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ROYAL AIRCRAFT ESTABLISHMENT

Technical Report 69054

March 1969

CCMPARISON OF AIR DENSITIES DERIVED FROM THE ORBITS OF 1966-51A, B AND C

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SUMMARY

On 9 June 1966 three satellites were launched into polar orbits with perigee heights near 180 km and apogees near 3600 km. They were Secor 6 (1966-51B), a rectangular box of mass 17 kg and length 0.33 m; ORS 2 (1966-51C), a regular octahedron of side 0.23 m and mass 5 kg; and the finalstage Agena rocket (1966-51A), of length at least 6 m. Analysis of the extensive orbital data now available on these three satellites gives 260 values of air density at heights near 190 km. Comparison of these values provides a severe test of the accuracy of the orbital data and methods of analysis, and shows that the accuracy is at least as good as has previously been estimated. The comparison also reveals that the effective cross-sectional area of Secor 6 oscillated with an amplitude of about 10%, with minima when perigee was near the equator and maxima at latitudes near 70°. As a consequence, the amplitude of the semi-annual variation in air density previously obtained from Secor 6 should be reduced from 1.45 to 1.40. Other findings are that the effective cross-sectional areas of the satellites never approached their extreme possible values, and that the effective area of 1966-51A changed by less than 10% from week to week.

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1 INTRODUCTION

The three satellites 1966-51A, B and C, launched on 9 June 1966, have yielded valuable information about the upper atmosphere. All three objects were in similar polar orbits, with perigees near 180 km and apogees initially near 3600 km. The decrease in inclination of all three orbits was used to determine the rotational speed of the atmosphere¹; the orbit of 1966-51B (Secor 6) was used^{2,3} in a detailed study of atmospheric density; and the orbit of 1966-51C (ORS 2) was calculated at R.A. 2. over a short time interval, and the variations in density analysed⁴.

The orbit of 1966-51A (Secor 6 rocket) has recently been determined² at R.A.E., mainly from visual observations. In the present paper, the values of orbital acceleration given in Ref.5, together with those derived from Spacetrack bulletins for 1966-51A and C, have been used to obtain values of air density for comparison with the results of Ref.2.

The comparison has three main purposes. First, it provides a severe test of the accuracy and reliability of determining density from Spacetrack orbital data: if the methods or the data are unreliable, the densities obtained from 1966-51B and C will differ significantly. Second, the results² from 1966-51B indicated an important semi-annual variation in air density, by a factor of 1.45 at 191 km: this is the only such evidence available for 1966-7 at heights below 200 km, and the value needs to be improved, if possible, because it has a crucial effect on the choice of upper-atmosphere models⁶. Third, the area/mass ratios for 1966-51B and C are known, with errors of about ±10%, and the results can be compared with those from 1966-51A, for which the size, shape and mass are unknown, to find the area/mass ratio of 51A and its variations, if any.

2 <u>CALCULATION OF ATMOSPHERIC DENSIT</u>

2.1 Orbital decay rate

The method used to obtain values of density is generally the same as that described in Ref.2; i.e. the Spacetrack values of mean motion n were differenced to give values Δn , which were then divided by the time interval Δt to obtain values of $\dot{n} = \Delta n/\Delta t$, which were assumed to apply at the midpoint of the time interval. Figs.1-3 show the values of \dot{n} for 1966-51B, C and A respectively, from Spacetrack and other sources.

The only difference in treatment here is that up to MJD 39347.8 (1966 Aug 10) the values of \dot{n} given by the R.A.E. orbit of 1966-51A were used instead of Spacetrack values of $\Delta n/\Delta t$. After this date the R.A.E. values were at irregular intervals and the Spacetrack values were used. As Fig.3 shows, there were no significant differences between the R.A.E. and Spacetrack values when both were available, so there should be no discontinuity at the change-over point.

2.2 Perigee height

The variation of perigee height for a polar crbit is given by

$$y_{p} = K - 9.2 \sin \omega + 21.4 \sin^{2} \omega - \frac{H}{2} \ln \frac{e_{o}}{e} km$$
, (1)

where ω is the argument of perigee, ε the eccentricity, and H (taken as 32 km) the density scale height at a height 3H/2 above perigee. The constant K was chosen to give the best fit to observed values of y_p ; the values used were 176.0 km, 174.4 km and 180.0 km for 1966-51A, B and C respectively.

The 'observed' values of y_p were taken from R.A.E. orbits for 51A and E, and from Spacetrack bulletins for 51C. The 3 R.A.E. values⁴ of y_p and the 12 NASA values shown in Fig.2 indicate that our chosen value of K for 51C is probably not more than 2 km in error. It should be noted that the Spacetrack values of y_p in Figs.1 and 2 require a correction of about -7 sin ω km to bring them into agreement with the usual definition of perigee height (for further explanation see Refs.2 and 7). In Fig.3 the value of K is chosen so that the curve passes through the group of values between MJD 39330 and 39350, which are much more accurate than the others⁵.

2.3 <u>Area/mass ratios</u>

The main difference between the three orbits arises from the different area/mass ratios of the three satellites, giving lifetimes of 6, 13 and 9 months for 1966-51A, B and C respectively. The orbital acceleration is proportional to $\delta = FSC_D/m$, where F = 1 for a polar orbit, S is the effective cross-sectional area, C_D is the drag coefficient, and m is the mass of the satellite.

For 1966-51B, a rectangular box measuring $33 \times 28 \times 23$ cm, the effective cross-sectional area, assuring $C_D = 2.2$, was between 0.078 m² and 0.115 m²,

and the mean value of S was taken² as 0.10 m², giving $\delta = 0.0129 \text{ m}^2/\text{kg}$ with an error of ±10% (s.d.).

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The shape of 1966-51C (erroneously described in Ref.4 as 'disc-like') is that of a regular octahedron of side 0.23 m, and choosing a value for S posed an interesting geometrical problem. The four principal projections of a regular octahedron of unit side are illustrated in Figs.4a-d: heavy lines represent edges which are perpendicular to the line of sight and therefore of unit length; other visible edges are represented by continuous light lines, and invisible edges by broken lines. The maximum and minimum values of effective cross-sectional area are shown by Figs.4a and 4b, giving possible extreme values of S for 1966-51C of 0.0529 m² and 0.0374 m² respectively, if the satellite were stabilized in either of these directions. It is more likely that the satellite was spinning, about an unknown axis, so that the best value of S is probably that given by Fig.4c, i.e. 0.0458 m², which is the same as for spin with random orientation, as well as being close to the mean of the two extremes quoted above. We therefore took S = 0.0458 m², with an error of $\pm 8\%$ (s.d.). With $C_{\rm D} = 2.2$ and m = 5.0 kg, this gives $\delta = 0.0202 \text{ m}^2/\text{kg}.$

As the size and weight of the rocket, 1966-51A, were not accurately known (it exhibited much sharper variations in brightness than a normal Agena rocket), it seemed best to choose a value of δ to give values of density consistent with those from the other two satellites. The value of δ used was 0.031 m²/kg, with a standard deviation which will be assessed in section 3.2.

2.4 Calculation of density

Values of density were calculated using the equations

$$\rho_{\rm A} = \frac{0.157 \ (1000 \text{n})}{10^6 \ \text{n}^2 \ \delta} \left(\frac{\text{e}}{\text{aH}^*}\right)^{\frac{1}{2}} \frac{(1-\text{e})^{1/2}}{(1+\text{e})^{3/2}} \left\{1 - \frac{\text{H}^*}{8\text{ae}} \left(1 - 8\text{e} + \frac{7\text{H}^*}{16\text{ae}}\right) + \frac{0.00335 \ \cos 2\omega}{\text{e}}\right\} \dots (2)$$

$$\rho_{\rm B} = \rho_{\rm A} \exp\left(\frac{y_{\rm p} - \bar{y}_{\rm p}}{{\rm H}'}\right) \tag{3}$$

where n is the mean angular motion in rev/day, a is the semi major axis in km, and H* is the value of H at perigee; \bar{y}_p is a chosen 'mean' perigee height and H' is the value of H at a height $\bar{y}_p + \frac{1}{2}$ H*. Equation (2) gives ρ_A with an error of 1% (s.d.) if, as here, e > 3H*/a. For all three

satellites \bar{y}_p was chosen as 177.5 km, while H* and H' were taken as 27 km and 30 km respectively. The resulting values of density, ρ_B apply to a height $\bar{y}_p + \frac{1}{2}$ H* = 191 km, and will be written as $\rho_{1.91}$.

3 <u>RESULTS</u>

3.1 Values of density

The resulting values of density at a height of 191 km, ρ_{191} , are shown in Fig.5, a complicated diagram which calls for extensive explanation.

Each of the 260 values of density obtained is plotted either at the epoch of the orbit determination (for the R.A.E. orbit), or at the mid-point of the time interval over which it applies (for Spacetrack values): squares, circles and triangles indicate 51A, B and C respectively. The points have been joined, by unbroken lines, in histogram form: since the Spacetrack values of $\dot{n} = \Delta n/\Delta t$ represent an average over the time interval Δt , the values of ρ_{191} are also averages over the time interval Δt , and a histogram is a more appropriate presentation than joining the points. The values of \dot{n} from the R.A.E. orbit are nominally 'instantaneous' values, at the epoch of the orbit determination; but in fact the variation of \dot{n} with time was taken as linear in most of the orbit determinations of 1966-51A, so that the 'instantaneous' value is in effect a mean value, over a time interval whose end points may conveniently be taken midway between the successive epochs of the orbit determinations.

Comparison between the values of density given by 1966-51A, B and C in Fig.5 is difficult, because the times at which the values are plotted are different, and so are the time intervals over which they are averaged. In view of these differences and the 'spiky' nature of the variations in density linked with geomagnetic activity - the peaks at MJD 39474 offer a clear example - comparisons are bound to be inexact. On the other hand, we are unlikely ever to have any better sets of data, unless satellites are specially launched for this purpose, so the comparisons are worth making.

3.2 Cross-sectional area of 1966-51A

We concentrate first on comparing the mean of the values from 51B and C with the values from 51A, to expose any large variations in the cross-sectional area of 51A. Taking the mean of B and C is easier said than done: we must average the values of ρ_{191} over time intervals that are, as nearly as possible, the same for both satellites. The values have therefore been combined into

groups, as shown by the broken lines in Fig.5. The grouping was made with the aim of ensuring that the end points of the corresponding time intervals for 51B and C should if possible not differ by more than 12 hours, and should never differ by more than 24 hours. This grouping of the values obscures some of the fine detail: but such a loss is inevitable if a valid comparison is to be made; and the fine detail has already been analysed in Ref.2, where it is noted, for example, that significant increases in density are evident at the times of 17 of the 19 geomagnetic disturbances during the life of 1966-51B.

7

The average values of density given by 1966-51B and C over each of the chosen time intervals are written between the two graphs in Fig.5. These values sometimes differ by as much as 30%, for reasons that will be discussed later. For the moment we assume that the arithmetic mean of the values from 51B and C gives the best indication of the values of density, and these mean values are superposed as a dot-dash histogram on the values of density from 1966-51A at the top of Fig.5. Although the time intervals have different endpoints the dot-dash histogram agrees remarkably well with the values of ρ_{191} from 1966-51A, shown by the histogram with unbroken lines. On only one occasion, between MJD 39400 and 39403, does the average of the values from 1966-51B and C differ by more than 10% from the average value given by 1966-51A over the same time interval.

This unexpectedly good agreement between satellites moving quite independently provides a conclusive empirical demonstration that (a) the errors inherent in the method are less than 10%, (b) the effective crosssectional area of 1966-51A did not change from week to week by as much as $\pm 10\%$, and (c) the assumed value of δ for 51A, 0.031 m²/kg, corresponding to $S/m = 0.0141 \text{ m}^2/\text{kg}$, was correct to within ±10%. If 51A had been a normal Agena rocket, of length 8 m and diameter 1.5 m, the expected maximum variations from the mean cross-section would have been ±13%, and since the actual variations are smaller, it seems very probable that any extra equipment aboard, responsible for the satellite's rather flashy appearance, did not greatly affect the cross-sectional area; i.e. the flat surfaces causing the flashes were probably integral with the body rather than on protruding vanes. If the size and shape of 51A were similar to a normal Agena, the mean crosssectional area would be about 10 m² and the mass about $10/0.0141 \simeq 700$ kg, which is close to the usual mass of an empty Agena rocket. Thus the flashing of 1966-51A was probably caused by fairly trivial alterations to the basic design of the Agena rocket.

3.3 Comparison of densities from 1966-51B and 51C

A comparison of the pairs of values of density obtained from 1966-51B and C, written on Fig.5, shows that the values from 51C are on average 16% greater than those from 51B. The possible sources of this bias are errors in (a) perigee height, (b) cross-sectional area and (c) mass. The perigee height of 1966-51B should be accurate to 11 km; the perigee height of 51C is based on rather more scattered observational values (see Fig.2), and the standard deviation is estimated as $1\frac{1}{2}$ km, corresponding to an error (s.d.) of 5% in ρ_{191} . The estimated errors (s.d.) in the cross-sectional areas of 1966-51B and C have already been quoted as 10% and 8% respectively. Recent data on the masses of the catellites suggest that the values used should not be in error by more than M. These three sources of error are enough to account for the 16% average difference between the densities found from 1966-51B and It is n ither necessary nor possible to specify how much each of the three С. errors contributes to the total; but perhaps the most likely explanation is that the area taken for 1966-51B was about 10% too large, and that the remaining 6% difference comes from errors in the perigee height, cross-section and mass of 51C.

3.4 Cross-sectional areas of 1966-51B and C

The variations in the ratio of the density given by 1966-51C, ρ_{51C} say, to the density given by 1966-51B, ρ_{51B} say, provide a record of the variations in the relative cross-sectional areas of the satellites, though we have to remember that the record may be slightly contaminated by error because the time intervals over which the density is evaluated are not exactly the same for the two satellites. We write

$$\mu = \frac{\rho_{51C}}{1 \cdot 16 \rho_{51B}} , \qquad (4)$$

where the factor 1.16 has been inserted so that μ has a mean value of 1; then, since ρ_{51C} is inversely proportional to the assumed cross-sectional area S_{51C} , and similarly for 51B,

$$\mu = K \frac{S_{51B}}{S_{51C}} = \frac{S_{51B}/S_{51B}}{S_{51C}/S_{51C}} \qquad (5)$$

8

where K is a constant which it is convenient to write in terms of the mean values of S_{51B} and S_{51C} , namely \overline{S}_{51B} and \overline{S}_{51C} respectively.

9

The 41 values of μ obtained from the values of ρ_{51C} and ρ_{51B} Fig.5 are plotted as circles in Fig.6. The variations in μ - between 0.86 and 1.14 - are smaller than might be expected. For if \bar{S}_{51B} was taken 10% too large, as we suspect, we have $\overline{S}_{51B} = 0.090 \text{ m}^2$, and from Ref.2 the limits for S_{51B} are 0.078 < S_{51B} < 0.115 m^2 . So, if \bar{S}_{51C} = 0.0458 m^2 , as assumed, and $0.0374 < S_{51C} < 0.0529 \text{ m}^2$, the extreme limits for μ are $0.75 < \mu < 1.57$. Since in fact μ remains between 0.86 and 1.14, our first conclusion from Fig.6 is that it provides strong observational support for the hypothesis that the cross-sectional areas of spinning satellites very rarely reach their extreme values. Indeed Fig.6 suggests that the variations in cross-sectional area are much less than the estimated bias error in the mean area, and that the s.d. of the variations may be taken as approximately $\frac{1}{3}$ of the difference between the mean and extreme values, instead of about $\frac{1}{2}$, as we usually assume. Taking the factor $\frac{1}{3}$ would give variations (s.d.) of 7% in S_{51B} and 6% in S_{54C} , i.e. a variation of 9% in μ : the actual variation (s.d.) in μ is 8%, so the factor $\frac{1}{3}$ provides an adequate margin of error.

The variations in μ in Fig.6 cannot be dismissed as mere scatter: they evidently have a significant structure. To remove distracting jaggedness, four pairs of points in Fig.6 have been averaged, and these average values, indicated by triangles, have been used when joining up the points with the unbroken lines. If we accept that the variations in μ , i.e. in the ratio of the cross-sectional areas of 1966-51B and C, have a significant structure, the variation in μ will be regular and continuous if, as is likely, the satellites were spinning and the changes in cross-section are caused by regular precession and nutation. A tentative curve has been drawn in Fig.6 to suggest a likely form for the variation. Our second conclusion from Fig.6 is, therefore, that it reveals a regular variation in S_{51B}/S_{51C} , as indicated approximately by the broken curve.

The third conclusion to be drawn from Fig.6 is concerned with the remarkably small scatter of the points about the broken curve, about 2% rms. It must be remembered that the values plotted in Fig.6 represent the ratios of values of density obtained at slightly different times, averaged over different time intervals, from satellites moving in independent orbits and having variable cross-sectional areas. Since the corresponding time intervals

for 51B and C often differ by as much as 20%, and the change in density between one interval and the next is also often as much as 20%, errors in μ of 4% could arise from this source alone; since the actual scatter in μ is only about 2%, errors inherent in the method of density determination must be less than 2%.

To illustrate the contribution of differences in the time intervals to the scatter, consider the point furthest from the broken curve in Fig.6, that at MJD 39470. As Fig.5 shows, the value of ρ_{51C} at this point is slightly 'contaminated' by the beginning of the magnetic storm at MJD 39478, whereas the effect of this storm on ρ_{51B} is concentrated in the <u>subsequent</u> value of density. Averaging of the two successive values of μ should noticeably reduce the scatter - and does so, as shown by the triangle in Fig.6 at day 39472. Examination of other points not very near the curve leads to similar conclusions.

3.5 Discussion of variations in μ

In considering the variations of μ , the latitude of perigee is relevant. The scales at the top of Fig.6 show the values of ω , the argument of perigee for 51B, and the latitude of perigee, ϕ_p . The variation of μ shows a consistent pattern: the four maxima in the broken curve occur at $\phi_p \simeq \pm 70^\circ$ and the three minima occur when perigee is near the equator $(\phi_p \simeq 0)$.

Details of the spin behaviour of the two satellites are not known, but it is believed that 1966-51C was spinning at rates close to its design value of 12 rev/min. If so, its cross-sectional area should not change by more than a few per cent, and the variations in μ would be primarily due to variations in S_{51R} .

Certainly 1966-51B, a rectangular box measuring $33 \times 28 \times 23$ cm, had more opportunities to change its cross-sectional area. The satellite was designed to be magnetically stabilized; but it entered an incorrect orbit on which the aerodynamic forces at perigee were about 10^8 times greater than on the intended orbit. On the actual orbit the magnetic stabilization is unlikely to have been fully successful; but it may have been partially effective, for the limited evidence from radar, radio and optical tracking is all consistent with a slow rotation rate, which may have been an angular oscillation rather than true spin. If so, the minimum cross-sectional area might have occurred when perigee was near the equator, when the longest side

(33 cm) would be horizontal if it was aligned with the magnetic field; S_{51B} could² then be as low as 0.078 m², as compared with the probable average value (see section 3.4) of 0.090 m². This may explain the minima in μ when perigee is near the equator. The maxima in S_{51B} at $\phi_p \simeq \pm 70^\circ$ cannot so easily be accounted for: the north and south magnetic poles are at latitudes of 74°N and 68°S; if they are responsible for the maxima, however, there should also be maxima at $\omega = 110^\circ$ and 250°, when perigee latitude is 70°. It is true that there are maxima in μ at MJD 39317 ($\omega \simeq 100^\circ$) and MJD 39401 ($\omega = 260^\circ$), though they might not be recognized except by those who were looking for them. Despite its deficiencies, this 'magnetic explanation' may well be correct.

If the variations in μ are caused by variations in the cross-sectional area of 1966-51B, the values of air density obtained from that satellite should be corrected to allow for the variations in S_{51B} . If this correction was made, the maxima and minima of the semi-annual variation in density (averaged over 30 days) would be as follows:

1966	July	minimum	3•7 × 1	0-10 1	kg/m ⁻	
1966	October	maximum	4•5	11	Ħ	
1967	January	minimum	3•3	"	tt	
1967	March	maximum	5•0	11		(unchanged)

In fact we cannot be sure that this full correction should be made, and the safest procedure is to take the mean of these and the original values: the amplitude of the semi-annual variation in density would then be reduced from the value of 1.45 given in Ref.2 to 1.40.

We also suspect (see section 3.4) that the mean value of S_{51B} should be rather smaller than was assumed in Ref.2. If so, all the values of density in Ref.2 should be slightly increased. We cannot specify the increase exactly, but it is unlikely to exceed 10%.

4 CONCLUSIONS

(1) The effective cross-sectional area of the Agena rocket 1966-51A changed by less than 10% from week to week, and the average area/mass was $0.014 \text{ m}^2/\text{kg}$. The size and mass were probably similar to a normal Agena, and the rather flashy appearance was probably caused by fairly minor alterations to the basic shape.

(2) The quantity μ plotted in Fig.6 represents the (normalized) ratio of the effective cross-sectional area of 1966-51B and 51C, i.e. S_{51B}/S_{51C} , as deduced from the variations in orbital decay rates for the two satellites. Although theoretically capable of ranging between 0.75 and 1.57, μ remains between 0.86 and 1.14, thus confirming the usual assumption that the effective cross-sectional areas of satellites rarely approach their extreme values.

(3) Fig.6 shows that μ undergoes regular oscillations with an amplitude of about 10%. This should probably be interpreted as a variation in S_{51B} , since S_{51C} is likely to have remained fairly constant. If so, S_{51B} had minimum values when perigee was at the equator and maximum values at latitudes near 70°. The most likely explanation is that the intended magnetic stabilization of 1966-51B, a rectangular box, was partially effective and that the maxima and minima in S_{51B} occur near the magnetic equator and poles.

(4) If S_{51B} varied in the manner indicated by μ , some small amendments should be made to the values of air density deduced in Ref.2 (in which S_{51B} was assumed constant). In particular the amplitude of the semi-annual variation in air density in 1966-7 at a height of 190 km should be changed from 1.45 (as given in Ref.2) to 1.40.

(5) The values of μ in Fig.6 are obtained from the ratio of orbital decay rates of 1966-51B and C, evaluated over slightly different time intervals, and errors of $\pm \mathcal{H}$ (s.d.) would be expected from this source, in addition to any errors due to inaccuracies in orbital data and the theory used. In fact the scatter of the values of μ about the curve is \mathcal{H} (s.d.): this severe test provides conclusive empirical confirmation that the errors inherent in the data and method of analysis give rise to errors of less than \mathcal{H} .

ACKNOWLEDGMENT

We thank Doreen Walker for calculating the values of n for 1966-51C.

REFERENCES

<u>No.</u>	Author	Title, etc.
1	D.G. King-Hele D.W. Scott	Further determinations of upper-atmosphere rotational speed from analysis of satellite orbits. <u>Planet. Space Sci.</u> , <u>15</u> , 1913-1931 (1967) R.A.E. Technical Report 67179 (1967)
2	D.G. King-Hele J.J. Hingston	Air density at heights near 190 km in 1966-7, from the orbit of Secor 6. <u>Planet. Space Sci., 16</u> , 675-691 (1968) R.A.E. Technical Report 67285 (1967)
3	R.H. Gooding R. Milner	The orbit of Secor 6 (1966-51B). R.A.E. Technical Report 67283 (1967)
4	D.W. Scott G.E. Cook	Analysis of the orbit of ORS 2 (1966-51C) in January 1967. R.A.E. Technical Report 67273 (1967)
5	D.W. Scott	The orbit of Secor 6 rocket (1966-51A). <u>J. Brit. Interplan. Soc.</u> (in the press) R.A.E. Technical Report 68170 (1968)
6	I. Harris W. Priester	On the semi-annual variation of the upper atmosphere. Unpublished paper (April 1968)
7	D.G. King-Hele D.M.C. Walker	Air density at heights between 130 and 160 km, from analysis of the orbit of 1968-59B. R.A.E. Technical Report 68260 (1968)



Fig.1 Orbital decay rate, \dot{n} , and perigee height, $y_{\rm F}$

A





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Fig.l





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Fig.3

Fig. 4 a-d



(a) Along axis of Symmetry (A=1)



(b) Along Plane of symmetry (A = $1/\sqrt{2} = 0.707$)



(c) Perpendicular to face ($A=\sqrt{3}/2 = 0.866$)



(d) Parallel to face (A = $\sqrt{2}/\sqrt{3}$ = 0.816)

Fig4a-d Visible areas A of a regular octahedron with edges of unit length, along four principal lines of sight



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Fig. 5 Values of p_{191} , air density at 0





at a height of 191 km, given by 1966-51A, 51C and 51B

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Fig. 6

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Fig. 6 Values of μ

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