (116) in the rest frame of the source. These equations are not explicit functions of the coordinates, but are expressed as functions of the coordinates, but are expressed as functions of two parameters which are obtained from the simultaneous solution of (90) and (102). The detailed computer calculations necessary to obtain a plot of the surface wave fields are in progress. However a preliminary study of (90) and (102) has been performed in an effort to find how the surface wave modes behave as the velocity increases. It was found that the criteria for existence of the ordinary surface wave is modified by the relative motion of the source and dielectric. As the relative velocity is increased, the ordinary surface wave ceases to be excited in all directions along the surface. The wave becomes restricted to a wedge shaped region ($0 \leq \text{wedge angle} < 2\pi$) behind the source. This is illustrated in figure 5 by the top three curves which show the wedge angle versus velocity.

In addition, for velocities greater than some critical velocity, a new type wave comes into existence. This wave is a surface type wave and radiates into a wedge shaped region behind the source. As the velocity increases, the wedge angle increases from zero degrees at the critical velocity to a larger value. This is illustrated for $\chi = 1.5$ by the lower curve in figure 5. From the work that has been done, it appears that this wave exists for all values of χ . This is contrasted by the ordinary surface wave which exists only for χ greater than two. It is anticipated that a further study of the surface wave modes will be made.

7. SUMMARY

The problem of a point electric dipole moving over a dispersive dielectric half-space has been studied. Expressions for the fields in both reference frames have been formulated for the reflected radiation fields for the case of an arbitrary, dispersive dielectric and for the lateral wave and surface wave fields for the case of a lossless plasma dielectric. In the rest frame of the source, it is found that all three waves exhibit the frequency of the source and that in the rest frame of the dielectric, they all exhibit some form of Doppler shift. It is found that the criteria for existence of the lateral wave is not modified by the relative motion of the source and dielectric, but that the criteria for existence of the surface wave is greatly modified by the relative motion. In addition, for velocities greater than some critical velocity, a new type of surface wave comes into existence.

ACKNOWLEDGEMENT

The work reported in this paper was sponsored in part by the Air Force Cambridge Research Laboratories, Office of Aerospace Research, under Contract F 19(628)-68-C-0126, but does not necessarily reflect endorsement by the sponsor.

REFERENCES

- Baños, A. (1966), Dipole Radiation in the Presence of a Conducting Half-Space, Pergamon Press, New York.
- Brekhovskikh, L. M. (1960), Waves in Layered Media, Academic Press, New York.
- Fujioka, H., T. Shiozawa and N. Kumagai (June, 1966), "Electromagnetic Radiation from an Electric Dipole Moving with Relativistic Velocity", Proceedings of Electronics and Communications Engineers of Japan, Vol. 49, pp. 89-95.
- Holmes, J. R. (1968), Scattering of Electric Dipole Radiation by a Moving, Dispersive Dielectric Half-Space, University of Washington Ph.D. Dissertation.
- Jackson, J. D. (1962), Classical Electrodynamics, John Wiley and Sons, New York.
- Pyati, V. P. (1966), Radiation Due to An Oscillating Dipole Over a Lossless Semi-Infinite Moving Dielectric Medium, University of Michigan Ph.D. Dissertation.
- Shiozawa, T. and K. Hazama (June, 1968), "General Solution to the Problem of Reflection and Transmission by a Moving Dielectric Medium", Radio Science, Vol. 3, pp. 569-576.
- Tai, C. T. (1965), "Reflection and Refraction of a Plane Electromagnetic Wave at the Boundary of a Semi-Infinite Moving Medium", paper presented at URSI Spring Meeting, Washington, D. C.
- Van Der Waerden, B. L. (1950), "On the Method of Saddle Points", Appl. Sci. Research, Vol. 2B, pp. 33-43.
- Yeh, C. (1967), "Reflection and Transmission of Electromagnetic Waves by a Moving Plasma Medium, II. Parallel Polarization", Journal of Applied Physics, Vol. 38, pp. 2871-2873.
- Yeh, C. (1966), "Reflection and Transmission of Electromagnetic Waves by a Moving Plasma Medium", Journal of Applied Physics, Vol. 37, pp. 3079-3082.
- Yeh, C. (1965), "Reflection and Transmission of Electromagnetic Waves by a Moving Dielectric Medium", Journal of Applied Physics, Vol. 36, pp. 3513-3517.

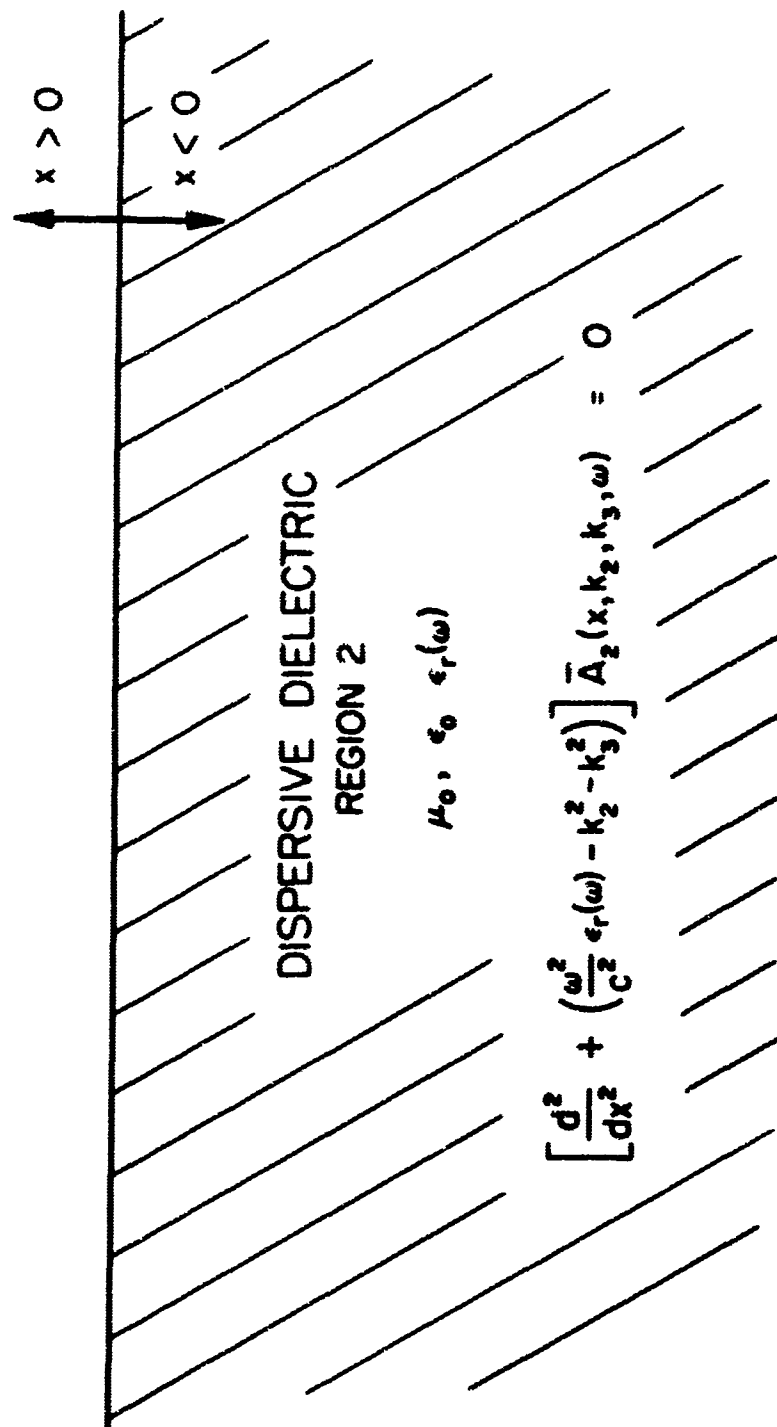
LIST OF FIGURES

- Figure 1. Boundary value problem, reference frame S.
- Figure 2. Variation with β of reflected wave fields, x-z plane, lossless plasma dielectric, $X = 5$.
- Figure 3. Relative frequency, reference frame S.
- Figure 4. Variation with β of lateral wave fields, y-z plane, lossless plasma dielectric, $X = 0.5$.
- Figure 5. Surface wave modes, lossless plasma dielectric.

FREE SPACE
REGION 1

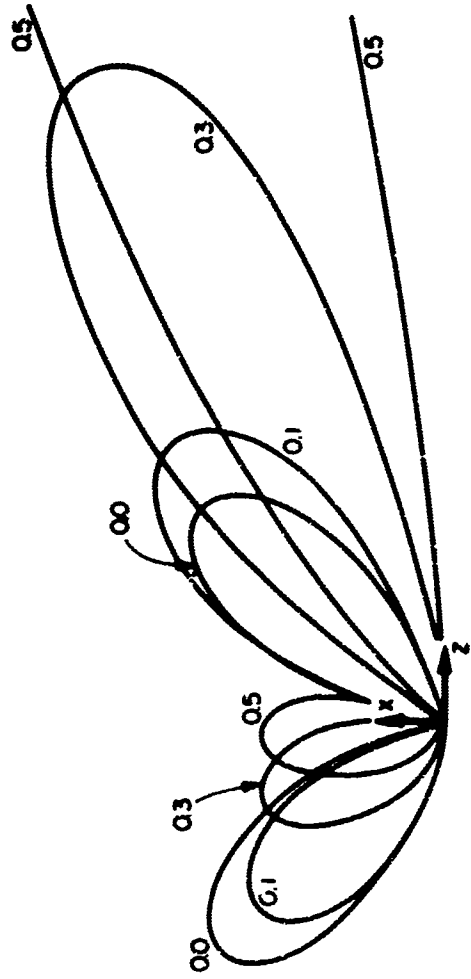
μ_0, ϵ_0

$$\left[\frac{d^2}{dx^2} + \left(\frac{\omega^2}{c^2} - k_2^2 - k_3^2 \right) \right] \bar{A}_1(x, k_2, k_3, \omega) = -\mu_0 \bar{J}(x, k_2, k_3, \omega)$$

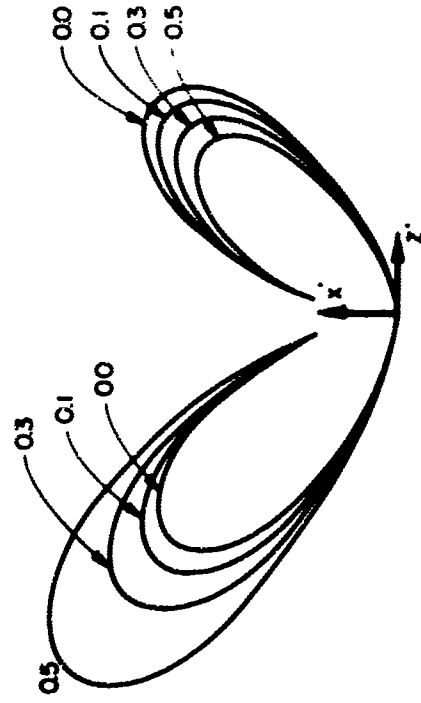


$$\left[\frac{d^2}{dx^2} + \left(\frac{\omega^2}{c^2} \epsilon_r(\omega) - k_2^2 - k_3^2 \right) \right] \bar{A}_2(x, k_2, k_3, \omega) = 0$$

REFLECTED RADIATION FIELDS

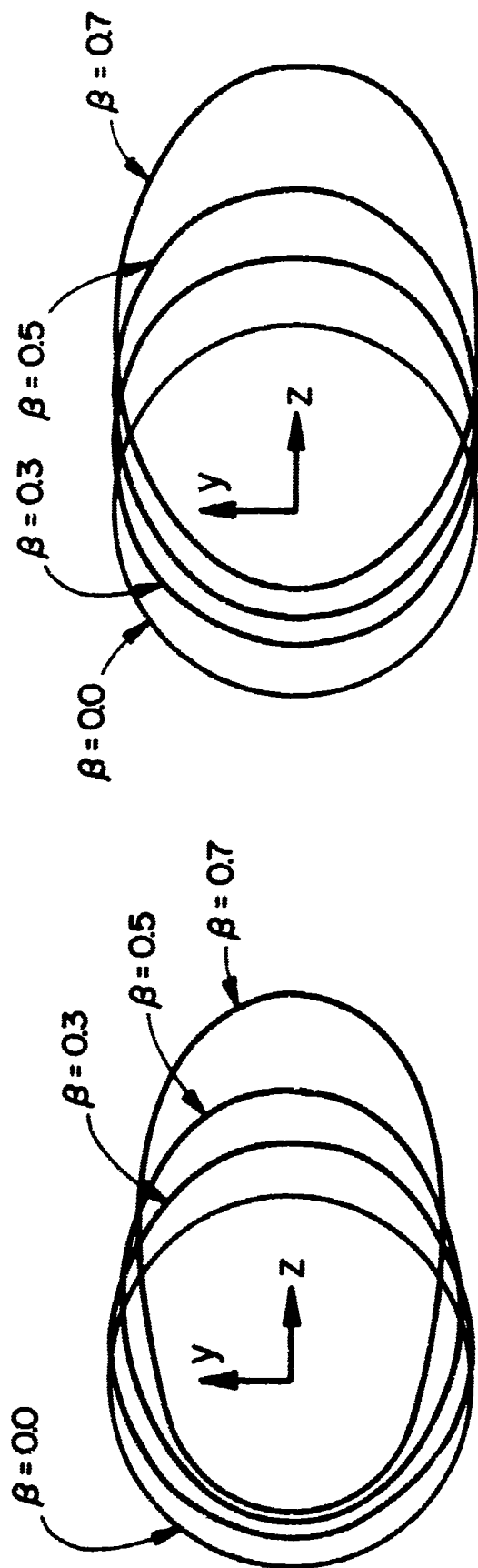


REFERENCE FRAME S



REFERENCE FRAME S'

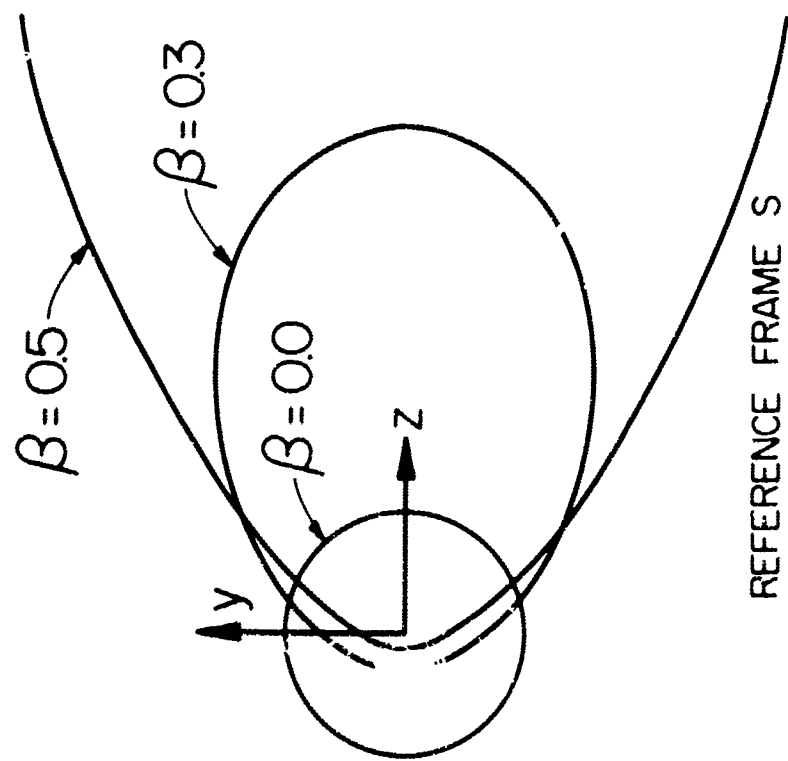
RELATIVE FREQUENCY, REFERENCE FRAME S



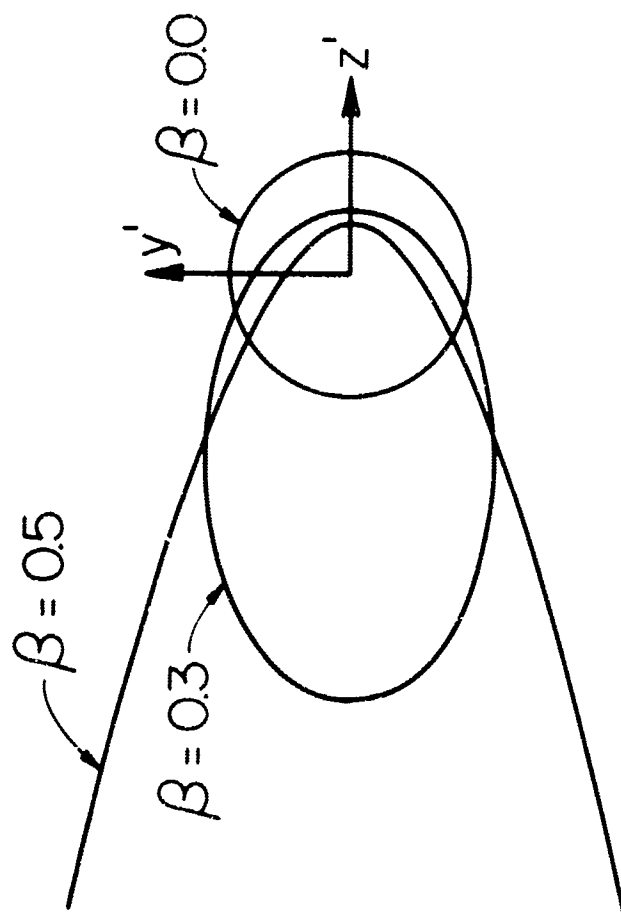
LATERAL WAVE

REFLECTED WAVE

LATERAL WAVE FIELDS



REFERENCE FRAME S



REFERENCE FRAME S'

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) University of Washington Department of Electrical Engineering Seattle, Washington 98105		2a. REPORT SECURITY CLASSIFICATION Unclassified	
3. REPORT TITLE SCATTERING OF ELECTRIC DIPOLE RADIATION BY A MOVING, DISPERSIVE DIELECTRIC HALF-SPACE			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Scientific Interim			
5. AUTHOR(S) (First name, middle initial, last name) James F. Holmes Akira Ishimaru			
6. REPORT DATE April 1969	7a. TOTAL NO OF PAGES 32	7b. NO OF PAGES 12	
8a. CONTRACT OR GRANT NO F 19(628)-68-C-0126 b. PROJECT NO 4600-10-01 c. DOD ELEMENT 62702F d. DOD SUBELEMENT 674600	9a. ORIGINATOR'S REPORT NUMBER(S) Technical Report No. 133 Scientific Report No. 3 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) AFCRL-69-0200		
10. DISTRIBUTION STATEMENT 1 - Distribution of this document is unlimited. It may be released to the Clearinghouse, Department of Commerce, for sale to the general public.			
11. SUPPLEMENTARY NOTES TECH, OTHER		12. SPONSORING MILITARY ACTIVITY Air Force Cambridge Research Laboratories (CRD) L. G. Hanscom Field Bedford, Massachusetts 01730	
13. ABSTRACT The problem of a point electric dipole moving over a dispersive dielectric half-space is studied. The dipole is located in the free space above the dielectric and is assumed to be time harmonic in its rest frame, oriented perpendicular to the interface and moving parallel to it. Previous work in this area has been mostly confined to the use of a plane wave as a source or a non-dispersive dielectric. Solutions for this problem are obtained using integral transform techniques. The integration is performed for the free space region by making an asymptotic expansion for one integration and then using the saddle point approximation for the remaining integration. The solution obtained in this manner for the vector potential is then used to generate the electric and magnetic fields in both the rest frame of the source and the rest frame of the dielectric. This yields the reflected radiation fields for the case of an arbitrary, dispersive dielectric and the lateral wave fields and surface wave fields for the case of a lossless plasma. The field patterns are distorted by the relative motion of the source and dielectric. In the rest frame of the source, all three waves exhibit the frequency of the source and in the rest frame of the dielectric, they all exhibit some form of Doppler shift (different from the primary wave). The criteria for existence of the lateral wave is not modified by the relative motion of the source and dielectric; but the criteria for existence of the surface waves is greatly modified by the relative motion. For velocities greater than some critical velocity, a new type of surface wave comes into existence.			

DD FORM 1473
1 NOV 65

Unclassified

Security Classification

Unclassified

Security Classification

14	KEY WORDS	LINK A		LINK B		LINK C	
		ROLE	WT	ROLE	WT	ROLE	WT
	Scattering Dipole Radiation Moving Media Moving Dispersive Dielectric Lateral Waves Surface Waves Doppler Shift						

Unclassified

Security Classification