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TECHNICAL REPORT 68299

A PROP3 USERS' MANUAL

by

R. H. Gooding

R. J. Tayler

MINISTRY OF TECHNOLOGY
FARNBOROUGH HANTS

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ERRATA

Page 12, line 19: for "e," read "e_1".

Page 13, lines 25-26: for "no provision has been introduced for presenting the computer with <u>fixed</u>" read "it will very rarely be required to read into the computer fixed".

Page 19, line 1: for "became" read "become".

Page 28, at the top: the 3 × 3 matrix should be post-multiplied by the column vector $(\cos\delta' \cos\alpha', \cos\delta' \sin\alpha', \sin\delta')^T$.

Page 28, last line: for "ICT" read "ICL".

Page 39, line 30: for "away" read "array".

Page 41, line 1: "0h" means, of course, "zero hours".

Page 42, line 14: for "job" read "run".

Page 42, line 18: for "use" read "use**".

Page 42, as a second footnote: "** The printing of block-data zeros as *000000.00 is a curious feature of 1907 Fortran".

Page 53, line 13: for "1950.0" read "1950.0**".

Page 53, as a second footnote: "** PROP3 treats INEQEQ values of 0 and 5 in the same way as for LOOXEE 666666 (q.v.). See also the large footnote on page 61."

In general: certain commas have failed to register.

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SUMMARY

PROP3 is the third version of a computer program for refining the orbital parameters of an earth satellite. A description of the program is given, together with full instructions on how to use it.

Departmental Reference: Space 288

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1 INTRODUCTION

PROP is the acronym for a computer program for the refinement of the orbital parameters of an artificial earth satellite. It was conceived in July 1965, as an eventual successor to the Pegasus programs[1,2,3] which were used at R.A.E. from 1962 to 1968. The new program has been written in the FORTRAN language (A.S.A. standard version[4]) so that it may be used on most large modern computers. It was developed on the Atlas computer of London University and is now working on the ICL 1907 computer.

PROP is based on Merson's analytical model[5] for the orbital motion of a satellite. The parameters of this motion are refined by use of observations of the satellite, and observations of many types may be used, such as range, right ascension and declination, direction cosines, range rate, etc. The present paper describes some of the structural features of the program and gives detailed instructions for using it.

In a preliminary version of the program - the unpublished PROP1 - it was only possible to determine a single set of orbital parameters from a single set of observations of one satellite; further determinations had to be set up ab initio. With a later version - PROP2 - it became possible to deal, in automatic succession, with sets of observations of a number of satellites. In the current version - PROP3 - a large number of fairly minor improvements has been introduced.

PROP3 consists of a main program and 38 subprograms, as listed in Appendix A. The structural relationship of the program units is illustrated at Fig.1 and a flow chart for the main program is provided at Fig.2. The program is available to interested parties either through the FORTRAN source listings or as a source deck of cards punched in the Atlas/IBM Fortran code, also known as the "bcd" code. A complementary program, known as PREP, is also available[6]; this is a program for reporting ephemeris position, based on the PROP dynamical model and incorporating most of the PROP subprograms.

This Manual may be read straight through, but is intended mainly as a work of reference. To this end an index has been provided. Also, comprehensive cross-references have been included; thus a reference to section 3 or section 6.1 would be indicated, respectively, by "(S3)" or "(S6.1)".

2 PROGRAM FUNCTION

The function of PROP3 may be summarized as follows: given estimates of the orbital elements of an earth satellite at some specified epoch, and given observations of the satellite over a period of some days, the program refines the estimates of some or all of these elements by an iterative least-squares differential-correction procedure. The initial elements are provided either directly by the user or else by the program itself, in the latter case by prediction from elements at some other epoch. By means of this prediction facility, orbital elements at a series of epochs can be generated during a single visit to the computer; a set of observations is provided for each epoch of the series, but estimated initial elements are required for the first epoch only.

2.1 Dynamical model

During the orbit refinement PROP takes account of perturbations due to the harmonic coefficients of the earth's gravitational field and to atmospheric drag. Formulae for the short-periodic, secular and long-periodic effects of the earth's zonal harmonics have been given by Merson[5]. The effect of the tesseral harmonic $J_{2,2}$ on the mean anomaly of the satellite is taken into account by the method described by Gooding[2].

The main effect of atmospheric drag on a satellite is to reduce the semi-major axis, a, and hence to increase the mean motion, n. This effect may be allowed for, in PROP, by taking the rate of change of mean motion, $\dot{n}$, as a parameter to be determined. The program calculates the small effects of drag on the elements e, i, Ω and ω (eccentricity, inclination, right ascension of the node and argument of perigee) in terms of $\dot{n}$.

No account is taken of perturbations due to the sun and moon. If, however, the secular effects of these can be calculated beforehand, constant contributions to the rates of change of the orbital elements can be fed into PROP. In the next version of PROP it is hoped to incorporate such calculations internally. It is remarked that constant rates of change are only valid for periods up to about three days, but that for close-earth satellites luni-solar perturbations are very small.

2.2 Observations and observed quantities

An observation consists of data obtained by a sensor (ground-station) at a particular time. It consists of one or more observed quantities, e.g. range rate or a pair of direction cosines. The differential-correction subprogram of PROP can handle the following 16 types of observation, but only Types 1, 2, 3, 4 and 7 are accepted by the PROP3 input subroutine.

Type	Observed quantities
1	ρ (range)
2	α, δ (right ascension and declination)
3	l, m (west-east and south-north direction cosines)
4	$\dot{\rho}$ (range rate)
5	$\dot{\alpha}$, $\dot{\delta}$ (R.A./dec. rates)
6	$\dot{l}$, $\dot{m}$ (direction cosine rates)
7	ρ, α, δ
8	ρ, l, m
9	$\dot{\rho}$, $\dot{\alpha}$, $\dot{\delta}$
10	$\dot{\rho}$, $\dot{l}$, $\dot{m}$
11	ρ, $\dot{\rho}$
12	α, δ, $\dot{\alpha}$, $\dot{\delta}$
13	l, m, $\dot{l}$, $\dot{m}$
14	ρ, α, δ, $\dot{\rho}$, $\dot{\alpha}$, $\dot{\delta}$
15	ρ, l, m, $\dot{\rho}$, $\dot{l}$, $\dot{m}$
16	x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$.

Type 16 does not relate to observations in the normal sense; it has been included in PROP for compatibility with the ephemeris-generation program PREP[6].

Observations are read from cards, each of which is punched in one of a number of possible formats and the program allocates the appropriate type number. Azimuth and elevation are converted to right ascension and declination, and so come under type 2, 7, 12 or 14.

The observation formats are associated with the sources of the observations, i.e. the organizations concerned, rather than with the observation types. Descriptions of the current formats in which observations can be punched are given in Appendix B. The input subroutine can easily be extended to cover further formats.

2.3 Observation weights

The differential-correction procedure uses the weighted least-squares technique. Every observed quantity is weighted according to an a priori estimate of its accuracy (one-sigma level) - the weight is in fact taken equal to the inverse square of this estimate.

For certain groups of sensors standard weights are used. Observations from sensors in these groups are automatically weighted from fixed values of the accuracy estimates; these values are set by the main program. The observations currently treated in this way are as follows:-

(i) Minitrack observations (l, m); the estimated accuracy of a direction cosine is 0.00029, equivalent to an angular accuracy of 1 min arc.

(ii) NASA range and range-rate observations (ρ, $\dot{\rho}$); the estimated accuracies are 150 m and 0.5 m/s respectively.

(iii) Moonwatch visual observations, as received on punched cards; the estimated accuracy is 0°.1, interpreted as 0.0017 radian.

For all other observations the estimate of accuracy must be punched for each observation as part of the format.

When more than one observed quantity is included in an observation, the quantities are regarded as independent. The estimated accuracy of both direction cosines of a pair is taken as the same. With α and δ, however, the estimated accuracy refers to δ; the estimated accuracy of α is given by $\sigma(\alpha) = \sigma(\delta) / \cos \delta$. It is because this weighting of right ascension and declination is equivalent to an analogous weighting of azimuth and elevation that azimuth/elevation observations are converted to right ascension/declination and not given their own type number.

Time is not treated as an observed quantity in PROP, but for observations which have a priori estimates of both timing and angular errors (σ_T, σ_A), the estimated accuracy in δ is assigned a compounded value given by:

$$\sigma(\delta) = [\sigma_A^2 + \tfrac{1}{2} \dot{\theta} \sigma_T^2]^{\frac{1}{2}} ,$$

where $\dot{\theta}$ is an approximation to the mean tracking rate, given by

$$\dot{\theta} = n / \sqrt{2} (1 + \tfrac{1}{2}e^2 - R/a) ,$$

where n, e and a are mean motion, eccentricity and semi-major axis of the orbit and R is the mean equatorial radius of the earth.

2.4 Sensors

The geographical location of sensors must be provided on punched cards, one card for each sensor. Instructions for punching sensor cards are given in section 8.6.

3 TIME AND COORDINATE SYSTEMS

The basic time system used by PROP is UT1 and epochs should normally be understood to be UT1 epochs. The times of many observations are given in the UT1 system, but some observations (e.g. Minitrack observations) refer to UTC, the system defined by WWV transmissions, and others (e.g. Baker-Nunn camera observations) to the atomic-time system A1. The input routines include the facility to correct WWV or A1 times to UT1.*

Calendar dates are reckoned in Modified Julian Days (MJD)[7]. These are related to (ordinary) Julian Days by the formula MJD = JD - 2400000.5. Thus during the present century each midnight is represented by a five-digit integer; e.g. 1960 JAN 1.0 is represented by 36934.

PROP uses a system of rectangular coordinates defined as follows. The origin O is at the centre of the earth and the axis Oz points towards the north pole. Ox lies in the plane of the equator of date, but instead of pointing towards the true equinox of date it points towards a projection of the mean equinox of the epoch 1950.0. More precisely, the position of Ox is obtained by rotation from the true equinox, by the amount of the precession and nutation in R.A. since 1950.0, but in the opposite sense. Oy completes the right-handed system Oxyz.

The above system is the one used by the Smithsonian Institution Astrophysical Observatory in the D.O.I. program[8]. It is remarked that 1950.0 above actually means 22h 09m 42s on 1949 DEC 31, i.e. 33281.9234 MJD.

For further information on systems of time and coordinates, in relation to satellite orbits, the comprehensive treatment by Veis[9] should be consulted.

4 ORBITAL ELEMENTS AND ORBITAL PARAMETERS

4.1 Orbital elements

In the orbital model used in PROP there are five basic elements: eccentricity, e; inclination, i; right ascension of the node, Ω; argument of perigee, ω; and mean anomaly, M. As in the D.O.I. program[8] these are mean elements in the sense that they are free of first-order short-periodic perturbations. Variation of the basic elements is represented by polynomials in time, with additional perturbation terms; thus

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*In orbits determined from Minitrack observations this facility has not so far been used; i.e. orbital parameters refer to the UTC system in which, it should be noted, there are sometimes 100 msec "jumps".

$$e = \bar{e} + de \quad ,$$

$$i = \bar{i} + di \quad ,$$

$$\Omega = \bar{\Omega} + d\Omega \quad ,$$

$$\omega = \bar{\omega} + d\omega$$

and

$$M = \bar{M} + dM \quad .$$

where $\bar{e}$ etc. are polynomials in time, i.e.

$$\bar{e} = \sum_{j \geq 0} e_j t^j \quad , \quad \text{etc.}$$

Here e_o, i_o, Ω_o, ω_o, M_o are values of the mean elements at epoch (when $t = 0$) and e, i, Ω, ω, M are values at time t, i.e. the perturbations de etc. are zero at epoch. (It is remarked that e_o, i_o etc. are <u>not</u> in general the values of the osculating eccentricity, inclination etc. at epoch, since the first-order short-periodic perturbations in the osculating elements are <u>not</u> defined to be zero at epoch - see also Appendix C.) The de etc. do not appear explicitly to the PROP user, who need not normally be concerned with them. In PROP3 they represent long-periodic perturbations due to the zonal harmonics of the earth's gravitational field and atmospheric drag, but they may be extended to cover the tesseral harmonics and luni-solar effects, in a later version of the program.

The degree of each polynomial is optional, between the values 0 and 5 (inclusive), except that the lower limit of the degree of $\bar{M}$ is 1, since M_1 - the mean motion at epoch - must always appear. The polynomials provide for the secular variation of the elements due to the earth's zonal harmonics and atmospheric drag. The set of coefficients e_o, e_1, ... i_o, ... etc. in the dynamical model, between 7 and 30 of them altogether, will be referred to as "the orbital elements" of the satellite.

The mean value of the mean motion, n, is a derived parameter given by the rate of change of $\bar{M}$; thus

$$n = \sum_{j \geq 0} n_j t^j \quad ,$$

where $n_j = (j + 1) M_{j+1}$.

The semi-major axis a is defined by

$$a = (\mu/n^2)^{\frac{1}{3}} - \tfrac{1}{2} J_2 R^2 (\mu/n^2)^{-\frac{1}{3}} (1 - 1\tfrac{1}{2} \sin^2 \bar{I})(1 - \bar{e}^2)^{-1\frac{1}{2}} ,$$

where μ is the earth's gravitational constant, J_2 is its second zonal harmonic coefficient and R is its mean equatorial radius.

4.2 Orbital parameters

The complete set of orbital elements, as just defined, are sometimes referred to as the orbital parameters of a satellite. With PROP, however, it is convenient to reserve the word "parameters" for a subset of the set of elements, namely, those elements which are refined by the differential-correction procedure. At least one element must be a parameter - since otherwise there would be nothing for PROP to do - and the maximum number of parameters allowed has arbitrarily been set at 20, though it would be unusual to use more than 10. It is possible for all elements to be parameters. Any that are not are regarded as being "held fixed", whereas the parameters may be thought of as "floating".

The situation may be clarified by consideration of the e polynomial, say $\bar{e} = \sum_{j=0}^{K-1} e_j t^j$, where $1 \leq K \leq 6$, in which case there are K elements associated with e. Then the first m, for any m such that $0 \leq m \leq K$, may be chosen by the PROP user to be parameters and the remainder, if any, to be "fixed elements". It is not possible to have a parameter with suffix higher than that for any of the fixed elements, e.g. to have e_1 as a parameter while holding e_0 fixed.

At the beginning of each PROP3 run a complete set of values of the orbital elements is made available to the computer, and the parameters are refined at the end of each iteration. The "fixed" elements are in general held fixed throughout the run, as the name suggests. In the case of e_1, i_1, Ω_1 and/or ω_1, however, contributions to these elements are computed afresh at the beginning of each iteration, from formulae which represent the theoretical effects of certain perturbations, so these elements will vary slightly from one iteration to the next (S4.3).

Although there is considerable flexibility in the possible choice of elements and parameters, there are two main ways in which PROP3 will be used and these are now summarized.

Standard model - observations are available over a period of a few days, usually not more than 4 or 5: $\bar{e}$, $\bar{i}$, $\bar{\Omega}$ and $\bar{\omega}$ are linear, while $\bar{M}$ is quadratic or cubic; e_o, i_o, Ω_o, ω_o, M_o, M_1, M_2 and M_3 (if $\bar{M}$ is cubic) are the parameters with M_2 (and M_3) providing an empirical representation of acceleration due to air drag; e_1, i_1, Ω_1 and ω_1 are "fixed" elements; there are 11 (or 12) elements, 7 (or 8) parameters.

Alternative model - observations over a much longer period, perhaps more than a month are to be combined: the polynomials are of high enough order to fit the observations, and all the elements are parameters.

4.3 Exclusive and inclusive elements

The elements e_1, i_1, Ω_1 and ω_1 will almost always be present in the dynamical model chosen for the orbit. They represent the secular rates of change of e, i, Ω and ω, and they receive special treatment by PROP. Consider the e polynomial, say. Whereas e_o and such of e_2, e_3, e_4 and e_5 as are in the dynamical model are either treated as parameters or else held absolutely fixed at their initial values, for e_1 the situation is more complicated. This element as used to evaluate the polynomial as a function of time, is formed by the addition of two components. One of these components has the same status as e_o, e_2 etc., in that an arbitrary value may be read into the computer initially and then treated as a parameter or held fixed. This component is called the "exclusive" e_1; it will normally be much the smaller of the two components and indeed will frequently - like e_2 etc. - be zero. The other component is computed as a function of the elements e_o, i_o, ω_o, M_1 and M_2 (assuming M_2 present in the model), at the beginning of every iteration of the PROP differential-correction procedure. This component has not been given its own name but its combination with the exclusive e_1 is called the "inclusive" e_1. For consistency of definition it has been arranged that the combination of a computed component of e_1 with the exclusive component is always carried out (assuming e_1 is present in the model), even when e_1 is being treated as a parameter.

The above explanation for e_1 applies also to i_1, Ω_1 and ω_1. The computed components of these elements allow, in PROP3, for secular perturbations due to the earth's zonal harmonics and atmospheric drag. The exclusive elements represent other secular perturbations; they may be treated as parameters, or set to zero, or given fixed values obtained by the user independently of PROP3 - for example to represent luni-solar secular effects.

The values of e_1 etc. on punched cards, both as input at the beginning of a run and as the final output provided by the computer, are exclusive values. The values given by the lineprinter output (S9) are inclusive values.

4.4 Further remarks on exclusive and inclusive elements

Two further points must be mentioned in order to provide a complete explanation of the system of elements used in PROP. They are rather technical and the rest of this section may be omitted, without loss, by the general reader.

First, it was decided at a late stage in the development of PROP3 to extend the distinction between exclusive and inclusive elements to e_j etc. with $j > 1$, the object being to improve the representation of drag. Prior to this modification the situation was simply that the drag sub-components of the computed components of e_1 etc. contained the acceleration element M_2 as a factor; i.e. they were proportional to the value of $\dot{n}$ at epoch. But a more natural procedure would have been to arrange for drag components of $\dot{e}$ etc. to be proportional to $\dot{n}$ all the time, i.e. to introduce computed components of e_2 etc., proportional to n_2, and so on for as many n's as are in the model. Thus the modification that was made was to introduce exclusive-to-inclusive computation for e_j etc., for as many j as there are both e_j and n_j in the chosen model. The zonal-harmonics sub-components still apply to $j = 1$ only, and no provision has been introduced for presenting the computer with fixed exclusive values of e_j etc., with $j > 1$, other than zero. So long as none of e, i, Ω and ω is represented by a quadratic or higher-degree polynomial, e.g. so long as the standard model is used, PROP will be unaffected by this modification.

Second, for reasons explained by Gooding[2], it would be unsatisfactory to permit the existence of exclusive elements Ω_j and ω_j $(j \geq 1)$ in the orbital model unless certain adjustments are made to the elements M_j. If the M_j presented to the computer are regarded as "exclusive", in a further extension of the meaning given in section 4.3, then the adjustment to each M_j consists of the subtraction of ω_j and $\Omega_j \cos i_o$, the adjusted M_j being regarded as "inclusive". Since this adjustment must apply to the mean anomaly but not to the mean motion, quantities n_j, equal to $(j + 1) M_{j+1}$, are computed and stored while the M_j are still in exclusive form. The Ω_j and ω_j are normally very small and hence there is usually not much difference between the exclusive and inclusive values of the M_j.

It is again stressed that the elements printed by PROP are inclusive, but that elements on cards are always exclusive.

5 UNITS AND CONSTANTS

Distances are measured in kilometres. Angles are expressed in degrees for input and output, but are held as radians inside the computer (though this is normally of no consequence to the user). Time is in days for the input and output of orbital elements (e.g. Ω_1 is in degrees/day and M_2 is in degrees/day/day), but otherwise in seconds (e.g. in the storage of orbital elements and the output of range-rate residuals).

Care has been taken over the specification of constants used in the program. The need to change the values of the geophysical and certain program constants has been anticipated and the current value of each constant is assigned once only, either in the main program or else in the block data subprogram; communication between program units* is by the FORTRAN variables to which these values have been assigned. To assign a new value to a constant it is normally necessary to recompile the main program or block data subprogram.

5.1 Geophysical constants

Current values of the geophysical constants are assigned in the block data subprogram and listed in every PROP output. Values of μ, J_2 and R (S4.1) are given - in terms of the FORTRAN variables used - as follows:-

EMU = 398602.0 (km^3/sec^2), EJ(2) = 1082.68 $\times$ 10^{-6} and ERAD = 6378.163 (km).

Values of J_3, J_4, ... J_{15} have been assigned from the results of King-Hele et al.[10,11] to EJ(3) etc. The maximum degree of the zonal harmonics allowed for, without recompiling, is given by L = 16. (To use a value $L_{max} > 16$ it is necessary to change the dimensions of the arrays EJ, in common block /ORBIT/, and ABCD, in common block /PRECON/. EJ must be given dimension L_{max} instead of 16 and ABCD must be given a first dimension of $L_{max} - 2$ instead of 14.)

The density scale height of the earth's atmosphere is currently set as DENSCH = 25.0 (km). The values of the tesseral harmonic $J_{2,2}$ and its

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*A "program unit" is either the main program or one of the three classes of subprogram: "subroutine", "function" and "block data".

longitude phase $\lambda_{2,2}$, used in the mean-anomaly correction^2, are given by **EJ22** = 0.0000018 and **ELAM22** = -0.31 (radian) respectively.

5.2 Program design constants

The initial value of ε (S6.6) is normally (S5.5) set, in the main program, by:- EPSLON = 200.0.

The maximum number of observations (MAXIM) and the maximum number of sensors (MAXSTA) which are allowed are set to 80 and 30 respectively. If it is desired to exceed these limits - or to reduce them to save storage - the main program must be recompiled; the statements MAXIM = 80 and MAXSTA = 30 must be altered and so must the items DATA(21,80) and STASHN(8,30) in the DIMENSION list.

Three accept/reject constants are set by: REJLEV(1) = 3.0, REJLEV(2) = 4.0, REJLEV(3) = 10.0. They have the function of dividing the observations, during each iteration of the differential-correction procedure, into four groups (S6.7).

5.3 Observation weights

The weights of NASA Minitrack, range and range-rate observations and of certain optical observations are given standard values (S2.3). The assigning statements are:- SIGMA(1) = 0.00029, SIGMA(2) = 0.15, SIGMA(3) = 0.0005 and SIGMA(4) = 0.0017.

5.4 Input/output constants

The card-reader, lineprinter and card-punch unit numbers are set in the block data subprogram. A further setting gives the number of lines on a lineprinter sheet, i.e. the maximum possible number of lines of printing which can occur, assuming no top or bottom margin. For running on Atlas these four input/output constants are set by: IR = 0, IP = 0, IPUNCH = 15 and ILINES = 51. On the 1907 they are set by: IR = 1, IP = 2, IPUNCH = 3 and ILINES = 66. Every input or output instruction in every program unit of PROP3 refers to its peripheral device by the appropriate one of these FORTRAN variables.

5.5 Constants that may be changed without recompiling

New values of four of the constants mentioned earlier can be set without recompiling (S8.2). The four are: L (must be ≯ 16), EJ22, DENSCH and EPSLON (initial value).

6 OVERALL PROGRAM DESIGN

6.1 Meanings of "run" and "job"

A "run" consists of the complete action of the program in refining orbital parameters for a particular satellite at a particular epoch; the epoch is specified by an integral number of days - held by the FORTRAN variable MJDOCH - so that epochs are always at 0h (midnight). A run starts with the reading of a control card (S8.2) and, if successful, ends with the output of the final values of the orbital elements and of the covariance matrix of the parameters. However, a run may end in failure due to one of a number of specified causes; for example the refinement process may not converge, or too many observations may be rejected at some stage. The cause of failure is printed, whenever this occurs. Besides success and one of the standard failures there is a further possibility. A run may break down without coming to an "end" at all; the most likely reason is that the data deck for the run has been prepared incorrectly.

A "job" consists of a sequence of runs carried out by PROP3, one after the other, without manual intervention.

6.2 Initial elements and prediction

For each run of a job PROP3 requires a set of initial orbital elements, including the estimated values of the parameters which are to be refined. These elements may be provided in one of two ways: either by the user in the data deck for the run, or else by the program itself, making use of the results of the previous run. In the latter case it must be true that (a) the run is not the first of the job and (b) the previous run was for the same satellite.

The situation just referred to involves the idea of "prediction"[6]. Given values of the orbital elements of a satellite at one epoch, PROP can be asked to predict their values at another epoch, using the normal formulae for orbital perturbations. Predictions can be either forwards to a later epoch or backwards to an earlier one.

The validity of the prediction formulae is limited by the difficulty in accounting for the long-term effects of drag. Prediction over a period of a week is usually satisfactory even for low-perigee satellites, since it is only initial elements which are predicted. Prediction over a number of weeks should normally be avoided, since the error in the (along-track) element M_0, in particular, may be so large as to cause the refinement process to diverge.

The usual application of prediction is when initial elements for a run are predicted automatically by PROP from the final elements of the previous run. A second application arises when the user provides PROP with an initial set of elements, but wishes the analysis to be based on a different epoch from the one associated with these elements. This happens when a series of runs is spread over more than one job and the results from the last run of a job are used in preparing the data for the first run of a new job.

The various possibilities are covered by the value of the FORTRAN variable NEWSAT, read from the control card at the beginning of each run (S8.2).

6.3 End of job

The most satisfactory end of a job occurs when all its runs have been dealt with successfully, i.e., a final (converged) set of orbital parameters has been printed, together with standard deviations, at the end of each run.

After the successful completion of any run, the next, if there is one, is automatically started. Otherwise, the action of the computer depends on whether the run came to a standard failure, or "broke down" (S6.1).

If a run comes to a standard failure then the control card for the next run is read. If this control card specifies the prediction of elements from the run that failed, then the new run does not proceed beyond this point; any further runs are abandoned and the job ends. If, however the new run supplies its own elements, then the failure of the preceding run is irrelevant and the job proceeds. Thus the success of the last run of a job does not imply the success of all preceding runs. The occurrence of a standard failure is marked by a printed statement about the failure (S9.4) and this always starts with the word "DISCONTINUE".

If a run breaks down the job is abandoned at once, no further run being started. Here "breaking down" covers two possibilities. The run may fail in a way which is regarded as non-standard, but yet which has been provided for by the program, since a suitable diagnostic is printed; e.g. "ORBITAL ELEMENTS ARE FOR WRONG EPOCH". Alternatively, a completely unanticipated failure may be reached; it is hoped this will be a rare occurrence.

6.4 "Mode" of refinement

If the initial estimates of the orbital elements are good, and if the coverage of the observations (i.e. their distribution around the orbit and in time) is adequate, it should be all right to start refining all the parameters together straight away. When the estimates are poor, however, it is better to start by refining one or two of the parameters only, temporarily holding the others as fixed elements; the latter can be introduced after one or more iterations with the restricted model. Two parameters which should be refined from the beginning are M_0 and M_1, in order to fit the mean-anomaly variation, i.e. to get the "timing" right.

The way in which the parameter model is restricted in PROP is by means of what is called the "mode" facility. There are three possible modes of operation, and the initial mode is indicated by the value of the FORTRAN variable MODE, read from the control card for the run (S8.2).

MODE = 0 gives the mode in which all parameters are refined immediately. MODE = 1 gives a mode in which at most the four parameters e_0, M_0, M_1 and M_2 are refined - "at most", because one or more of these elements may not be parameters to be refined at all. MODE = 2 gives a mode in which at most M_0 and M_1 are refined. For MODE > 2 the mode is the same as for MODE = 2, but the process will stay longer in this mode.

At the end of each iteration, if no observations have been rejected (S6.7) and if MODE > 0, the value of MODE is reduced by unity; otherwise, the next iteration is carried out in the same mode. Since a run is only permitted to complete its convergence in the zero mode, all successful runs will eventually reach this mode, with all restrictions on the parameter model removed.

6.5 Singularities

Steps have been taken to minimise the effects of the well-known singularities in the orbital elements - those at $\sin^2 i = 0.8$ (the critical inclinations), at $e = 0$ (circular orbits), and at $i = 0°$ or $180°$ (equatorial orbits). The formulae of Merson[5] for the position and velocity of a satellite at arbitrary time have been so expressed that none of these singularities causes any trouble in the part of PROP which computes position and velocity. It should be remarked, though, that the orbital model may represent the perturbations in the satellite motion less accurately near the singularities. Thus Cook[12] has shown that near the "circular" singularity perturbations are more accurately expressed in terms of elements $e \cos \omega$ and $e \sin \omega$ than in terms of the normal e and ω.

There are real difficulties associated with the differential-correction part of PROP, however, at both the "circular" and the "equatorial" singularities.

These difficulties arise from the fact that ω and M became indeterminate at the circular singularity, and Ω and ω at the equatorial one. The effects are two-fold: first the convergence of the process may be adversely affected - it may break down altogether - due to the use of inappropriate elements near a singularity; second the accuracy of the computer's basic matrix-inversion operation may be affected by the fact that partial-derivative matrices become ill-conditioned due to the presence of almost-equal columns. Either or both of these effects may be minimised if the PROP user gives an appropriate value to the control parameter (FORTRAN variable) JELTYP (S9.2).

Satisfactory convergence is restored by a temporary transformation of orbital parameters at just one part of the program. Consider, for example, a nearly equatorial orbit for which both i_o and Ω_o are (floating) parameters for which corrections Δi_o and $\Delta\Omega_o$ have just been computed. Then if JELTYP has been appropriately set the computer does not correct i_o and Ω_o directly, but forms corrections to the temporary parameters $\sin i_o \cos \Omega_o$ and $\sin i_o \sin \Omega_o$ instead, using

$$\Delta(\sin i_o \cos \Omega_o) = \Delta i_o \cos i_o \cos \Omega_o - \Delta\Omega_o \sin i_o \sin \Omega_o$$

and

$$\Delta(\sin i_o \sin \Omega_o) = \Delta i_o \cos i_o \sin \Omega_o + \Delta\Omega_o \sin i_o \cos \Omega_o \quad .$$

It then computes new values of i_o and Ω_o from the corrected values of the temporary parameters. Similarly, for a nearly circular orbit the parameters $e_o \cos \omega_o$, $e_o \sin \omega_o$ and $\omega_o + M_o$ temporarily replace e_o, ω_o and M_o. For an orbit which is both nearly equatorial and nearly circular the parameters e_o, i_o, ω_o, Ω_o and M_o are temporarily replaced by $\sin i_o \cos \Omega_o$, $\sin i_o \sin \Omega_o$, $e_o \cos \tilde{\omega}_o$, $e_o \sin \tilde{\omega}_o$ and $\tilde{\omega}_o + M_o$; here $\tilde{\omega}_o = \omega_o + k\,\Omega_o$, where k is a fixed number equal to $\cos i_o$ and it is assumed that e_o etc. are indeed all parameters under refinement.

The other aspect of the singularities, ill-conditioning, is also controlled by a transformation of parameters. Suppose that ω_o and M_o are both parameters and that we are near the circular singularity. Then partial derivatives with respect to these parameters are almost equal. However if these parameters are replaced by ω_o and $M_o + \omega_o$ the derivatives with respect to ω_o became very small and the matrix of partial derivatives is no longer ill-conditioned. The computer now obtains corrections to ω_o and $M_o + \omega_o$ from which the correction to M_o is of course immediate. Similarly, near the equatorial singularity derivatives with respect to Ω_o and ω_o are replaced by derivatives with respect to Ω_o and $\omega_o + k\,\Omega_o$, where k is the same as in the last paragraph. (Remember - a partial derivative with respect to Ω_o, keeping ω_o fixed, is not the same as a partial derivative with respect to Ω_o, keeping $\omega_o + k\,\Omega_o$ fixed!) Finally, when near to both singularities the transformation is from Ω_o, ω_o, M_o to Ω_o, $\omega_o + k\,\Omega_o$, $M_o + \omega_o + k\,\Omega_o$. It may be objected that it does not help to replace an ill-conditioned matrix by a well-conditioned matrix containing a nearly zero column, since matrix inversion will then lead to floating-point overflow. This only occurs when e_o or i_o is almost exactly equal to zero, however, and in this remote contingency PROP will fail.

Correction for both the above aspects of the singularities is by setting JELTYP (S8.2). The two aspects have been kept separate to facilitate the experimental use of PROP, but it will be normal of course, if the user considers he is near the eccentricity singularity, say, to set JELTYP to deal with both aspects together.

The effect on the computer output must be mentioned. For the line-printer output there is no effect at all on the final results (assuming that final results could be obtained if the two aspects of the singularities were not tackled!); in particular the correlation matrix (S9.3) still gives the correlations between standard parameters. For the card-punch output, tackling the first aspect has no effect. Tackling the second, however, leads to the punching of a non-standard covariance matrix; thus if partial derivatives with respect to Ω_o, $\omega_o + k\,\Omega_o$ and $M_o + \omega_o + k\,\Omega_o$ have been introduced, the punched-card covariance matrix relates to these parameters.

6.6 Convergence

The aim of the PROP refinement procedure is to adjust the orbital parameters in such a way as to minimize the sum of the squares of the weighted residuals, a residual being the difference between an observed quantity and the corresponding quantity calculated from the dynamical model; symbolically the aim is to minimize $\Sigma\, w\, R^2$, where $R = O - C$ and w, R, O, C denote respectively weight, residual, observed quantity and calculated quantity. The procedure is an iterative one and a test for convergence is made at the end of each iteration.

The convergence test is based on the value of a quantity ε, stored as the FORTRAN variable EPSLON. This is the estimated standard deviation of an observation of unit weight; if the total number of observed quantities is n and the number of orbital parameters under refinement is N, then

$$\varepsilon = \{\Sigma w R^2 / (n - N)\}^{\frac{1}{2}} ,$$

there being $n - N$ degrees of freedom.

If ε decreases from one iteration to the next, the refinement procedure is considered to be converging. When the decrease is less than one per cent the convergence is deemed to be complete, so long as the process has reached the zero mode (S6.4) and observations are not still being rejected. If the dynamical model is a good representation of the actual satellite motion, and if the observations have been given realistic weights, then the final value of ε should lie between about 0.7 and 1.5. A value in excess of 3 or 4 should be viewed with some disquiet.

Due to the non-linearity of the system, ε may increase instead of decrease. If this happens twice in succession the refinement procedure is considered to be divergent and is discontinued. However, it is possible for the procedure neither to converge nor to diverge; for example, ε may oscillate. It is essential, therefore, that the PROP user set a maximum to the number of iterations that will be allowed. This is done by one of the parameters - MAXITN - on the control card.

6.7 Rejection

A major problem in the processing of observational data in any field is the rejection of poor data. With orbit determination the basic difficulty is that a good orbit cannot be obtained until bad observations have been eliminated, while they cannot be judged "bad" with any certainty until a good orbit is available. An additional difficulty with PROP is that rejection has had to be made entirely automatic - it cannot be left to the user to decide at the end of one iteration how to reject during the next iteration.

The PROP3 rejection mechanism has the following three main features:

(i) its basis is the ratio of the weighted residual associated with an observed quantity to the current value of ε (where an observation consists of more than one observed quantity it is the numerically largest of the weighted residuals which is considered);

(ii) the decision as to which observations to reject is, as far as possible, postponed until the end of the iteration;

(iii) rejection of an observation is never irrevocable - in subsequent iterations it may be accepted if its residuals are smaller.

To implement features (i) and (ii) the observations are divided into four groups, or levels, during any iteration. Currently (S5.2) the magnitudes of the weighted residuals for all observations at the lowest level are less than 3ε, the magnitude of the largest weighted residual for each observation at the second level is between 3ε and 4ε, at the third level it is between 4ε and 10ε, and at the highest level it exceeds 10ε. Observations at each level will be either all accepted or all rejected. The lowest level is always accepted and the highest always rejected. The program decides at the end of the iteration whether to accept one or both of the two middle levels; the basis for this will not be described here.

Implementation of feature (iii) leads to the slight risk of an oscillation, against which the PROP user must be warned. A small set (say two or three) of observations may be rejected during an iteration, and the effect on the orbital parameters and ε may be such that on the next iteration they have to be accepted, after which they are rejected again etc.

If too many observations have been rejected at any stage of the refinement process it becomes pointless to continue. The PROP user should specify, by the control parameter MINOBS, the minimum number of observations with which the process will be allowed to continue at any stage.

6.8 The "previous-orbit" facility

It sometimes happens that, after the determination of a set of orbital parameters at a given epoch, further observations for the same period become available. It would be possible to make a fresh determination, using the old and new observations together, but this would be inefficient. The same results could be obtained by using the new observations only, if the PROP user

(a) sets the control parameter NOTHER to a non-zero value (as explained below),

(b) takes the final elements of the previous run as initial elements for the new run, and

(c) supplies the computer with "previous-orbit" cards, which give the final ε, the number of degrees of freedom and the covariance matrix of orbital parameters from the previous run (S8.5).

If NOTHER = 0 the computer will not read previous-orbit cards. If NOTHER = 1 it reads them, taking the covariance matrix as in standard form. If NOTHER > 1 the computer will interpret the covariance matrix as having been derived from a non-standard setting of JELTYP in the previous run (S6.5 and S8.2).

If the previous-orbit facility is used MODE should always be set to zero since the initial parameters must be "good". If MODE is set greater than zero PROP will in fact ignore the previous-orbit data until MODE has dropped to zero.

There is one situation when running a fresh determination with all the observations may lead to slightly different results from using the previous-orbit facility. In the former case the presence of new observations may cause a different subset of the old observations to be rejected in comparison with the subset previously rejected. In the latter case the subset of (implied) rejections cannot vary.

7 PRELIMINARY PROCESSING OF OBSERVATIONS

The two main stages in any PROP run are the input of observations - subroutine OBSIN - and the differential correction of orbital parameters - subroutine DIFCOR. In between these is a subsidiary stage during which certain preliminary processing of the observations is carried out. This subsidiary stage organizes the rotation of station coordinates from an earth-fixed coordinate system to the standard PROP system, and is also responsible for the correction of the observations for refraction, light-time, and precession and nutation. The subroutine responsible for all this is called PROCES.

7.1 Rotation of station coordinates

The coordinates of each sensor are rotated from an earth-fixed system of axes, such that the x-axis points towards the Greenwich meridian and the z-axis towards the north pole, to the standard PROP system (S4). The rotation is about the z-axis, by an amount $\hat{\theta}$ where $\hat{\theta}$ is the "modified sidereal angle", given by

$$\hat{\theta} = 100^\circ.075542 + 360^\circ.985612288\,(d - 33282.0) ;$$

here d refers to UT1 time, measured in MJD.

No account is taken of the polar motion, i.e. of the motion of the earth's rotation axis relative to its crust. The resulting error should be less than 15 m in station position, i.e. less than 0".5 in direction of the local vertical.

7.2 Refraction correction

Since all methods of observation depend on the transmission of electromagnetic radiation, and since all such radiation is refracted by the atmosphere, it is necessary to make appropriate corrections to the observations. For observations of right ascension and declination it is not appropriate to make the total correction from observed line of sight to true line of sight because the observations are made relative to stars which are themselves seen by refracted radiation; only the "parallactic correction" is appropriate in this case. Whether the observed quantities were originally azimuth and elevation or right ascension and declination, it is elevation which must be corrected, so that in the case of original right ascension and declination an initial transformation is made.

The formulae used for the correction of elevation are the same as were used in the old Pegasus program[1] but are repeated in the next two paragraphs for convenience. The correction, whether of total refraction or parallactic only, is a function of the observed elevation itself and the geocentric distance of the satellite. For the latter quantity a value is computed from the initial values of the orbital elements, and this should normally be sufficiently accurate.

For total refraction let E' be the observed elevation, E the corrected elevation and let r and R be the geocentric distances to the satellite and station respectively. Let η be given by

$$8.4 \tan 2\eta = \cot E'$$

and let

$$E_{\infty} = E' - (a \tan \eta + b \tan^3 \eta + c \tan^5 \eta + d \tan^7 \eta) .$$

where a = 956".513, b = 657".638, c = 179".1 9 and d = 248".314.
Let γ and θ be derived from

$$\cos \gamma = 1.000276454 \, (R/r) \cos E'$$

and

$$\theta = \gamma - E_{\infty} \quad .$$

Then E is given by

$$\tan E = \cot \theta - (R/r) \operatorname{cosec} \theta \quad .$$

For parallactic refraction E_{∞} is observed. Let η be given by

$$8.4 \tan 2\eta = \cot E_{\infty}$$

and let

$$E' = E_{\infty} + a \tan \eta + b \tan^3 \eta + c \tan^5 \eta + d \tan^7 \eta$$

where $a = 956''.426$, $b = 564''.095$, $c = 175''.239$ and $d = 9''.216$.
Then the formulae for γ, θ and E are the same as above.

For radio observations the amount of refraction, $E' - E$, is multiplied by a factor of 5. (This factor was obtained from Ref.[8] and is presumably related to a particular frequency.)

In the present version of PROP no refraction correction is made for observations of other types. In particular, range, range rate and direction cosines are not corrected. It is believed that Minitrack observations (direction cosines) and some range observations are corrected for refraction before distribution.

7.3 Light-time correction

As with the old Pegasus program[1] the finite speed of light is allowed for by correcting the reported time of each observation back to the time at which the light (or radio) waves left the satellite. This requires an approximate value of the topocentric range of the satellite to be known. If range is one of the observed quantities it is used directly. Otherwise, the approximate range is computed from the initial orbital elements.

7.4 Correction of right ascension and declination for precession and nutation

Whether right ascension and declination are given relative to the mean equator and equinox at one of the epochs 1855.0, 1875.0, 1900.0 or 1950.0 (the epochs of the principal star catalogues), or whether they are given relative to the equator and equinox of date, a transformation to the standard PROP coordinate system (S3) is carried out. Let α_o and δ_o be the quoted values of right ascension and declination, assumed relative to a standard epoch, let α' and δ' be their values relative to the mean equator and equinox of date, and let α and δ be their values relative to the standard PROP system. The transformation from α_o and δ_o to α and δ is made in three stages. (Only the last stage is involved if the angles are given relative to the true equator and equinox of date.)

Stage 1 The first transformation is from α_o and δ_o to α' and δ', i.e. it corrects for precession between the date of the star catalogue and the date of the observation.

Let k be the number of days from the catalogue epoch to 1950.0, i.e. 34698, 27393, 18262 and 0 for the epochs of 1855.0, 1875.0, 1900.0 and 1950.0 respectively. Let d be the days from 1950.0 to the time of the observation and let $d' = k + d$. (For an observation quoted relative to the mean equator and equinox of date $d' = 0$ and $k = -d$.)

Then the transformation may be expressed in terms of the standard[13] precessional quantities ζ_o, z and θ, given in radians by:-

$$\zeta_o = (3.059537 \times 10^{-7} - 5.0781 \times 10^{-15} k)\, d' + 1.0986 \times 10^{-15} d'^2 \quad ,$$

$$z = \zeta_o + 2.8809 \times 10^{-15} d'^2 \quad \text{and}$$

$$\theta = (2.66040 \times 10^{-7} + 3.1011 \times 10^{-15} k)\, d' - 1.5507 \times 10^{-15} d'^2 \quad .$$

The required transformation is given by:-

$$\begin{pmatrix} \cos\delta' \cos\alpha' \\ \cos\delta' \sin\alpha' \\ \sin\delta' \end{pmatrix}$$

$$= \begin{pmatrix} \cos z \cos\theta & -\sin z & -\cos z \sin\theta \\ \sin z \cos\theta & \cos z & -\sin z \sin\theta \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\delta_o & \cos(\alpha_o + \zeta_o) \\ \cos\delta_o & \sin(\alpha_o + \zeta_o) \\ & \sin\delta_o \end{pmatrix} .$$

Stage 2 The next transformation is from α' and δ' to values relative to the true equator and equinox. These may be denoted by α'' and δ, since the true equator is the final PROP equator required. This transformation corrects for nutation.

Let Ω_M, L_M and F_M be arguments of the moon's orbit such that, in radians,

$$\Omega_M = 0.2114 - 9.2422 \times 10^{-4}\, d \quad .$$

$$2L_M = 3.4935 + 3.440558 \times 10^{-2}\, d \quad \text{and}$$

$$2(F_M + \Omega_M) = 2.2473 + 4.599431 \times 10^{-1}\, d \quad .$$

Then the principal terms of $\Delta\psi$ the nutation in longitude and $\Delta\varepsilon$, the nutation in the obliquity are given by:-

$$\Delta\psi = 10^{-5} (-8.35 \sin\Omega_M - 0.62 \sin 2L_M + 0.10 \sin 2\Omega_M - 0.10 \sin 2(F_M + \Omega_M)) \text{ and}$$

$$\Delta\varepsilon = 10^{-5} (4.46 \cos\Omega_M + 0.27 \cos 2L_M - 0.04 \cos 2\Omega_M + 0.04 \cos 2(F_M + \Omega_M)) \quad .$$

The required transformation is given by:-

$$\begin{pmatrix} \cos\delta\ \cos\alpha'' \\ \cos\delta\ \sin\alpha'' \\ \sin\delta \end{pmatrix} = \begin{pmatrix} 1 & -\Delta\psi\ \cos\varepsilon & -\Delta\psi\ \sin\varepsilon \\ \Delta\psi\ \cos\varepsilon & 1 & -\Delta\varepsilon \\ \Delta\psi\ \sin\varepsilon & \Delta\varepsilon & 1 \end{pmatrix} .$$

where $\sin\varepsilon = 0.39788121$ and $\cos\varepsilon = 0.91743694$.

<u>Stage 3</u> The final transformation is from α'' to α. This arises from a rotation within the true equator from the true equinox to the pseudo-equinox used by PROP.

Define

$$\psi_\varepsilon = 6.119 \times 10^{-7}\ d \text{ (in radians)} .$$

Then

$$\alpha = \alpha'' - \psi_\varepsilon - \Delta\psi \cos\varepsilon .$$

8 THE DATA DECK

8.1 Overall description

This section explains how to prepare a data deck of punched cards for PROP3, i.e. the complete set of cards read under various input formats during the various runs of a job. The listing of a typical data deck is attached, for reference, as Fig.4. It is not possible to give instructions for the preparation of a complete job deck because this depends on the job description cards required by the particular computer being used whether "source" or "binary" cards are being read for the program itself, whether some or all of the program is held on magnetic tape etc. (On the 1907 computer if the program is run from magnetic tape as overlaid in binary, then no job description cards at all are required.)

Instructions for punching will not pre-suppose any knowledge of FORTRAN. However, the actual FORTRAN input formats are always given for completeness. It is remarked that, according to A.S.A. rules, the character "blank" (indicated in what follows by the letter b) is completely equivalent to zero. However Atlas is known to object to embedded blanks; thus it is better to punch 680202 (for the date 1968 FEB 2, read as a 6-digit integer) than to punch 68b2b2. The punching of plus signs, though optional, should be eschewed, since "plus" (unlike "minus") differs as between the Atlas/IBM and ICT card codes.

The order of the data cards for a run is:-

(i) control card,

(ii) epoch/identity card + orbital elements (normally on five cards).

(iii) previous-orbit cards.

(iv) sensor cards + blank card.

(v) time-system correction cards.

(vi) observation cards + blank card.

It will be seen in due course that any of these six items, except the first, may be omitted in appropriate circumstances. The blank card in item (iv) is actually read as a sensor card and indicates that no more sensor cards follow. With item (vi) the situation is more complicated. A 'set' of observation cards is made up of one or more subsets, as explained in section 8.8; each subset ends with a blank card and so item (vi) ends with two blank cards. Thus a run ends in two blank cards whenever item (vi) is present (but in no blank cards if both items (iv) and (vi) are absent).

The complete data deck for a job consists of the data cards for the various runs, followed by a blank card; this is read as a control card and indicates that there are no further runs. Assuming item (vi) is included in the last run, a complete data deck must end with three consecutive blank cards.

A general warning on punching must be made. Integers must always be punched as far right as possible in their respective fields, since they will otherwise be misinterpreted. A mixed number, on the other hand, may be punched anywhere in its field, provided that a decimal point is included.

8.2 The control card

The control card for each run contains twelve parameters (all integer), which control the working of the program during that run. It also contains optional re-settings of four of the geophysical and program constants (one integer and three real). The input format is (13I5, 3F5.0) which, since every field is five characters in width, makes it easy to punch. The interpretation of the twelve control parameters and four optional constants is given below, with some indication of the values that should normally be punched. The parameters and constants are referred to by their FORTRAN-variable names as used in the program. One possible punching, for reference is as follows:

```
2222411114bbbb0bbbb1bbbb3bbbb7bbb20bbb11bbbb2bbbb0bbbb2bbbb1bbbb9bb2.1bbbbbbbbbb
```

KKKKK (cols.1-5) This indicates which elements are present in the orbital model. The five digits of KKKKK are associated, in order, with the polynomials which represent $\bar{e}$, $\bar{I}$, $\bar{\Omega}$, $\bar{\omega}$ and $\bar{M}$; for each digit K, the degree of the corresponding polynomial is K - 1. E.g. if KKKKK = 22224, then $\bar{e}$, $\bar{I}$, $\bar{\Omega}$, $\bar{\omega}$ are linear and $\bar{M}$ is cubic; the elements present in this case would be e_o, e_1, i_o, i_1, Ω_o, Ω_1, ω_o, ω_1, M_o, M_1, M_2, M_3. The standard model (S4.2) gives a KKKKK of 22223 or 22224. (N.B. If account is taken of the first point discussed in section 4.4 it is logical to make the degrees of the e, i, Ω and ω polynomials one less than the degree of the M polynomial, so that the standard values of KKKKK become 22223 and 33334. The difference, in orbit determination over a few days, is negligible.)

MMMMM (cols.6-10) This indicates, similarly, the elements which are (floating) parameters to be refined. E.g. if MMMMM = 11114 then e_o, i_o, Ω_o, ω_o, M_o, M_1, M_2 and M_3 are parameters; if MMMMM = 10102 then only e_o, Ω_o, M_o and M_1 are parameters. The standard model gives an MMMMM of 11113 or 11114, corresponding to the standard values of KKKKK. Clearly no m, where m is a digit of MMMMM, may exceed the corresponding K, and at least one m must be non-zero.

MODE (cols.11-15) This indicates the mode in which refinement is to take place on the first iteration of the differential-correction process. If the initial estimates of the orbital parameters are believed to be good, punch MODE = 0; if they are uncertain in quality, punch 1; if they are believed to be rather poor, punch 2, etc. (S6.4).

NEWSAT (cols.16-20) This indicates the way in which the epoch and initial elements for the run are provided. There are five possibilities, as follows:-

NEWSAT = 0: epoch and elements will be taken directly from the six cards which immediately follow the control card; i.e. epoch/identity and element cards cannot be omitted.

NEWSAT = 1: epoch/identity and the five element cards are present, as in the previous case, but the run is actually to be associated with a different epoch (usually a few days later) and the input elements are to be used to predict initial elements at the new epoch.

NEWSAT = 2: no epoch/identity or element cards are provided; initial elements for the run are to be obtained by prediction from elements stored in the computer at the end of the run just completed.

NEWSAT = 3: this is the same as the previous case except that a special card is provided from which new values of the exclusive elements e_1, i_1, Ω_1 and ω_1 are taken.

NEWSAT = 4: this is similar to the first case (NEWSAT = 0), but after the epoch/identity card appears a single card, containing the six components of initial position and velocity, instead of the five element cards; these will at once be converted to mean orbital elements with any additional (exclusive) elements in the model set to zero.

(N.B. (1) If NEWSAT = 2 or 3 there must be at least one preceding run in the data deck, and this must be for the same satellite as the new run, since otherwise prediction from the preceding run would have no meaning. If NEWSAT = 0, 1 or 4 there may or may not be preceding runs and, if there are, the new run may be for the same satellite or a "new satellite".

(2) There is no NEWSAT value which covers position and velocity plus prediction.)

MJDINC (cols.21-25) This is used if prediction is required (NEWSAT = 1, 2 or 3). It gives the increment in Modified Julian Days, to be applied to the current epoch (MJDOCH) to bring it to the epoch required for the new run. E.g. to predict forward three days punch 3: to predict back a week punch -7.

MAXITN (cols.26-30) This specifies the maximum number of iterations that will be allowed by the refinement process. E.g. to stop a particular run after ten iterations, whether the process has converged or not, punch 10. (If the PROP refinement process is going to converge it will rarely require more than ten iterations.)

MINOBS (cols.31-35) This specifies the minimum number of observations with which the refinement process will be allowed to continue at any stage. Thus it might be thought intolerable to have more than a third of the observations rejected at any stage, in which case with 30 observations initially, say, MINOBS would be 20.

JELTYP (cols.36-40) This indicates whether the standard elements e_o, i_o, Ω_o, ω_o, M_o are used throughout the run, or whether non-singular elements are introduced at some point. JELTYP is best thought of as a two-digit number, of which the first, or tens digit (which appears in column 39) may be denoted by JEL1 and the second. or units digit (which appears in column 40) may be denoted by JEL2. JEL1 and JEL2 are treated independently by the program and since each can have the value 0. 1. 2 or 3 there are 16 possibilities for JELTYP. JEL1 is associated with the transformation of parameters to avoid ill-conditioned matrices (S6.5) as follows:-

JEL1 = 0 : no transformation;

JEL1 = 1 : transformation to ω_o and $M_o + \omega_o$, when e_o is close to zero;

JEL1 = 2 : transformation to Ω_o and $\omega_o + k\,\Omega_o$. when i_o is close to 0° or 180°;

JEL1 = 3 : transformation to Ω_o, $\omega_o + k\,\Omega_o$ and $M_o + \omega_o + k\,\Omega_o$. when both e_o and i_o are near to singular values.

JEL2 is associated with the temporary transformation of elements to improve the convergence of the PROP refinement process (S6.5) as follows:-

JEL2 = 0 : no transformation;

JEL2 = 1 : transformation to $e_o \cos \omega_o$ $e_o \sin \omega_o$ and $M_o + \omega_o$. when e_o is close to zero;

JEL2 = 2 : transformation to $\sin i_o \cos \Omega_o$ and $\sin i_o \sin \Omega_o$. when i_o is close to 0° or 180°;

JEL2 = 3 : transformation appropriate when both e_o and i_o are near to singular values.

Experience shows that the choice of a suitable JEL2 is more important than JEL1; however, a simple procedure is to choose the same value for both, so that JELTYP is 0, 11, 22 or 33.

<u>NOTHER</u> (cols.41-45) This indicates whether or not previous-orbit cards are included in the data deck for the run:

NOTHER = 0 : previous-orbit cards are not included;

NOTHER = 1 : previous-orbit cards are included and JEL1 = 0 for both the previous and new runs;

NOTHER = 2 : previous-orbit cards are included and JEL1 = 1 for both the previous and new runs;

NOTHER = 3 : previous-orbit cards are included and JEL1 = 2 for both the previous and new runs;

NOTHER = 4 : previous-orbit cards are included and JEL1 = 3 for both the previous and new runs.

N.B. previous-orbit cards may only be included if NEWSAT = 0.

<u>ITIMEC</u> (cols.46-50) This indicates what time-system correction cards if any, are included in the deck.

ITIMEC = 0 : no time-correction cards are included for this run;

ITIMEC = 1 : a single WWV time-correction card is included;

ITIMEC = 2 : a pair of WWV time-correction cards is included;

ITIMEC = 10 : an A1 time-correction card is included;

ITIMEC = 11 : an A1 and one WWV card are included;

ITIMEC = 12 : an A1 and two WWV cards are included.

<u>ISENSR</u> (cols.51-55) This relates to the reading and printing of sensor-card data.

ISENSR = 0 : sensor cards are present, but information is <u>not</u> to be printed after the reading of each card.

ISENSR = 1 : sensor cards are present and information about each card <u>is</u> to be printed after reading.

ISENSR = 2 : sensor cards are not present, and the subroutine for reading sensor cards is to be by-passed in the present run; PROP will use sensor data provided by an earlier run in the same job. (This option is quite independent of the NEWSAT option; so long as the MAXSTA limit is not exceeded (S5.2) it is possible to put all sensor-cards for a job into the first run and set ISENSR = 2 for all the other runs.)

<u>IOBSNS</u> (cols.56-60) This relates to the reading and printing of data from observation cards.

IOBSNS = 0 : observation cards are present, but information is <u>not</u> to be printed after the reading of each card;

IOBSNS = 1 : observation cards are present, and information about each card <u>is</u> to be printed after reading.

IOBSNS = 2 : observation cards are not present and the subroutine for reading them is to be by-passed in the present run; PROP will use the same observations as for the preceding run in the same job; preliminary processing will be restricted to resetting the initial rejection levels. (This option will rarely be selected, but occasionally it is desirable to run the same observational data twice in succession with, say, two different orbital models.)

IOBSNS is the last control parameter on the control card. The remaining columns are used for such of the constants, described below, as it is desired to reset. A blank (or zero) field for any of these constants means that the "standard value" will be used.

L (cols.61-65) This is the index of the last zonal harmonic to be used by PROP; i.e. J_2, J_3, ... J_L will be used. The standard value is 15 and the maximum permissible value, unless the program is recompiled (S5.1) is L = 16. A new specification will apply to the current run only and PROP will simply use the BLOCK DATA J's up to J_L.

EJ22 (cols.66-70) This is $10^6 J_{2,2}$ where $J_{2,2}$ is the standard tesseral harmonic. The present standard value is 1.8. If it is required to be zero a very small non-zero value must be punched. During most of a PROP run EJ22 holds the value of $J_{2,2}$ itself, instead of $10^6 J_{2,2}$.

DENSCH (cols.71-75) This designates H, the value of the density scale height of the earth's atmosphere at a height $\frac{1}{2}$H above perigee, in km. The PROP3 standard value is 25 (km) and alternative values are available from Fig.3, taken from Ref.14.

EPSLON (cols.76-80) This specifies the initial value of ε (S6.6), the convergence parameter. The standard value is 200.0. The three most likely reasons for setting a different value are:-

(i) The observations to be used in the run are of very high nominal accuracy (e.g. Baker-Nunn observations) and hence their weighted residuals will be numerically large in comparison with their unweighted residuals. If the initial values of the orbital parameters are poor the initial ε must be much larger than 200 or else all the observations may be rejected.

(ii) The initial values of the orbital parameters are thought to be so good (because they have been obtained from prediction, for example) that it would be unfortunate if an observation with a weighted residual of 50, say, were accepted on the first iteration. To ensure rejection of such observations a much smaller initial ε is required. The obvious application of this is to prevent wasted iterations when there is, say, a single "rogue" observation which will be rejected at the end anyway, but there is another application which should be mentioned. If there is a large number, more than 35% say, of poor observations the rejection process may fail completely and the run may end in "convergence" to an unacceptable final ε with none of the poor observations rejected. This can sometimes be avoided by starting with an ε as low as 2.0, say. Then all or most of the poor observations are immediately rejected at the highest level (S6.7); any observations that start by being wrongly rejected will be accepted in later iterations if all goes well.

(iii) It is desired to effect convergence after only one iteration. As an example of when this may occur, suppose that a perfectly successful run has been carried out, ending with, say, EPSLON = 1.231. It is then realised that due to use of the wrong value of JELTYP the output covariance cards are worthless. To repeat the run, with minimum waste of computer time, it is only necessary to start with the final elements just obtained and an initial EPSLON of, say, 1.235, since the convergence criterion will then be satisfied (S6.6) after a single iteration. (N.B. It would be necessary in this case to omit all the observations which were rejected by the final iteration of the earlier run, since otherwise their fresh rejection would prevent single-iteration convergence.)

It is probably sound practice, when analysing the orbit of the same satellite at regular intervals, to start all runs with an EPSLON of 5.0, 10.0 or 20.0.

8.3 The epoch/identity card

An epoch/identity card is required when the control parameter NEWSAT has the value 0, 1 or 4. It contains three items: the epoch, stored as the (integer) variable MJDOCH; the satellite identification number, stored as the (integer) variable IDENT; and the satellite name, stored as the (integer-Hollerith) array NAME(4). The input format is (I6, 3X, I7, 4A4) and a typical punching for reference is b39891bbb6704201-bARIELb3bbbbbbb etc. b.

MJDOCH (cols.1-6) This specifies the Modified Julian Day number of the epoch at which the orbital elements of the satellite - which will be punched on the five cards (or single card if NEWSAT = 4) following the epoch/identity card - are defined. If NEWSAT = 0 this is the same as the epoch with which the run is to be associated. MJDOCH may be punched as the MJD number itself, e.g. b39891 for 1968 FEB 5, or as a year-month-day number, 680205. (The computer interprets the number read by checking whether it is smaller or larger than 100000.)

IDENT (cols.10-16) This is the 7-digit designation used by the Smithsonian Astrophysical Observatory. It is used by PROP solely for checking that SAO Baker-Nunn (and certain other) observations are for the right satellite. Denoting the 7 digits by YYLLLNN YY gives the year of launch, LLL the number of the launch within that year, and NN the number of the object within that launch. E.g. the satellite Ariel 3 (1967-42A) is designated 6704201 and the rocket of that satellite (1967-42B) is 6704202.

NAME(4) (cols.17-32) This may be punched arbitrarily, or omitted. Its only function is one of reference, and the 16 allocated columns are simply copied to the printed output. A natural punching would be for example. -bARIELb3bbbbbbb or -bARIELb3bROCKET.

8.4 Orbital elements

8.4.1 Five-card orbital elements*

These cards, which must always be preceded by an epoch/identity card, are associated, one each in order, with the basic elements e, i, Ω, ω and M. They are read by the subroutine ELREAD. The read format is (I5, 1X, I1, 1X, 2F12.0, 4E12.4) and this is fully compatible with the punched-card output of (refined) elements which takes place at the end of a PROP run. This fact is very useful when, for example, orbital elements for a given satellite are being computed at, say, three-day intervals, with jobs prepared from batches of half-a-dozen runs; element cards which have been output at the end of the last run of one job may be put straight into the data deck for the first run of the next job, with NEWSAT = 1 and MJDINC = 3 on the new control card of course.

In the explanation below it is convenient to take M as a typical element and to describe the punching of the associated card. A typical punching is

```
39801/5bbbb-115.6244bbb5431.0448bbbbbb0.0492etc(blank) .
```

Cols.1-5 contain MJDOCH, which denotes the same epoch as on the epoch/identity card (S8.3) and must be punched in the MJD form (or else omitted - see below); col.6 contains a solidus (optional); col.7 contains N, the serial number (1 to 5) of the element card, e.g. 5 in the case of the M card, (or is omitted if MJDOCH is); and col.8 is always blank. These first eight columns permit a program check that the element cards are in the right order and that they correspond to the epoch given by the epoch/identity card. If the check

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*These are quite different from the five-card elements issued by the U.S.A.F. in their Spacetrack bulletins.

fails, the run - and indeed the rest of the job - is abandoned. The check will be by-passed for any card for which cols.1-8 are blank; the PROP user may take advantage of this fact, but this is not recommended.

Cols.9 onwards of the M card contain, in fields of width 12, the K_M coefficients in the polynomial (of degree $K_M - 1$) which represents M in the orbital model. Here K_M is given by the fifth digit of the control parameter KKKKK. E.g. if KKKKK = 22223, then $K_M = 3$, and if the quadratic polynomial representing M is

$$\bar{M} = -115.6244 + 5431.0448\,t + 0.0492\,t^2 \quad ,$$

the M card could be punched as in the typical example above. Here t is time in days from epoch and M is in degrees.

Punching of the e, i, Ω and ω cards is similar, but it must be remembered that the dominant terms of e_1, i_1, Ω_1 and ω_1 must not be included; i.e. the punched values are of the exclusive elements (S4.3). These exclusive elements will often be zero, in which case cols.21-32 may be left blank, since a blank field is read as 0.0.

Units are degrees (for angles) and days (for time); e.g. e_o is non-dimensional, i_1 is in deg/d and Ω_2 is in deg/d^2.

8.4.2 One-card position and velocity

This card provides an alternative to the input of five-card elements and is used when NEWSAT = 4. It must always be preceded by an epoch/identity card.

The main quantities on the card are the three coordinates of epoch position (x, y, z) in km and the three coordinates of velocity $(\dot{x}, \dot{y}, \dot{z})$ in km/sec. The read format is (I5, 3X, 6F12.0); cols.1-5 provide an optional check of MJDOCH, as in section 8.4.1, and cols.9-20, 21-32 etc. are used for x, y etc.

The card is read by the subroutine PVREAD, which converts the coordinates of position and velocity into the standard mean elements e_o, i_o, Ω_o, ω_o, M_o and M_1.

8.4.3 One-card orbital elements

This card is required in the special situation which arises when NEWSAT = 3. There will be no epoch/identity card preceding it.

It is recalled that NEWSAT = 3, like NEWSAT = 2, involves the prediction of elements from values held in the computer before the beginning of the run. If non-zero values of the exclusive elements e_1, i_1, Ω_1 and ω_1, computed outside PROP, are being supplied for each run, these have to be read in to supersede predicted values. This is the purpose of the NEWSAT = 3 option. It is emphasised that an epoch/identity card is not required for this case.

The one-card elements are simply the exclusive values of e_1, i_1, Ω_1, and ω_1. The format is (4F20.0) and the units are degrees (for angles) and days (for time).

RESDEL(1) (cols.1-20) This is the exclusive e_1.

RESDEL(2) (cols.21-40) This is the exclusive i_1.

RESDEL(3) (cols.41-60) This is the exclusive Ω_1.

RESDEL(4) (cols.61-80) This is the exclusive ω_1.

8.5 Previous-orbit cards

The significance of these cards has been explained in section 6.8. They consist of a single card, described in the next paragraph, plus a set of cards which contain the elements of the covariance matrix as punched at the end of the previous run. (N.B. 'Previous run' has the meaning implied by section 6.8 and does not relate to the preceding run, if any, in the deck of cards for the current job.) The orbital model must be the same as in the previous run and contains, say, ℓ parameters given by summing the digits of the control parameter MMMMM. The covariance cards are read under format (8E10.3); if $\ell \leq 8$ there will be ℓ such cards, if $9 \leq \ell \leq 16$ there will be 2ℓ such cards, and if $17 \leq \ell \leq 20$ (its maximum possible value) there will be 3ℓ cards.

The card which precedes the covariance cards contains two quantities, read under the format (I5, F15.0).

NOODOF (cols.1-5) This if the number of degrees of freedom for the last iteration of the "previous run", given by the printed output for that run.

OEPSLN (cols.6-20) This is the final value of ε (S6.6) for the previous run.

8.6 Sensor cards

Up to 30 sensor (observing-station) cards may be included in the data deck for a run. (This upper limit may be changed if desired; see section 5.2.) They are read by the subroutine SENSIN, assuming the control parameter ISENSR is less than 2. The sensor cards must be followed in the deck by a blank card; PROP exits from SENSIN as soon as this is encountered.

A sensor card contains information about the sensor's location (in either geographical or Cartesian coordinates), its reference number and its name. The format is (I1, I9, 2X, 4A4, F12.0, 4F10.0) and typical examples of sensor cards are

```
                                  (22)
2bbbbbbb12bbWINKFIELDbbbbbbbb......bbb3983.130bbb-48.404bb4964.711
```

and

```
                                                                     (20)
bbbbbb1000bbABAKANbbbbbbbbbbbbb91.43833bbb53.72167bbb247.0bbb.......b
```

The interpretation of the quantities to be punched on a sensor card is given below.

MARKER (col.1) This specifies the type of information provided by the rest of the card. There are four possibilities:-

MARKER = 0: geographical coordinates (longitude, latitude and height) are supplied and they refer to the Fischer spheroid[15].

MARKER = 1: geographical coordinates are supplied and information which defines the reference spheroid is also supplied.

MARKER = 2: Cartesian coordinates are supplied and they refer to the Fischer spheroid.

MARKER = 3: both geographical coordinates (longitude and latitude only) and Cartesian coordinates are supplied. (This effectively means that the sensor need not be located anywhere near the surface of the earth.)

NOSTAT (cols.2-10) This is the station number. It should adhere to a recognized numbering system such as the COSPAR system. E.g. for Villa Dolores punch bbbbb9011.

NAMEST (cols.13-28) This is the station name, stored as an away of four Hollerith elements. Its only function is to appear with the output printing (if ISENSR = 1) for reference purposes. Punch, for example, VILLAbDOLORESbbb.

STALON (cols.29-40) If MARKER ≠ 2 this is the longitude in degrees, positive to the east of Greenwich. If MARKER = 2 it is ignored.

STALAT (cols.41-50) If MARKER ≠ 2 this is the latitude in degrees, positive for the northern hemisphere. If MARKER = 2 it is ignored.

STAHGT (cols.51-60) If MARKER = 0 or 1 this is the height in metres above the reference spheroid. If MARKER = 2 or 3, it is X in km, where X is the station's coordinate measured from the centre of the earth towards the Greenwich meridian.

AEARTH (cols.61-70) If MARKER = 0 this is ignored. If MARKER = 1 it gives the semi-major axis, in km, of the required reference spheroid, e.g. 6378.388 for the Hayford spheroid. If MARKER = 2 or 3 it gives Y, where Y is measured towards the 90°E meridian.

BEARTH (cols.71-80) If MARKER = 0 this is ignored. If MARKER = 1 it gives the semi-minor axis of the reference spheroid, e.g. 6356.912 for the Hayford spheroid. If MARKER = 2 or 3 it gives Z, measured towards the north pole.

8.7 Time-system correction cards

Up to three cards for correcting the time system (S3) may be called for by the program, depending on the value of the control parameter ITIMEC (S8.2).

If an A1 (atomic time) correction card is required, it must come first. It contains a single quantity, as follows, read under format (F10.0).

A1TC (cols.1-10) This gives the correction to A1 time to give UT1; i.e. it is the epoch value of UT1-A1. It should be punched as a time in seconds, with decimal point included.

If a single WWV correction card is required, this contains a single quantity, read under format (F10.0), as follows.

WWVTC (cols.1-10) This gives the correction to WWV time to give UT1. Its punching is the same as for A1TC.

If a pair of WWV cards is required, the first simply contains WWVTC as above. The second card defines a discontinuity in the WWV system and contains two quantities, read under format (I6, F10.0), as follows.

<u>JMPDAY</u> (cols.1-6) This gives the date of the discontinuity, 0h understood. Like an epoch (S8.3) it may be punched in two possible ways.

<u>WWVTC2</u> (cols.7-16) This gives the correction to be applied <u>after</u> the discontinuity, the correction before the discontinuity being given by WWVTC from the previous card. Note that if the discontinuity occurs before epoch, i.e. JMPDAY < MJDOCH, then the correction applied at the instant of epoch will be WWVTC2.

8.8 Observation cards

PROP is flexible with regard to the variety of observation formats it is capable of accepting. It can deal with the standard formats of such organizations as the Smithsonian Astrophysical Observatory (SAO), the National Aeronautics and Space Administration (NASA) and the Radio and Space Research Station (RSRS). It can also be very easily augmented to cope with new formats, by adding extra sections to the basic observations-input subroutine OBSIN. (Sections can equally easily be removed, for computer storage economy, if they are not required.)

Details of the formats covered by PROP3 are given in Appendix B. Here we shall merely explain how a complete set of observation cards is made up.

The subroutine OBSIN reads an observations-format or "LOOXEE" card, so called from the name of the corresponding Fortran variable. This card causes control to be directed to a section of OBSIN which reads observations in a specific format. The several formats available do not, in general, correspond to particular types of observation, but rather to the styles which various organizations have adopted in publishing satellite observations. So, for example, the Moonwatch format for presenting observations in right ascension and declination is quite different from the RSRS format.

Each such section of OBSIN reads observation cards until a blank card is encountered, after which OBSIN interprets the next card as a new LOOXEE card. A complete set of observation cards for a given run will thus contain as many subsets as desired; each subset consists of a LOOXEE card followed by observations in the corresponding format and concluded with a blank card. Exit from OBSIN occurs when a blank LOOXEE card is read. This means that the last observation of the complete set must be followed by two blank cards; the first of these concludes the last subset, and the second concludes the set as a whole. These two blank cards must be present even if all the observations were read under the same LOOXEE.

Each LOOXEE card contains a single quantity, read under format (I6), as follows:

LOOXEE (cols.1-6)	This specifies the format under which succeeding cards are to be read, as explained above and detailed in Appendix B.

9 DESCRIPTION OF OUTPUT

The output of PROP has been designed to be as nearly self-explanatory as possible. The purpose of this section is to make such comments as seem necessary upon the output of a typical run*, reproduced as Fig.5. This is the output of the run for which Fig.4 lists the input data.

9.1 Preliminary output

The first three computer sheets give certain preliminary information relating to the whole run.

The title is followed, in brackets, by the date and time at which the computer started processing the job. This is a non-standard facility, added to the ICL 1907 version of PROP, which has been found useful in the day-to-day organization of orbit determinations on the R.A.E. computer.

No comment is needed on the listing, for reference, of the control parameters and of the geophysical constants in current use. Note the remark which draws attention to the total number of zonal harmonics actually to be used (as specified on the control card). In fact a remark appears whenever any of the constants L, EJ22 and DENSCH (S8.2) is reset.

Printing of the epoch date and satellite identification is preceded, in the case of prediction (S6.2), by a listing of the orbital elements used for the prediction.

The listing of sensor locations is straightforward. Note that the stations numbered 1 to 18 (which are, in fact, NASA stations) have MARKER = 2, indicating that the input coordinates were X, Y, Z. The remaining stations have MARKER = 0, indicating the the input coordinates were latitude, longitude, height.

The listing of observations needs a few comments. Each observation is assigned a serial number (in the example they run from 1 to 37), which is used for reference within the run only. If an observation card is rejected due to

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*In fact this run is not entirely "typical". The visual observations are of poorer quality than usual, possibly because the observers were concentrating on flash assessment.

an anticipated error (i.e. one which does not cause deletion of the job), then an error message is printed by the program, the information from that card is not stored, and no serial number is assigned to it.

The type number refers to one of the types listed in section 2.2; 3 indicates direction-cosines and 2 indicates R.A./declination observations; if R.A. and declination have been converted from azimuth and elevation this is indicated by an asterisk before the type number. Note that information from a pair of direction-cosine cards (see Fig.4) read under LOOXEE 444444 (see Appendix B) is printed on a single line.

The information given at the end of the line for every observation, e.g. -27 REV + .27 (.41), gives a rough measure of the spread of the observations. The quantity -27 REV + .27 gives the difference between the time of the epoch for the run and the time of the particular observation, expressed in complete revolutions plus a positive fraction of a revolution. The fraction in brackets, e.g. (.41), is the argument of latitude (i.e. argument of perigee plus true anomaly) of the satellite's position at the time of the observation.

The five column headings from OBS. NO. to TIME relate to all the observations. Other column headings are generated each time a LOOXEE card is read, and refer to a particular type of observation. If a group of observations do not have individual accuracies assigned to them but are given standard weights - e.g. with direction-cosines read under LOOXEE 444444 - then the appropriate information is given after all the observations in that group.

The transformation of direction-cosines to azimuth and elevation is done only for the purpose of having a printed record of the observation in this form; within the computer the observation is used in its original direction-cosines form. The rule that observed quantities are always used in the form in which they are given does not hold for azimuth/elevation observations; these are converted to R.A./declination, and the computer holds a record of the original form only until 'processing of observations' (S7) is complete.

With angular observations both the right ascension and declination and the azimuth and elevation are listed. Note that the listing is given by subroutine OBSIN, so that the processing, by subroutine PROCES, has not yet been incorporated, and the conversion of angles from one form to the other is carried out relative to the standard PROP axes (S3). This means that if an observation starts as right ascension and declination the listed azimuth and elevation are only approximate, particularly if the observation relates to the equinox of 1855, 1875 or 1900.

The angular accuracy is given, in seconds of arc, under S (DEC). This is a compounded value containing a component derived from the timing error (S2.3). Moonwatch observations read under LOOXEE 555555 do not have individual accuracies attached to them, and they are all assigned a standard a priori standard deviation by the program (S2.3).

The preliminary output is concluded by a listing of the initial estimates of elements, whether these were obtained directly as input data or indirectly from prediction. The elements are laid out in standard tabular form corresponding to the 'five-card orbital elements' (S8.4.1), with the addition that the derived semi-major axis (in km) is printed before the line containing the e-elements.

N.B. This listing of initial elements, though conveniently thought of as concluding the preliminary output, is actually provided at the beginning of the differential-correction subroutine.

9.2 Output during the differential-correction process

Each iteration of the process is numbered. If the mode of refinement (S6.4) has changed prior to the current iteration, a statement of the new mode is printed at the head of the table of residuals. The residuals for each observation, both the weighted and the unweighted ones, are printed, and the units of the unweighted residuals are specified. For observations of right ascension and declination a further quantity is printed; this is a "total" unweighted residual given by $\{(\cos\delta\,\Delta\alpha)^2 + (\Delta\delta)^2\}^{\frac{1}{2}}$, where $\Delta\alpha$ and $\Delta\delta$ are the residuals in right ascension and declination.

The marking of each observation with three, two, one or no asterisks relates to the separating of observations into the four 'rejection levels' (S6.7). Those marked with three asterisks will invariably be rejected at the end of the iteration, and those with no asterisks will be accepted. The acceptance or rejection of observations marked with one or two asterisks depends on rather complicated criteria which need not be set out here; the program always prints a comment as to which levels are being rejected. Note that 'residual' has the sense 'observed' minus 'computed'.

The text at the end of each iteration is self-explanatory. Note, however, that, although reference is made to the number of "fresh rejections", there is no information about "fresh acceptances", i.e. about observations rejected on the previous iteration but now accepted. Prediction of the value of ε after the next iteration is based on assumption of linearity.

9.3 Concluding output

Once convergence has been achieved, standard deviations for the final set of elements are printed, laid out in the same way as the corresponding orbital elements. If there are orbital elements which are not parameters then these have not varied during the refinement process and the corresponding numbers in the standard-deviations layout are zero.

The symmetric matrix which is the final block of data printed consists of the correlations of the orbital parameters in the order (of those present):- e_0, e_1, e_2, ..., i_0, i_1, ..., Ω_0, ..., ω_0, ..., M_0.... .

At this final stage there is also some punching of cards. Final elements are punched on five cards; these elements differ from the printed set in being exclusive (S4.3) elements. They constitute the definitive set of elements for the epoch. The covariance matrix of the orbital parameters is punched on ℓ (or 2ℓ or 3ℓ (S[illegible].5)) cards. Note that the punched covariance matrix, unlike the printed correlations, refers to non-standard parameters if these have been used during the differential correction (S6.5). Also note that the units are those used inside the computer (S5); e.g. the printed standard deviation for M_1 is 0.00057°/day, but the punched-card variance is 1.33×10^{-20} (radians/sec)^2.

9.4 Output in the event of failure

Program failures may be considered to be of three types. (Rejection of an observation card due to an anticipated error (S9.1) is not counted as a failure.)

First there are the standard failures referred to in section 6.1. A failure of this type can only occur after input of all the data for a given run. In PROP3 there are six possible standard failures (the first occurs in the main program and the others in subroutine DIFCOR) and the corresponding output is as follows:-

```
DISCONTINUE - TOO FEW OBSERVATIONS INPUT AND ACCEPTED
DISCONTINUE - TOO FEW OBSERVATIONS LEFT
DISCONTINUE - ELEMENT TYPE INCONSISTENT WITH USE OF PREVIOUSLY-DETERMINED ORBIT
DISCONTINUE - DEGREES OF FREEDOM NEGATIVE
DISCONTINUE - LIMIT ON ALLOWED NUMBER OF ITERATIONS REACHED
DISCONTINUE - DIVERGENCE ON TWO SUCCESSIVE ITERATIONS.
```

Second there are failures which, though they lead to deletion of the program (via a Fortran STOP instruction), have been anticipated and provide explanatory printing before the deletion. These all occur (or may occur) before input of the complete data for the run, and this is why it has not been arranged for the next run to be started.

Finally there are unanticipated failures which will leave no direct explanation of their cause. These will arise from breaking a fundamental rule of the computer, e.g. through overflow or a data card punched inconsistently with the associated format statement, and lead to deletion of the program via the computer's supervisor or executive.

10 PRESENTATION OF RESULTS

It will be normal for the PROP3 user to present his accumulated results in tabular form, particularly when a number of orbit determinations for a given satellite have been carried out at regular intervals. It may also be appropriate to illustrate the variation of certain orbital elements, for example the eccentricity, in graphical form, but this section considers only the preparation of tables.

All PROP3 orbit determinations relate to midnight epochs, and an obvious first quantity to tabulate is the date of epoch. It is a good practice to give this both as a normal date and as an MJD number. The remaining quantities to be tabulated will normally include the orbital parameters, with their standard deviations, some useful derived parameters (functions of the official parameters) such as semi-major axis and perigee height, also with standard deviations, and other quantities, such as the number of observations used.

It is suggested that values of the orbital parameters be rounded such that standard deviations, when similarly rounded, are given to one significant figure, or at most two. Thus it is absurd to refer to an eccentricity value of 0.018423 ±0.001264; with an obvious space-saving convention one would round to 0.0184 13. However this principle raises problems when certain of the parameters are highly correlated. There are two possible reasons for high correlations: one or more of the parameters are inherently ill-defined due to proximity to one of the singularities in e and i (S6.5); or else the coverage of the orbit by the available observations is poor.

To exemplify the problem just referred to, consider a satellite for which $e_o = 0.001$. A given orbit determination may yield standard deviations including $\sigma(\Omega_o) = 0°.003$ and $\sigma(\omega_o) = \sigma(M_o) = 2°$; i.e. Ω_o is worth quoting to three decimal places, but ω_o and M_o only to the nearest degree. However, ω_o and M_o will have a correlation very close to -1.0, and the composite parameter $M_o + \omega_o$ may be known as accurately as Ω_o. To conform to the general principle of the last paragraph, while retaining real accuracy, the PROP user should quote ω_o to the nearest degree and $M_o + \omega_o$ to three decimal places, not quoting M_o itself at all (- there is no problem here with M_1 etc.). The standard deviation of the composite parameter would have to be quoted instead of $\sigma(M_o)$, of course, and this is given by

$$\sigma^2(M_o + \omega_o) \;=\; \sigma^2(\omega_o) + 2\,\mathrm{cov}(\omega_o, M_o) + \sigma^2(M_o) \quad .$$

An attempt to calculate $\sigma(M_o + \omega_o)$ using the value of $\mathrm{cov}(\omega_o, M_o)$ on an output covariance-matrix card will fail, since these cards do not give enough significant figures to cover the high-correlation situations, but this should be unnecessary anyway: the user should have chosen a JELTYP value (S8.2) such that $\sigma^2(M_o + \omega_o)$ is given directly (S6.5).

It is suggested that $M_o + \omega_o$ be quoted, instead of M_o, whenever a satellite's eccentricity is less than about 0.05. This is only a guide, of course; one should not change parameters from line to line of a table, just because e is oscillating about 0.05.

When i_o is near 0° or 180° (but assuming, first, that e_o is not small) there is a similar problem for ω_o (M_o being now all right) and $\omega_o + \Omega_o$ or $\omega_o - \Omega_o$ should be quoted instead of ω_o. It will be recalled (S6.5) that, with the appropriate choice of JELTYP, the covariance cards will give $\sigma^2(\omega_o + k\,\Omega_o)$, but this will certainly give $\sigma(\omega_o + \Omega_o)$ or $\sigma(\omega_o - \Omega_o)$ to sufficient accuracy, since k will be near to +1 or -1. When, in addition, e_o is small it will be necessary to quote parameters Ω_o, $\omega_o + \Omega_o$ and $M_o + \omega_o + \Omega_o$, or else Ω_o, $\omega_o - \Omega_o$ and $M_o + \omega_o - \Omega_o$; no new principle is involved.

Returning to general advice on the quantities to tabulate, e_1, i_1 etc. will of course be tabulated if they are parameters. If they are merely elements, for which the exclusive values (S4.3) are zero, it may be preferred to omit them. If e_1, i_1 etc. are quoted it will certainly be necessary to remind readers of the difference between inclusive and exclusive values.

Other quantities which it may be desired to tabulate are:- number of observations used, counting any rejected during analysis; number of observations rejected; number of days spanned by the observations (e.g. 3.8); and the final value of ε (e.g. 1.8).

The best way to prepare an error-free table is to use the output cards from each run. A Fortran program TOP (- Tabulation of Orbital Parameters) has been written which reads in as many sets of these as desired and prints the table, one row for each set of output cards. As the output cards cover only elements and covariances, an extra card is required with each set, giving the number of observations, etc. TOP has been written with formats suitable for the listing of orbital parameters for Ariel 3, as determined from Minitrack observations, but it can easily be modified for use with other satellites.

11 CONCLUSIONS

We have described in detail the operation of PROP, a Fortran program for refining the orbital parameters of an earth-satellite. Although the program is not capable of refining station coordinates or geophysical constants, its flexibility in other ways makes it an excellent tool in the analysis of the orbits of many close earth-satellites. However, certain limitations in the dynamical model should be kept in mind, to avoid using PROP for tasks for which it is unsuited.

(i) The program does not contain formulae for representing luni-solar perturbations. This fact imposes an upper limit on the value of a satellite's semi-major axis, beyond which the program should not be used. (As a very rough rule-of-thumb this upper limit might be taken to be two earth-radii.) Some allowance can be made for luni-solar effects by calculating the main secular terms in e, i, Ω and ω before using PROP; these terms can then be fed in as fixed elements in the model. It is planned to include this computation internally in the next version of PROP.

(ii) The representation of atmospheric-drag effects is not adequate for a satellite strongly affected by drag. This imposes a lower limit on the value of a satellite's perigee distance, below which PROP should not be used. This lower limit depends on so many factors - including the satellite's drag coefficient and the level of solar activity - that no clear criterion can be suggested for assessing it; the best guide is probably the value of M_2.

In the next version of PROP it is planned to deal with the high-drag situation by a standard procedure: to alter the weighting of the observations used in a given orbit determination so that the along-track component of each observation (the component most affected by drag) receives less weight than the cross-track components.

(iii) The effects of all tesseral harmonics have been neglected, with the exception of the dominant $J_{2,2}$ effect. This leads to errors of the order of $\frac{1}{4}$ km in the position of a satellite as computed from a given set of orbital elements. It is hoped to improve this situation in the next version of PROP.

The authors have attempted to be comprehensive in preparing this manual, but it is unlikely that there are no errors of omission or commission in the text. In anticipation of the probability that it will be revised and reissued when PROP4 is available we invite all readers of the document, and in particular those who actually use the program, to bring any queries or criticisms to our attention.

ACKNOWLEDGEMENTS

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Appendix A

PROP3 PROGRAM UNITS

BLOCK DATA	Sets certain constants.
Main prog.	Sets other constants and supervises flow - MASTER PROP in 1907 Fortran.
(a)ACOS	Arc-cosine.
ALTELS	Alters orbital elements ("exclusive" to and from "inclusive").
AZEL	Az/el/range from topocentric satellite position.
CONVER	Tests refinement process for convergence.
CONVRT	Converts between az/el, R.A./dec and direction cosines.
COTOEL	Coordinates to osculating elements.
DHMS	Given days, or degrees, introduces hours, minutes and seconds.
DIFCOR	Differential correction - the refinement process.
EAFKEP	Eccentric anomaly from Kepler's equation.
(b)ECENTR	Geocentric-to-topocentric conversion of satellite position etc.
ELPRIN	Prints (and/or punches) orbital elements.
ELREAD	Reads orbital elements.
EPARAS	Abstracts list of parameters from the array of elements.
GEOCOR	Geographical coordinates from geocentric Cartesians.
(c)MA01A	Matrix division or inversion (single-length).
MODJD	Expresses date as MJD number.
NEWELS	Sets new values of parameters after each refinement.
(a)NINT	Nearest integer.
NUPOLY	New polynomial coefficients after shift of origin.
OBSIN	Input of observations in various formats.
PARSHL	Partial derivatives of (geocentric) satellite coordinates.
PRDATE	Prints date, given MJD number.
PRELON	Sets time-independent quantities for perturbations.
PRENUT	Correction of R.A./dec observations for precession and nutation.
PROCES	Processing of observations prior to refinement processes.
PVREAD	Reads position and velocity.
REFCOR	Refraction correction of observed elevation.
REJECT	Decision as to observations to be rejected.
SATELS	Position and velocity from mean orbital elements.
SATXYZ	Perturbed geocentric position and velocity.
SDPRIN	Prints standard deviations of elements.
SENSIN	Input of sensor locations.
SHOPER	Computes short-periodic perturbations.

SIDANG	Sidereal angle relative to 1950.0.
SINXOX	Sin x / x, $(1 - \cos x) / x^2$ and $(x - \sin x) / x^3$.
TRINV	Polar coordinates from Cartesians (two-dimensional).
XLONG	Computes long-periodic perturbations.

Notes

(a) The ACOS and NINT functions, though not included in A.S.A. Fortran, are provided by both the Atlas and the 1907 computers and need not be provided separately for these computers.

(b) ECENTR is called as the dummy-argument subroutine CENTRE, this being organized by an EXTERNAL statement in the main program. For orbiting bodies other than earth satellites a different CENTRE may be used.

(c) MA01A, similarly, is called as MA01. The 'A' version gives single-length matrix inversion. If double-length inversion is required, MA01B should be used instead, and references to MA01A in the main program replaced by references to MA01B.

Appendix B

OBSERVATIONS-FORMATS CURRENTLY PROVIDED FOR BY THE SUBROUTINE OBSIN

Each observations-format is associated with an appropriate identifier. Before any group of observations is read, the appropriate identifier is read into the Fortran variable LOOXEE, and it is convenient to list the formats currently available under the corresponding LOOXEE numbers.

B.1 LOOXEE 111111

This format reads Baker-Nunn observations in the standard SAO format[8], implemented* in OBSIN by:

```
FORMAT (I7, 5X, I5, I6, 2I2, F6.4, 1X, 2I2, F5.3, A1, 2I2, F4.2, I1, I2, 1X,
        I1, 7X, F7.4)   .
```

The 17 quantities input are read into the following Fortran variables:

JDENT (cols.1-7)	This is the satellite identification according to the international designation; e.g. satellite 1967-48B is punched as 6704802.
NOSTAT (cols.13-17)	This is the number of the observing-station; e.g. Maui, Hawaii, is b9012.
MDAY (cols.18-23)	This is the date in 6-digit form; e.g. 1968 May 22 is 680522.
MHR (cols.24-25 MIN (cols.26-27) SEC (cols.28-33)	These give the time of the observation; e.g. 13h 4m 29.631s is punched as 130429631b. (Note that column 33 must be blank unless the time is given to 0.1 msec.)
MRAH (cols.35-36) MRAM (cols.37-38) RAS (cols.39-43)	These give the satellite's right ascension, e.g. 5h 44m 19s is punched as b54419bbb. A decimal point is implicit between columns 40 and 41.
MSIGN (col.44) MDECD (cols.45-46) MDECM (cols.47-48) DECS (cols.49-52)	These give the satellite's declination, a decimal point being implicit between columns 50 and 51; e.g. -5° 29' 32".41 is punched as -05293241 and +5° 29' 32".41 is punched as bb5293241. The minus sign (if any) must appear in column 44.

---

*Only S.A.O. R.A./dec observations are covered at present. It is hoped to cover other S.A.O. formats (ℓ/m, A/E etc.) in an extended LOOXEE 111111 in a later version of PROP.

NTSD (col.53)	This is a code digit indicating the timing accuracy of the observation[8]; zero (or blank) signifies that there is no estimate of timing accuracy, in which case the subroutine sets an arbitrary accuracy of $0^{s}.003$; 1 signifies an accuracy of $0^{s}.0001$; 2 signifies an accuracy of $0^{s}.001$; 3 signifies an accuracy of $0^{s}.003$; and 4 signifies an accuracy of $0^{s}.01$ *.
NASD (cols.54-55)	This is the angular accuracy of the observation in seconds of arc.
INEQEQ (col.57)	This is a code digit indicating the date of the equator and equinox to which the observations are referred; the digits 0, 1, 2, 3 and 4 refer respectively to the equator and equinox of date, 1855.0, 1875.0, 1900.0 and 1950.0.
TIMCR (cols.65-71)	This is the time correction to be subtracted from the time in the A1 system to give time in UT1; e.g. a difference of $42^{s}.217$ is punched as b42217b. The decimal point is implicit between columns 67 and 68.

The card may contain other quantities , e.g. a reference number in cols.8-12, which are not used by PROP.

B.2 LOOXEE 222222

This format reads direction-cosine (Minitrack) or range or range-rate observations in a particular form. At R.A.E. it has been used only for the satellite OGO-2 and may be abandoned in future versions of PROP. The format is implemented in OBSIN by:

```
FORMAT (I2, 8X, I5, F9.9, 10X, I1, 2(4X, I1, E15.0))   .
```

The 10 quantities input are read into the following Fortran variables:

NOSTAT (cols.1-2)	This is the number of the observing-station. Note that this is a 2-digit code, covering a possible maximum of 99 stations, and is inconsistent with the international 4-digit code for station numbers. (When used for OGO-2 the observations were from the following stations, numbered 01 to 15: College, Blossom Point, Fort Myers, Grand Forks, Johannesburg, Lima, Mojave, Newfoundland, Woomera, Quito, Santiago, Winkfield, Rosman (U.S.A.), Tananarive (Malagasy Republic) and Carnarvon (Australia).)

---

*Ref.8 also covers values of 4 to 9, but these are not covered in PROP3.

NTIMHR (cols.11-15)	The number of complete hours elapsed since MJD 36933.0 (1960 January 0.0) to the time of the observation; e.g. 1200 hours on 1965 November 9 (viz. MJD 39073 + 12 hours) is 2140 days and 12 hours from MJD 36933.0, so it is punched as 51372.
TIM (cols.16-24)	The fraction of an hour elapsed from NTIMHR to the tim of the observation; e.g. 27m 43.27^{s} is punched as 462019444. The decimal point is implicit before column 16.
NOBS (col.35)	The number of observed quantities given on the card: 1 for a range observation or a range-rate observation, 2 for a direction-cosines observation.
NTYP1 (col.40)	A code digit indicating the type of observed quantity which is read into OBS1 : 3, 4 and 7 refer respectively to range, a range-rate and an east-west direction cosine.
OBS1 (cols.41-55)	The first observed quantity; range must be in units of half-earth-radii, where the earth's radius is taken to be 6378.165 km; range-rate must be in units of earth-radii/hour; direction-cosines are, of course, dimensionless. A decimal point and/or exponent should be present.
NTYP2 (col.60)	A code digit indicating the type of quantity read into OBS2; only one value is recognised: 8 for a north-south direction cosine.
OBS2 (cols.61-75)	The north-south direction cosine.

Note: If NOBS = 1, indicating a range or range-rate observation, the fields of NTYP2 and OBS2 will be blank.

B.3 LOOXEE 333333

This format reads range or range-rate observations in the standard form supplied by NASA. It is implemented in OBSIN by:

```
FORMAT (7X, 2A3, 1X, I6, 1X, 2I2, 1X, F6.4, 16X, F9.3, F7.5)   .
```

The 7 quantities input are read into the following Fortran variables:

NAMEST(2) (cols.8-13)	The station name, in a 6-digit alphanumeric code. NAMEST must identify with one of the station names held (as text) in the array MONICA(36). Each of these names is held in two words of storage; thus the first name is held

in MONICA(1) and MONICA(2), and the n^{th} name is held in MONICA(2n-1) and MONICA(2n). If NAMEST is the n^{th} name, station coordinates will be taken from a sensor card for which the station number is n.

The 18 code-names currently held in MONICA are, in order, COLEG6, BPOIN6, FTMYR6, GFORK6, JOBUR6, LIMAP6, MOJAV6, NEWFL6, OOMER6, QUITO6, SNTAG6, WNKFL6, ROSRAN, MADGAR, CARVON, ULASK6, MADGA6 and ORORA6. Thus, for example, the station name QUITO6 refers to a sensor-card numbered 10.

MDAY (cols.15-20)	The date in 6-digit form; e.g. 1968 May 22 is punched as 680522.
MHR (cols.22-23) MIN (cols.24-25)	The hours and minutes components of the UTC time of the observation; e.g. 07h 28m is punched as b728.
SEC (cols.27-32)	The seconds component of the time of the observation; e.g. $17^s.819$ is punched as 17819b. The decimal point is implicit between columns 28 and 29.
RANGE (cols.49-57)	The satellite's range from the station, in kilometres, the decimal point implicit between columns 54 and 55; e.g. 1936.363 km is punched as bb1936363.
RGEDOT (cols.58-64)	The rate of change of range, in km/sec, the decimal point implicit between columns 59 and 60; e.g. 4.14812 km/sec is punched as b414812.

N.B. Only one of RANGE and RGEDOT may be present on the same card. The other field must be left blank. (PROP reads both fields, but interprets the observation as of "range" or "range rate" according to whether RANGE is not, or is, zero.)

B.4 LOOXEE 444444

This format reads standard Minitrack direction-cosine cards, and is implemented in OBSIN by:

```
FORMAT (7X, 2A3, 1X, I6, 1X, 2I2, 1X, F5.3, 28X, F8.6, 4X, I1)   .
```

Pairs of cards are looked for, and single cards are rejected. The quantities on the first card of a pair are read into the following Fortran variables:

NAMEST(2) (cols.8-13)	The station name, in a 6-digit alphanumeric code, as described under LOOXEE 333333.
MDAY (cols.15-20)	The date, in 6-digit form; e.g. 1968 May 7 is punched as 680507.
MHR (cols.22-23) MIN (cols.24-25)	The hours and minutes components of the UTC time of the observation; e.g. 16h 55m is punched as 1655.
SEC (cols.27-31)	The seconds component of the time of the observation; e.g. $54^s.43$ is punched as 5443b. The decimal point is implicit between columns 28 and 29.
DCL (cols.60-67)	The east-west direction-cosine, with the decimal point implicit between columns 61 and 62.
NP (col.72)	This is for checking that DCL is, in fact, the east-west direction-cosine. If NP ≠ 2 the check fails and the card is rejected.

The first four quantities on the second card of the pair are read into the Fortran variables: NAMEST, MDAY, MHR, MIN exactly as before; in fact these quantities overwrite those which were set from the first card. The remaining quantities are read into the following Fortran variables:-

SEC2 (cols.27-31)	This must be within 5 msec of SEC, otherwise the pair of data-cards is rejected.
DCM (cols.60-67)	The north-south direction-cosine, the decimal point implicit between columns 61 and 62.
NP (col.72)	This is for checking that DCM is, in fact, the north-south direction-cosine. If NP ≠ 3, the check fails; but a further check is made: if NP = 2 the subroutine assumes that the second card of a pair has been lost. It therefore sets DCL = DCM and SEC = SEC2 and assumes that this card is the first of a pair. If NP is neither 3 nor 2 the card is rejected (and so, of course, is the previous one of the pair).

Note: NASA Minitrack cards contain a 5-digit satellite identification (cols.2-6), but this is not used by PROP.

B.5 LOOXEE 555555

This format reads angular observations on standard Moonwatch cards as supplied by the S.A.O. The description will be abbreviated here, since the PROP user will not normally be punching cards in this format. If he wishes to punch optical or radar observations, the format described in section B.6 below will be found to be more flexible and comprehensive, and its use is recommended.

The format is implemented in OBSIN by:

```
FORMAT (I7, 5X, I5, I6, 2I2, F6.4, I3, I2, F2.0, 3X, A1, 2I2, F2.0, 6X, I1) ,
```

where the quantities are read into the following Fortran variables:

JDENT (cols.1-7)	Satellite's international designation.
NOSTAT (cols.13-17)	Observing station's number.
MDAY (cols.18-23)	Date of the observation, in 6-digit form.
MHR MIN SEC (cols.24-33)	Time of the observation, the decimal point implicit between columns 29 and 30; e.g. 19h 49m $33^s.6$ is punched as 1949336bbb.
M1 M2 AM3 (cols.34-40)	Hours, minutes and seconds components of right ascension, or degrees, minutes and seconds components of azimuth. E.g. an R.A. of 21h 50m 12s is punched as b215012, an azimuth of 113° 46' 18" as 1134618.
MSIGN1 N1 N2 AN3 (cols.44-50)	Sign, degrees, minutes and seconds components of declination or of elevation. E.g. a declination of -74° 17' is punched as -7417bb, an elevation of 44° 6' 14" as b440614.
INEQEQ (col.57)	Code digit indicating the type of observation and the date of the equator and equinox to which it is referred; zero (or blank) implies azimuth and elevation; 1, 2, 3 and 4 imply right ascension and declination, and refer respectively to the equator and equinox of 1855.0, 1875.0, 1900.0 and 1950.0.

B.6 LOOXEE 666666

This format reads range and/or angular observations on cards punched according to the format agreed between R.A.E. (Farnborough), RSRS (Slough), the Royal Greenwich Observatory (Herstmonceux), the Royal Observatory, Edinburgh, and the Sub-Committee on Optical Tracking of the British National Committee for Space Research. It is therefore the standard British format for reporting satellite observations. It is implemented in OBSIN by:

FORMAT (I7, I4, 5I2, F6.4, F5.4, 1X, I1, 2F8.0, F4.0, I1, F8.3, F5.3) ,

where the quantities are read into the following Fortran variables:

JDENT (cols.1-7)	The satellite designation in the 7-digit international code; e.g. satellite 1967-48B is punched as 6704802.
NOSTAT (cols.8-11)	The normal 4-digit number assigned to a station by a coordinating centre on behalf of COSPAR; e.g. Horsebridge is 3657.
MY MM (cols.12-17) MD	The date of the observation, in 6-digit form; e.g. 1968 May 7 is 680507.
MHR MIN (cols.18-27) SEC	The time of the observation (UT1); e.g. 16h 4m 6.234s is punched as 160406234b. Note that the decimal point is implicit between columns 23 and 24.
TSD (cols.28-32)	The timing accuracy in seconds of time; the decimal point is implicit between columns 28 and 29. E.g. $0^{s}.5$ is punched as b5bbb.
NCODE (col.34)	A code digit which indicates the format and units of the angular part of the observation (see below).
RAAZ (cols.35-42) DECEL (cols.43-50)	The first and second position coordinates, i.e. the angular part of the observation, according to the following table:

NCODE value	First position coordinate	Second position coordinate
0	Right ascension measured in hours, minutes and seconds, a decimal point implicit between columns 40 and 41, e.g. 4h 19m 38.4s is punched as b4193840b	Declination measured in degrees, minutes and seconds of arc, a decimal point implicit between columns 49 and 50, e.g. -8° 12' 7".7 is punched as -0812077
	The observation has been corrected for both components (astronomical and parallactic) of refraction.	
1	Format identical to that for NCODE = 0, but the observation has been corrected for astronomical refraction only.	
2	Right ascension measured in hours and minutes, a decimal point implicit between columns 38 and 39, e.g. 4h 19.64m is punched as b41964bb	Declination measured in degrees and minutes of arc, a decimal point implicit between columns 47 and 48, e.g. -8° 12'.128 is punched as -0812128
	The observation is corrected for astronomical refraction only.	

NCODE value	First position coordinate	Second position coordinate
3	Right ascension measured in hours and minutes, as with NCODE = 2, e.g. 4h 19.64m is punched as b41964bb	Declination measured in degrees, the decimal point implicit between columns 45 and 46, e.g. -8°.20233 is punched as -0820233
	The observation has been corrected for astronomical refraction only.	
4	Azimuth measured in degrees, minutes and seconds, a decimal point implicit between columns 41 and 42, e.g. 194° 3' 17".4 is punched as 19403174	Elevation measured in degrees, minutes and seconds, a decimal point implicit between columns 49 and 50, e.g. 46° 19' 46".6 is punched as b4619466
	The observation has been corrected for both components of refraction.	
5	Azimuth measured in degrees and minutes, a decimal point implicit between columns 39 and 40, e.g. 194° 3'.29 is punched as 1940329b	Elevation measured in degrees and minutes, a decimal point implicit between columns 47 and 48, e.g. 46° 19.777 is punched as b4619777
	The observation has been corrected for both components of refraction.	
6	Azimuth measured in degrees, a decimal point implicit between columns 37 and 38, e.g. 194°.05483 is punched as 19405483	Elevation measured in degrees, a decimal point implicit between columns 45 and 46, e.g. 46°.32963 is punched as b4632963
	The observation has been corrected for both components of refraction.	
7, 8, 9	Format and units the same as for 4, 5, 6, respectively, but the observations are completely uncorrected for refraction.	

ASD (cols.51-54) The angular accuracy of the observation; units and format depend upon the value of NCODE: for NCODE = 0, 1, 4 or 7 ASD is in seconds of arc with a decimal point implicit between columns 53 and 54; if NCODE = 2, 5 or 8 ASD is in minutes of arc with a decimal point implicit between columns 52 and 53; if NCODE = 3, 6 or 9 ASD is in degrees with a decimal point implicit between columns 51 and 52.

INEQEQ (col.55)	A code digit indicating the date of the equator and equinox to which right ascension and declination observations are referred; 0 refers to the true equator and equinox of date, and 1, 2, 3, 4 and 5 refer respectively to the mean equator and equinox of 1855.0, 1875.0, 1900.0, 1950.0 and date. This digit is ignored with observations of azimuth and elevation.
RANGE (cols.56-63)	The range of the satellite, measured in kilometers, the decimal point implicit between columns 60 and 61; e.g. 587.32 km is punched as bb58732b.
RSD (cols.64-68)	The range accuracy, measured in kilometres, a decimal point implicit between columns 65 and 66; e.g. 1.5 km is punched as b15bb.

Notes: (i) Other columns on the card are used by RSRS (e.g. "brightest magnitude" in columns 69-71), but only those detailed above are read by PROP.

(ii) Either the angular or the range part of the observation may be omitted, the appropriate field being left blank. The corresponding "accuracy fields" should also be left blank, of course.

(iii) If no estimate of angular or range accuracy has been given for an observation, the appropriate field should be left blank when punching the card. A standard value will be set from SIGMA(4) for an angular observation, or from SIGMA(2) for a range observation (see section 5.3 for values of SIGMA).

(iv) The zero value of NCODE (corresponding to a right ascension/declination observation, fully corrected for refraction) was not included in the format as originally agreed between the various organisations concerned. It was added at a late stage in the development of PROP3 and thus represents an addition to the agreed standard format.

(v) Observations of right ascension and declination, because they are evaluated relative to a star background, have been automatically corrected for astronomical refraction; this is why there is no NCODE value for "uncorrected right ascension and declination".

B.7 LOOXEE 777777

Concurrently with the final revision of the draft of this Manual a seventh observations format is being added to PROP. This provides for input of range, azimuth and elevation for a named (instead of numbered) station. The format is (2A3, 2X, 5I2, F8.6, 4X, 3E15.0) and the quantities on a card, in order, are station name, date, time, range (ft), azimuth and elevation (degrees).

Footnote on LOOXEE 111111 and LOOXEE 555555

These two formats are almost identical and in Ref.8 there is in fact a single format for S.A.O. observations. As stated in the footnote on page 52 it is hoped to extend LOOXEE 111111 to be in complete conformity with the S.A.O. format; when this has been done LOOXEE 555555 will be dropped. An initial extension of LOOXEE 111111 is almost complete and should be incorporated in PROP3 by the end of March, 1969. The format will be modified to

FORMAT (I7, 5X, I5, I6, 2I2, F6.4, I3, I2, F5.3, A1, 2I2, F4.2, I1, I2, 2I1, 7X, F7.4)

and the following remarks supply the necessary amending information to section B.1.

(1) The format covers observations of azimuth and elevation as well as right ascension and declination. Thus MRAH (which now occupies col.34 as well as cols.35-36) may be degrees of azimuth, instead of hours of right ascension, etc.

(2) An additional variable, NCODE, is read from col.56. This may be 0 (indicating an observation of right ascension and declination), 1 (for azimuth and elevation, corrected for refraction) or 3 (azimuth and elevation, not corrected).

(3) NTSD (col.53) and NASD (cols.54-55) have been extended to cover all the values permitted in Ref.8. Zero values are interpreted by $0^s.05$ and the value of SIGMA(4), respectively. NASD is equal to the estimated accuracy in seconds of arc for values up to 21.

This extension will enable standard Moonwatch cards to be read under LOOXEE 111111, with estimated accuracies used when available. LOOXEE 555555 will have to be used only for the non-standard cards in which azimuth and elevation cards are indicated by an A in cols.41 and 51, and a zero in col.57, instead of by a 1 or 3 in col.56.

Appendix C

INTERPRETATION OF THE SIX MAIN ORBITAL ELEMENTS

It may be important to the user of PROP to know precisely how to interpret the six main elements of the PROP model, viz. e_o, i_o, Ω_o, ω_o, M_o and n_o (where n_o denotes the exclusive element M_1). It has happened in the past that an organization has published accurate orbital elements of a number of satellites but that the usefulness of these elements has been limited by the failure of the organization to publish definitions of the elements.

Roughly speaking PROP elements are osculating elements with short-periodic perturbations removed. This statement may be made more precise by giving the formulae for the differences between PROP elements and osculating elements, and this is done in the next paragraph. It must be stressed, however, that these formulae are for reference only. They are not (apart from the formula in i) used directly in PROP, which applies short-periodic perturbations to a set of six quantities which are intermediate between elliptic elements and Cartesian coordinates - a set chosen to make all the formulae as simple as possible[5]. It should also be stressed that PROP elements have not been freed from long-periodic perturbations. In fact long-periodic perturbations in PROP (unlike those of the Smithsonian Astrophysical Observatory for instance) are defined so as to be zero at epoch. This means that the only harmonics involved in the following formulae are J_2 (for the short-periodic terms) and $J_{2,2}$ (for the along-track correction referred to in Ref.2 and in section 2.1 of the present paper). Also it is convenient to regard Ω_{osc} as defined relative to the true equinox of date and hence to include a term for transformation of equinox in the formula connecting Ω_o and Ω_{osc}.

Let

$$k_1 = (J_2/32)(R/p)^2 \quad \text{and} \quad k_2 = J_{2,2}\,(R/p)^2\, n_o \,/\, (\dot{\theta} - \dot{\Omega}) \quad ,$$

where $p = a_o(1 - e_o^2)$ and θ is the modified sidereal angle. To give first-order expressions it is legitimate and convenient to drop the zero suffix when writing functions of e_o, i_o etc; also for convenience we introduce $f = \sin^2 i$. Then, if $\lambda_{2,2}$ is the longitude of one extremity of the major axis of the earth's equator and v is the true anomaly corresponding to the epoch value M_o of the mean anomaly, the required first-order expressions are:-

$$\begin{aligned}
e_{osc} - e_o = k_1 \big[& 6(4+e^2)(2-3f) \cos v + 3(4+11e^2)f \cos(2\omega+v) \\
& + (28+17e^2)f \cos(2\omega+3v) + 12e(2-3f) \cos 2v \\
& + 6ef \{10 \cos(2\omega+2v) + 3 \cos(2\omega+4v)\} \\
& + 3e^2 f \{\cos(2\omega-v) + \cos(2\omega+5v)\} + 2e^2(2-3f) \cos 3v \\
& + 18ef \cos 2\omega + 4e(2-3f)\{5 + 2(1-e^2)/(1 + (1-e^2)^{\frac{1}{2}})\}\big] \quad ,
\end{aligned}$$

$$i_{osc} - i_o = 4k_1 \sin 2i \, [3 \cos(2\omega+2v) + 3e \cos(2\omega+v) + e \cos(2\omega+3v)]$$

$$\begin{aligned}
\Omega_{osc} - \Omega_o = 8k_1 \cos i \, [& 6(M_o - v - e \sin v) + 3 \sin(2\omega+2v) + 3e \sin(2\omega+v) \\
& + e \sin(2\omega+3v)] \\
& + 6.119 \times 10^{-7} (\text{MJD} - 33281.9234) + \text{Equation-of-Equinoxes}
\end{aligned}$$

$$\begin{aligned}
\omega_{osc} - \omega_o = k_1 \big[& 24(4-5f)(v-M_o) + 6e^{-1}\{4(2-3f) + e^2(14-17f)\} \sin v \\
& - 3e^{-1}\{4f + e^2(8-15f)\} \sin(2\omega+v) \\
& + e^{-1}\{(28f - e^2(8-19f)\} \sin(2\omega+3v) \\
& + 12(2-3f) \sin 2v - 12(2-5f) \sin(2\omega+2v) + 18f \sin(2\omega+4v) \\
& - 18f \sin 2\omega - 3ef \sin(2\omega-v) + 2e(2-3f) \sin 3v \\
& + 3ef \sin(2\omega+5v)\big] \quad ,
\end{aligned}$$

$$\begin{aligned} M_{osc} - M_o = \; & k_1 (1-e^2)^{\frac{1}{2}} [f \{ 18 \sin 2\omega + 3e^{-1} (4+5e^2) \sin (2\omega+v) \\ & - e^{-1} (28-e^2) \sin (2\omega+3v) - 18 \sin (2\omega+4v) \\ & + 3e \sin (2\omega-v) - 3e \sin (2\omega+5v) \} \\ & - (2-3f) \{ 6e^{-1} (4-e^2) \sin v + 12 \sin 2v + 2e \sin 3v \}] \\ & + \{ 9 k_2 n/(2\dot{\theta}-2\dot{\Omega}) \} f \sin 2(\theta-\Omega+\lambda_{2,2}) \end{aligned}$$

and

$$\begin{aligned} n_{osc} - n_o = \; & -3 k_1 n(1-e^2)^{-1} [f \{ 12 (2+3e^2) \cos (2\omega+2v) + 9e (4+e^2) \cos (2\omega+v) \\ & + 9e (4+e^2) \cos (2\omega+3v) + 18e^2 \cos (2\omega+4v) \\ & + 3e^3 \cos (2\omega-v) \\ & + 3e^3 \cos (2\omega+5v) + 18e^2 \cos 2\omega \} \\ & + (2-3f) \{ 6e (4+e^2) \cos v + 12e^2 \cos 2v \\ & + 2e^3 \cos 3v + 4(2+3e^2) \}] \quad . \end{aligned}$$

It must be emphasised again that these formulae are given only as a means of defining, to first-order, the PROP elements; the omitted second-order term in $n_{osc} - n_o$ is unfortunately not insignificant. The e^{-1} terms in $\omega_{osc} - \omega$ and $M_{osc} - M$ arise because perigee is not well-defined for near-circular orbits. This singularity is unimportant in practice since PROP never computes osculating elements, but from mean elements goes directly to Cartesian coordinates by a procedure which is free of singularities. The best method of obtaining osculating elements from mean elements, or vice versa, is in fact always to compute coordinates as an intermediate step[6].

Some users of PROP may wish to know how PROP elements relate to the Pegasus elements previously used[1,2]. To indicate the relationship we now give a complete set of formulae by means of which a set of Pegasus elements may be obtained from a set of PROP elements. These formulae include conversions for changes of units.

Let e_o, i_o, Ω_o, ω_o, M_o, M_1 ($= n_o$), M_2 etc. be a set of PROP elements, and let a', e', i', Ω', ω', M_o', n_1' etc. be the equivalent set of Pegasus elements at the same epoch. If the epoch is not an ascending node (as it always was for Pegasus computer orbit determinations) the dashed elements must be interpreted not as true smoothed elements, as originally defined by Merson,[16] but as mean smoothed elements (related to true smoothed elements in the same way that PROP elements are to osculating elements).

Taking angles in degrees we require a constant k ($= 57.2957795$) for conversion from radians. We also require conversions in time since M_2 is in °/day/day but n_1' is in °/(100 days)^2. Let k_1 be the same constant as before. As in the previous formulae zero suffices are omitted when possible.

Then a complete set of first-order formulae is:-

$$a' = \{\mu\ (86400\ k/M_1)^2\}^{\frac{1}{3}}, \quad \text{where} \quad \mu = 398602 ,$$

$$k_1 = (J_2/32)(R\ /\ a'(1-e^2))^2 ,$$

$$f = \sin^2 i ,$$

$$e' = e_o + 8k_1\ (2-3f)(1-e^2)\ e^{-1}\ \{(1+e^2) - (1-e^2)^{\frac{1}{2}}\}^{*} ,$$

$$i' = i_o - 4\,k\,k_1 \sin 2i\ (3 + 4\ e \cos \omega) ,$$

$$E_o = -2k \tan^{-1} \{(1-e)^{\frac{1}{2}}\ (1+e)^{-\frac{1}{2}} \tan \tfrac{1}{2}\omega\} , \quad \text{with} \quad |E_o + \omega| < 180 ,$$

$$M_{oo} = E_o - k\ e \sin E_o ,$$

$$\Omega' = \Omega_o + 16k_1 \cos i\ \{3(\omega + M_{oo}) + 4k\ e \sin \omega\}$$
$$+ 3.506 \times 10^{-5}\ (\mathrm{MJD} - 33281.9234) + E/240 ,$$

where E is the equation of equinoxes[13],

$$\omega' = \omega_o + 4k_1\ \{k\ e^2\ (2-3f) \sin 2\omega - 2(4-5f)(3\omega + 3\ M_{oo} + 2k\ e \sin \omega)\} ,$$

---

*If e is small it is better to replace $e^{-1}\{(1+e)^2 - (1-e^2)^{\frac{1}{2}}\}$ by the non-singular truncated expression $\frac{1}{2}e\ (3 + \frac{1}{4}e^2 + \frac{1}{8}e^4)$.

$$M_o' = M_o \quad ,$$

$$n_1' = 2 \times 10^{-4} M_2 \quad \text{etc} \quad .$$

INDEX

INDEX (Contd.)

INDEX (Contd.)

REFERENCES

No.	Author(s)	Title, etc.
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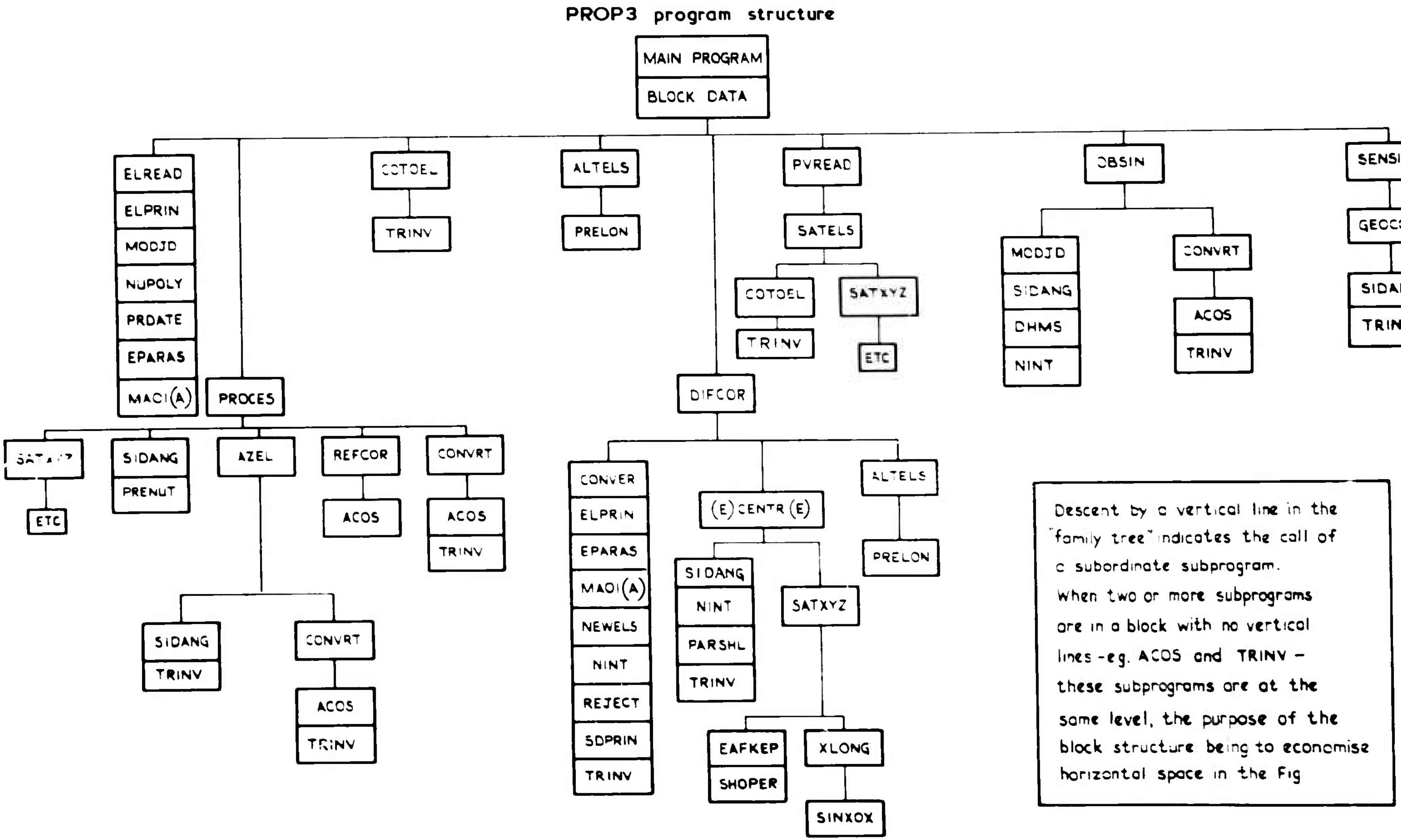
16	R.H. Merson	The perturbations of an earth satellite in an axi-symmetric gravitational field. R.A.E. Technical Note Space 26 (1963)



Descent by a vertical line in the "family tree" indicates the call of a subordinate subprogram.
When two or more subprograms are in a block with no vertical lines - eg. ACOS and TRINV - these subprograms are at the same level, the purpose of the block structure being to economise horizontal space in the Fig

Fig.1 Calling structure for PROP3 program units

Fig.2

004 901774

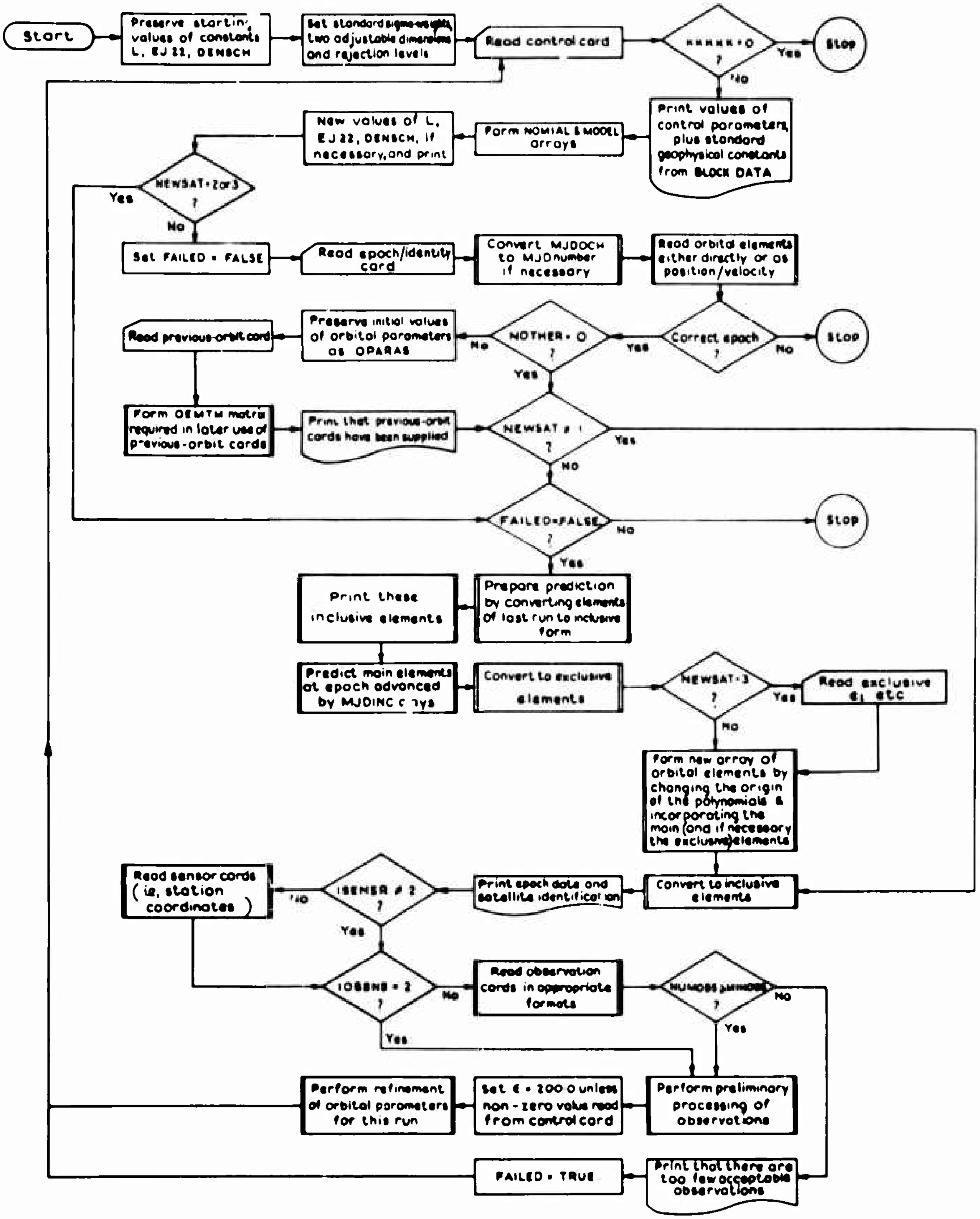


Fig. 2 Flow chart for Prop 3 main program

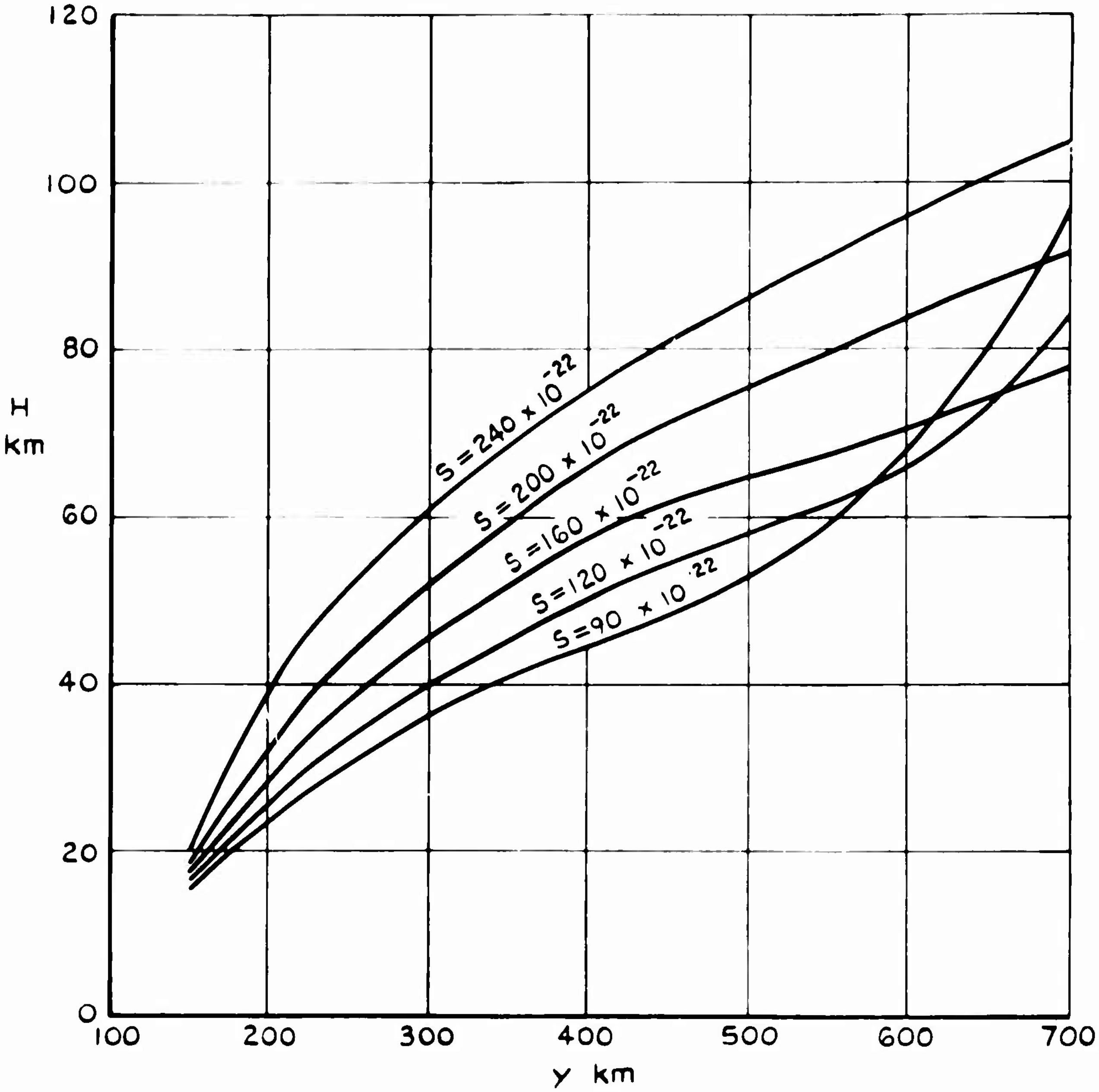


Fig. 3 Variation of density scale height, H, with altitude (S = 10·7 cm solar radiation in $Wm^{-2}\ Hz^{-1}$)

```
2222311113   0   1   3   9   9   10   0   1   1   1   9        5.0
 39670   6704201 -ARIEL 3-
39670/1     0.0075574  0.00000000
39670/2      80.18235    0.00000000
39670/3     100.81009    0.00000000
39670/4       8.78265    0.00000000
39670/5    -15.74406 5421.880745  2.6248E-02
2          1  COLLEGE, ALASKA                          -2299.236 -1445.730  5751.857
2          2  BLOSSOM POINT                             1118.063 -4876.358  3943.016
2          3  FORT MYERS                                 807.885 -5652.020  2833.549
2          4  GRAND FORKS                               -521.676 -4242.083  4718.764
2          5  JOHANNESBURG                              5084.798  2670.474 -2768.164
2          6  LIMA, PERU                                1388.818 -6088.429 -1293.207
2          7  MOJAVE, CALIF.                           -2357.214 -4646.369  3668.363
2          8  NEWFOUNDLAND                              2602.801 -3419.184  4697.694
2          9  WOOMERA                                  -3977.166  3725.708 -3303.141
2         10  QUITO, ECUADOR                            1263.617 -6255.010   -68.856
2         11  SANTIAGO, CHILE                           1769.707 -5044.642 -3468.192
2         12  WINKFIELD                                 3983.130   -48.404  4964.711
2         13  ROSMAN, CAROLINA                           647.215 -5178.372  3656.185
2         14  MALAGASY                                  4091.411  4434.227 -2065.889
2         15  CARNARVON, AUS.                          -2328.116  5299.752 -2669.482
2         16  ULASKA                                   -2282.332 -1452.667  5756.942
2         17  MADAGASCAR                                4091.903  4434.373 -2064.537
2         18  ORRORAL                                  -4447.361  2677.215 -3695.209
      2212  BEXHILL1            0.4672     50.8539     24.
      2521  LITTLE BADDOW       0.57264    51.74361     0.
      2526  POYNTON            -2.11390    53.34470   100.
      2344  THAMES DITTON     359.6550     51.3817     11.
      2525  CRAWLEYRIDGE       -0.72833    51.33750   115.
      2528  ALDERSHOT          -0.76390    51.23530    75.
      2574  WETTEREN            3.92222    51.00444    23.
      2541  MALTA              14.4419     35.8358    134.
      2265  FARNHAM           359.1847     51.2138    110.
      2512  MOUNTCASTLE       356.8653     55.9569     22.

 0.0185
444444
 67421 FTMYR6 670630 0524 24380                                  -0392682    P2
 67421 FTMYR6 670630 0524 24380                                   0037197    P3
 67421 QUITO6 670630 1729 02389                                  -0600756    P2
 67421 QUITO6 670630 1729 02389                                  -0012579    P3
 67421 ORORA6 670701 0118 08402                                   0664319    P2
 67421 ORORA6 670701 0118 08402                                   0013610    P3
 67421 FTMYR6 670701 0519 28380                                  -0521084    P2
 67421 FTMYR6 670701 0519 28380                                   0005014    P3
 67421 QUITO6 670701 0526 47389                                  -0581484    P2
 67421 QUITO6 670701 0526 47389                                   0048938    P3
 67421 LIMAP6 670701 0529 48393                                  -0599271    P2
```

Fig.4 Input cards for typical PROP run

NEG.NO.C5495

```
67421 LIMAP6 670701 0529 48393                                 0054512   P3
67421 WNKFL6 670701 1250 57295                                -0088600   P2
67421 WNKFL6 670701 1250 57295                                 0012667   P3
67421 NEWFL6 670701 1601 14285                                 0285177   P2
67421 NEWFL6 670701 1601 14285                                -0013121   P3
67421 WNKFL6 670701 2245 04295                                 0559435   P2
67421 WNKFL6 670701 2245 04295                                -0055560   P3
67421 FTMYR6 670702 0514 28380                                -0625880   P2
67421 FTMYR6 670702 0514 28380                                 0009679   P3
67421 QUIT06 670702 0521 51389                                -0681459   P2
67421 QUIT06 670702 0521 51389                                 0008141   P3
67421 LIMAP6 670702 0525 00393                                -0686299   P2
67421 LIMAP6 670702 0525 00393                                -0062409   P3
67421 NEWFL6 670702 1556 13285                                 0142309   P2
67421 NEWFL6 670702 1556 13285                                -0011269   P3
67421 WNKFL6 670702 2239 58295                                 0461626   P2
67421 WNKFL6 670702 2239 58295                                -0008141   P3
67421 ORORA6 670703 0108 03402                                 0418855   P2
67421 ORORA6 670703 0108 03402                                 0003179   P3
67421 QUIT06 670703 0516 58389                                -0753723   P2
67421 QUIT06 670703 0516 58389                                -0058006   P3
67421 LIMAP6 670703 0519 52393                                -0765027   P2
67421 LIMAP6 670703 0519 52393                                 0002236   P3
67421 JOBUR6 670703 0908 58413                                 0744795   P2
67421 JOBUR6 670703 0908 58413                                -0011112   P3
67421 NEWFL6 670703 1551 11285                                -0008718   P2
67421 NEWFL6 670703 1551 11285                                -0012946   P3
67421 WNKFL6 670703 2234 57295                                 0362822   P2
67421 WNKFL6 670703 2234 57295                                -0015829   P3

666666
670420122126706302248019   01     304151     6860    01   4
670420125216706302249191   02     321199     6230    005  4
670420125266706302250263   01     320353     2942    02   4
670420123446706302250165   01     3202735    3020    001  4
670420125256706302251010   3      320100     1100    2    4
670420125286707012244214   03     321500     6466    02   4
670420125286707012244293   03     321310     6192    02   4
670420125286707012244587   03     320300     4194    02   4
670420125746707012245456   01     318215     2175    01   4
670420122126707022237501   01     305137     7335    005  4
670420125286707022238520   03     323420     7716    02   4
670420125266707022240570   01     319241     0294    01   4
670420125416707022242220   0025   631897     2361    005
670420122656707032233386   02     302402     8125    005  4
670420122126707032233594   01     300480     8765    005  4
670420125266707032234320   02     3193305    4230    003  4
670420125126707032235035   02     3185935    0830    005  4
```

(THREE BLANK CARDS)

Fig.4

NEG.NO.C5496

```
ORBIT DETERMINATION BY PROP   (29/01/69 AT 08/32/21)
--------------------------

CONTROL PARAMETERS ARE     22223 (KKKKK),     11113 (MMMMM),    0 (MODE),    1 (NEWSAT),    3 (MJDINC),
         9 (MAXITN),    9 (MINOBS),   10 (JELTYP),    0 (NOTHER),    1 (ITIMEC),    1 (ISENSR),    1 (IOBSNS)


                  GEOPHYSICAL CONSTANTS ARE 398602.0 (EMU), 6378.163 (ERAD), 1.8 (EJ22*E6), -18.0 (ELAM22 - DEGS),
                   15 (L),+000000.00, 1082.68, -2.53, -1.61, -0.22,  0.71, -0.41,  0.13,  0.09,  0.09, -0.14, -0.31,
                    0.29,+000000.00, -0.40,+000000.00 (16 ZONAL HARMONICS IN E-6 UNITS), 25.0 (DENSCH)

USE ZONAL HARMONICS UP TO J 9 ONLY

RUN BASED ON PREDICTION FROM THE FOLLOWING ELEMENTS - FOR MJD EPOCH 39670.0 -

                    6927.7959  (DERIVED SEMI-MAJOR AXIS)
                     0.007557       -0.0000046
                    80.1824         -0.00001
                   100.8101         -1.27296
                     8.7826         -3.19086
                   -15.7441       5421.88074          0.026248



     EPOCH IS  1967 JUL  2.0 (39673.0), SATELLITE IDENTIFICATION IS 6704201  -ARIEL 3-

LISTING OF SENSOR LOCATIONS

STA.NO.      STATION NAME        LONG.(E)       LAT.(N)      HEIGHT(M)       X KM)      Y(KM)      Z(KM)           MARKER
      1    COLLEGE, ALASKA       212.1612       64.8718        189.05      -2299.236  -1445.730   5751.857           2
      2    BLOSSOM POINT         282.9137       38.4305          5.77       1118.063  -4876.358   3943.016           2
      3    FORT MYERS            278.1546       26.5483          8.70        807.885  -5652.020   2833.549           2
      4    GRAND FORKS           262.9692       48.0226        249.85       -521.676  -4242.083   4718.764           2
      5    JOHANNESBURG           27.7079      -25.8836       1564.20       5084.798   2670.474  -2768.164           2
      6    LIMA, PERU            282.8498      -11.7764         34.14       1388.818  -6088.429  -1293.207           2
      7    MOJAVE, CALIF.        243.1002       35.3302        921.51      -2357.214  -4646.369   3666.363           2
      8    NEWFOUNDLAND          307.2796       47.7414        111.87       2602.801  -3419.184   4697.694           2
      9    WOOMERA               136.8697      -31.3917        157.56      -3977.166   3725.708  -3303.141           2
     10    QUITO, ECUADOR        281.4210       -0.6224       3577.58       1263.617  -6255.010    -68.856           2
     11    SANTIAGO, CHILE       289.3313      -33.1490        680.08       1769.707  -5044.642  -3468.192           2
     12    WINKFIELD             359.3038       51.4460         87.39       3983.130    -48.404   4964.711           2
     13    ROSMAN, CAROLINA      277.1241       35.1960        876.24        647.215  -5178.372   3656.185           2
     14    MALAGASY               47.3026      -19.0198       1384.44       4091.411   4434.227  -2065.889           2
     15    CARNARVON, AUS.       113.7153      -24.9041         50.79      -2328.116   5299.752  -2669.482           2
     16    ULASKA                212.4761       64.9771        512.25      -2282.332  -1452.667   5756.942           2
     17    MADAGASCAR             47.3001      -19.0070       1360.86       4091.903   4434.373  -2064.537           2
     18    ORRORAL               148.9530      -35.6271        945.91      -4447.361   2677.215  -3695.209           2
   2212    BEXHILL1                0.4672       50.8539         24.00       4034.547     32.899   4923.344           0
   2521    LITTLE BADDOW           0.5726       51.7436          0.00       3937.220     39.552   4985.216           0
   2526    POYNTON                -2.1139       53.3447        100.00       3813.453   -140.759   5093.644           0
   2344    THAMES DITTON         359.6550       51.3817         11.00       3988.892    -24.019   4960.193           0
   2525    CRAWLEYRIDGE           -0.7283       51.3375        115.00       3992.548    -50.755   4957.203           0
   2528    ALDERSHOT              -0.7639       51.2353         75.00       4001.362    -53.352   4950.061           0
   2574    WETTEREN                3.9222       51.0044         23.00       4012.259    275.092   4933.899           0
   2541    MALTA                  14.4419       35.8358        134.00       5013.236   1291.089   3713.535           0
   2265    FARNHAM               359.1847       51.2158        110.00       4003.199    -56.968   4948.590           0
   2512    MOUNTCASTLE           356.8653       55.9569         22.00       3573.492   -195.704   5261.802           0
WWV TIME CORRECTION HAS BEEN SUPPLIED
```

NEG.NO C5497

Fig.5 Output for typical PROP run

Fig.5(cont'd)

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LISTING OF OBSERVATIONS

OBS.  STA. TYPE    DATE        TIME
 NO.   NO.

                                            E/WCOSINE  N/SCOSINE   AZ.    EL.

   1     3    3 1967 JUN 30    5 24 24.4  -0.392682  0.037197  275.4  66.8                              -27 REV + .27 (.41)
   2    10    3 1967 JUN 30   17 29  2.4  -0.600756 -0.012579  268.8  53.1                              -20 REV + .85 (.99)
   3    18    3 1967 JUL  1    1 18  8.4   0.664319  0.013610   88.8  68.4                              -15 REV + .76 (.89)
   4     3    3 1967 JUL  1    5 19 28.4  -0.521084  0.005014  270.6  58.6                              -12 REV + .28 (.42)
   5    10    3 1967 JUL  1    5 26 47.4  -0.581484  0.048938  274.8  54.3                              -12 REV + .36 (.49)
   6     6    3 1967 JUL  1    5 29 48.4  -0.599271  0.054512  275.2  53.0                              -12 REV + .39 (.52)
   7    12    3 1967 JUL  1   12 50 57.3  -0.088600  0.012667  278.1  84.9                               -7 REV + .00 (.14)
   8     8    3 1967 JUL  1   16  1 14.3   0.285177 -0.013121   92.6  73.4                               -6 REV + .99 (.13)
   9    12    3 1967 JUL  1   22 45  4.3   0.559435 -0.055560   95.7  55.8                               -1 REV + .22 (.36)
  10     3    3 1967 JUL  2    5 14 28.4  -0.625880  0.009679  270.9  51.2                                3 REV + .29 (.43)
  11    10    3 1967 JUL  2    5 21 51.4  -0.681459  0.008141  270.7  47.0                                3 REV + .37 (.50)
  12     6    3 1967 JUL  2    5 25  0.4  -0.686299 -0.062409  264.8  46.4                                3 REV + .40 (.56)
  13     8    3 1967 JUL  2   15 56 13.3   0.142309 -0.011269   94.5  81.8                               10 REV + .00 (.14)
  14    12    3 1967 JUL  2   22 39 58.3   0.461626 -0.008141   91.0  62.5                               14 REV + .22 (.36)
  15    18    3 1967 JUL  3    1  8  3.4   0.418855  0.003179   89.6  65.2                               15 REV + .77 (.91)
  16    10    3 1967 JUL  3    5 16 58.4  -0.753723 -0.058006  265.6  40.9                               18 REV + .38 (.51)
  17     6    3 1967 JUL  3    5 19 52.4  -0.765027  0.004236  270.2  40.1                               18 REV + .41 (.54)
  18     5    3 1967 JUL  3    9  8 58.4   0.744795 -0.011112   90.9  61.9                               20 REV + .80 (.94)
  19     8    3 1967 JUL  3   15 51 11.5  -0.008718 -0.012946  214.0  89.1                               25 REV + .01 (.15)
  20    12    3 1967 JUL  3   22 34 57.3   0.362822 -0.01[illegible]29   92.5  68.7                               29 REV + .23 (.37)

A PRIORI STANDARD DEVIATION FOR ABOVE OBSERVATIONS IS 0.00029

                                              R.A.        DEC.         AZ.    EL.      S(DE[illegible].)   RANGE  S(R)

  21  2212    2 1967 JUN 30   22 48  1.9    4 15  6.0  68 36  0     7.00 30.10    387.3                 -16 [illegible] + .19 (.33)
  22  2521    2 1967 JUN 30   22 49 19.1   21 19 54.0  62 18  0    47.89 57.52    337.4                 -16 REV + .20 (.34)
  23  2526    2 1967 JUN 30   22 50 26.3   20 35 18.0  29 25 12  102.93 46.61    734.0                 -16 REV + .2[illegible] (.33)
  24  2344    2 1967 JUN 30   22 50 16.5   20 27 21.0  30 12  0  103.51 49.83    147.2                 -16 REV + .21 (.35)
  25  2525    2 1967 JUN 30   22 51  1.0   20 10  0.0  11  0  0  124.04 37.01   8376.7                 -16 REV + .22 (.36)
  26  2528    2 1967 JUL  1   22 44 21.4   21 50  0.0  64 39 36    41.97 53.58    837.7                  -1 REV + .2[illegible] (.35)
  27  2528    2 1967 JUL  1   22 44 29.3   21 31  0.0  61 55 12    47.34 55.11    837.7                  -1 REV + [illegible] (.35)
  28  2528    2 1967 JUL  1   22 44 58.7   20 30  0.0  41 56 24    86.89 56.42    837.7                  -1 REV + .22 (.3[illegible])
  29  2574    2 1967 JUL  1   22 45 45.6   18 21 30.0  21 45  0   159.66 59.52    38[illegible]                  -1 [illegible]
  30  2212    2 1967 JUL  2   22 37 50.1    5 13 42.0  73 21  0     0.54 34.21    [illegible]                  16 REV + .2[illegible] (.34)
  31  2528    2 1967 JUL  2   22 38 52.0   23 42  0.0  77  9 36    19.31 [illegible].12    837.7                  16 REV + [illegible]
  32  2526    2 1967 JUL  2   22 40 57.0   19 24  6.0   2 56 24   [illegible] 32.82    587.3                  16 [illegible] (.37)
  33  2541   *2 1967 JUL  2   22 42 22.0   11 50  3.5  52 38 29   [illegible].97 23.61    183.3                  [illegible] .2[illegible] (.39)
  34  2265    2 1967 JUL  3   22 33 38.6    2 40 12.0  81 15  0     7.59 44.10    337.4                  20 [illegible] + .22 (.36)
  35  2212    2 1967 JUL  3   22 33 59.4    0 48  [illegible].0  87 3[illegible]  0     3.39 49.93    229.7                  [illegible] + .22 (.36)
  36  2526    2 1967 JUL  3   22 34 32.0   19 33 [illegible]  42 [illegible]  0   100.50 63.96    305.2                  [illegible] + .23 (.37)
  37  2512    2 1967 JUL  3   22 35  3.5   18 59 2[illegible]  [illegible] 18  0   143.96 37.44    337.4                  [illegible] REV + .23 (.37)
```

Fig.5(cont'd)

```
INITIAL ELEMENTS ARE

                    6927.6636  (DERIVED SEMI-MAJOR AXIS)
                     0.007346        -0.0000045
                    80.1823          -0.00001
                    96.9913          -1.27303
                    -0.6828          -3.19105
                    50.0273         5422.03614          0.026248
```

Fig 5(cont'd)

NEG NO C5499

NEG NO C5500

```
FIRST ITERATION                     ITERATE IN MODE 0

   1      TYPE  3               2   -15                          0.00050 -0.00440  (DCS)
   2      TYPE  3               8    26                          0.00241  0.00748  (DCS)
   3      TYPE  3              27    31                          0.00785  0.00895  (DCS)
   4      TYPE  3               4   -29                          0.00118 -0.00846  (DCS)
   5      TYPE  3               3   -29                          0.00099 -0.00841  (DCS)
   6      TYPE  3               4   -30                          0.00109 -0.00868  (DCS)
   7      TYPE  3              16    68                          0.00459  0.01390  (DCS)
   8      TYPE  3              12    68                          0.00350  0.01391  (DCS)
   9      TYPE  3              10   -39                          0.00495 -0.01127  (DCS)
  10      TYPE  3               5   -41                          0.00140 -0.01191  (DCS)
  11      TYPE  3               5   -39                          0.00159 -0.01139  (DCS)
  12      TYPE  3               8   -60                          0.00237 -0.01174  (DCS)
  13      TYPE  3      *       19    66                          0.00548  0.01923  (DCS)
  14      TYPE  3              17   -55                          0.00485 -0.01586  (DCS)
  15      TYPE  3      *      -17    68                         -0.00486  0.01982  (DCS)
  16      TYPE  3               7   -65                          0.00216 -0.01294  (DCS)
  17      TYPE  3               6   -65                          0.00163 -0.01318  (DCS)
  18      TYPE  3               6    53                          0.00167  0.01540  (DCS)
  19      TYPE  3      *       23    74                          0.00666  0.02135  (DCS)
  20      TYPE  3              19   -60                          0.00565 -0.01735  (DCS)
  21      TYPE  2              -4    -1                     -0.653   -0.076   (DEGS)        TOTAL =  0.660
  22      TYPE  2               0    -3                     -0.062   -0.244   (DEGS)        TOTAL =  0.247
  23      TYPE  2     **       30    83                      6.112   16.968   (DEGS)        TOTAL = 18.017
  24      TYPE  2              -1    -6                     -0.029   -0.233   (DEGS)        TOTAL =  0.235
  25      TYPE  2               0     0                      0.645    0.267   (DEGS)        TOTAL =  0.698
  26      TYPE  2               2     7                      0.567    1.712   (DEGS)        TOTAL =  1.804
  27      TYPE  2               4    12                      0.962    2.811   (DEGS)        TOTAL =  2.971
  28      TYPE  2              -2    -4                     -0.565   -0.992   (DEGS)        TOTAL =  1.131
  29      TYPE  2               4   -24                      0.618   -2.577   (DEGS)        TOTAL =  2.610
  30      TYPE  2     **      -50   103                     -3.191    6.590   (DEGS)        TOTAL =  7.322
  31      TYPE  2              -5    -2                     -1.154   -0.407   (DEGS)        TOTAL =  1.224
  32      TYPE  2               0    -4                     -0.060   -0.420   (DEGS)        TOTAL =  0.422
  33      TYPE  2               5    -2                      0.257   -0.119   (DEGS)        TOTAL =  0.283
  34      TYPE  2              -7     2                     -0.613    0.230   (DEGS)        TOTAL =  0.655
  35      TYPE  2              -2    -1                     -0.127   -0.086   (DEGS)        TOTAL =  0.153
  36      TYPE  2              -1    -9                     -0.095   -0.801   (DEGS)        TOTAL =  0.807
  37      TYPE  2              -1    -5                     -0.051   -0.470   (DEGS)        TOTAL =  0.473

 ** REJECTION OF  2 OBSERVATION(S) - 4 FRESH - WITH RESIDUAL ABOVE  80
THERE ARE 63 DEGREES OF FREEDOM
EPSILON HAS CHANGED FROM   20.000 TO   27.570          (AND ITS NEXT VALUE IS PREDICTED TO BE    6.108)

 ELEMENTS BECOME

                 6927.0456  (DERIVED SEMI-MAJOR AXIS)
                  0.007336         -0.0000037
                 80.1783           -0.00001
                 96.9926           -1.27357
                 -0.6793           -3.19066
                 50.0975         5622.05715          0.021700

IMPORTANT - ANY OF THE FIRST FOUR SUB-1 ELEMENTS PRESENT HAS ITS FULL VALUE. THE MAIN SECULAR COMPONENT
            OF WHICH HAS BEEN COMPUTED INTERNALLY. THE EXTERNAL COMPONENTS, AND ONLY THESE APPEAR IN PUNCHED
            CARD FORMAT, ARE, RESPECTIVELY, 0.0000000.   0.00000.   0.00000.   0.00000

PROCESS CONTINUES
```

Fig 5(cont'd)

```
FURTHER ITERATION - NUMBER  2

   1      TYPE  3              0    -2              0.00003 -0.00052  (DCS)
   2      TYPE  3              5    -1              0.00133 -0.00024  (DCS)
   3      TYPE  3             23     0              0.00001  0.00000  (DCS)
   4      TYPE  3              1     3              0.00016  0.00081  (DCS)
   5      TYPE  3              0     2              0.00005  0.00044  (DCS)
   6      TYPE  3              0     0              0.00006 -0.00006  (DCS)
   7      TYPE  3             -1     2             -0.00046  0.00056  (DCS)
   8      TYPE  3             -1     0             -0.00037 -0.00006  (DCS)
   9      TYPE  3              2     1              0.00056  0.00035  (DCS)
  10      TYPE  3              1     1              0.00039  0.00042  (DCS)
  11      TYPE  3              2     1              0.00048  0.00033  (DCS)
  12      TYPE  3              2     0              0.00058 -0.00012  (DCS)
  13      TYPE  3              0     2              0.00010  0.00059  (DCS)
  14      TYPE  3              3    -1              0.00099 -0.00034  (DCS)
  15      TYPE  3            -29     1             -0.00046  0.00040  (DCS)
  16      TYPE  3              2     0              0.00063 -0.00014  (DCS)
  17      TYPE  3              2    -2              0.00054 -0.00072  (DCS)
  18      TYPE  3             -1    -1             -0.00017 -0.00024  (DCS)
  19      TYPE  3              1     1              0.00030  0.00030  (DCS)
  20      TYPE  3              4     1              0.00111  0.00047  (DCS)
  21      TYPE  2             -3    -2          -0.303  -0.216   (DEGS)      TOTAL =  0.372
  22      TYPE  2              2     1           0.188   0.140   (DEGS)      TOTAL =  0.234
  23      TYPE  2    *        30    84           6.203  17.212   (DEGS)      TOTAL = 18.296
  24      TYPE  2              3     4           0.103   0.145   (DEGS)      TOTAL =  0.178
  25      TYPE  2              0     0           0.724   0.537   (DEGS)      TOTAL =  0.901
  26      TYPE  2              4    10           0.976   2.255   (DEGS)      TOTAL =  2.457
  27      TYPE  2              8    15           1.340   3.405   (DEGS)      TOTAL =  3.659
  28      TYPE  2             -1    -1          -0.303  -0.330   (DEGS)      TOTAL =  0.448
  29      TYPE  2              3   -18           0.340  -1.907   (DEGS)      TOTAL =  1.937
  30      TYPE  2    *       -48    98          -3.047   6.265   (DEGS)      TOTAL =  6.966
  31      TYPE  2             -2    -1          -0.518  -0.194   (DEGS)      TOTAL =  0.553
  32      TYPE  2              0     0           0.032   0.008   (DEGS)      TOTAL =  0.033
  33      TYPE  2             -1     0          -0.069   0.018   (DEGS)      TOTAL =  0.071
  34      TYPE  2              0    -1          -0.029  -0.086   (DEGS)      TOTAL =  0.091
  35      TYPE  2             10     0           0.625  -0.032   (DEGS)      TOTAL =  0.626
  36      TYPE  2              1     2           0.105   0.190   (DEGS)      TOTAL =  0.217
  37      TYPE  2              0     1           0.000   0.061   (DEGS)      TOTAL =  0.061

 * REJECTION OF  2 OBSERVATION(S) - 0 FRESH - WITH RESIDUAL ABOVE  83
THERE ARE 65 DEGREES OF FREEDOM
EPSILON HAS CHANGED FROM   27.570 TO    0.067          (AND ITS NEXT VALUE IS PREDICTED TO BE    0.066)

 ELEMENTS BECOME

              6927.6458  (DERIVED SEMI-MAJOR AXIS)
                0.007335           -0.0000057
               80.1784             -0.00001
               96.0925             -1.27355
               -0.6825             -3.19065
               50.1307            5422.05603              0.041775

IMPORTANT - ANY OF THE FIRST FOUR SUB-1 ELEMENTS PRESENT HAS ITS FULL VALUE. THE MAIN SECULAR COMPONENT
          OF WHICH HAS BEEN COMPUTED INTERNALLY. THE EXTERNAL COMPONENTS. AND ONLY THESE APPEAR IN PUNCHED
          CARD FORMAT. ARE. RESPECTIVELY. 0.0000000.   0.00000.   0.00000.   0.00000

PROCESS CONTINUES
```

Fig 5(cont d)

NEG NO C55[illegible]

NEG NO C5502

TR 68209

```
FURTHER ITERATION - NUMBER  3

   1     TYPE  3              0   -1                       -0.00001 -0.00042  (DCS)
   2     TYPE  1              4   -1                        0.00130 -0.00035  (DCS)
   3     TYPE  3   *         23    0                        0.00458 -0.00008  (DCS)
   4     TYPE  3              0    3                        0.00214  0.00085  (DCS)
   5     TYPE  3              0    2                        0.00003  0.00048  (DCS)
   6     TYPE  3              0    0                        0.00065  0.00001  (DCS)
   7     TYPE  3             -1    2                       -0.00026  0.00052  (DCS)
   8     TYPE  3             -1    0                       -0.00037 -0.00009  (DCS)
   9     TYPE  3              2    1                        0.00054  0.00034  (DCS)
  10     TYPE  3              1    1                        0.00038  0.00040  (DCS)
  11     TYPE  3              2    1                        0.00047  0.00032  (DCS)
  12     TYPE  3              2    0                        0.00057 -0.00013  (DCS)
  13     TYPE  3              0    2                        0.00012  0.00063  (DCS)
  14     TYPE  3              3   -1                        0.00097 -0.00041  (DCS)
  15     TYPE  3   **       -29    2                       -0.00848  0.00045  (DCS)
  16     TYPE  3              2   -1                        0.00063 -0.00019  (DCS)
  17     TYPE  3              2   -3                        0.00054 -0.00077  (DCS)
  18     TYPE  3             -1   -1                       -0.00018 -0.00018  (DCS)
  19     TYPE  3              1    1                        0.00043  0.00044  (DCS)
  20     TYPE  3              4    1                        0.00110  0.00028  (DCS)
  21     TYPE  2             -3   -2                    -0.300   -0.216   (DEGS)       TOTAL =  0.370
  22     TYPE  2              2    2                     0.188    0.144   (DEGS)       TOTAL =  0.237
  23     TYPE  2   ***       30   84                     6.204   17.216   (DEGS)       TOTAL = 18.298
  24     TYPE  2              3    4                     0.102    0.148   (DEGS)       TOTAL =  0.180
  25     TYPE  2              0    0                     0.724    0.538   (DEGS)       TOTAL =  0.902
  26     TYPE  2              4   10                     0.974    2.256   (DEGS)       TOTAL =  2.457
  27     TYPE  2              6   15                     1.339    3.405   (DEGS)       TOTAL =  3.659
  28     TYPE  2             -1   -1                    -0.305   -0.330   (DEGS)       TOTAL =  0.450
  29     TYPE  2              3  -18                     0.338   -1.908   (DEGS)       TOTAL =  1.937
  30     TYPE  2   ***      -48   98                    -3.046    6.266    DEGS)       TOTAL =  6.967
  31     TYPE  2             -2   -1                    -0.521   -0.194   (DEGS)       TOTAL =  0.556
  32     TYPE  2              0    0                     0.030    0.007   (DEGS)       TOTAL =  0.031
  33     TYPE  2             -1    0                    -0.067    0.017   (DEGS)       TOTAL =  0.069
  34     TYPE  2              0   -1                    -0.032   -0.083   (DEGS)       TOTAL =  0.089
  35     TYPE  2             10    0                     0.620   -0.031   (DEGS)       TOTAL =  0.621
  36     TYPE  2              1    2                     0.102    0.183   (DEGS)       TOTAL =  0.210
  37     TYPE  2              0    1                    -0.001    0.058   (DEGS)       TOTAL =  0.058

** REJECTION OF  3 OBSERVATION(S) - 1 FRESH - WITH RESIDUAL ABOVE  24
THERE ARE 61 DEGREES OF FREEDOM
EPSILON HAS CHANGED FROM    6.067 TO    4.094          (AND ITS NEXT VALUE IS PREDICTED TO BE    4.026)

 ELEMENTS BECOME

                    6927.6456  (DERIVED SEMI-MAJOR AXIS)
                     0.007322         -0.0000037
                    80.1843           -0.00001
                    96.9987           -1.27279
                    -0.7501           -3.19130
                    53.1690         5422.03733           0.021628

IMPORTANT - ANY OF THE FIRST FOUR SUB-1 ELEMENTS PRESENT HAS ITS FULL VALUE. THE MAIN SECULAR COMPONENT
           OF WHICH HAS BEEN COMPUTED INTERNALLY. THE EXTERNAL COMPONENTS, AND ONLY THESE APPEAR IN PUNCHED
           CARD FORMAT, ARE, RESPECTIVELY, 0.0000000,   0.00000,   0.00000,   0.00000

PROCESS CONTINUES
```

Fig 5(cont d)

Fig.5(cont'd)

```
FURTHER ITERATION - NUMBER  4

   1      TYPE  3              -4   -2                     -0.00100 -0.00071  (DCS)
   2      TYPE  3               2   -2                      0.00052 -0.00068  (DCS)
   3      TYPE  3      **      20   -1                      0.00570 -0.00028  (DCS)
   4      TYPE  3              -3    2                     -0.00082  0.00064  (DCS)
   5      TYPE  3              -2    1                     -0.00056  0.00040  (DCS)
   6      TYPE  3              -1    0                     -0.00037  0.00000  (DCS)
   7      TYPE  3               0    1                     -0.00010  0.00018  (DCS)
   8      TYPE  3              -1   -1                     -0.00048 -0.00040  (DCS)
   9      TYPE  3              -2   -1                     -0.00045 -0.00018  (DCS)
  10      TYPE  3              -2    1                     -0.00044  0.00044  (DCS)
  11      TYPE  3               0    1                     -0.00003  0.00047  (DCS)
  12      TYPE  3               1    0                      0.00040 -0.00007  (DCS)
  13      TYPE  3               0    1                      0.00009  0.00037  (DCS)
  14      TYPE  3              -1   -3                     -0.00025 -0.00096  (DCS)
  15      TYPE  3      **     -35    1                     -0.01014  0.00024  (DCS)
  16      TYPE  3               1   -1                      0.00022 -0.00023  (DCS)
  17      TYPE  3               1   -3                      0.00023 -0.00080  (DCS)
  18      TYPE  3              -3   -1                     -0.00089 -0.00033  (DCS)
  19      TYPE  3               1    1                      0.00028  0.00048  (DCS)
  20      TYPE  3              -1   -1                     -0.00036 -0.00038  (DCS)
  21      TYPE  2              -2   -2                  -0.259   -0.193   (DEGS)          TOTAL =  0.323
  22      TYPE  2               1    2                   0.129    0.182   (DEGS)          TOTAL =  0.223
  23      TYPE  2     ***      30   84                   6.151   17.209   (DEGS)          TOTAL = 18.275
  24      TYPE  2               1    4                   0.039    0.153   (DEGS)          TOTAL =  0.158
  25      TYPE  2               0    0                   0.669    0.532   (DEGS)          TOTAL =  0.855
  26      TYPE  2               4   10                   0.923    2.302   (DEGS)          TOTAL =  2.480
  27      TYPE  2       *       6   15                   1.281    3.445   (DEGS           TOTAL =  3.676
  28      TYPE  2              -2   -1                  -0.376   -0.312   (DEGS)          TOTAL =  0.488
  29      TYPE  2       *       2  -18                   0.252   -1.922   (DEGS)          TOTAL =  1.938
  30      TYPE  2     ***     -47   98                  -2.996    6.271   (DEGS)          TOTAL =  6.950
  31      TYPE  2              -2   -1                  -0.536   -0.123   (DEGS)          TOTAL =  0.550
  32      TYPE  2               0    0                  -0.029   -0.003   (DEGS)          TOTAL =  0.029
  33      TYPE  2              -1   -1                  -0.059   -0.027   (DEGS)          TOTAL =  0.065
  34      TYPE  2               0    0                   0.009   -0.022   (DEGS)          TCTAL =  0.024
  35      TYPE  2              10    1                   0.626    0.050   (DEGS)          TOTAL =  0.628
  36      TYPE  2               0    2                   0.009    0.186   (DEGS)          TOTAL =  0.186
  37      TYPE  2              -1    0                  -0.070    0.046   (DEGS)          TOTAL =  0.083

 ** REJECTION OF  4 OBSERVATION(S) - 1 FRESH - WITH RESIDUAL ABOVE  20
THERE ARE 50 DEGREES OF FREEDOM
EPSILON HAS CHANGED FROM    4.894 TO    3.970          (AND ITS NEXT VALUE IS PREDICTED TO BE    3.903)

 ELEMENTS BECOME

                    6927.0448  (DERIVED SEMI-MAJOR AXIS)
                     0.007324         -0.0000037
                    80.1819           -0.00001
                    96.9961           -1.27311
                    -0.7066           -3.19103
                    50.1253         5422.05812            0.021592

IMPORTANT - ANY OF THE FIRST FOUR SUB-1 ELEMENTS PRESENT HAS ITS FULL VALUE, THE MAIN SECULAR COMPONENT
          OF WHICH HAS BEEN COMPUTED INTERNALLY. THE EXTERNAL COMPONENTS, AND ONLY THESE APPEAR IN PUNCHED
          CARD FORMAT, ARE, RESPECTIVELY, 0.0000000,   0.00000,   0.00000,   0.00000

PROCESS CONTINUES
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Fig 5(cont'd)

Fig.5(cont'd)

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FURTHER ITERATION - NUMBER  5

  1    TYPE  3              -2   -3                  -0.00057 -0.00095  (DCS)
  2    TYPE  3               3   -1                   0.00086 -0.00025  (DCS)
  3    TYPE  3    **        21    0                   0.00411 -0.00005  (DCS)
  4    TYPE  3              -1    2                  -0.00038  0.00061  (DCS)
  5    TYPE  3              -1    1                  -0.00048  0.00035  (DCS)
  6    TYPE  3              -1    0                  -0.00017 -0.00006  (DCS)
  7    TYPE  3               0    2                  -0.00011  0.00050  (DCS)
  8    TYPE  3              -1    0                  -0.00048 -0.00013  (DCS)
  9    TYPE  3               0    0                  -0.00005  0.00002  (DCS)
 10    TYPE  3               0    1                  -0.00009  0.00035  (DCS)
 11    TYPE  3               1    1                   0.00019  0.00037  (DCS)
 12    TYPE  3               1    0                   0.00036  0.00000  (DCS)
 13    TYPE  3               0    1                   0.00010  0.00042  (DCS)
 14    TYPE  3               1   -2                   0.00021 -0.00059  (DCS)
 15    TYPE  3    **       -33    0                  -0.00047  0.00010  (DCS)
 16    TYPE  3               1    0                   0.00038 -0.00001  (DCS)
 17    TYPE  3               1   -2                   0.00035 -0.00058  (DCS)
 18    TYPE  3              -2   -2                  -0.00060 -0.00047  (DCS)
 19    TYPE  3               1    0                   0.00029  0.00010  (DCS)
 20    TYPE  3               1    1                   0.00016  0.00018  (DCS)
 21    TYPE  2              -3   -2                -0.281   -0.198   (DEGS)      TOTAL =  0.344
 22    TYPE  2               2    2                 0.147    0.155   (DEGS)      TOTAL =  0.213
 23    TYPE  2   ***        30   84                 6.170   17.205   (DEGS)      TOTAL = 18.278
 24    TYPE  2               2    3                 0.061    0.142   (DEGS)      TOTAL =  0.154
 25    TYPE  2               0    0                 0.689    0.529   (DEGS)      TOTAL =  0.869
 26    TYPE  2               4   10                 0.942    2.282   (DEGS)      TOTAL =  2.469
 27    TYPE  2     *         6   15                 1.303    3.428   (DEGS)      TOTAL =  3.667
 28    TYPE  2              -1   -1                -0.348   -0.320   (DEGS)      TOTAL =  0.472
 29    TYPE  2    **         3  -18                 0.288   -1.916   (DEGS)      TOTAL =  1.937
 30    TYPE  2   ***       -47   98                -3.016    6.267   (DEGS)      TOTAL =  6.955
 31    TYPE  2              -2   -1                -0.525   -0.150   (DEGS)      TOTAL =  0.546
 32    TYPE  2               0    0                -0.004    0.006   (DEGS)      TOTAL =  0.007
 33    TYPE  2              -1    0                -0.065   -0.007   (DEGS)      TOTAL =  0.065
 34    TYPE  2               0   -1                 0.000   -0.051   (DEGS)      TOTAL =  0.051
 35    TYPE  2              10    0                 0.635    0.019   (DEGS)      TOTAL =  0.635
 36    TYPE  2               1    2                 0.050    0.401   (DEGS)      TOTAL =  0.207
 37    TYPE  2               0    1                -0.041    0.060   (DEGS)      TOTAL =  0.073

 ** REJECTION OF  5 OBSERVATION(S) - 1 FRESH - WITH RESIDUAL ABOVE  10
THERE ARE 57 DEGREES OF FREEDOM
EPSILON HAS CHANGED FROM    3.970 TO    3.172          (AND ITS NEXT VALUE IS PREDICTED TO BE    3.168)

 ELEMENTS BECOME

                   6927.0651  (DERIVED SEMI-MAJOR AXIS)
                    0.007321          -0.0000037
                   80.1818            -0.00001
                   96.9961            -1.27311
                   -0.7251            -3.19132
                   50.1436          5422.05786           0.021794

IMPORTANT - ANY OF THE FIRST FOUR SUB-1 ELEMENTS PRESENT HAS ITS FULL VALUE. THE MAIN SECULAR COMPONENT
         OF WHICH HAS BEEN COMPUTED INTERNALLY. THE EXTERNAL COMPONENTS, AND ONLY THESE APPEAR IN PUNCHED
         CARD FORMAT, ARE, RESPECTIVELY, 0.000000,  0.0000,  0.00000,  0.00000

PROCESS CONTINUES
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Fig 5(cont'd)

Fig.5(cont'd)

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FURTHER ITERATION - NUMBER  6

   1      TYPE  3               -2    -3                  -0.00057 -0.00088  (DCS)
   2      TYPE  3                3    -1                   0.00084 -0.00038  (DCS)
   3      TYPE  3     **        21    -1                   0.00610 -0.00015  (DCS)
   4      TYPE  3               -1     2                  -0.00037  0.00035  (DCS)
   5      TYPE  3               -1     1                  -0.00047  0.00031  (DCS)
   6      TYPE  3               -1     0                  -0.00016 -0.00009  (DCS)
   7      TYPE  3                0     2                  -0.00011  0.00054  (DCS)
   8      TYPE  3               -1     0                  -0.00026 -0.00008  (DCS)
   9      TYPE  3                0     0                  -0.00004 -0.00009  (DCS)
  10      TYPE  3                0     1                  -0.00007  0.00024  (DCS)
  11      TYPE  3                1     1                   0.00021  0.00029  (DCS)
  12      TYPE  3                1     0                   0.00038 -0.00007  (DCS)
  13      TYPE  3                0     2                   0.00012  0.00031  (DCS)
  14      TYPE  3                1    -2                   0.00045 -0.00072  (DCS)
  15      TYPE  3    ***       -33     0                  -0.00946  0.00008  (DCS)
  16      TYPE  3                1     0                   0.00040 -0.00008  (DCS)
  17      TYPE  3                1    -2                   0.00037 -0.00004  (DCS)
  18      TYPE  3               -2    -2                  -0.00059 -0.00049  (DCS)
  19      TYPE  3                1     1                   0.00030  0.00016  (DCS)
  20      TYPE  3                1     0                   0.00019  0.00010  (DCS)
  21      TYPE  2               -3    -2              -0.282   -0.198   (DEGS)      TOTAL =  0.345
  22      TYPE  2                2     2               0.146    0.153   (DEGS)      TOTAL =  0.212
  23      TYPE  2    ***        30    84               6.170   17.206   (DEGS)      TOTAL = 18.277
  24      TYPE  2                1     3               0.061    0.140   (DEGS)      TOTAL =  0.153
  25      TYPE  2                0     0               0.689    0.528   (DEGS)      TOTAL =  0.868
  26      TYPE  2      *         4    10               0.939    2.276   (DEGS)      TOTAL =  2.462
  27      TYPE  2     **         6    15               1.300    3.422   (DEGS)      TOTAL =  3.660
  28      TYPE  2               -2    -1              -0.349   -0.326   (DEGS)      TOTAL =  0.478
  29      TYPE  2     **         3   -18               0.289   -1.022   (DEGS)      TOTAL =  1.064
  30      TYPE  2    ***       -47    98              -3.017    6.270   (DEGS)      TOTAL =  6.958
  31      TYPE  2               -2    -1              -0.330   -0.152   (DEGS)      TOTAL =  0.332
  32      TYPE  2                0     0              -0.004    0.003   (DEGS)      TOTAL =  0.005
  33      TYPE  2               -1     0              -0.002   -0.008   (DEGS)      TOTAL =  0.008
  34      TYPE  2                0    -1              -0.003   -0.050   (DEGS)      TOTAL =  0.050
  35      TYPE  2      *        10     0               0.031    0.018   (DEGS)      TOTAL =  0.036
  36      TYPE  2                1     2               0.049    0.197   (DEGS)      TOTAL =  0.203
  37      TYPE  2                0     1              -0.061    0.038   (DEGS)      TOTAL =  0.071

 ** REJECTION OF  6 OBSERVATION(S) - 1 FRESH - WITH RESIDUAL ABOVE  13
THERE ARE 55 DEGREES OF FREEDOM
EPSILON HAS CHANGED FROM    3.172 TO    2.420          (AND ITS BEST VALUE IS PREDICTED TO BE    2.420)

 ELEMENTS BECOME

                     6927.6451  (DERIVED SEMI-MAJOR AXIS)
                      0.007323          -0.0000037
                     80.1810            -0.00001
                     90.0961            -1.27310
                     -0.7180            -3.10103
                     50.1375          5422.05780            0.021713

IMPORTANT - ANY OF THE FIRST FOUR SUB-1 ELEMENTS PRESENT HAS ITS FULL VALUE, THE MAIN SECULAR COMPONENT
          OF WHICH HAS BEEN COMPUTED INTERNALLY. THE EXTERNAL COMPONENTS, AND ONLY THESE APPEAR IN PUNCHED
          CARD FORMAT, ARE, RESPECTIVELY, 0.0000000.    0.00000.    0.00000.    0.00000

PROCESS CONTINUES
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Fig 5(cont'd)

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FLATTED ITERATION - NUMBER  7

  1    TYPE 3           -2   -3                          -0.00036 -0.00087  (SCS)
  2    TYPE 3            3   -1                           0.00385 -0.00035  (SCS)
  3    TYPE 3   **      21    0                           0.00418 -0.00011  (SCS)
  4    TYPE 3           -1    2                          -0.00036  0.00058  (SCS)
  5    TYPE 3           -1    1                          -0.00048  0.00033  (SCS)
  6    TYPE 3           -1    0                          -0.00017 -0.00080  (SCS)
  7    TYPE 3            1    2                          -0.00411  0.00058  (SCS)
  8    TYPE 3           -1    0                          -0.00047 -0.00014  (SCS)
  9    TYPE 3            1    0                          -0.00004 -0.00004  (SCS)
 10    TYPE 3            1    1                          -0.00008  0.00048  (SCS)
 11    TYPE 3            1    1                           0.00148  0.00031  (SCS)
 12    TYPE 3            1    0                           0.00037 -0.00006  (SCS)
 13    TYPE 3            1    2                           0.00011  0.00040  (SCS)
 14    TYPE 3            1   -2                           0.00023 -0.00006  (SCS)
 15    TYPE 3   ***    -33    0                          -0.00047  0.00013  (SCS)
 16    TYPE 3            1    0                           0.00038 -0.00008  (SCS)
 17    TYPE 3            1   -2                           0.00036 -0.00005  (SCS)
 18    TYPE 3           -2   -2                          -0.00008 -0.00045  (SCS)
 19    TYPE 3            1    1                           0.00038  0.00010  (SCS)
 20    TYPE 3            1    0                           0.00010  0.00018  (SCS)
 21    TYPE 2           -3   -2                      -0.281   -0.100   (DEGS)       TOTAL =  0.366
 22    TYPE 2            2    2                       0.167    0.155   (DEGS)       TOTAL =  0.216
 23    TYPE 2   ***     31   84                       0.178   17.265   (DEGS)       TOTAL = 18.278
 24    TYPE 2            2    3                       0.081    0.142   (DEGS)       TOTAL =  0.155
 25    TYPE 2            0    0                       0.000    0.528   (DEGS)       TOTAL =  0.009
 26    TYPE 2   **       4   10                       0.048    2.270   (DEGS)       TOTAL =  2.400
 27    TYPE 2   **       8   15                       1.302    3.425   (DEGS)       TOTAL =  3.004
 28    TYPE 2           -1   -1                      -0.360   -0.323   (DEGS)       TOTAL =  0.478
 29    TYPE 2   **       3  -17                       0.288   -1.030   (DEGS)       TOTAL =  1.061
 30    TYPE 2   ***    -47   98                      -3.816    0.289   (DEGS)       TOTAL =  0.037
 31    TYPE 2           -2   -1                      -0.520   -0.155   (DEGS)       TOTAL =  0.550
 32    TYPE 2            0    0                      -0.004    0.063   (DEGS)       TOTAL =  0.006
 33    TYPE 2           -1    0                      -0.003   -0.038   (DEGS)       TOTAL =  0.003
 34    TYPE 2            0   -1                      -0.002   -0.056   (DEGS)       TOTAL =  0.056
 35    TYPE 2   **      10    0                       0.032    0.010   (DEGS)       TOTAL =  0.032
 36    TYPE 2            1    2                       0.040    0.187   (DEGS)       TOTAL =  0.283
 37    TYPE 2            0    1                      -0.042    0.058   (DEGS)       TOTAL =  0.071

** REJECTION OF  8 OBSERVATION(S) - 2 [illegible] - WITH RESIDUAL ABOVE  10
THERE ARE 51 DEGREES OF FREEDOM
EPSILON HAS CHANGED FROM    2.420 TO    1.494          (AND ITS NEXT VALUE IS PREDICTED TO BE    1.482)

 ELEMENTS BECOME

                6927.0444  (DERIVED SEMI-MAJOR AXIS)
                 0.007324        -0.0000030
                80.1810          -0.00001
                96.9961          -1.27310
                -0.7054          -3.19104
                53.1236        5422.03801           0.041916

IMPORTANT - ANY OF THE FIRST FOUR SUB-1 ELEMENTS PRESENT HAS ITS FULL VALUE, THE MAIN SECULAR COMPONENT
         OF WHICH HAS BEEN COMPUTED INTERNALLY. THE EXTERNAL COMPONENTS, AND ONLY THESE APPEAR IN PUNCHED
         CARD FORMAT, ARE, RESPECTIVELY, 0.00 0000,   0.0.000,   0.00000,   0.00000

PROCESS CONTINUES
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Fig 5 (cont d)

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FURTHER ITERATION - NUMBER  8

   1      TYPE  3              -2    -3                      -0.00061 -0.00077  (DCS)
   2      TYPE  3               3    -1                       0.00009 -0.00026  (DCS)
   3      TYPE  3    ...       21     0                       0.00418  0.00004  (DCS)
   4      TYPE  3              -1     2                      -0.00048  0.00061  (DCS)
   5      TYPE  3              -1     1                      -0.00038  0.00032  (DCS)
   6      TYPE  3              -1     0                      -0.00018 -0.00011  (DCS)
   7      TYPE  3               0     2                      -0.00009  0.00049  (DCS)
   8      TYPE  3              -1     0                      -0.00027 -0.00014  (DCS)
   9      TYPE  3               0     0                      -0.00004  0.00000  (DCS)
  10      TYPE  3               0     1                      -0.00010  0.00031  (DCS)
  11      TYPE  3               1     1                       0.00018  0.00029  (DCS)
  12      TYPE  3               1     0                       0.00035 -0.00010  (DCS)
  13      TYPE  3               0     1                       0.00011  0.00042  (DCS)
  14      TYPE  3               1    -2                       0.00021 -0.00058  (DCS)
  15      TYPE  3    ...      -33     1                      -0.00067  0.00023  (DCS)
  16      TYPE  3               1     0                       0.00037 -0.00006  (DCS)
  17      TYPE  3               1    -2                       0.00036 -0.00042  (DCS)
  18      TYPE  3              -2    -1                      -0.00062 -0.00042  (DCS)
  19      TYPE  3               1     0                       0.00027 -0.00004  (DCS)
  20      TYPE  3               0     1                       0.00012  0.00033  (DCS)
  21      TYPE  2              -3    -2                   -0.279   -0.200   (DEGS)       TOTAL =  0.343
  22      TYPE  2               2     2                    0.149    0.159   (DEGS)       TOTAL =  0.218
  23      TYPE  2    ...       30    84                    6.171   17.206   (DEGS)       TOTAL = 18.280
  24      TYPE  2               2     4                    0.063    0.144   (DEGS)       TOTAL =  0.158
  25      TYPE  2               0     0                    0.690    0.530   (DEGS)       TOTAL =  0.870
  26      TYPE  2     ..        4    10                    0.943    2.281   (DEGS)       TOTAL =  2.468
  27      TYPE  2     ..        6    15                    1.306    3.427   (DEGS)       TOTAL =  3.667
  28      TYPE  2              -1    -1                   -0.347   -0.321   (DEGS)       TOTAL =  0.473
  29      TYPE  2    ...        3   -18                    0.288   -1.917   (DEGS)       TOTAL =  1.939
  30      TYPE  2    ...      -67    98                   -3.015    6.266   (DEGS)       TOTAL =  6.954
  31      TYPE  2              -2    -1                   -0.524   -0.150   (DEGS)       TOTAL =  0.545
  32      TYPE  2               0     0                   -0.006    0.003   (DEGS)       TOTAL =  0.007
  33      TYPE  2              -1     0                   -0.065   -0.007   (DEGS)       TOTAL =  0.066
  34      TYPE  2               0    -1                    0.007   -0.056   (DEGS)       TOTAL =  0.056
  35      TYPE  2     ..       10     0                    0.643    0.022   (DEGS)       TOTAL =  0.643
  36      TYPE  2               1     2                    0.051    0.210   (DEGS)       TOTAL =  0.216
  37      TYPE  2               0     1                   -0.061    0.046   (DEGS)       TOTAL =  0.076

 * REJECTION OF  8 OBSERVATION(S) - 0 FRESH - WITH RESIDUAL ABOVE   6
THERE ARE 59 DEGREES OF FREEDOM
EPSILON HAS CHANGED FROM    1.464 TO    1.462          (AND ITS NEXT VALUE IS PREDICTED TO BE    1.462)

 ELEMENTS BECOME

                    6927.0669  (DERIVED SEMI-MAJOR AXIS)
                       0.007329        -0.0000038
                      80.1810          -0.00001
                      96.9961          -1.27310
                      -0.7056          -3.19106
                      50.1237        5622.05806            0.021976

IMPORTANT - ANY OF THE FIRST FOUR SUB-T ELEMENTS PRESENT HAS ITS FULL VALUE. THE MAIN SECULAR COMPONENT
          OF WHICH HAS BEEN COMPUTED INTERNALLY. THE EXTERNAL COMPONENTS, AND ONLY THESE APPEAR IN PUNCHED
          CARD FORMAT, ARE, RESPECTIVELY, 0.0000000,   0.00000,   0.00000,   0.00000

      (CONCLUDED OVERLEAF)
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Fig 5(cont d)

NEG NO C5467

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CONVERGENCE ACHIEVED, STANDARD DEVIATIONS AND CORRELATIONS OF THE ELEMENTS ARE AS FOLLOWS
    (PUNCHED CARD OUTPUT IS OF FINAL ELEMENTS AND COVARIANCE MATRIX)

                       0.0005       (S.D. FOR DERIVED SEMI-MAJOR AXIS)
                     0.000014          0.0000000
                     0.0011            0.00000
                     0.0010            0.00000
                     0.0494            0.00000
                     0.0497            0.00057          0.000508


   1.000     0.033    -0.189     0.238    -0.248    -0.088    -0.006
   0.033     1.000     0.179     0.026    -0.026     0.008     0.004
  -0.189     0.179     1.000    -0.025     0.025     0.009    -0.004
   0.238     0.026    -0.025     1.000    -1.000     0.034    -0.012
  -0.248    -0.026     0.025    -1.000     1.000    -0.031     0.001
  -0.088     0.008     0.009     0.034    -0.031     1.000    -0.338
  -0.006     0.002    -0.004    -0.012     0.001    -0.338     1.000
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Fig 5(concl'd)