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FOREIGN TECHNOLOGY DIVISION

ELECTROMAGNETIC DEVICES OF AUTOMATION

by

Ye. I. Yurevich

(Vol I of 2 Volumes)

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Ye. I. Yurevich

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ABSTRACT

This book describes the operating principles and cites the fundamentals in the design of electromagnetic automation equipment. In addition to the basic electromagnetic elements, discrete action ferromagnetic equipment, magnetic amplifiers, special magnetic elements of digital mathematical equipment, magnetic generators and frequency converters, voltage and current stabilizers, elements using the Hall effect and magnetic resistance, dielectric amplifiers and relays are examined. The book is a textbook in the course "Electromagnetic Equipment" for the specialities "Automation and Remote Control" and "Electrometrology" of polytechnic institutes.

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The book describes the principles of operation and design of electromagnetic devices used in automation. In addition to the basic electromechanical elements, it also deals with magnetic amplifiers, ferromagnetic devices of discrete operation, special magnetic elements of digital computers, magnetic frequency generators and converters, voltage and current stabilizers, elements using the Hall effect and magnetic resistors, dielectric amplifiers and relays, etc.

The book serves as a manual for the course of lectures "Electromagnetic Devices" in the special subjects "Automation and Remote Control" and "Electrical Measurement Techniques" studied at polytechnical institutes.

PREFACE

The book examines the principles of operation, design and construction of the basic magnetic and electromechanical components used in automatic and remote control devices. The book is based on a course of lectures read by the author to the students specializing in the subjects "Automation and Remote Control" and "Electrical Measurement Techniques" at the Electromechanical Faculty of the Leningrad Kalinin Polytechnical Institute.

The main purpose of the book is to serve as a manual for the subjects just mentioned, for students taking the course "Electromagnetic Devices" and for those planning their courses. In conformity with the program of this course, the book examines only the most important electromagnetic elements of automation. It does not deal with inductive, induction-type and magneto-elastic mechanical-to-electrical transducers, since these components are considered in the "Electrical Measurements" course. On the other hand, dielectric amplifiers and other dielectric devices are examined in the book, since, although they are not in the class of electromagnetic elements, it is more convenient from a methodical point of view to consider these devices in the same course as magnetic amplifiers.

Remarks and comments about the book should be addressed to: Leningrad, D-41, Marsovo Pole 1, Leningrad Branch of "Energiya" Press.

The Author

INTRODUCTION

Any automatic system consists of individual elements that differ in their utilization, static characteristic, physical nature, etc.

With regard to utilization, the elements can be grouped into sensing, actuating, and intermediate elements.

Sensing (detecting) elements serve for detecting some variable quantity as a function of which the automatic system operates. In general, this quantity is converted by the sensing element into a quantity of different type, more convenient for subsequent use in the system.

Actuating elements are intended for controlling the plant (process), this control being in fact the purpose of the automatic system as such.

Ranging between the sensing and actuating elements are the elements of intermediate type. They serve for the amplification or conversion of signals obtained from the sensing elements. An amplifying element (amplifier) is intended for increasing the signal power, and a converting element (converter) is intended for signal conversion (changing the physical nature of signals) such as the conversion of an electrical signal into a mechanical signal or of an ac-signal into a dc-signal (or vice versa).

Let us note that although signal conversion is also common in sensing and actuating elements, these are merely side-effects which do not necessarily occur. One and the same element can be used as a sensing, intermediate or actuating element. Depending on the function it fulfils, the requirements imposed on this element as well as its design will be different. Let us consider, for example, an electromagnet. It converts an electrical signal into a displacement, and is often utilized as an intermediate transducer. The principal requirements to be met in this case by the device are linearity and stability of the static "input-output" characteristic. If the electromagnet is used as an actuating element (for example, in the control of shutters), the essential feature will be its efficiency. Finally, if the electromagnet serves as a sensing element, its most important properties will be sensitivity and null stability.

To the class of intermediate elements belong also the so-called compensating elements, incorporated into automatic systems for the purpose of improving their dynamic performance. As compensating elements one can use very simple differentiating, integrating, or lag-type elements.

Finally, a typical auxiliary element of automatic systems is the stabilizer, used for improving the quality of the power supply of the automatic system. We can have voltage or current stabilizers.

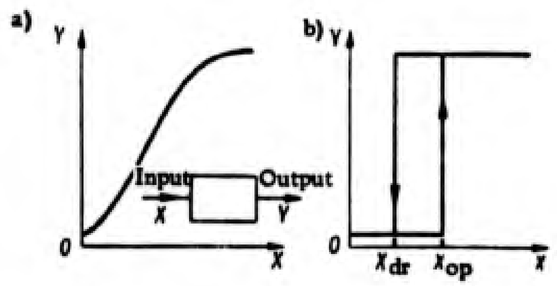


Fig. 0.1. Static input-output characteristics of elements of continuous (a) and discrete (b) operation.

From the point of view of the static input-output characteristic, the elements of automatic systems can be grouped into elements of continuous and of discrete operation.

In Fig. 0.1 we present the simplest static characteristics Y(X) of these two types of elements. In the case of an element of continuous operation (Fig. 0.1a), a smooth variation of the input X will be accompanied by a smooth variation of the output Y, the correspondence between the values of X and Y being one-to-one.

The static properties of an element of continuous operation, i.e., the dependence of the output on the input under steady-state conditions, can be described analytically by the transfer function

$$K = \frac{\Delta Y}{\Delta X}.$$

In the case of an amplifier, this ratio is called the gain. The transfer function specifies the slope of the static characteristic of the element at the particular point. If the static characteristic is linear, this function will be constant in the entire range of variation of the input variable. In the general case the transfer function varies with the input variable. The dynamic properties of elements of continuous operation are described analytically by a differential equation specifying the relationship between the output and input during the transient process caused by the variation of the input.

In the simplest element of discrete operation (Fig. 0.1b), the output variable does not change as long as the input variable X increases from 0 to X_{op}; but as soon as the latter value is reached, the output variable experiences a jump-like increase to its maximum value, which it retains even if X goes on increasing. If X decreases, the output variable again drops to its minimum value as soon as X passes through the value X_{dr}. Thus the full range of variation of the input variable corresponds to only two discrete steady-state values of the output variable. Such a simple element of discrete operation is called a relay. In the general case a discrete element can have more than two discrete steady-state values of the output variable and they occur in a jump-like manner when the input variable varies continuously.

The input signal threshold values X_{op} and X_{dr} are called the operating and dropout signals respectively. The ratio $K_r = X_{op}/X_{dr}$ is called the reset ratio. It is always smaller than unity. For reliable operation of an element it is necessary that the signal applied to it (the driving signal) X_{imp} be slightly higher than X_{op}. The ratio $K_{saf\ op} = X_{imp}/X_{op}$ is called the operating safety factor. Similarly we have the dropout safety factor $K_{saf\ dr} = X_{dr}/X_q$, where X_q is a so-called quiescent signal, at which the dropout is reliable. The two safety factors $K_{saf\ op}$ and $K_{saf\ dr}$ are evidently larger than unity.

The dynamic performance of a discrete element can be estimated by two parameters, the operate time t_{op} and the dropout time t_{dr}, where t_{op} is the time required by the element to operate (change the output variable Y from its minimum to its maximum value) after the signal X_{imp} has been applied to the input t_{dr} is the time required for dropout of the element after complete removal and of the input signal.

In Fig. 0.2 we show another type (important in practice) of static characteristics of elements of continuous and discrete operation. In this case a change in the polarity of the input signal is accompanied by a corresponding change in the polarity of the output signal. Elements with such characteristics are therefore called reversible or polarized. The first term is normally applied to elements of continuous operation and the second term to elements of discrete operation. The elements whose characteristics are presented in Fig. 0.1 are not sensitive to the polarity of the input signal, responding in the same way to signals of either polarity therefore, if the characteristics of Fig. 0.1 are extended into the second quadrant, they will be symmetrical about the ordinate axis. Elements with static characteristics of this type are called nonreversible, unpolarized, or neutral.

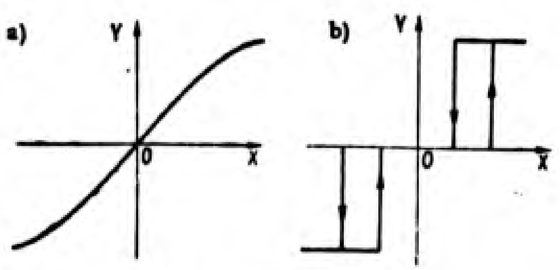


Fig. 0.2. Static characteristics of reversible (polarized) elements of continuous (a) and discrete (b) operation.

From the point of view of their physical nature, the elements of automatic systems can be electrical, mechanical, pneumatic, etc. The electrical elements, in turn, may be further divided into the following main groups: electrodynamic, electromechanical, ferromagnetic, dielectric, electronic, and semiconductor. In this book we are considering electromechanical, ferromagnetic, and dielectric elements of automatic systems.

From a historical point of view, the electromechanical elements (together with the electrodynamic elements) were the first elements of automatic systems. They date back to last century and served for a long time as the only elements of electrical automation.

To electromechanical elements belong all sorts of electromechanical relays and contactors, various converters of electrical signals and energy into mechanical displacements and forces (electromagnetic, magnetoelectric and similar systems, electromechanical clutches), induction-type and other converters of displacements and forces into electrical signals.

With the advent of electronics at the beginning of this century, the choice of elements for automatic and remote-control systems was broadened by electronic and ionic elements, thereby opening new possibilities for development of automation. In particular, they led to creation of the first high-speed digital computers, which were closely related to development of the concept of cybernetics.

Ferromagnetic elements are static devices, i.e., without moving parts. Among these elements we have first of all magnetic amplifiers and contactless magnetic elements of discrete operation. Although these elements have been known as long as electronic elements, they have come into extensive use only fairly recently—after the second world war, i.e., after the peak of vacuum-tube electronics development had been reached and the development of semiconductor electronics had just started. The merits of ferromagnetic elements are their unusual reliability and durability. Moreover, discrete magnetic elements are simple and cheap. With the ever increasing complexity of automatic systems (particularly, computers), the primary problem was to ensure reliability. Indeed, the more elements a system contains, the higher the probability of breakdown of one of these elements, i.e., the lower the reliability of the system as a whole as compared to the reliability of its individual components. It is precisely for this reason that electronic tubes, used in radio receivers, and electromechanical relays, used in the automatic protection of systems, were found to be unsuitable as basic elements of even small computers. The basic elements of present-day digital computers and control systems are magnetic elements of discrete operation. These elements are replacing the electromechanical relay also in simple automatic and remote control devices. By virtue of their reliability, magnetic amplifiers are being more and more utilized instead of electrodynamic, electronic and ionic amplifiers.

Dielectric elements have many similarities with ferromagnetic elements, yet their use is only just beginning.

Thus we shall be considering in this book the electrical elements that were historically the first, as well as the new elements that were invented only very recently. We are continuously witnessing the appearance of new elements of automation and remote control. Yet this does not mean that the "old" elements are in danger of being displaced by the newer elements. Although an increase in the number of types of various elements brings about a decrease in the relative importance of each of these types in the components produced, we have nevertheless a situation in which the steadily increasing demand for these components has the result that the production of any of these devices (such as electromechanical relays), far from being curtailed, is even expanded, as in other branches of our economy. The decrease in the relative importance of individual types of equipment leads to greater specialization, these types being used only in those combinations with other types in which all the merits of the particular element are exploited to the full, whereas its shortcomings are insignificant.

PART ONE

ELECTROMECHANICAL DEVICES

Chapter 1

NEUTRAL ELECTROMAGNETS

§ 1.1 Basic Concepts Relating to Electromagnets and Their Classification

Electromagnets are the most common converters of electrical signals into displacements and mechanical force. There exist also many other types of electromechanical converters, such as magnetoelectric, electrodynamic, induction, electrostatic, thermal, etc. All these elements are well known, since the corresponding electrical measuring instruments are based on them. Yet as independent elements of automatic devices they are much less common than electromagnets.

The only exception is the magnetoelectric transducer, which owing to the light weight of its moving system (loop) is faster than an electromagnet; for this reason it often replaces the electromagnet whenever a particularly fast electromechanical low-power transducer is required. Yet the principal use of these and other types of electromechanical transducers is to serve as component elements of electromechanical relays. For this reason they will be examined in Chapter 4 under the same heading as these relays.

Thus among all electromechanical transducers we shall consider separately electromagnets only.

Various types of neutral electromagnets are shown in Fig. 1.1. When an electrical input signal is applied to the coil of the electromagnet, the armature of the latter is attracted. The armature is mechanically linked to devices such as shutters, slide valves, etc. The task of the electromagnet is to operate such actuating devices.

The armature of an electromagnet is subjected to two resultant forces: the pull force F_p produced by the electromagnet itself, and the opposing force F_{opp}, which is the sum of all the forces counteracting the attraction of the armature to the electromagnet and mainly determined by the actuating device. In order that the application of a current to the coil of the electromagnet should cause the armature to move, it is necessary that the pull force be stronger than the opposing force. When the armature moves, these two forces vary. The dependence of the pull force on the position of the armature (with a specified input signal) is called the pull characteristic, whereas the dependence of the opposing force of the armature position is called the opposing characteristic.

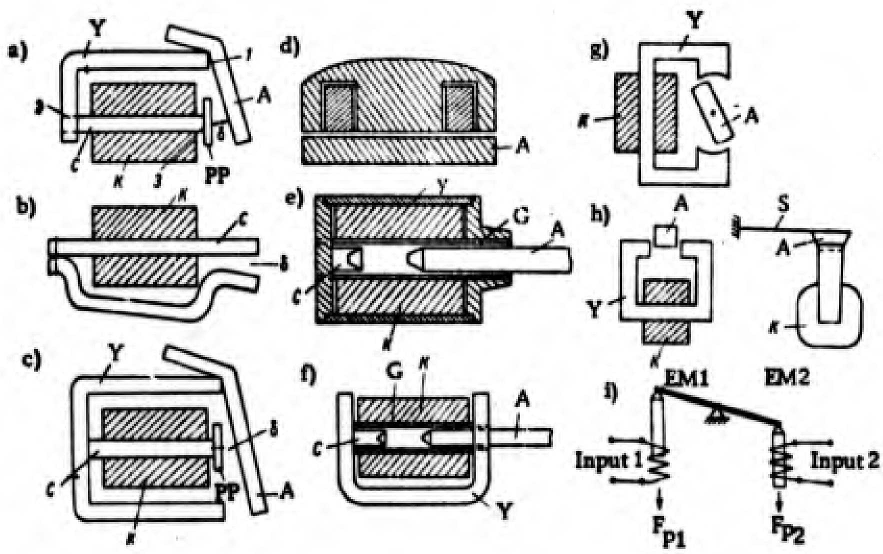


Fig. 1.1. Neutral electromagnets: a, b, c, d—bar (clapper) type (a—with pi-shaped magnetic circuit; b—same, but with flat armature-yoke; c—with three-legged magnetic circuit; d—with cylindrical magnetic circuit); e and f—plunger-type (pull-in armature); g and h—with transversely moving armature; i—balanced electromagnetic system.

A—armature; Y—yoke; C—core; PP—pole piece; S—spring; G—guide frame; K—coil.

In Fig. 1.2 we show examples of these characteristics. They are plotted here not as a function of the path traversed, but as a function of the air gap δ (see, for example, Fig. 1.1a), i.e., of the path still remaining. In order that the armature be completely attracted by the electromagnet, it is necessary to feed to its coil a current I such that the pull characteristic $F_p = f(\delta)$ should entirely lie above the opposing characteristic $F_{opp} = f(\delta)$. This condition is met by the characteristic 3 in Fig. 1.2.

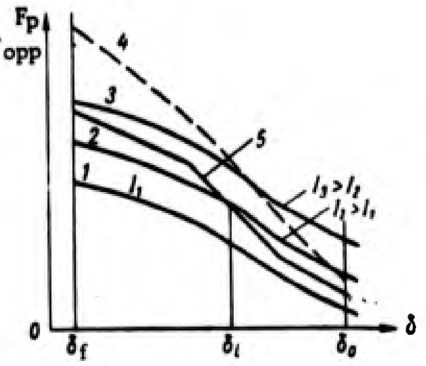


Fig. 1.2. Electromagnet characteristics.

1, 2, 3, 4—pull; 5—opposing.

With a coil current $I = I_1$ (characteristic 1), the armature does not move (for $\delta = \delta_0$). With $I = I_2$ the armature starts moving, but stops at half-way, when $\delta = \delta_i$. The pull and opposing characteristics are static characteristics of the forces acting in an electromagnet under steady-state conditions; they do not comprise the forces acting only during the motion of the armature and depending on its velocity and acceleration.

An electromagnet can serve as an element of continuous or discrete operation. In the first case, i.e., with continuous conversion of an electrical signal into motion, we have for each value of the input signal a particular position of the armature. In order to achieve such operation it is necessary that the opposing and pull characteristics be matched to a certain extent, i.e., it is necessary that when the coil current increases the point of intersection between the pull and the opposing characteristic should move in the same proportion to the left (i.e., towards smaller δ). This implies first of all that the opposing characteristic be steeper than the pull characteristic (as in the case of Fig. 1.2). Such a proportional transducer can be realized, for example, with the aid of an electromagnet equipped with a counteracting spring whose deformation increases with the input signal.

In the case of an electromagnetic element of discrete operation under steady-state conditions, the armature can occupy only two positions: Either the initial position (with $\delta = \delta_0$ in Fig. 1.2), when the input signal is below some threshold value, or the final position, when the input signal exceeds the threshold value. This discrete type of electromagnet operation corresponds to a pull characteristic which is steeper than the opposing characteristic (the dashed line 4 in Fig. 1.2).

Electromagnets are being mainly used in systems of discrete operation.

Let us now distinguish between types of electromagnets irrespective of their function as discrete or continuous elements.

With respect to applications, electromagnets can be grouped into portative, tractive, and special types.

Portative electromagnets have the simple purpose of holding ferromagnetic bodies. Among these are lifting-crane electromagnets, intended for the lifting of ferromagnetic objects, electromagnetic plates for holding workpieces on surface-grinding and other metal-cutting machines, and various electromagnetic holders and coupling tools. Portative electromagnets do not perform any work. They must only deliver a certain force, for which they are rated.

Tractive electromagnets have the purpose of moving various actuating devices, such as drops, shutters, track switches, etc. Tractive electromagnets are hence performing a certain amount of work, and they are rated to produce a certain force and displacement.

Tractive electromagnets have often also the function of portative electromagnets, i.e., they must move the actuating device and then hold it in the new position with a prescribed force. Of this type are the electromagnets in electromagnetic friction clutches (see § 3.2), and in relays and contactors (see Chapter 4).

Special electromagnets are used in electron optics, elementary particle accelerators, medical equipment, etc.

With regard to the coil current, we distinguish between dc electromagnets and ac electromagnets. The dc electromagnets, in turn, may be of the neutral and of polarized types. Neutral electromagnets are not sensitive to the polarity of the signal and thus respond in the same way to a signal of either polarity. In polarized electromagnets the direction of the force acting on the armature changes as a function of the signal polarity.

From the point of view of design we distinguish between the following types of electromagnets: a) bar (clapper) type (also called electromagnets with attracted external armature; Figs. 1.1a, 1.1b, 1.1c and 1.1d); b) plunger type (also called electromagnets with pull-in armature, or solenoidal electromagnets; Figs. 1.1e and 1.1f); c) with transversely moving armature (rotary electromagnets), the motion of the armature being perpendicular to the axis of the poles of the magnetic circuit (Figs. 1.1g and 1.1h).

In bar-type electromagnets the armature moves only a small distance (a few millimeters), yet owing to the small working air gap δ_0 these electromagnets are powerful and sensitive.

Plunger-type electromagnets are distinguished by a much longer stroke of the armature. They are also smaller and faster than bar-type electromagnets, though inferior to the latter in sensitivity.

Plunger-type electromagnets are designed in two versions: with a fixed core ("stop," Figs. 1.1e and 1.1f), and without a core (i.e., the coil axis is hollow throughout, as shown by the dashed lines in Fig. 1.1e). Fixed-core electromagnets develop greater strength, which sharply increases when the armature approaches the core. On the other hand, an electromagnet without a fixed core makes it possible to obtain a longer stroke of the armature (many decimeters) in view of the elongated coil. The pull force of a plunger-type electromagnet without fixed core varies little over most of the path traversed by the armature; it gradually decreases to zero when the armature enters completely into the solenoid.

Electromagnets with transversely moving armature (Figs. 1.1g and 1.1h) are less economical, but on the other hand they facilitate the obtaining of a pull characteristic of any form desired (increasing, horizontal, or decreasing, with any slope desired); this can be achieved by appropriate choice of the armature profile. In those cases in which it is required that the motion of the magnet be rotatory and not translatory, it is very convenient to use the electromagnet with transversely moving armature, shown in Fig. 1.1g.

With the aid of two electromagnets of any of the types described above it is possible to construct a balanced electromagnetic system, as shown schematically in Fig. 1.1i. The electromagnets of this system act on the actuating device in opposite directions. Therefore the direction of the resultant force and of the motion is determined by the electromagnet to which the stronger signal is applied.

An electromagnetic system with two counteracting inputs can be also obtained by using only one electromagnet which is equipped with two coils. The polarities of the signals applied to these coils are adjusted in such a way that the magnetizing forces of the coils act in opposite directions. The magnitude of the force acting on the armature will be determined in this case by the difference between the input signals (just as in a balanced system), and hence it will also be equal to zero when the input signals are equal. An electromagnet with such differentially connected coils is also called a differential electromagnet. It differs from a balanced electromagnetic system by the fact that in it the direction of the resultant force does not change when the sign of the input-signal difference changes (as in the case of a balanced system).

The design types considered above are basic, and they relate to neutral dc electromagnets. The peculiar features of ac electromagnets and polarized electromagnets will be considered below.

With regard to speed of operation, we have high-speed, normal, and delayed electromagnets. The operate time of a high-speed electromagnet in discrete operation is not more than 0.05 sec, and for a delayed electromagnet it is above 0.15 sec. The normal operate time (0.05 - 0.15 sec) is obtained in neutral electromagnets if no special measures are taken for changing the speed in either direction.

§ 1.2. Types of Electromagnets

The available types of electromagnets are shown in Fig. 1.1. Let us examine them in greater detail. An electromagnet consists of a magnetic circuit and a coil.

The magnetic circuit. In dc electromagnets it is made of one piece, mostly of low-carbon transformer steel (grades E, A, etc.) and of high-quality structural steel (grades 0, 1, 2, etc.). For high-sensitivity electromagnets one uses also the low-nickel permalloys 45NP and 50NP. The magnetic circuits of high-speed electromagnets are made of silicon steels (grades E11, E21, etc.). These steels have high electric resistance, which reduces the eddy currents that slow down the operation of electromagnets.

Electromagnets of minimum size can be obtained by using iron-cobalt alloys (Permendur grade) due to their high saturation induction (see Appendix AP-1).

In ac electromagnets the magnetic circuit is made of laminated plates that are stamped from sheet material 0.3-0.5 mm thick, this material being hot-rolled or cold-rolled steel or permalloy. Sometimes the magnetic circuit of dc electromagnets is also laminated, in order to remove the eddy currents during the transient processes of operation and drop-out. This increases the speed of the electromagnet (see § 1.6) and its sensitivity (by preventing signal power dissipation due to eddy currents). Moreover, this might be useful for component standardization, i.e., to manufacture both ac and dc electromagnets on the basis of the same magnetic system.

Small ac electromagnets, on the other hand, are sometimes made (for reasons of simplicity) of compact material. The parameters of the principal magnetically soft materials used for magnetic circuits are presented in Appendix AP-1.

A compact (one-piece) magnetic circuit consists of a core on which the coil, yoke and moving armature are placed.

In a laminar magnetic circuit the yoke and the core are joined (pressed) together. Yet in order to be able to wind the coil, it is sometimes necessary that laminar magnetic circuits be also made of several parts.

A bar-type electromagnet with a compact pi-shaped or three-legged magnetic circuit has an L-shaped (Fig. 1.1a) or pi-shaped (Fig. 1.1c) yoke. The latter is made of strip material of rectangular cross-section. The core is made of cylindrical material, since the coil to be wound on it should preferably have circular cross-section (see below). In order to reduce the resistance to the magnetic flux, the core is dead-ended to the yoke, either by fitting it through pressing or by unrivetting the end of the core.

In order to reduce the magnetic resistance of the working gap δ, the core is equipped with a pole piece, normally fixed with a screw. But sometimes the pole piece is turned together with the core. In this case the core will be of replaceable type, fixed to the yoke on a screw thread, in order to be able to wind the coil.

The armature of a bar-type electromagnet is made (just as the yoke) of sheet material. The armature rotates on a pivot (Figs. 1.1a and 1.1c), and in some cases on a shaft (which is more complicated). Figure 1.1b shows a version of a bar-type electromagnet with a flat armature-yoke designed in the form of a single, trough-shaped, component. In the case of a bar-type electromagnet with a cylindrical magnetic circuit (Fig. 1.1d) the armature is disk shaped.

Plunger-type electromagnets with a compact magnetic circuit have a tubular (Fig. 1.1e) or U-shaped (Fig. 1.1f) yoke. The armature (plunger) is circular. The guide frame for the armature motion is made of brass.

In the case of a laminar magnetic circuit the yoke is made of pi-shaped plates; the core is also laminar and has rectangular cross-section. Hence the coil is rectangular, too.

Electromagnets with transversely moving armature have a similar design.

The coil. We have coils with bobbins and coils without bobbins. With regard to shape, we have circular (in most cases) and rectangular coils.

A bobbined coil consists of a bobbin and a coil with leads. The rating plate fixed on the coil lists the principal coil parameters (voltage, number of turns, wire diameter, and coil resistance). The coil of one bobbin may consist of several sections, connected in series or superposed. The bobbins are made of plastic or porcelain (for high voltages), or they are metallic, pasted, or composite.

Bobbins made of pressed plastic and porcelain are shown in Fig. 1.3a. Plastic bobbins are made of Tenazit (a mixture of sawdust and bakelite resin). These bobbins are manufactured only under mass production conditions (when the expenses on the pressing mold are worth while).

Figure 1.3b shows a composite bobbin. It consists of parts stamped from micarta or similar materials. Figure 1.3c shows another type of composite bobbin, consisting of two disks 1 (normally of micarta) and a spool 2 of the same material.

The ends 3 of this spool are hot rolled (under a die). Sometimes the disks are pasted to the spool by means of linen strips. Such bobbins are called pasted bobbins.

In metallic bobbins the spools and the disks are made of brass. They are connected by rolling and brazing. Eddy currents are eliminated by making a longitudinal cut in the spool.

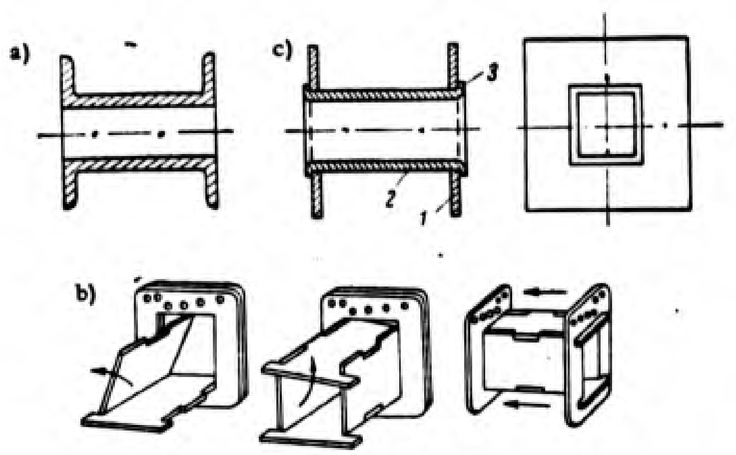


Fig. 1.3. Coil bobbins: a—pressed plastic; b and c—composite.

1—disk; 2—spool; 3—rolled end of spool.

The coil is wound on the bobbin on a winding machine.

The wire used for the windings is copper wire. For adequate mechanical strength it is recommended to use wire of diameter not below 0.07 mm, yet in high-sensitivity electromagnets one sometimes uses also thinner wire. In the case of wires of more than 10 mm^2 cross-sectional area, one uses leads of rectangular cross-section. The type most frequently used is enameled wire (PEL grade, which is an enameled varnish-fast wire, PET grade, which has heat stability, and PEV, which is a high-strength grade). This type is the cheapest and it has the thinnest insulation. Yet in the case of wires of large cross-section the mechanical strength of the enamel insulation greatly diminishes and therefore in the case of wires of more than 1 mm diameter one uses silk insulation (PShO and PShD grades, which are wires with single and double silk insulation), or one uses a wire that is cheaper but of much poorer quality—a wire with cotton insulation (PBD, which is a grade with double cotton insulation). If the insulation must meet higher demands, one uses wire with combined insulation (PELShO and PELShD grades, which are varnish-fast enameled grades with single and double silk insulation).

The permissible heating temperature is 90°C for silk and cotton insulation (Class 0), 105°C for enamel insulation (Class A), and 120°C for combined insulation.

By impregnating the silk and cotton insulation with organic varnish or oil it is possible to increase the permissible heating temperature to 105°C, whereas by compounding it can be increased to 120°C. Compounding signifies impregnation with a special material (the compound is a mixture of bitumina and vegetable oils, etc.) at a temperature of up to 150°C and a pressure of about 7 atm.

Greatest heat stability is possessed by glass insulation. Wire with glass-fiber insulation is impregnated with organic silicon varnish. This raises the permissible heating temperature to 180°C.

Coils intended for very high currents are made of a copper bushbar of rectangular cross-section. By winding such a bar in the form of a spiral (as shown in Fig. 1.4a), we obtain a helical coil. If the bar is wound in the form of a disk (as in the case of magnetic tape on a tape-recorder reel), we obtain a disk coil. The turns are insulated from each other by strip insulation, which is wound simultaneously with the bar.

A bobbinless coil is simpler than a bobbined coil. Moreover, the absence of a bobbin makes it possible to make fuller use of the window aperture under the coil. Here

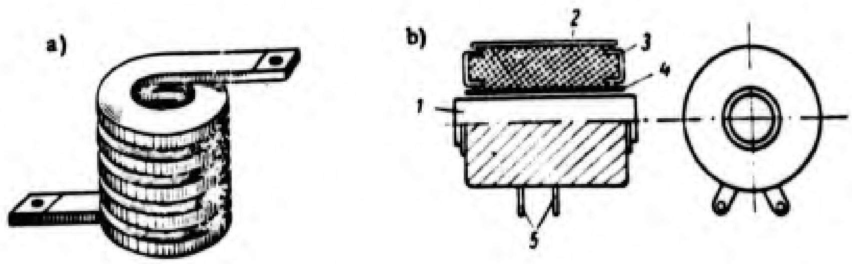


Fig. 1.4. Helical (a) and bobbinless (b) coils.

1—core; 2—rim; 3—linen strip; 4—insulation slug; 5—coil leads.

the coil is wound directly on the core of the magnetic circuit or on a form, whence it is transferred to the core. After winding the coil, its mechanical strength is increased in most cases by means of a rim, i.e., by winding around it a strip (normally of taffeta), followed by compounding. In Fig. 1.4b we show a model of such a bobbinless coil, wound on a core.

The coil leads are made either from the same wire or from thicker wire. They are enclosed in linoxyn or vinyl-chloride sheaths. One also uses leads that are in the form of stiff brass strips (flag-shaped or angled).

§ 1.3. The Pull Force of Electromagnets

When a current is fed to the coil of an electromagnet, a pull force begins to act on the armature. This force can be interpreted as being the result of the attraction exerted by the tubes of magnetic induction produced by the coil of the electromagnet. The movement of the armature is brought about by this force.

Let us derive the formula for the electromagnet pull force F_p. For definiteness we shall consider a bar-type electromagnet (Fig. 1.1a). The following simplifying assumptions are introduced:

1) The coil current remains constant during the operation of the electromagnet.

2) The magnetic circuit is not saturated (its magnetic reluctance is constant, i.e., it does not depend on the magnitude of the magnetic flux).

3) There is no spread of the magnetic flux beyond the edge of the working gap (i.e., all the flux passes through the working gap).

From theoretical electrical engineering we know that the force produced in a magnetic field is equal to the derivative (with respect to the coordinate) of the field energy acted upon by this force, i.e.,

$$F_p = \left(\frac{\partial W_M}{\partial X}\right)_{i=\text{const}} = -\left(\frac{\partial W_M}{\partial \delta}\right)_{i=\text{const}}, \tag{1.1}$$

where W_M is the energy of the magnetic field and X is displacement of the armature; it is evident that $dX = -d\delta$ (when X increases the gap δ decreases).

The energy of the magnetic field produced by the coil current is

$$W_M = \frac{1}{2} i\Psi,$$

where i is the current and Ψ is the total flux.

Since there is no leakage, we have

$$\Psi = W\Phi,$$

where W is the number of turns of the coil, $\Phi = F/R_M = FG_M$ is the magnetic flux, $F = iW$ is the magnetizing force, and R_M and G_M are the magnetic reluctance and conductance of the magnetic circuit.

Thus we have

$$W_M = \frac{1}{2} i\Psi = \frac{1}{2} iW \frac{F}{R_M} = \frac{1}{2} \cdot \frac{F^2}{R_M},$$

where only the magnetic reluctance R_M varies when the electromagnet operates. Hence

$$F_p = -\left(\frac{\partial W_M}{\partial \delta}\right)_{i=\text{const}} = \frac{F^2}{2R_M^2} \cdot \frac{dR_M}{d\delta} = \frac{\Phi^2}{2} \cdot \frac{dR_M}{d\delta}. \tag{1.2}$$

The reluctance R_M consists of the reluctances of the individual portions of the magnetic circuit: the working gap, the ferromagnetic parts of the magnetic circuit (the core, yoke and armature), and the small air gaps at the junction of these parts—the so-called idle gaps. If we assume that a displacement of the armature is accompanied by a variation of the reluctance of the working gap only (as a result of a variation in its length), then

$$\frac{dR_w}{d\delta} = \frac{dR_W}{d\delta}, \tag{1.3}$$

where R_W is the reluctance of the working gap.

This assertion is true if (as we stipulated above) the magnetic system is unsaturated, and hence the magnetic reluctance of its parts does not vary with increasing flux (decreasing δ) when the electromagnet operates. Moreover, in the case of a bar electromagnet we are neglecting a certain dependence of the idle air gap between the yoke and the moving armature on δ.

By substituting (1.3) into formula (1.2) we obtain

$$F_P = \frac{\Phi^2}{2} \cdot \frac{dR_W}{d\delta}.$$

With the substitution $R_W = 1/G_W$ one can rewrite this formula as follows:

$$F_P = -\frac{\Phi^2}{2G_W^2} \cdot \frac{dG_W}{d\delta} = -\frac{F^2}{2} \cdot \frac{dG_W}{d\delta}. \tag{1.4}$$

Now let us remove the restrictions introduced at the very beginning. First of all, if we take into account the losses of the magnetizing force when the magnetic flux passes through the idle gaps and the ferromagnetic material, i.e., the ferromagnetic parts of the magnetic system, then the ratio Φ/G_W in the derivation of (1.4) will no longer constitute the entire magnetizing force F of the coil (as assumed in formula (1.4)), but only a portion of this force, spent on carrying the flux through the working gap. When the losses of the magnetizing force are taken into account, it is therefore necessary to rewrite formula (1.4) as follows:

$$F_P = -\frac{F_W^2}{2} \cdot \frac{dG_W}{d\delta}. \tag{1.5}$$

Here F_W is the effective magnetizing force, i.e., the magnetizing force contained in the working gap. The total magnetizing force produced by the coil is

$$F = iW = F_W + F_l, \tag{1.6}$$

where F_l is the portion of the magnetizing force spent on carrying the flux through the parts of the magnetic circuit other than the working gap, thus representing the losses in magnetizing force.

Formula (1.5) is final and will be used by us below for the designing of electromagnets. Although no allowance was made, in the derivation of this formula, for the coil current variations during the operation of the electromagnet, and the leakage of the magnetic flux and the saturation of the magnetic circuit were also not taken into account, a more rigorous derivation that takes into account all these complications yields practically the same result for most types of actual electromagnets [1, 2, 3].

With the substitution $G_w = 1/R_w$, formula (1.5) can be also rewritten as follows:

$$F_p = \frac{\Phi_w^2}{2} \cdot \frac{dR_w}{d\delta}. \tag{1.7}$$

Here $\Phi_w = F_w G_w$ is the magnetic flux in the working gap; the subscript "w" below Φ has been introduced in order to account for the leakage in case the flux in the working gap is smaller than the total flux Φ produced by the coil.

In the particular case that the working gap is formed by parallel planes and is uniform (i.e., when its distortion at the edges can be neglected), we have

$$R_w = \frac{\delta}{\mu_0 S_w}, \tag{1.8}$$

where S_w is the cross-sectional area of the working gap, and μ_0 is the premeability of air.

By substituting (1.8) into (1.7) we obtain the well-known Maxwell formula

$$F_p = \frac{\Phi_w^2}{2\mu_0 S_w} = \frac{B_w^2 S_w}{2\mu_0}, \tag{1.9}$$

where $B_w = \Phi_w/S_w$ is the magnetic induction in the working gap.

This formula is often used in approximate calculations of electromagnetic systems.

There are types of electromagnets in which the pull force acting on the armature is produced not only by the flux in the working gap, but also by the leakage flux. An example of such an electromagnet is the bar-type electromagnet with flat armature-yoke, shown in Fig. 1.1b. In this case the formula for the pull force assumes the form:

$$F_p = -\left(\frac{F_w^2}{2} \cdot \frac{dG_w}{d\delta} + \int_0^l \frac{F_{lk}^2}{2} \cdot \frac{dg_{lk}}{d\delta} dl\right), \qquad (1.10)$$

where F_{lk} is the drop in magnetizing force over the path of the leakage flux (between the core and the yoke in Fig. 1.1c), g_{lk} is the specific conductance for the leakage flux (the conductance per unit length of the magnetic circuit, i.e., unit width of the leakage flux), and l is the length of the portion of the magnetic circuit along which there exists a leakage flux.

Formula (1.10) is the most general formula, whereas (1.5) is a particular case of (1.10), namely the case when the conductance for the leakage current does not depend on the position of the armature ($dg_{lk}/d\delta = 0$).

In the derivation of all the formulas we are using (both here and in the following) the international system of units SI, according to which

$$\mu_0 = 0.4\pi \cdot 10^{-6} \text{ ohm} \cdot \text{sec/m}.$$

The physical quantities utilized above have the following dimensionalities: G_M (ohm·sec); Φ (v·sec), the unit of measurement is the weber (wb); B (v·sec/m^2), the unit of measurement is the tesla (tl); the magnetizing force F (amp); the mechanical force F_p (kg·m/sec^2 = j/m), the unit of measurement is the newton (n).

In Appendix AP-2 we present a conversion table for the units of the SI system and of the other systems met in the literature.

§ 1.4. Design of the Magnetic Circuit of Electromagnets

Formula (1.5) and the more general formula (1.10) establish a relationship between the pull force of the electromagnet and the magnetizing force of the coil. Yet in order to be able to utilize this formula it is necessary to determine the conductance G of the air gaps and the losses F_l of the magnetizing force, which connect (by formula (1.6)) the effective magnetizing force F_w and the total magnetizing force F. These design problems of the magnetic circuit of an electromagnet are examined in the present section.

At first let us determine the conductances of the air gaps in the magnetic circuit. In addition to the working air gap, an electromagnet contains also idle air gaps at the junction of the individual parts of the magnetic circuit. For example, the magnetic circuit of Fig. 1.1a has three idle gaps: 1—between the armature and the yoke, 2—between the yoke and the core, and 3—between the core and the pole piece (if the latter is dismountable). The conductance of the idle gaps must be known in order to be able to determine the losses of the magnetizing force in these gaps.

For the determination of the conductance of an air gap it is irrelevant whether this gap is a working or an idle gap, but we must know whether the gap lies between equipotential or between non-equipotential surfaces. In the first case all the points of each of these surfaces have the same magnetic potential, whereas in the second case the potential varies along the surfaces.

In the magnetic circuit depicted in Fig. 1.1a the working gap and all three idle gaps 1, 2 and 3 lie between equipotential surfaces. On the other hand, the gap between the core and the yoke wall (along the coil), through which the leakage flux passes, is formed by non-equipotential surfaces: the difference of the magnetic potentials between them increases in moving across this gap from the base of the core towards the pole piece. The gap between the core and the flat yoke-armature in Fig. 1.1b and the gap along the armature in the plunger-type electromagnet of Figs. 1.1e and 1.1f are also bounded by non-equipotential surfaces.

A. Determination of the Magnetic Conductance of the Air Gap Between Equipotential Surfaces

The magnetic conductance of the air gap between equipotential surfaces can be determined by any of the following methods:

1) The analytic method.

2) Empirical formulas.

3) The method of splitting the field pattern into simple (typical) shapes.

4) The method of construction of the actual field pattern.

The analytic method is based on the use of the formulas of field theory; it can be used only for gaps of simplest shape, and only in those cases when the field distortion at the edges can be neglected.

As an example let us determine the conductance of the air gap between planes that are at an angle (Fig. 1.5). Such a shape is possessed by the working gap of the bar-type electromagnet depicted in Fig. 1.1a. Let us assume that the tubes of magnetic flux in the gap travel in circular arcs. The conductance of an elementary path, shown in the figure, is

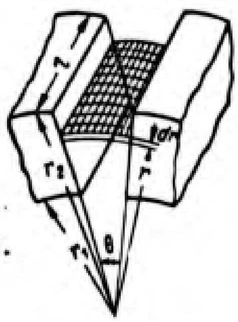


Fig. 1.5. Air gap between mutually inclined planes.

$$dG = \mu_0 \frac{l\,dr}{\Theta r}.$$

By integrating this expression in the interval from r_1 to r_2, we obtain the sought-for formula for the total conductance between the entire planes:

$$G = \int_{r_1}^{r_2} dG = \frac{\mu_0 l}{\Theta} \int_{r_1}^{r_2} \frac{dr}{r} = \frac{\mu_0 l}{\Theta} \ln \frac{r_2}{r_1}. \tag{1.11}$$

Formula (1.11) holds only for an angle $\Theta < 5°$. With $\Theta > 5°$ it yields a large error in view of the fact that the edge effect is not taken into account.

For the gap between parallel planes of area S we previously used the analytically derived formula

$$G = \mu_0 \frac{S}{\delta}.$$

For various gap shapes, empirical formulas were proposed for the magnetic conductance. Sometimes these formulas are simply corrections (to the analytic formulas) that account for the edge effect. For example, for the gap between the inclined surfaces of Fig. 1.5, with $5° < \theta < 12°$, it is recommended to introduce into formula (1.11) a correction factor 1.2-1.3 that accounts for the edge flux passing outside the cross-sectional area $S = l(r_2 - r_1)$. Numerous empirical data permitting the determination of the conductance of gaps can be found in [3].

The method involving the splitting of the field pattern into simple shapes is as follows. The field in the gap is represented in simplified fashion as a set of tubes of magnetic induction of simplest shape (parallelepipeds, cylinders, portions of cylinders and spheres). The gap conductance G is expressed in the form of the sum of the conductances G_i of the individual tubes, i.e.,

$$G = \sum_i G_i.$$

The conductance of each tube is specified in terms of its mean length $\delta_{m\,i}$ and mean cross-section $S_{m\,i}$:

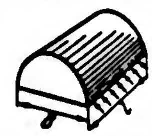
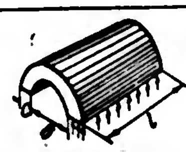
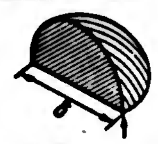
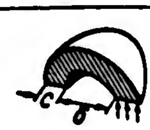
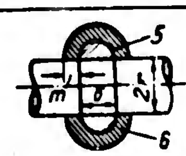
$$G_i = \mu_0 \frac{S_{m\,i}}{\delta_{m\,i}}. \tag{1.12}$$

In Table 1.1 we present the conductance formulas for various simple shapes obtained in this way. The arrows indicate the direction of assumed entry of the flux in each case. For example, in a semi-cylinder the flux enters along the edge (across the line), whereas in a spherical quadrant it enters at an angle. The conductance of the quarter cylinder and sphere (not shown in the table) are twice as large as the values (listed in the table) of the conductances of the halves of these figures.

In Fig. 1.6 we give an example of splitting the field in the gap into simple shapes. The entire field is divided into elementary induction tubes in the form of a rectangular parallelepiped 1, four cylindrical quadrants 2, four quarters of a hollow cylinder 3, four eighths of a sphere 4, and four eighths of a hollow sphere 5. In the construction one assigns the quantity m, which (on the basis of experiment) is taken as equal to (1 - 2) δ.

Table 1.1

Conductance Formulas For Gaps of Simplest Shape

Gap shape	Schematic	Conductance formula
Semi-cylinder		$G_1 = \mu_0 \cdot 0.26l$
Semi-annulus (half of hollow cylinder)		$G_2 = \mu_0 \dfrac{0.64l}{\frac{\delta}{C}+1}$
Spherical quadrant		$G_3 = \mu_0 \cdot 0.077\delta$
Quadrant of spherical shell		$G_4 = \mu_0 \cdot 0.25C$
Body of revolution		$G_5 = \mu_0 \cdot 1.63r$; $G_6 = \mu_0 \dfrac{4r}{\frac{\delta}{m}+1}$

Since the expressions for the conductances G_i of the individual shapes are given in general algebraic form, this method yields an expression for the total conductance of the gap as a function of δ that is also of general form.

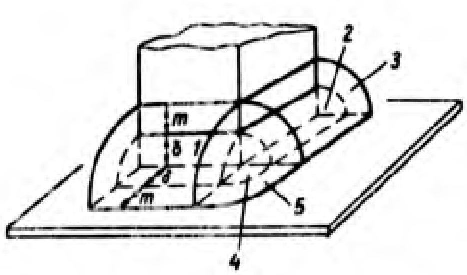


Fig. 1.6. Example of splitting the field into simple shapes.

The method of determination of the conductance of the air gap by constructing the actual field pattern is based on the exact construction of the induction tubes and of the equipotential surfaces. In Fig. 1.7 we present an example of the field pattern in the gap between the armature and the stop of a plunger-type electromagnet.

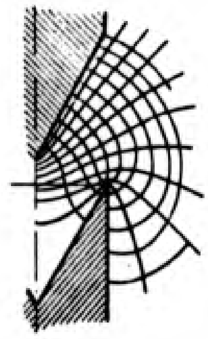


Fig. 1.7. Field pattern in the working gap of a plunger-type electromagnet.

In designing the electromagnet, one begins with the more uniform part of the field, constructing simultaneously the induction lines and the equipotential lines. It is necessary, first, that these lines be mutually perpendicular and, secondly, that the elementary curvilinear sections into which the field is divided by these lines should be similar, i.e., they must have the same length-to-width ratio (in the case of a plane field pattern) or the same cross-sectional area (S_i) to length (δ_i) ratio (in the general case of a three-dimensional field pattern). The latter condition signifies that all these elements have the same conductance $G_i = \mu_0 S_i/\delta_i$. Hence the conductance of the entire gap can be expressed by the simple formula

$$G = \frac{m}{n} G_i, \tag{1.13}$$

where m is the number of induction tubes, n is the number of gaps between equipotential surfaces, and G_i is the conductance of an individual element of the field.

The quantity G_i is specified, for an element that lies in the most uniform part of the field, in terms of the mean cross-section and the mean length, i.e., by formula (1.12).

This method of determination of G is used in the case of more complicated gaps, when other methods are inapplicable. In addition to being more laborious, this method has a major shortcoming, since it yields directly a numerical value of G for given numerical values of the gap dimensions. If, therefore, we must know, for example, the value of the conductance for several values of the air-gap length δ, we must repeat the construction of the field pattern for each of the values of δ. More details about this method can be found in [3].

In Appendix AP-3 we present formulas for the conductances of the more common air gaps.

B. Determination of the Reduced Magnetic Conductance of the Air Gap Between Non-Equipotential Surfaces

Let us consider the air gap between the core and the yoke wall in a bar-type electromagnet (Fig. 1.8a).

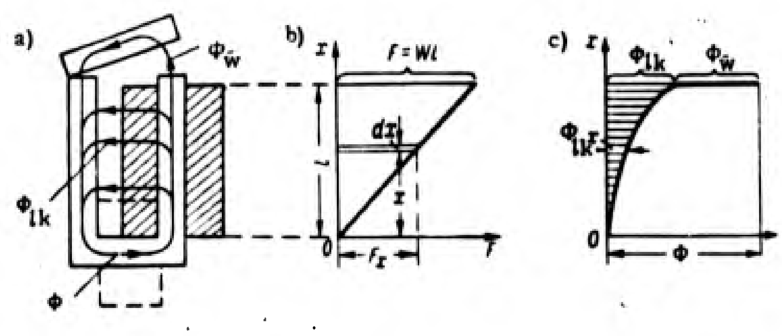


Fig. 1.8. Magnetic flux distribution in a bar-type electromagnet.

The flux passing through the working gap is not the entire flux Φ produced by the coil, but only the effective portion of the flux Φ_w; the other portion—the leakage flux Φ_{lk}—goes through the yoke, without passing the armature.

Let us determine the leakage flux Φ_{lk}. The gap through which this flux passes is formed by non-equipotential surfaces: the magnetizing force is distributed along the core, varying from 0 to its full value F in the working gap, as shown in Fig. 1.8b. In Fig. 1.8c we plotted the leakage flux Φ_{lk} as a function of the length of the core.

The leakage flux emanating from a core element dx at a height x is equal to

$$d\Phi_{lk} = F_x \, dG_{lk}$$

Here $F_x = Fx/l$ is the magnetizing force of the coil at the level x, assumed to be constant in the section dx; F_x specifies the magnetic potential difference between the core and the yoke at the height x, provided that the drop in the magnetizing force in the ferromagnetic material itself is neglected; dG_{lk} is the conductance of an elementary leakage gap of height dx.

The quantity dG_{lk} can be defined as

$$dG_{lk} = g\,dx,$$

where g is the specific magnetic conductance of the path traversed by the leakage flux, i.e., the conductance per unit height of the core calculated on the basis of the geometrical dimensions of the gap, the surfaces bounding the gap being regarded as equipotential.

The leakage flux at a height x is specified by the formula

$$\Phi_{lk.x} = \int_0^x d\Phi_{lk} = \int_0^x F_x\,dG_{lk} =$$
$$= \int_0^x F\frac{x}{l}\,g\,dx = \frac{F}{l}g\int_0^x x\,dx = Fg\frac{x^2}{2l}.$$

This formula has been plotted by the graph $\Phi_{yx} = f(x)$ in Fig. 1.8c. The expression for the total leakage flux can be obtained by setting $x = l$:

$$\Phi_{lk} = F\frac{gl}{2} = \frac{FG_{lk}}{2}, \tag{1.14}$$

where $G_{lk} = gl$ is the magnetic conductance of the leakage gap, determined from its geometrical dimensions, i.e., by assuming its bounding surfaces to be equipotential.

It is more convenient to write formula (1.14) as follows:

$$\Phi_{lk} = FG_{lk\,red}, \tag{1.15}$$

where $G_{lk\,red}$ is the so-called reduced leakage conductance.

In the magnetic circuit under consideration we have

$$G_{lk\,red} = \frac{G_{lk}}{2}, \tag{1.16}$$

i.e., half the value obtained in the case of equipotential surfaces. The latter case can occur, for example, in the same bar-type system if the coil is fixed along the base of the yoke instead of on the core (this case is represented in Fig. 1.8a by the dashed line).

In deriving formula (1.16) we neglected the drop in magnetizing force in the ferromagnetic material. This is permissible in the case of an unsaturated magnetic circuit, when the permeability of the ferromagnetic material, and hence also the magnetic conductance, are large. In a saturated bar-type system, the value of $G_{lk\,red}$ decreases considerably when the drop in magnetizing force in the ferromagnetic material is taken into account. Calculations show that in this case

$$G_{lk\,red} = \frac{1}{3} G_{lk} \tag{1.16a}$$

In a similar way one can determine the reduced conductance for the leakage flux, for example, the leakage flux between the armature and the yoke or between the stop and the yoke or between the stop and the yoke in a plunger-type electromagnet (Fig. 1.1e).

C. The Leakage Coefficient

In designing a magnetic circuit we must know the flux distribution in the entire magnetic circuit. Since the pull force of the electromagnet is determined by the flux in the working gap (Φ_w), we must as a rule be able to determine the magnitude of the flux in all the other sections of the magnetic circuit on the basis of the known value of Φ_w. For this purpose one uses the leakage coefficient σ_{lk}. Thus, in designing a bar-type system with an L-shaped yoke (Fig. 1.8a), we introduce the leakage coefficient

$$\sigma_{lk} = \frac{\Phi}{\Phi_w}.$$

If we know this coefficient we can find by means of the known flux Φ_w the total flux passing through the base of the core, i.e., $\Phi = \sigma_{lk}\Phi_w$. The quantity σ_{lk} is evidently always larger than unity.

Let us derive the formula for σ_{lk}:

$$\sigma_{lk} = \frac{\Phi}{\Phi_w} = \frac{\Phi_w + \Phi_{lk}}{\Phi_w}.$$

If the losses of the magnetizing force in spreading the flux through the ferromagnetic material are not taken into account, whereas the drop in magnetizing force in the idle gap 1 between the armature and the yoke is taken into account (Fig. 1.1a), then Φ_w will be expressed in terms of the total magnetizing force F as follows:

$$\Phi_w = F \frac{G_w G_{id\,1}}{G_w + G_{id\,1}},$$

where the co-factor of F is the total conductance of series-connected working (G_w) and idle ($G_{id\,1}$) gaps.

By expressing Φ_{lk} likewise in terms of F and using the reduced leakage conductance, we obtain by virtue of formula (1.15) the sought-for expression:

$$\sigma_{lk} = \frac{\Phi_w + \Phi_{lk}}{\Phi_w} = 1 + \frac{\Phi_{lk}}{\Phi_w} = 1 + G_{lk\,red} \frac{G_w + G_{id\,1}}{G_w G_{id\,1}} = \tag{1.17}$$
$$= 1 + G_{lk\,red}\left(\frac{1}{G_w} + \frac{1}{G_{id\,1}}\right).$$

In the designing of more complex magnetic circuits one utilizes several leakage coefficients. For example, in a bar-type electromagnet with a pi-shaped yoke (Fig. 1.1c), the determination of the fluxes in the two walls of the yoke requires two leakage coefficients. The same is true for a plunger-type system with a stop.

Yet in all the cases the expressions for the leakage coefficients σ_{lk} are obtained in the same manner (as was shown above), i.e., in terms of the gap conductances, while neglecting the losses of the magnetizing force in the ferromagnetic material.

D. Determination of the Losses of the Magnetizing Force in the Ferromagnetic Material and in the Idle Gaps

In order to determine the total magnetizing force F to be produced by the coil, we must know according to formula (1.6) not only the drop in the magnetizing force in the working gap (F_w), but also the losses of the magnetizing force in the other parts of the magnetic circuit (F_l). These losses are given by

$$F_l = F_{id} + F_{\Phi M},\tag{1.18}$$

where F_{id} is the drop in magnetizing force in the idle gaps, and $F_{\Phi M}$ is the drop in magnetizing force in the ferromagnetic portions of the magnetic circuit (in the ferromagnetic material).

Let us examine how to determine these two components of F_l. We shall assume that the fluxes in all the portions of the magnetic circuit are known (having been determined with the aid of the leakage coefficients in terms of the known value of Φ_w, specified by the required pull force).

The drop in magnetizing force in one (the i-th) idle gap is specified by the formula

$$F_{id.i} = \frac{\Phi_i}{G_{id.i}}.$$

When the gaps are connected in series (for example, in a bar-type electromagnet with an L-shaped yoke (Fig. 1.1a), where we have three idle gaps) the total losses in all the idle gaps are

$$F_{id} = \sum_{i=1}^{n} F_{id.i}.$$

In determining $G_{id\,i}$, the magnitude of the idle gap at the junction of the fixed components is taken equal to 0.05 mm. If the components have an anti-corrosion coating (for example, zinc), we must add to this value also the thickness of two non-magnetic layers, equal to 0.08 mm [2].

The drop $F_{\Phi M}$ of the magnetizing force in the ferromagnetic portions of the magnetic circuit is likewise obtained by adding together the losses of this force ($F_{\Phi M\,i}$) in the individual sections of the magnetic circuit.

In the magnetic circuit of the electromagnet depicted in Fig. 1.1a, for example, we have four sections: the armature, the core, the yoke base and the yoke wall. (The yoke is divided into two parts, since the fluxes passing through them are not equal).

The drop in magnetizing force in a section along which the flux does not vary (the armature and the yoke base in Fig. 1.1a) can be expressed as follows:

$$F_{\text{фм}.i} = H_i l_i,$$

where H_i is the magnetic field strength, and l_i is the length of the section.

H_i is determined with the aid of the magnetization characteristic of the ferromagnetic material, in terms of the induction:

$$B_i = \frac{\Phi_i}{S_i},$$

where Φ_i is the magnetic flux, and S_i is the cross-sectional area of this portion.

The magnetization curves of magnetically soft materials are presented in Appendix AP-1.

The drop in magnetizing force in a section along which the flux and hence also the magnetizing force vary in magnitude (the core and the yoke walls in Fig. 1.1a) can be expressed in terms of the mean value of the magnetizing force:

$$F_{\text{фм}.i} = H_{im} l_i,$$

where

$$H_{im} = \frac{H_{ii} + H_{if}}{2},$$

H_{ii} and H_{if} being the values of the magnetic field strength at the beginning and at the end of the section. H_{ii} and H_{if} are also found with the aid of the magnetization curves from the values of the induction at the beginning and at the end of the section; the latter two are expressed in terms of the flux:

$$B_{ii} = \frac{\Phi_{ii}}{S_i}; \qquad B_{if} = \frac{\Phi_{if}}{S_i}.$$

If all the sections of the magnetic circuit are connected in series, forming a single circuit (as for example in the electromagnet shown in Fig. 1.1a), then the total losses of the magnetizing force in the ferromagnetic part of the magnetic circuit are obtained in the form of the sum of the losses in the individual sections, i.e.,

$$F_{\text{фм}} = \sum_{i=1}^{n} F_{\text{фм}.i}.$$

In the case of a bifurcated magnetic circuit, as for example in the bar-type electromagnet with pi-shaped yoke of Fig. 1.1c, the losses of the magnetizing force $F_l = F_{id} +$ $+ F_{\Phi M}$ are determined for each flux circuit separately. The total magnetizing force of the coil is

$$F = F_{w1} + F_{l1} = F_{w1} + F_{w2} + F_{l2},$$

where F_{w1} and F_{w2} are the magnetizing force in the middle and in the outer working gaps; F_{l1} and F_{l2} are the losses of the magnetizing force in the circuit formed by the core and one wall of the yoke, and in the circuit formed by the core and the other wall of the yoke.

E. Construction of the Magnetization Curves and of the Pull Characteristic of an Electromagnet

Owing to the saturation of the ferromagnetic portions of the magnetic circuit, the working flux Φ_w, which determines the pull force, and the magnetizing force are connected by a nonlinear relationship.

In view of the fact that for the designing of electromagnets we must know this dependence, which constitutes the magnetization characteristic of the magnetic circuit, it will be constructed by us graphically. Yet in design calculations it is more convenient to use instead of the characteristic $\Phi_w = f(F)$, i.e., of the dependence of Φ_w on the total magnetizing force F, the characteristic $\Phi_w = f(F_l)$, i.e., the dependence of Φ_w on the losses of the magnetizing force, which constitute a part of F. The characteristic $\Phi_w = f(F)$ can be easily constructed from the characteristic $\Phi_w = f(F_l)$.

Let us examine the construction of the magnetization curve $\Phi_w = f(F_l)$ (Fig. 1.9). The curve is deformed when the magnitude of the working gap δ varies. For a thorough knowledge of the dependence $\Phi_w = f(F_l)$ it is therefore necessary to construct several curves: for the initial position of the armature, for several intermediate positions, and for the final position. In each case one assigns several values of Φ_w and one determines the corresponding values of F_l. For this purpose one finds at first the leakage coefficient σ_{lk} for each value of the working gap; σ_{lk} decreases with decreasing δ from its initial to its final value, i.e., from 2-3 to 1.1 and below. Then one assigns successively

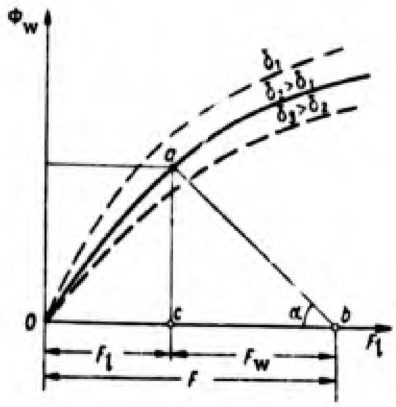


Fig. 1.9. Magnetization curve $\Phi_w = f(F_l)$ of magnetic circuit.

a number of values of Φ_w. With the aid of the leakage coefficients one finds for each of these values the magnitude of the flux in every section of the magnetic circuit ($\Phi_i = \Phi_w \sigma_{lki}$), and by means of the flux one finds the losses of the magnetizing force in these sections and the total losses F_l (see § 1.4, A-D).

With the aid of the curve $\Phi_w = f(F_l)$ it is very easy to determine graphically the value of Φ_w for assigned δ from the given value of the total magnetizing force F or, conversely, to determine on the basis of Φ_w the required value of F. Such a construction is shown in Fig. 1.9. In order to find F from Φ_w, one drops from the point a with ordinate Φ_w a line ab at an angle $\alpha = \mathrm{arctg}\, G_w$ on the abscissa. The abscissa of the point of intersecti-n with the abscissa axis—the point b—is the sought-for value of F. Indeed, since $G_w = \Phi_w/F_w$, the segment bc is equal to $F_w = \Phi_w \mathrm{ctg}\, \alpha$ and hence the segment Ob is equal to $F = F_l + F_w$.

Conversely, in order to determine Φ_w from F, one plots on the abscissa the quantity F, thus finding the point b, and then one draws from this point a line at an angle $\alpha = \mathrm{arctg}\, G_w$. The ordinate of the point of intersection of this line with the curve $\Phi_w = f(F_l)$—the point a—is the sought-for value of Φ_w.

This curve can be also used for determining F from F_l and conversely. With the aid of such a family of magnetization curves it is easy to construct the pull characteristic $F_p = f(\delta)$ of an electromagnet. Thus, for each value of δ for which the curve $\Phi_w = f(F_l)$ exists we find the pull force F_p by formula (1.5); F_w is determined with the aid of the curve $\Phi_w = f(F_l)$ on the basis of known F or by formula (1.7)—in terms of Φ_w instead of of F_w.

In order to determine the quantity $dG_w/d\delta$ entering in the formula for F_p, one must find at first the dependence of the working-gap conductance G_w on the gap length δ, and by differentiating this dependence one finds the dependence of $dG_w/d\delta$ on δ. From this relationship one takes the values of the derivative $dG_w/d\delta$ for the value of δ for which the pull force is being determined. The dependence of $dG_w/d\delta$ on δ is most easy to find if the function $G_w = f(\delta)$ is given in analytic form. If, on the other hand, $G_w = f(\delta)$ is given graphically, as for example when G_w is determined by constructing the actual field pattern, then the differentiation of this dependence must also be performed graphically.

§ 1.5. Design of the Coil of Electromagnets

In this section we examine the design of dc coils. The peculiar features of ac coil design will be considered in § 1.9 below, which is devoted to ac electromagnets.

A. The Coil Heating Equation

The passage of a current through a coil is accompanied by the liberation of heat in the coil, the heat power being

$$P = i^2R,$$

where R is the electrical resistance of the coil, and i is the current.

Thus the coil temperature increases.

If the coil temperature increases by $\Delta\Theta$ degrees during a time Δt, then the amount of heat producing such an increase is equal to $cG\Delta\Theta$, where $\Delta\Theta = \Theta - \Theta_0$, Θ is the coil temperature at the particular instant, Θ_0 is the ambient temperature (the original temperature of the coil), c is the specific heat of the coil (for copper we have c = 390 w·sec/kg deg), and G is the weight of the copper of the coil (in kg).

Hence by going over to the limit, we obtain for the power dissipated on heating the formula

$$P_h = CG\frac{d\Theta}{dt}.$$

With increasing coil temperature and the appearance of a temperature gradient $\Delta\Theta$ between the coil and the ambient medium, heat is being transferred to the latter. The power dissipated thereby is

$$P_{dis} = K_T S_{cool} \Delta\Theta,$$

where S_{cool} is the cooling surface of the coil (in m^2), and K_T is the heat-transfer coefficient, equal to $(10 - 14) \cdot 10^{-8}$ w/m^2 deg (it increases with S_{cool}).

S_{cool} is specified by the formula

$$S_{cool} = S_{c\ out} + \eta_T S_{c\ in} \tag{1.19}$$

where $S_{c\ out}$ and S_{cin} are the outer and the inner surfaces of the coil (the ends are not taken into account in view of their stronger heat insulation); η_T is a coefficient that characterizes the efficiency of heat transfer from the inner surface of the coil, i.e., to the magnetic circuit.

According to [4] we have $\eta_T = 0.9$ for bobbinless rimmed coils, whereas for coils with metallic bobbins we have $\eta_T = 1.7$; for coils that are wound directly on the body of the magnetic cirucit (optimum heat-transfer conditions) we have $\eta_T = 2.4$.

The coil heat balance equation is

$$P_h + P_{dis} = P,$$

i.e.,

$$cG\frac{d\Theta}{dt} + K_T S_{cool} \Delta\Theta = P,$$

or

$$(Tp + 1)\Delta\Theta = \frac{1}{K_T S_{cool}} P, \tag{1.20}$$

where $p = d/dt$ is the symbol of differentiation, and $T = cG/K_T S_{cool}$ is the coil heating time constant (in sec).

The solution of this equation is

$$\Delta\Theta(t) = \frac{P}{K_T S_{cool}}(1 - e^{-\frac{t}{T}}) = \Delta\Theta_{st}(1 - e^{-\frac{t}{T}}), \qquad (1.21)$$

where $\Delta\Theta_{st} = P/K_T S_{cool}$ is the steady-state value of $\Delta\Theta$.

The curve $\Delta\Theta$ (t) is plotted in Fig. 1.10 (curve 1). In the absence of heat transfer to the ambient medium all the heat $P = i^2R$ liberated in the coil would be used for its heating and the coil temperature would increase without bounds along the tangent 2. Yet since we have heat leakage proportional to the temperature gradient $\Delta\Theta$, it follows that an increase in the coil temperature will be accompanied by a decrease in the fraction of the power released in the coil that contributes to its heating, viz. $P_h = P - P_{dis}$. As a result, the increase in coil temperature becomes slower and slower (the curve $\Delta\Theta$ (t) becomes less and less steep), until it ceases altogether, when the quantity P_{dis}, which increases with the temperature, becomes equal to P. At this moment the steady-state thermal regime of the coil sets in: all the power liberated in the coil is dissipated in the ambient medium and the coil temperature no longer changes. The duration of the transient process, which follows an exponential curve, can be assumed to be equal to 3-4 T. Since the coil time constant T is normally of the order of tens of minutes, the duration of the coil heating process is on the average about one hour.

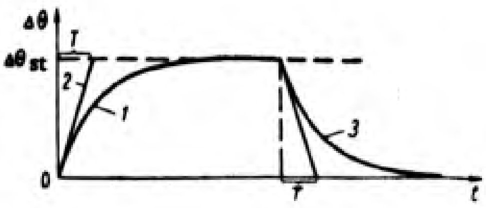


Fig. 1.10. The time characteristics $\Delta\Theta$ = f(t) for heating and cooling of the coil.

The equation for coil cooling after the current has been switched off can be obtained from formula (1.20) by introducing into it the value P = 0 with the conditions $\Delta\Theta = \Delta\Theta_{st}$. The solution of this equation

$$\Delta\Theta = \Delta\Theta_{st}\, e^{-\frac{t}{T}}$$

is shown in Fig. 1.10 in the form of the exponential curve 3.

B. Coil Heating Under Various Operating Conditions

From the thermal point of view, we can have three different modes of operation of the coil, depending on the length of time during which the signal fed to the coil is switched on switched off. These modes of operation are: long-time operation of the coil, short-time operation of the coil, and pulsed (periodic short-time) operation.

In long-time operation the duration of coil excitation is sufficiently long for the coil to be heated to its steady-state temperature, i.e., when the coil operating time

$$t_{op} > 4T.$$

In this mode the maximum coil temperature according to formula (1.20) is equal to

$$\Delta\Theta_{lt} = \Delta\Theta_{st} = \frac{P}{K_T S_{cool}}. \tag{1.22}$$

From (1.22) one can determine the maximum permissible value of the power liberated in the coil over a prolonged period on the basis of the condition that the coil is heated to the permissible temperature:

$$P_{lt\,per} = K_T S_{cool} \Delta\Theta_{per} \tag{1.23}$$

where $\Delta\Theta_{per} = \Theta_{per} - \Theta_0$, Θ_{per} is the permissible coil heating temperature, determined by the insulation of the wire, and Θ_0 is the ambient temperature, normally taken as equal to 35°C.

Formula (1.23) can be rewritten as

$$P_{lt\,per} = \frac{S_{cool}}{\sigma_{per}}, \tag{1.23a}$$

where $\sigma_{per} = 1/K_T \Delta\Theta_{per}$ is the minimum cooling surface required per 1 watt of power liberated in the coil.

The values of σ_{per} are listed in Table 1.2 [3], where l_{cl}/D_{cl} is the ratio of the coil length to its outer diameter.

The dependence of σ_{per} on l_{cl}/D_{cl} accounts for the heat transfer via the ends of the coil. Since the values of σ_{per} are known, it is much simpler to use the formula (1.23a) as compared to (1.23), yet the former is less exact (let us note that the values of σ_{per} are given above in cm^2/w).

Table 1.2

The Values of the Coefficient σ_{per} (in cm^2/w) for $\Delta\Theta_{per} = 70°C$

$\frac{l_{cl}}{D_{cl}}$	<1	1	>1
σ_{per}	8	10	12

In rough calculations one uses a technique that is even simpler, being based not on the permissible power, but on the maximum permissible coil current density in long-time operation ($j_{lt\,max}$). The formula for this quantity can be obtained from (1.23). Just as the permissible power, the permissible density likewise depends on the coil dimensions and on the values of K_T and $\Delta\Theta_{per}$. Yet this dependence proves to be much weaker; in the first approximation it is therefore possible to assume that $j_{lt\,max}$ is constant, being equal for low-power electromagnets to

$$j_{lt\,max} = 2 - 4 \text{ amp/mm}^2.$$

When the coil power increases, the value of $j_{lt\,max}$ decreases somewhat, as illustrated by the curve of Fig. 10.6.

In short-time operation the coil operating time t_{op} is insufficient for steady-state heating $\Delta\Theta_{st}$, and the time interval (pause) t_p before the next excitation of current is much larger, being sufficient for the coil to cool down completely to the ambient temperature, i.e.,

$$t_{op} < 4T < t_p.$$

In this mode of operation the maximum coil temperature $\Delta\Theta_{cr}$ is smaller than $\Delta\Theta_{st}$ and it depends on the coil operating time t_{op}.

According to formula (1.21) we have

$$\Delta\Theta_{cr} = \frac{P}{K_T S_{cool}}\left(1 - e^{-\frac{t_{op}}{T}}\right).$$

Hence we obtain for the maximum power permissible from the viewpoint of coil heating the formula

$$P_{cr\,per} = K_T S_{cool} \Delta\Theta_{per}\left(1 - e^{-\frac{t_{op}}{T}}\right)^{-1}$$

or, by using (1.23),

$$P_{\mathrm{cr\ per}} = P_{lt\ \mathrm{per}}(1 - e^{-\frac{t_{\mathrm{op}}}{T}})^{-1}.$$

This formula can be more convenientl.y rewritten as

$$P_{\mathrm{cr\ per}} = \zeta_{\mathrm{cr}} P_{lt\ \mathrm{per}} \tag{1.24}$$

where $\zeta_{cr} = (1-e^{-t_{op}/T})^{-1}$ is the thermal overloading coefficient, which specifies by how many times the power supplied to the coil in short-time operation can exceed the power supplied in long-time operation under equal heating conditions.

In the particular case $t_{op} << T$ we have

$$\zeta_{\mathrm{cr}} = \frac{T}{t_{\mathrm{op}}}, \tag{1.24a}$$

i.e., according to (1.23),

$$P_{\mathrm{cr\ per}} = cG\Delta\Theta_{\mathrm{per}}/t_{\mathrm{op}} \ . \tag{1.25}$$

Let us determine by formula (1.25) the permissible current density. For this purpose we shall introduce into this formula the expressions

$$G = \gamma l_w W q \quad \text{and} \quad P = i^2 R,$$

where $R = \rho\, l_w W/q$ and $i = jq$.

Here γ is the specific weight of the wire material (for copper we have $\gamma = 8.9$ g/cm^3), l_w is the mean length of a coil turn, W is the number of coil turns, q is the wire cross section of a copper coil, and ρ is the resistivity of the wire material.

The values of ρ for copper are listed in Appendix AP-4C for various temperature values.

As a result of this substitution we obtain

$$j_{\mathrm{cr\ per}} = \sqrt{\frac{c\gamma\Delta\Theta_{\mathrm{per}}}{\rho t_{\mathrm{op}}}} = \frac{j_{\mathrm{cr\ max\ (1\ sec)}}}{\sqrt{t_{\mathrm{op}}}}. \tag{1.26}$$

Here $j_{cr\,max}$(per 1 sec) = $\sqrt{c\gamma\Delta\Theta_{per}/\rho}$ is the maximum permissible current density for t_{op} = 1 sec.

For $\Theta_0 = 35°C$ and a wire insulation with $\Theta_{per} = 90 + 120°C$ we have

$$j_{cr\,max}\,(\text{per 1 sec}) = 100 - 120\ \text{amp/mm}^2.$$

Formula (1.25) and hence also formula (1.26) will be the more exact, the smaller the value of t_{op} as compared to the coil time constant T. It can be roughly assumed that these formulas are applicable as long as t_{op} is not larger than one or two tenths of a second.

The physical meaning of the replacement of the exact expression for ξ_{cr} in formula (1.24) by the approximate expression $\zeta_{cr} = T/t_{op}$ is tantamount to neglecting the heat transfer to the ambient medium (P_{dis}), i.e., to the assumption that when the coil is heated the temperature increase follows the tangent 2 in Fig. 1.10, and not the exponential curve 1.

Formula (1.25) can be directly obtained from the heat-balance equation by setting $P_{dis} = 0$. Thus the condition that t_{op} is small as compared to T signifies that the deviation of the exponential curve 1 from the tangent 2 is small.

Pulsed operation involves periodic short-time switching on and off of the signal, when the duration t_{op} of a single switching operation is not sufficient for attaining the steady-state temperature, whereas the pause t_p between switchings is too small for complete cooling of the coil, i.e., t_{op} and t_p are smaller than 4T.

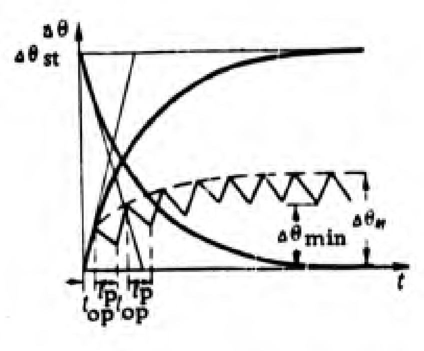


Fig. 1.11. Coil temperature variation in the pulsed mode.

In Fig. 1.11 we plotted the coil temperature variation in such a mode of operation. As can be seen, in the end there establishes itself in this case a temperature regime such that the temperature gradient $\Delta\Theta$ fluctuates between fixed maximum and minimum values $\Delta\Theta_M$ and $\Delta\Theta_{min}$; the mean value of the temperature will be the larger, the larger t_{op} and the smaller t_p. The initial increase in the mean value of the temperature is due to the fact that as a result of the smallness of $\Delta\theta$ the amount of heat liberated during a pause t_p is smaller than the heat quantity liberated during the operating interval t_{op}. But since the temperature increase at the end of any particular operating interval t_{op} is accompanied by a corresponding increase in the heat dissipation during the next pause, a thermal equilibrium finally establishes itself, i.e., the amount of heat stored in the coil during the time t_{op} is equal to the amount of heat dissipated during the time t_p.

By writing down the differential equations for coil heating and cooling, successively for every interval t_{op} and t_p, and by eliminating the intermediate values of $\Delta\theta$ and going over to the limit $t \to \infty$, we obtain the following expression (fairly obvious from a physical point of view) for the coefficient of thermal overloading under pulsed conditions:

$$\xi_{pulse} = \frac{P_{pulse\ per}}{P_{lt\ per}} = \frac{1 - e^{-\frac{t_{op}+t_p}{T}}}{1 - e^{-\frac{t_{op}}{T}}}. \tag{1.27}$$

By setting $t_p = 0$ (or $t_{op} = \infty$) in this formula, which signifies permanent operation, we obtain $\xi_{pulse} = 1$, as could be expected. In the other limiting case, i.e., with short-time switching, when $t_p \gg T$, we have $t_{op}+t_p/T \gg 1$, and therefore (by neglecting the term $e^{-t_{op}+t_p/T}$ in the numerator) we indeed find that

$$\xi_{pulse} = \left(1 - e^{-\frac{t_{op}}{T}}\right)^{-1} = \xi_{cr}.$$

In the case that the full period of operation $(t_{op} + t_p)$ is small as compared to the coil heating time constant T, we have

$$\xi_{pulse} \approx \frac{t_{op}+t_p}{t_{op}}. \tag{1.27a}$$

This is equivalent to a linear temperature variation during t_{op} and t_p.

C. The Relationship Between the Electrical and the Geometrical Parameters of a Coil

The coil resistance is

$$R = \rho \frac{l_w W}{q}.$$

Let us substitute into this formula the expression

$$q = K_{f\,cl} \frac{S_{cl}}{W},$$

where S_{cl} is the cross-sectional area of the coil, $K_{f\,cl} = qW/S_{cl}$ is the filling factor (with copper) of the cross section; this factor depends on the wire diameter — it decreases when the latter decreases. Hence

$$R = \frac{\rho l_w W^2}{K_{f\,cl} S_{cl}} = CW^2, \tag{1.28}$$

where $C = \rho l_w / K_{f\,cl}\, S_{cl}$ is the so-called coil constant.

For a circular coil we have

$$C = \frac{\rho}{K_{f.cl}} \cdot \frac{\pi (D_{cl} - h_{cl})}{h_{cl} l_{cl}},$$

where h_{cl} is the height (thickness) of the coil, and l_{cl} is the coil length.

From formula (1.28) one can see that with given coil dimensions the coil resistance is directly related to the number of turns W (it increases as W^2). By introducing the formula (1.28) into the expression for the power $P = i^2R$, we obtain

$$P = \frac{\rho l_w}{K_{f.cl} S_{cl}} F^2 = CF^2, \tag{1.29}$$

i.e., with a given magnetizing force $F = iW$ to be generated by the coil, the necessary input power P will be inversely proportional to the coil cross section S_{cl}. Thus the only way to increase the sensitivity of an electromagnet (i.e., to reduce the value of P) for a prescribed F, is to increase the coil dimensions. Thereby it is more convenient to increase the coil length l_{cl}, and not the height h_{cl}, since this does not involve an increase in the mean length of a turn l_w. The choice of the wire cross section has little effect on the value of P (only to the extent to which the factor $K_{f\,cl}$ depends on the wire cross section).

Since $K_{f\,cl}$ increases with q, it is desirable from this point of view to take a wire of as large a cross section as possible.

Let us now proceed directly to the designing of coils. It involves the determination of the diameter d and of the number of turns W on the bases of the assigned magnetizing force F and the permissible heating temperature Θ_{per}, determined by the type of insulation.

In addition to the value of F, specified by the pull force to be developed by the electromagnet, one normally assigns also the input signal, fed to the coil in the form of a voltage U or current i. Sometimes one assigns the coil resistance R instead. Finally, one can prescribe the input power P or (what amounts to the same) any two of the following three quantities: U, I and R.

The principal design rule is as follows. For a given F, a decrease in the coil cross section S_{cl} will be accompanied, on the one hand, by a decrease in the sensitivity of the electromagnet (the input power P increases), and on the other hand by an increase in the coil temperature, since the cooling surface decreases, as shown by formula (1.21). Thus the minimum possible coil dimensions are obtained by designing the coil for the maximum permissible heating temperature. This maximizes the input signal power. Conversely, by designing the coil with maximum possible cross-section, i.e., when the window in the magnetic circuit is completely filled, we obtain minimum power P, but maximum copper expenditure. The coil temperature will also be ninimal. As will be shown below (§ 1.7), an increase in the coil dimensions is also accompanied by an increase in the lag of the electromagnet.

D. Coil Design With Specified Window (Maximum Sensitivity)

On the basis of a specified window S_0 in the magnetic circuit one can determine the coil cross-section S_{cl}. It is smaller than the full cross section of the window by the cross-sectional area of the bobbin, the outer insulation of the coil, and a possible clearance between the coil and the magnetic circuit. On the average, S_{cl} amounts to 10–40% of S_0. In Appendix AP-4A we have listed the geometrical dimensions of the insulation materials used for the coils of electromagnets.

From the obtained maximum possible (for the given dimensions of the magnetic circuit) value of S_{cl} and the assigned value of the magnetizing force F one determines the number of turns W and the wire cross-section q. Then one checks the heating. The calculations will be different, depending on the particular input signal parameter (the voltage U or the current i) assigned.

1. Suppose that the voltage U is assigned. We have

$$F = iW = \frac{U}{R} W = \frac{Uq}{\rho l_w},$$

since

$$R = \frac{\rho l_w W}{q}.$$

Hence

$$q = \frac{\rho l_w F}{U}. \tag{1.30}$$

By formula (1.30) we find ρ and, by rounding off to the next highest value available in the batch of wires of the grade selected by us, we choose the wire diameter d.

If l_w in formula (1.30) is expressed in meters, then in order to obtain q in mm^2 the dimensionality of ρ must be ohm.mm^2/m. The values of ρ for copper can be found in Appendix AP-4C.

Then we determine the number of turns by the formula

$$W = \frac{K_{f.cl} S_{cl}}{q}. \tag{1.31}$$

The experimentally obtained values of $K_{f\,cl}$ for a PEL-grade wire of various diameter are listed in Appendix AP-4. Moreover, in wire handbooks one normally lists for each wire diameter the number of turns per 1 cm^2 coil cross-section, i.e., the quantity W/S_{cl}, from which one can find at once W if we know S_{cl}, without having to use formula (1.31). In this case, however, we must deduct from S_{cl} the cross-sectional area of the turn-to-turn insulation, if the use of the latter is envisaged.

Then we find $R = \rho l_w W/q$ and, finally, the current $i = U/R$ and the power $P = Ui$.

The checking of the coil heating involves the setting up of the inequality

$$P \leqslant P_{per},$$

where P_{per} is determined by formula (1.23) in the case of long-time switching of the coil; in the case of short-time switching it is determined either by formula (1.24) or, if the switching time does not exceed about 20 sec, quite simply -- by formula (1.26); in the case of pulsed operation it is determined by formula (1.27).

In approximate calculations the heating check-up is performed by means of the current density, whereby we must have

$$j = \frac{i}{q} \leqslant j_{per},$$

where j_{per} depends on the operating conditions of the coil. In the case of prolonged operation the quantity j_{per} is taken from the curve of Fig. 10.6, whereas in the case of short-time operation it is determined by formula (1.26).

2. Suppose that the current i is assigned.

The number of turns can be determined at once:

$$W = \frac{F}{i}.$$

Then one determines by formula (1.31) the wire cross-section q. Next one finds R, U and P by the same method as above and then checks the heating.

E. Coil Design With Maximum Heating (Minimum Dimensions)

In order to determine the minimum coil dimensions, we must equate the formula (1.29) for the power liberated in the coil to the formula for the maximum permissible power from the point of view of heating:

$$P_{per} = \xi K_T S_{cool} \Delta\Theta_{per},$$

where ξ is the thermal overloading factor; in the case of prolonged operation we have $\xi = 1$. In other modes of operation we must replace ξ by ξ_{cr} or ξ_{pulse}. We obtain

$$\frac{\rho l_w}{K_{f.cl} S_{cl}} F^2 = \xi K_T S_{cool} \Delta\Theta_{per}.$$

Let us express l_w, S_{cl} and S_{cool} by the coil dimensions:

$$l_w = \frac{\pi}{2}(D_{cl} + d_{cl}),$$

where d_{cl}, is the inner diameter of the coil, is determined by the core on which the coil is wound;

$$S_{cl} = l_{cl} h_{cl};$$

$$S_{cool} = S_{cl.\ out} + \eta_r S_{cl.\ in},$$

by formula (1.19), where

$$S_{cl.\ out} = \pi D_{cl} l_{cl};$$
$$S_{cl.\ in} = \pi d_{cl} l_{cl}.$$

For simplicity we take $\eta_r = 1$, which roughly corresponds to a bobbinless rimmed coil. By substituting these expressions into the above equation, we obtain

$$h_{cl} = \frac{\rho F^2}{2\xi K_1 K_{f.\ cl} \Delta\theta_{per} l_{cl}^2}. \qquad (1.32)$$

Here the coil length is taken to be the maximum possible length for the given dimensions of the magnetic circuit.

By assuming in the general case that $\eta_r \neq 1$, we obtain for h_{cl} a quadratic equation whose positive root is the sought-for value of h_{cl}.

From the thus obtained dimensions of the coil, one then determines q and W as in the previous calculation when the window S_{cl} is assigned.

In rough calculations one proceeds from the permissible current density j_{per}. Thereby it is easy to determine the minimum required coil cross-section:

$$S_{cl} = \frac{F}{K_{f.\ cl} j_{per}}, \qquad (1.33)$$

since $F = iW = jqW = jK_{f\ cl} S_{cl}$.

Then one determines in the usual way the quantities q and W. If instead of the voltage U we assigned the current i, it is more correct to determine at first the quantity $q = i/j_{per}$, and then $S_{cl} = K_{f\ cl} qW$, where $W = F/i$, since in this case it is no longer necessary to assign approximately the quantity K_{cl}, but one can determine it exactly from a known q.

§ 1.6 Dynamics of Electromagnets

A. The Time Parameters Of Electromagnets

If an electromagnet is regarded as an element of discrete operation whose armature has two positions in the steady-state regime -- an initial position (when $\delta = \delta_0$) and a final position (when $\delta = \delta_f$), then its dynamic properties can be characterized by two time parameters: the operate time t_{op} and the dropout time t_{dr}.

The operate time t_{op} is the time interval between the instant at which a voltage is applied to the coil and the instant at which the armature is completely pulled, when $\delta = \delta_f$. The dropout time t_{dr} is the time interval between the instant at which the input signal is switched off and the instant at which the armature is completely released, i.e., when it returns to its initial position, with $\delta = \delta_0$.

The delay in the movement of the armature following the variation of the input is determined, firstly, by the electromagnetic lag of the coil circuit of the electromagnet which causes the coil current variation to lag behind the variation of the impresses voltage and, secondly, by the mechanical lag of the moving parts of the electromagnet (the armature and the other parts connected to it).

Hence, t_{op} and t_{dr} can be expressed in the form

$$\left.\begin{aligned} t_{op} &= t'_{pk} + t'_{mt}; \\ t_{dr} &= t''_{pk} + t''_{mt}, \end{aligned}\right\} \qquad (1.34)$$

where t'_{pk} and t''_{pk} are the pickup time for operation and dropout respectively, this time being equal to the time interval between the instant of variation (switch on or switch off) of the input voltage and the beginning of the motion of the armature. This portion of t_{op} and t_{dr} is entirely determined by the electromagnetic lag of the coil circuit. t'_{mt} and t''_{mt} are the armature motion times when the electromagnet operates and drops out respectively.

According to the magnitude of t_{op} and t_{dr} the electromagnets can be divided into fast (t_{op}, $t_{dr} < 0.05$sec), normal (0.05 sec $> t_{op}$, $t_{dr} < 0.15$sec), and delayed

(t_{op}, t_{dr} > 0.15 sec). Since t_{op} and t_{dr} are not equal, one and the same electromagnet can be, for example, fast in operation and slow in dropping out, etc.

B. Determination Of Operate Time

Let us determine at first the operation pickup time t_{pk}.

The coil circuit equation is

$$U = Ri + \frac{d\Psi}{dt}, \tag{1.35}$$

where Ψ = Li, L is the coil inductance, and R is the coil resistance.

If the magnetic circuit is not saturated, as is normally the case in operation, since the magnetic flux increases with the current from zero, we have a constant coil inductance ($L = L_0$ const) and equation (1.35) can be written as

$$U = Ri + L_0 \frac{di}{dt},$$

or

$$(T_0 p + 1) i = \frac{U}{R}, \tag{1.36}$$

where p ∎ d/dt, and $T_0 = L_0/R$ is the electromagnetic time constant of the coil for the initial position of the armature ($\delta = \delta_0$) in sec.

In the case of a step variation of the input voltage from 0 to U the solution of equation (1.36) will be

$$i(t) = I_{st}\left(1 - e^{-\frac{t}{T_0}}\right), \tag{1.37}$$

where I_{st}=U/R is the steady-state value of the coil current, i.e., the exponential curve (curve 1) of Fig. 1.12.

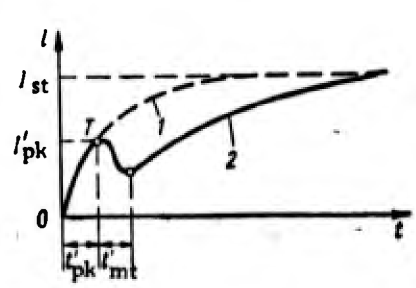


Fig. 1.12. The transient characteristic i = f(t) of an electromagnet.

The armature of the electromagnet begins to move when the pull force, which increases together with the coil current, exceeds the opposing force at $\delta = \delta_0$ (see Fig. 1.2). Let us denote by I'_{pk} the pickup current for operation, i.e., the value of the coil current at which the armature begins to move.

Then the sought-for value of the pickup time t'_{pk} can be found from the expression (1.37) for $i(t)$ by introducing into it the quantity $i = I'_{pk}$:

$$t'_{pk} = T_0 \ln \frac{I_{st}}{I_{st} - I'_{pk}}, \qquad (1.38)$$

or in terms of the magnetizing force:

$$t'_{pk} = T_0 \ln \frac{F}{F - F'_{pk}}. \qquad (1.38a)$$

If the pull characteristic is steeper than the opposing characteristic, i.e., when these characteristics are closest at $\delta = \delta_0$ (Fig. 1,2), then

$$I_{pk} = I_{op}. \qquad (0.11)$$

Thereupon formula (1.38) can be rewritten as

$$t'_{pk} = T_0 \ln \frac{K_{s\,op}}{K_{s\,op} - 1}, \qquad (1.38b)$$

where $K_{saf\,op} = I_{st}/I_{op}$ is the operating safety factor.

In the case of a compact magnetic circuit made of a material with a small resistivity ρ the current rise time in the coil will be greatly increased as compared to the value found by formula (1.38), this being due to the eddy currents in the magnetic circuit. Their effect is similar to that of a short-circuited coil, used for slowing down the electromagnets (see § 1.7). The effect of eddy currents, just as the effect of a short-circuited coil, can be taken into account by increasing the time constant of the input coil by the value of the time constant of the eddy-current circuit. The magnitude of this time constant is determined by the geometrical dimensions of the magnetic circuit and the electrical resistance of the material [2]. Thus in accounting for the eddy currents one must replace in the formula for t'_{pk} the quantity T_0 by $(T_0 + T_{e0})$, where T_{e0} is the time constant of the eddy-current circuit, determined at $\delta = \delta_0$.

When the armature begins to move (point T on the current curve in Fig. 1.12) the current will no longer vary along the exponential curve (1.37) (the dashed-line prolongation of curve 1 in Fig. 1.12), but according to some other law, since from this instant on the coil inductance L is no longer constant and equal to L_0, but it begins to increase. Indeed, if we neglect the leakage of the magnetic flux, i.e., we assume that $\Psi = W\Phi$, then

$$L = \frac{\Psi}{i} = \frac{W\Phi}{i} = \frac{WF}{iR_M} = \frac{W^2}{R_M}.$$

Since the main contribution to the magnetic resistance of the magnetic circuit is made by the resistance of the working gap ($R_M \approx R_w$), it can be assumed that

$$L = \frac{\Psi}{i} \approx \frac{W^2}{R_w} = W^2 G_w, \tag{1.39}$$

where G_w increases with decreasing working gap δ. The smaller the value of δ, the larger will be (for a given current i) the magnetic flux, and hence also the inductance $L = \Psi/i$.

Thus, by taking into account the dependence of L on δ, the coil circuit equation (1.35) assumes the following form when the armature begins to move:

$$U = Ri + L(\delta)\frac{di}{dt} + \frac{dL(\delta)}{dt}i,$$

or, by setting $\frac{dL(\delta)}{dt} = \frac{dL(\delta)}{d\delta} \cdot \frac{d\delta}{dt}$, we obtain

$$U = Ri + L(\delta)\frac{di}{dt} + \frac{dL(\delta)}{d\delta} \cdot \frac{d\delta}{dt} i. \tag{1.40}$$

The current variation follows the curve 2 in Fig. 1.12. The temporary drop in the current after the armature has begun to move is due to the appearance of an emf generated by the variation of the coil inductance and described by the third term in the right-hand side of (1.40).

Since we are considering the process of operating an electromagnet, the coefficient in front of i in this term is positive, and the appearance of this term in the equation must indeed cause a drop in the current, as though the R of the coil would have temporarily increased.

After the termination of the armature motion the current continues to increase until it reaches its steady-state value I_{st}; this increase likewise follows an exponential law, but with a different time constant $T_f = L_f/R$, which is larger than the time constant $T_0 = L_0/R$ prior to armature pickup, this being due to an increase in coil inductance when we go over from δ_0 to δ_f.

In order to determine the second component of the operate time — the motion time t'_{mt}, we must utilize the equation of motion of the armature:

$$m\frac{d^2\delta}{dt^2}+k\frac{d\delta}{dt}=F_p(\delta,\ i)-F_{opp}(\delta), \tag{1.41}$$

where m is the mass of the armature and of the totality of moving parts, and $k\frac{d\delta}{dt}$ is a term that accounts for the forces proportional to the velocity of motion (viscous friction forces, etc.).

In the right-hand side of equation (1.41) stands the resultant force acting on the armature. It is equal to the difference between the static pull force F_p and the static opposing force F_{opp}. The opposing force depends on δ alone, whereas the pull force depends also on the current i.

We must find the time during which δ varies from δ_0 to δ_f. This time will be precisely the time t'_{mt}. Equation (1.41) contains, in addition to the variables δ and t, also a third variable i, which is related to δ and t by the coil-circuit equation (1.40). For the determination of t'_{mt} we must therefore jointly solve the system of two equations (1.40) and (1.41). Elimination of the current from this system yields the dependence δ (t), whence one can determine the sought-for time t'_{mt}. Conversely, by elimination of δ we obtain the dependence i (t), represented in Fig. 1.12 by the curve 2.

Owing to the nonlinearity of these equations it is very difficult to solve them; for this purpose one uses approximate grapho-analytic integration, performed by the method of successive intervals [5]. In practical calculations, however, one frequently confines oneself to an approximate determination of t'_{mt} by the equation of motion

(1.41) alone, assuming the current to be fixed and equal to I_{st} (for this current value one normally has a pull characteristic constructed beforehand). The simplest way to determine t'_{mt} is as follows.

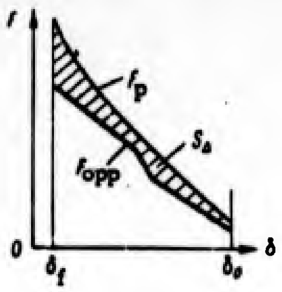


Fig. 1.13. Concerning the determination of the armature motion time of an electromagnet during operation

At first let us neglect the term $k.\,d\delta/dt$ Thereupon equation (1.41) assumes the form:

$$m\frac{d^2\delta}{dt^2} = F_p(\delta) - F_{opp}(\delta) = \Delta F(\delta). \quad (1.41a)$$

The right-hand side of this equation contains the resultant force ΔF, determined by the space between the pull characteristic at $I = I_{st}$ and the opposing characteristic (Fig. 1.13). Let us replace the force $\Delta F(\delta)$, which varies with δ, by its average value $\Delta F_{av} = S_\Delta/(\delta_0 - \delta_f)$, where S_Δ is the area enclosed between the pull and the opposing characteristics in the interval of δ from δ_0 to δ_f (it has the dimensionality of a force multiplied by a length). Thereupon we obtain the equation of uniformly accelerated motion:

$$m\frac{d^2\delta}{dt^2} = \Delta F_{av}.$$

Hence,

$$t'_{mt} = \sqrt{\frac{2m(\delta_0 - \delta_f)}{\Delta F_{av}}} = (\delta_0 - \delta_f)\sqrt{\frac{2m}{S_\Delta}}. \quad (1.42)$$

Such a simplified determination of t'_{mt} by formula (1.42) is good for small fast electromagnets only, for which t'_{mt} is smaller than t_{pk}. For such electromagnets we have

$$t'_{mt} \approx (0.1 - 0.4)\, t'_{pk}.$$

C. Determination Of Dropout Time

Let us start, as before, with the pickup time. The dropout of an electromagnet can be brought about either by disconnecting the voltage U from the coil by opening the coil circuit by means of contacts (Fig. 14a), or by short-circuiting the coil (Fig. 1.14b) or by closing the coil circuit across a small resistance.

In the second case, in order to prevent the simultaneous short-circuiting of the voltage supply U, we must insert a series resistance R_{aux} into the circuit (Fig. 1.14b).

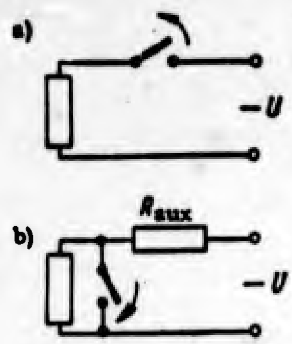


Fig. 1.14. Concerning the determination of the dropout time of an electromagnet.

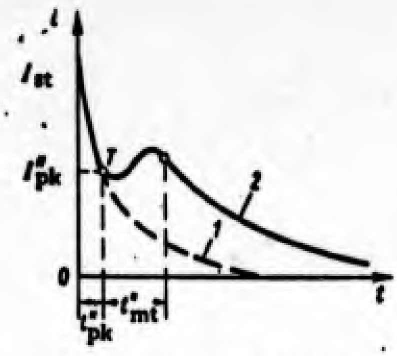


Fig. 1.15. The transient characteristic i = f (t) for electromagnet dropout.

When the voltage U is switched off (Fig. 1.14a), the coil current drops almost instantaneously from its initial steady-state value I_{st} to zero. It is true, though, that the energy stored in the magnetic field of the coil produces a short transient process, by maintaining the current for some time by means of a spark or even arc discharge between the contacts disconnecting the coil circuit (see § 4.2). During this process most of the energy stored in the magnetic field is converted into heat. Yet this process is of short duration, and in releasing the electromagnet by switching off the input signal U we can therefore assume that $t''_{pk} \approx 0$ as compared to the subsequent motion time.

In the case of dropout by means of short-circuiting the coil, the time variation of the current will be described by equation (1.36) in which we set U = 0, i.e., by the equation

$$(T_f p + 1) i = 0,$$

$$(T_f p + 1) i = 0,$$

assuming that at $t = 0$ we have $i = I_{st}$.

Here $T_f = L_f/R$ is the electromagnetic time constant of the coil for $\delta = \delta_f$.

The solution of this equation

$$i(t) = I_{st} e^{-\frac{t}{T_f}}, \qquad (1.43)$$

is an exponential curve (curve 1 in Fig. 1.15).

The armature of the electromagnet begins to move away from the core when, as a result of the current drop, the pull force becomes weaker than the opposing force (Fig. 1.2). Let us denote by I''_{pk} the release pickup current, i.e., the value of the current at which the armature begins to move away. Then, by substituting into formula (1.43) for the current $i(t)$ its value $i = I''_{pk}$, we obtain the formula for the release pickup time

$$t''_{pk} = T_f \ln \frac{I_{st}}{I''_{pk}} = T_f \ln \frac{F}{F''_{pk}}. \qquad (1.44)$$

In order to account for the retarding action of the eddy currents, and to determine the motion time for operation, one must add to the coil time constant in formula (1.44) the time constant $T_{ed\,f}$ of the eddy-current circuit, determined for $\delta = \delta_f$.

If the electromagnet is released by switching off the voltage U, while taking into account the eddy currents, the quantity t''_{pk} will no longer be equal to zero; it must be determined by formula (1.44), substituting into it for T_f the time constant of the eddy-current circuit, which of course remains in existence when the coil circuit is opened; as a result, the time constant T_f does not vanish.

Curve 2 in Fig. 1.15 represents the current variation up to the very end of the transient process. One can see that when the armature begins to move during the release stage, the current curve has a spike (as in the operate stage) which is due to the variation of the coil inductance from its maximum (L_f) to its minimum (L_0) value. After the value $\delta = \delta_0$ has been reached, the current continues to drop exponentially with a time constant $T_0 = L_0/R$.

The motion time for release t''_{mt} is determined in the same way as for operation, i.e., in the general case by jointly solving two equations (the equation of the coil circuit and the equation of motion of the armature), whereas in the case of high-speed electromagnets one obtains an approximate solution with the aid of the equation of motion alone. In the latter case the coil current is assumed to be fixed and equal to I''_{pk}.

§ 1.7. Methods Of Altering The Time Parameters Of Electromagnets

A. Design Methods Of Altering The Time Parameters Of Electromagnets

If the values of the time parameters t_{op} and t_{dr} of electromagnets are not taken into account in the design calculations, these values are normally of the order of 0.05 to 0.15 sec. In the designing of high-speed electromagnets, with smaller values of t_{op} and t_{dr}, or, conversely, of delayed electromagnets, one must take special measures for appropriately changing the time parameters. Let us examine these measures.

The values of t_{op} and t_{dr} can be diminished in the following manner. Firstly, one reduces in every possible way the eddy currents in the magnetic circuit, which increase the pickup time. For this purpose the materials used for the magnetic circuit must have a high electrical resistance (silicon steels, low-nickel Permalloy), or, which is even more effective, though more complicated — the magnetic circuit is laminated, as in the case of ac electromagnets. Secondly, in order to reduce the motion time one diminishes the armature stroke δ_0 and the mass of the armature and of the other moving parts. Thirdly, in order to diminish the pickup time and the motion time one takes a sufficiently large value of the operating safety factor $K_{sap\,op}$. This is borne out by the equations (1.38b) and (1.42), where an increase in $K_{sap\,op}$ is accompanied by an increase in ΔF_{op}. For high-speed electromagnets it is recommended to take $K_{sap\,op} = 1.5 - 3$. Fourthly, in order to reduce the dropout time one additionally increases the final value δ_f of the working air gap, in order to eliminate the so-called armature "sealing" effect. This effect can be described as follows:

Even after the signal has been switched off, the armature remains pulled to the core due to the magnetic flux produced by the remanent magnetism of the magnetic circuit. If the idle air gaps between the individual parts of the magnetic circuit are found to be insufficient, one introduces into the magnetic circuit a special nonmagnetic gasket — a so-called unblocking pin. This gasket is normally placed in the working gap and fixed on the core.

Finally, in the designing of high-speed electromagnets one imposes also certain restrictions on the coil. According to formulas (1.39) and (1.28) the coil time constant T_0 for $\delta = \delta_0$ is equal to

$$T_0 = \frac{L_0}{R} \approx \frac{G_w}{C} = \frac{K_{f.\,cl}}{\rho l_w} G_w S_{cl}. \tag{1.45}$$

Hence follows that T_0 is proportional to the coil cross-section S_{cl}, and in order to obtain a minimal time constant of the coil the coil dimensions must be minimized, i.e., designed for maximal heating (see § 1.5E). Yet this would not minimize at all the operate time of the electromagnet. As a matter of fact, not only the coil time constant depends on the value of S_{cl}, but also the magnetizing force produced by the coil; according to (1.29), this force is equal to

$$F = \sqrt{\frac{P}{C}} = \sqrt{\frac{K_{f.\,cl} S_{cl}}{\rho l_w} P},$$

where P is the signal power.

As can be seen from this formula, a decrease in S_{cl} will be accompanied by a decrease in the magnetizing force F. But this affects (on the one hand) the pickup time in the sense of increasing it, since for $F \to F_{pk}$ we have by virtue of (1.38a) the value $t'_{pk} \to \infty$, and (on the other hand) the motion time is also increased, since the pull force is diminished. Thus whereas a decrease in S_{cl} is accompanied at first by a net decrease in the operate time, the latter will start increasing again afterwards. Hence, there exist optimum coil dimensions that minimize t_{op}. With a given signal voltage and current we thereby unambiguously determine the corresponding optimum number of turns W and the coil wire cross-section q. The simplest way to obtain the optimum coil parameters that minimize t_{op} is to find at first the optimum value L_{0opt} of the coil inductance, which is then used for determining W_{opt} and q_{opt}. Such a technique is described in [6]. The minimum values of t_{op} and t_{dr} that can be obtained by using all the methods considered above amount to a few milliseconds. The smallest values of t_{op} and t_{dr}, down to fractions of a millisecond, can be obtained by using polarized electromagnets, which will be examined in Chapter 2.

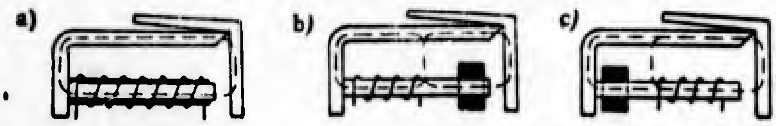


Fig. 1.16. Different layouts of a short-circuited coil (slug).

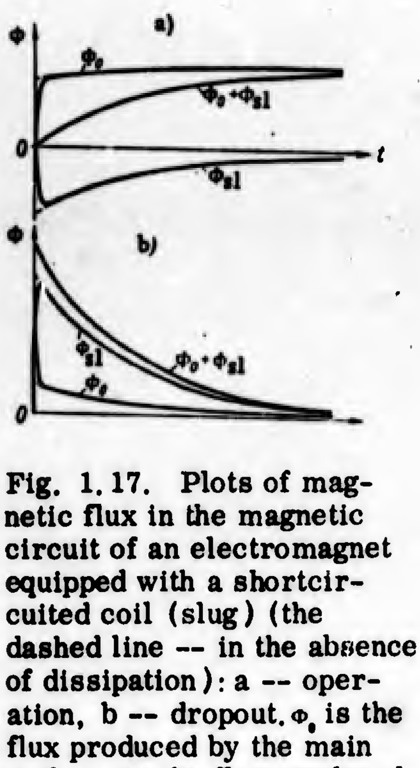


Fig. 1.17. Plots of magnetic flux in the magnetic circuit of an electromagnet equipped with a shortcircuited coil (slug) (the dashed line -- in the absence of dissipation): a -- operation, b -- dropout. Φ_0 is the flux produced by the main coil, Φ_{sl} is the flux produced by the slug, and $\Phi_0 + \Phi_{sl}$ is the resultant flux.

In order to increase t_{op} and t_{dr} it is evidently necessary to proceed in a manner opposite to the one used for diminishing t_{op} and t_{dr}, i.e., to increase the stroke of the armature, the coil dimensions, etc.

Yet in addition to these measures, one can achieve great time delays by using short-circuited coils (electromagnetic damping) and mechanical means of slowing down the armature movement (mechanical damping).

A short-circuited coil is normally designed in the form of a single turn, i.e., in the form of a compact copper bushing. It is frequently called a slug. In Fig. 1.16 we show different positions of such a slug on the core of a pi-shaped magnetic circuit.

The increase in t_{op} and t_{dr} resulting from the insertion of a short-circuited coil is due to the fact that this coil retards the variation of the magnetic flux produced by the main coil. Such a retarding effect of a short-circuited coil is determined by the law of electromagnetic induction, according to which any variation of the magnetic field linked to any particular coil generates in this coil an emf whose current produces a magnetic flux directed in such a way that it tends to preserve unchanged the resultant magnetic flux permeating this coil.

In Fig. 1.17 we plotted the variation of the flux in a magnetic circuit in the absence (Φ_0) and in the presence ($\Phi_0 + \Phi_{sl}$) of a short-circuited coil.

Let us examine the process of operating an electromagnet equipped with a short-circuited coil. Prior to armature pick-up the equations of the main coil and of the short-circuited coil are

$$\left.\begin{aligned} U &= Ri + W\frac{d\Phi}{dt}; \\ 0 &= R_{sl}i_{sl} + W_{sl}\frac{d\Phi}{dt}, \\ \text{where } \Phi &= \frac{Wi + W_{sl}i_{sl}}{R_M}. \end{aligned}\right\} \quad (1.46)$$

For simplicity we shall neglect the dissipationof the magnetic flux and assume that the entire flux Φ permeates the entire main coil as well as the entire short-circuited coil. The quantities relating to the short-circuit (the slug) are marked by the subscript "sl".

The armature begins to move when the flux Φ, which grows from zero, attains a value such that the pull force (which grows with the flux) exceeds the opposing force. By eliminating from (1.46) the currents, we obtain the equation for the flux:

$$[(T_0 + T_{sl0})p + 1]\Phi = \Phi_{st}, \qquad (1.47)$$

where $\Phi_{st} = \frac{U}{R}\frac{W}{R_{M0}}$ is the steady-state value of the magnetic flux, specified for $\delta = \delta_0$; $T_0 = \frac{L_0}{R}$ is the time constant of the main coil for $\delta = \delta_0$; and $T_{sl0} = \frac{L_{sl0}}{R_{sl}}$ is the time constant of the short-circuited coil for $\delta = \delta_0$.

In the absence of a short-circuited coil ($R_{sl} = \infty$) we have $T_{sl0} = 0$ and equation (1.47) is directly obtained from the earlier considered equation (1.36), by converting in this equation from the current to the flux by multiplication by W/R_{M0}. (Here we converted to the flux, since the latter, and hence also the transient process, is no longer specified by a single current, but by two currents i and i_{sl}).

The pickup time can be easily determined by solving the equation (1.47) and by introducing into this solution the value of the pickup flux:

$$\Phi'_{pk} = \frac{I'_{pk}W}{R_{M0}}.$$

Thus the effect of a short-circuited coil on the pickup time t'_{pk} can be taken into account by adding in the formula (1.38) for t'_{pk} to the time constant T_0 of the main coil the time constant T_{sl0} of the short-circuited coil. This means that the increase in the operate time due to the short-circuited coil takes place up to the value of t_{op} that would be obtained by fully utilizing as the main coil the entire cross-section occupied by both coils.

Yet if the short-circuited coil is designed in the form of a solid slug, its utilization yields a much greater delay as compared to an increase in the cross section of the main coil by the value of the cross section of this slug. This is due to the fact that the copper filling factor of a solid slug $K_{f.cl} = 1$, i.e., roughly twice as high as in the case of an ordinary wire coil, and we know [see formula (1.45)] that the coil time constant is proportional to $K_{f.cl}$.

In the release process, initiated by short-circuiting the main coil, the increase in the pickup time obtained as a result of using a special short-circuited coil can be determined (as in the case of operating process) by adding the time constant T_{slf} of this coil to the time constant T_f of the main coil (for $\delta = \delta_f$).

Here, too, the increase in t''_{pk} as compared to the case when the entire window under the main coil is used is due only to the fact that the window has been better filled with copper. Yet in releasing the electromagnet by disconnecting its coil circuit, the use of a short-circuited coil permits t''_{pk} to be increased by tens of times. In this case t''_{pk} is determined by T_{slf} alone, since $T_f = 0$ ($R = \infty$).

The manner of accounting for the effect of a short-circuited coil on the pickup time by increasing the time constant of the main coil by the value of the time constant of the short-circuited coil is approximate, since it does not allow (among others) for the dissipation of the magnetic flux. In such an analysis all the positions of the short-circuited coil (slug) shown in Fig. 1.16 are equivalent. Yet in actual fact there is a considerable difference between them. When the slug is placed near the working gap (Fig. 1.16b), the pickup time for operation is much more increased as compared to the case when the slug is located along the entire core (Fig. 1.16a). This is due to the fact that in operation the flux Φ_{sl} produced by the slug counteracts the flux Φ_0 produced by the main coil and displaces it, so that a closed dissipation flux (shown in Fig. 1.16b by the dashed line) is formed outside the working gap.

Conversely, a slug placed at the other end of the core at the yoke (Fig. 1.16c) has a weaker effect on the pickup time for operation as compared to a slug along the entire core. This is due to the fact that in operation the effect of the slug is tantamount here to the displacing of the flux from the end of the core and the base of the yoke into the air (as shown in Fig. 1.16c by the dashed line). Yet this does not prevent the passage of the flux through the working gap. The flux simply closes itself outside the slug, leaving it aside.

In the release process all three positions of the slug have roughly the same effect on the pickup time, since the working gap is small in this case and the dissipation is altogether weak.

The use of a short-circuited coil permits the obtaining of time delays for operation and release that are greater than one second.

In order to obtain even greater time delays (up to several tens of seconds) one utilizes diverse nethods of slowing down the movement of the armature, i.e., of increasing t_{mt}. For this purpose one fixes to the armature a damping device that produces a counter force proportional to the rate of displacement (in the first or higher order).

Such delaying devices can be mechanical (for example, a retarding device using mechanical friction, or a starting mechanism as in clocks), pneumatic (air bellows), hydraulic (a dashpot in the form of an oil-filled cylinder with piston), and electromagnetic (a metal disk placed in the field of a permanent magnet).

In order to increase the counter force produced by such devices, the latter are normally fixed to the armature via a step-up gear transmission.

B. Circuit Methods Of Altering The Time Parameters Of Electromagnets

The time parameters of a ready electromagnet can be varied within certain limits by varying the magnitude of the input signal or by connecting in series or in parallel with the coil diverse combinations of active and reactive linear and nonlinear resistors, i.e., by altering the circuit diagram of the coil of the electromagnet.

Various circuit methods of altering the time parameters of electromagnets are shown in Fig. 1.18.

All these methods involve changes in the pickup time. As it follows from the formulas (1.38) and (1.44) for t'_{pk} and t'_{pk}, the pickup time can be altered by changing the steady-state value I_{st} of the coil current and the time constant of the coil circuit (T_0 for operation and T_f for dropout. In order to increase the pickup time for operation t'_{pk}, and hence the time t_{op} as a whole, one must diminish I_{st} and increase T_0; the inverse procedure must be followed when it is required to diminish t'_{pk}. In order to increase the pickup time for dropout t''_{pk} one must increase both I_{st} and T_f, whereas for reducing it one must proceed in the opposite way.

At first let us examine circuit methods of increasing the operate time t_{op}. This time can be increased by reducing the value of the input voltage U, since this leads to a decrease in $I_{st} = U/R$.

Theoretically we have for $I_{st} \to I_{op}$ the value $t_{op} \to \infty$.

Yet in practice the possibilities are limited, since reliable operation of an electromagnet presupposes a sufficiently large operating safety factor

$$K_{saf\,op} = \frac{I_{st}}{I_{op}} > 1.$$

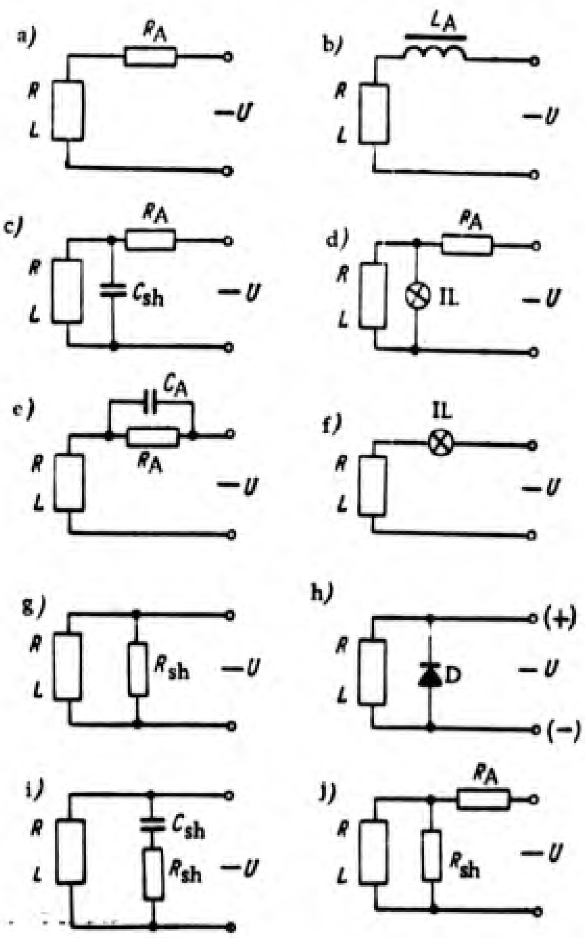


Fig. 1.18. Circuit methods of alteration of the time parameters of electromagnets.

In the particular case of fixed U, t_{op} can be diminished by connecting in series with the coil an auxiliary resistor R_A (Fig. 1.18a). It is true, though, that in this case the increase in t_{op} will be even smaller, since R_A reduces not only the value of

$$I_{st} = \frac{U}{R + R_A},$$

but also the time constant

$$T_n = \frac{L_0}{R + R_A},$$

which has the opposite effect on t_{op} by weakening the effect due to a reduction in I_{st}.

The operate time can be increased by putting auxiliary inductance L_A in series with the coil (Fig. 1.18b). This increases the time constant of the coil circuit

$$T_0 = \frac{L_0 + L_A}{R},$$

without altering I_{st}.

In the circuit of Fig. 1.18c, operation is delayed by means of an RC lag network inserted in front of the coil. The coil voltage, equal to the voltage across the shunt capacitor C_{sh}, lags behind the input voltage U and increases gradually with the charging of the capacitor.

Figure 1.18d shows a model of an operation delay circuit using a nonlinear element. This element is here in the form of an incandescent lamp IL. The operation of the circuit is based on the fact that the resistance of the lamp filament is small in the cold state, while strongly increasing (tenfold) when the filament is heated by the current passing through it. At the first instant after the application of U, the coil voltage, equal to the voltage drop across the lamp, is low. Then, when the filament becomes hotter and its resistance increases, the voltage gradually increases and reaches in the end a value that is sufficient for operating the electromagnet. Thus the time delay is determined in this case by the thermal lag of the filament. Instead of an incandescent lamp one can also use other types of nonlinear lag resistors. If the magnitude of such a resistance increases with the current, it must be connected in parallel with the coil (as in the case of an incandescent lamp). If, on the other hand, the resistance decreases with increasing current, such a resistor must be connected in series with the coil.

Let us now examine the methods of diminishing t_{op}. First, one can increase I_{st} by increasing U. Yet such a possibility is limited by the maximum value of I_{st} compatible with heating. The pickup time for operation can be infinitely reduced provided that when U is increased we are also inserting a resistor R_A (Fig. 1.18a) which ensures that $I_{st} = U/(T+R_A)$ remains constant. Here the increase in speed is achieved by diminishing the time constant $T_0 = L_0/(R+R_A)$. Yet one must take into account that this decrease in T_0 is obtained at the expense of a proportional increase in the power dissipated in R_A, i.e.. at the cost of diminishing the sensitivity of the circuit as a whole.

The speed can be even further increased by connecting in parallel to R_A a capacitor C_A (Fig. 1.18e). The forcinf of the coil current obtained in this circuit is due to the fact that at the first instant after application of the voltage U the yet-uncharged capacitor offers an additional path for the current, shunting the resistor R_A. As a result, the coil current increases faster owing to the charging current of the capacitor. (Let us note that the circuit of Fig. 1.18e permits forcing as compared to the circuit of Fig. 1.18f, i.e., with R_A but without C_A, and not as compared to the circuit in which the same value of U is applied directly to the coil without any R_A).

Figure 1.18 f shows a circuit that diminishes the operate time with the aid of an incandescent lamp IL. The steady-state value of the coil current I_{st} is determined by the resistance of the incandescent filament. At the first instant following the application of U we obtain current forcing as in the previous circuit, since the resistance of the filament is small at first. Instead of an incandescent lamp it is possible to use also in this case various other nonlinear lag resistors.

Let us now examine circuit methods of altering the pickup time for release. With regard to diminishing this time, it can be reduced to practically zero (as was already mentioned above) if the electromagnet is released by opening the coil circuit.

In order to increase the release time one must insert in parallel with the coil a resistor R_{sh} (Fig. 1.18g). which forms together with the coil a closed circuit with a time constant

$$T_f = \frac{L_f}{R + R_{sh}},$$

which specifies the value of the pickup time for release in accordance with formula (1.44).

The shortcoming of this circuit lies in the fact that power losses occur in the resistor R_{sh} as long as a voltage is applied to the coil. In order to increase the release time, R_{sh} must be diminished, and this is accompanied by a corresponding increase in the losses. Therefore it is better to use instead of this circuit the circuits of Figs. 1.18h and 1.18i.

In the first of these circuits the resistor R_{sh} has been replaced by a semiconductor rectifier D. When the electromagnet is released, its coil is short-circuited across the small forward resistance of D, whereas the power losses in the rectifier due to the supply voltage U are negligible in the steady state, since U is applied to the rectifier in the reverse direction, in which its resistance is high.

In the circuit of Fig. 1.18i the steady-state losses have been eliminated by inserting a capacitance C_{sh} in series with R_{sh}.

In the case of a small resistance $(R+R_{sh})$ of such an LC circuit, the transient process may have periodic character. This is not permissible, since it would lead to repeated operation and release of the electromagnet. An aperiodic transient process can be ensured by taking a sufficiently large value of R_{sh}.

The time delay in this circuit increases with the capacitance C_{sh}. (If this capacitance is sufficiently large the transient process of coil current decay will be mainly determined by the energy stored earlier in the capacitor C_{sh}; the coil inductance L can be neglected, regarding the release process as a process of discharging the capacitor C across the resistor $R+R_{sh}$).

In those cases in which it is necessary to alter both the operate and the release time, one utilizes a combination of the circuits described above. As an example we show in Fig. 1.18j a combination of the circuits of Fig. 1.18a and Fig. 1.18g. In order to still further diminish t_{op} we can insert here a capacitor in parallel to R_A, as in Fig. 18e. Instead of R_{sh} one can insert a rectifier, as in the circuit of Fig. 1.18h, or an $R_{sh}C_{sh}$ network, as in Fig. 1.18i, etc.

The circuits considered above make it possible to reduce the operate time of electromagnets of normal speed to a few milliseconds, or to increase the pickup time to a few seconds. In principle it is possible to obtain also greater delays, but this involves an unpermissible drop in the stability of the time parameters.

§ 1.8 The Order Of Designing An Electromagnet

The basic design data are:

a) The operating parameters of the electromagnet, the principal ones being the voltage and the current of the input signal applied to the coil, the operating conditions, the time parameters, the opposing characteristic, the dimensions, the weight, and the cost.

b) The principal service conditions: the ambient temperature, humidity, dustiness, possibility of vibrations, and magnitude of vibrations.

c) The manufacturing conditions, such as the number of electromagnets that have to be produced, the availability of ready equipment for the manufacture of similar devices, etc.

As a result of the calculations one must determine the design parameters of the electromagnet, i.e., select the type of electromagnet (Fig. 1.1), the material of the magnetic circuit and of the other parts of the electromagnet, determine the geometrical dimensions of the magnetic circuit and of the coil, as well as the electrical parameters of the latter.

The criterion of optimality of the design type can be minimum weight, size, or cost — depending on which of these factors is more important in the particular case; it can also be a compromise between these indicators. If basically new types are designed, it is necessary to construct several models which are then compared. On the other hand, the selection of electromagnets of existing types is facilitated by the availability of ready recommendations [2, 3, 5] concerning the choice of the principle design parameters of these electromagnets, thereby achieving an optimum model. In this case it is unnecessary to design several versions.

The order of designing is as follows.

1. The choice of the type of eletromagnet and of its kinematic circuit connection with the actuating mechanism to be operated, which constitutes the first design stage.

In selecting any particular type of electromagnet among those presented in Fig. 1.1, we must proceed from a comparison of their properties (§ 1.1). It is desirable that power (high-strength) electromagnets be of plunger-type (Fig. 1.1e), since this minimizes the dimensions of the device, and hence also the weight and cost. Only in the case of a very small stroke of the armature (as for example in electromagnet friction clutches and in various sorts of holding devices) it is more convenient to use a bar-type electromagnet with a cylindrical magnetic circuit (Fig. 1.1d).

In order to minimize the weight (volume) of a plunger-type electromagnet, the shape of its armature and of the fixed core must be selected on the basis of a prescribed relationship between the values of the pull force and of the armature stroke.

Table 1.3

Optimum Values of Design Factor $\sqrt{F_{p0}}/\delta_0$ $[N^{1/2}/m]$ for Various Electromagnets

Type of electromagnet	$\sqrt{F_{p0}}/\delta_0$ $[N^{1/2}/m]$
Plunger-type electromagnet without fixed core	400
Same, with fixed core and an armature that has a conical end with an apex angle of 60° .	400—1600
Same, with an apex angle of 90°	1600—5200
Same, with an armature that has a flat end (apex angle of 180°)	5200—29000
Bar-type electromagnet with a cylindrical magnetic circuit.	2900

In Table 1.3 we present the recommended optimum core-shapes of a plunger-type electromagnet as a function of the ratio

$$\frac{\sqrt{F_{p0}}}{\delta_0},$$

where δ_0 is the initial value of the working air gap, and F_{p0} is the value of the required force at the initial position of the armature, i.e., at $\delta = \delta_0$.

The ratio $\sqrt{F_{p0}}/\delta_0$ is called the design factor. It is used as a criterion for selecting the optimum relationships between the principal geometrical dimensions of the magnetic circuit, not only of plunger, but also of all the other types of electromagnets. The reason for selecting the optimum shape of an electromagnet on the basis of such an indicator is the circumstance that the ratio $\sqrt{F_{p0}}/\delta_0$ is proportional to the ratio of the transverse dimension of the core to its length, which practically determines the shape of the electromagnet as a whole. Indeed, the pull force of an electromagnet is proportional to the core cross-section [see, for example, the formula (1.9)], i.e., to the square of its transverse linear dimension. Hence, with other conditions being equal, the transverse dimension of the core is proportional to the root square of the pull force. With regard to the core length, the latter increases as the magnetizing force to be developed by the coil, and it can be assumed that this force is proportional to the maximum value δ_0 of the working air gap.

Low-power electromagnets, for which high sensitivity is of primary importance, and not minimum weight (volume or dimensions), are of bar type (as shown in Figs. 1.1a, 1.1b and 1.1c). For these electromagnets the optimum value of the design factor $\sqrt{F_{p0}}/\delta_0$ ranges from 800 to 8000 $N^{1/2}/m$. It is desirable to achieve these values by appropriate selection of the transmission ratio between the kinematic circuit diagram of the electromagnet and the actuating device, for a prescribed counter-force characteristic of the latter and prescribed dimensions of the pole piece.

Depending on the required shape of the pull characteristic it might be more convenient to use one of the electromagnets with transversely moving armature (Figs. 1.1g and 1.1h).

With regard to possible shapes of the pull characteristic of diverse types of electromagnets and of the dependence of this shape on the design of the magnetic circuit and of the armature, see [1].

II. The material of the magnetic circuit is selected in conformity with the principles stated in § 1.2.

III. The preliminary design of an electromagnet is carried out in two stages: At first one performs preliminary calculations in order to determine in the first approximation the principal geometrical dimensions of the electromagnet; thereby the dissipation of the magnetizing force and the leakage of the magnetic flux are taken into account only approximately. Then the design of the electromagnet is carried out exactly. Now the dimensions of the magnetic circuit, found in the preliminary calculations, make it possible to take into account with the necessary accuracy the saturation of the magnetic circuit, the dissipation of the magnetizing force, and the flux leakage. During these calculations one checks the optimality of the electromagnet dimensions found above, one corrects them, and one determines the final values of all the service parameters of the electromagnet.

The order of the preliminary calculations for a plunger-type electromagnet with a fixed core and for a bar-type electromagnet is as follows:

1. One selects the value of the induction in the working gap (B_{w0}) for the initial gap value. An increase in induction is accompanied by a decrease in the dimensions of the magnetic circuit; but the sensitivity decreases also, since the dissipation of the magnetizing force and the flux leakage are increasing. It is recommended to take $B_{w0} = 0.06 - 1$ tl, this value being the larger, the larger the design factor $\sqrt{F_{p0}}/\delta_0$. In Fig. 1.19 we plotted the optimum values of B_{w0} versus $\sqrt{F_{p0}}/\delta_0$.

2. The cross section of the portions of the magnetic circuit can be determined with the aid of a greatly simplified formula for the pull force. Thus for a bar-type

electromagnet or a plunger-type electromagnet with a flat armature one can use Maxwell's formula (1.9).

From this formula we find the working gap cross-section

$$S_w = \frac{2\mu_0 F_{p0}}{B_{w0}}$$

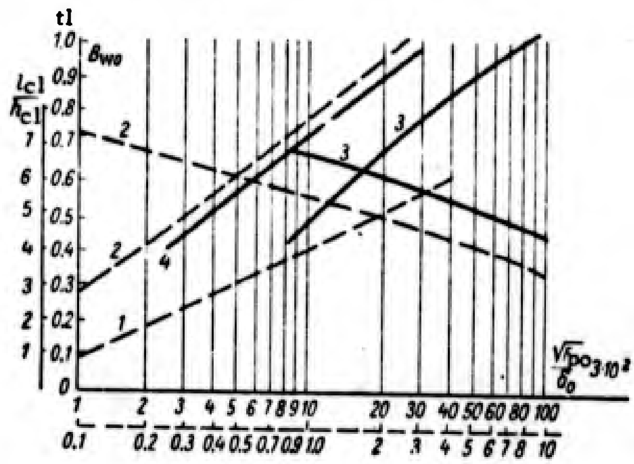


Fig. 1.19. Optimum values of B_{w0} and l_{cl}/h_{cl} versus the ratio $\sqrt{F_{p0}}/\delta_0$. 1 — bar-type electromagnet; 2 — plunger-type electromagnet with flat armature; 3 — same, with armature having a conical end with a 45° apex angle; 4 — same, with a 60° apex angle. The curves B_{w0} are increasing, and the curves l_{cl}/h_{cl} are decreasing. For the solid lines the abscissa scale is taken along the upper axis, and for the dashed lines it is taken along the lower axis.

Here the required value of the pull force F_{p0} is taken by 15-25% higher than the given opposing force for $\delta = \delta_0$.

For plunger-type and bar-type electromagnets without pole pieces, this value of the working gap cross section S_w is equal to the sought-for core cross section S_c. In a bar-type electromagnet with a pole piece (see Figs. 1.1a and 1.1c), this will be the pole piece cross section S_{pp}. In the latter case, we can use for the determination of S_c also the value B_{c0} of the induction at the core base and the value σ_{lk0} of the leakage coefficient. It is recommended to take $B_{c0} = 0.4 - 1.5$ tl, i.e., more than B_{w0}, and $\sigma_{lk0} = 1.1 - 2.5$ (σ_y increases with δ_0, i.e., with decreasing design factor).

$$S_c = \frac{\Phi_0}{B_{c0}},$$

where $\Phi_0 = \sigma_{lk0}\Phi_{w0} = \sigma_{lk0}B_{w0}S_w$.

The cross section of the other portions of the magnetic circuit is assumed to be proportional to the magnitude of the magnetic flux in them.

For plunger-type electromagnets with a conical armature it is likewise possible to utilize Maxwell's formula. In this case the value of S_w, obtained by this formula, specifies the area of the end surface of the armature. The armature cross section and the core cross section S_c can be unambiguously determined from the obtained value of S_w and the shape of the end surface of the armature.

3. Determination of the magnetizing force F. The drop in magnetizing force in the working gap for $\delta = \delta_0$ is

$$F_{w0} = H_w\delta_0 = \frac{B_{w0}}{\mu_0}\delta_0.$$

Assuming that the losses of the magnetizing force $F_{l0} \approx 0.2$—$0.5\ F_{w0}$, we obtain for the total magnetizing force of the coil the expression

$$F = (1.2\text{-}1.5)F_{w0}.$$

4. Determination of the coil length and thickness. At first we must assign the coil length-to-thickness ratio, i.e., l_{cl}/h_{cl}. When this ratio increases the copper expenditure decreases and the sensitivity increases, but the expenditure of ferromagnetic material also increases. For the bar-type electromagnets shown in Figs. 1.1a, 1.1b and 1.1c it is recommended to take $l_{cl}/h_{cl} = 1\text{-}8$, this value being the larger, the smaller the design factor $\sqrt{F_{p0}}/\delta_0$.

The optimum values of l_{cl}/h_{cl} for plunger-type electromagnets lie in the interval 5-8 and are plotted in Fig. 1.19 as a function of $\sqrt{F_{p0}}/\delta_0$.

The coil dimensions are determined in the manner indicated in § 1.5, dealing with coil design; thus, for example, in determining the coil heating we are using the formula (1.32). The filling factor entering in this formula can be tentatively taken as $K_{f\,cl} \approx 0.5$.

IV. Exact calculation

1. After having found by a preliminary calculation the dimensions of the magnetic circuit, we determine for several values of the working gap (as a rule, for three values — the initial value δ_0, an intermediate value, and the final δ_f) the conductances G of the working and idle air gaps and the leakage coefficients σ_{lk} (see § 1.4).

2. We construct the magnetization curves $\Phi_w = f(F_l)$ for the values of the working gap δ used above (see § 1.4e).

3. We check the optimality of the earlier-selected value of the working-gap cross section S_w; if necessary, we make an appropriate correction.

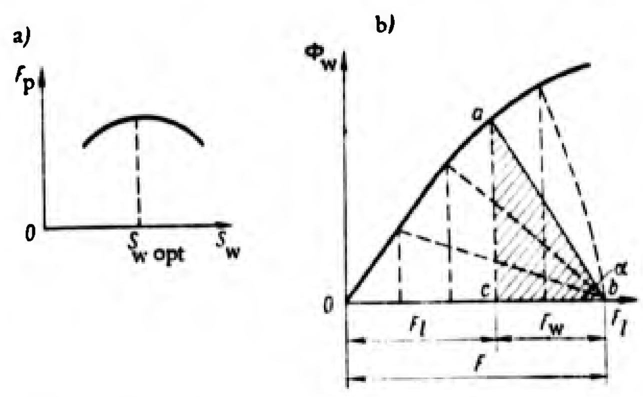


Fig. 1.20. Determination of optimum working-gap cross section.

As a matter of fact, if we alter S_w for a fixed value of the magnetizing force of the coil, the pull force of the electromagnet will also change, and (as can be seen from Fig. 1.20a) at a certain value of S_w it will be maximal. This value of S_w is precisely the optimal value, since it maximizes the pull force of the electromagnet, and hence also the work performed (with a fixed input-signal power and fixed dimensions of the electromagnet).

The value of S_w is found in the preliminary calculation as a result of selecting the value of the induction in the working gap (B_{w0}). Therefore the optimality of S_w is determined by the optimality of the value of B_{w0} selected by us. The values of B_{w0}, plotted in Fig. 1.19, according to which B_{w0} is selected in the preliminary calculation, have in fact be determined on the basis of the condition of maximization of the work performed by the electromagnet. Now, in the exact design stage, we must check these preliminary calculations; for this purpose we can use the magnetization curve $\Phi_w = f(F_l)$ for $\delta = \delta_0$ (Fig. 1.20b).

The area of the triangle abc is equal to $1/2\ \Phi_w F_w = 1/2\ \Psi_w I = W_M$, which is the energy of the magnetic field in the gap. This energy determines the magnitude of the work performed by the electromagnet.

With a fixed value of the magnetizing force F the area of the triangle abc is determined by the value of the angle α, equal to

$$\alpha = \operatorname{arctg} G_{w0},$$

where in the case of a uniform field between parallel planes we have $G_{w0} = \mu_0 \delta_0 / S_w$, i.e., it is determined by the working gap cross section S_w.

Thus the optimum value of S_w is determined by the value of the angle α that maximizes the area of the triangle abc.

The thus obtained value $S_{w.opt}$ is taken as the final value. If it does not coincide with the value found earlier in the preliminary calculation, the latter must be repeated, since a change in S_w is tantamount to a change in B_{w0}, i.e., it involves a corresponding alteration of the cross sections and of the other parts of the magnetic circuit. Only in the case of a bar-type electromagnet with a pole piece does a change in S_w involve a change in the dimensions of the pole piece alone, without affecting the dimensions of the core and of the other parts of the magnetic circuit.

4. We construct the pull characteristic with the aid of the earlier-constructed magnetization curves (see § 1.4E), and we perform the final correction of the value of the coil magnetizing force F if it is found to be useful to vertically move the pull characteristic with respect to the opposing-force characteristic.

5. We perform the coil design calculations (see § 1.5).

6. We determine the time parameters and the other service parameters of the electromagnet.

§ 1.9. Alternating Current Electromagnets

A. The Peculiar Features of Operation of an AC Electromagnet

An electromagnet can be operated (i.e., it attracts the armature) not only by a direct current, but also by an alternating current. This is due to the fact that by formula (1.5) the pull force of an electromagnet is proportional to the square of the magnetizing force, and hence also to the square of the coil current. For this reason the sign of the pull force remains unchanged although an alternating current changes periodically its direction. In order to convince ourselves of this we shall utilize Maxwell's formula (1.9):

$$F_p = \frac{\Phi_w^2}{2\mu_0 S_w}.$$

If the coil current is alternating, the magnetic flux Φ_w produced by this current in the working gap will also be variable, i.e.,

$$\Phi_w = \Phi_{w.м} \sin \omega t.$$

By substituting this expression into Maxwell's formula we obtain

$$F_p = F_{p.м} \sin^2 \omega t, \tag{1.48}$$

where $F_{p.м} = \frac{\Phi_{w.м}^2}{2\mu_0 S_w}$.

The time variation of the pull force is plotted in Fig. 1.21.

The armature will be attracted under the action of the mean value of the pull force, i.e., of its dc component $F_{p.mean}$. The quantity $F_{p\ mean}$ can be determined from formula (1.48) by introducing into it

$$\sin^2 \omega t = \frac{1}{2}(1 - \cos 2\omega t).$$

Hence

$$F_p = \frac{F_{p.M}}{2}(1 - \cos 2\omega t) = F_{p.mean} - F_{p\sim}, \qquad (1.49)$$

where $F_{p.mean} = \frac{F_{p.M}}{2} = \frac{\Phi^2_{W.M}}{4\mu_0 S_W}$ is the dc component of the pull force, and $F_{p\sim} = \frac{F_{pM}}{2}$ $\cos 2\omega t$ is the ac component.

The dc component of the pull force of an ac electromagnet is much smaller than the pull force of a dc electromagnet of the same size, this being due to the less efficient utilization of the material of the magnetic circuit.

Thus if $\Phi_{W.M} = \Phi_{W-}$, where Φ_{W-} is the flux in the case of a direct coil current (i.e., if we assume same saturation of the magnetic circuit for alternating and direct coil currents), then the mean values of the pull force produced by an alternating current are only half as much (according to formula [(1.49)]) as the values obtained for a direct current.

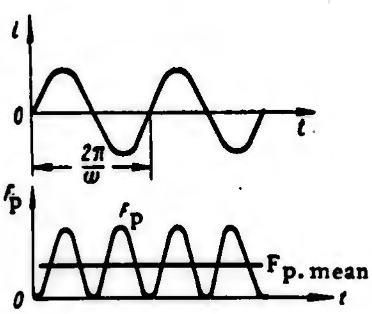


Fig. 1.21. Plots of coil current i and of pull force F_p of an ac electromagnet.

The shape of the pull characteristic of an ac electromagnet likewise differs from the shape of the pull characteristic for a direct current. This is due to the fact that the magnitude of the alternating current in the coil of the electromagnet depends not only on the coil resistance as in the case of a direct current, but also on its inductive reactance, which varies with the working-gap length δ. Indeed, in the case of a dc signal the coil current

$$I = \frac{U}{R} = \text{const}$$

being independent of the position of the armature, i.e., of δ. In the case of an ac signal, on the other hand, the effective value of the coil current is

$$I = \frac{U}{\sqrt{R^2 + X^2}},$$

where, according to formula (1.39), the inductive reactance of the coil

$$X = \omega L \approx \omega W^2 G_w.$$

Thus whereas according to Maxwell's formula (1.9) the pull force of a dc electromagnet increases sharply with decreasing δ (the curve F_{p-} in Fig. 1.22), since this is accompanied by an increase in the flux $\Phi = FG$, where $F = IW = \text{const}$, we have in the case of an alternating current a much smaller increase in the pull force with decreasing δ, since this is accompanied by a decrease in the magnetizing force F (the curve $F_{p.\,mean}$ in Fig. 1.22). In the limit, when the coil resistance can be neglected as compared to the inductive reactance, the entire input voltage U will be balanced by the self-induction emf alone:

$$U = \omega W \Phi \text{ (we are neglecting the leakage)},$$

i.e., the flux

$$\Phi = \frac{U}{\omega W} = \text{const}$$

being independent of δ.

According to Maxwell's formula the pull characteristic of the electromagnet will be horizontal in this case (the characteristic $F_{p.\,mean}$, $R = 0$ in Fig. 1.22).

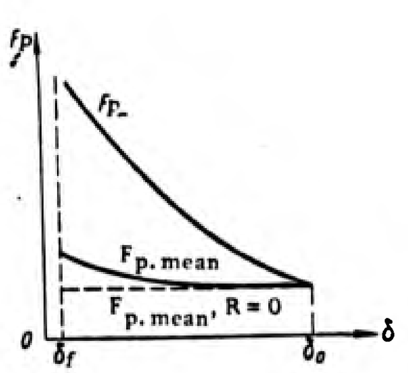


Fig. 1.22. Pull characteristics of an ac electromagnet.

Thus, whereas the operating conditions of the coil of a dc electromagnet are characterized by the relations $U = \text{const}$, $I = \text{const}$, $\Phi = \text{var}$, we have for an electromagnet the opposite relations, i.e., $\Phi = \text{const}$ and $I = \text{var}$.

Thus, in this manner it has

been established that the shape of the pull characteristic of an ac electromagnet depends on the relative magnitude of the inductive reactance of the coil with respect to its impedance. Thus it follows also that the shape of the pull characteristic of an ac electromagnet depends also on the circuit diagram of the coil: if an auxiliary impedance Z_A is connected in series with the coil, the pull characteristic will not only drop lower, as in the case of a direct current, but also become steeper. (When $Z_A \to \infty$, the pull characteristic of an ac electromagnet approaches the pull characteristic for the dc case).

So far we examined the mean value of the pull force of an ac electromagnet. Yet, as was shown above, its instantaneous value is not equal to its mean value. The former is not constant in time, but pulsates with a double frequency 2ω and vanishes twice during a period of the input signal. Therefore the armature of the electromagnet may vibrate, being periodically repulsed from the core by the counter force. In order to eliminate armature vibrations in ac electromagnets one takes special design measures; these will be considered in Section B below.

As a result of these peculiar features of operation of ac electromagnets, the design of such electromagnets differs considerably from that of dc electromagnets.

In fact, the performance of ac electromagnets is much poorer than that of dc electromagnets, since (assuming equal size of the two types) the former produce less pull, have lower sensitivity, and much poorer parameter stability. Moreover, they are more complex and hence also more costly in view of the need to use a laminated magnetic circuit in the case of an alternating current and to take special design measures for the prevention of armature vibrations.

For this reason, although alternating current is much more common than direct current, dc electromagnets are nevertheless much more often used than ac electromagnets; in particular, it is often more convenient to use instead of an ac electromagnet a dc electromagnet with an input rectifier.

B. Methods of Preventing Armature Vibrations

The armature vibrations of an ac electromagnet can be avoided by using any of the following three methods:

1) A weighted armature;

2) A multiphase electromagnet;

3) A short circuited winding (slug), enclosing a portion of the core cross section.

In case of a weighted armature there are no vibrations, since the great lag of the armature makes it impossible for it to vibrate at a frequency 2ω, i.e., it has no time to move away from the core at the instants when the coil current passes through zero. Since an increase in the armature dimensions is accompanied by a decrease in the sensitivity of the electromagnet and by an increase in its size and overall weight, such a method of preventing vibrations is seldom used. Yet it comes into effect by itself in those cases in which the actuating mechanism, driven by the electromagnet, has sufficient lag for cancelling the vibrations of the moving parts of the electromagnet.

A multiphase electromagnet — normally biphase or triphase, consists of two or three electromagnets supplied by phase-shifted currents and having a single common armature. Since the currents pass through zero at different instants of time, the resultant force acting on the armature never vanishes, and with an appropriate choice of the phase shoft between the currents this force will be constant altogether.

In a biphase electromagnet the resultant pull force is constant, i.e., it does not contain a variable component if the coil currents of the two electromagnets are shifted by $\pi/2$. Indeed, in this case the resultant pull force [by virtue of formula (1.48)] is

$$F_p = F_{p1} + F_{p2} = F_{p1M} \sin^2 \omega t + F_{p2M} \sin^2 \left(\omega t + \frac{\pi}{2}\right)$$

and when $F_{p1M} = F_{p2M} = F_{p.M}$ it is equal to

$$F_p = F_{p.M} (\sin^2 \omega t + \cos^2 \omega t) = F_{p.M} = \mathrm{const},$$

where

$$F_{p.M} = \frac{\Phi^2_{W.M}}{2\mu_0 S_W}.$$

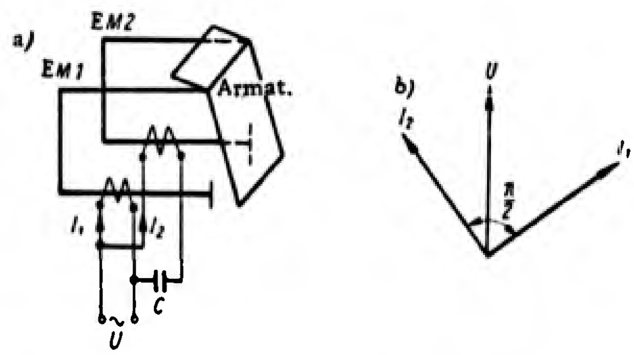


Fig. 1.23. Biphase electromagnet: a — design principle of device: b — vector diagram for coil currents.

Let us note that in these, just as in other ac electromagnets, the pull force is only half as strong as it would be if a direct current, equal to the maximum value of the alternating current, is passed through the coil.

Biphase electromagnets are supplied by a single input voltage, applied in parallel to the coils of the two electromagnets. In order to obtain a phase shift between the coil currents, a capacitor C is connected in series with one of the coils (Fig. 1.23).

The coils of a triphase electromagnet are supplied from a triphase ac source. In the case of a symmetrical coil current system, when the currents are equal in magnitude and shifted by $2\pi/3$ in phase, the resultant force is equal [according to formula (1.49)] to

$$F_p = F_{p1} + F_{p2} + F_{p3} = \frac{3}{2} F_{p.m} + \frac{F_{p.m}}{2}\Big[\cos 2\omega t + \cos 2\left(\omega t + \frac{2}{3}\pi\right) + \cos 2\left(\omega t + \frac{4}{3}\pi\right)\Big] = \frac{3}{2} F_{p.m} = \text{const},$$

i.e., it is constant in time, since the sum of the three harmonics in square brackets, shifted with respect to one another by $2\pi/3$, is equal to zero.

Triphase electromagnets are used as power electromagnets only.

The last method of avoiding armature vibrations is to use a short-circuited winding (slug) that encloses a portion of the end of the core (Fig. 1.24a). This method is the one most widely used and can be characterized as follows. The core

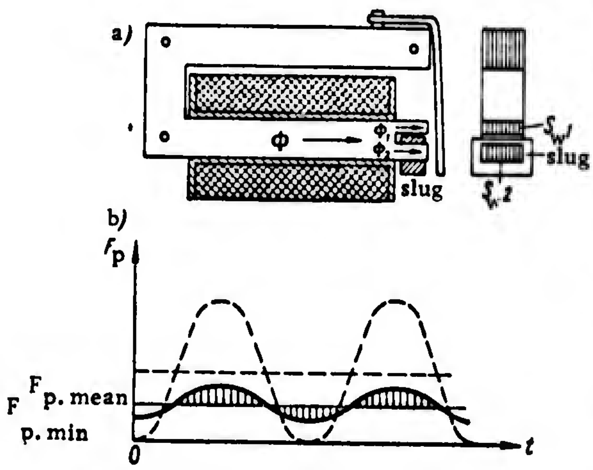


Fig. 1.24. An ac electromagnet with a short-circuited winding (slug) : a — sketch of magnetic system; b — time variation of pull force (the dashed line is the corresponding plot without a slug).

end is split into two parts, one of which is enclosed by a short-circuited winding (slug). In examining the techniques of changing the time parameters of electromagnets (§ 1.7), we already learned that a short-circuited winding counteracts changes in the flux permeating it, retarding this flux. Therefore a variable flux Φ_2 that passes through a section S_{w2} of the core end which is enclosed by a slug, lags in phase behind the flux Φ_1 that passes through another section S_{w1} of the core end. As a result, the armature is under the action of two phase-shifted magnetic fluxes. The total pull force generated by the two fluxes is

$$F_p = F_{p1} + F_{p2} = \frac{\Phi^2_{M1}}{2\mu_0 S_{w1}} \sin^2 \omega t + \frac{\Phi^2_{M2}}{2\mu_0 S_{w2}} \sin^2 (\omega t - \varphi) = F_{p.\,mean} + F_{p\sim},$$

$$F_{p.\,mean} = \frac{F_{p1M} + F_{p2M}}{2}; \tag{1.50}$$

$$F_{p\sim} = \frac{F_{p1M} \cos 2\omega t + F_{p2M} \cos 2(\omega t - \varphi)}{2};$$

$$F_{p1M} = \frac{\Phi^2_{1M}}{2\mu_0 S_{w1}};$$

$$F_{p2M} = \frac{\Phi^2_{2M}}{2\mu_0 S_{w2}}.$$

F_p never vanishes, since the two fluxes pass through zero at different instants of time. This is illustrated by Fig. 1.24b. For the total elimination of the variable component of the pull force it is necessary:

1) that $F_{p1M} = F_{p2M}$, or, according to formula (1.50), that

$$\frac{\Phi^2_{1M}}{\Phi^2_{2M}} = \frac{S_{w1}}{S_{w2}}; \tag{1.51}$$

2) that the phase angle between the fluxes Φ_1 and Φ_2 should be

$$\varphi = \frac{\pi}{2}. \tag{1.51a}$$

The second condition cannot be fully met, since φ can be equal to $\pi/2$ only if the resistance of the short-circuited winding is equal to zero. In practice one can obtain $\varphi = 60° - 80°$. Thus the pull force will contain a variable component. As can be seen from Fig. 1.24b, the mean value of the pull force ($F_{p.mean}$) is smaller in the presence of a slug as compared to the case of no slug.

C. The Peculiar Features of Design of Ac Electromagnets

The general order of design is the same in this case as in the case of dc electromagnets (§1.3). One starts again with a preliminary calculation, permitting the determination of the dimensions of the magnetic circuit.

In such a calculation one assigns the same values of the induction in the working gap as were recommended above for a dc electromagnet; yet here these will be the amplitude values of $B_{w.M}$. In the formula for the design factor, on the other hand, one must introduce double the value for the force, i.e., take it in the form $\sqrt{2F_{p0}}/\delta_0$.

Thus one takes into account that the same electromagnet produces twice the force when it is supplied by a direct current as compared to the case of an alternating current, the saturation of the magnetic circuit being the same. The optimum ratio l_{cl}/h_{cl} is smaller for ac electromagnets as compared to dc electromagnets; for this reason the coils are shorter and thicker [2, 5]. This is due to the fact that in an ac electromagnet we have input signal power dissipation in the form of eddy currents and hysteresis in the magnetic circuit, these losses being proportional to the volume of the magnetic circuit. In order to diminish these losses we must reduce the length of the core, i.e., the quantity l_{cl}. Such a decrease is limited by the losses in the copper of the coil, which are increasing thereby. Indeed, when the coil is shortened, its cross section increases proportionally, thus causing the coil resistance to increase owing to an increase in the average length of its turns. The optimum value of l_{ch}/h_{cl}, corresponding to minimum total power losses, is hence smaller than in a dc electromagnet, which has only losses in the copper.

In the final calculations one takes into account the flux leakage and the losses of the magnetizing force. Yet in the case of an ac electromagnet there arises also the need to determine the hysteresis power losses P_h of the input signal and the eddy current losses in the magnetic circuit (P_E) and in the slug (P_S), if the latter is used:

$$P_l = P_h + P_E + P_S \tag{1.52}$$

The quantity P_S is determined by the current in the slug and by its resistance [2], whereas P_h and P_E are specified by the formulas

$$P_h = \sigma_h \frac{fB_M^2}{100} V_c\gamma \text{ [watt]}; \tag{1.53}$$

$$P_E = \sigma_E \left(\frac{fB_M}{100}\right)^2 V_c\gamma \text{ [watt]}; \tag{1.54}$$

where B_M is expressed in $V \cdot sec/m^2$ = tl, and f in cps; V_c is the volume of the magnetic circuit in m^3, γ is the specific weight of the material of the magnetic circuit in kg/m^3, and δ_h and δ_E are electrical coefficients that depend on the grade of the material and the thickness of the slug [2].

Since P_h and P_E depend on the induction B_M, and since the latter varies in the different portions of the magnetic circuit, the hysteresis and eddy-current losses must be calculated for the individual portions of the magnetic circuit in the same way as the losses of the magnetizing force in the magnetic circuit, with subsequent summation of the individual losses.

As a result of the power losses in the magnetic circuit of an ac electromagnet, the design of its coil is more complicated than in the case of a dc electromagnet.

The number of turns of the coil can be determined by the formula

$$U_M = \omega W \Phi_M,$$

whence

$$W = \frac{U_M}{\omega \Phi_M}.$$

The wire diameter in coil heating calculations is determined on the basis of the total current

$$I = \sqrt{I_l^2 + I_\mu^2}, \qquad (1.55)$$

where I_l is the loss current and I_μ is the magnetizing current.

For illustration we present in Fig. 1.25 the equivalent electric circuit of an ac electromagnet.

If the voltage drop across the coil resistance R and the leakage flux are neglected, we obtain according to Fig. 1.25 for the loss current

$$I_l = \frac{P_l}{U}.$$

The magnetizing current I_μ, which produces the magnetic flux in the magnetic circuit, can be calculated from the magnetizing force

$$I_\mu = \frac{F}{W},$$

where F can be determined from the magnetic circuit.

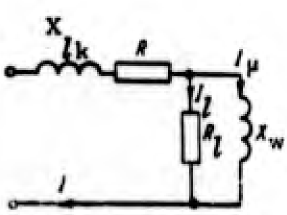


Fig. 1.25. Equivalent electric circuit of ac electromagnet. R — coil resistance; Xw — component of inductive reactance of coil due to working flux; X_{lk} — component of inductive reactance of coil due to leakage flux; R_l — resistance due to losses in the magnetic circuit.

When the losses of the magnetizing force are neglected we have

$$I_\mu = \frac{\Phi_W}{W G_W}.$$

In designing an electromagnet with a short-circuited winding (slug), the preliminary calculations can be performed according to $F_{p.\,mean}$ without taking into account the slug, whereas the final calculations must be performed according to $F_{p.\,min}$ (see Fig. 1.24b). In order to eliminate the armature vibrations it is thereby necessary that $F_{p.\,min}$ should exceed the opposing force F_{opp}. The peculiar feature of designing an electromagnet with a short-circuited winding consists in the need to calculate this winding and to select the ratio S_{w2}/S_{w1} [2, 5]. Here it is of primary importance to obtain as large as possible a value of $F_{p.\,min}$, since it is precisely this value that is compared with F_{opp}, and hence it determines the magnetizing force of the coil. The optimum ratio $S_{w2}/S_{w1} = 2 - 4$, and it depends on the manner in which the signal is fed to the coil. In any case, as we can see, the cross section enclosed by the short-circuited winding (S_{w2}) is taken larger than S_{w1}.

In determining the time parameters of the electromagnet, the pickup time is neglected, since it normally does not exceed a quarter of the period of the input voltage (at a frequency of 50 cps this amounts to 5 msec), i.e., it is much smaller than in the case of a dc electromagnet.

Chapter 2

POLARIZED ELECTROMAGNETS

In contrast to the neutral electromagnets considered above, the direction of the pull force in a polarized electromagnet depends on the polarity of the dc signal fed to its coil. Let us consider, for example, one of the common models of polarized electromagnets, shown schematically in Fig. 2.1a. In addition to the working flux Φ_w produced by the input coil (in the given case it consists of two halves), we have here also the polarizing flux Φ_p produced by a permanent magnet. Thus the pull force acting on the armature is determined by the two fluxes Φ_w and Φ_p jointly. The change in the direction of the pull force accompanying a change in the polarity of the coil current is due to the fact that this involves a change in the direction of the working flux Φ_w with respect to the polarizing flux Φ_p.

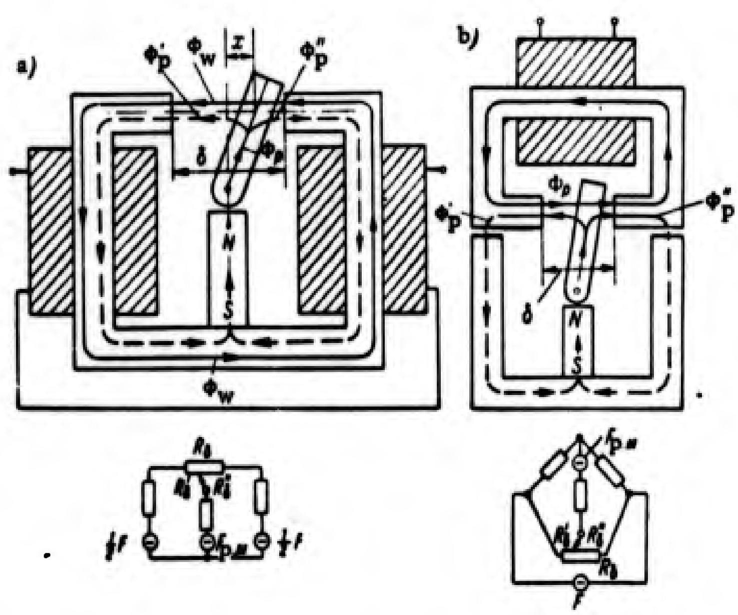


Fig. 2.1. Polarized electromagnets: a — differential; b — bridge-type.

The polarizing flux Φ_p emerging from the armature branches off into two parts Φ_p' and Φ_p'' in accordance with the conductances of the air gaps to the right and to the left of the armature. Depending on the polarity of the input signal, the working flux Φ_w is added to Φ'_p in the gap to the left of the armature and subtracted from Φ''_p to the right (as shown in Fig. 2.1a), or conversely. The resultant pull force acting on the armature will be directed towards the gap in which the fluxes are added.

Besides the dependence of the direction of the pull force on the polarity of the input signal, which constitutes the main difference between polarized and neutral electromagnets, polarized electromagnets have also another distinctive feature: their sensitivity, speed, and transfer ratio are higher than in the case of neutral electromagnets. The maximum sensitivity of polarized electromagnets is characterized by an operating power of up to 10^{-5}w. Their operate time can be reduced to a few milliseconds. All these advantages are obtained at the price of a more complicated design and of increasing severalfold the dimensions and the cost as compared to neutral electromagnets.

Polarized electromagnets are utilized as low-power reversible transducers of electrical signals into displacement and force (both for continuous and relay action). Thus they are used for driving the shutters and valves of hydraulic and pneumatic devices. But the principle utilization of polarized electromagnets is in polarized electromagnetic relays and in dc-to-ac and ac-to-dc vibroconverters (see §4.4 B, C).

Polarized electromagnets can be of differential and bridge type. An example of a differential polarized electromagnet is the already considered electromagnet, depicted in Fig. 2.1a. In Fig. 2.1b we show a model of a bridge-type polarized electromagnet. Its action is also based on the fact in one air gap (at one side of the armature) the working flux is in the same direction as the polarization flux, whereas in the other air gap it is in the opposite direction. The designations "differential" and "bridge" stem from the type of equivalent circuit of the electromagnets, presented in the same figure.

Although bridge-type polarized electromagnets are more complex than differential ones, they have better parameters (smaller size and higher sensitivity and parameter stability).

The polarizing flux must not necessarily be produced by a permanent magnet, but can be generated by a special coil, fed by a direct current. For example, in the electromagnet depicted in Fig. 2.1a the permanent magnet can be replaced by a core of magnetically soft material with the same coil, producing a constant magnetizing force. Yet in most cases one uses a permanent magnet, since this is simpler to do.

Let us consider the pull force of a polarized electromagnet. For definiteness we shall examine the same electromagnet, depicted in Fig. 2.1a.

At first let us express Φ'_p and Φ''_p in terms of the displacement of the armature from its middle position x. We have

$$\Phi'_p + \Phi''_p = \Phi_p;$$

$$\frac{\Phi'_p}{\Phi''_p} = \frac{R''_M}{R'_M} = \frac{\frac{\delta''}{\mu_0 S}}{\frac{\delta'}{\mu_0 S}} = \frac{\delta''}{\delta'} = \frac{\frac{\delta}{2} - x}{\frac{\delta}{2} + x},$$

where S is the working gap cross-section.

Hence we obtain the evident formulas:

$$\left.\begin{aligned} \Phi'_p &= \frac{\frac{\delta}{2} - x}{\delta}\Phi_p; \\ \Phi''_p &= \frac{\frac{\delta}{2} + x}{\delta}\Phi_p. \end{aligned}\right\} \qquad (2.1)$$

If we take into account the direction of Φ_w, indicated in Fig. 2.1a, then the resultant fluxes occuring in the gaps to the left and to the right of the armature will be expressed as

$$\Phi' = \Phi'_p + \Phi_w;$$
$$\Phi'' = \Phi''_p - \Phi_w.$$

The force produced by each of these fluxes will be according to Maxwell's formula:

$$F'_p = \frac{\Phi'^2}{2\mu_0 S};$$
$$F''_p = \frac{\Phi''^2}{2\mu_0 S}$$

The resultant pull force acting on the armature:

$$F_p = F_p'' - F_p' = \frac{\Phi''^2}{2\mu_0 S} - \frac{\Phi'^2}{2\mu_0 S} = \frac{1}{2\mu_0 S}[(\Phi_p'' - \Phi_w')^2 - (\Phi_p' + \Phi_w)^2] =$$
$$= \frac{1}{2\mu_0 S}[(\Phi_p''^2 - \Phi_p'^2) - 2\Phi_w(\Phi_p' + \Phi_p'')].$$

By introducing into this formula the expressions (2.1) for Φ_p' and Φ_p'', and by replacing $(\Phi_p' + \Phi_p'')$ by Φ_p, we finally obtain

$$F_p = \frac{\Phi_p}{\mu_0 S}\left(\Phi_p \frac{x}{\delta} - \Phi_w\right). \quad (2.2)$$

The flux occurring in formula (2.2) is expressed as

$$\Phi_w = FG_M,$$

where F is the magnetizing force produced by the coil current, and G_M is the magnetic conductance of the path of the flux Φ_w.

Thus the pull force of a polarized electromagnet consists of two components, i.e.,

$$F_p = F_{p.p} + F_{p.w} = Ax + BF, \quad (2.3)$$

where $F_{p.p} = \frac{\Phi_p^2}{\mu_0 S}\frac{x}{\delta} = Ax$ is a force, proportional to the armature displacement x, that has been produced by the polarizing flux alone, and $F_{p.p} = -\frac{\Phi_p \Phi_w}{\mu_0 S} = -\frac{\Phi_p G_M}{\mu_0 S}F = BF$ is a force which is proportional to the magnetizing force of the coil, i.e., to the coil current; hence this force exists only in the presence of the input signal.

In Fig. 2.2 we plotted the pull characteristics of a polarized electromagnet corresponding to formula (2.3).

In order to be able to use a polarized electromagnet as a continuous current-to-displacement transducer, it is necessary to generate a counter force F_{opp} that increases linearly with x and is stronger than $F_{p.p}$, i.e., a force which can be represented in the form of the dashed line in Fig. 2.2. (Such a counter force is normally produced by a spring). In the absence of a coil current (F = 0), the armature will be in this case at its middle position (x = 0), being held there by the counter force.

When a current is applied to the coil the armature will move by a distance x, proportional to the strength of the current. The equilibrium equation of the armature will be in this case:

$$F_p = F_{opp}, \tag{2.4}$$

where $F_{opp} = Cx$, and C is the stiffness of the spring.

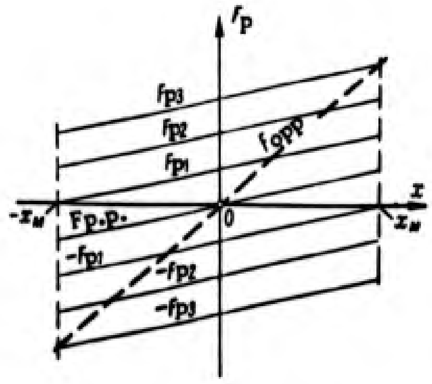


Fig. 2.2 Pull characteristics of a polarized electromagnet.

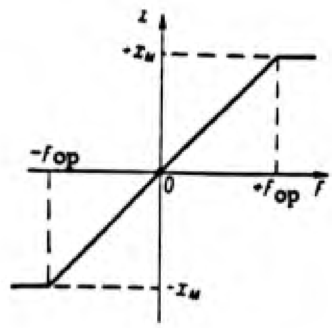


Fig. 2.3 Static characteristic of a polarized electromagnetic transducer.

By introducing into this formula the expression (2.3) for F_p, we obtain the equation of the static characteristic of the transducer:

$$x = \frac{B}{C - A} F. \tag{2.5}$$

This characteristic is plotted in Fig. 2.3.

When the counter force, produced by the actuating mechanism driven by the transducer, is taken into account, this force must be added to the right-hand side of equation (2.4). As a result it will reduce the slope of the characteristic x (F), and when this force depends nonlinearly on the displacement, the linearity of the characteristic will be disturbed.

In the absence of a counter force F_{opp} or when $F_{opp} < F_{p.p}$, a polarized electromagnet acts like a relay element. In the steady state, its armature can occupy either of two limiting positions, i.e., at one or at the other pole of the electromagnet ($x = \pm x_M$).

When the input signal is switched off, the armature is held at one of the extreme positions by the force $F_{p.p}$, since it is larger than F_{opp}. By substituting $\Phi_w = 0$ into (2.2), we find the value of this force for $x = x_M$:

$$F_{p.p.M} = \frac{\Phi_p^2 x_M}{\mu_0 S \delta}. \tag{2.6}$$

(The quantity x_M is determined by the stops limiting the travel of the armature; these stops can be also the poles of the electromagnet).

In order to throw the armature from one extreme position to another, we must feed a current to the coil whose direction is such that the pull force $F_{p.w}$ is opposite to $F_{p.p}$; the magnitude of the operate current is specified as

$$F_{p.w.op} = F_{p.p.M}.$$

By introducing into this formula the expression for $F_{p.w}$ from (2.3) and the expression for $T_{p.p.M}$ from (2.6), we obtain the magnetizing force for operation:

$$F_{op} = \frac{\Phi_p x_M}{G_M \delta}.$$

If the losses of the magnetizing force are neglected, by assuming that

$$G_M = \frac{\delta}{\mu_0 S},$$

then

$$F_{op} = \frac{\Phi_p x_M}{\mu_0 S}. \tag{2.7}$$

By comparing the formulas (2.7) for F_{op} and (2.6) for $F_{p.p.M}$, one can see that the sensitivity of the electromagnet and the force produced by it involve contradictory requirements: in order to increase the sensitivity (diminish F_{op}) one must reduce the armature stroke x_M, whereas in order to increase the pull force one must increase x_M. For this reason highly sensitive polarized electromagnets that serve as relays are designed as low-power devices. and an increase in the force developed by these electromagnets is invariably accompanied by a loss in sensitivity. Incidentally. such a contradiction will be encountered in one or another form also for the other elements examined in this book.

As a relay element one can also use the polarized electromagnet with

$$F_{opp} = Cx > F_{p.p},$$

considered by us above and which has a linear static characteristic $x = f(F)$. For this purpose we must apply to it a signal that produces a magnetizing force which exceeds the value corresponding to the end of the linear section of the static characteristic (F_{op} in Fig. 2.3). In contrast to a two-position relay mode of operation with $F_{opp} < F_{p.p}$, we obtain in this case, i.e., with $F_{opp} > F_{p.p}$, a three-position relay mode. Thus the armature can occupy one of the following three positions: in the absence of a signal it is at the middle position; in the presence of a signal and $F > F_{op}$ it is at one of its extreme positions ($x = \pm x_M$), depending on the polarity of the signal; after the signal has been switched off, the armature returns to the middle position.

Details about polarized electromagnets and their utilization can be found in [7].

Chapter 3

ELECTROMECHANICAL CLUTCHES

§ 3.1. Utilization and Classification of Electromechanical Clutches

A clutch is a device used for the coupling of two shafts, i.e., for the transmission of torque from one shaft (the driving shaft) to another (the driven shaft). The driving shaft is run by a motor, whereas the driven shaft is connected to the load. A clutch is said to be electromechanical if the mechanical torque is transmitted by electrical processes. The clutch is controlled in this case with the aid of an electrical signal; thus it serves as a converter of an electrical signal into a mechanical torque. Since the power developed at the driven shaft is much greater than the power of the controlling signal, an electromechanical clutch is at the same time increasing the signal power. The motor turning the driving shaft has from an energy point of view the same function as the supply source in an ordinary electric amplifier. In either case the electrical input signal controls the power flow from this source towards the load, just as a water-pipe cock controls the flow of water.

There are relay clutches which perform rigid coupling of the shafts when a signal is applied, and continuous-duty clutches which perform flexible coupling, when the speed of the driven shaft depends on the magnitude of the input signal and this dependence can be represented by a smooth static "driven-shaft speed vs. input signal" characteristic. The clutches of primary interest for automatic systems are clutches with flexible coupling.

Electromechanical clutches are used as actuating elements. They replace controlled motors with variable speed of rotation, thus permitting the use of uncontrolled electric motors as driving motors. Such a replacement generally simplifies the circuitry and instrumentation of the automatic system as a whole and increases its reliability.

Yet the main advantage of electromechanical clutches over ordinary controlled electrical motors is their greater speed. This is due to the fact that the moment of inertia of the rotating parts of a clutch, that are connected to the driven shaft, can be much smaller than the moment of inertia of a driving motor, which determines the speed of the load velocity control system acting on the motor and not on the clutch.

Electromechanical clutches are used at powers ranging from a few watts (for instance in low-power instrument-type servomechanisms) to tens of thousands of kilowatts (in the electrical motors of rolling mills and metal-cutting machines, in the motors of the propeller screws in ships).

According to their operating principle, electromechanical clutches are of two main types: electromechanical friction clutches and electromechanical slider clutches (induction type).

§ 3.2. Electromechanical Friction Clutches

In friction clutches the torque at the driven shaft is produced by a frictional force. Electromechanical friction clutches are subdivided into dry-friction clutches and ferropowder (magnetic emulsion) friction clutches.

A. Electromechanical Dry-Friction Clutches

The operating principle of dry-friction clutches is illustrated in Fig. 3.1a. The clutch consists of two members 1 and 2 (semi-clutches), on which the friction disks 3 (made of friction material) are fixed. One clutch member can move along its shaft on a key, whereas the other is rigidly fixed to its shaft. If the two parts of the clutch are pressed against each other, the driven shaft which was hitherto at rest begins to rotate together with the driving shaft, i.e., the shafts are engaged as a result of the friction force, thus preventing slippage of the friction disks.

Figure 3.1b illustrates the operating principle of a reversible friction clutch that makes it possible to change the direction of rotation of the driven shaft. It constitutes a combination of two simple nonreversible clutches: the driven disk 1 is placed between the two driving disks 2 and 3 which rotate in different directions.

The direction of rotation of the driven disk can be altered by pressing it against either of the two driving disks. The driving disks are started by two separate motors or by one common motor. In the latter case one of the disks is connected to the motor via a gear that changes its direction of rotation as compared to the other disk.

If fast braking is required, the driven shaft of the clutch is combined with a brake. Such a combined clutch looks like the reversible clutch of Fig. 3. 1b, apart from the fact that the second driving disk is being replaced by a fixed disk, rigidly attached to the base of the clutch. The driven disk is braked by pressing it against this fixed disk.

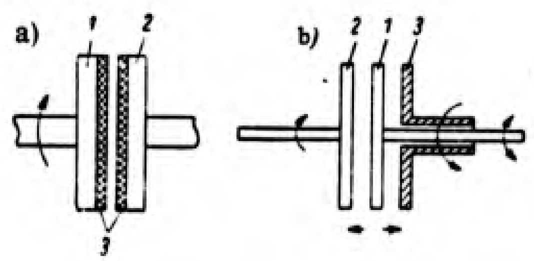


Fig. 3. 1. Operating principle of dry-friction clutches: a — nonreversible; b — reversible

The two parts of a clutch can be pressed against each other by a hydraulic or a pneumatic servomotor. In this case we refer to a hydraulic or pneumatic friction clutch (a friction clutch with hydraulic or pneumatic control). If the parts of the clutch are pressed against each other by means of an electromechanical transducer, we talk about an electromechanical friction clutch.

The most widely used electromechanical friction clutches are those with electromagnetic control, in which the two members of the clutch are engaged by an electromagnet. The fastest electromechanical friction clutches have magnetoelectric control. In this case the electromagnet is replaced by a magnetoelectric transducer of an electrical signal into displacement and force.

Figure 3. 2a shows a schematic of the simplest nonreversible friction clutch with electromagnetic control. It is in the form of an electromagnet consisting of the armature 1 and the magnetic circuit 2 with coil 3. The current is supplied to the coil by means of two contact rings with brushes. The magnetic circuit and the armature are lined with the friction disks 4. The armature can move along one shaft on a key. The

magnetic circuit is rigidly fixed on the other shaft. The armature is pulled away from the magnetic circuit by a restoring spring (not shown in the figure). Therefore the driven shaft is at rest in the absence of a current through the coil. When an input signal is fed to the coil the armature is attracted to the magnetic circuit and the shafts are engaged.

Friction clutches with electromagnetic control are of two types: single-disk (Fig. 3.2a) and multiple-disk (Fig. 3.2b). Powerful clutches are of multiple-disk type, so as to permit their dimensions to be reduced.

Figure 3.3 shows a model of a friction clutch with electromagnetic control and a fixed coil. Such clutches are not only free from sliding contacts for supplying the current to the coil, but they are also faster (the coil constant is smaller). On the other hand their coil dimensions are greater owing to the larger air gap.

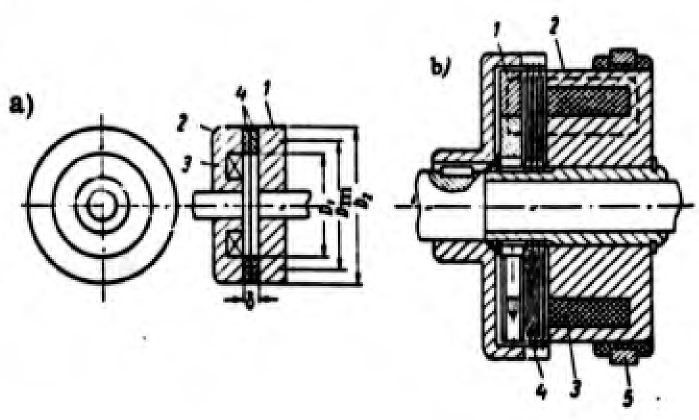


Fig. 3.2 Friction clutch with electromagnetic control: a — single-disk; b — multiple-disk
1 — armature; 2 — magnetic circuit; 3 — coil;
4 — friction disks; 5 — contact ring.

In electromagnetic friction clutches one uses dc electromagnets as a rule, since the latter have certain advantages over ac electromagnets. In the presence of an ac signal, the dc electromagnets are equipped with a rectifier.

The designing of friction clutches with electromagnetic control involves the designing of the friction surfaces and of the electromagnet. The initial data are the value of the torque to be transmitted by the clutch, and the dimensions of the clutch. Let us consider the order of designing a clutch on the example of a disk clutch.

The torque to be transmitted by the clutch is

$$M = m\int_{R_1}^{R_2} dM = m\int_{R_1}^{R_2} R\,dF_{fr} = 2\pi mpK_{fr}\int_{R_1}^{R_2} R^2 dR = \frac{2\pi}{3} mpK_{fr}(1-K_R^3)R_2^3. \quad (3.1)$$

Here m is the number of friction surfaces, determined by the number of friction disks; R_1 and R_2 are the inner and outer radii of the friction disks; $K_R = \frac{R_1}{R_2}$; K_{fr} is the friction coefficient, which depends on the material of the disks and on the state of their surfaces; p is the pressure by which the disks are pressed together by the electromagnet; $dF_{fr} = K_{fr}\, p 2\pi\, R dR$ is the frictional force on an elementary ring of width dR and radius R.

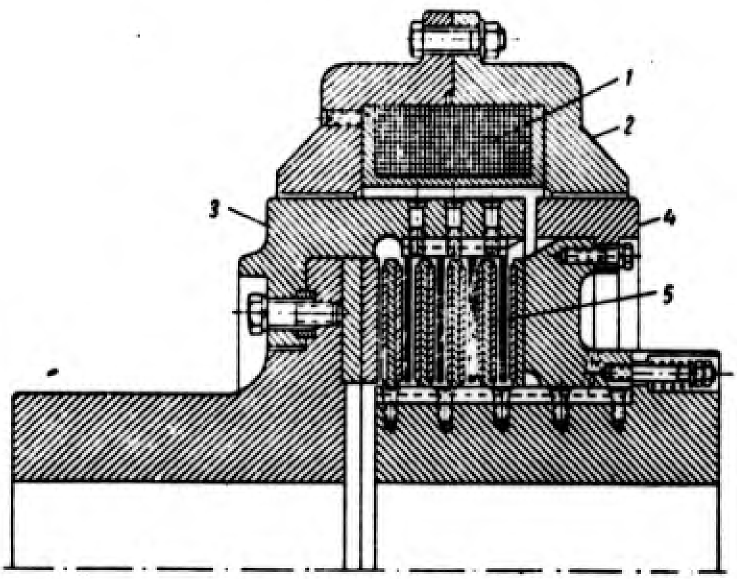


Fig. 3.3. Friction clutch with electromagnetic control and fixed coil. 1 — coil; 2 — magnetic circuit; 3 and 4 — clutch members; 5 — friction disks.

By assigning the outer radius R_2 of the friction disks, we can determine by formula (3.1) the number m of friction surfaces required for transmitting a prescribed torque M, or, conversely, with m assigned we can find R_2.

The pull force to be produced in this case by the electromagnet is

$$F_p = p\pi (R_2^2 - R_1^2) = p\pi (1 - K_R^2) R_2^2. \quad (3.2)$$

Then we perform the design calculations for the electromagnet. In order to reduce the coil dimensions, limited by the permissible heating, powerful clutches are endowed with liquid cooling. Less powerful clutches are equipped with ribs, which increase the cooling surface.

The value of the friction coefficient K_{fr} entering in formulas (3.1) and (3.2), as well as of the ratio $K_R = R_1/R_2$ and of the pressure p, and also a detailed calculation of electromechanical friction clutches can be found in [8]. The reader can get an idea about these quantities from the following values. The coefficient of friction K_{fr} for bodies of steel, cast iron and bronze is approximately 0.1-0.2; for special friction materials of type Ferrodo, etc., the coefficient K_{fr} is much higher (up to 0.4-0.8). The ratio $K_R = R_1/R_2$ is taken from 0.3 to 0.6. The pressure p = 2-3 kg/cm^2.

Let us consider the dynamics of operation of electromechanical friction clutches. The transient process from the instant at which the voltage is applied to the coil of the clutch (when the driven shaft is at rest) until the driven shaft attains its constant speed of rotation consists of the following three stages. The first stage, lasting from the switching on of the input signal to the instant at which the friction surfaces touch, is the stage of operating the electromagnet. Its duration is equal to the operate time t_{op} of the electromagnet. If it is necessary to reduce it, one uses the ordinary methods of changing the time parameters of electromagnets (see §1.7). The second stage terminates when the friction surfaces are fully engaged. This is the stage during which the slippage of the driven shaft with respect to the driving shaft ceases. Yet the transient process does not terminate at this point, since the engaging of the clutch members is accompanied by the application of the load, represented by the driven shaft, to the motor turning the driving shaft that was running idle before. Therefore the second stage is followed by the third stage, which constitutes the transient process for the driving motor that is due to the application of the load.

The total operate time of a clutch with electromagnetic control, equal to the sum of the three stages of the transient process, is of the order of tenths or hundredths of a second.

The friction clutches under consideration are mainly used for rigid coupling of shafts. Yet with their aid one can also smoothly vary the steady-state value of the speed of the driven shaft by pulsed control of the clutch. In this case the input signal will consist of voltage pulses that succeed each other at a constant frequency. During a pulse the clutch operates and the driven shaft is accelerated; during a pause between pulses the clutch is released and the shaft is slowed down. Thus the speed of the driven shaft fluctates about a mean value at the frequency of repetition of the control pulses.

The magnitude of this mean value of the speed can be controlled by changing the duty ratio of the input pulses.

B. Ferropowder Friction Clutches

These clutches are mainly used for the flexible coupling of shafts, although they are also utilized for rigid coupling.

The basic difference between a ferropowder friction clutch and an electromagnetically controlled dry-friction clutch like that considered above consists in the fact that the air gap between the clutch members is filled with ferromagnetic powder. The driven member, just as the driving member, are rigidly connected to their shafts. Thus the gap between the clutch members does not vary. The ferromagnetic filler is a mechanical mixture of ferromagnetic (normally iron) powder with a liquid or solid dielectric. The liquid dielectric most frequently used is oil of various kinds, and the solid dielectric is in the form of graphite powder or talc.

If such a ferromagnetic filler is placed in a magnetic field, the individual ferromagnetic particles begin to mix, attracting each other and forming entire chains directed along the field. Hence when the magnetic field increases the liquid filler becomes thicker, passing at first through a jelly-like state and then solidifying altogether.

Similarly, the solid filler goes over in the end from a quick powder into a solid body whose cohesion has been brought about by the forces of the magnetic field.

This property of ferromagnetic fillers to solidify in a magnetic field is being utilized in ferropowder friction clutches. In the absence of a signal in the coil of the electromagnet, only an insignificant torque is transmitted from the driving shaft to the driven shaft, this being due to the initially low viscosity of the ferromagnetic filler. When a current is passed through the coil, the magnetic flux produced in the gap between the clutch members causes an increase in the viscosity of the filler. As a result, the torque transmitted to the driven shaft increases. An increase in the coil current is accompanied by an increase in the torque of the driven shaft, this increase going on until this torque reaches the value of the total developed by the driving shaft, which occurs when the two members are rigidly coupled by the solidified filler.

Thus the torque on the driven shaft is determined by the coil current and is practically independent of the speed of rotation of the driving shaft. Figure 3.4a shows a typical static "output torque versus input signal" characteristic of a ferropowder friction clutch. Figure 3.4b shows a model of a ferropowder friction clutch of cylindrical type. There exist also disk-type clutches of this sort, like the dry-friction clutch considered above.

Ferropowder friction clutches are much more complicated in design as compared to dry-friction clutches. The main difficulty arises from the need to manufacture reliable seals that do not permit the powder to fall into the bearings (see Fig. 3.4b) and to prevent the escape of ferromagnetic powder from the filler under the action of the centrifugal force when the clutch rotates and its escape due to precipitation when the clutch is at rest.

Let us now examine the dynamics of ferropowder clutches. In view of the fact that in this case we have no moving armature (this refers, of course, to translatory motion) as in dry-friction clutches, ferropowder clutches are distinguished by a much greater speed. It can be assumed that the torque on the driven shaft during the transient period does not lag the magnetic flux in the gap. Therefore the lag of the viscosity variation of the filler

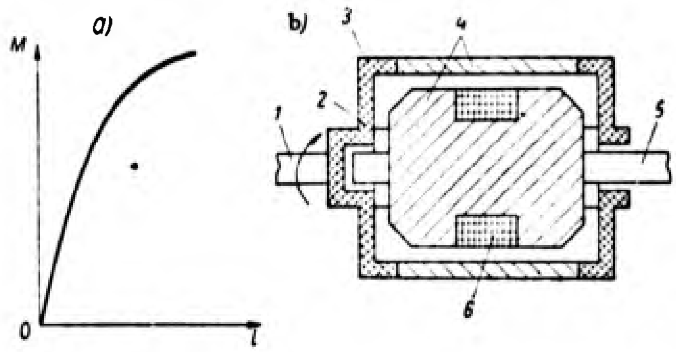


Fig. 3.4 Ferropowder friction clutch: a — static characteristic of clutch; b — model of a clutch of cylindrical type. 1 — driving shaft; 2 — seal; 3 — flange of nonmagnetic (diamagnetic) material; 4 — ferromagnetic parts; 5 — driven shaft; 6 — coil.

can be neglected. On the other hand, the instantaneous value of the magnetic field is determined by the coil current, i.e., the characteristic $M = f(i)$ presented in Fig. 3.4a holds also in the transient period. The coil current i is related to the input voltage U by the equation (1.36):

$$(Tp + 1)\,i = \frac{U}{R},$$

where $T = L/R$ is the time constant of the coil circuit, and $p \equiv d/dt$.

Thus the dynamic equation of a ferropowder friction clutch that relates the torque on the driven shaft to the coil voltage has the form:

$$(Tp + 1)\,M = KU, \qquad (3.3)$$

where $K = \frac{K_{Mi}}{R}$, $K_{Mi} = \frac{\Delta M}{\Delta i}$ being the transfer ratio that can be determined from the static characteristic of Fig. 3.4a.

For determining the speed of the driven shaft we require also the equation of motion of this shaft:

$$Jp\omega = M - M_r, \qquad (3.4)$$

where ω is the speed of rotation of the driven shaft, J is the moment of inertia, reduced to the driven part of the clutch, and M_r is the resistive load torque (this torque can depend on the speed).

Thus when a step voltage U is applied to the coil, the torque M on the driven shaft will increase exponentially according to equation (3.3). As long as $M < M_r$, the driven shaft remains at rest; at the instant when the increasing torque M attains the value M_r the driven shaft begins to rotate with an acceleration $p\omega$, specified by equation (3.4).

The equations (3.3) and (3.4), which describe the dynamics of a clutch, are in general nonlinear. The first equation is nonlinear if we take into account the saturation of the magnetic circuit, since this involves a decrease in the circuit time constant $T = L/R$ when the signal increases (owing to the fact that L decrease together with μ). The second equation is nonlinear owing to the presence of the load torque M_r. Moreover, M_r can be also a nonlinear function of the velocity.

If we confine ourselves to small deviations of the input signal U from its value corresponding to some initial steady-state speed of rotation of the load, it will be possible to linearize the equations (3.3) and (3.4). Indeed, the time constant T in equation (3.3) can simply be regarded as fixed, determining it by the value of μ at the steady-state point; hence the equation for the increments of the variables can be rewritten as

$$(Tp+1)\Delta M = K\Delta U, \qquad (3.3a)$$

where $T = L_{st}/R$ is determined by the value of μ at the steady-state point.

The equation (3.4) can also be replaced by a linear differential equation, if we rewrite it for the increments of the variables. At the same time we linearize also the dependence of the load torque M_r on the speed ω (if it is nonlinear) by representing it as

$$M_r = M_{r.st} + \left(\frac{\partial M_r}{\partial \omega}\right)_{st} \Delta\omega, \qquad (3.5)$$

where $M_{r.st}$ is the steady-state value of M_r, $\left(\frac{\partial M_r}{\partial \omega}\right)_{st}$ is the tangent of the angle of inclination of the static characteristic $M_r(\omega)$ at the steady-state point, and $\Delta\omega$ is the increment of the speed above its steady-state value.

By substituting the formula (3.5) into equation (3.4) and by taking into account that under steady-state conditions we have $M_{st} = M_{r.st}$, we obtain the equation

$$Jp\omega = \Delta M - \left(\frac{\partial M}{\partial \omega}\right)_{st} \Delta\omega,$$

where $\Delta M = M - M_{st}$ is the increment of the torque, or, in other form:

$$(T_M p + 1)\Delta\omega = K_M \Delta M, \qquad (3.6)$$

where $T_M = \frac{J}{\left(\frac{\partial M_r}{\partial \omega_{st}}\right)}$ is the time constant in seconds.

(Let us note that $p\omega \equiv p\Delta\omega$).

The linear differential equations (3.3a) and (3.6) jointly determine the clutch dynamics in the case of small deviations of the input signal from its previous steady-state value. By eliminating from these equations the intermediate variable (the torque), we obtain one second-order equation

$$(Tp + 1)(T_M p + 1)\Delta\omega = KK_M \Delta U,$$

or

$$[TT_M p^2 + (T + T_M)p + 1]\Delta\omega = KK_M \Delta U. \qquad (3.7)$$

The quantity KK_M specifies the dependence of the speed of the driven shaft on the coil voltage of the clutch under steady-state conditions, i.e., the slope of the static characteristic $\omega = f(U)$ of the clutch at the steady-state point, to which the deviations of the variables are referred.

§ 3.3. Electromechanical Slider Clutches

These clutches, used for flexible coupling of shafts, consist of two parts: an inductor and an armature.

Figure 3.5 shows a model of a slider clutch. The inductor is a dc electromagnet whose magnetic flux closes itself across the armature. Suppose, for example, that the inductor is connected to the driving shaft. and the armature to the driven shaft (we can also have the opposite). When the inductor rotates its magnetic field will rotate with respect to the armature. Currents are thus induced in the armature whose interaction with the field of the inductor gives rise to a torque that causes the the armature to move after the inductor.

Hence such a clutch is induction controlled. Its principle of operation is the same as that of an asynchronous motor, apart from the fact that here the rotating magnetic field is produced by the rotation of the poles of a dc electromagnet and not by a multiphase alternating current.

In contrast to ordinary asynchronous machines, the clutch depicted in Fig. 3.5 has an outer armature, whereas the inductor is inside the clutch. Yet there exist also clutches with inner armature.

In slider clutches, just as in asynchronous motors, the armature is designed in the form of a short-circuited coil ("squirrel cage") of a compact rotor, whereas in fast low-power clutches it is in the form of a hollow rotor (cup).

Figure 3.6 shows typical static characteristics of slider clutches, i.e., plots of the speed of the driven shaft versus the torque acting on this shaft for various values of the coil current of the inductor. If we know the dependence of the load torque on the speed of rotation, $M_r = f(\omega)$ (the dashed line in Fig. 3.6), then by plotting this dependence in the same coordinate system we can construct through the points of intersection between the characteristics of the clutch and of the load the resultant static dependence of the load's rotational speed on the inductor coil current ($\omega = f(i)$) or on the coil voltage.

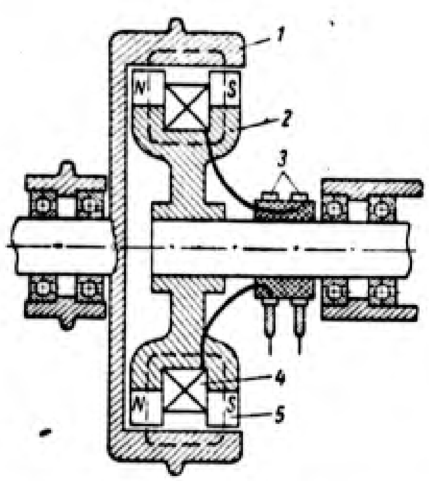


Fig. 3.5. Electromechanical slider clutch. 1 — armature; 2 — inductor; 3 — contact rings for feeding the current to the coil; 4 — inductor coil; 5 — inductor poles

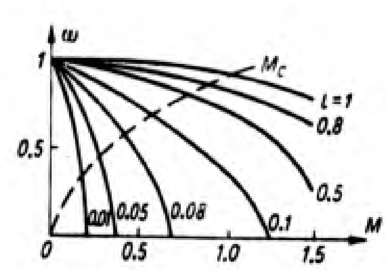


Fig. 3.6. Static characteristics of electromechanical slider clutch. All the variables are expressed in relative units (fractions of their nominal value).

Slider clutches permit the speed of the load to be adjusted in a range of approximately 1:10. (It is clear that this range depends on the load characteristic $M_r = f(\omega)$). Such clutches are designed at powers from tens of watts to thousands of kilowatts. Just as friction clutches, they can be also of reversible type.

In addition, there exist also composite clutches, incorporating a slider clutch and a friction clutch. The latter serves for rigidly coupling the shafts after the load has been accelerated by means of the slider clutch.

The dynamics of slider clutches are described by the same equations (3.3) and (3.4) as were used for ferropowder friction clutches, with the sole difference that the torque M of a slider clutch depends on the speed (see Fig. 3.6). In linearizing the equation (3.4) by going over to the increments of the variables, it is therefore necessary to represent the clutch torque M in the same form (3.5) as the load torque M_r, i.e.,

$$M = M_{st} + \left(\frac{\partial M}{\partial \omega}\right)_{st} \Delta\omega, \qquad (3.8)$$

where $\left(\frac{\partial M}{\partial \omega}\right)_{st}$ is the tangent of the angle of inclination of the slope of the static characteristic $M = f(\omega)$ of the clutch at the steady-state point.

As a result, the coefficients of equation (3.6)

$$(T_M p + 1)\Delta\omega = K_M \Delta M$$

assume the form

$$\left.\begin{aligned} T_M &= \frac{J}{\left(\frac{\partial M_r}{\partial \omega}\right)_{st} - \left(\frac{\partial M}{\partial \omega}\right)_{st}}; \\ K_M &= \frac{1}{\left(\frac{\partial M_r}{\partial \omega}\right)_{st} - \left(\frac{\partial M}{\partial \omega}\right)_{st}}. \end{aligned}\right\} \qquad (3.9)$$

The transfer ratio K_M specifies the dependence of the speed of the driven shaft on the load torque under steady-state conditions for a fixed coil current of the inductor.

In conclusion let us note that the dynamic equations of ferropowder and slider clutches have been derived on the assumption that the speed of rotation of the driving shaft is constant.

Chapter 4

ELECTROMECHANICAL RELAYS AND CONTACTORS

§ 4.1. Classification of Relays

The definition of a relay has been given in the introduction. In contrast to elements of continuous duty, a relay is an element whose output variable in the steady state can assume only one of two limiting values. The transition from one limit to another is jumplike, taking place as soon as the input signal exceeds a certain well-defined threshold value, for the particular relay. In the introduction, in Figs. 0.1b and 0.2b, we showed the static characteristics of a neutral and of a polarized relay. There we also listed the principal static parameters of relays (the operate signal, the dropout signal, the reset ratio, and the safety factor), as well as the dynamic parameters (the operate time and the dropout).

Let us now examine in more detail the types of relays, their classification. We shall be interested in electrical relays only, although there exist also nonelectrical relays, such as hydraulic, pneumatic, optical, chemical, etc.

The first major subdivision of relays is into contact relays and contactless relays.

Any element of continuous duty can be made to operate as a relay. Such an element will be a contactless relay. For example, any amplifier can be made to operate as a relay by imposing on it a strong positive feedback. In this case a smooth variation of its input signal will be accompanied by a jumplike change of its output from its minimum to its maximum value in accordance with the static relay characteristic. On the other hand, there exist contactless relay elements that by virtue of their principle of operation can serve only in the relay mode. Contactless relays using ferromagnetic elements are considered in Chapter 11.

Electric contact relays consist of a signal-to-displacement transducer and of contacts. The transducer has the purpose of closing or opening the contacts when an input signal of a certain magnitude is applied. The contacts are inserted in the electrical actuating circuit. Thus with the aid of such a relay it is possible to connect and disconnect this circuit by applying a signal to the relay. One relay can have many contacts (up to several tens), and hence it can simultaneously control many actuating circuits.

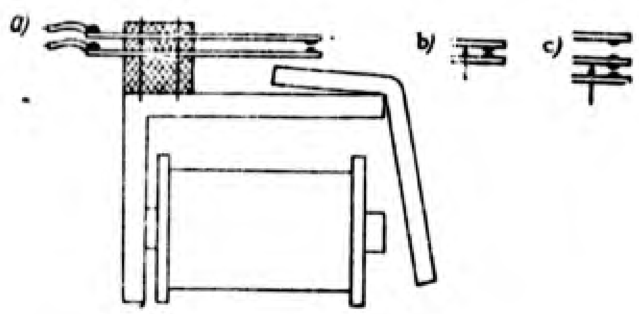


Fig. 4.1. Electromagnetic bar-type relay:
a — with make contact; b — break contact;
c — make-break (transfer) contact.

Figure 4.1a shows a model of a contact relay in which the electromechanical transducer is an electromagnet. When a signal, sufficient for attracting the armature, is applied to the coil of the electromagnet, the armature experts a pressure on the lower contact spring, whose deformation causes the contact to close — the relay operates. When the signal is switched off the armature drops again, the contact is opened, and the relay is released.

The contact depicted in Fig. 4.1a is open when the relay armature is in a released state and it closes when the relay operates. Such contacts are called make (or normally open) contacts. It might be required, however, that the contacts be closed as long as the relay does not operate, and that they open when it operates. For this purpose the design of the relay of Fig. 4.1a must be modified as follows: the armature must act on the upper contact spring, withdrawing it from the lower one when the relay operates, as shown in Fig. 4.1b (the arrow indicates the spring upon which the armature acts). Such contacts are called break (or normally closed) contacts.

Finally, there exist also make-break (transfer) contacts. They are a combination of contacts of the first two types with one common middle contact spring (Fig. 4.1c).

According to the operating principle of the electromechanical transducer used, electric contact relays can be divided into electromagnetic, magnetoelectric, electrodynamic, induction, thermal, etc. Here, the basis of the classification of electric contact relays is the same as that used for electrical measuring instruments. As a matter of fact, electrical contact relays differ in principle from the corresponding electrical measuring instruments only by the fact that the moving system of the former actuates contacts and not a pointer.

According to applications, we can have control relays, communication (telephone, code) relays, aviation relays, protective relays, etc. The type most widely used and of most general purpose are the control relays; they are widely used in communications, protective circuits, and aviation. The most special relays are the protective relays.

Prior to examining the individual types of electrical relays, let us consider separately the contact relays, since they are the same for all types of relays.

§4. 2. Contacts

A. The Contact Parameters

The designing or selection of contacts is performed on the basis of the following service parameters: the maximum permissible current I_{br} through the closed contacts, the maxim permissible voltage U_{br} across the open contacts, the maximum power interrupting capacity P_{br} of the contacts (when they are opened), and, finally, the maximum number of switchings.

The numerical values of the service parameters of the contacts are determined by the actuating circuit into which they will be inserted, and by the external operating conditions.

On the basis of the service parameters one determines the design parameters of the contacts: the shape, dimensions and material of the contacts, the pressure exerted upon them when they are closed, the distance they move apart in opening, and special means for arc and spark quenching.

Let us examine the relationship between the service parameters and the design parameters. The maximum permissible current I_{br} is determined by the permissible temperature of contact heating at which the contacts do not become yet soft. The heating of the contacts is due to the power (dissipated in the form of heat)

$$P_c = I^2 R_c,$$

where R_c is the resistance of the contacts.

The steady-state temperature of the contacts is proportional to P_c and inversely proportional to the cooling surface of the contacts (see formula (1.22) of §1.5, devoted to coil design of electromagnets). For increasing the maximum permissible current I_{br} it is therefore necessary to reduce the contact resistance R_c and to increase the dimensions (cooling surface) of the contacts; one must also take other measures for better cooling of the contacts.

The resistance of the contacts R_c is determined by the transition resistance at the point of junction of the contacting bodies, and not by the resistance of these bodies themselves. Therefore R_c depends not on the dimensions of the contacts, but on the contact force F_c — the force which presses the two contacting bodies against each other.

As a matter of fact, even in the case of very careful finishing of the surfaces of the contacting bodies, the contact between them occurs at individual points only (as shown in Fig. 4.2). When the contacting bodies are pressed against each other, they are flattened at the points of contact, and thus the number of these points increases. Hence an increase in the contact force will be accompanied by an increase in the effective contact surface of the bodies, and hence by a decrease in the contact resistance R_c. In Fig. 4.3 we plotted the resistance R_c versus the contact force F_c.

Thus in order to diminish the contact resistance R_c we must increase the contact force F_c. In any case, F_c must not be less than the value F_{c0}, indicated in Fig. 4.3.

On the other hand, the larger we take F_c, the lower will be the sensitivity of the relay, since its electromechanical transducer (for example, its electromagnet) will have to develop a larger force.

In sensitive relays the contact force used is equal to a few gram, in highly sensitive relays it is equal to fractions of a gram. Thus the current in the contacts amounts to fractions of an ampere. For contacts operating at currents of up to 3 amperes the contact force used will 30 grams, and at currents of up to 10 amperes it will be 100 grams and more.

The value of R_c is of the order of 10^{-3} - 10^{-5} ohm (the latter value for more powerful contacts).

The second operating parameter — the <u>maximum permissible voltage</u> U_{br} — is determined by the breakdown voltage of the gap between the open contacts and the breakdown voltage of the insulation of the contacts. The latter must be calculated not merely on the basis of the voltage of the actuating circuit, but also with allowance for the maximum possible overvoltage that may appear at the contacts when they are opened (see below).

The third operating parameter of contacts is the <u>maximum interrupting capacity of the contacts</u> P_{br}. It constitutes the power of the actuating circuit $P_{br} = (UI)_{br}$ that the particular contacts can interupt without causing the formation of a stable arc when they move apart to their maximum distance. In other words, the quantity P_{br} specifies the condition of extinction of the arc formed between the contacts after they are opened.

Let us examine the process of opening the contacts [1]. At the first instant, when the moving apart of the contacts leads to the formation of a gap that increases from zero, a <u>gas discharge</u> normally occurs in this gap. It can be either a spark discharge or an arc discharge.

An <u>arc</u> discharge occurs when the current of the interrupted circuit is sufficiently large. The minimum value of the current at which an arc appears depends on the material

[1] See footnotes p. 152

of the contacts. For copper and silver, for example, it is equal to about 0.4 amp under normal atmospheric conditions. The voltage applied to the air gap must also be not below a certain limiting value. Yet this value is sufficiently small and is normally exceeded. Thus for copper and silver it is about 12 V. If the current in the interrupted circuit is lower than the value indicated above, the gas discharge can be only in the form of a spark discharge. Its occurrence necessitates, however, a voltage of not less than ~ 300 V (under normal atmospheric conditions). In the absence of such a voltage the opening of the contact will not be accompanied at all by the occurrence of a gas discharge.

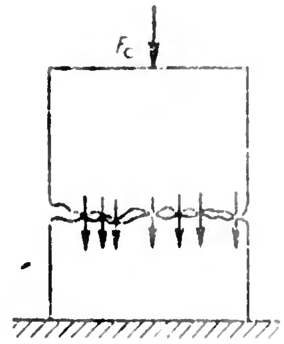


Fig. 4.2 Contact surfaces that are increasing

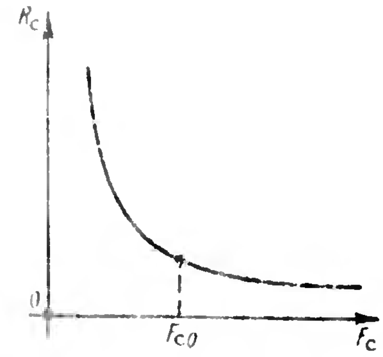


Fig. 4.3. Contact resistance R_c plotted versus contact force F_c.

It must be taken into account that if the interrupted circuit contains an inductance, which is normally the case, then the voltage across the contacts at the instant of opening the circuit may exceed by many times (by an entire order of magnitude) the supply voltage of this circuit, owing to the appearance of a self-induction emf. Indeed, the equation of the transient process in the interrupted RL circuit has the form:

$$U = L\frac{di}{dt} + Ri + U_c, \qquad (4.1)$$

where U_c is the voltage across the contacts.

Hence

$$U_c = (U - Ri) - L\frac{di}{dt}. \qquad (4.2)$$

When the circuit is opened the current i decreases. Hence the derivative $\frac{di}{dt}$ will be negative and the self-induction emf $L\frac{di}{dt}$ will be added together with the supply voltage U. The faster the contacts move apart, tending to break the current i, and hence the faster this current drops, the larger will be the emf $L\frac{di}{dt}$ and the greater will be the overvoltage at the contacts. It is impossible to open instantaneously the current circuit in view of the fact that at this instant the voltage U_c would become infinitely large.

Thus in the case of sufficiently fast opening of an inductive circuit (even of low voltage), a spark discharge will occur at the instant at which the contact is opened.

The physical reason for the occurrence of an overvoltage is as follows. If a direct current flows through a circuit containing an inductance, a certain amount of energy will be stored in the magnetic field of this inductance. When the current circuit is opened this energy must vanish, being converted into other forms. Therefore an energy avalanche develops between the moving-apart contacts, causing an overvoltage and arcing in the air gap thus formed.

Any gas discharge accompanying the opening of a contact is harmful, since it involves the transfer of metal particles from one contact to the other, thus polluting their surface. Particularly harmful is the arc discharge; in this case the transport of metal is larger, since the current is stronger, and, moreover, the arc heats the contacts to incandescence, causing their oxidation.

Thus the moving apart of the contacts is accompanied by the appearance of a gas discharge. It will be in the form of an arc discharge if the circuit current is sufficiently strong. In order to stop the discharge the contacts must be moved apart at a distance such that the voltage applied to them be insufficient for sustaining the discharge. Conversely, with a fixed value Δ of the final distance between the contacts, there corresponds to each value of the circuit voltage U a limiting value of the current I that can be interrupted by these contacts. At a larger value of the current an arc will be formed between the contacts that does not become extinct even after the contacts have moved apart to the limiting distance, which of course is not permissible. The higher the voltage U, the smaller will be the value of the current I at which such a stable arc will appear. The relationship between the limiting (according to the condition of self-extinction of the arc) values of U and I for Δ = const is plotted in Fig. 4.4. When the final distance Δ between the contacts increases, the curves are shifted upwards. These plots are almost hyperbolic, i.e., $UI \approx$ const.

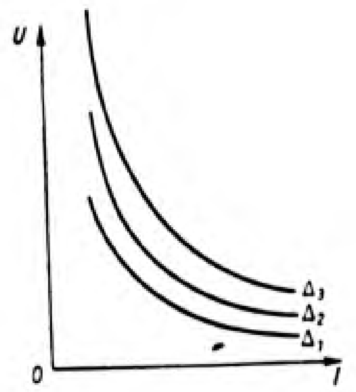


Fig. 4.4. Limiting current-voltage curves of a contact. $\Delta_3 > \Delta_2 > \Delta_1$.

This constitutes in fact the maximum power interrupting capacity $P_{br} = UI$. For given contacts (of specified design and material), there corresponds to each value Δ of their maximum distance a particular value of $P_{br} = (UI)_{br}$ such that a stable arc is formed between the opened contacts when this value is exceeded. The individual values of U and I are not important; what matters is their product. (If U or I are smaller than the above-mentioned minimum values necessary for the arc to appear, the latter will not appear at all even at the initial instant of opening the contact).

Relays of highest sensitivity have low-power contacts with a distance Δ of the order of fractions of a millimeter, and hence with a maximum interrupting capacity P_{br} of several watts. In medium-power relays the distance Δ is of the order of 1 mm, whereas P_{br} is of the order of tens or hundreds of watts. In order to obtain a large value of P_{br}, one increases Δ accordingly.

When an ac circuit is opened the arc is easier to quench than in the case of a direct current, since in an ac circuit the current decreases periodically to zero. Therefore the maximum ac power that can be interrupted by the contacts is two to three times higher than the dc P_{br}.

A gas discharge appears not only when the contacts are opened, but also when they are closing. But since in the latter case the arc appears at the end, just before the contacting bodies touch, the harm due to such a discharge is much smaller than in the case of contact opening. It can be assumed that the maximum power connected by the contacts is two to three times higher than the maximum power P_{br} disconnected by the contacts, provided only that the closing of the contacts is not accompanied by chatter, i.e., multiple bouncing.

Instead of P_{br}, the relay adjustment sheet often lists separately the actual limiting break-capacity values of U and I at which the number of relay switchings indicated in the sheet is assured. This number is usually of the order of tens and hundreds of millions. In order to determine the maximum current I interrupted by the particular contacts at a different value of U, the quantity P_{br} can be specified as the product of the values of U and I listed in the sheet. Yet if thereby the current of the circuit to be interrupted is larger than the value indicated in the sheet, then the service life of the contacts may decrease owing to greater wear.

B. The Material and the Design of Contacts

Material used for contacts must satisfy very high requirements: It must be mechanically strong, have a sufficiently high melting point, be corrosion and erosion proof, and have good heat and electrical conductivity. It must also be amenable to the necessary machining operations.

The corrosion of contacts is promoted by their heating. As a result of the chemical interaction with the oxygen and sulfur of the ambient atmosphere, the metal surface may be coated by an oxide or sulfide film that poorly conducts electricity. This is not permissible. The erosion of contacts (their destruction and wear under the action of electricity) is a result of the gas discharge during opening and closing.

In view of these requirements the contacts must be made of noble metals. Low-power contacts are usually made of silver, and less frequently of platinum, gold and their alloys (as a rule, with iridium). The main advantage of noble metals is their resistance to corrosion. On the other hand, they are not resistant to erosion; in view of this and of their high cost, the noble metals are not used for more powerful contacts. For contacts designed for currents of a few amperes (1-10 amp) one utilizes more refractory and harder metals, such as tungsten, molybdenum and their alloys with platinum, iridium, etc. High-power contacts are made of copper.

Contacts widely used are those made of compositions. A composition is a mechanical mixture of powders of several metals. The mixture is hot-pressed, so the components are sintered. By this method it is possible to obtain high-quality contacts from all sorts of combinations of metals that are not inter-fusible.

Common shapes of contacts are shown in Fig. 4.5. The contacts intended for low currents (of up to 1 amp approximately) are in the form of a cone on a plane (a), those for higher currents are in the form of a hemisphere on a plane (b), and those for even higher currents are in the form of a plane on a plane (c).

Tentative values of the parameters of various contacts are listed in Table 4.1.

Table 4.1

Tentative Parameter Values of Various Contacts

Shape of contact in Fig. 4.5	Gap between contacts Δ, mm	Contact force F_c, grams	Maximum continuous current I_{br}, amp	Maximum interrupted power P_{br}, w
a		Fractions of a unit and units		Units and tens
b		Up to 30	Up to 3	Tens and hundreds
c				Hundreds and more

There exist also contacts of special type, such as vacuum and mercury (Fig. 4.6). The placing of contacts in vacuo impedes arcing and thus greatly increases the maximum interrupting capacity P_{br}. Moreover, such contacts can be used in explosive, chemically agressive media and in the most unfavorable atmospheric conditions (dust, moisture).

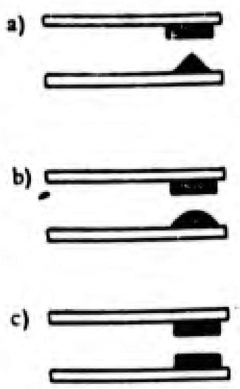


Fig. 4.5. Contact shapes.

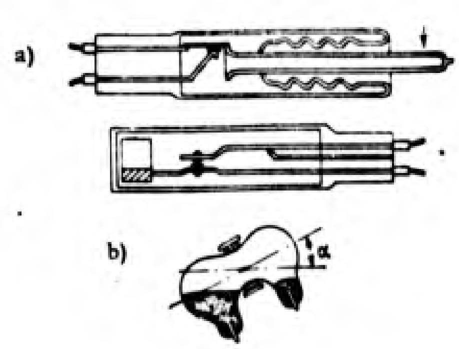


Fig. 4.6. Special contacts: a — vacuum; b — mercury.

In order to move the movable (upper) contact in the vacuum contact of Fig. 4.6a, one must bend the corrugated glass tube by applying to it a force as indicated by the arrow.

Mercury contacts (Fig. 4.6b), just as vacuum contacts, are not connected to the ambient atmosphere and posses therefore all the advantages of these contacts. In order to increase P_{br}, the envelope of a mercury contact is filled with hydrogen. The opening of the contact takes place between the mercury particles when the envelope is inclined at an angle α, whereas the closing takes place when the envelope returns to its previous position. Mercury contacts are designed for higher power as compared to vacuum contacts. Let us note that the value of P_{br} of a mercury contact can be estimated by the volume of the mercury, i.e., 1 cm^3 mercury corresponds to approximately 500 w power interrupted.

C. Methods of Spark and Arc Suppression

The main reason for the destruction of contacts, which determines their service life, is the gas discharge primarily occurring when the contacts are opened. A partic- strong and protracted discharge appears when we open a circuit which contains a strong inductance, this being due to the overvoltage generated thereby at the contacts.

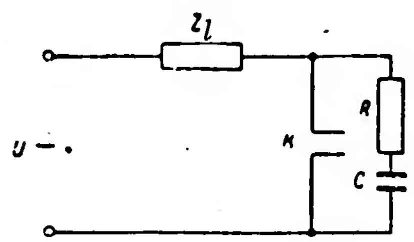


Fig. 4.7. Spark-suppression dc circuit

In the case of low-power contacts the gas discharge in a dc circuit can be strongly suppressed and even eliminated altogether by inserting an RC network in parallel to the contacts K (as shown in Fig. 4.7). This RC network offers a by-pass (outside the contacts) for the load current decaying after the contacts have been opened, and also for the dissipation of the energy stored in the magnetic field of the load. The transient process in the load circuit reduces to the charging of the capacitor C to the full supply voltage U. Hence after the opening of the contacts, the current in the circuit will gradually drop to zero while the capacitor C is charged from zero to U. As a result, the opening of the contacts will not be accompanied by the appearance of an overvoltage, i.e., the voltage across the contacts, just as the voltage across the capacitor C, gradually increases to the value U. The current, too, flows outside the contacts, through the RC network. Thus there will be no gas discharge. Yet in order that all the processes should take place as described, it is necessary that the capacitance C be sufficiently large, that it should be charged sufficiently slowly, and hence that the voltage across the contacts should increase sufficiently slowly as compared to the moving apart of the contacts.

The reasons for inserting a resistor R are twofold: first, in order to ensure that the current decay process in the circuit containing the load inductance and the capacitance C will not be periodic and, secondly, in order to limit the discharge current of C that flows through the contacts when they are closed.

The above RC spark-suppression circuit can be equally well connected in parallel with the load.

This method of spark suppression is used only for low-power contacts, when the value of C required is still fairly small.

Contacts designed for the breaking of larger powers (from hundreds of watts to tens of kilowatts) are equipped with special arc-suppression devices in the form of arc-suppression chambers with magnetic blowout and deionizing grids (Fig. 4.8).

The arc-suppression chamber 2 (Fig. 4.8) is fitted from above onto the horizontal contacts 1. The flow of air heated by the arc blows the latter upwards; this is called thermal blowout. The chamber enhances this blowout, since the pipe increases the pull in the stove. As a result, the arc is stretched over a distance considerably exceeding the gap between the contacts, thus being quenched. (In the absence of blowout it would be necessary to correspondingly increase the gap between the contacts).

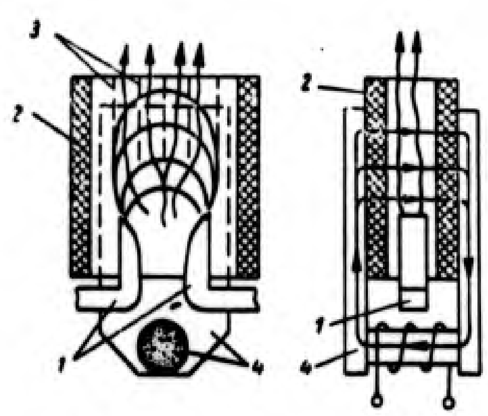


Fig. 4.8. Arc-suppression chamber. 1 — contacts; 2 — arc-suppression chamber; 3 — deionizing grids; 4 — electromagnet for magnetic blowout.

Magnetic blowout enhances the upward blowing of the arc, thus supplementing the thermal blowout. It is effected with the aid of an electromagnet 4 which produces a magnetic field that is perpendicular to the direction of the arc. An arc (just as any other current-carrying conductor) which lies in a magnetic field is subjected to a force that will be directed upwards in the chamber if the directions of the field and of the current are matched. The coil of the electromagnet is connected in series with the contacts, forming a controlled circuit, or (less frequently) parallel to the line voltage. Magnetic blowout can be used with both direct and alternating currents. In the case of dc controlled circuits one can use a permanent magnet instead of an electromagnet.

In order to speed up arc extinction, one sometimes fixes in the arc-suppression chambers transverse metal plates — so-called deionizing grids[2] (marked by 3 in Fig. 4.8).

An arc struck across these grids is more rapidly quenched, since it is broken up into individual contiguous parts; as a result, the overall voltage drop across the arc increases, and the outflow of heat from the arc via the grids increases, thus enhancing the deionization of the air gap.

For the interruption of very powerful circuits the contacts are either placed in oil, or one uses high-pressure air blowout.

Hence the devices used for the breaking of such circuits are called oil or air circuit breakers.

§ 4.3. The Construction of the Opposing Characteristic of a Contact System

As a rule, a relay has more than one (up to 20-30) contacts. Only the most sensitive and, conversely, the most powerful, relays are made with a single contact.

All the contacts of a relay are grouped in a single unit, called the contact system.

In Fig. 4.9 we present two types of electromagnetic relays in which the contact systems are clearly seen. A contact system consists of the contacts proper, the contact springs on which the contacts are fixed, the parts connecting the moving contacts with the armature of an electromechanical transducer, and the mounting elements.

The designing of a contact system involves the choice of the contacts and the construction of the opposing restraining characteristic $F_{opp} = f(\delta)$ of the system as a whole. Then one uses this characteristic for designing the electromechanical transducer of the relay, for example an electromagnet.

The opposing characteristic of a contact system is obtained by combining the opposing characteristics of contacts of various types — make contacts, break contacts, and transfer contacts.

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[2] See footnotes p. 152

In Fig. 4.10 we plotted typical opposing characteristics of various types of contacts. In addition to the contact springs 1 and 2 (Fig. 4.10a and 4.10b), we have also the spring 3, which serves as a support for the contact spring and is not subjected to the pull force F_p. The support reduces the stroke of the armature required for obtaining the necessary contact force and for moving apart the contacts by the necessary distance; it also diminishes the vibrations of the contacts and it improves the process of closing and opening the contacts. The contact system is adjusted by choosing the initial deformation of the support springs 3.

The initial vertical section 4 of the characteristic shown in Fig. 4.10a is determined by the initial force exerted by the spring 2 on the armature or on the support (not shown in the figure). The section 4-5 corresponds to the deformation of the spring 2, and the point 5 — to the touching of the contacts. The vertical section of the characteristic at this place is determined by the removal of the spring 1 from the support 3 (in actual fact this is not a vertical, but a very steep section). Then there occurs the joint deformation

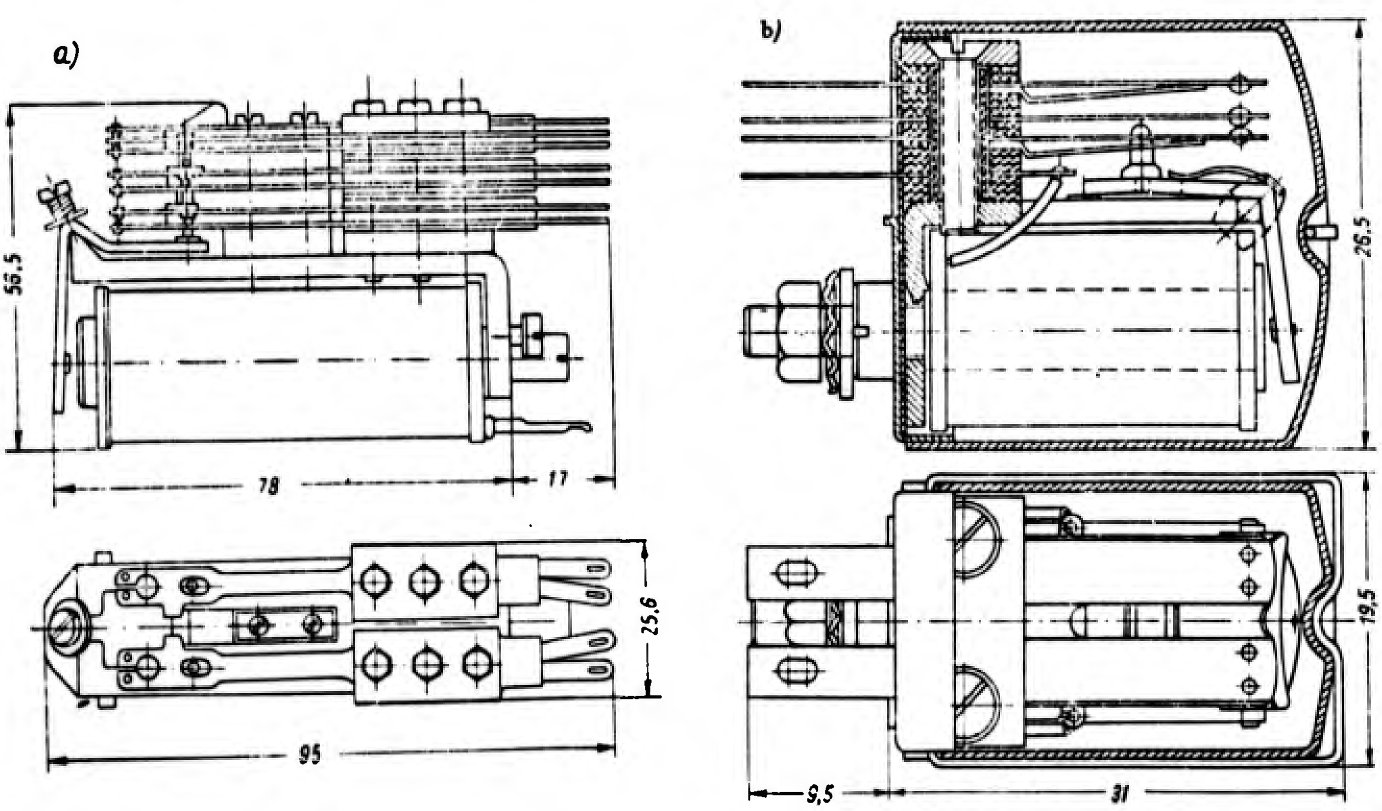

Fig. 4.9. Electromagnetic relays: a — RKN; b — RES—6.

of the springs 1 and 2 until the necessary contact force F_c is obtained. The slope of section 5-6 is correspondingly larger than the slope of section 4-5, since the stiffness of the contact system has been increased by connecting to the spring 2 the spring 1.

On the characteristic of break contacts (Fig. 4.10b), on the other hand, the slope of section 7-8 is greater than the slope of the next section 8-9, since the former is subjected to two springs (1 and 2), whereas the latter is subjected to only one (1). The jump exhibited by the characteristic at the point 8 corresponds to the instant at which the spring 2 lies on the support 3, which causes the force, exerted by the spring 2 on the spring 1, to be transmitted to the support 3.

The characteristic of the transfer contacts shown in Fig. 4.10c is a combination of the two previous characteristics.

The opposing characteristics can be calculated by the well-known formulas for the deformation of an elastic beam, rigidly fixed at one end, that is subjected to a concentrated force F. Details about the method of calculation can be found in [2] and [5].

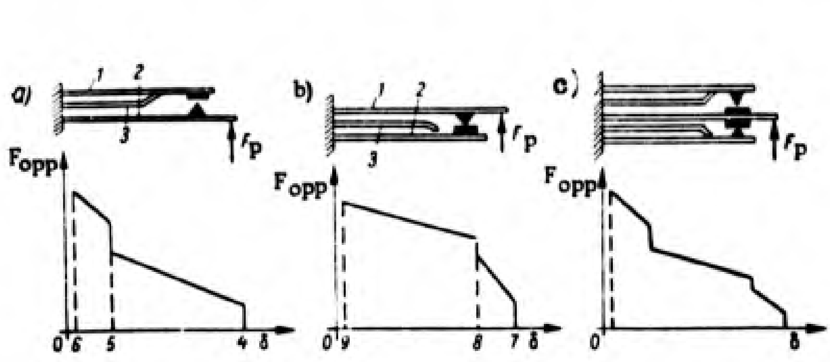

Fig. 4.10. Opposing characteristics of contacts: a —make; b — break; c — transfer.

§ 4.4. Electromagnetic Relays

An electromagnetic relay consists of a contact system and of an electromagnet that serves as an electromechanical transducer (Fig. 4.1). This is the principal type of electrical contact relays.

According to the type of electromagnet one distinguishes between neutral and polarized relays. As a matter of fact, the classification of these relays is the same as in the case of electromagnets.

A. Neutral Electromagnetic Relays.

The majority of neutral electromagnetic relays have a bar-type electromagnet, and less often — a plunger-type electromagnet. The maximum power P_{br} controlled by these relays (i.e., the power interrupted by the contacts) ranges from a few watts to several kilowatts.

Powerful electromagnetic relays which can switch a power of several kilowatts or more ($I_{br} > 20$ amp) are called contactors. They are always equipped with an arc-suppression chamber. A contactor with a time delay is called a timetactor.

The power which must be fed to the coil of the relay for the latter to operate is called the operating power; it depends on the power of the contacts; for neutral electromagnetic relays we have $P_{op} = 10^{-3}$-10^{-2} watt.

The ratio P_{br}/P_{op}, called the control ratio (similar to the transfer ratio of a continuous-duty network), is of the order of a few hundred in the case of neutral relays.

The operate and release time of these relays, just as in the case of electromagnets, varies from a few milliseconds to fractions of a second. Relays with special delay, in which the operate or release time exceeds 1 sec, are called time relays. The methods of retardation have been examined in § 1.7.

The number of contacts of relays such as the ones shown in Fig. 4.9 may reach thirty, and it is evident that one and the same electromagnet can serve as a basis for relays with different numbers of contacts. The smaller their power, the greater the number of such contacts that can be controlled by the electromagnet.

Electromagnetic relays are often designed with a hermetic case. The smallest of these relays have a volume of less than 1 cm^3 and a weight of 1-2 gram at $P_{op} \approx 0.1$ watt and a single contact.

In most cases the electromagnetic relays, just as the electromagnets themselves, are of dc type; ac electromagnets are equipped either with a short-circuit winding (Fig. 1.24) or with a semiconductor rectifier. In this case they are called rectifier relays.

A description of various types of domestic and foreign relays can be found in [5].

B. Polarized Electromagnetic Relays

Figure 4.11a shows a polarized relay based on the differential polarized electromagnet depicted earlier in Fig. 2.1. Polarized relays can be of three-position, two-position, and one-position type.

Figure 4.11a shows a two-position relay, using a two-position polarized electromagnet. Depending on the polarity of the dc input signal, one of the two contacts of the relay closes, remaining closed after the signal has been switched off. In order to move the armature to the other fixed contact, one must apply to the relay coil a signal of opposite polarity.

If one of the fixed contacts is shifted to the neutral line passing through the axis of the armature (as shown in Fig. 4.11b), we obtain a one-position relay. In the absence of a signal, one particular contact (in Fig. 4.11b it is the right contact) must be closed in such a relay. In order to close the other contact one must apply a signal of well-defined polarity. Yet after this signal has been switched off the armature returns anew to its original position at the other contact. This is due to the fact that the armature remains all the time at the same side of the neutral line, and hence in the absence of a signal the pull force produced by the polarizing flux always exists in the direction away from this line (as shown by the arrow in Fig. 4.11b).

In a three-position relay one uses a three-position electromagnet that has a restoring spring which returns the armature to the neutral (vertical) position after the signal has been switched off. Thus in the absence of a signal both contacts are open. In order to close one of them it is required to have an input signal of appropriate polarity.

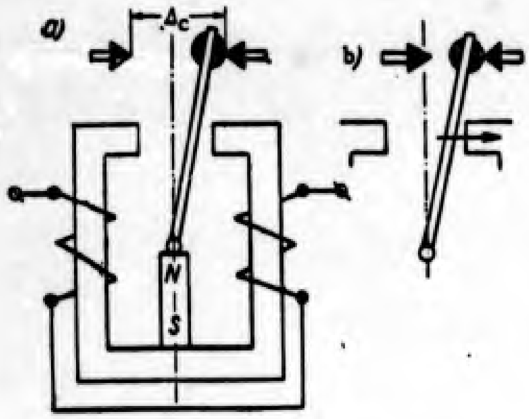

Fig. 4.11. Polarized electromagnetic relays: a — two-position; b — one-position

Polarized electromagnetic relays differ from neutral relays not only by the fact that they sense the polarity of the input signal. They have also much higher sensitivity, control ratio, and speed; their power P_{op} is as low as a few microwatts (as compared to a few milliwatts in the case of neutral relays) with a control ratio $P_{br}/P_{op} \approx 10^4$ (as compared to $\sim 10^2$ for neutral relays). The operate time of polarized relays can be as low as 1 μsec, and in very fast relays it can be as low 0.1 μsec.

In polarized relays it is normally possible to vary the distance between the fixed contacts (Δ_c in Fig. 4.11a). The circumstance that these contacts are in most cases fixed on a screw thread makes it possible to vary in a wide interval the operating parameters of the relay: when this distance decreases the sensitivity and speed are increasing, but the power of the contacts P_{br} and the maximum current I_{br} are decreasing (the latter decreases as a result of a decrease in the contact force).

Polarized relays are high-power, high-sensitive and high-speed relays. For this reason they are designed with the use of not more than two contacts. They are more expensive and less reliable in operation as compared to neutral relays; they should be used only in those cases in which one cannot dispense with them or when they yield major advantages in instrumentation.

C. Vibroconverters

Vibroconverters, or vibrators, are used for dc-to-ac and ac-to-dc conversion, i.e., they serve as modulators and demodulators.

Such devices are widely used for the amplification of direct currents and dc voltages. As a matter of fact, owing to the zero drift of dc amplifiers weak dc signals must be amplified with the aid of the following circuit: modulator - ac amplifier - demodulator.

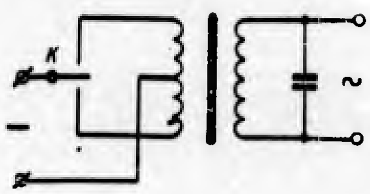

Fig. 4.12. Model of dc-to-ac conversion circuit using a vibrator

Moreover, it is often necessary to encrease the dc supply voltage obtained from a low-voltage source, for example a storage battery. In this case, too, one converts at first this dc voltage into an ac voltage, which is then stepped up by a transformer, and afterwards rectified.

A vibroconverter is a special electromagnetic relay, intended for service under vibration conditions. Its armature is permanently fluctuating between fixed contacts (it vibrates), this being due to the fact that the coil is either connected to an ac source or to a dc source via a break contact of the vibroconverter itself (i. e., a contact which opens when the armature moves under the action of the pull force produced by the coil current).

Figure 4.12 shows the most common dc-voltage - to - ac voltage conversion circuit. Here K denotes the vibrator contacts, which alternatively connect (at a given frequency) one of the input terminals to the beginning and to the end of the primary winding of a transformer, whereas the second terminal is permanently connected to the midpoint of this winding. Thus an emf is induced in the secondary winding with a frequency equal to the armature vibration frequency, whereas the magnitude of this emf is determined by the transformer ratio.

Vibrators are usually designed in the form of polarized electromagnetic relays, since they are faster than neutral relays, which is particularly important for improving the conversion performance. Figure 4.13a shows a polarized vibroconverter of type VP. It is hermetically closed and fixed on a base similar to the base of an electronic tube. Its magnetic circuit is differential (see Fig. 4.13a, where the permanent magnet is represented in the form of two halves of its magnetizing force).

Ordinary polarization relays can also serve as vibroconverters. Yet a special vibrator has a much better conversion performance, and a much lower noise level and higher sensitivity. In order to reduce the noises due to all sorts of stray currents the vibrator is

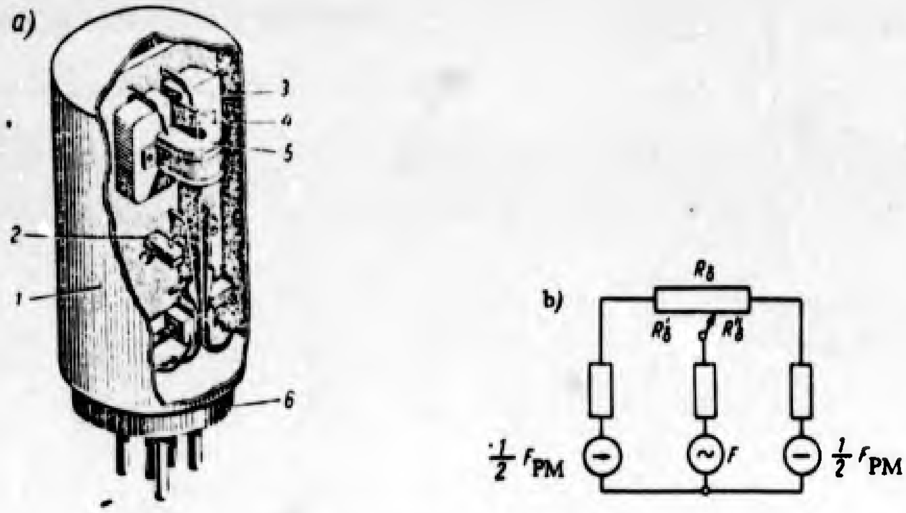

Fig. 4.13. Type-VP vibrator: a — external view; b — equivalent circuit of magnetic circuit of vibrator
1 — shell; 2 — fixed contact; 3 — permanent magnet; 4 — armature; 5 — coil; 6 — base.

carefully shielded. The material selected for the contacts is such that their heating in service will not be accompanied by the generation of thermal emf and of a contact potential difference due to the touching of different metals. The contacts are usually made of gold and of its alloys with nickel.

In order to diminish the travel time, which causes conversion errors, and to increase the sensitivity of the armature, the gap between the contacts is made as small as possible—down to small fractions of a millimeter.

D. Tuned Electromagnetic Relays

A tuned relay responds to an ac frequency. It operates when the frequency lies in a certain region. Tuned relays are used, for example, in remote control systems for the frequency selection of a certain object among several.

Figure 4.14a shows a tuned relay with a reed. The operation of the relay is based on the resonance between the frequency of natural oscillations of the reed and the frequency of the coil current. (The same principle is used in electromagnetic resonance frequency meters, which contain an entire set of reeds of different natural frequencies.)

The pull force produced by a polarized electromagnet causes the reed to vibrate at the frequency of the coil current. When this frequency approaches the natural frequency of the

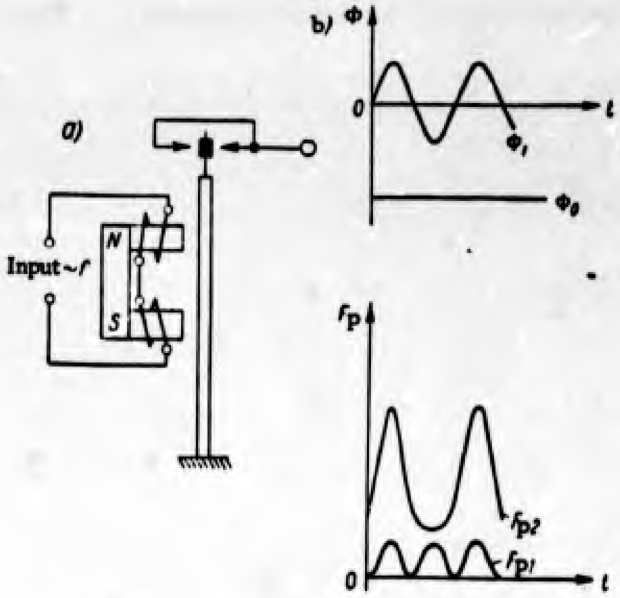

Fig. 4.14. Tuned electromagnetic relay

reed, the oscillation amplitude increases. If the current frequency is sufficiently near to the natural frequency of the reed, the relay contacts will be periodically closed. Instead of a reed one sometimes uses a tuning fork. The utilization of a permanent magnet is not mandatory, but it permits the input signal power to be reduced (Fig. 4.14b). Indeed, in the absence of a permanent magnet the variable magnetic flux $\Phi_1 = \Phi_M \sin\omega t$ produces (according to formula (1.9)) a pull force

$$F_{p1} = \frac{\Phi_W^2}{2\mu_0 S_W} = \frac{\Phi_M^2 \sin^2\omega t}{2\mu_0 S_W} = \frac{\Phi_M^2}{\mu_0 S_W}(1 - \cos 2\omega t),$$

whose ac component has an amplitude $\Phi_M^2/\mu_0 S_W$ and a frequency 2ω.

When a permanent magnet is inserted that produces a flux Φ_0, the pull force will be

$$F_{p2} = \frac{(\Phi_0 + \Phi_M \sin\omega t)^2}{2\mu_0 S_W} = \frac{\Phi_0^2}{2\mu_0 S_W} + \frac{\Phi_M \Phi_0}{\mu_0 S_W}\sin\omega t + \frac{\Phi_M^2 \sin^2\omega t}{2\mu_0 S_W}.$$

Since we normally have $\Phi_M < \Phi_0$, the last term in this formula can be neglected. The amplitude of the variable component of the pull force is equal in this case to $\Phi_M \Phi_0/\mu_0 S_W$, and the frequency is equal to ω. Thus the larger the value of Φ_0 as compared to Φ_M, the greater will be the increase in the amplitude of the variable component of the pull force. Let us also note that the insertion of a permanent magnet increases also by twofold the resonant frequency (relay operating frequency) with the same reed. In the absense of a permanent magnet the reed vibrates at the double frequency — like the armature of an ac electromagnet, whereas in the presence of such a magnet it vibrates at the signal frequency.

The designation of tuned electromagnetic relay is due to its principle of operation; from the point of view of its application, such a device is a narrow-band frequency relay.

The same function can be also fulfilled by an ordinary electromagnetic dc relay if a band filter (with a rectifier) is connected to its input. Yet if a very narrow passband is required, a tuned relay will be simpler.

In addition to tuned frequency relays there exist also other types of frequency relays, such as maximum frequency relays, minimum frequency relays and difference frequency relays. All these devices are special-purpose relays. They are designed on the basis of the electromechanical induction systems considered in §4.6C, or by combining an ordinary electromagnetic dc relay with a frequency detector in the form of an electrical circuit that yields a direct output current which is proportional to the deviation of the frequency from its assigned value [1].

§ 4.5. Stepping Switches

A step selector, or stepping switch, is a device of discrete operation, similar to a relay, though more complicated. Like a relay, it has contacts that switch the actuating circuits when a control signal is applied. In the simplest case the actuating part of a stepping switch has one input terminal and n output terminals, as shown in Fig. 4.15a. Under the action of a control signal, applied to a special input, the moving contact K scans the field of the fixed output contacts 1, 2, . . . , n. In order to scan the entire field, it is necessary to successively apply n control pulses. After that the 1-st contact closes again. In order to connect the input terminal to the i-th output it is necessary to apply i control pulses.

The need to successively switch a number of actuating circuits is often encountered in automation, and in particular in computer engineering, remote control, and communications. For this reason stepping switches are being widely used as typical elements of automatic devices.

Figure 4.16 shows a simplified model of an electromagnetic stepping switch. The movement of the mobile contact — the wiper 1 — along the contacts 2 is effected with the aid of the electromagnet 3, to which the control pulses are applied. The armature 4 of the electromagnet is connected to the moving contact via the ratchet mechanism 5. The moving contact makes one step when the electromagnet is released, under the action of the restoring spring 6, after the control pulse has been switched off. Such a selector is called a <u>reverse-travel</u> selector. There exist also <u>forward-travel</u> selectors in which the displacement of the movable contact takes place when the electromagnet operates, i.e., when the armature is attracted.

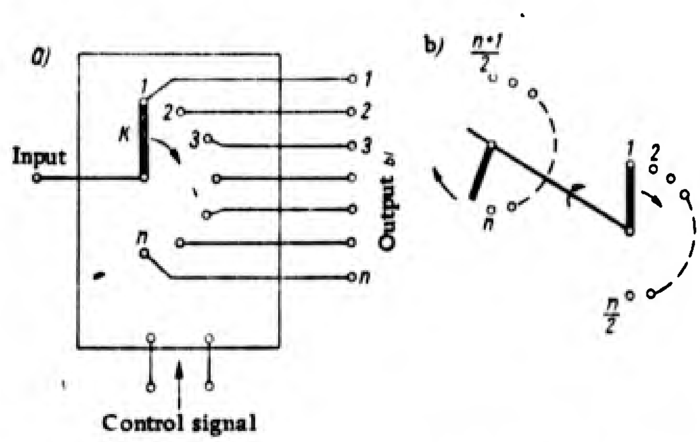

Fig. 4.15. Operating principle of stepping switch.

From the point of view of design, a stepping switch normally incorporates several fields of fixed contacts with their moving contacts that are driven simultaneously by a single electromagnet. The contact fields do not comprise an entire circle, but are the form of a semicircle (180°) or of a third of a circle (120°). If it is necessary that the moving contact should have an unlimited angle of rotation, so that it should return directly from the last fixed contact to the first, one utilizes either two fields of 180° or three fields of 120°. The moving contacts are also mutually shifted and connected to the common input terminal (Fig. 4.15b). As a result, only one of the fields is active on each portion of the circle: when one moving contact leaves the last contact of its field, another moving contact arrives at the first contact of another field.

The number of contacts in one field ranges normally from 11 to 50. The scanning speed is limited by the lag of the electromagnet, and it varies from one to several tens of steps per second.

In addition to electromagnetic stepping switches there exist also other types of selectors, such as electromechanical and electron-beam selectors.

An electromechanical (motor) selector is driven not by an electromagnet, but by an ordinary electrical motor of continuous rotation of by a step motor. These selectors have higher speed as compared to electromagnetic switches.

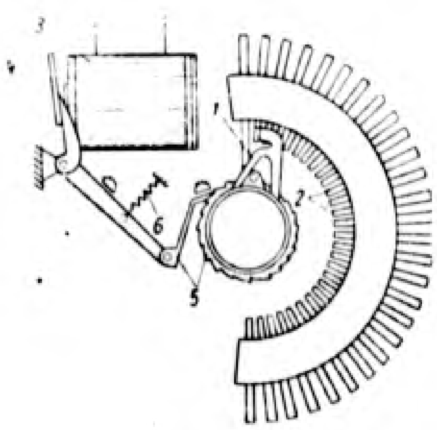

Fig. 4.16. Model of electromagnetic stepping switch.

An electron-beam selector is an electronic instrument that has the form of a special electron-beam tube. The bulb, from which the air has been evacuated, contains a hot cathode and the field of the electrodes (contacts). The cathode serves as an input contact, and the electron beam emitted by the cathode serves as the moving contact that scans the field of the fixed contacts — the electrodes. The cyclic motion of the electron beam over the contact field is effected with the aid of an electrostatic or magnetic control (deflection) system. The main advantage of electron-beam selectors is their practically unlimited speed.

§ 4.6. Other Types of Electromechanical Relays

The electromagnetic relays considered above are the most common relays of automation. To a much lesser extent one uses relays that are based not on electromagnets, but on other electromechanical transducers. The main types of these relays will be examined in this section. They are — magnetoelectric, electrodynamic, induction, thermal, and magnetostriction relays.

We are using here the same nomenclature as for electrical measuring instruments. Indeed, all the electromechanical transducers used in measuring instruments are also utilized in relays. Yet there exist many other types of relays, which have no counterpart among measuring instruments, since relays are simpler and can be designed on the basis of electromechanical transducers that are entirely unfit for measuring instruments. This is understandable, since a measuring instrument of continuous duty requires the same type of transducer with a more or less linear and stable static "input-output" characteristic, which is not mandatory for a relay.

A. Magnetoelectric Relays

The operating principle of this relay is the same as for the corresponding electrical measuring instruments, with the sole difference that instead of a pointer we have a moving contact (Fig. 4.17). The magnetoelectric signal-to-displacement transducer used here is one of the most sensitive transducers of this type. It is for this reason that the most sensitive electrical measuring instruments are using precisely this transducer (let us mention the galvanometer and the magnetoelectric vibrator for oscilloscopes). Its moving part can execute any angular (Fig. 4.17a) or translatory (Fig. 4.17b) motion. Instead of the loop, a permanent magnet can serve as the moving part.

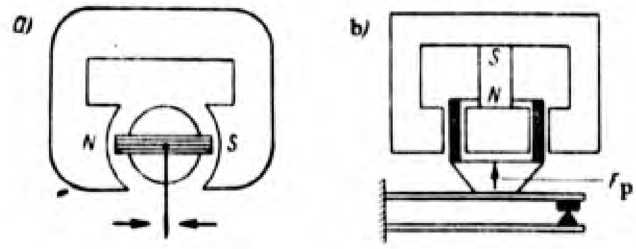

Fig. 4.17. Magnetoelectric relay with angular (a) and translatory (b) motion of the loop.

The pull force in a magnetoelectric transducer is represented by the force of interaction between the loop current and the magnetic field of the permanent magnet. If the conductors of the loop are perpendicular to the direction of the field, this force will be expressed as

$$F_p = 2B_w lWI, \tag{4.3}$$

where B_w is the induction of the magnetic field in the working air gap, I is the loop current, W is the number of turns of the loop, and l is the length of the loop.

The factor 2 accounts for the fact that the force acts on each of the two sides of the loop.

In the case of angular motion of the loop (Fig. 4.17a), one determines instead of the force the moment

$$M_p = F_p r = 2B_w lWIr, \tag{4.4}$$

where r is the loop radius.

As it follows from the formula for the pull force, a magnetoelectric transducer is a reversible or polarized device, since the direction of its pull force varies with the polarity of the loop current. A magnetoelectric relay is therefore also a polarized dc relay.

Among all electromechanical relays the magnetoelectric relay is the most sensitive one. It has also a many times higher control ratio P_{br}/P_{op}. The operating power of the most sensitive magnetoelectric relays is only about 10^{-10}w (as compared to at least 10^{-6}w for electromagnetic polarized relays) with a control ratio $P_{br}/P_{op} = 10^4 - 10^9$ (as compared to at most 10^5 for electromagnetic polarized relays).

Yet the magnetoelectric relays have the disadvantage of being slow. Their operate time is normally of the order of 0.1 sec, and their shortest operate time is of the order of fractions of a second. Thus with regard to speed these relays are inferior even to neutral electromagnetic relays, let alone polarized relays.

The development of magnetoelectric relays began fairly recently, being prompted by the needs of aviation automation, and the main applications of these relays lie still ahead. At present they are beginning to replace electromagnetic polarized relays wherever this is compatible with their lag.

B. Electrodynamic Relays

If the permanent magnet in the magnetoelectric relay shown in Fig. 4.17 is replaced by an electromagnet, such a relay is said to be electrodynamic. Hence it will possess two coils: a moving coil as in a magnetoelectric relay, and a fixed coil of the electromagnet that is placed on the magnetic circuit.

The inputs of the relay are formed by both coils. One can apply to them either two individual signals, or one common signal.

By replacing the permanent magnet by an electromagnet excited by a control signal, the high sensitivity characterizing magnetoelectric relays is lost. Yet in exchange the relay acquires entirely new properties. Let us examine these properties.

An electrodynamic relay has the same expression (4.3) for the pull force. Yet in this formula the magnetic induction B_W will no longer be constant, but will be determined by the magnetizing force of the fixed coil, i.e., if dissipation is neglected, we have

$$B_W = \frac{\Phi_2}{S_W} = \frac{F_2}{R_M S_W} = \frac{W_2 I_2}{R_M S_W},$$

where Φ_2 and F_2 are the magnetic flux and the magnetizing force produced by the fixed coil (coil 2), R_M is the magnetic reluctance of the magnetic circuit, and S_W is the cross-sectional area of the magnetic flux in the working air gap containing the moving coil (coil 1).

By introducing this expression into formula (5.3) we obtain

$$F_p = K I_1 I_2, \tag{4.5}$$

where

$$K = \frac{2 l W_1 W_2}{R_M S_W}.$$

The direction of F_p depends on the direction of the currents I_1 and I_2 in the two windings of the relay, i.e., F_p is positive if the currents have the same direction, and negative otherwise.

Electrodynamic relays can operate on direct and on alternating current, but their main domain of applicability is based on alternating current.

Let

$$i_1 = I_{1M} \sin \omega t;$$
$$i_2 = I_{2M} \sin (\omega t + \Psi).$$

Owing to the finite lag of the moving system of a relay, its motion will be determined by the mean value (over one period of the alternating current) of the pull force $F_{p.m}$, and not by its instantaneous value F_p.

According to formula (4.5) this mean value is equal to

$$F_{p.m} = \frac{1}{2\pi}\int_0^{2\pi} K I_{1m} I_{2m} \sin\omega t \cdot \sin(\omega t + \Psi)\, d(\omega t) = \frac{K}{2} I_{1m} I_{2m} \cos\Psi, \qquad (4.6)$$

since

$$\sin\omega t \cdot \sin(\omega t + \Psi) = \frac{\cos\Psi - \cos(2\omega t + \Psi)}{2},$$

and

$$\int_0^{2\pi} [\cos\Psi - \cos(2\omega t - \Psi)]\, d(\omega t) = 2\pi\cos\Psi.$$

Or, by going over to the effective values of the currents,

$$F_{p.m} = K I_1 I_2 \cos\Psi. \qquad (4.6a)$$

Formula (4.6) shows that an electrodynamic relay is phase sensitive, i.e., its pull force depends on the phase shift between the coil currents. Hence such a relay can be used as a phase-shift relay that operates at a certain value of Ψ, and as an ac or dc power relay (in this case a current is fed to one of the coils, and the voltage of the circuit whose power is switched by the particular relay is applied to the other coil). If both coils are connected in series, i.e., one common signal is applied to them, we obtain $F_p = KI^2$, i.e., the electrodynamic relay assumes the properties of an electromagnetic relay whose pull force is likewise proportional to the current squared.

If an electrodynamic relay is designed without a ferromagnetic magnetic circuit, i.e., if it consists simply of two coils (a fixed and a moving coil), it can be used at high frequencies like the corresponding measuring instruments. It is clear that this involves a sharp drop in sensitivity and pull force, i.e., in contact power.

The electrodynamic relays considered above that contain a ferrodynamic magnetic circuit are also called ferrodynamic relays (like the corresponding measuring instruments), in contrast to electrodynamic relays without a ferromagnetic part.

Electrodynamic relays, including ferrodynamic, are not extensively used.

C. Induction Relays

The operating principle of these relays is the same as for the corresponding measuring instruments, i.e., their operation is based on the force of interaction between the alternating current induced in the conductor and the variable magnetic flux. Hence this type is an ac relay.

Induction relays are being widely used, mainly as automatic protection relays in ac circuits (current relays, voltage relays, power relays, frequency relays, and resistance relays). Therefore they are designed for operation with measuring current and voltage transformers. The extensive use of induction relays as protective relays is due to the ease with which their pull and time characteristics can be varied, and their easy adaptability to the most diverse requirements imposed on protective relays.

Induction relays consist of two main parts — a stator and a rotor, just as in an electric motor. The stator has one or two ac coils. The magnetic fluxes produced by them induce alternating currents in the rotor (which can be solid or equipped with a coil). The interaction between the currents induced in the rotor and the magnetic field of the stator produces the necessary pull force.

From the point of view of design, induction relays can be of three types: with a rotor that has the form of a short-circuited loop, or a disk, or a cup.

An induction relay with a short-circuited loop (Fig. 4.18) resembles a ferrodynamic relay. Here, too, the pull force used is the force acting on the loop, whose current is produced by the field of the coil denoted in Fig. 4.18a by 2. The only difference consists in the fact that in an induction relay the current in the loop is induced not directly, but by transformation from the special coil 1. Hence this relay, just as an electrodynamic relay, has two inputs — the coils 1 and 2. The first serves for producing the current I in the short-circuited loop, and the second — for producing the magnetic flux Φ_2 in the air gap containing the loop.

The instantaneous value of the pull moment acting on the loop can be determined by formula (4.4):

$$M_p = 2B_{w.inst} l W i r,$$

or

$$M_p = \frac{2lrW}{S_w}\Phi_{2inst} i,$$

where $i = I_M \sin\omega t$ is the loop current, and $\Phi_{2inst} = \Phi_{2M}\sin(\omega t+\Psi)$ is the magnetic flux produced by the coil 2.

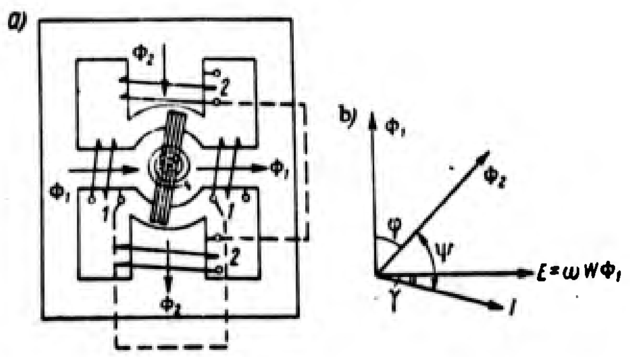

Fig. 4.18. Induction relay with short-circuited loop (the contact system is not shown).

The mean value (over a period) of the pull moment is

$$M_{p.m} = \frac{1}{2\pi}\int_0^{2\pi}\frac{2lrW}{S_w}\Phi_{2M} I_M \sin\omega t\cdot\sin(\omega t+\Psi)\,d(\omega t),$$

or, by effecting the same transformation as in the derivation of formula (4.6), we obtain

$$M_{p.m} = C\Phi_{2M} I_M \cos\Psi, \tag{4.7}$$

where

$$C = \frac{lrW}{S_w}.$$

By introducing into (4.7) the expression

$$I_M = \frac{E_M}{Z} = \frac{\omega W}{Z}\Phi_{1M},$$

we can represent the pull moment as follows

$$M_{p.m} = C'\Phi_{1M}\Phi_{2M}\cos\Psi, \tag{4.8}$$

where

$$C' = \frac{C\omega W}{Z}.$$

From the vector diagram of Fig. 4.18b, it follows that the phase angle between Φ_2 and I is

$$\Psi = \frac{\pi}{2} + \gamma - \varphi = \frac{\pi}{2} - (\varphi - \gamma),$$

where φ is the phase angle between Φ_1 and Φ_2, or, what amounts to the same, between the currents in the coils 1 and 2; γ is the phase angle between the emf E of the loop and the current I produced by it.

By introducing this expression for Ψ into formula (4.8) we obtain

$$M_{p.m} = C'\Phi_{1m}\Phi_{2m}\sin(\varphi - \gamma). \tag{4.9}$$

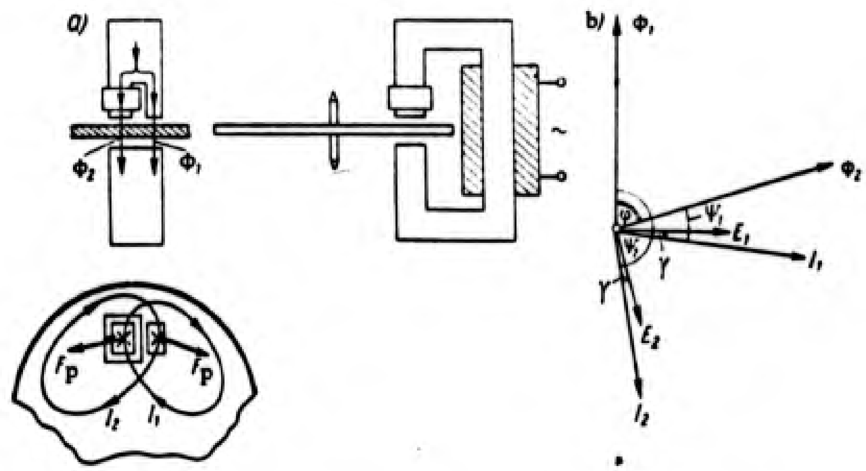

Fig. 4.19. Induction disk relay (the contact system is not shown).

Hence an induction loop relay is phase sensitive: when φ changes sign, its pull moment also changes sign. Therefore such a relay, just as an electrodynamic relay, can be used not only as a current and voltage relay, but also as a phase, power, frequency, and resistance relay. The relay is equipped with a helical restoring spring.

Fig. 4.19a shows the simplest induction disk relay. It has only one input coil and can therefore be used only as a current or voltage relay. The relay has a short-circuited winding (slug) that encloses part of the pole cross-section. As in the case of ac electromagnets (§ 1.9), it serves for producing in the air gap two phase-shifted magnetic fluxes Φ_1 and Φ_2.

The rotor in which the currents are induced is here in the form of an aluminum disk. Each of the two fluxes produces in it an alternating current that closes itself, as schematically shown in the lower projection of Fig. 4.19a. For the purpose of illustration we projected here both halves of the upper pole onto the plane of the disk; the crosses indicate the instantaneous directions of the fluxes Φ_1 and Φ_2. Through the portions of the disk, permeated by either of the fluxes, passes the current induced by the other flux. Therefore these portions of the disk are under the action of the pull forces F_{p1} and F_{p2}, shown in Fig. 4.19a.

The mean value of the moments, produced by the forces F_{p1} and F_{p2}, is specified by the formulas

$$M_{p1m} = \frac{1}{2} C'\Phi_{2m}\Phi_{1m}\cos\Psi_1,$$
$$M_{p2m} = \frac{1}{2} C'\Phi_{1m}\Phi_{2m}\cos\Psi_2.$$

Here

$$C' = \frac{lr\omega}{S_w Z};$$

l is the length of the portion of the current-carrying disk that lies in the magnetic field, r is the distance from the axis of rotation of the disk to the line of action of the pull force, S_w is the cross-sectional area of the flux in the working gap, and Z is the electric impedance of the current path in the disk.

The angles Ψ_1 and Ψ_2, shown in the vector diagram of Fig. 4.18b, are expressed as follows:

$$\Psi_1 = \frac{\pi}{2} + \gamma - \varphi; \quad \Psi_2 = \frac{\pi}{2} + \gamma + \varphi.$$

Hence

$$M_{p1m} = \frac{1}{2} C'\Phi_{2m}\Phi_{1m}\sin(\varphi - \gamma);$$
$$M_{p2m} = -\frac{1}{2} C'\Phi_{1m}\Phi_{2m}\sin(\varphi + \gamma).$$

whence we obtain for the resultant pull moment

$$M_{p.m} = M_{p.m} - M_{p.m} = C'\Phi_{1m}\Phi_{2m}\cos\gamma\cdot\sin\varphi. \tag{4.10}$$

The direction of the pull moment is <u>towards the delayed flux</u> (in Fig. 4.19a — to the left). As it follows from formula (4.10), in order to increase the pull moment of the relay it is necessary that φ be as close as possible to $\pi/2$, and γ to 0. The first condition requires the generation of two fluxes whose phase is shifted by an angle that is as close as possible to $\pi/2$; this condition can be met with the aid of a short-circuited winding (slug).

We have been considering the simplest induction relay with a disk, one magnetic system, and a short-circuited winding (slug) for the generation of two phase-shifted fluxes. But there exist also induction disk relays with two independent magnetic systems that produce their own magnetic fluxes. An example of such an induction system is the widely used ac energy counter system.

The pull force of induction disk relays with two magnetic systems is specified by the same formula (4.10), i.e., it is also proportional to the sine of the angle between the fluxes Φ_1 and Φ_2; yet in this case the fluxes are produced by two independent coils. Therefore such a relay is phase sensitive.

The induction cup relay (with cylinder or drum), shown in Fig. 4.20, differs from a disk-type relay with two coils only by the shape of the rotor. It operates in the same way and the formula for the pull force is the same. Each of the two fluxes interacts with the current induced by the other flux.

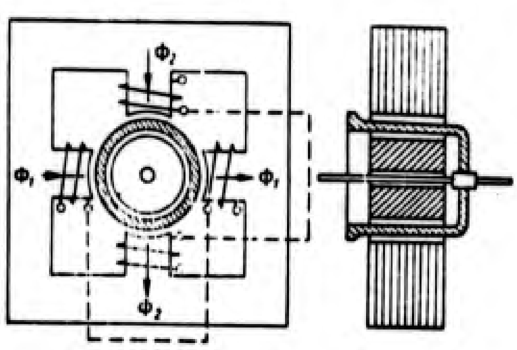

Fig. 4.20. Induction cup relay (the contact system is not shown).

This induction system is similar to a two-phase asynchronous motor with a hollow rotor from the point of view of the operating principle as well as design.

Owing to the small radius of the cup, and hence also of its moment of inertia as compared to that of a disk, an induction relay with a cup, just as a relay with a short-circuited loop, has a much smaller operate time than that of a disk-type relay (of the order of one period of the signal frequency, i.e., about 0.02 sec at 50 cps).

D. Thermal Electromechanical Relays

The operation of these relays is based on the thermal action of the electrical current. One uses thermal effects such as an increase in the dimensions of bodies, melting, and variations in permeability.

The simplest thermal relay is a fuse link.

A type, widely used, is the bimetallic thermal relay (Fig. 4.21). The principle component of such a relay is a thermal bimetallic sheet, i.e., a sheet consisting of two rigidly fixed (by welding or brazing) metal strips whose heat expansion coefficients differ substantially. When such a sheet is heated, it bends towards the metal with the smaller expansion coefficient. This effect is used in the relay. The heating is produced by means of a heater spiral (as shown in Fig. 4.21a), or by passing the current directly through the thermal bimetallic sheet. The sensitivity of the relay can be increased by increasing the length of its sheet, by shaping it, for example, in the form of a spiral (Fig. 4.21b).

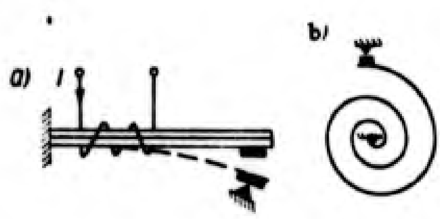

Fig. 4.21. Bimetallic relay with plane (a) and helical (b) bimetallic sheet.

Bimetallic relays are used as protective relays and as time relays.

As protective relays they serve for the protection of all sorts of electrical devices (electrical motors, transformers, etc.) against overheating due to overloads. These relays are therefore known as overload relays. The use of bimetallic relays as overload relays is based on the fact that their operation is determined not by the instantaneous value of the current, but by their heating. In order that the relay should operate precisely at the moment when the temperature of the object to be protected reaches a certain limit, the heating time constant of the relay must be matched with the heating time constant of the object.

As a result of their thermal lag, bimetallic relays can be also used as time relays with a time delay ranging from a few seconds to a few minutes. The time delay can be set by varying the distance between the contacts. The accuracy of these relays is not high, since their operate time depends on the control current; on the other hand they are very simple, reliable and small in size. For this reason bimetallic relays are utilized when it is required to obtain time delays without particular acuracy. (For example, the automatic switching on of the anode voltage of thyratrons with a time delay following the switching on of their filament current necessary for heating the instruments.)

Thermal bimetallic relays serve also as a base for relays used for the direct indication of the temperature — temperature relays. Such relays are known as thermostats. The input variable of these relays is the ambient temperature. In addition to bimetallic thermostats, there exist also other types of thermostats.

E. Magnetostriction Relays

These relays use the magnetostriction effect, i.e., the change in the dimensions of ferromagnetic bodies in a magnetic field. This effect is most strongly exhibited in iron-nickel alloys.

In design, magnetostriction relays are similar to thermal relays using the thermal expansion of bodies, such as bimetallic relays (Fig. 4.21).

§4.7. Typical Electromechanical Relay Circuits

Figure 4.22 shows the graphic symbols for relays and contactors. In order to distinguish between the main (power) contacts of a contactor from its low-power auxiliary contacts, called block contacts, the main contacts are represented by thicker lines, generally used to distinguish power circuits from control circuits. (Block contacts serve for the switching of control circuits as a function of the position of the contactor.)

Let us consider some typical relay circuits. In Fig. 4.23a we show a so-called economical relay circuit.

In order to keep the armature of an already-operated relay in the pulled-up position one requires much less current in the relay coil as compared to the operate current. For this reason, if the relay must remain for a sufficiently long time in the operate state, it is more convenient to reduce (after the relay has operated) the coil current to the minimum value necessary for keeping the armature in this position. Thus one prevents superfluous energy losses in the coil circuit and one diminishes the heating of the coil. This problem is solved by the circuit of Fig. 4.23a. After the relay P has operated, for example, as a result of depressing the key K as shown in the figure (instead of this key we can have a contact of any other relay), the resistor R is inserted in the coil circuit of the relay; before the relay operated this resistor was short-circuited by the opening contact of this same relay. As a result, the coil current of the relay drops to the necessary minimum.

Figure 4.23b shows a self-locking relay circuit. After briefly depressing the key K1 the relay operates and by its make contact closes the coil supply circuit independently of the key K1. As a result, the subsequent opening of the key K1 does not cause relay dropout. (The efficiency of the circuit can be increased by connecting a resistor R in series with the relay contact.) Relay dropout is effected by disconnecting the common supply circuit, which can be done with the aid of the key K2.

Figure 4.23c shows a mutual-locking circuit of two relays. It prevents the simultaneous switching of both relays by way of inserting into the coil circuit of each of them a break contact of the other relay. The notations of each contact indicate the relay to which this contact belongs. The need for mutual locking arises quite often. For example, it protects against damage in the case that one relay serves for starting a motor in a certain direction of rotation, whereas the other relay starts the motor in the other direction.

Figure 4.23d shows a pulse generator circuit using two relays (a pulse pair). When the K is depressed the relay P1 operates and closes by its make contact the coil circuit of the relay P2. The latter operates and opens by its break contact the coil of the relay P1, which drops out. Thereupon the relay P2 drops out too, but as a result the coil circuit of the relay P1 is again closed (if the key K is closed). The relay P1 operates and the whole process is repeated. Thus the two relays are periodically switched on and off, and the circuit generates pulses whose frequency will be the lower, the greater the lag of the relay. The circuit goes on operating as long as the key K is closed. Such circuits are used as low-frequency pulse sources (for example in remote control). The load R, to which these

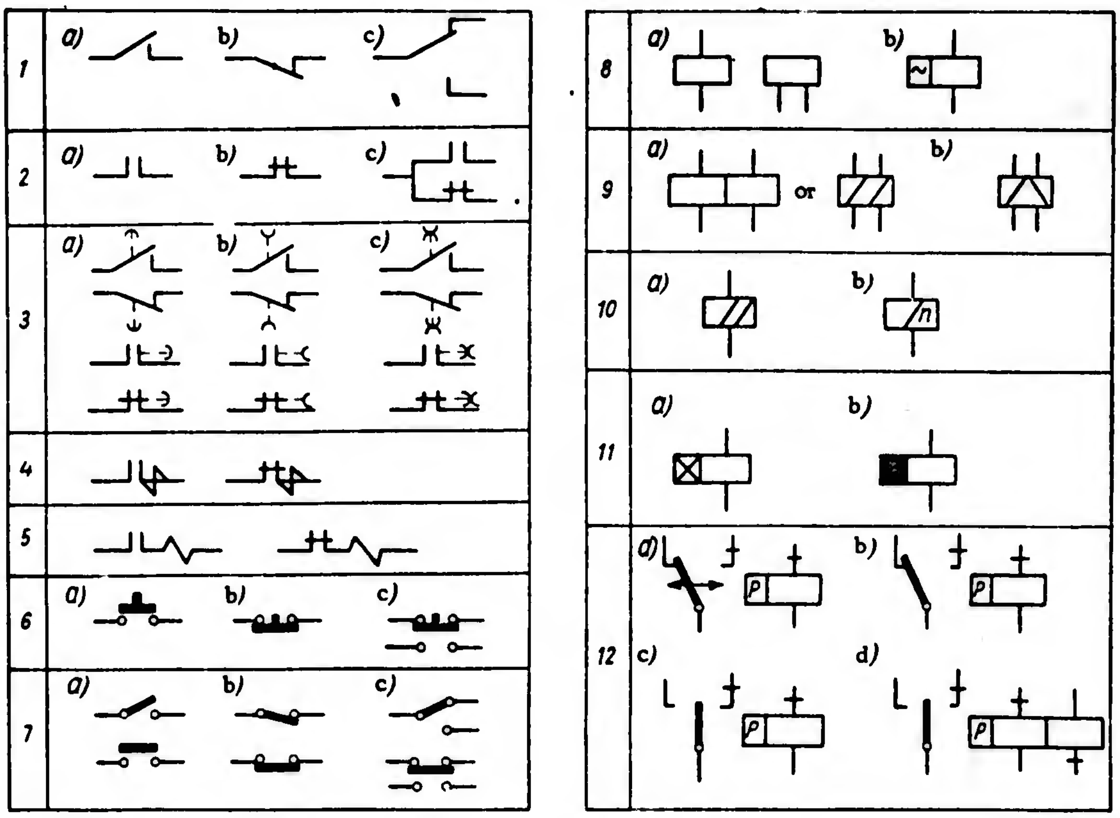

Fig. 4.22. Graphic symbols for relays and contactors (GOST 7624-62). 1 — relay contacts: a) make (NO), b) break (NC), c) transfer (MB); 2 — contactor contacts: a) NO, b) NC, c) MB; 3 — contact with time delay: a) in closing, b) in opening, c) in closing and opening; 4 — contact with quenching; 5 — contact with electromagnetic quenching (magnetic blowout); 6 — keys: a) with NO, b) with NC, c) with MB; 7) switch-on and switch-off contacts: a) NO, b) NC, c) MB; 8 — coil: a) of relay, contactor, magnetic starter, electromagnet; b) same, indicating also the type of current (~); similarly one specifies a current coil (I), voltage coil (U), etc; 9 — coils: a) of two-coil relay, b) of differential relay; 10 — one coil: a) of two-coil relay, b) of n-coil relay; 11 — coil of electromagnetic relay with time delay: a) for operation, b) for release; 12 — polarized relay: a) two-position, b) two-position, with left contact predominant, c) three-position, d) same, but two-coil type; the contact, marked by a bar, closes (when a signal is applied to the coil) with the plus towards the output marked by a bar.

pulses are applied, is connected to a dc voltage across the contact of one of the relays, as shown in the figure.

Such a pulse generator can be also designed with a single relay, by inserting a break contact into its coil supply circuit.

Figure 4.24 shows the control circuit of an ac motor that permits the motor to be s started and stopped with the aid of the two buttons K1 and K2. Such circuits are designed in the form of individual devices, called magnetic starters. The circuit under consideration consists of a contactor with three main contacts and one block contact which serves for self-locking of the contactor after it has been operated by depressing the key K1 (start), and of two bimetallic relays PT1 and PT2 which are overload relays. For motor reversal the magnetic station is supplemented by an additional contactor by means of which the phase terminals of the motor are connected to the line phases in inverse order.

Magnetic starters for controlling dc motors are similar in design.

Figure 4.25 shows a diagram of a relay selector.

This circuit is used for the same purpose as the stepping switch described in §4.5. It has one input — the coil of the relay T to which the pulses are fed, and several outputs — the actuating circuits (not shown in the figure) to which the contacts of the other relays (1, 2, 3, . . .) are connected. Their number is not limited. These relays operate in succession, upon arrival of the input pulses. In contrast to a stepping switch, the relay switch shown in Fig. 4.25 makes one "step" during a pulse, as well as during a pause: during a pulse the odd-numbered relays are operating, whereas during a pause the even-numbered

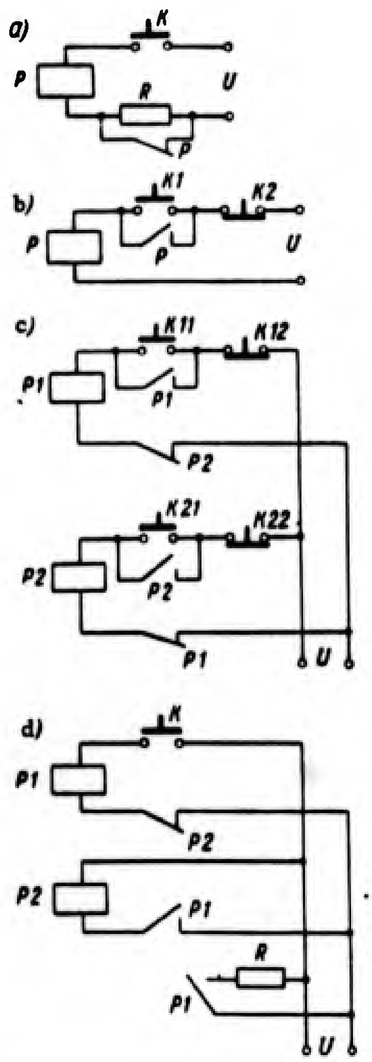

Fig. 4.23. Typical relay circuits: a — economic relay circuit; b — self-locking relay; c — mutual locking of two relays; d — pulse generator (pulse pair).

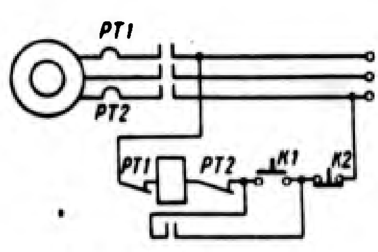

Fig. 4.24. Schematic of magnetic station for controlling the starting and stopping of an ac motor.

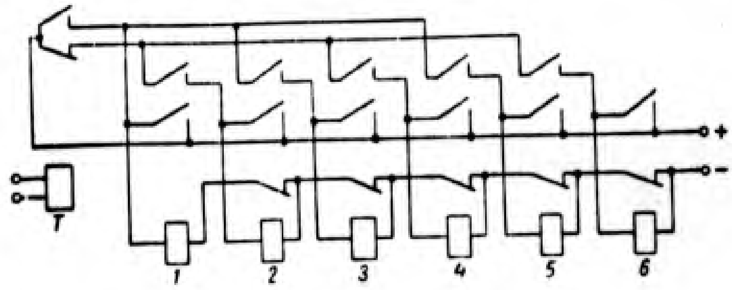

Fig. 4.25. Relay Selector

relays operate. The circuit functions as follows. In the original position all the relays are in drop-out position. When the first input pulse arrives at the relay T, its closing contact causes the relay 1 to operate. By one contact it becomes self locked, whereas by the other contact it makes ready the coil circuit of relay 2. As a result, the break contact of the relay T causes the relay 2 to operate during the pause following the first input pulse. Relay 2, which is likewise self-locking, makes ready the coil circuit of relay 3, while at the same time opening the circuit of relay 1 by its break contact. The second input pulse operates the relay 3, while making ready the coil circuit of the (next) relay 4 and opening the circuit of the (preceding) relay 2, etc.

The network can be closed into a cycle; in this case when the last relay operates the relay 1 will be in a ready state. Such a circuit will operate cyclically, as a stepping switch with unlimited motion of the moving contacts.

There exist many other versions of relay selectors, for example those operating only during a pulse or only during a pause, analogously to a stepping switch.

An interesting application of electromechanical relays are the so-called vibration amplifiers. They are dc amplifiers of continuous duty based on electromechanical (normally polarized) relays operating in the vibration mode.

Vibration amplifiers are either externally excited or self-excited.

The simplest model of a non-reversible vibration amplifier with external excitation is shown in Fig. 4.26a. It consists of a polarized relay P with two coils — the exciting coil W_E and the control coil W_C, and also of a dc voltage supply. Under the action of an

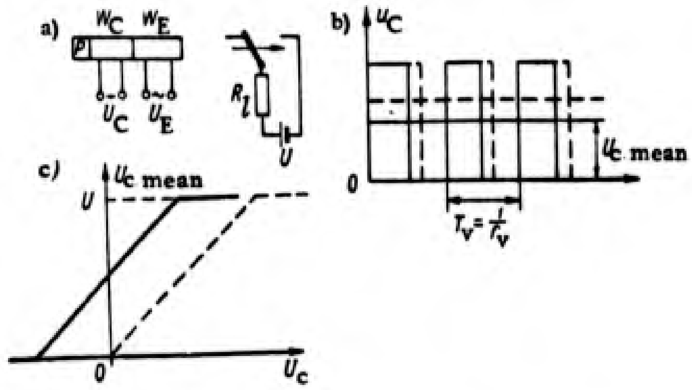

Fig. 4.26. Non-reversible vibration amplifier with external excitation.

ac voltage U_V the armature of the relay vibrates at the frequency f_V of this current, while voltage pulses (shown in Fig. 4.26b) are applied to the load R_l. The duration of the pulses is equal to the time during which the contact of the relay P is closed. When a dc input signal U_C is applied to the control coil W_C, the pulse duration, and hence also the mean value of the load current are varying (the dashed line in Fig. 4.26b) Figure 4.26c shows the static "input-output" characteristic of the amplifier. If the input signal U_C is sufficiently strong, the relay contact will be permanently closed under the action of U_C with a certain polarity, and U_E will be unable to open it. As a result, the load will be permanently connected to the supply voltage, i.e., $U_l = U$. Conversely, with the other polarity of U_C the relay contact will be permanently open, and $U_l = 0$ (see Fig. 4.26c).

Like in other types of amplifiers (such as electronic, magnetic, etc.) one uses also in vibration amplifiers a dc bias, feedbacks, the coupling of two non-reversible amplifiers in a reversible configuration, and cascade connections. For example, if it is necessary to shift the static characteristic, so as to ensure that it passes through the origin of coordinates (as shown by the dashed line in Fig. 4.26c), one uses a dc bias. It can be realized by applying a direct bias current, obtained, for example, from the same supply source of the load circuit (U) and fed to a special bias coil or to one of the coils already used (W_C and W_E). Finally, the shifting of the static characteristic can be also achieved by mechanical means – by readjusting a polarized relay (shifting its movable contacts).

Vibration amplifiers are used for the amplification of dc signals of low frequency. For the undistorted amplification of an ac signal it is necessary that its frequency be at least five times smaller than the armature vibration frequency f_v. For ordinary polarized relays the maximum armature vibration frequency is 200 cps.

In accordance with the magnitude of the control ratio of polarized relays, the power gain of vibration amplifiers may attain several thousand. The maximum output power of a vibration amplifier is determined by the maximum power that can be switched by the contacts. For example, by using RP-type polarized relays it is possible to control a dc motor with a nominal voltage of 24 v and a current of 1 amp.

Vibration amplifiers have the highest efficiency among all the amplifiers available. Their load-circuit losses are negligible, being contact losses; the excitation losses are also small, in view of the high control ratio of polarized relays.

The size and weight of vibration amplifiers are small. Thus 1 w output power correspond to roughly 1 cm^3 of the total amplifier volume.

The shortcomings of vibration amplifiers are obvious: the presence of movable parts, contacts, and hence their short service life (without adjusting the relays it amounts to 200-250 hours).

Vibration amplifiers are being mainly used in control circuits of low-power dc motors (with a power of up to several tens of watts).

In Figs. 4.27a and 4.27b we present two models of a reversible vibration amplifier with independent excitation. Their static characteristic is shown in Fig. 4.27c. In the circuit of Fig. 4.27a, with $U_C = 0$, the armature of a polarized relay vibrates symmetrically, closing each of the two contacts for the same period of time. As a result, the current flowing through the load will be purely alternating, i.e., the dc component of the load voltage is equal to zero ($U_l = 0$). With $U_C \neq 0$ the relay functions nonsymmetrically, and a dc voltage U_l appears across the load that is proportional to the difference between the times during which the contacts are closed. The circuit of Fig. 4.27b is characterized, on the one hand, by the fact that it connects one voltage supply U, and, on the other hand, by the fact that with

$U_C = 0$ there is no alternating current flowing through the load. In contrast, it uses two relays.

With $U_C = 0$ the contacts of the two relays are simultaneously switched and R_l is not connected to U. With $U_C \neq 0$, as a result of the fact that the control coils of the relays are opposing - connected (whereas the exciting coils are aiding-connected), the two relays will function nonsymmetrically: in one relay the closing time of the first contact increases, whereas in the other relay the closing time of the second contact increases (or conversely), depending on the polarity of the signal. The load R_l will thus be periodically connected to U.

The resistor R and the capacitor C, connected in parallel to the control coils, have the purpose of producing a phase shift between the currents flowing in these coils, and hence also between the positions of the relay armatures. This eliminates the dead zone of the amplifier due to the finite travel time of the armatures.

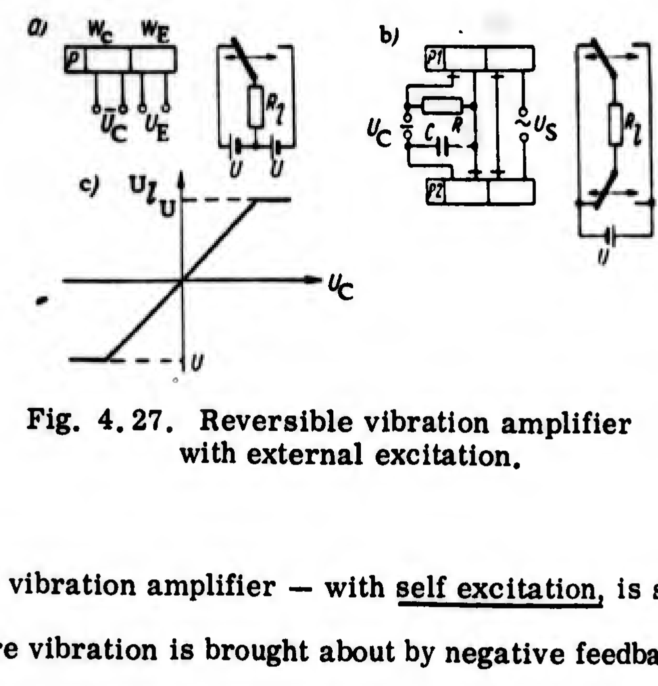

Fig. 4.27. Reversible vibration amplifier with external excitation.

Another type of vibration amplifier — with <u>self excitation</u>, is shown in Fig. 4.28. In this case the armature vibration is brought about by negative feedback. A similar self-excitation regime occurs in relay pulse generators.

The feedback is produced with the aid of the winding W_{fb}. In the circuit of Fig. 4.28a the winding W_{fb} is connected in such a way that when any contact of the relay is closed, the pull force produced by it tends to move the armature in the opposite direction, i.e., we obtain negative feedback.

As a result, with $U_C = 0$ the armature of the relay P vibrates symmetrically, closing the two contacts for the same period of time. With $U_C \neq 0$ this symmetry is violated and a dc voltage proportional to U_C appears at the load.

The circuit of Fig. 4.28b uses two relays, but only one supply source. With $U_C = 0$ the feedback windings fix the armatures of the two relays in the same position in such a way that either both left contacts are closed (as shown in Fig. 4.28b) or both right contacts. (When this position is disturbed, one connects the circuit of the windings W_{fb} and as a result of these two windings acting in the same direction the armatures will likewise assume the same position.) Since the windings W_C are opposing - connected, the armature of one of the relays (when $U_C \neq 0$), in which the signal in W_C acts counter to W_{fb}, will vibrate analogously to the case represented earlier in Fig. 4.28a.

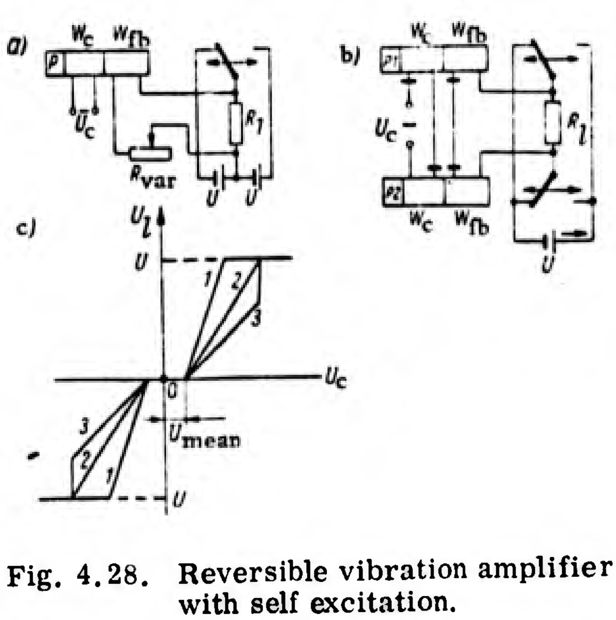

Fig. 4.28. Reversible vibration amplifier with self excitation.

The gain of an amplifier with self excitation depends on the magnitude of the feedback (it decreases when the latter increases). In Fig. 4.28c we show the static characteristics of an amplifier for three values of the resistance R_{var}, which determines the strength of the feedback (the smaller the value of R_{var}, the stronger the feedback); to a larger number corresponds a larger value of R_{var}.

Above we assumed that the amplifier circuit with self excitation used two-position relays. One can also used three-position relays. The difference between these configurations consists in the fact that with $U_C = 0$ the armature of each relay is at rest and that both its contacts are opened. Vibration will set in only with the appearance of the input signal U_C.

In contrast to vibration amplifiers with independent excitation, in amplifiers with self excitation the vibration frequency varies with the signal (in the case of maximum signal variation the frequency varies by a factor of 2-4). Further details about vibration relays can be found [7].

Footnotes

p. 112. [1] The word "contact" has two meanings: the point at which the contacting bodies are in touch is called a contact, and, for brevity, the bodies themselves are also called contacts.

p. 121. [2] From the word deionization.

PART TWO

FERROMAGNETIC DEVICES

Chapter 5

Principle of Operation and Basic Properties of Magnetic Amplifiers

§ 5.1. The Principle of Operation of Magnetic Amplifiers

The operation of a magnetic amplifier (MA) is based on the saturation effect of ferromagnetic bodies in a magnetic field, i.e., on the nonlinearity of their magnetization curves B = f(H) (Fig. 5.1a).

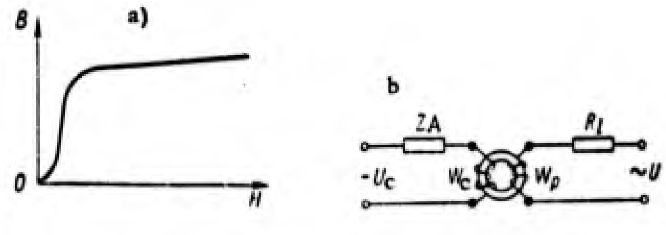

Fig. 5.1. Simplest model of magnetic amplifier

The simplest model of a magnetic amplifier is shown schematically in Fig. 5.1b. The ferromagnetic core has two windings: the control winding W_C and the load (output) winding W_p. The signal U_C to be amplified is applied to the control winding. A resistor R_l, which serves as the amplifier output, is connected in series with the load winding. The load-winding circuit (output circuit) is supplied by an ac voltage source U, of 50 cps for example. The inductive impedance Z_A, additionally inserted into the control circuit. has the purpose of suppressing the alternating current due to the emf induced in the control winding by the load winding.

The output current is expressed as

$$I = \frac{U}{\sqrt{X_p^2 + R^2}},$$

where R is the total resistance of the output circuit, including the load resistance R_l and the resistance of the load winding, X_p is the inductive reactance of the load winding,

$X_p = \omega L_p$, ω ω is the angular frequency of the load circuit supply, L_p is the inductance of the load winding, $L_p = \frac{W_p^2 S_c}{l_c}\mu$. S_c is the cross-sectional area of the core, l_c is the mean length of the path of the magnetic flux in the core, and μ is the dynamic permeability of the core.

The load current depends on the inductive resistance of the load winding, which is proportional to the dynamic permeability μ of the core.

If a direct current I_C is applied to the control winding W_C, a dc magnetic flux will appear in the core that is superimposed on the ac magnetic flux produced by the load winding. Owing to the nonlinearity of the magnetization curve the core saturation increases with increasing I_C. As a result, its permeability μ decreases, and hence also the load-winding inductance L_p. A decrease in the inductance L_p is accompanied by an increase in the load current I. In Fig. 5.2 we plotted the inductance L_p and the current I versus the control current I_C. Both curves are symmetrical with respect to the ordinate, since the magnetic state of the core does not depend on the direction of I_C.

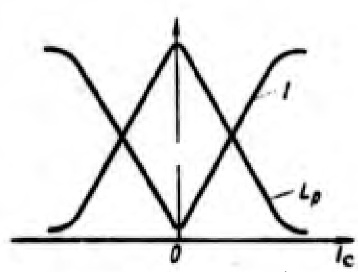

Fig. 5.2. Characteristic of magnetic amplifier.

Thus, by magnetizing the core by means of a direct current it is possible to control the inductance of the load winding, and hence also the alternating current in its circuit.

The power dissipated in the resistance of the control circuit is much lower than the power generated in the load R_l. Hence the circuit under consideration can be used as a power amplifier.

The decrease in the permeability of a ferromagnetic core for an ac magnetic flux when the core is magnetized by a dc field — an effect utilized in magnetic amplifiers — occurs also (among others) in the chokes of rectifier output filters and in output transformers of electronic amplifiers. But in such devices this constitutes a harmful effect that is fought against (by means of air gaps in the cores).

The variable inductance controlled by magnetizing the core by a direct current is often called a saturable reactor (SR). In addition to serving as a component of magnetic amplifiers, it is also widely used as a variable inductive reactance.

The operating principle of a magnetic amplifier can be easily visualized by means of the graphs of Fig. 5.3. Let us consider at first an amplifier whose load circuit does not contain a resistance R. In this case the instantaneous value of the supply voltage will be fully balanced by the self-induction emf e_p of the load circuit, i.e.,

$$u = -e_p = W_p \frac{d\Phi_{inst}}{dt} =$$
$$= W_p S_c \frac{dB_{inst}}{dt}.$$

Here Φ_{inst} and B_{inst} are the instantaneous values of the variable magnetic flux and of the variable induction in the core.

Hence follows that in the case of a sinusoidal variation of the voltage $u = U_M \sin t$, the induction will also have a sinusoidal variation, though lagging the voltage by a quarter period. The induction amplitude will be

$$B_M = \frac{U_M}{\omega W_p S_c}.$$

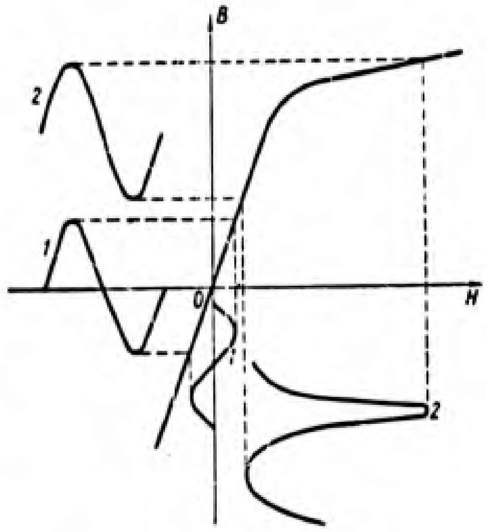

Fig. 5.3. Graphic description of principle of operation of a magnetic amplifier.

Thus when we know the value of the voltage applied to the load winding, it is easy to determine the core induction. In Fig. 5.3 we constructed on the basis of the induction curve B_{inst} the curve for the ac magnetic field

$$H = \frac{W_p i}{l_c}.$$

The curves were plotted for two cases: 1 — in the absence of a dc magnetic field, and 2 — in the presence of such a field.

From Fig. 5.3 one can see that when the dc magnetization increases, the amplitude of the ac magnetic field increases strongly (as a result of core saturation). This means that the amplitude of the output current, which is proportional to the field amplitude, will also increase:

$$I_м = \frac{H_м l_c}{W_p}.$$

§5.2. Higher Harmonics in a Magnetic Amplifier

Figure 5.3 shows that when a sinusoidal voltage is applied to the load winding, the current in this winding will not be sinusoidal (as a result of the nonlinearity of the magnetization curve), i.e., it will contain higher harmonics. Let us note that that in the absence of dc magnetization, the curve H_{inst} and hence also the current i will contain only odd harmonics (the third, etc.). This is a consequence of the full symmetry of the two half-waves H_{inst} with respect to the time axis. In the presence of dc magnetization, on the other hand, the half-waves become nonsymmetrical, and hence they will contain also even harmonics[1].

It is likewise easy to see that when the polarity of the dc magnetic field changes, the phase of the even harmonics is shifted by 180°.

In the above analysis the induction of the ac field was sinusoidal in view of the fact that the load-circuit resistance R was not taken into account, assuming that the entire supply voltage U is balanced by the self-induction emf of the load circuit.

In the presence of a resistor R in the load circuit, a voltage drop will occur across this resistor and therefore the voltage across the load winding will be non-sinusoidal, since the voltage across the resistor R is non-sinusoidal.

Thus in an actual amplifier, both the induction B_{inst} and the field strength H_{inst} are non-sinusoidal. The larger the load-circuit resistance R, the more distorted will be the curve B_{inst} and the closer to a sinusoid the curve H_{inst}.

Let us show that among the two limiting cases – sinusoidal induction and sinusoidal ac field – the first mode of operation yields greater amplifcation as compared to the second. The curves B_{inst} and H_{inst} were constructed in Fig. 5.4 for the two modes. The curves 1 correspond (as in Fig. 5.3) to sinusoidal induction, and the curves 2 to a sinusoidal field.

[1] See footnote p. 163.

The curves have been constructed for two cases: in the absence and in the presence of dc magnetization. The order of construction is as follows. By assigning the dc magnetic induction and the amplitude of the ac sinusoidal magnetic induction, we construct on the basis of the magnetization characteristic the ac field curve H_{inst}. In the presence of dc magnetization the H_{inst} curve will be nonsymmetrical with respect to the time axis, i.e.. it contains even harmonics. From the condition that the areas of the two half-waves must be equal, we find the dc component H_{C1} of the field generated by the control winding, i.e..

$$H_{c1} = \frac{W_c I_{c1}}{l_c}.$$

Next, while retaining the same limiting values of the field strength as in the first case, we assign the sinusoidal H_{inst} curve and we construct on its basis the induction curve (curve 2'). The dc component of the field in the second case (H_{C2}) is found to be larger than in the first case. Hence in order to change the ac field strength, and consequently also the alternating current in the load circuit by the same magnitude, the dc magnetization H_C will be smaller in the case of a sinusoidal induction as compared to the case of a sinusoidal field strength. This means that the gain of the amplifier will be greater in the first case as compared to the second.

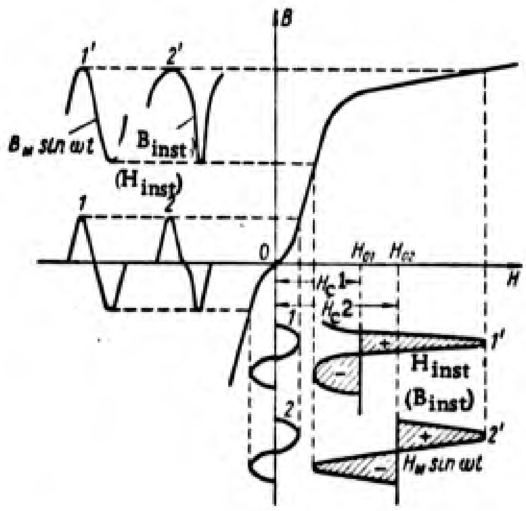

Fig. 5.4. Illustrating the magnetizing effect of even harmonics.

The smaller magnitude of H_{C1} as compared to H_{C2} is due to the presence of even harmonics in the field-strength curve in the first case. The even harmonics have an effect similar to that of additional dc magnetization of the core by a value of $(H_{C2}-H_{C1})$. In this sense one refers to the "magnetizing effect of the even harmonics of the field strength". As will be shown below, this positive effect of the even harmonics is widely used in magnetic amplifiers for improving their performance.

§ 5.3. Magnetic Amplifiers With Output Rectification

The magnetic amplifier represented in Fig. 5.1 not only amplifies the input signal but also converts it into an ac signal whose frequency is equal to the load-circuit supply frequency ω, i.e., such an amplifier performs amplitude modulation, serving as a modulator. Indeed, by applying a direct current I_C to the amplifier input, we obtain at the output an alternating current of frequency ω, whose amplitude varies as the instantaneous value of I_C. This is illustrated in Fig. 5.5.

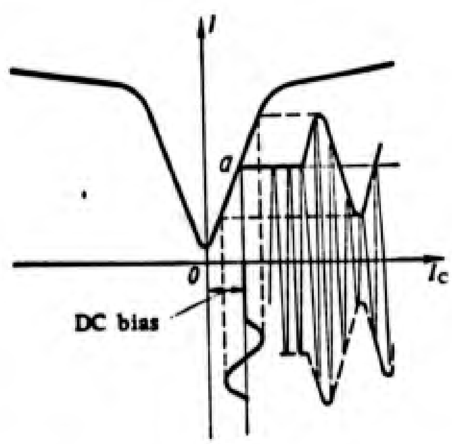

Fig. 5.5. Amplitude modulation in a magnetic amplifier.

In order that a magnetic amplifier should properly respond to signals of different polarity and be able to amplify ac signals, one introduces a dc bias into the amplifier (as shown in Fig. 5.5). This bias is produced by means of a special bias winding, through which a direct current is passed. The magnitude of the current is selected in such a way that in the absence of a signal the amplifier will operate at the middle of the linear section of its characteristic (the point a in Fig. 5.5). The bias circuit is normally supplied from the load voltage source U via a rectifier.

Undistorted amplification of the input signal presupposes that the envelope of the load current I should follow exactly after all the variations of the input current I_C. This is the case if the input signal varies sufficiently slowly, when it has no time to undergo a strong variation during one period of the alternating current I. Undistorted amplification requires therefore the fulfillment of the condition

$$f_{C.M} \leqslant (0.1 - 0.2) f,$$

where $f_{C.M}$ is the maximum frequency of the ac component of the signal, and f is the supply frequency of the load circuit.

By analogy with amplitude modulation in radio engineering, the operating (fundamental) frequency f is often called the carrier frequency.

Thus, for example, with a supply frequency f = 50 cps, a magnetic amplifier can be used for the amplification of signals that vary with a frequency of not more than 5-10 cps.

In order to obtain an unmodulated signal at the output of a magnetic amplifier, the load is connected to the circuit via a rectifier, which performs the inverse conversion, i.e., demodulation (rectification). In Fig. 5.6 we present two models of a magnetic amplifier with a dc load and output rectification.

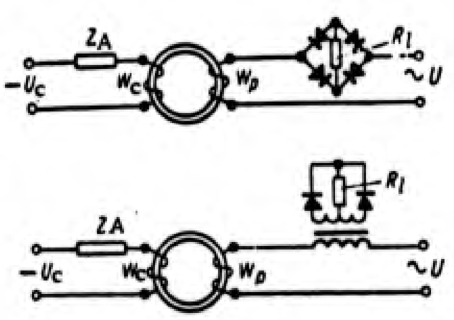

Fig. 5.6. Magnetic amplifier with dc load and output rectification.

§ 5.4. Control of Magnetic Amplifier by Fundamental-Frequency AC Signals

The load current of the amplifier is controlled by altering the core saturation by magnetizing the core by a direct current. The same effect is obtained when we apply to the control winding an ac voltage of operating frequency that is matched in phase with the load-circuit supply voltage. An increase in the control current of operating frequency will be accompanied by an increase in the load current, this being due to an increase in core saturation. Yet such a circuit lacks in principle the property of power amplification (just as an ordinary transformer), and is therefore not to be used as an amplifier [9].

Power amplication of a fundamental-frequency signal is possible in magnetic amplifiers only by means of strong positive feedback, as will be shown below in §§ 7.11 and 7.12.

In a number of cases, a fundamental-frequency alternating current is used in magnetic amplifiers for producing the bias. This makes it possible to circumvent the use of rectifiers in the bias circuit. The bias winding is directly connected to the supply source of the load circuit across a large (inductive) reactance which ensures that the alternating bias current is independent of the control signal.

§5.5. Basic Properties and Domains of Application of Magnetic Amplifiers

The very simple circuit examined above makes it possible to understand the following basic properties of magnetic amplifiers, constructed on the basis of this circuit.

1) High reliability. Magnetic amplifiers are static devices (without movable parts) of great mechanical strength. They can operate in a wide range of variation of temperature, humidity and pressure. Magnetic amplifiers do not require periodic maintenance. They are explosion- and fire proof, since they do not contain sparking sources and elements with a high temperature, such as filaments of electronic tubes.

2) Long life. The service life of a magnetic amplifier is determined by the rectifiers incorporated in its circuit. The high performance of present-day semiconductor rectifiers, as well as the possibility to create easier operating conditions for them in a magnetic amplifier, permit us to assume a practically unlimited life for magnetic amplifiers.

3) Instant readiness to operate after the supply has been switched on. A magnetic amplifier has no starting time, as for example an electronic amplifier (its heating time) or electromechanical amplifier.

4) The capability to amplify very weak dc signals. Magnetic amplifiers are better suited to amplify weak dc signals as compared to electronic and semiconductor amplifiers. Their sensitivity threshold for dc signals is as low as 10^{-19} watt. One of the merits of magnetic amplifiers of direct currents and low frequencies is the possibility to supply them directly from an ac line of commercial frequency (i.e., no supply current rectifier is required as in the case of an electronic amplifier).

5) The possibility of summation of a large number of output signals. In this respect a magnetic amplifier can be compared to an electronic tube with many (up to several tens) control grids, the magnetic amplifier being characterized by the absence of resistance coupling between the input circuits.

6) Easy matching with low-ohmic load and signal supply. In this respect, too, magnetic amplifiers compare favorably with electronic and semiconductor amplifiers.

7) Unlimited output power. There exist, for example, magnetic amplifiers with a power of hundreds of thousands of kilowatts.

8) Higher efficiency than electronic amplifiers. This is due to the fact that the load current is controlled by varying the inductive reactance of the load winding, which is lossless. The losses occur only in the load winding resistance and in the rectifiers (in an electronic amplifier the load current is controlled by varying the internal resistance of the tube).

9) The power gain attains 10^3-10^6 per stage.

10) As will be shown below, the size and weight of a magnetic amplifier are inversely proportional to the supply frequency. Beginning with a frequency of about 400 cps and higher, magnetic amplifiers are lighter and more easily portable than electronic amplifiers.

11) The range of operation of magnetic amplifiers is confined to low frequencies, not higher than a few hundred kilocycles. This is due to the fact that an increase in supply frequency is accompanied by a deterioration in the magnetic properties of the ferromagnetic cores.

Magnetic amplifiers have considerable lag, which increases with increasing supply frequency. The time constant of magnetic amplifiers ranges from a few tens of periods to a few periods of the load circuit supply.

The above properties of magnetic amplifiers have resulted in their extensive use in various domains of engineering.

Magnetic amplifiers became known at the beginning of the present century, at about the same time as electronic amplifiers. Like the latter, magnetic amplifiers were developed in response to the needs of radio engineering, and acquired only much later an independent status as a branch of science and technology. The first major application of magnetic amplifiers was their use as audio-frequency modulators in radio stations. Yet for a considerable time magnetic amplifiers failed to gain wide acceptance. This is due, on the one hand, to the momentous development of electronics (the advent of high-performance electron tubes) and to the progress in electromechanical amplifiers. On the other hand, at the time we did not yet possess high-quality semiconductor rectifiers, which are an important element of magnetic-amplifier circuits. The rapid development of magnetic amplifiers dates back to the period of the second world war. This constituted in fact their rebirth.

At present magnetic amplifiers are widely used in automation and remote control devices, in electrical motors, and in computer and measurement technology.

Magnetic amplifiers are used as variable inductors (saturable reactors) in thyratron phase control circuits, in current and voltage stabilizers, and for the frequency modulation of electronic frequency generators (in radio engineering and remote control).

Magnetic amplifiers are widely used as dc-to-ac signal converters-amplifiers in highly sensitive amplifiers of dc signals obtained from strain gauges and other sensors, photocells, thermocouples, and null indicators (magnetic modulators); they are also used as power amplifiers in the control of ac motors.

Magnetic amplifiers with dc output are used as dc and low-frequency amplifiers, for example in the control of dc motors and of the excitation of synchronous generators, and also in protective and checking relay circuits.

Magnetic amplifiers are used as pulse amplifiers, for example in mathematical machines of discrete operation.

Magnetic amplifiers serve as a basis for contactless magnetic relays that are being more and more used in automation, remote control and computer technology.

Like other types of amplifiers, magnetic amplifiers make extensive use of all sorts of feedback, reversible circuits, and cascade connections.

Footnotes

p. 156. [1] A half-period of the fundamental frequency contains an even number of even-harmonic periods and an odd number of odd-harmonic periods. Therefore, when higher harmonics are superimposed on the fundamental frequency, the non-symmetrical character of the half-waves can be due to the even harmonics only.

Chapter 6

Nonreversible Magnetic Amplifiers Without Feedback

§ 6.1. Two-Core Magnetic Amplifiers

The very simple one-core magnetic amplifier, examined above, is seldom used, since it has the following shortcomings:

1) The need to insert into the control circuit a high auxiliary inductive impedance Z_A for the suppression of the alternating current induced from the load circut greatly increases the lag of the amplifier while diminishing its gain. If the control-circuit resistance to this current is small, the control winding will constitute a short-circuited loop. Thus the inductive impedance of the load circuit will be negligibly small and weakly dependent on the direct control current.

2) Strong distortion of the load current waveform (see Fig. 5.3). For these reasons one-core magnetic amplifiers are used only in individual cases, in the form of amplifiers with strong positive feedback (see § 7.11), when the W_C/W_p ratio is small (of the order of 0.1-0.01), and hence the stray currents in the control circuit are small. They are mainly used in push-pull circuits and in amplifiers of operating-frequency signals.

The circuits depicted in Fig. 6.1 are not beset by the above shortcomings, inherent in the simplest model of a magnetic amplifier. The beginning of the winding is marked by black dots. The two circuits consist of two entirely similar cores and differ only by the connection of the load windings. The load can be connected to an alternating current, as well as to a direct current, i.e., across a rectifier.

Let us consider at first the circuit of Fig. 6.1a. The control windings are series-aiding connected, whereas the load windings are series-opposing connected. We can also have the converse. With either of these connections of the windings the amplifier input will not carry an operating-frequency voltage, since the operating-frequency emf's induced in the control windings are in anti-phase and balance each other. The arrows in Fig. 6.1a show schematically the directions of the magnetic fluxes produced by the windings at a particular instant of time. During one half-period the ac and dc magnet fluxes act in the same direction in the first core, and in opposite directions in the second core. During the next half-period we have the converse. Thus during two half-periods the two load windings act together on the load circuit in exactly the same way. For this reason the two half-waves of the load current will be symmetrical (without even harmonics), i.e., the current waveform will be less distorted than in a single-core circuit.

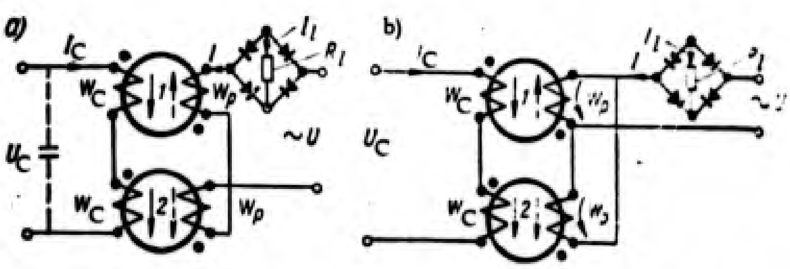

Fig. 6.1. Two-core magnetic amplifier circuits.

Yet if the load current curve, and hence also the ac field strength are symmetrical, then (as was shown above) in the presence of dc magnetization the variable-induction curve (see curves 2' in Fig. 5.4) will be nonsymmetrical, i.e., with even harmonics. As a result, even harmonics will be contained in the emf's induced in the control windings. But whereas the odd harmonics of the emf are phase-shifted by 180°, the even harmonics will coincide in phase, i.e., add up. Thus the input of the circuit depicted in Fig. 6.1a will be completely without a variable voltage only if $U_C = 0$. When an input signal is applied, a variable voltage will appear at the input terminals that is composed of even harmonics of the fundamental frequency (mainly the second harmonic) and increases as U_C.

This voltage is undesirable, since it can disturb the normal operation of the source of the signal U_C. On the other hand it can be also made good use of for the purpose of improving the amplifier performance. Above we mentioned the magnetizing effect of even harmonics of the field, whose presence can increase the gain of a magnetic amplifier.

The even-harmonic voltage induced in the control winding can be used for producing such a magnetizing effect. For this purpose it is necessary to make sure that the control circuit has a low resistance to the current generated by this voltage, so that this current, and hence also the field produced by it in the cores, will be as large as possible. For this purpose one normally connects in parallel to the control windings of the amplifier a capacitance which constitutes a small resistance to currents of even harmonics of the fundamental frequency (shown in Fig. 6.1a by the dashed line). For illustration we present in Fig. 6.2. the characteristic $I_l = f(I_C)$ of a magnetic amplifier in the case of a large and of a small control-circuit resistance to even harmonics.

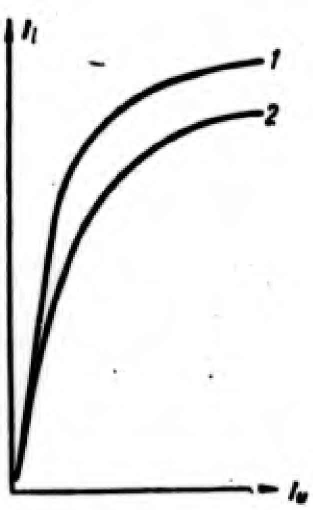

Fig. 6.2. Magnetizing effect of even harmonics of the field. 1— the resistance of the control circuit to even harmonics is low; 2— this resistance is high.

Since the magnetization characteristic establishes a one-to-one correspondence between the induction and field-strength curves it follows that a decrease in the control-circuit resistance to even harmonic- hence an increase in the even harmonics in the field curve will be accompanied by a decrease in the even harmonics in the induction curve.

Figure 6.1b show the second possible version of a two-core magnetic amplifier circuit. Here the load windings are connected in parallel, but (as before) in such a way that the operating frequency emf's induced in the control windings should mutually balance. This circuit has the advantage that the high-frequency components (including even harmonics) are completely absent at the input, both in the case of an input signal and without such a signal. Indeed, in the circuit under consideration the load current is equal to the sum of the components flowing through each of the two output windings. In the presence of dc magnetization each of these components contains even harmonics, like in the single-core circuit. Since the directions of the magnetizing field and of the variable field in the cores are different, it follows that the even current harmonics in the common load circuit must be shifted by 180°, i.e., these harmonics will be absent. The even harmonic current will circulate in the loop formed by the output windings. The curved arrows in Fig. 6.1b indicate the directions of the even harmonic currents in the output windings. The induction curve, and hence also the emf's induced in the control windings, do not have even harmonics.

A shortcoming of an amplifier with parallel-connected output windings is its high lag. This is due to the fact that the short-circuited loop of the output windings, acting like a screen, delays the variation of the magnetizing field of the control windings. From the circuit of Fig. 6.1b it is easy to see that the dc emf's, induced in the ou. .t windings when the control current varies, are acting in the same direction. For this reason one mainly uses the circuit · h series-connected output windings.

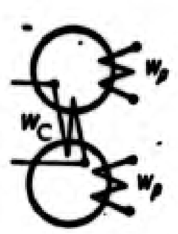

Fig. 6.3. Two-core magnetic amplifier with a common control winding.

Instead of two series-connected control windings one can use a single common winding that encloses both cores, as schematically shown in Fig. 6.3. In the common control winding only a small even-harm. ic emf is induced (and this only when the output windings are series-connected). Therefore the coil insulation can be greatly diminished as compared to separate windings. The copper expenditure decreases by 60-80% when a common control winding is used. The resistance of the control circuit will also decrease, and hence the power amplification will increase.

Magnetic amplifier models with a common control winding on cores of various shape are shown in Fig. 6.4.

The magnetic amplifiers depicted in Figs. 6.1a and 6.1b can be also designed with a single three-legged core, as shown in Fig. 6.5. Owing to its simplicity of design, such

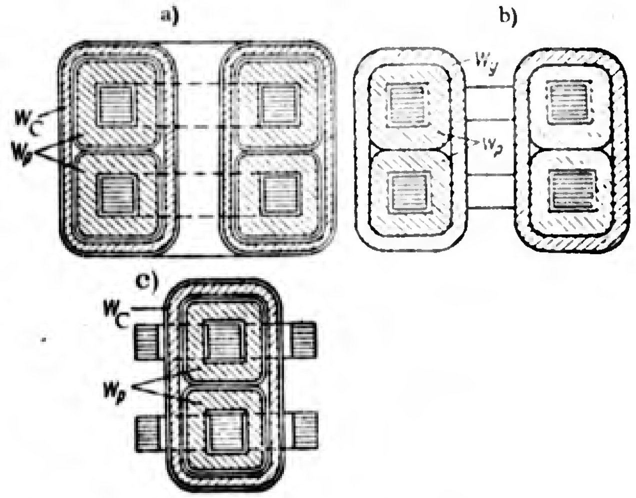

Fig. 6.4. Magnetic amplifier models with toroidal (a), pi-shaped (b) and three-legged (c) cores.

a model is being used as high-power amplifier. Yet is necessitates a greater expenditure of core material, and its characteristic is less linear than in the case of other types of amplifiers.

Two-core magnetic amplifiers (Figs. 6.1 and 6.3) are called transducers in the foreign literature.

§6.2. Magnetic Amplifiers with Parallel Load

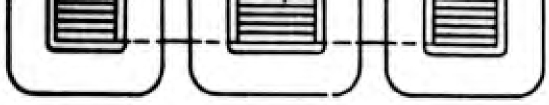

Fig. 6.5. Magnetic amplifier with a single three-legged core.

In the magnetic amplifier circuit of Fig. 6.6a the load R_l is connected in parallel with the output windings, and not in series. In this case the output circuit is not fed from a current source. The figure represents this as a voltage source U with a series-connected large ballast resistor R_b, which determines the magnitude of the current I in the nonbranching portion of the output circuit irrespective of the magnitude of the impedances of the output windings and of the load. The static characteristic

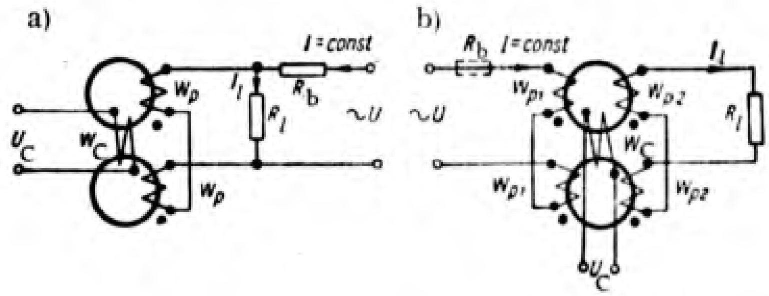

Fig. 6.6. Magnetic amplifier with parallel load.

$I_l = f(I_C)$ of the amplifier is plotted in Fig. 6.7. It is the reciprocal of the characteristic of an amplifier with a series-connected load. When the input signal increases, the load I_l decreases as a result of a decrease in the output winding resistance which shunts the load. Figure 6.6b shows a modification of the previous circuit — a transformer amplifier. Since I = const, the voltage across the primary windings W_{pl} is determined by their resistance, which varies with the magnetization. The merit of transformer magnetic amplifiers is the absence of resistance coupling between the load and the supply source and the possibility to obtain any current at the amplifier output, irrespective of the supply current (voltage).

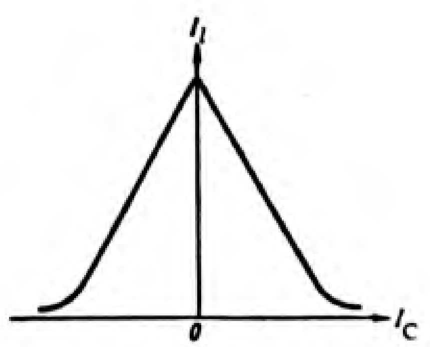

Fig. 6.7. $I_l = f(I_C)$ characteristic of a magnetic amplifier with parallel load.

Single-ended amplifiers with parallel load are seldom used, and when they are utilized it is mainly in those cases in which a current source is available, as for example in protective circuits and automatic control circuits that are supplied by current transformers. To supply such an amplifier from a voltage source with the use of a ballast resistor R_b is not profitable, in view of the high losses in such a resistor. Transformer amplifiers are widely used as elements of push-pull type magnetic amplifiers (see Chapter 8).

§ 6.3. Steady-State Operation of an Ideal Magnetic Amplifier

Owing to the complex operation of actual magnetic amplifiers, various simplifications are introduced into the pertinent mathematical analysis. The principal simplified theories of magnetic amplifiers:

1) The theory of linearized magnetic amplifiers.

2) The theory of ideal magnetic amplifiers.

The theory of linearized amplifiers is based on the assumption that the core permeability remains constant during a supply period. Thus, the magnetic amplifier is regarded as a linear variable inductor. In this case, the actual non-sinusoidal B_{inst}, H_{inst}, u, and i curves are replaced by equivalent sinusoidal curves, and the ordinary design method of linear electrical networks is used. Such an approach is not feasible if we must know the actual waveforms of the fields and currents, i.e., the harmonics. The theory of linearized magnetic amplifiers was used by us in the description of the principle of operation of such amplifiers. The concept of inductive reactance of the output winding

$$X_p = \omega L_p = \frac{\omega W_p^2 S_c}{l_c} \mu$$

used on this occasion makes sense only in the case of sinusoidal voltages and currents.

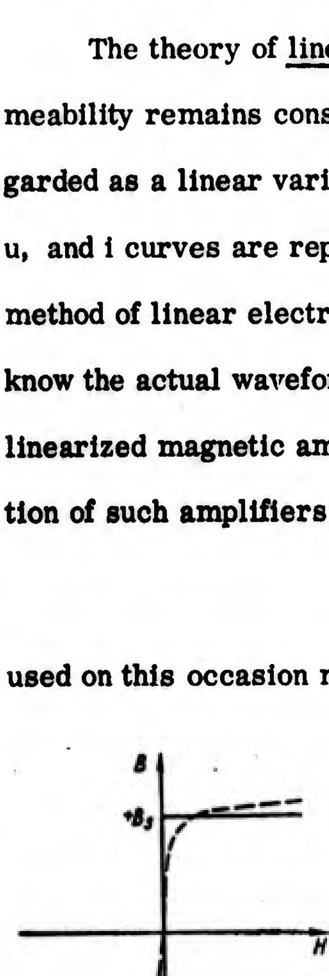

Fig. 6.8. Magnetization curve of ideal core. The characteristic of an actual core.

The theory of ideal magnetic amplifiers is based on the assumption that the core is ideal, i.e., that its magnetization characteristic has the form shown in Fig. 6.8. The differential permeability μ_d = dB/dH an ideal core can assume only two limiting values

$$\mu_d = \begin{cases} \infty & \text{for } B < B_s; \\ 0 & \text{for } B \geq B_s. \end{cases}$$

In contrast to the theory of linearized magnetic amplifiers, the theory of ideal magnetic amplifiers makes it possible to estimate the waveforms representing the amplifier variables. It also yields very simple and clear formulas for the relationships between the basic parameters of a magnetic amplifier.

Let us consider the operation of the magnetic amplifier whose circuit is represented in Fig. 6.1a (having series-connected output windings and a dc load), assuming to be ideal. The operation of the amplifier is illustrated by the curves of Fig. 6.9. The curve in Fig. 6.9a represents the supply voltage

$$u = U_м \sin \omega t. \qquad (6.1)$$

If we neglect the resistance of the control circuit to the alternating current induced in it, then the equation of this circuit for alternating current will be

$$e_{y1} + e_{y2} = -W_y S_c \left(\frac{dB_{1\,inst}}{dt} - \frac{dB_{2\,inst}}{dt} \right) = 0, \qquad (6.2)$$

where e_{C1} and e_{C2} are the emf's induced in the control windings of the first and second cores by the ac magnetic flux; $B_{1\,inst}$ and $B_{2\,inst}$ are the instantaneous values of the induction of the ac field in the cores.

Integration of (6.2) yields

$$B_{1\,inst} = B_{2\,inst} + 2B_0, \qquad (6.3)$$

where $2B_0$ is the integration constant, i.e., B_0 is the dc component of the induction in each core.

Formula (6.3) signifies that the core inductions vary according to the same law, differing from each other by the constant $2B_0$. Hence, it follows that the emf's induced in the load windings will also be the same, i.e.,

$$e_{p1} = e_{p2}, \qquad (6.4)$$

where

$$e_{p1} = -W_p S_c \frac{dB_{1\,inst}}{dt};$$
$$e_{p2} = -W_p S_c \frac{dB_{2\,inst}}{dt}.$$

Let us first examine the operation of the system in the absence of an input signal, i.e., with $U_C = 0$. If in this case the value of the supply voltage is such that both cores are unsaturated, i.e., $|B_1| < B_s$ и $|B_2| < B_s$, then the inductive impedance of the output windings is equal to infinity (since $\mu_1 = \mu_2 =$) and the output current is equal to zero. Hence, the equation for the output circuit will be

$$u = -(e_{p1} + e_{p2}).$$

By virtue of (6.4) this means that the supply voltage is evenly distributed between the output windings, i.e.,

$$e_{p1} = e_{p2} = -\frac{u}{2} = -\frac{U_m}{2}\sin\omega t. \tag{6.5}$$

Let us introduce into (6.5) the formula (6.4) for the emf:

$$W_p S_c \frac{dB_{1\,inst}}{dt} = W_p S_c \frac{dB_{2\,inst}}{dt} = \frac{U_m}{2}\sin\omega t. \tag{6.6}$$

Integration of (6.6) yields

$$B_{1\,inst} = B_{2\,inst} = -B_m \cos\omega t, \tag{6.7}$$

where

$$B_m = \frac{U_m}{2\omega W_p S_c}.$$

The variation of core induction in the absence of dc magnetization is represented in Fig. 6.9b by the dashed line.

Suppose now that a signal is applied to the amplifier input, i.e., $U_C \neq 0$. The direct current generated by U_C in the control circuit produces a dc magnetic field whose direction differs from that of the ac field in the various cores. According to formula (6.3), its induction is B_0. Hence, as can be seen from Fig. 6.9b, the induction curve in one (the first) core lies higher than in the case $U_C = 0$, whereas in the other (second) core it lies lower. The sinusoidal variation of the induction ceases in this case when the value $+B_s$ of the saturation induction is reached. The saturation of the first core is preserved up to the beginning of the next supply half-cycle, when as a result of a supply-voltage sign change the sign of $dB_{1\,inst}/dt$ also changes, so that the induction begins to decrease. Yet as long as $B_{1\,inst} = +B_s = \text{const}$, the induction $B_{2\,inst}$ in the second core will also remain constant (by virtue of formula (6.3)), as indicated in Fig. 6.9b. From a physical point of view this is due to the fact that when $B_{1\,inst} = +B_s$ the quantity $e_{C1} = 0$, and hence the control winding W_{C2} of the second core is found to be short-circuited. But if the induction in the cores is constant, then by virtue of formula (6.4) the self-induction emf in the output windings vanishes. Hence, the entire supply voltage is found to be applied to the load, through which a current $i = u/R$ flows. Here R is the total resistance of the output circuit.

Thus, from the beginning of a half-cycle and until the instant of saturation of the first core at $\omega t = \alpha_s$ the entire supply voltage is applied to the output windings, whereas during the remaining part of the half-cycle (from α_s to π) it is applied to the load, which is illustrated in Fig. 6.9a.

The process just described is repeated during the next half-cycle, with the sole difference that the cores are interchanged, i.e., when the angle α_s is reached the second core saturates (in this core the ac field induction has now the same direction as the dc field induction).

Figure 6.9c shows the current curves of the output windings and Fig. 6.9d shows the current curves of the load.

The angle α_s, which specifies the length of the current waveform, is called the <u>saturation</u> (firing) <u>angle</u>; this angle is determined by the magnitude of the constant induction B_0, and hence by the signal U_C producing this induction. When $U_C = 0$ we have $B_0 = 0$, the angle $\alpha_s = \pi$, and no current flows through the load. When U_c increases, the induction B_0 also increases and α_s decreases accordingly. This mode of operation of magnetic amplifiers resembles the operation of phase-controlled thyratrons. The saturation angle corresponds to the firing angle of a thyratron.

An ideal magnetic amplifier operates like a switch (synchronous commutator) that periodically connects the load to the supply voltage at instants of time that are fixed with respect to the beginning of a supply half-cycle and are determined by the magnitude of the control signal.

Thus, a magnetic amplifier can be regarded not only as a variable inductor (in conformity with the theory of linearized magnetic amplifiers), but also as a magnetic switch (in conformity with the theory of ideal magnetic amplifiers).

Let us derive the analytic relationships between α_s and B_0 and between I_m and α_s that will be needed in subsequent discussion.

According to formula (6.4), the induction in a saturable core is

$$B_{1\,inst} = -\frac{1}{W_p S_c}\int_0^t e_1\,dt + B_{10},$$

where B_{10} is an integration constant. In the case under consideration this is the value of the induction $B_{1\,inst}$ at $t = 0$, i.e., at the beginning of the particular half-cycle.

Using formula (6.5), we obtain

$$B_{1\,inst} = \frac{1}{W_p S_c}\int_0^t \frac{U_M}{2}\sin\omega t\,dt + B_{10} = B_{10} + B_M(1 - \cos\omega t).$$

Here B_M is specified by formula (6.7a).

By virtue of (6.3), we have

$$B_{10} = B_{20} + 2B_0 = 2B_0 - B_s,$$

since $B_{20} = -B_s$. The latter follows from the fact that we are considering steady-state conditions. In this case, assuming that during the half-cycle under consideration the first core saturates, it follows that during the previous half-cycle the second core was saturated. Thus we obtain

$$B_{1\,inst} = 2B_0 - B_s + B_M(1 - \cos\omega t). \tag{6.8}$$

By introducing $B_{1\,inst} = +B_s$ into formula (6.8), we find the instant at which the first core saturates:

$$B_M(1 - \cos\alpha_s) = 2(B_s - B_0);$$
$$\cos\alpha_s = 1 - 2\frac{B_s - B_0}{B_M}, \tag{6.9}$$

or, with $B_M = B_s$, which (as will be shown below) can be normally achieved by selecting the supply voltage,

$$\cos\alpha_s = 1 - 2 + 2\frac{B_0}{B_s} = 2\frac{B_0}{B_s} - 1. \tag{6.10}$$

In Table 6.1 we list the values of α_s corresponding to various values of B_0.

The mean value of the output current, i. e., of the load current (after the rectifier) is

$$I_l = I_m = \frac{1}{\pi}\int_{\alpha_s}^{\pi} i\, d(\omega t) = \frac{U_m}{\pi R}(1 + \cos\alpha_s). \tag{6.11}$$

Table 6.1

Firing angle α_s as a function of B_0

B_0	$\cos\alpha_H$	α_H
0	−1	π
$+B_s/2$	0	$\pi/2$
$+B_s$	+1	0

Let us derive the equation of the static characteristic (load characteristic) $I_l = f(I_C)$ for an ideal amplifier. As can be seen from the magnetization curve of an ideal amplifier, plotted in Fig. 6.8, for an unsaturated core we have H = 0. In the magnetic amplifier circuit under consideration the two cores are saturated in alternation. During each half-period one of the cores must be unsaturated. Hence for this core we have the equation

$$H = \frac{iW_p}{l_C} + \frac{i_C W_C}{l_C} = 0.$$

For example, in the mode of operation illustrated in Fig. 6.9 the core unsaturated during the first half-period is core 2, whereas during the second half-period it is core 1. Thus at any time the instantaneous values of the currents in the control and output windings are related by the formula

$$i_C W_C = |i|\, W_p. \tag{6.12}$$

This means that the current in the control circuit is in the form of pulses of double frequency, as shown in Fig. 6.9e. From a physical point of view, this is due to the fact that when one core is saturated the other (unsaturated) core will act as an ideal transformer with a short-circuited secondary winding.

The curve i_C can be decomposed into a dc component I_C (the mean value of the current) produced by the input signal U_C (the dashed line in Fig. 6.9e), and an ac component induced by the magnetic flux of the cores. This ac component has a

Let us derive the analytic relationships between α_s and B_0 and between I_m and α_s that will be needed in subsequent discussion.

According to formula (6.4), the induction in a saturable core is

$$B_{1\,inst} = -\frac{1}{W'_p S_c}\int_0^t e_{11}\,dt + B_{10},$$

where B_{10} is an integration constant. In the case under consideration this is the value of the induction $B_{1\,inst}$ at t = 0, i.e., at the beginning of the particular half-cycle.

Using formula (6.5), we obtain

$$B_{1\,inst} = \frac{1}{W'_p S_c}\int_0^t \frac{U_M}{2}\sin\omega t\,dt + B_{10} = B_{10} + B_M(1-\cos\omega t).$$

Here B_M is specified by formula (6.7a).

By virtue of (6.3), we have

$$B_{10} = B_{20} + 2B_0 = 2B_0 - B_s,$$

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By introducing $B_{1\,inst} = +B_s$ into formula (6.8), we find the instant at which the first core saturates:

$$\begin{aligned} &B_M(1-\cos\alpha_s) = 2(B_s - B_0);\\ &\cos\alpha_s = 1 - 2\frac{B_s - B_0}{B_M}, \end{aligned} \tag{6.9}$$

or, with $B_M = B_s$, which (as will be shown below) can be normally achieved by selecting the supply voltage,

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$$H = \frac{iW_P}{l_C} + \frac{i_C W_C}{l_C} = 0.$$

For example, in the mode of operation illustrated in Fig. 6.9 the core unsaturated during the first half-period is core 2, whereas during the second half-period it is core 1. Thus at any time the instantaneous values of the currents in the control and output windings are related by the formula

$$i_C W_C = |i|\,W_P. \tag{6.12}$$

This means that the current in the control circuit is in the form of pulses of double frequency, as shown in Fig. 6.9e. From a physical point of view, this is due to the fact that when one core is saturated the other (unsaturated) core will act as an ideal transformer with a short-circuited secondary winding.

The curve i_C can be decomposed into a dc component I_C (the mean value of the current) produced by the input signal U_C (the dashed line in Fig. 6.9e), and an ac component induced by the magnetic flux of the cores. This ac component has a fundamental frequency 2f, which conforms with our earlier analysis concerning higher harmonics in a magnetic amplifier.

Formula (6. 12), which relates the instantaneous values of the currents, holds also for their mean values, i. e.,

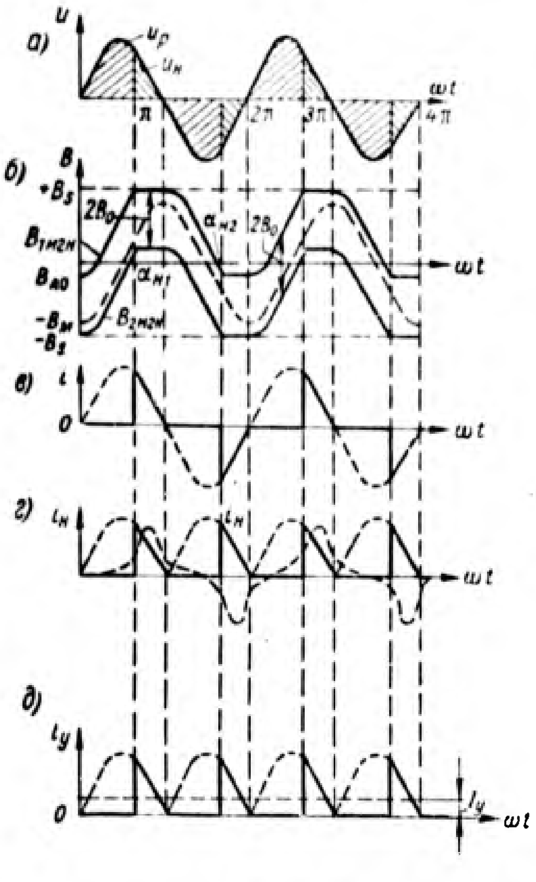

Fig. 6. 9. Diagrams illustrating the operation of an ideal magnetic amplifier.

$$I_{\text{н}}W_{\text{р}} = I_{\text{ср}}W_{\text{р}} = I_{\text{у}}W_{\text{у}},$$

or

$$H_{\text{ср}} = H_{\text{у}}. \qquad (6.13)$$

This formula, which resembles the equation of an ideal transformer, represents the equation for the static characteristic $I_l = f(I_C)$ of an ideal magnetic amplifier. In Fig. 6. 10 this characteristic is represented by the straight line 1. The point K corresponds to $\alpha_s = 0$, when both cores are simultaneously saturated, and hence equation (6. 13) ceases to be valid. The corresponding maximum value of the mean current is

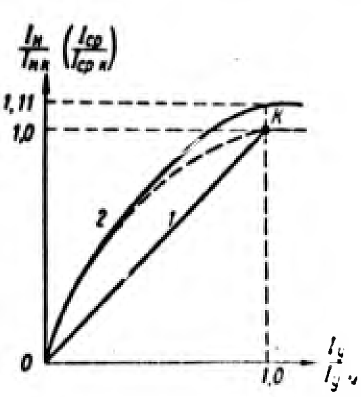

Fig. 6. 10. Static (load) characteristics of an ideal magnetic amplifier.

$$I_{\text{н. к}} = I_{\text{ср. к}} = \frac{U_{\text{ср}}}{R} = \frac{2U_{\text{м}}}{\pi R}. \qquad (6.14)$$

By virtue of formula (6. 13), the current gain, voltage gain, and power gain are

$$K_I = \frac{I_{\text{н}}}{I_{\text{у}}} = \frac{W_{\text{у}}}{W_{\text{р}}}; \qquad (6.15)$$

$$K_U = \frac{U_{\text{н. ср}}}{U_{\text{у}}} = \frac{I_{\text{н}}R_{\text{н}}}{I_{\text{у}}R_{\text{у}}} = K_I\frac{R_{\text{н}}}{R_{\text{у}}} = \frac{W_{\text{у}}R_{\text{н}}}{W_{\text{р}}R_{\text{у}}}; \qquad (6.16)$$

$$K_P = \frac{P_{\text{н}}}{P_{\text{у}}} = \frac{I_{\text{н}}U_{\text{н}}}{I_{\text{у}}U_{\text{у}}} = K_IK_U = K_I^2\frac{R_{\text{н}}}{R_{\text{у}}} = \frac{W_{\text{у}}^2R_{\text{н}}}{W_{\text{р}}^2R_{\text{у}}}. \qquad (6.17)$$

As can be seen from formula (6.15), the gain K_I, and hence also the operating section of the static characteristic of a magnetic amplifier, depend neither on the load resistance nor on the supply voltage and frequency. Figure 6.11 shows the static characteristics of an ideal magnetic amplifier for various values of U and R. According to formula (6.7), the limiting value of the supply voltage, at which the cores do not yet saturate in the absence of an input signal, is equal to $U_s = 2\omega W_p S B_s$. For U > U_s the characteristic is in the form of curve 3, i.e., in this case a current flows through the load circuit even if $U_C = 0$. By taking $U = U_s$, one obtains maximum possible output power under the condition that the characteristic passes through the origin of coordinates.

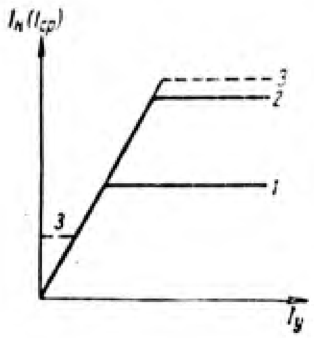

Fig. 6.11. Static characteristics of an ideal magnetic amplifier for various values of the supply voltage U and of the output circuit resistance R.
1—U_{M1}, R_1; 2—$U_{M2} = 2U_{M1}$ or $R_2 = 0.5R_1$; 3—$U_{M3} > U_s = 2\,W_p S_c B_s$.

The variation of the load resistance has the opposite effect on the magnitude of the power gain

$$K_p = \frac{W_C^2 R_C}{W_p^2 R_C}$$

and of the maximum output power

$$P_{l.м} = \frac{U_{C.м}^2}{R_l} = \frac{U_m^2}{R_l}.$$

The product

$$K_p P_{l.м} = \frac{U_m^2 W_C^2}{R_C W_p^2} = \text{const}, \tag{6.18}$$

i.e., it does not depend on R_l and is a constant for the particular amplifier.

We have examined the operation of an ideal magnetic amplifier with series-connected output windings, a dc active load, and a low ac resistance of the control circuit. Let us now consider the peculiar features of operation of other types of magnetic amplifier circuits.

A. Amplifiers with parallel-connected output windings

As we showed above, in this case even-harmonic currents do not flow in the control circuit, while flowing through the output windings. Since the cores saturate also here in alternation, the equality $H_m = H_C$ remains valid. But since in the case of parallel-connected output windings the output current is equal to half the load current, the equation for the static characteristic assumes the form

$$I_l W_p = 2 I_C W_C. \qquad (6.19)$$

The formulas for the gain factors are modified accordingly.

B. The case of an ac load (amplifier without output rectifier)

In this case we are interested in the effective value of the current, i.e., $I_l = I = K_f I_m$, where K_f is the form factor of the output current.

Hence the gain factors will be expressed as

$$K_{I\sim} = K_f K_I; \quad K_{U\sim} = K_f K_U; \quad K_{P\sim} = K_f^2 K_P. \qquad (6.20)$$

The characteristic of an amplifier without a rectifier (for the effective value of the output current) is represented by curve 2 in Fig. 6.10. The nonlinearity of the characteristic is determined by the dependence of k_f on α_s. For $\alpha_s = 0$ we evidently have $k_f = 1.11$ and this value increases with α_s.

The presence of an inductance in the ac load smooths the current curve and causes the current to lag behind the voltage, as shown by the dashed line in Fig. 6.9d.

C. An inductive load at the rectifier output

The characteristic $I_l = f(I_C)$ for this case is plotted by the dashed line in Fig. 6.10. As we can see, the maximum value of the load current does not depend on the presence of an inductance after the rectifier. Yet the intermediate values of I are found to be higher than the mean value I_m of the output-circuit current. This is due to the fact that the load current I_l does not cease at the end of each half-period, but continues to flow (owing to the self-induction emf) for some time through the load circuit, passing along two parallel routes via all four arms of the rectifying loop.

D. A load with a parallel capacitor at the rectifier output

If a capacitor is connected in parallel to the load at the rectifier output, the voltage across the load will not drop to zero at the end of each half-period, but maintain itself owing to the energy stored in the capacitor. The maximum value $U_{l,M}$ will be equal to the amplitude value of the supply voltage U_M (less the voltage drop across the resistance of the output windings and in the rectifier). Hence by connecting a capacitor in parallel to the load it is possible to increase $U_{l.M}$ and $I_{l.M}$ by a factor of $\pi/2$ in the best case, i.e., to increase the maximum output power of the amplifier by a factor of $(\pi/2)^2$ 2.5. In the general case we can write

$$I_{l.M} \doteq \gamma \frac{U_m}{R},$$

where R is the total resistance of the output circuit, and $\gamma = 1$ to $\pi/2$ is a factor that specifies by how many times $I_{l.M}$ increases when a capacitor is connected.

In Appendix AP-5 we have plotted curves permitting the determination of γ as a function of R , R and the capacitance C. These curves are also valid for an active inductive load.

E. An amplifier with a high resistance to even current harmonics

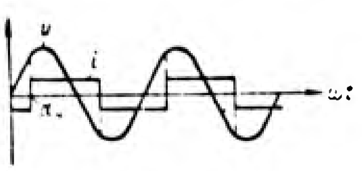

Fig. 6.12. Output current curve I for an ideal amplifier without even harmonics.

This case can occur only if the output windings of the amplifier are series-connected. When the control circuit has an infinitely high resistance to even harmonics, as for example when it is supplied by a current transformer, only a direct current I_C without even harmonics will flow in the control circuit. Since the cores saturate one after another, i.e., the relation (6.13) remains valid, the load current waveform will be rectangular, as shown in Fig. 6.12. Moreover, since $K_f = 1$, the amplifier characteristic for the effective value of the output current will be under these conditions also linear, just as for its mean value.

With a finite value of the control-circuit resistance the form of the load current curve will be intermediate between those of the curves shown in Figs. 6.9 and 6.12. In practice, however, the ordinary mode of operation of magnetic amplifiers is the first one, also called the mode of operation with free even-harmonic currents. As we noted above, this mode yields higher gain.

§ 6.4. Dynamics of an Ideal Magnetic Amplifier

The voltage across the load of a magnetic amplifier lags (in its variations) behind the variations of the input voltage U_C, i.e., a magnetic amplifier has a certain lag. Of predominant importance in this respect is the time lag between I_C and U_C in the control circuit, which is a dc RL circuit with a high inductance. The lag between I and I_C, i.e., the transient process in the ac circuit, can normally be neglected, assuming that equation (6.13) holds also in the transient period.

The equation for the transient process in the control circuit is

$$U_C = R_C I_C + W_C S_C\left(\frac{dB_{1\,inst}}{dt} - \frac{dB_{2\,inst}}{dt}\right),$$

where R_C is the total resistance of the control circuit, including the resistance of the control windings and the output resistance of the signal source U_C. Or, by virtue of formula (6.3), we have

$$U_C = R_C I_C + 2W_C S_C \frac{dB_0}{dt} \tag{6.21}$$

In order to obtain the sought-for equation connecting I_C and U_C, let us express B_0 in terms of cos α_s by means of equation (6.9), i.e.,

$$B_0 = B_s - \frac{1}{2} B_m (1 - \cos \alpha_s). \tag{6.22}$$

On the other hand, by virtue of (6.11) we have

$$\cos \alpha_s = \frac{\pi R}{U_м} I_m - 1 \tag{6.23}$$

After effecting substitutions into formula (6.23), we obtain on the basis of (6.7a) the expression

$$U_м = 2\omega W_p S_c B_м = 4\pi f W_p S_c B_м$$

Formula (6.13) yields

$$I_m = \frac{W_C}{W_p} I_C$$

By substituting the latter two expressions into formula (6.23) for cos α_s, and the latter into (6.22), we obtain the following expression for B_0 in terms of I_C:

$$B_0 = B_s - B_м + \frac{RW_C I_C}{8fS_c W_p^2} \tag{6.24}$$

By substituting this equation into (6.21), we obtain the equation for the transient process in the control circuit in the following form:

$$T_C \frac{dI_C}{dt} + I_C = \frac{U_C}{R_C} \tag{6.25}$$

where

$$T_C = \frac{RW_C^2}{R_y W_p^2} \cdot \frac{1}{4f} \text{ [sec]} \tag{6.26}$$

is the time constant of the control circuit, which specifies its lag. As can be seen from (6.26), T_C depends not only on the parameters of the control circuit (R_C and W_C), but also on the parameters of the output circuit (R, W_p and f). This is due to the fact that these parameters affect the rate of change of the dc component of the induction B_0 accompanying variations in U_C, i.e., they affect the quantity dB_0/dt occurring in equation (6.21).

By neglecting the transient process in the ac (output) circuit, we obtain, by substituting into (6.25) the expression

$$I_C = \frac{I_m}{K_I} = \frac{U_l}{R_l K_I},$$

the following dynamic equation for magnetic amplifiers:

$$T_C \frac{dU_l}{dt} + U_l = K_U U_C \tag{6.27}$$

In the general case of a magnetic amplifier with several control windings, its equation assumes the form

$$\left(\sum_{i=1}^{n} T_i\right)\frac{dU_l}{dt} + U_l = \sum_{i=1}^{n} K_{Ui} U_{yi}. \tag{6.28}$$

Let us note that in deriving this equation, the quantity $W_C I_C$ in formula (6.24) and the other formulas is replaced by $\sum_{i=1}^{n} W_{ci} I_{ci}$, this sum being taken over all the control windings.

The sum of time constants $\sum_{i=1}^{n} T_i$ contains the time constants of not only the control-winding circuits, but also of the dc bias winding circuits; moreover, in the case of parallel-connected output windings, it contains also the time constant of the short-circuited loop formed by these windings (T_p). The introduction of the quantity T_p into the equation accounts for the delay in the transient process due to the screening effect of the circuit formed by parallel-connected output windings (discussed earlier in § 6.1).

If the load, connected after the rectifier, is not purely active, the load current I_l will lag the output voltage U_l of the rectifier, and hence also the alternating current in the output circuit. In this case the equation (6.28) must be supplemented by the equation for the transient process in the dc circuit of the load:

$$T_l \frac{dI_l}{dt} + I_l = \frac{U_l}{R_l}, \tag{6.29}$$

where T_l is the time constant of the load circuit.

Thus a magnetic amplifier with such a load can be described by a system of two first-order differential equations (6.28) and (6.29).

Yet such a description is approximate, since it is based on the assumption that in the case of a complex load at the rectifier output we have also in the transient period $U_l = I_m R_l$, like in the case of a purely active load, and that U_l varies instantaneously with I_C. In actual fact this is not so. If the load is inductive, then at the first instant following a sharp increase in U_C the voltage U_l will briefly assume a value exceeding the value it would take in the case of a purely active resistance, i.e., $I_m R_l$. This is due to the fact that with a rapidly varying voltage across the rectifier, the impedance of the inductive load will be higher than its resistance (as a result of the self-induction emf). Such a forcing of the load voltage U_l during the transient period may reduce by several times the delay in the load circuit as compared to the time constant T_l calculated for it.

In the case of a capacitive load we obtain the converse effect of a protracted transient process.

These effects must be taken into account whenever T_l exceeds by at least several times the supply period.

§ 6.5. The Figure of Merit of Magnetic Amplifiers

An important parameter of magnetic amplifiers is the quantity

$$D = \frac{K_p}{T_c},$$

called the figure of merit, or speed of response of a magnetic amplifier. The larger the figure of merit, the smaller will be the amplifier lag for a given value of the power gain.

By virtue of (6.17) and (6.26) we have

$$D = \frac{K_p}{T_c} = 4f\eta \tag{6.30}$$

where $\eta = R_l/R$ is the efficiency of the load circuit of the amplifier.

For an amplifier with an ac output we have

$$D_{\sim} = \frac{K_{p\sim}}{T_c} = \frac{K_i^2 K_p}{T_c} = K_i^2 D. \tag{6.31}$$

From (6.30) follows that with given values of f and η an increase in K_p will be accompanied by a proportional increase in T_C and conversely. In practice the figure of merit of an amplifier can be increased only by increasing the supply frequency. Thus with f = 50 cps the maximum possible figure of merit, corresponding to $\eta = 1$, is equal to 200 sec^{-1}. This means, for example, that when $K_p = 100$, the time constant $T_C = 0.5$ sec. When the frequency increases tenfold, i.e., with f = 500 cps, the maximum figure of merit will be 2000 sec^{-1}. With the same value of K_p as before, this yields $T_C = 0.05$ sec.

According to formula (6.30) the control-circuit time constant T_C can be diminished without bounds at the expense of reducing K_p. Yet it does not follow from this that the overall amplifier lag can be reduced as much as we like, since the lag of the ac output circuit (neglected by us earlier) still remains. The duration of the transient process in the output circuit ranges from 1/2 to 1.0 of a supply period T = 1/f. Therefore the overall duration of the transient process in the amplifier cannot be made smaller than this quantity. Thus at f = 50 cps this limiting value of the duration of the transient process is equal to 0.01-0.02 sec. If T_C and T_l are commensurable with the supply period T, the equation (6.28) cannot be used and one must take into account the transient process in the ac output circuit. Thus the safest way of reducing as much as we like the lag of a magnetic amplifier is to increase the supply frequency.

§ 6.6. The Parameters of Actual Magnetic Amplifiers

In Fig. 6.13 we present for comparison the static characteristics of an actual and an ideal magnetic amplifier. As before, the main difference consists in the presence of a no-load (quiescent) current I_{l0} at $I_C = 0$ in the actual amplifier, this current being caused by the finite value of the permeability of the core material. The relative magnitude of the no-load current can be estimated by the form factor $K_k = I_{lk}/I_{l0}$.

Magnetic amplifiers with cores of low-grade materials (hot-rolled steels) have $K_k = 5$-20. When materials of higher permeability are used (Permalloy) the factor K_k can exceed 100.

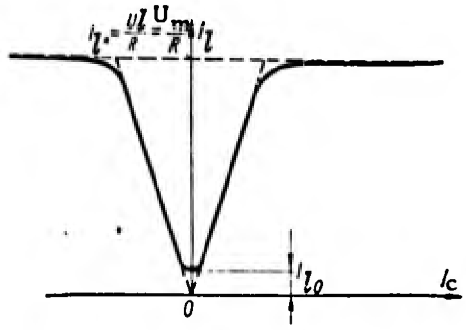

Fig. 6.13. Static characteristics of an actual (solid line) and of an ideal (dashed line) magnetic amplifier.

The static characteristic of an actual magnetic amplifier is nonlinear. With increasing I_C it gradually deviates from the characteristic of an ideal amplifier. Hence the gain of actual amplifiers decreases somewhat with increasing input signal. The operating section of the characteristic terminates at roughly $I_l = 0.8I_{l\,.\,k}$. The poorer the magnetic properties of the core material, the greater the nonlinearity of the characteristic. The nonlinearity of the characteristic increases also with the load resistance R_l (since the form factor of the current decreases).

The figure of merit $D = K_P/T_C$ of actual magnetic amplifiers, even if they have low-grade cores, differs little from the one calculated by formula (6.30).

In actual magnetic amplifiers without feedback the gain K_P has values ranging from a few tens to hundreds of units, whereas the values of T_C range from a few units to tens of supply periods $T = 1/f$.

§ 6.7. Grapho-Analytic Method of Construction of the Static Characteristic of a Linearized Magnetic Amplifier

In the case of cores made of low-grade material the static characteristic must be constructed on the basis of linearized theory, and not of the theory of idealized magnetic amplifiers. The basic data will be the experimental characteristics of the simultaneous magnetization of the core material by ac and dc fields, $B = f(H, H_{-})$. Here B and H are the induction and the strength of the ac magnetic field, whereas H_{-} is the strength of the magnetizing dc field. The characteristics $B = f(H, H_{-})$ are plotted in general form in Fig. 6.14. In the design of magnetic amplifiers they have the same role as the characteristics of electronic tubes in the design of electronic amplifiers. Yet in the case of magnetic amplifiers matters are more complicated as a result of the fact that the characteristics B = f(H, H_) depend strongly not only on the material of the core, but also on its shape and dimensions, the operating frequency, the circuit connection of the windings (series- or parallel connected output windings W_p), and the magnitude of the control-circuit resistance. It is not permissible, for example, that characteristics belonging to toroidal cores are utilized for the design of a magnetic amplifier that has a three-legged core. In Appendix AP-1 we present the characteristics B = f(H, H_) for a number of materials and core types.

In taking the characteristics B = f(H, H_), the quantities H and B are specified by the formulas

$$H = \frac{W_p I}{l_c}; \qquad B = \frac{E_p}{2\omega W_p S_c}$$

(in the case of series-connected output windings). Here E_p is the emf in the output windings. Since the formula for B refers to sinusoidal B and E_p, we shall linearize this formula by going over to equivalent sinusoids for the emf and the current in the output circuit.

Frequently utilized are the magnetization characteristics for amplitude values, $B_M = f(H_M, H\_)$. In this case B_M and H_M are defined as the amplitude values of the equivalent sinusoids, i. e.,

$$B_M = \sqrt{2}B; \quad H_M = \sqrt{2}H.$$

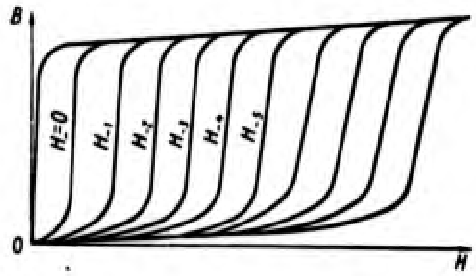

Fig. 6.14. The characteristic B = f(H, H_) of simultaneous magnetization of the cores.

For a magnetic amplifier with a dc output one utilizes the magnetization characteristics for H_m, i. e., $B_M = f(H_m, H\_)$, where H_m is determined on the basis of the mean value I_m of the output current. The quantity H_m cannot be determined on the basis of the effective value of the field H with the aid of a form factor, since the field (current) waveform is not known.

Graphically, the equation (6.34) represents an ellipse with axes B_0' and H_k'. The points of intersection between the ellipse and the magnetization characteristics yield the sought-for function $H = f(H_-)$, represented by curve 1 in Fig. 6.15. Curve 2 in this figure represents the static characteristic of an ideal amplifier.

In the case of a complex load the construction is more complicated. In addition to the ellipse we draw from the origin of coordinates a ray α at an angle

$$\alpha = \operatorname{arctg} \frac{X_l l_c}{2\omega W_p^2 S_c}, \tag{6.36}$$

where X_l is the load reactance.

By subtracting from the ordinates of the ellipse the ordinate of the ray α, we obtain a new curve, depicted in Fig. 6.15 by the dashed line. The points of interaction between this curve and the magnetization curves specify the sought-for function $H = f(H_-)$ for an inductive load. In the case of a capacitive load the ordinates of the ellipse and of the ray the ray are not subtracted, but added together.

When H_k' is very large, it is more convenient to construct the ellipse from points. For this purpose one assigns a number of values of I and one calculates the corresponding values of

$$H = \frac{W_p I}{l_c} \quad \text{and} \quad B = \frac{\sqrt{U^2 - I^2 R^2}}{2\omega W_p S_c}.$$

As we already noted, in the case of a dc load the static characteristic must be constructed by means of the magnetization curves taken for the mean value of H, i.e., the curve $B = f(H_m, H_-)$.

If a capacitor is connected in parallel to the load, the magnitude of the ellipse axis H_k' must be increased by γ times as compared to the case of an active load, the quantity γ being determined from the graph presented in Appendix AP-5.

If the dc load has an inductance, the latter will not affect the value of H_k, i.e., it does not alter $I_{l \cdot M}$. Yet the intermediate values of the direct current in the load will be larger than the mean value I_m of the alternating current in the output-winding circuit, a value determined by H_m (see § 6.3, C). Hence, the quiescent current I_{l0} in the load will

Let us examine the construction of the static characteristic of a linearized magnetic amplifier with a fundamental-frequency output. The load is considered to be purely active. For the output circuit we can thereupon write down the following equation

$$E_p^2 + I^2R^2 = U^2, \tag{6.32}$$

where

$$R = R_l + R_p.$$

In the case of series-connected output windings we have

$$\left.\begin{aligned} E_p &= 2\omega W_p S_c B; \\ I &= \frac{Hl_c}{W_p}. \end{aligned}\right\} \tag{6.33}$$

By substituting these expressions into (6.32), we obtain the equation

$$\left(\frac{B}{B_e'}\right)^2 + \left(\frac{H}{H_\kappa'}\right)^2 = 1, \tag{6.34}$$

where we introduced the notations

$$\left.\begin{aligned} B_e' &= \frac{U}{2\omega W_p S_c}; \\ H_\kappa' &= \frac{W_p}{l_c} \cdot \frac{U}{R}. \end{aligned}\right\} \tag{6.35}$$

The equation (6.34) for the output circuit interrelates the two variables B and H. These variables are also connected by the magnetization characteristics B = f(H, H_).

By eliminating B, it is possible to obtain by a joint graphic solution the sought-for function H = f(H_), which represents the static amplifier characteristic I = f(I_C) in a different scale. This construction is shown in Fig. 6.15.

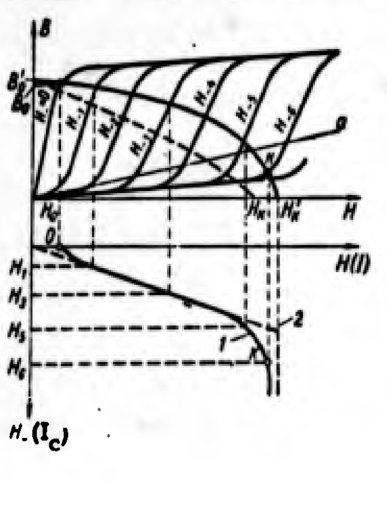

Fig. 6.15. Construction of the static characteristic of a linearized magnetic amplifier.

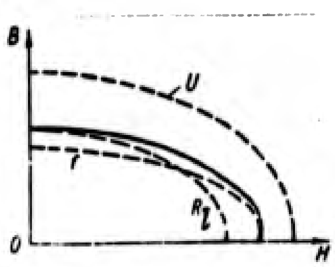

Fig. 6.16. The effect which an increase in U, f and R_1 has on the ellipse of the output circuit.

The solid line represents the ellipse for the original values of U, f and R_1.

be larger than the mean value I_0 of the alternating quiescent current in front of the rectifier, this value being determined by the minimum value H_0 of the ac field. The quantity I_{l0} can be expressed in terms of I_0 by the formula

$$I_{l0} = \gamma I_0,$$

where $\gamma = 1 - \pi/2$ can be determined by the graph presented in Appendix AP-5, with the aid of the curve $\eta = 1$, replacing ωCR_1 by $R_1/\omega L_1$.

This method of construction of the static characteristic of a magnetic amplifier makes it very easy to estimate the effect of the individual parameters of an amplifier on its static characteristic. For this purpose we must only take into consideration how a particular parameter affects (in conformity with formula (6.35)) the magnitude of the ellipse axes B_0' and H_k'. Figure 6.16 shows how the ellipse is distorted when U, f and R vary.

These curves show that the higher the quality of the core, i.e., the steeper its magnetization characteristics, the weaker will be the effect of variations in U, f and R on the static characteristic of the amplifier. This effect is completely absent in the case of an ideal core, when its magnetization characteristics are strictly vertical.

Chapter 7

NONREVERSIBLE MAGNETIC AMPLIFIERS WITH FEEDBACK

§ 7.1. The Use of Feedback in Magnetic Amplifiers

Feedback signifies that in addition to the input signal to be amplified, one feeds to the amplifier input also a signal which is a function of the output signal of the amplifier. This illustrated in Fig. 7.1.

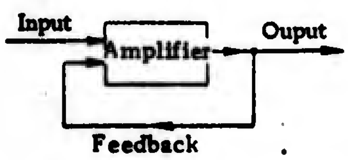

Fig. 7.1. The principle of feedback in an amplifier.

Feedback can be rigid or flexible. In the case of rigid feedback the feedback signal is proportional to the output signal of the amplifier. In the case of flexible feedback the feedback signal is determined by the rate of change of the output signal (or, in principle, by derivatives of any order of the output signal). Flexible feedback is achieved by means of a differentiating network. It is used for improving the dynamic performance of amplifiers.

Feedback (rigid or flexible) can be positive or negative. In the first case the feedback signal is aiding the input signal, whereas in the second case it is in opposition to the input signal.

Rigid negative feedback diminishes the amplifier gain, but on the other hand it increases the stability of the static characteristic of the amplifier and it reduces the noise level. Rigid positive feedback has the opposite effect, i. e., it increases the gain, but the other properties mentioned above are impaired.

Finally, the feedback can be without lag (ideal) or with lag. All these types of feedback are used in magnetic amplifiers, just as in other types of amplifiers. Yet a special feature of magnetic amplifiers is the very extensive use of rigid positive feedback for the purpose of increasing the figure of merit D. The possibility of using positive feedback in magnetic amplifiers is a consequence of the high stability of their static characteristics, so that even very strong positive feedback will not have any harmful effect on the static characteristic. This is a distinctive feature of magnetic amplifiers as compared to other amplifiers, such as electronic, semiconductor, electromechanical, etc.

In a magnetic amplifier (like in an electromechanical amplifier) the feedback can be of two types: electric feedback and magnetic feedback.

In the case of electric feedback the feedback signal is algebraically added to the input signal and their sum is applied to the control winding of the amplifier.

In the case of magnetic feedback the feedback signal is applied to a separate "feedback" winding (as will be shown in § 7.7 below, the output windings of the amplifier can serve as such a feedback winding). In this case the summation of the feedback signal and of the input signal amounts to the summation of the magnetic field strengths produced by the corresponding windings. This is the reason why such a feedback is called magnetic.

Unlike electric feedback, magnetic feedback has no effect on the input impedance of the amplifier, since in this case the feedback signal is not applied at all to the control winding. For this reason rigid positive feedbacks are always magnetic, so that the input impedance of the amplifier is not diminished, whereas negative feedbacks are as a rule electric (see § 7.14 below). Magnetic feedback can be external or internal. In the first case the feedback is produced with the aid of a special feedback winding, whereas in the second case one uses the output windings of the amplifier.

Since in the following we shall always refer to rigid feedback, the word "rigid" will be omitted.

§ 7.2. Magnetic Amplifier Circuits with External Magnetic Feedback

External feedback can be either voltage or current feedback. A model of a magnetic amplifier with current feedback is shown in Fig. 7.2a. Here, as in the following, the core of the magnetic amplifier is represented by a straight-line segment. The arrows indicate the direction of the magnetic field, produced by the individual windings. The direction of the current in the feedback windings W_{fb} is constant, being determined by the polarity of connecting these windings to the rectifier. Therefore the feedback is positive only for one polarity of the input signal U_C shown in Fig. 7.2a (aiding action of the W_C and W_{fb} windings), whereas for the other polarity the feedback is negative. This circuit is used for the amplification of signals of the first polarity. (Let us note that, as will be shown in § 7.5 below, in going over from a positive U_C to a negative one, the sign change of the feedback corresponds exactly to $U_C = 0$ only if the amplifier is ideal.) In the case of an ac load the rectifier serves only for producing the feedback. Sometimes one uses a separate rectifier for the feedback also in the case of a dc load. This is done in order to increase the stability of the characteristic of the magnetic amplifier. As a matter of fact, the stability of an amplifier with positive feedback depends to a large extent on the stability of the feedback. The use of a special feedback recifier under light-load conditions permits an increase in the stability of the parameters of the feedback circuit. Since the feedback rectifier is much smaller than the load rectifier, this does not lead to a great increase in the overall size and cost of the amplifier. Moreover, as will be shown below, the use of a separate feedback rectifier considerably improves the linearity of the amplifier characteristic in the case of a load which is not purely active.

The degree of feedback can be regulated by means of a variable resistor which shunts a portion (about 10-15%) of the feedback winding W_{fb}.

A magnetic amplifier circuit with voltage feedback is shown in Fig. 7.2b.

In this case one applies the voltage of the load R_l to the feedback winding W_{fb}. If the load resistance R_l is linear and time-constant, the two types of feedback — current and voltage — are equivalent. Normally, current feedback is more useful. Voltage feedback is sometimes utilized in high-power amplifiers (when the load current is large), enabling the size of the feedback rectifiers to be reduced. In the case of a variable load the stability of the characteristic $I = f(I_C)$ in the sense of being independent of

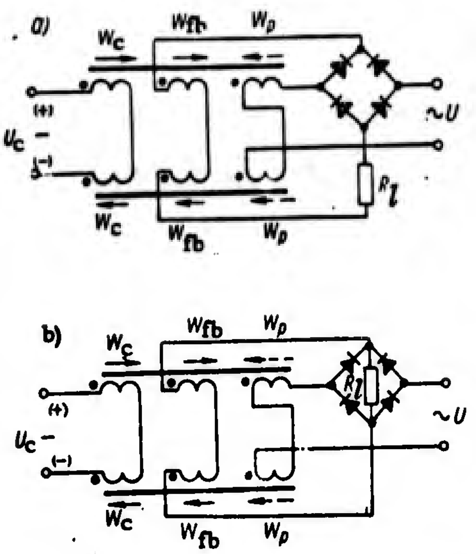

Fig. 7.2. A magnetic amplifier with external magnetic current feedback (a) and voltage feedback (b).

the magnitude of the load resistance, will be higher with positive voltage feedback, whereas the stability of the characteristic $U_l = f(U_C)$ will be higher with positive current feedback. This is due to the fact that whereas negative feedback increases the stability of the static characteristic for the variable with respect to which the feedback is applied, positive feedback has the opposite effect.

In order to regulate the degree of voltage feedback, a variable resistor is inserted in series with the feedback winding.

§ 7.3. Ideal Magnetic Amplifiers with External Feedback

A. Statics of an Ideal Magnetic Amplifier with Feedback

In the presence of feedback the equation (6.13) for the static characteristic of an ideal magnetic amplifier must be rewritten as follows

$$H_m = H_- .$$

Here $H_- = H_C \pm H_{fb}$ is the strength of the dc magnetic field produced in the given case by the control winding and the feedback winding. The upper (plus) sign corresponds to positive feedback, and the lower (minus) sign — to negative feedback.

The field strength produced by the feedback winding is

$$H_{fb} = \frac{W_{fb} I_{fb}}{l_c} .$$

In the case of current feedback, assuming that the entire rectified current of the output circuit flows through the feedback winding, we have $I_{fb} = I_m$. Hence

$$H_{fb} = K_{fb} H_m, \tag{7.1}$$

where $K_{fb} = W_{fb}/W_p$ is the feedback factor. The higher this factor, the stronger the feedback. Thus we obtain the following final equation for the static characteristic of an ideal magnetic amplifier with feedback:

$$H_m = \frac{H_C}{1 \pm K_{fb}}. \tag{7.2}$$

Here we use the same convention as before, i.e., the upper sign corresponds to positive feedback, and the lower sign to negative feedback.

From (7.2) follows

$$K_I = \frac{I_l}{I_C} = \frac{\frac{W_c}{W_p}}{1 \mp K_{fb}} = \frac{K_{I0}}{1 \mp K_{fb}};$$

$$K_U = K_I \frac{R_l}{R_y} = \frac{K_{U0}}{1 \mp K_{fb}}; \tag{7.3}$$

$$K_P = K_I^2 \frac{R_l}{R_C} = \frac{K_{P0}}{(1 \mp K_{fb})^2}.$$

Here K_{I0}, K_{U0} and K_{P0} denote the gain factors of a magnetic amplifier without feedback.

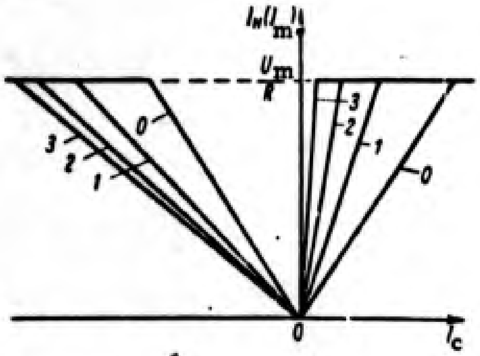

Fig. 7.3. Static characteristics of an ideal magnetic amplifier with feedback for various values of the feedback factor.

The characteristic 0 corresponds to K_{fb}=0, and the characteristics 1, 2 and 3 correspond to $K_{fb1} < K_{fb2} < K_{fb3}$.

Thus positive feedback is increasing the current and voltage gain by a factor of $(1-K_{fb})$, and the power gain by a factor of $(1-K_{fb})^2$. When $K_{fb} = W_{fb}/W_p$ increases, tending to 1, the gain factors are also increasing, tending to ∞.

Figure 7.3. shows a family of static characteristics of a magnetic amplifier for various values of K_{fb}.

When $K_{fb} > 1$, the magnetic amplifier is converted to a contactless magnetic relay (see §§ 7.6 and 11.2).

B. Time Constant and Figure of Merit of an Ideal Magnetic Amplifier with Feedback

In § 6.4 we considered the dynamics of an ideal magnetic amplifier without feedback. Like in the case of the gain factors, one must replace also in analysis of the dynamics of an amplifier with feedback the quantity H_C by $H_- = H_C \pm H_{fb}$.

Taking into account that $H_- = H_m$ and using the expression (7.2) for H_m (with current feedback), we obtain

$$H_- = \frac{H_C}{1 \mp K_{fb}}. \tag{7.4}$$

In deriving the equation for the transient process in the control circuit of a magnetic amplifier, we replace in formula (6.24) for B_0 the quantity $W_C I_C$ by the quantity $W_C I_C/(\mp K_{fb})$. As a result we obtain the final equation for the transient process with the following time constant

$$T_C = \frac{T_C}{1 \mp K_{fb}} \text{ [sec]}. \tag{7.5}$$

Here $T_{C0} = \frac{RW^2_C}{R_C W^2_p}\frac{1}{4f}$ is the time constant of the control circuit of a magnetic amplifier without feedback.

According to formulas (7.3) and (7.5), the figure of merit of a magnetic amplifier with feedback is

$$D = \frac{K_p}{T_C} = \frac{4\eta f}{1 \mp K_{fb}} = \frac{D_0}{1 \mp K_{fb}}, \tag{7.6}$$

where $D_0 = 4\eta f$ is the figure of merit of an amplifier without feedback.

Formula (7.6) shows that apart from increasing the supply frequency f, a powerful means for improving the figure of merit of a magnetic amplifier is to use positive feedback.

It is evidently necessary that the feedback factor K_{fb} should be as close as possible to 1, insofar as this is permitted by the instability and nonlinearity of the amplifier characteristic (which are increasing with the strength of the positive feedback).

From (7.3) and (7.5) follows yet another interesting formula:

$$\frac{K_U}{T_C} = 4\pi f \frac{W_p}{W_C} \qquad (7.7)$$

i.e., the ratio K_U/T_C does not depend on the presence and magnitude of feedback in the amplifier.

In the case of a magnetic amplifier with ac output (without rectification), one must introduce a form factor K_f into the above formulas, as in the case of an amplifier without feedback.

§ 7.4. Methods of Changing the External Feedback Factor

The feedback factor $K_{fb} = W_{fb}/W_p$ is determined by the number of feedback turns W_{fb}. In order to be able to alter the K_{fb} of a ready amplifier, the feedback winding is made with taps. For the smooth adjustment of K_{fb} one uses variable resistors R_{var}, that are connected in parallel to W_{fb} in the case of current feedback (Fig. 7.4), and in series in the case of voltage feedback (Fig. 7.4b).

The shunting of W_{fb} increases the lag of the amplifier, since a closed circuit is thus formed which delays the transient process in the control circuit. In the case of current feedback, one therefore connects the resistor R_{var} in parallel to only a small portion of the W_{fb} winding (and not to the entire winding), amounting to 10-20% of this winding (W_{fb2} in Fig. 7.4a).

In the presence of a shunt resistance R_{var} we have

$$K_{fb} = K_{fb1} + K_{fb2} = \frac{W_{fb1}}{W_p} + \frac{W_{fb2}}{W_p} \cdot \frac{R_{var}}{R_{fb2} + R_{var}},$$

or

$$K_{fb} = \frac{W_{fb}}{W_p} K_{var}, \tag{7.8}$$

where

$$W_{fb} = W_{fb1} + W_{fb2};$$

$$K_{var} = \frac{W_{fb1}}{W_{fb}} + \frac{W_{fb2}}{W_{fb}} \cdot \frac{R_{var}}{R_{fb2} + R_{var}}.$$

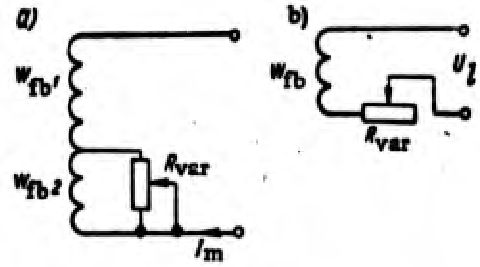

Fig. 7.4. Current feedback (a) and voltage feedback (b) regulation circuits.

The range of variation of K_{fb} can be doubled if the winding W_{fb2} is made separate, and not tapped from the common coil W_{fb}. In this case the plus sign in the expression for K_{var} in formula (7.8) must be replaced by $\mp$, so as to account for the possibility of interchanging the ends of W_{fb2}, i.e., of connecting this winding in an aiding direction with W_{fb1}, as well as in an opposing direction with the latter.

For voltage feedback (Fig. 7.4b) we have

$$I_{fb} = \frac{U_l}{R_{fb} + R_l} = \frac{R_l}{R_{fb} + R_l} I_m.$$

Hence

$$K_{fb} = \frac{H_{fb}}{H_m} = \frac{W_{fb}}{W_p} \cdot \frac{R_l}{R_{fb} + R_l} = \frac{W_{fb}}{W_p} \cdot \frac{R_l}{R_{fb}} K_{var}, \tag{7.9}$$

where

$$K_{var} = \frac{R_{fb}}{R_{fb} + R_l}.$$

Thus, whatever the feedback (current or voltage), the variable resistor can be accounted for by introducing into the expression for K_{fb} the factor K_{var}, specified by formulas (7.8) and (7.9).

§ 7.5. The Parameters of Actual Magnetic Amplifiers with Feedback

In Fig. 7.5 we present the characteristics of an actual amplifier with various K_{fb}. As can be seen from Fig. 7.5, with increasing K_{fb} the characteristics are shifted to the left, and hence the load current at $I_C = 0$ is not equal to its minimum value I_0, but also increases with K_{fb}. This is due to the fact that in actual amplifiers the current in the output circuit is not equal to zero when $I_C = 0$. Therefore, in the presence of feedback this current (flowing through W_{fb}) produces a dc magnetization, shifting the amplifier characteristic.

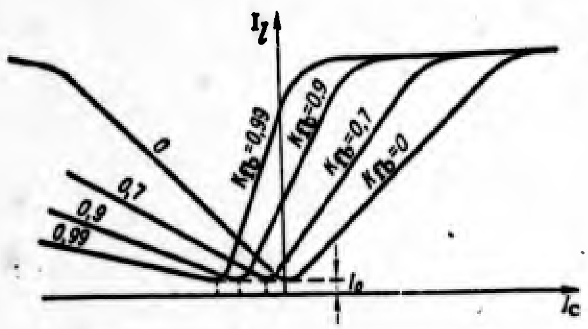

Fig. 7.5. Static characteristics of an actual magnetic amplifier with feedback.

In order to obtain $I_l = I_0$ when $I_C = 0$, amplifiers with feedback are equipped with a special bias winding. The field strength produced by this winding must be equal in magnitude to the field strength produced by the feedback winding when a current I_0 flows through it, i.e.,

$$H_b = \frac{W_{fb} I_0}{l_c},$$

and it must be opposite in direction. In this case the bias winding fully compensates the magnetizing effect produced by the feedback winding for $I_C = 0$.

When K_{fb} increases and approaches unity, the instability and nonlinearity of the magnetic amplifier characteristic increase. Likewise increasing is its dependence on the actual shape of the core magnetization characteristic $B = f(H)$ and on the rectifier characteristic (see § 7.13). The instability, due to the nonconstancy of the temperature, supply voltage and frequency, and load resistance, is also increasing. Therefore the above formulas for an ideal amplifier with feedback yield a greater error as compared to the analogous formulas for an amplifier without feedback. This error increases with K_{fb}, and when $K_{fb} > 0.8$-0.9 these formulas are no longer applicable.

In an actual magnetic amplifier with $K_{fb} = 1$ the quantities K_I and D will not be equal to infinity, as it follows from formulas (7.3) and (7.6). They will be finite and their value will be the higher, the better the quality of the amplifier cores and of the rectifiers. Thus by using germanium instead of selenium rectifiers, and toroidal cores instead of composite pi-shaped and three-legged cores, the figure of merit increases severalfold. On the basis of the linearity and stability requirements towards the characteristic of a mangetic amplifier, the quantity K_{fb} is normally taken not higher than 0.95-0.97. If, however, one uses high-grade cores and rectifiers, and in particular if the temperature fluctuations are small (room-temperature conditions), the latter affecting primarily the parameters of the rectifiers, then K_{fb} can be taken as large as 0.98-0.99, while preserving the linearity and stability of the amplifier characteristic.

The figure of merit of actual amplifiers with feedback is normally $D = (100$-$1000)f$. Other conditions being equal, D increases with the amplifier power.

§ 7.6. Graphic Construction of the Static Characteristic of a Magnetic Amplifier with Feedback

The static characteristic of a magnetic amplifier with feedback is easy to construct graphically on the basis of the characteristic of such an amplifier without feedback. This makes it possible to take into account the nonlinearity of the actual characteristic of the magnetic amplifier. This is very important, since the nonlinearity of the characteristic increases with the magnitude of the positive feedback, and at large values of K_{fb} the characteristic of an ideal amplifier can no longer be used.

The construction is presented in Fig. 7.6, where we proceed from the characteristic of an amplifier without feedback, $H_m = f(H_-)$.

In the presence of feedback we have $H_- = H_C \pm H_{fb}$, where $H_{fb} = K_{fb}H_m$. It is required to construct the function $H_m = f(H_C)$. The function $H_{fb} = f(H_m)$ is graphically represented by a ray emanating from the origin of coordinates at an angle $\alpha = \text{arc tg } K_{fb}$ towards the ordinate. In order to obtain a certain value H_{m1} we must (in accordance with the characteristic $H_m = f(H_-)$ of an amplifier without feedback) produce a dc field with a strength H_{-1} (see Fig. 7.6). Part of this field strength $H_{fb1} = K_{fb}H_{m1}$ is produced by the feedback winding. In order to obtain H_{m1}, it is therefore necessary that the control winding should produce the field strength

$$H_{C1} = H_{-1} \mp H_{fb1}.$$

Hence in the case of positive feedback (the first quadrant in Fig. 7.6), the quantity H_C, corresponding to some value H_m, is determined graphically as the difference between the abscissas of the characteristics $H_m = f(H_-)$ and $H_{fb} = f(H_m)$, i.e., as the magnitude of the distance between them. For a field H_C of opposite polarity (the second quadrant of Fig. 7.6), the sought-for value of H_C is determined, on the other hand, as the sum of the abscissas of the characteristics $H_m = f(H_-)$ and $H_{fb} = f(H_m)$, since in this case the feedback will negative, i.e., H_{fb} is directed opposite to H_C and must be compensated by the control winding:

$$H_C = H_- + H_{fb}.$$

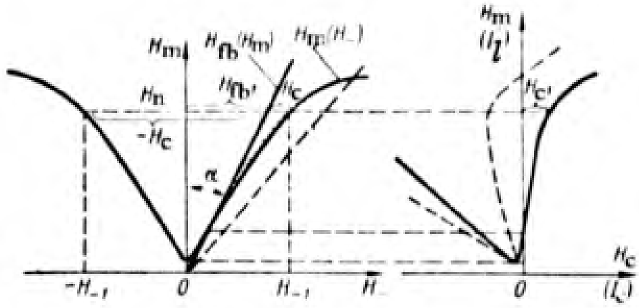

Fig. 7.6. Graphic construction of the static characteristic of a magnetic amplifier with feedback.

The above construction shows clearly that positive feedback increases the nonlinearity of the characteristic, whereas negative feedback reduces it. Indeed, the characteristic of an amplifier with positive feedback (the right-hand branc · of the entire characteristic) is obtained by deducting from the original characteristic $H_m = f(H_{\_})$ of an amplifier without feedback the linear characteristic $H_{fb} = f(H_m)$ of the feedback circuit. As a result, the nonlinearity present in the original characteristic becomes more emphasized owing to the decrease in the linear component of $H_m = f(H_{\_})$, and hence the degree of nonlinearity of the characteristic increases. In the case of negative feedback, on the other hand, the sought-for characteristic $H_m = f(H_C)$ is obtained by adding to the original function $H_m = f(H_{\_})$ the linear dependence $H_{fb} = f(H_m)$. As a result, the relative importance of the nonlinearity contained in the original characteristic decreases.

In Fig. 7.6 the construction of the static characteristic of a magnetic amplifier with $K_{fb} > 1$ is represented by a dashed line. The characteristic becomes distorted altogether, exhibiting a section with a negative slope, characteristic of trigger (relay) circuits. Magnetic amplifiers with $K_{fb} > 1$ are used as contactless magnetic relays (magnetic flip-flops), in the same way as electronic and semiconductor dc amplifiers with strong positive feedback (which causes them to operate in the relay mode) are utilized as electronic and semiconductor contactless relays (flip-flops). Relay circuits based on magnetic amplifiers will be considered in § 11.2.

The accuracy of the graphic construction of the characteristic of a magnetic amplifier with feedback decreases when K_{fb} approaches unity, i. e., when the angle between the characteristics $H_m = f(H_{\_})$ and $H_{fb} = f(H_m)$ decreases. With $0.8 < K_{fb} < 1.2$, this construction yields only the approximate shape of the static characteristic of a magnetic amplifier.

§ 7.7. Magnetic Amplifier Circuits with Internal Feedback

Let us consider the principle of operation of a magnetic amplifier with internal feedback on the example of the very simple circuit depicted in Fig. 7.7. The impedance Z_A in the control winding circuit serves for limiting the current induced from the output circuit. This configuration is characterized by the presence of a rectifier D in the output circuit. Therefore a half-wave rectified current is flowing through W_p. Its dc component produces a core magnetization that is proportional to the load current R_l. Thus an effect is obtained which is similar to the action of the feedback in a circuit with external feedback. Therefore such a circuit is called a magnetic amplifier with internal feedback or a magnetic amplifier with auto-excitation.[1]

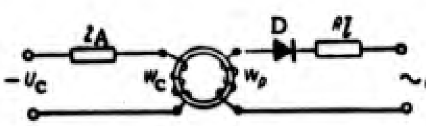

Fig. 7.7. Simplest magnetic amplifier with internal feedback.

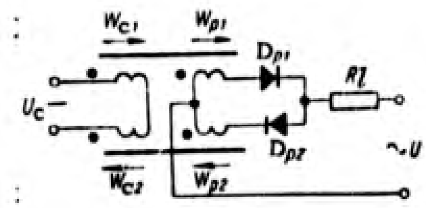

Fig. 7.8. Two-core magnetic amplifier with internal feedback.

A magnetic amplifier with internal feedback has a factor $K_{fb} = H_{fb}/H_m = 1$; the output winding W_p serves also as the feedback winding, so that the dc component of the current flowing in W_p is equal to the mean load current I_m.

[1] See footnotes p. 243.

Figure 7.8 shows a two-core amplifier with internal feedback. The use of two cores makes it possible to obtain a purely alternating load current and to eliminate from the control circuit the operating-frequency current induced by the output circuit. The direction of dc magnetization is schematically indicated in Fig. 7.8 by arrows. If the rectifiers are removed from this system, we obtain an ordinary magnetic amplifier without feedback that has parallel-connected output windings.

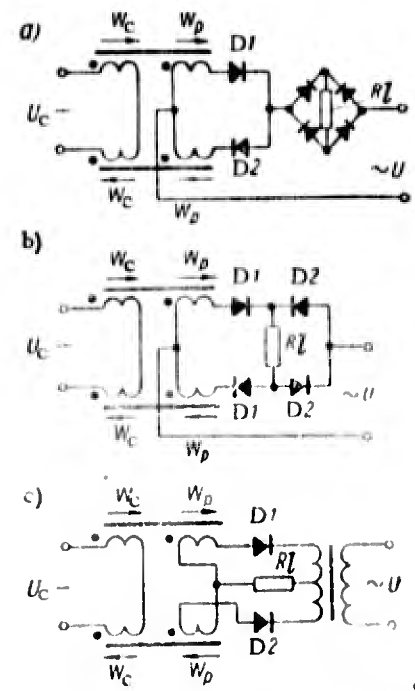

Fig. 7.9. Magnetic mpli-fiers with internal feedback and output rectification.

Figures 7.9a, b and c show three amplifier circuits with internal feedback and output rectification (the load R_l is dc connected). The circuit that gained widest acceptance is that of Fig. 7.9b, which is the simplest. The circuit with greatest stability and linearity of the amplifier characteristic is that of Fig. 7.9a. This is due to the fact that this circuit uses separate rectifiers for the load and the feedback (see § 7.13).

Let us consider methods of altering K_{fb} in a magnetic amplifier with internal feedback.

Figure 7.10 shows circuits permitting K_{fb} to be gradually increased or decreased with respect to unity. The output winding is divided into two parts W_{p1} and W_{p2}. In Fig. 7.10a the feedback is produced by using only part (W_{p1}) of the output winding $W_{p1} + W_{p2}$. Hence

$$K_{fb} = \frac{W_{fb}}{W_p} = \frac{W_{p1}}{W_{p1} \pm W_{p2}}. \tag{7.10}$$

The plus sign in the denominator corresponds to an aiding connection of the W_{p1} and W_{p2} windings ($K_{fb} < 1$), and the minus sign corresponds to an opposing connection ($K_{fb} > 1$).

The circuit of Fig. 7.10b differs from the circuit of Fig. 7.10a only by the fact that the W_{p2} windings are connected in series, and not parallel.

Since in this case the total load current will be flowing through the W_{p2} windings, we have

$$K_{fb} = \frac{W_{p1}}{W_{p1} \pm 2W_{p2}} . \tag{7.11}$$

For the circuit of Fig. 7.10c we have

$$K_{fb} = \frac{W_{fb}}{W_p} = \frac{W_{p1} \mp W_{p2}}{W_{p1} \pm W_{p2}} . \tag{7.12}$$

The upper signs in formula (7.12) hold for $K_{fb} < 1$ and correspond to opposing-connected windings W_{p1} and W_{p2} (see the arrows in Fig. 7.10c), whereas the lower signs hold for $K_{fb} > 1$ and correspond to aiding-connected windings.

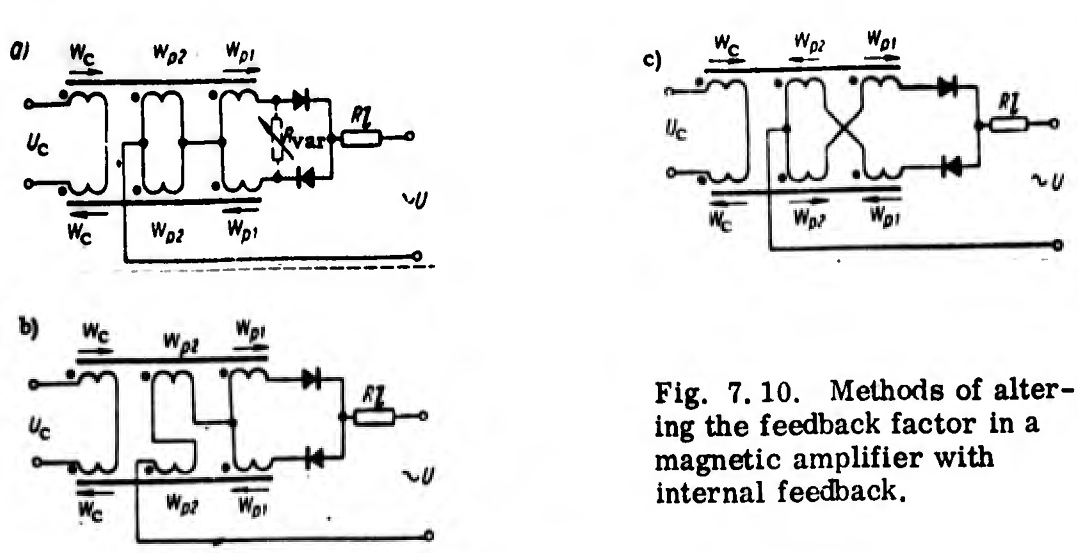

Fig. 7.10. Methods of altering the feedback factor in a magnetic amplifier with internal feedback.

In order to smoothly vary (regulate) the feedback factor K_{fb} it is necessary to shunt the rectifiers by the variable resistor R_{var}, as shown by the dashed line in Fig. 7.10a. If $R_{var} = \infty$, then $K_{fb} = 1$; if, on the other hand, $R_{var} = 0$, then $K_{fb} = 0$;

a finite value of R_{var} corresponds to $0<K_{fb}<1$. Let us present without deduction the formula for K_{fb} (see [9], § 7.12):

$$K_{fb} = \frac{R_{var} - R_{D\,for}}{R_{var} + R_{D\,for} + 2R_p}, \tag{7.13}$$

where $R_{D\,for}$ is the forward resistance of the rectifier, and R_p is the resistance of the W_p winding.

If the amplifier load is dc connected, the adjustment of K_{fb} can be performed with the aid of a small variable external feedback, since this does not necessitate the introduction of a new rectifier into the circuit.

§ 7.8. Comparison Between Magnetic Amplifiers with Internal and External Feedbacks

Figure 7.11 shows magnetic amplifier circuits with internal (Fig. 7.11a) and external (Fig. 7.11b) feedback. In the case of internal feedback the output windings act on their cores in succession, i.e., during odd supply half-cycles of the output circuit the loop W_p of the first core is formed, whereas during even half-cycles the loop of the second core is formed.

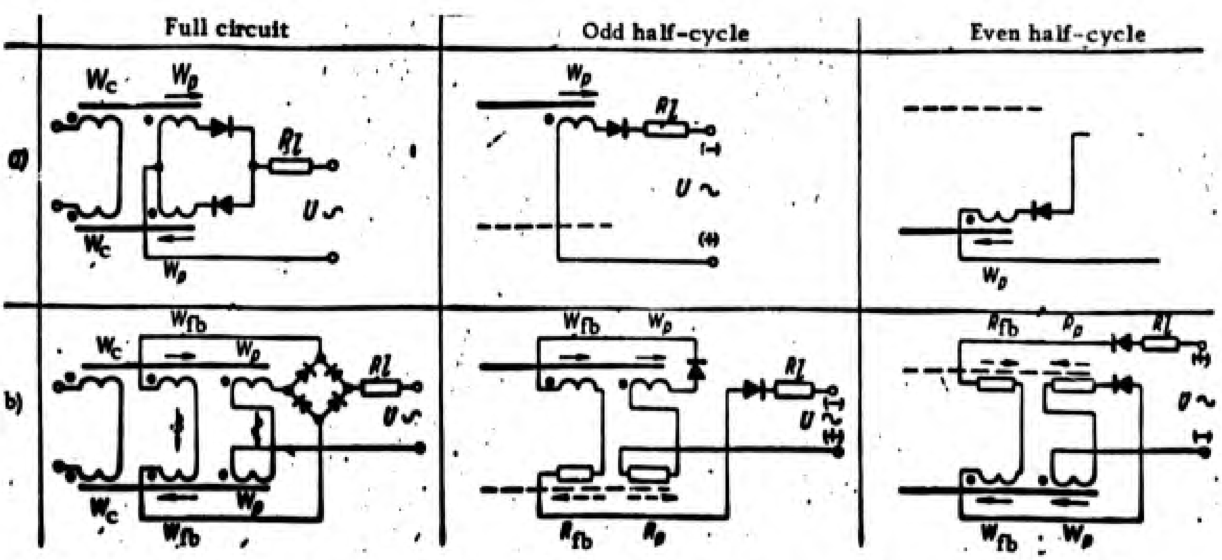

Fig. 7.11. Comparison of magnetic amplifiers with internal (a) and external (b) feedback for $K_{fb} = 1$.

Therefore the load circuit of an amplifier with internal feedback for the individual half-cycles can be represented in the form shown in Fig. 7.11a.

Let us now examine the circuit with external feedback (Fig. 7.11b). The direction of the magnetizing force of the windings in the two supply half-cycles is unchanged, being determined by the polarity of connecting these windings to the rectifier. The magnetizing force of the output windings changes its direction together with the current during each half-cycle. Let us recall that in the case of aiding-connected W_{fb} (as well as W_C) windings, the W_p windings are connected in opposition. For this reason, during each half-cycle the magnetizing forces of the W_p and W_{fb} windings are summed for one of the cores, whereas for the other core they are subtracted. If $K_{fb} = 1$, we have $W_{fb} = W_p$, and hence the resultant action of the W_p and W_{fb} windings in one core will be equal to double the action of W_p, whereas in the other core it will be equal to zero, since the W_{fb} winding fully compensates the action of the W_p winding ($H_{fb\ inst} = H_{inst}$). In the next half-cycle the cores change roles.

As a result, it is possible to represent the output circuit of an amplifier with external feedback and $K_{fb} = 1$ in the individual supply half-cycles in the form shown in Fig. 7.11b.

Thus if the losses in the output circuit are not taken into account, an amplifier with external feedback and $K_{fb} = 1$, i.e., $W_{fb} = W_p$, will be fully equivalent to an amplifier with internal feedback and an output winding that has $W_p + W_{fb} = 2W_p$ turns. This means that if instead of a circuit with external feedback and $W_{fb} = W_p$ we design on the same cores a circuit with internal feedback, using as the output winding the series-connected W_p and W_{fb} windings of the previous circuit, then all the parameters and characteristics of the amplifier remain unchanged to within the losses in the output circuit.

Yet in the case of internal feedback the losses in the output circuit are smaller than in the case of external feedback. This is evident from Fig. 7.11. In the output circuit of an amplifier with external feedback the resistances of the two output windings and of the feedback winding are all the time connected, whereas in an equivalent amplifier with internal feedback the losses during each half-cycle are occurring only in the principal output winding with a number of turns equal to $W_p + W_{fb}$.

Thus a magnetic amplifier with internal feedback has a higher efficiency $\eta = R_l/R$, and hence also a higher figure of merit as compared to an amplifier with external feedback. With same core dimensions, an amplifier with internal feedback has about 40% higher maximum output power. As an example, we plotted in Fig. 7.12 the static characteristics of an actual amplifier with $K_{fb} = 1$, designed at first with external feedback, and then with internal feedback (and hence with double the number of turns of the output winding).

The circuit with internal feedback has yet another advantage over the circuit with external feedback, namely it requires fewer rectifiers (see Fig. 7.11). More convenient from the point of view of rectifiers is also the circuit with internal feedback and output rectification (Fig. 7.9b). Although we again have four rectifiers, let us note that whereas in the circuit with external feedback (Fig. 7.2a and 7.11b) all the four rectifiers must operate on the same reverse voltage, for example equal to the supply voltage, the rectifiers D1 in Fig. 7.9b, just as the rectifiers in Fig. 7.8, are operating at a reverse voltage not exceeding 20-30% of the supply voltage. The reverse voltage for these rectifiers is equal to the voltage drop across the resistance of the output winding and the forward resistance of the other rectifier D1.

The above comparison of magnetic amplifiers with internal and external feedback shows that as a rule internal feedback is more advantageous. External feedback is used only in small-size low-power amplifiers, when the efficiency and

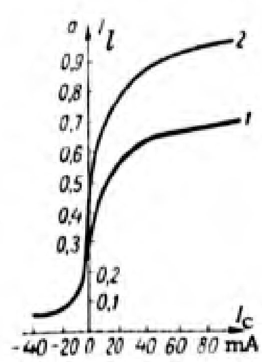

Fig. 7.12. Static characteristics of equivalent amplifiers with external (1) and internal (2) feedback.

size are not important. In this case external feedback is more convenient, since the factor K_{fb} is easier to adjust.

§ 7.9. Steady-State Operation of a Magnetic Amplifier with Internal Feedback

In § 7.3 above, we examined the statics of a magnetic amplifier with feedback and we derived the formulas for the gain factors of such an amplifier. Since the amplifier was considered to be ideal, the accuracy of these formulas decreases with increasing K_{fb}. The error is determined by the non-ideal character of the core magnetization curve, and also by the non-ideal character of the rectifiers. The lower the quality of the cores and rectifiers, the higher will be the error and the smaller will be the limiting value of K_{fb} down to which it is permissible to utilize the formulas of an ideal amplifier with feedback.

The formulas derived above are entirely inapplicable to an amplifier with $K_{fb} = 1$. Therefore this section is especially devoted to the steady-state operation of a magnetic amplifier with internal feedback and $K_{fb} = 1$. Here the core magnetization curve can no longer be considered as ideal, like in Fig. 6.8.

We shall take into account the value of the permeability of an unsaturated core (when $|B| < B_s$) and the presence of hysteresis, i.e., we shall proceed from the characteristic shown in Fig. 7.13.

Let us re-examine the very simple one-core amplifier circuit with internal feedback (Fig. 7.14). During each supply cycle of the output circuit, this circuit is formed only during one negative half-cycle, when the rectifier D is conducting. Therefore this half-cycle will be called output half-cycle. During the other half-cycle the rectifier D in the output circuit is cut off by the negative supply-voltage half-wave, and hence only the control circuit is acting on the core. This half-cycle is therefore called the control half-cycle.

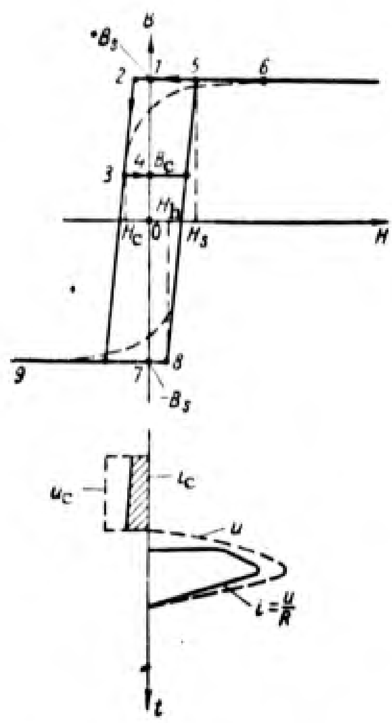

Fig. 7.13. Dynamic hysteresis loop.

Let us consider the operation of the circuit during the individual half-cycles.

During the output half-cycle, under the action of the positive half-wave of the voltage applied to the output circuit, the core induction increases (the core is magnetized) up to the value $+B_s$, when core saturation sets in.

During the control half-cycle the negative signal, applied to the control winding, reduces the core induction and the core is demagnetized. Let us denote the core induction at the end of the control half-cycle by B_C (point 4 in Fig. 7.13).

The variation of the induction during the control half-cycle will be the greater (and point 4 in Fig. 7.13 the lower), the higher the amplifier input voltage U_C. Indeed, if we neglect the resistance of the control circuit, the equation for this circuit will be

$$U_C = W_C S_c \frac{dB_{inst}}{dt}. \tag{7.14}$$

Therefore the variation of the induction during a control half-cycle will be

$$\Delta B_c = \frac{1}{W_c S_c} \int_0^{\frac{T}{2}} u_c \, dt = \frac{1}{W_c S_c} U_{c.m} \frac{T}{2} = \frac{1}{2W_c S_c f} U_{c.m} \tag{7.15}$$

where $T = 1/f$ is the supply period, and $U_{C \cdot m}$ is the average (over the control half-cycle) value of the input voltage.

Since we are considering steady-state operation with $u_C = U_C = \text{const}$, we must set $U_{C \cdot m} = U_C$, so that

$$\Delta B_c = \frac{1}{2W_c S_c f} U_c. \tag{7.16}$$

During the output half-cycle the core operation is entirely determined by the action of the output winding. During this half-cycle the input signal U_C, present in W_C, has no effect whatsoever on the core in view of its smallness as compared to the supply voltage U (when U_C and U have been referred to the same number of turns). This is illustrated by the equivalent circuit of Fig. 7.14b.

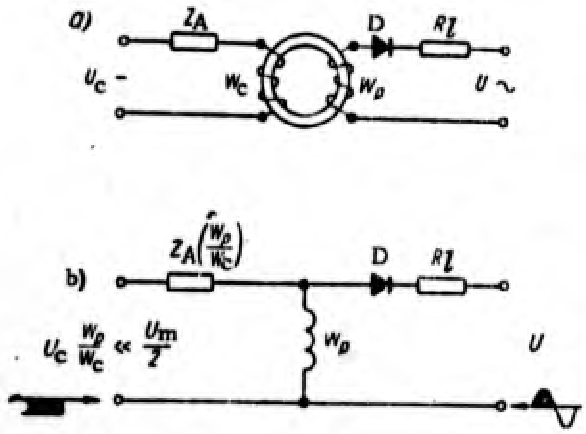

Fig. 7.14. Basic (a) and equivalent (b) diagrams of a very simple magnetic amplifier with internal feedback.

As long as the core is not saturated during the output half-cycle, the equation for the output circuit (if its resistance in comparison to the self-induction emf is neglected) will be

$$u = W_p S_c \frac{dB_{inst}}{dt}. \tag{7.17}$$

In other words, whereas the rate of decrease of the induction from $+B_s$ to B_C during the control half-cycle is proportional to the signal voltage U_C, the rate of increase of the induction from B_C back to $+B_s$ during the output half-cycle is proportional to the supply voltage U of the output circuit. The total variation of the induction during the output half-cycle is $\Delta B_p = -\Delta B_s$. The instant at which the induction reaches the value $+B_s$ (point 5 in Fig. 7.13) depends on the value of B_C; the lower B_C, i.e., the lower the point 4 in Fig. 7.13, the later the onset of core saturation during the output half-cycle. From the instant of core saturation and until the end of the output half-cycle (the trajectory 5-6-1 in Fig. 7.13) the self-induction emf of the output winding is found to be equal to zero and the entire supply voltage is applied to the resistance R of this circuit (see the lower part of Fig. 7.13). Thus the mean value of the load voltage U_l will depend on the instant of saturation of the core during the output half-cycle, i.e., the firing angle α_s.

Figure 7.15 illustrates the above-described process of variation of the core induction of an amplifier.

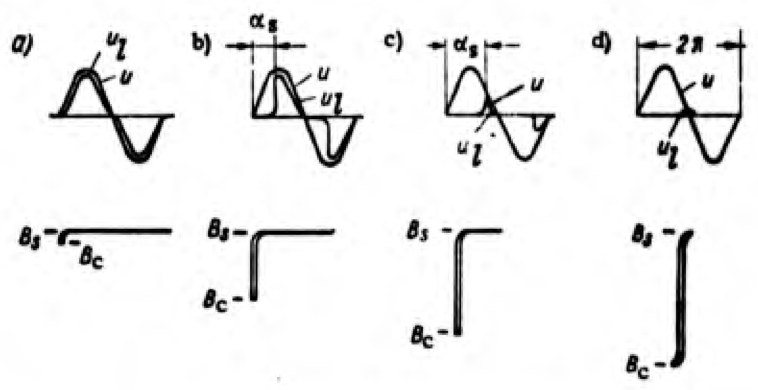

Fig. 7.15. Plots describing the operation of a magnetic amplifier with internal feedback.

Figures a, b, c and d correspond to various values of the control current, from its minimum (a) to its maximum (d) value.

Figure 7.16 (curve 1) shows the static amplifier-characteristic $U_l = f(U_C)$.

When $U_C = 0$ we have $\Delta B_C = 0$, i.e., $B_C = +B_s$, and we obtain

$$\alpha_s = 0 \text{ and } U_s = U_{l \cdot m} = \frac{U_m}{2} \eta,$$

where $\eta = R_l/R$.

The quantity $U_{l \cdot m}$ is determined by half the mean value of the supply voltage of the output circuit, since a half-wave voltage is applied to the resistance of this circuit.

When $U_C = U_{C \cdot M}$ we have $B_C = -B_s$ and

$$\alpha_s = \pi \text{ and } U_l = 0.$$

Just as in the case of an amplifier without feedback, the output circuit supply voltage is taken equal to

$$U_m = U_s = \omega W_p S_c B_s \tag{7.18}$$

In the case of a maximum control signal $U_C = U_{C \cdot M}$, corresponding to $B_C = -B_s$, the core is remagnetized along the full hysteresis loop (the trajectory 1-2-7-8-5-1 in Fig. 7.13), achieving saturation only at the end of the output half-cycle. In this case we have $\alpha_s = \pi$ and $U_l = 0$. Indeed, according to formula (7.17) we obtain, when $U_M = U_s$ and (7.18) is taken into account, the expression

$$\Delta B_p = \frac{1}{W_p S_c} \int_0^{T/2} u\,dt = \frac{1}{W_p S_c} \int_0^{T/2} U_s \sin \omega t\, dt = 2B_s.$$

The corresponding value $U_{C \cdot M} = U_{C \cdot s}$ is obtained by introducing $\Delta B_C = 2B_s$ into formula (7.16):

$$U_{C \cdot M} = U_{C \cdot s} = 4fW_C S_c B_s. \tag{7.19}$$

In the case of a larger value of U, operation with $U_l = 0$ ($\alpha_s = \pi$) is impossible, since with $B_C = B_s$ the core will have time to saturate during the output half-cycle before its termination ($\alpha_s < \pi$) (characteristic 2 in Fig. 7. 16). A smaller value of U is not advantageous, since this would involve a corresponding decrease in the maximum output power of the amplifier, determined by the quantity $U_{l \cdot M} = U_m/2$ (characteristic 3 in Fig. 7.16). In this case the core will be remagnetized along a partial cycle and not along the full hysteresis loop (for example, along the trajectory 1-2-3-4-5-1 in Fig. 7.13). The value of $U_{C \cdot M}$, necessary for obtaining $\alpha_s = \pi$, will also decrease accordingly.

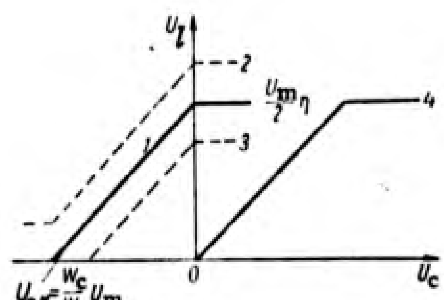

Fig. 7.16. Static characteristic $U_l = f(U_C)$ of a magnetic amplifier with internal feedback.

On the basis of the obtained formulas for the input and output voltages of the investigated one-core amplifier with $R_C = 0$ it is easy to derive the formula for the voltage gain:

$$K_U = \frac{\Delta U l}{\Delta U_c} = \frac{U_{l.M}}{U_{c.M}} = \frac{\frac{U_m}{2}\eta}{U_{c.M}}.$$

By substituting into this formula the expression

$$U_m = \frac{2}{\pi} U_M,$$

where $U_M = U_s$ according to (7.18) and $U_{C \cdot M} = U_{C \cdot s}$ according to (7.19), we obtain

$$K_U = \frac{W_p}{2W_c}\eta. \tag{7.20}$$

Let us now consider a two-core circuit (Figs. 7.8 and 7.9). As before, we shall neglect the control circuit resistance R_C, assuming that it is small (operation with free even harmonics). Since the output windings of the two cores operate during different half-cycles, each supply half-cycle will be an output half-cycle for one core and a control half-cycle for the other core. When the output winding of the core for which the particular half-cycle is of output type magnetizes its core to the induction $+B_s$, the other core is demagnetized, under the action of its control winding, to the induction B_C. Here, in contrast to a one-core circuit, the control winding of each core is (during its control half-cycle) not only under the action of the input voltage U_C, but also of the emf induced into the control winding of the second core by its output winding, this action continuing until the induction in the second core attains the value $+B_s$. For this reason the induction in the first core will vary at the end of the particular half-cycle (which is a control half-cycle for it) by a larger value ΔB_C as compared to the case when only the input voltage U_C would act on its control winding. As a result of such a positive feedback between the cores, produced by their control windings, the voltage gain of a two-core circuit is found to be many times higher than that of a one-core circuit.

Let us consider the static characteristic $I_l = f(I_C)$ of a two-core amplifier with $K_{fb} = 1$ (when the resistance of the control circuit is small).

According to formula (6.11), the mean value of the load current is

$$I_l = I_m = \frac{1}{\pi} \int_{\alpha_s}^{\pi} i d(\omega t) = \frac{U_M}{\pi R}(1 + \cos \alpha_s). \tag{7.21}$$

Let us express α_s in terms of B_C. By formula (7.17) we have for the instantaneous value of the induction in an output half-cycle:

$$B_{inst} = \frac{1}{W_p S_c} \int_0^t U_M \sin \omega t \, dt = B_c + \frac{U_M}{\omega W_p S_c}(1 - \cos \omega t).$$

Here B_C is an integration constant, namely the value of B_{inst} at t = 0, i.e., a the end of the previous control half-cycle.

By virtue of (7.18) we have

$$B_{inst} = B_c + B_s(1 - \cos \omega t). \tag{7.22}$$

By introducing $B_{inst} = +B_s$ into (7.22), we find the instant of time when core saturation sets in during the output half-cycle, i.e., we determine the firing angle:

$$\cos \alpha_s = \frac{B_c}{B_s}. \tag{7.23}$$

By substituting (7.23) into (7.22), we obtain a formula for I_l as a function of B_C:

$$I_l = \frac{U_M}{\pi R}\left(1 + \frac{B_c}{B_s}\right). \tag{7.24}$$

In order to determine the dependence of I_l on I_C, we must express B_C in terms of I_C. The direct current I_C in the control circuit that corresponds to a particular value of B_C in formula (7.24) can be determined in terms of the field strength by means of the dynamic hysteresis loop (Fig. 7.13).

The instantaneous value of the field, produced by the control winding, varies during the control half-cycle (for the particular core), increasing from zero (point 1 in Fig. 7.13) to a final value H_C, determined by the value of B_C (point 3 in Fig. 7.13). The value of the direct current I_C corresponding to a given value of B_C is determined by the mean value (the dc component) of the field produced by the W_C winding during a half-cycle.

Thus the overall dependence $B_C = f(I_C)$ is determined by the left branch of the dynamic hysteresis loop $B = f(H)$, i. e., the shape of the static amplifier characteristic $I_l = f(I_C)$ resembles to a considerable extent the shape of this branch of the hysteresis loop. This is illustrated by Fig. 7.17. Here curve 1 is the static characteristic corresponding to the squared loop of Fig. 7.13.

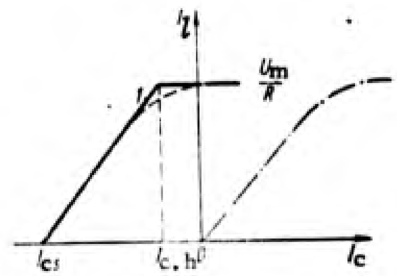

Fig. 7.17. Static characteristic of a magnetic amplifier with internal feedback and $K_{fb} = 1$.

The control current $I_{C\cdot h}$ in Fig. 7.17 corresponds to the field strength H_h in Fig. 7.13. When $0 < |I_C| < I_{C\cdot h}$, the induction will not vary during the control half-cycle, but remain equal to $+B_s$; for this reason the load current I_l will also remain unchanged. The dashed line in Fig. 7.17 corresponds to the actual hysteresis loop, represented in Fig. 7.13 by a dashed line too. Thus in order to increase the linearity of the static characteristic of an amplifier with internal feedback it is necessary that the core material should have a hysteresis loop which is as linear as possible.

Let us determine the current gain. For a linear characteristic we have

$$K_I = \frac{\Delta I_l}{\Delta I_c} \approx \frac{\Delta I_{l.м}}{\Delta I_{c.м}}.$$

Here

$$\Delta I_{l.м} = I_{m.м} = \frac{U_m}{R} \cdot \frac{2U_м}{\pi R}$$

is the maximum increase of the mean value of the load current, $\Delta I_{C\cdot M}$ is the corresponding maximum increase of the dc control current;

$$\Delta I_{c.м} = \frac{(H_s - H_h) l_c}{2W_c} = \frac{B_s l_c}{\mu_d W_c},$$

where $\frac{H_s - H_h}{2}$ is the mean value of the increase of the control field-strength corresponding to a load current increase from zero to maximum.

Thus,

$$K_I = \frac{U_m W_c}{l_c B_s R} \mu_d = \frac{2U_M W_c}{\pi l_c R B_s} \mu_d. \qquad (7.25)$$

For $U_M = U_s = \omega W_p S_c B_s$ we can rewrite this formula as follows:

$$K_I = \frac{2\omega S_c W_p W_c}{\pi l_c R} \mu_d = \frac{2X_{p.M}}{\pi R} \cdot \frac{W_c}{W_p}, \qquad (7.26)$$

where

$$X_{p.M} = \omega L_{p.M} = \omega \frac{S_c W_p^2}{l_c} \mu_d$$

is the inductive resistance of the output winding in the case of an unsaturated core.

As can be seen from formula (7.25), the current gain of an amplifier with internal feedback is directly proportional to μ_d, i.e., to the slope of the left branch of the dynamic hysteresis loop.

The static characteristic of an amplifier with $K_{fb} = 1$, obtained in this section, lies mainly in the second quadrant of the system of coordinates. This conforms entirely with the graphic construction of the static characteristic of an amplifier with feedback, described in § 7.6, i.e., in its construction, the closer the value of K_{fb} to unity, the more the curve is shifted to the left, and hence I_l increases when $I_C = 0$.

In concluding this section let us examine the effect of the control-circuit resistance on the amplifier parameters. Hitherto this effect was neglected. We shall now consider another limiting case, when the control-circuit resistance R_C is so high that the control-circuit current does not depend on the emf induced into the control windings, and we simply have $I_C = U_C/R_C = \text{const}$.

In this case the current in the control windings, just as the field produced by them, will not vary (as in the case $R_C = 0$) during each half-cycle from zero to their maximum value, determined on the hysteresis loop by the quantity B_C. To each value of B_C, and hence of I_l, there correspond now values of the field H_C and current I_C that are constant during the entire half-cycle. For this reason the quantity B_C in equation (7.24) for I_l must be replaced directly by the functions $B_C = f(H_C)$ or $B_C = f(I_C)$ specified by the left branch of the hysteresis loop; we must not determine I_C, corresponding to a certain value of B_C, by the mean value of the field strength during a half-cycle, as we did in the case of $R_C = 0$. Thus the first effect of R_C is as follows: When R_C increases, the static characteristic of the amplifier resembles more and more in shape the left branch of the hysteresis loop. The second and more important effect of R_C consists in the circumstance that the gain factors K_I and K_U decrease with increasing R_C.

The decrease in K_I is due to the fact that in its definition by formula (7.25) we must now introduce the quantity $\Delta I_{C \cdot M}$, determined by the total increment (H_s-H_h), and not by the average (over a half-cycle) increment $(H_s-H_h)/2$ of the field strength, as we did in the case $R_C = 0$. Therefore an increase in R_C is accompanied by a decrease in K_I to half its value (in the limit).

The decrease in K_U with increasing R_C is due to the fact that this weakens the positive feedback between the cores, which is effected via the control windings and brings about an increase in the demagnetization of one core as a result of the emf induced in the control winding of the other core.

In the case of a large R_C the amplifier decomposes into two single-core amplifiers that operate independently at a common load. Hence its K_U will simply be twice as large as in the case of one single-core amplifier, i.e., according to formula (7.20) it will be

$$K_U = \frac{W_p}{W_c}\eta. \tag{7.27}$$

§ 7.10. Dynamics of a Magnetic Amplifier with Internal Feedback

At first let us consider a single-core amplifier (Fig. 7.14).

The firing angle α_s depends on the value of B_C, whereas B_C is determined (according to formula (7.15)) by the mean value, over a control half-cycle, of the voltage u_C. Thus the mean value, over a given output half-cycle, of the load voltage is determined by the mean value, over the previous control half-cycle, of the input voltage. If the input voltage undergoes a jumplike change at the beginning of a given control half-cycle, the corresponding variation of the output voltage (current) will not appear at once, but with a time lag equal to a supply half-cycle. When the input voltage varies at the beginning of an output half-cycle, this lag will increase up to a full cycle. Finally, if the input variation occurs in the middle of the control half-cycle, it will have only a partial effect on the induction B_C at the end of this half-cycle, whereas its full effect will manifest itself during the next cycle. In other words, in this case the delay in the output response of the amplifier will amount to 1-1.5 of the supply period, depending on the instant at which U_C changes jumplike.

Thus a single-core amplifier with internal feedback is from a dynamic point of view not an ordinary lag network, but a network with a pure time delay of 0.5-1.5 supply periods, depending on the instant at which the input signal varies with respect to the beginning of the cycle.

The dynamic equation of such an amplifier is

$$\Delta U_l(t) = K_U \Delta U_c(t-\tau), \tag{7.28}$$

where τ = (0.5-1.5)/f is the delay time.

Equation (7.28) signifies that

$$\Delta U_l(t) = \begin{cases} 0 & \text{for} \quad t < \tau; \\ K_U \Delta U_c & \text{for} \quad t > \tau. \end{cases}$$

By applying to (7.28) the direct Laplace transform, we can write this expression in operator form

$$\Delta U_l(p) = K_U e^{-\tau p} \Delta U_c(p). \tag{7.29}$$

Now let us consider a two-core amplifier (Figs. 7.8 and 7.9). If the resistance of the control circuit is high, the cores of the amplifier will not affect each other and therefore such an amplifier is from a dynamic point of view likewise a network with a pure time delay τ, described by equation (7.28) or (7.29). The only difference consists in the fact that K_U is here twice as large, whereas τ does not exceed one period, i.e., τ = (0.5-1)/f (since the cores operate in succession).

Matters are different in an ordinary two-core amplifier with internal feedback and a small resistance of the control circuit. In the case of an instantaneous jump of the input voltage, α_s and U_l do not at once assume (after the completion of a given half-cycle) the corresponding new steady-state values, but will reach these values gradually, during several supply periods. The reason for such a protraction of the transient process is the mutual effect of the cores via the control windings. Indeed, the variation ΔB_1 of the induction during the demagnetization of one core is determined not only by the mean value (over the given half-cycle) of the input voltage U_C, but also by the mean value of the emf e_{C2}, induced into the control winding of the second core which is magnetized during this half-cycle. On the other hand, e_{C2} is determined by the variation ΔB_{p2} of the induction in the second core ($\Delta B_{p2} = \Delta B_{C2}$), i. e., by the variation of the induction in the second core when it is demagnetized during the previous half-cycle. Thus ΔB_{C1} and hence also α_{s1} will depend on the values of ΔB_{C2} and α_{s2}, whereas the latter depend on the values of ΔB_{C1} and α_{s1} occurring in the previous period. In other words, owing to the fact that the cores operate in succession and that they are interrelated, the transition from one steady state of the amplifier to another is delayed by several periods.

Suppose, for example, that in an amplifier which is in a steady state at $U_C = U_{C \cdot s}$, so that $U_l = 0$ ($\alpha_s = \pi$), the input voltage decreases instantaneously to zero at the transition between half-cycles. Let us consider the transient process towards the new steady state with $U_C = 0$ and $U_l = U_{l \cdot M} = U_m \eta$ ($\alpha_s = 0$). During the first half-cycle following the switching off of U_C the core (for which this was an output half-cycle), having been completely demagnetized during the previous half-cycle to the induction $-B_s$, is being magnetized to the induction $+B_s$ at the end of the half-cycle, like during the previous steady-state mode, i. e., its firing angle $\alpha_s = \pi$. During this time the second core should be demagnetized to the level $-B_s$, i. e., get ready for operation during the next half-cycle. If the cores were not interrelated, then at the end of this half-cycle the induction in the second core would remain unchanged and equal to $+B_s$ (since we have $U_C = 0$). As a result, this core would at once go over into the new steady state with $\alpha_s = 0$. But since we are neglecting the resistance of the control circuit, the second core (despite the fact that $U_C = 0$) will nevertheless be demagnetized owing to the emf induced into the W_C winding of the first core when the latter is magnetized. It is true, though, that the variation of the induction during a half-cycle will be smaller than during the previous steady state, i.e., the core is not fully demagnetized (not to $-B_s$). During the next half-cycle the cores change roles. The second core will be magnetized by the output winding. In this case, since we now have $B_C < -B_s$, core saturation will set in before the end of the half-cycle, i.e., $\alpha_s < \pi$. The first core is demagnetized under the action of the emf induced into the W_C of the second core. This emf will be smaller, however, than the emf induced earlier into the W_C of the first core; therefore the demagnetization of the first core during this half-cycle will also be smaller than the demagnetization of the second core during the previous half-cycle. Thus the demagnetization of the cores and hence the firing angle will gradually (from half-cycle to half-cycle) decrease to zero.

Hence in the case of a jumplike variation of U_C the mean value of the load voltage U_l will gradually vary from half-cycle to half-cycle, in accordance with the variation of α_s.

The transient process will be described by the same first-order differential equation (6.27) as in the case of an amplifier without feedback (see § 6.4):

$$T_c \frac{d(\Delta U_l)}{dt} + \Delta U_l = K_U \Delta U_c. \tag{7.30}$$

This equation is written for the voltage increments, since for $U_C = 0$ the absolute value of U_l will depend on the shape of the static amplifier characteristic $U_l = f(U_C)$ and on the dc bias in particular.

The duration of the transient process is determined by the time constant T_C. The constant T_C of an amplifier with internal feedback ($K_{fb} = 1$) can be expressed in terms of K_U by formula (7.7), which does not depend on K_{fb}, and which is therefore valid for any value of this factor, up to unity:

$$T_c = \frac{W_c}{4\eta f W_p} K_U.$$

On the basis of this formula it is possible to obtain an expression for the figure of merit of an amplifier with internal feedback ($K_{fb} = 1$):

$$D = \frac{K_p}{T_c} = \frac{K_U}{T_c} K_I = 4\eta f \frac{W_p}{W_c} K_I. \tag{7.31}$$

Thus, indeed, the figure of merit as well as the current gain of an amplifier with $K_{fb} = 1$ are not infinite, as it follows from formulas (7.3) and (7.6) for an ideal magnetic amplifier by substituting $K_{fb} = 1$ into these formulas, but finite and proportional to the actual value of the core permeability.

Equation (7.30) can be used when the duration of the transient process is by one order of magnitude greater than the supply period, i.e., $T_C \gg T$. If, on the other hand, T_C is commensurable with T, it is necessary to take into account also the pure time lag (existing in an amplifier with $K_{fb} = 1$) between the instant at which α_s begins to vary and the instant at which U_C varies; this lag amounts to (0-0.5)T, depending on the instant at which U_C varies. When U_C undergoes a jump, the value of α_s during the given half-cycle will be still determined by the old value of the input signal, i.e., the variation of α_s begins at the next half-cycle. With allowance for this lag, the dynamic equation of an amplifier with $K_{fb} = 1$ assumes the form

$$T_c \frac{d\Delta U_l(t)}{dt} + \Delta U_l(t) = K_U \Delta U_c(t-\tau), \tag{7.32}$$

or in operator notation

$$(T_c p + 1)\Delta U_l(p) = K_U e^{-\tau p} \Delta U_c(p). \tag{7.32a}$$

Here $\tau = T/4$ is the delay time; τ is taken as equal to a quarter supply period, as the average between 0 and 0.5 T.[2]

The aforesaid is illustrated by Fig. 7.18. The curves 1 and 2 represent the transient characteristics $\Delta U_l(t)$ corresponding to equation (7.30) for two values of T_C ($T_{C2} < T_{C1}$).

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[1] See footnotes p. 243.

The jump in the value of U_C is assumed to take place at the beginning of the period. The stepped lines 3 and 4 represent formally the actual jumplike variation of the mean value of U_l from half-cycle to half-cycle, determined by the jumplike variation of α_s. The dashed curves 5 and 6 are the characteristics ΔU_l (t) with allowance for the lag corresponding to equation (7.32). These curves differ from 1 to 2 solely by a T/4-shift to the right, and describe more exactly the actual step process of variation of ΔU_l (t). When the control circuit resistance R_C increases, the time constant T_C decreases. The importance of the pure lag for the dynamics of the amplifier increases accordingly. This is clearly evident from Fig. 7.18. In the limit, when R_C is very large ($R_C \to \infty$, $T_C \to 0$), the amplifier goes over (as stated above) into a network with a pure time lag (the line 7 in Fig. 7.18).

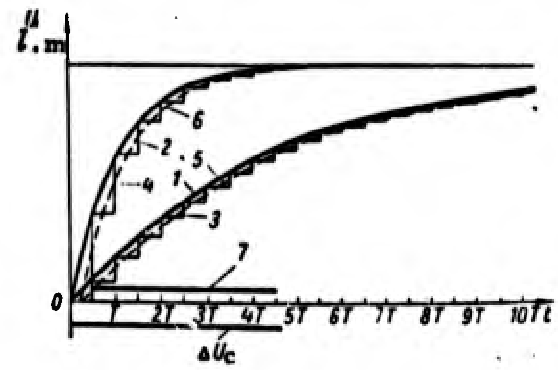

Fig. 7.18. Transient characteristics of a magnetic amplifier with internal feedback.

Let us show that the same conclusion follows also from the formulas derived above. Thus from formula (7.7) we obtain

$$T_c = \frac{W_c}{4\eta f W_p} K_U.$$

By substituting into this relation the expression (7.27) for the K_U of an amplifier with a large R_C, we obtain

$$T_c = \frac{W_c}{4\eta f W_p} \cdot \frac{W_p \eta}{W_c} = \frac{1}{4f} = \frac{T}{4},$$

i.e., the transient process in such an amplifier reduces indeed to a single jump in α_s — the amplifier constitutes a pure-lag network.

In examining above the dynamics of an amplifier with internal feedback, we took into account only the effect of a single control circuit. If the amplifier has several control circuits and other loops that are magnetically coupled to the former, the equations (7.30) and (7.32) must be rewritten like the equation (6.28) for a magnetic amplifier without feedback, replacing T_C with the sum of the time constants of all these circuits. In particular, in the case of the networks depicted in Figs. 7.8 and 7.9a, we must introduce, in the analysis of the transient process brought about by a jumplike variation in U_C and causing a decrease in U_l, into the sum of the time constants the time constant T_p of the circuit formed by the output windings via the rectifiers D_{p1} and D_{p2}. For a jump in U_C of opposite sign this circuit will be open, since the emf's induced into the output windings are acting in this case in the opposite direction.

In concluding this section let us note that the above analysis regarding an amplifier with internal feedback is entirely valid also for an amplifier with external feedback and $K_{fb} = 1$.

§ 7.11. Various Methods of Controlling a Magnetic Amplifier with Internal Feedback

The load current of a magnetic amplifier with internal feedback (Fig. 7.14) can be controlled by demagnetizing the core, i.e., reducing its induction from $+B_s$ to B_C during the control half-cycle, when the output circuit does not operate. In an ordinary amplifier circuit the variation of B_C takes place under the action of the dc voltage applied to the control winding. Yet owing to the division of the amplifier operating cycle into a control and an output half-cycle, there can be in principle also other methods of controlling a magnetic amplifier with internal feedback.

Let us examine these methods, which are illustrated in Fig. 7.19. The first system (Fig. 7.19a) is an ordinary dc amplifier.

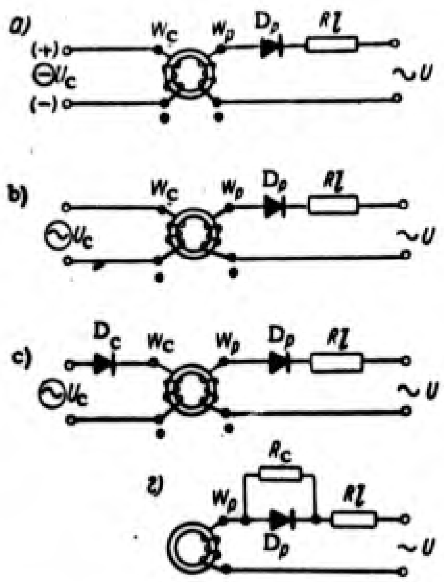

Fig. 7.19. Methods of control of a magnetic amplifier with internal feedback.

A fundamental-frequency ac magnetic amplifier is shown in Fig. 7.19b.

Earlier (in § 5.4) we already noted that it was possible, in principle, to control a magnetic amplifier by an ac signal of the fundamental frequency. Let us now examine how this process takes place in the case of an amplifier with internal feedback (Fig. 7.19b). Let u_C be a voltage whose frequency and phase are the same as those of the supply voltage U of the load circuit. Thereupon a negative voltage half-wave u_C will be applied to W_C during each control half-cycle that demagnetizes the core in the same way as would be done by a dc signal. The fact that the voltage u_C changes its polarity during the output half-cycle has no significance whatsoever, since the control winding does not act on the core during that time.

According to formula (7.15) the variation of the core induction during the control half-cycle will proportional to the mean value of the control voltage $U_{C \cdot m}$:

$$\Delta B_c = \frac{1}{W_c S_c}\int_0^{T/2} u_c\,dt = \frac{1}{W_c S_c}\int_0^{T/2} U_{c.m} \sin\omega t\,dt =$$
$$= \frac{1}{W_c S_c} U_{c.m} \frac{T}{2} = \frac{U_{c.m}}{2W_c S_c f}.$$

Hence the fundamental-frequency ac voltage gain, determined for the mean value of the voltage, will be equal to the dc voltage gain, i. e., by formula (7. 20) we have

$$K_U = \frac{\Delta U_l}{\Delta U_{c.m}} = \frac{W_p \eta}{2W_c}. \tag{7. 33}$$

The control of an amplifier with internal feedback by fundamental-frequency signal can be effected not only by varying the signal amplitude, but also by varying the signal phase relative to the supply voltage of the output circuit, i. e., in the form of phase control, similar to the phase control of thyratrons.

The variation of the core induction during the control half-cycle is equal to

$$\Delta B_c = \frac{1}{W_c S_c} \int_0^{T/2} U_{c.m} \sin(\omega t + \varphi)\, dt,$$

where φ is the phase angle between u_C and u.

As we can see, ΔB_C depends on the control voltage amplitude $U_{C \cdot M}$ and phase φ. Above we referred to control by varying $U_{C \cdot M}$. If, on the other hand, we vary the phase φ at a fixed amplitude, the quantity ΔB_C will vary from its maximum value to zero when φ varies from 0 to $\pi/2$. The load current will vary conversely, i. e., from zero to its maximum value. Such phase control can be effected with the aid of the same phase-shifting devices as those used in thyratron control. Let us note that the phase control of magnetic amplifiers has not come into practical use so far. We merely pointed out the feasibility, in principle, of such a method of control.

A magnetic amplifier of unipolar pulses of fundamental frequency is shown in Fig. 7. 19c.

Since the control winding acts on the core only during the control half-cycle, it follows that the control of the amplifier by means of a fundamental frequency voltage will not be disturbed if we insert a rectifier D_C into the control circuit (as shown in Fig. 7. 19c).

It is evident that a one-core amplifier with internal feedback can also directly amplify unipolar pulses of fundamental frequency that are directly applied to W_C during the control half-cycle.

Since the output signal of such an amplifier is also in the form of unipolar pulses (just as the input signal), it is convenient to connect these amplifiers in series in multistage systems.

A model of such a three-stage amplifier is shown in Fig. 11.11b. Whereas the odd cores are operating in the output half-cycle, this half-cycle represents the control half-cycle for the even cores. Since the characteristic $U_l = f(U\ C.m)$ for each stage has the form of graph 1 in Fig. 7.16, i.e., $U_l = U_{l \cdot M}$ for $U_C = 0$, it follows that the entire multistage amplifier will also have such a characteristic in the case of an odd number of stages, and a characteristic like the one represented by line 4 in Fig. 7.16 — in the case of an even number of stages.

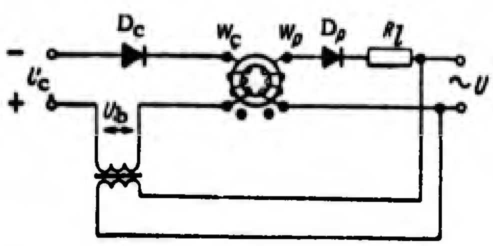

Fig. 7.20. One-core magnetic amplifier with internal feedback and bias.

Figure 7.20 shows another version of the circuit of Fig. 7.19c that has a bias which ensures that $U_l = 0$ when $U_C = 0$ (the static characteristic 4 in Fig. 7.16). The bias is produced not with the aid of a separate bias winding, but by means of the control winding to which the voltage difference $U_b - U_C$, where $U_b = (W_C/W_p)U$, is applied.

The circuit of Fig. 7.20 can be controlled not only by an ac signal or by unipolar pulses, but also by a dc signal. Another interesting feature of the circuit is the fact that the input signal does not produce the current in the control circuit, but only cuts off the rectifier D_C during a portion of the control half-cycle. The direction of the current in the W_C circuit is determined by U_b, i.e., opposite to the polarity of U_C. The amplifiers having the configuration of Fig. 7.20 are also convenient to connect in multistage systems, as shown in Fig. 11.11a.

The utilization of the circuits of Fig. 7.19c and Fig. 7.20 as pulse amplifiers in devices of discrete operation is considered in Chapter 11.

In Fig. 7.19d we show a magnetic amplifier, controlled with the aid of a variable resistor.

This method of control of a magnetic amplifier with internal feedback is very simple. The demagnetization of the core during the control half-cycle is effected in this case by means of the output winding itself. When the variable resistance $R_C = 0$, the core induction varies during the two half-cycles by its maximum value and $I_l = 0$, like in an amplifier without feedback in the absence of a signal. When R_C increases, the core demagnetization during the control half-cycle decreases and the load current increases accordingly. As R_C one can use various controlled resistors, such as electronic amplifier tubes, semiconductor triodes, photoresistors, etc. The input signal changes the magnitude of the resistance R_C, and hence also the load current.

The above methods of control of magnetic amplifiers with internal feedback can be used for the design of two-core circuits, i.e., of circuits with a two half-cycle output. An amplifier, controlled by a variable resistor, can be designed, for example, in the form of the circuit of Fig. 7.10a. The role of the controlled resistor is played here by R_{var}.

Two-core fundamental-frequency ac amplifiers, designed on the basis of two circuits of Fig. 7.19b, differ from the circuits of Figs. 7.8 and 7.9 of an ordinary amplifier with internal feedback only in one respect, namely that the terminals of the control or output windings of one of the cores are interchanged. Indeed, if in a two-core circuit the input signal demagnetizes during one half-cycle the first core, for which this is the control half-cycle, this signal must demagnetize during the next half-cycle the second core. But if the input signal is an alternating current of fundamental frequency, it will change its polarity during this half-cycle, and hence in order that it should correctly operate on the second core it is necessary to interchange the terminals of the control (or output) windings of this core as compared to the circuit for a dc signal.

Yet this causes such a fundamental frequency ac amplifier to have a major shortcoming as compared to an ordinary dc amplifier, namely a very low power gain, not exceeding a few tens. This is due to the fact that as a result of winding reversal the emf, induced into the control winding of the core for which the half-cycle under consideration is an output half-cycle, will no longer be aiding the input signal to demagnetize the second core during its control half-cycle (as in an ordinary amplifier), but act in the opposite direction. For obtaining the same value of B_C it is therefore necessary that the fundamental-frequency signal-voltage U_C be much higher than the dc voltage in an ordinary amplifier. Hence such a fundamental-frequency ac amplifier gives a low voltage gain.

The current gain and the overall static characteristic $I_l = f(I_{C \cdot m})$ of this amplifier are the same as for an ordinary dc amplifier, since the input current $I_{C \cdot m}$ is determined in either case from the dynamic hysteresis loop in terms of the field strength.

The overall power gain $K_P = K_I K_U$ of the fundamental-frequency ac amplifier under consideration is small, owing to the smallness of K_U. In other words, the low power gain of this amplifier is due to its high input resistance to an alternating current, resulting from the counter emf's induced into the control windings by the output circuit. In order to limit these counter emf's one must take the ratio $W_C/W_p \leq 0.1 - 0.01$, like in a one-core amplifier.

The transient process in a two-core fundamental-frequency ac amplifier is completed during one supply cycle, i.e., from a dynamic point of view such an amplifier is a network with a pure time lag, just as all the other amplifiers considered in this section.

Owing to their low gain, fundamental-frequency signal amplifiers are not widely used. They can be of interest in those cases when the required ac signal amplification is fairly low, as in the case of rotating transformers, ac tachogenerators, the previously considered magnetic amplifier, and similar elements and ac generators. The direct amplification of an ac signal with the aid of such an amplifier, supplied together with the signal generator from a common source, may enable us to reduce in such a case the number of rectifiers in the amplifier circuit, since no preliminary signal rectification is needed as in the case of an ordinary dc amplifier.

§ 7.12. High-Speed Magnetic Amplifiers

High-speed magnetic amplifiers are a special group of magnetic amplifiers which represent from a dynamic point of view a pure time-delay network. This delay, equal to 0.5-1.5 of the fundamental-frequency period, is rigidly fixed for the particular circuit and in principle it does not depend on the numerical values of the circuit parameters.

The simplest high-speed magnetic amplifier, on which all the circuits of this group of amplifiers are based, is the one-core magnetic amplifier with internal feedback, represented in all its versions in Fig. 7.19.

An ordinary two-core magnetic amplifier with internal feedback is not of high-speed type, although it can also be converted into a pure lag network by increasing the resistance of the control circuit (diminishing its time constant).

Yet in such an ordinary amplifier the quantity fixed with respect to the supply frequency is not the lag (delay), as in a high-speed amplifier, but the figure of merit $D = K_P/T_C$. The lag can vary in wide intervals as a result of gain variations.

As we showed in the case of a one-core amplifier with internal feedback, high-speed magnetic amplifiers do not differ in their figure of merit from ordinary amplifiers. Their very low lag is achieved at the expense of a corresponding decrease in gain.

Therefore their nomenclature must be regarded as arbitrary. The specific feature of high-speed amplifiers, the one that distinguishes them from ordinary amplifiers, is not their high speed, but their fixed time delay. This peculiar feature has resulted in their wide use in all sorts of control and computation devices of discrete operation that operate with sampled signals, i.e., at fixed instants.

As compared to ordinary amplifiers, high-speed magnetic amplifiers have the same property, i.e., they can amplify both dc and ac signals of the fundamental frequency.

Moreover, in a number of cases high-speed amplifiers may prove to be simpler and more convenient in tuning and service, as compared to ordinary amplifiers [10].

In the present section we are considering high-speed two-core magnetic amplifiers. An ordinary two-core amplifier with internal feedback, although consisting of two individual high-speed amplifiers, is not high-speed, owing to the positive coupling of the cores, produced via the control windings. This coupling increases sharply the gain as compared to a one-core circuit, but the lag also increases correspondingly (the figure of merit remains practically unchanged). Thus in order that a two-core amplifier should be a pure lag-network, as the individual one-core amplifiers of which it consists, it is necessary to eliminate the coupling between the control windings. The high-speed amplifiers considered below are in fact two-core amplifiers with internal feedback in which this coupling has been eliminated by circuit methods.

As we already stated, the same result can be achieved also in an ordinary two-core amplifier by simply increasing the resistance of the control circuit. Hence the gain obtained in high-speed amplifiers will be the same as in an ordinary amplifier with high R_C, i.e., K_I as in an ordinary amplifier, and K_U much smaller, being specified by formula (7.27).

A. High-Speed Magnetic Amplifiers with DC Output

From the point of view of the load circuit these amplifiers do not differ from the ordinary amplifiers with internal feedback and dc output, shown in Figs. 7.9b and 7.9c. For definiteness we shall represent them in the version of Fig. 7.9b.

Figure 7.21 shows two models of the high-speed dc amplifier circuit, first proposed in 1951 by R. A. Ramey. The operating principle of the circuit is illustrated in Fig. 7.21a. Here K is a switch that operates synchronously with the supply voltage.

The coupling between the control windings is eliminated by connecting them one after another to the input voltage U_C only during their control half-cycle. The position

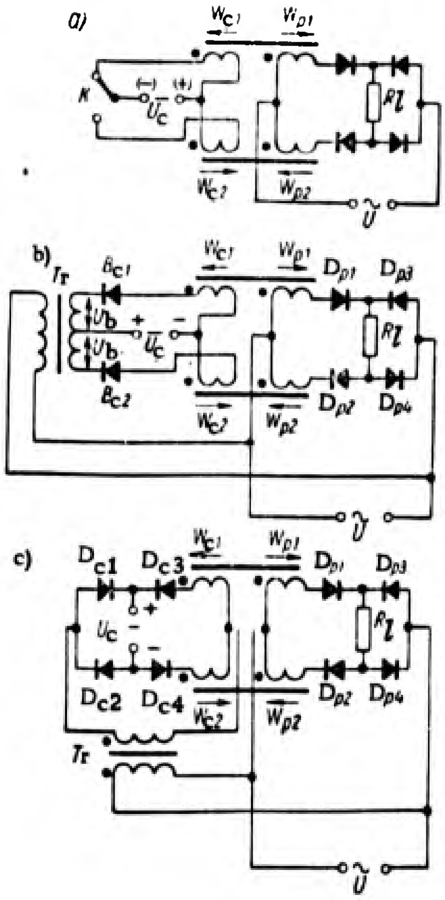

Fig. 7.21. Ramey high-speed magnetic amplifier.

of the switch K in Fig. 7.21a corresponds to a half-cycle which is of control type for the first core and of output type for the second core. During the next half-cycle the switch occupies another position.

In the actual circuit depicted in Fig. 7.21b, the switching in the control circuit is effected with the aid of the rectifiers D_{C1} and D_{C2}, which are (one after another) cut off synchronously with the supply voltage, under the action of the voltage U_b. During one half-cycle the rectifier D_{C2} is cut off in the W_{C1} circuit, but the circuit of the control winding W_{C1} of the first core is closed via the rectifier D_{C1}. During the other half-cycle, we have the converse — D_{C1} is cut off and D_{C2} is conducting. The voltage U_b, which commutates the rectifiers, is at the same time also a bias voltage, i.e., when $U_C = 0$ it causes maximum core demagnetization, ensuring that $I_l = 0$ ($\alpha_s = \pi$). The input signal U_C acts counter to U_b, reducing the core demagnetization. Therefore the characteristic of such an amplifier has the form of the dash-dotted line of Fig. 7.17.

This amplifier is a combination of two circuits of Fig. 7.20 into a single full-wave circuit.

In Fig. 7.21c we show another model of the circuit, with the use of only the bias voltage U_b (the secondary winding of the bias transformer Tr). It is precisely this model of a high-speed magnetic amplifier that was initially proposed by Ramey. Its shortcoming consists in the large number of rectifiers in the control circuit. In it, during one half-cycle the W_{C1} circuit is closed via the rectifiers D_{C2} and D_{C3}, whereas during the other half-cycle the W_{C2} circuit is closed via the rectifiers D_{C1} and D_{C4}.

Ramey's amplifiers can be utilized for the amplification of a signal of the fundamental frequency, as well as for the simultaneous amplification of dc and ac signals of the fundamental frequency. The voltage of the fundamental-frequency signal must be applied either in series with U_b or instead of the latter, in which case the amplifier will have a maximum output signal in the absence of input signals.

In Fig. 7.22 we show the high-speed magnetic amplifier proposed in 1953 by D. Scorgie. Here the control circuits of the cores are decoupled with the aid of negative external feedback. At first let us consider the circuit of Fig. 7.22a, which serves for dc signal amplification. During each supply half-period the control circuit is under the action of the input signal U_C as well as of the feedback voltage u_{fb} and the emf e_C, induced into the control winding of the core for which this half-cycle is of output type. The emf e_C is present from the beginning of the half-cycle and until the core saturates, whereas u_{fb} is present during the remaining time, i.e., from the instant of core saturation to the end of the half-cycle:

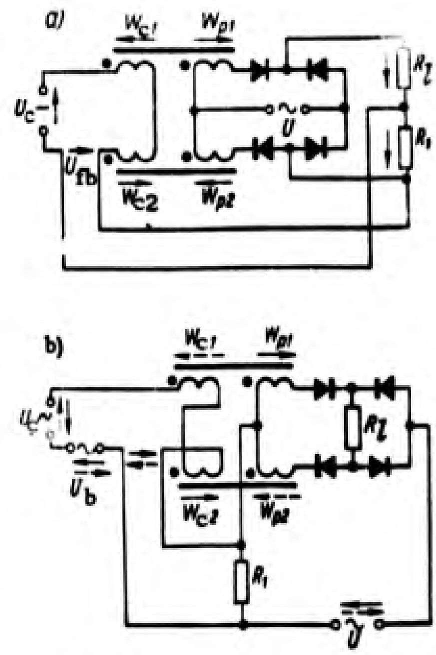

Fig. 7.22. Scorgie's high-speed magnetic amplifier.

$$e_c = \frac{W_c}{W_p} U_m \sin \omega t,$$

where

$$0 < \omega t < \alpha_s,$$

$$u_{fb} = R_1 i_l = R_1 \frac{U_m}{R} \sin \omega t,$$

where

$$\alpha_s < \omega t < \pi.$$

By selecting the resistance R_1 in such a way that

$$\frac{R_1}{R} = \frac{W_C}{W_p}, \tag{7.34}$$

we obtain

$$e_C + u_{fb} = \frac{W_C}{W_p} U_m \sin \omega t \quad \text{for} \quad 0 < \omega t < \pi,$$

i.e., u_{fb} supplements e_C to a complete half-wave of sinusoidal voltage that is proportional to the supply voltage and does not depend on α_s. Thus the voltage $e_C + u_{fb}$, added to the input signal U_C, does not depend on U_C and plays hence the same role as the bias voltage U_b in Ramey's circuit of Fig. 7.21. Consequently, Scorgie's circuit operates in the same way and has the same characteristics as Ramey's circuit. It has however the major advantage of not requiring rectifiers in the control circuit.

Condition (7.34) imposes a restriction on the magnitude of the ratio W_C/W_p. Since $R_1 < R$ (R_1 is included in R), it follows that necessarily $W_C < W_p$, and in order to increase the efficiency of the output circuit by reducing R_1 it is desirable that $W_C \ll W_p$.

In Fig. 7.22b we show a model of the investigated amplifier for an ac signal of fundamental frequency. Here the solid arrows correspond to one supply half-period, whereas the dashed arrows correspond to the other half-period. Since u_C is an ac voltage, the terminals of one of the control windings are reversed, and the feedback is produced with respect to the alternating current in the output circuit. As we already saw in the analysis of an ordinary fundamental-frequency ac amplifier, such a reversal has the result that the induced emf's e_C are no longer aiding the voltage u_C, but acting in opposition to it, i.e., aiding the magnetization of the other core. Hence the sum $e_C + u_{fb}$ will also be a positive voltage, i.e., it cannot serve as a negative bias ensuring that $I_l = 0$ when $U_C = 0$. A separate bias u_b is therefore provided in the circuit of

Fig. 7.22b; it compensates the action of $e_C + u_{fb}$ and therefore ensures that $I_l = 0$ when $U_C = 0$, i.e.,

$$u_b = 2(e_C + u_{fb}) = 2\frac{W_C}{W_p} U_M \sin \omega t. \tag{7.35}$$

The possibility, in principle, of converting an ordinary amplifier into a pure-lag network by using strong negative feedback is evident also in view of the basic fact that negative feedback diminishes the amplifier's time constant just as an increase in the input circuit resistance R_C does. Like an increase in R_C, an increase in negative feedback permits reducing the time constant to fractions of the supply period; if we take into account the discrete character of the transient process in an amplifier with internal feedback (see Fig. 7.18), this permits the entire transient process to be reduced to a single jump of α_s (in Fig. 7.18 the transition from curve 3 to curve 4 and then to curve 7).

Figure 7.23 shows a high-speed magnetic amplifier, proposed in 1954 by H. W. Lord. The high speed is obtained in this model as a result of the fact that the circuit consists of two one-core amplifiers that are not coupled at the input and operate independently at a common load R_l during different half-cycles. Since the circuits of the W_{C1} and W_{C2} windings are not coupled, they do not affect each other, which is the reason for the lag of ordinary amplifiers. The input signal is applied only to the W_{C1} winding of the first amplifier. The second amplifier serves as a repeater, i.e., its half-wave output voltage repeats the output voltage of the first amplifier, but during the other half-cycle. In order to obtain such a mode of operation, the second amplifier has an ac bias U_b of fundamental

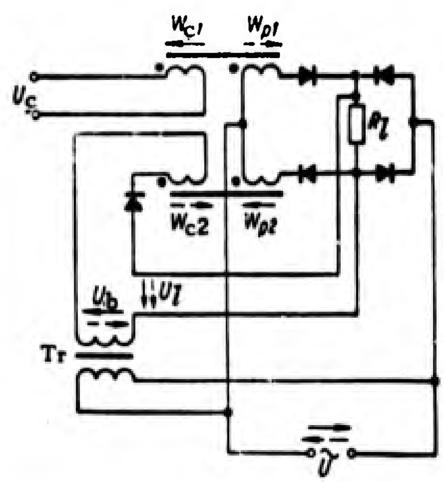

Fig. 7.23. Lord's high-speed magnetic amplifier.

frequency. The second amplifier is controlled from the output of the first amplifier. The voltage half-waves are applied to W_{C2} only during the half-cycle which is an output half-cycle for the first amplifier and a control half-cycle for the second amplifier. The mean value of the voltage across W_{C2} during this half-cycle, which determines the magnitude of the decrease in the induction of the second core (ΔB_{C2}), is

$$U_{c2m} = U_{b.m} - U_l.$$

In the case of a maximum input signal $U_C = U_{C \cdot M}$ in W_{C1}, when $\alpha_{s1} = \pi$ and $U_l = 0$, the mean value U_{C2m} will be maximal and equal to $U_{b.m}$. This will correspond to $\alpha_{s2} = \pi$. Conversely, with $U_C = 0$ the load voltage $U_l = U_{l \cdot M}$ and $U_{C2s} = U_{bs} - U_{l \cdot M} = 0$.

Like a one-core amplifier with internal feedback, the amplifier under consideration can amplify dc, ac and unidirectional signals. It has the merit that it can be used as a basis for simple multistage amplifiers with a full-wave output, in which, however, all the stages except output stage (which is designed in conformity with the circuit of Fig. 7.23) are half-wave stages, designed for example in the form of the circuit of Fig. 7.20.

B. High-Speed Magnetic Amplifiers with AC Output

These amplifiers differ from the above-considered high-speed amplifiers with dc output only in their load circuit. High-speed amplifiers with dc output do not differ with respect to the load circuit from ordinary amplifiers with internal feedback. Matters are more complicated in the case of amplifiers with ac output. The load circuit of an ordina ordinary amplifier with internal feedback in the configuration of Fig. 7.8 cannot be used directly in a high-speed amplifier, in view of the fact that (as we noted already in § 7.10) when U_C undergoes a jump in the direction corresponding to a decrease in U_l, a short-circuited loop is formed in the output circuit by the output windings W_{p1} and W_{p2} and the rectifiers D_{p1} and D_{p2}, this being yet another reason for the lag of such an amplifier.

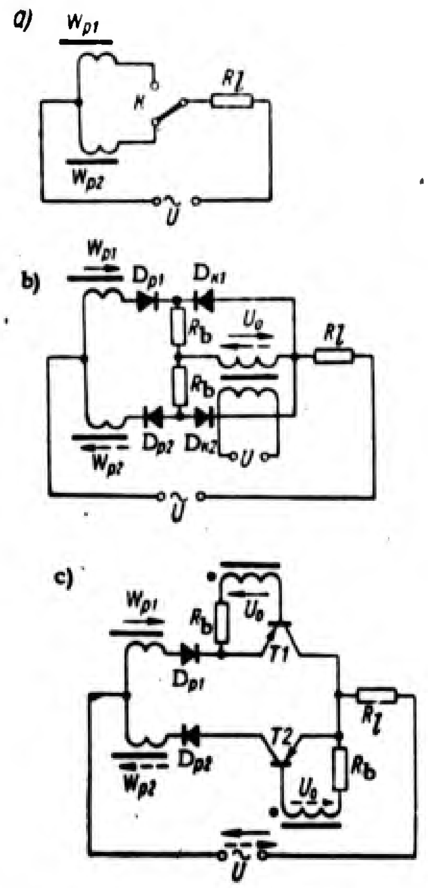

Fig. 7.24. Diagrams of output circuit of high-speed magnetic amplifiers with ac output.

In order to eliminate this short-circuited loop one introduces controlled rectifiers into the output circuit of high-speed amplifiers with ac output. They connect one after another the load to the supply voltage via the output winding of the core for which the particular half-cycle is an output half-cycle. The operating principle of such a circuit is similar to that of the control circuit in Ramey's high-speed amplifier and the like.

Figure 7.24 shows the output circuit of a high-speed magnetic amplifier with ac output. The operating principle of these circuits is illustrated in Fig. 7.24a, where K is a switch that operates synchronously with the supply frequency. In Fig. 7.24b the commutation of the output circuit is effected with the aid of the rectifiers D_{k1} and D_{k2}, that are cut off in turn by the voltage $U_0 \geq U$. For the proper operation of the circuit it is necessary that the current (produced by U_0) through D_k should not be smaller than the maximum value of the output current. Therefore the power dissipated in the ballast resistors is equal to the maximum load power of the amplifier. Although this power is not taken from the amplifier, but from the line, the overall efficiency of the circuit is below 50% in this way. In some cases, though, one can take as R_b some useful load. Figure 7.24c shows the model of a circuit in which the transistors T_1 and T_2 are used as the commutating elements.

In this case the losses in the ballast resistors R_b are negligible.

§ 7.13. Methods of Increasing the Stability and Linearity of the Characteristics of Magnetic Amplifiers with Feedback

A. Effect of Nonideal Rectifiers on the Characteristic of an Amplifier with Feedback

Figure 7.25 shows the actual characteristic of a semiconductor rectifier. Its reverse resistance is not infinitely large. Therefore the rectified current I_l in the rectifier circuit will not be equal to the mean value I_m of the rectified alternating current, but smaller than the latter by the value of the reverse current, i.e.,

$$I_l = K_D I_m,$$

where K_D is the rectification factor:

$$K_D = \frac{I_l}{I_m} = \frac{I_{for.m} - I_{rev.m}}{I_{for.m} + I_{rev.m}} < 1. \qquad (7.36)$$

Here $I_{for.m}$ is the mean value of the forward current through the rectifier, and $I_{rev.m}$ is the mean value of the reverse current through the rectifier.

The magnitude of the rectification factor K_D depends on the quality of the rectifier. For an ideal rectifier we have $K_D = 1$.

The nonideal character of the rectifier leads to a reduction in the feedback factor, since it diminishes the feedback current as compared to I_m in the output circuit. This effect will be the stronger, the larger the value of K_{fb}. When $K_{fb} \geqslant 0.8$, it can be no longer neglected. With allowance for the nonideal character of the amplifier, we have

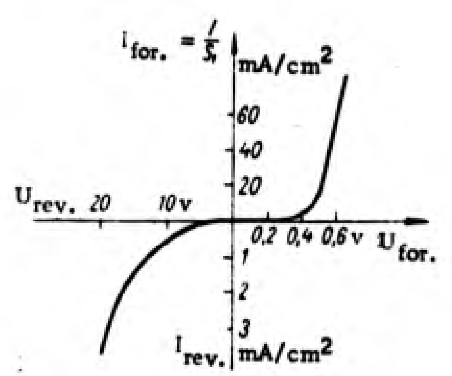

Fig. 7.25. Current-voltage curve of a selenium rectifier. The coordinate axes are in different scales.

$$K_{fb} = \frac{W_{fb}}{W_p} K_{var} K_D. \qquad (7.37) \qquad (7.37)$$

This is precisely the formula to be used for specifying the feedback factor K_{fb} in the case of strong feedback. For the calculation of K_D it is more convenient to express the latter in terms of the resistances, and not of the currents as in formula (7.36). In this case,

$$K_D = \frac{R_{rev} - R_{for}}{R_{rev} + R_{for} + 2R_{l.D}}, \tag{7.38}$$

where R_{for} and R_{rev} are the forward and reverse resistances of the rectifier in this mode of operation, and $R_{l \cdot D}$ is the load resistance of the rectifier.

In Appendix AP-8 we listed the values of R_{for} and R_{rev} for various rectifiers.

Formula (7.38) holds for an individual rectifier, as well as for a bridge rectifier circuit. According to this formula, we have for an amplifier with external feedback and dc output (Fig. 7.2a) the expression

$$K_D = \frac{R_{rev} - R_{for}}{R_{rev} + R_{for} + 2(R_l + R_{fb})}. \tag{7.39}$$

If the load is ac coupled or connected via a separate rectifier, the quantity R_l in the denominator of (7.39) vanishes, and hence the value of K_D increases. For an amplifier with internal feedback in the configuration of Fig. 7.8 or 7.9a we have

$$K_D = \frac{R_{rev} - R_{for}}{R_{rev} + R_{for} + 2R_p}, \tag{7.40}$$

whereas in the configuration of Fig. 7.9b we have

$$K_D = \frac{R_{rev} - R_{for} - Rl}{R_{rev} + R_{for} + 2R_p}. \tag{7.41}$$

Here R_l is added to R_{for}.

The decrease in the feedback factor, due to the nonideal character of the rectifiers, can be compensated by increasing W_{fb}. But this is accompanied by a decrease in the efficiency η, and hence also of the figure of merit of the amplifier.

The most important effect of nonideal rectifiers consists in the deterioration of the linearity and stability of the static characteristic of amplifiers. This is accompanied by a decrease in the maximum permissible value of K_{fb}, i.e., likewise to a decrease in the figure of merit of the amplifier.

The decrease in the linearity and stability of the static amplifier-characteristic is due to the nonlinearity and instability of the rectifier characteristic. This nonlinearity involves the dependence of K_D on the current passing through the rectifier (Fig. 7.25), whereas the instability consists in the fact that K_D is not constant in time. The instability of the rectifier parameters is mainly caused by their strong temperature dependence, and in the case of selenium rectifiers it is also due to aging. On the graph of the static characteristic of a magnetic amplifier with feedback (§ 7.6) this manifests itself in the fact that the actual feedback characteristic $H_{fb} = f(H_m)$ is nonlinear and unstable. This, of course, is also reflected by the amplifier characteristics obtained.

How can one combat the harmful effect of the nonideal character of the rectifiers contained in the feedback loop?

First of all, for high-performance amplifiers with strong positive feedback one must use also high-performance rectifiers with a high feedback resistance (silicon and germanium rectifiers). In such rectifiers the R_{rev}/R_{for} ratio attains 10^5-10^6. An effective means for increasing the linearity and stability of the static characteristic of an amplifier with feedback is the use of separate rectifiers for the load and the feedback. In the case of internal feedback, for example, this amounts to going over from the circuit of Fig. 7.9b to that of Fig. 7.9a. Thus one increases, in the first place, the value of K_D, since in the formulas (7.39) and (7.41) the quantity R_l vanishes; secondly, it becomes possible to use elements of higher quality in the feedback rectifiers as compared to those used in the load rectifiers, these elements being rated for operation at a reverse voltage that is at a lower (roughly half) value than the nominal voltage. As can be seen from Fig. 7.25, a reduction in the reverse voltage permits a strong increase in R_{rev} (i.e., a reduction in the reverse current). The use of a separate rectifier for the feedback is particularly effective in the case of an inductive or variable load.

Finally, the performance of the rectifiers, their ideal character and stability can be considerably increased by using instead of semiconductor diodes, semiconductor triodes (transistors). The transistor operates in this case as a synchronous gate, controlled by the supply voltage of the amplifier. As an example, we present in Fig. 7.26a a model of a nonreversible amplifier with internal feedback that is similar to the circuit of Fig. 7.9c, though it uses the transistors T1 and T2 instead of the diodes D1 and D2. The transistors, in a common collector configuration (emitter follower), are blocked one after another during one supply half-period, and completely unblocked during the other half-cycle by means of the voltage applied to their collectors from the separate windings of the supply transformer Tr.

Curve 1 in Fig. 7.26b represents the initial section of the I-V characteristic of the transistor operating as a controlled rectifier. For comparison we plotted by curve 2 the typical characteristic of a semiconductor diode. We can see that the transistor characteristic approaches the characteristic of an ideal rectifier.

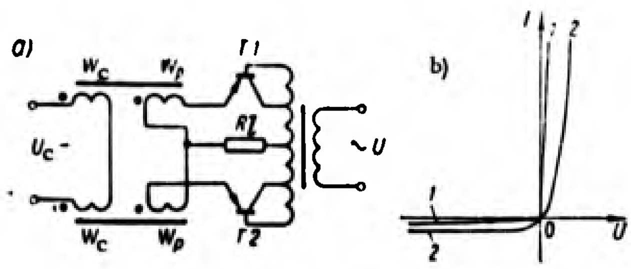

Fig. 7.26. Magnetic amplifier with internal feedback and transistors serving as rectifiers.

The use of transistors instead of diodes permits the obtaining of good rectification (high K_D) down to very low voltages, when diodes can be no longer used in practice. Therefore the use of transistors instead of diodes is particularly effective in circuits with a low supply voltage, of the order of a few volts.

B. Effect of DC Load Inductance on the Static Characteristic of an Amplifier with Feedback

In the case of an inductive load at the rectifier output, the rectified current I_l will exceed the mean value I_m of the alternating current as a result of the self-induction emf of the load, and (what is most undesirable) the dependence of I_l on I_m is strongly nonlinear. Therefore the static characteristic of an amplifier with an inductive dc load and without feedback will be nonlinear (§ 6. 3 C). In the presence of positive feedback this nonlinearity is greatly increased.

The characteristic will be especially distorted if the feedback is produced not with the aid of a separate rectifier, but directly by means of the load current I_l, since in this case the feedback characteristic itself ($I_{fb} = f(I_m)$) will also be strongly nonlinear. Owing to the fact that $I_l > I_m$, the amplifier can operate in the relay mode with $K_{fb} < 1$.

In this case the best means for improving the amplifier characteristic (apart from using a separate rectifier for the feedback) is to insert a large capacitance in parallel with the load, so that the load becomes capacitive.

This not only linearizes the static characteristic, but it also leads to a considerable increase (by two to four times) in the maximum output current of the amplifier.

C. Methods of Reducing the Effect of Temperature, Voltage and Supply-Frequency Fluctuations on the Amplifier Parameters

The factor most strongly affecting the characteristic of a magnetic amplifier are temperature variations. These are mainly due to the temperature instability of the rectifiers.

In principle there exist two methods of eliminating (or at least reducing) the effect of external factors, such as temperature and supply voltage and frequency, on the amplifier parameters: compensation and negative feedback.

Compensation of the effect of external factors consists in the following. A special compensation winding is inserted in the amplifier that produces a magnetization proportional to the variation of the particular external factor (for example, the temperature) whose effect we desire to eliminate. The shift in the static amplifier-characteristic, due to any variation of this factor, is compensated by an equal and opposite shift under the action of the compensation winding. Thus, in order to compensate the effect of temperature variations, one connects in series with the compensation winding (which, like an ordinary bias winding, is supplied by a dc voltage) a resistor that has a temperature coefficient which ensures the necessary current variation in this coil as a function of the temperature variation. The effect of voltage or frequency fluctuations can be compensated by introducing reactances into the compensation-winding circuit that depend on the magnitude of the voltage or frequency. Often it is possible to forgo the use of a special compensation winding, whose role is played in this case by an ordinary bias or feedback winding in whose circuit one inserts a corresponding variable resistor that depends on the external factor to be compensated.

The most universal method for increasing the stability and linearity of the amplifier characteristic, irrespective of the reasons causing its distortion and shift, is negative feedback, described in the next section.

§ 7.14. Negative Feedback and other Feedback Types in Magnetic Amplifiers

Negative feedback is used in magnetic amplifiers for increasing the stability and linearity of the characteristics, for reducing the output noise level, for changing (most often — increasing) the input resistance, and, finally, for diminishing the lag. As a rule, the negative feedback is electrical. The shortcomings of magnetic negative feedback will be examined below.

In Fig. 7.27 we illustrate various possibilities of producing electric negative feedback in an amplifier with dc output. Series load-voltage and load-current feedback is illustrated in Figs. 7.27a and 7.27b, whereas parallel feedback is illustrated in Figs. 7.27c and 7.27d.

In the case of series feedback the input signal and the feedback signal are summed in the form of voltages, so that the voltage applied to the control winding is $U_C = U_{in} - U_{fb}$ (minus in the case of negative feedback). In the case of parallel feedback the currents are summed at the input, i. e., the control-winding current $I_C = I_{in} - I_{fb}$.

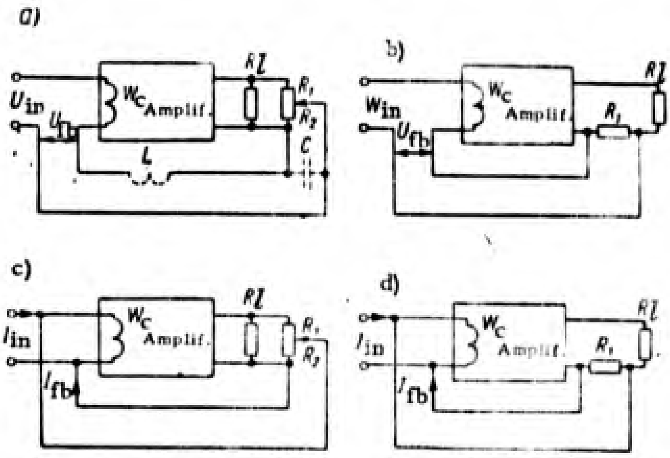

Fig. 7.27. Types of electric feedback in an amplifier: series voltage feedback (a), series current feedback (b); parallel voltage feedback (c), parallel current feedback (d).

A. Series Electric Negative Feedback

This feedback (Figs. 7 27a and 7 27b) leads to a decrease in K_U, K_P and T_C, and to an increase in the input resistance R_{in} by a factor of $(1+K_{U0}K_{fbU})$. Here K_{U0} is the voltage gain in the absence of feedback, and $K_{fbU} = U_{fb}/U_l$ is the series feedback factor. For the circuit of Fig. 7.27a we have $K_{fbU} = R_1/(R_1 + R_2)$, whereas for the circuit of Fig. 7.27b we have $K_{fbU} = R_1/R_l$.

Indeed, $U_l = K_{U0}U_C$, where in the presence of feedback we have

$$U_c = U_{in} - U_{fb};$$
$$U_{fb} = K_{fb}U_l.$$

Whence,

$$U_l = \frac{K_{U0}}{1 + K_{U0}K_{fb\,U}} U_{in},$$

i.e., the voltage gain in the presence of feedback is

$$K_U = \frac{U_l}{U_{in}} = \frac{K_{U0}}{1 + K_{U0}K_{fb\,U}}. \qquad (7.42)$$

The introduction of series feedback left, of course, the current gain unchanged, i.e., in order to obtain a particular load current we require the previous value of the current in the control circuit W_C, determined by the static characteristic $I_l = f(I_C)$ of an amplifier without feedback. On the other hand, for obtaining such a current in W_C it is now necessary to apply to the input a signal voltage that is higher by a factor of $(1+K_{U0}K_{fbU})$. This means that the amplifier input resistance $R_{in} = U_{in}/I_{in}$ (where $I_{in} = I_C$) has increased by a factor of $(1+K_{U0}K_{fbU})$. This is accompanied by a proportional decrease in the input-circuit time constant $T_{in} = L_C/R_{in}$ of the amplifier.

The power gain

$$K_P = K_U K_I = \frac{K_{U0}K_I}{1+K_{U0}K_{fb\,U}} = \frac{K_{P0}}{1+K_{U0}K_{fb\,U}},$$

where $K_{P0} = K_{U0}K_{I0}$ is the power gain of an amplifier without feedback.

Since the power gain and the time constant decreased by the same amount, the figure of merit of the amplifier does not change as a result of the introduction of feedback.

Series negative feedback is used for increasing the input resistance and stability and linearity of the static characteristic of an amplifier, as well as its gain. The increase in the stability of the input-output characteristic by means of feedback is based on the same principle as the action of an automatic-control system operated by the deviation of the controlled bariable from the prescribed value. In our case the "controlled" variable is the load voltage (for the circuit of Fig. 7.27a) or the load current (for the circuit of Fig. 7.27b), whereas the "driving" variable is the input-signal voltage U_{in}. Thus negative feedback stabilizes the static dependence between the input and output variables of the amplifier that are compared in the control circuit. In the case of series feedback the input variable to be compared is the input voltage, whereas the output variable is the load voltage or load current, depending on the type of feedback (voltage or current). Hence in the circuit of Fig. 7.27a the stabilized characteristic is $U_l = f(U_{in})$, whereas in the circuit of Fig. 7.27b it is $I_l = f(U_{in})$.

If the load resistance is constant ($U_l/I_l = R_l$ = const), then there is evidently no difference between these circuits from the point of view of stability and linearity of the characteristics. Yet it is easy to see that in the case of a nonconstant or nonlinear load the feedback (for example, voltage feedback), while improving the characteristic $U_l = f(U_{in})$, leads to an increase in the instability and nonlinearity of the characteristic I_l , $f(U_{in})$ and conversely. This must be taken into account when the type of feedback is selected.

Since negative feedback reduces the amplifier gain, it is normally used in conjunction with positive magnetic feedback. At first one makes the positive feedback maximally strong ($K_{fb} \approx 1$), so that the amplifier operates at the boundary of the relay mode, i. e., with $K_I \approx \infty$ and $K_U \approx \infty$, and then the voltage gain is diminished by the application of negative feedback. In this case, if $K_{fbU} = 1$ (one-hundred percent feedback), then formula (7.42) yields

$$K_U = \frac{K_{U0}}{1 + K_{U0} K_{fb\,U}} = \frac{1}{\frac{1}{K_{U0}} + K_{fb\,U}} \approx 1,$$

assuming that $K_{U0} \gg 1$.

This means that the feedback voltage U_{fb} balances completely the input voltage U_{in}($U_l = K_U U_{in} \approx U_{in}$, and $U_{fb} = K_{fbU} U_l = U_l = U_{in}$). Therefore such amplifiers are also called compensation-type amplifiers. In this case the input signal generates practically no current in the control circuit ($K_I \approx \infty$), i. e., the input resistance of the amplifier is very high. As a result, such an amplifier with one-hundred percent positive ($K_{fb} \approx 1$) and series negative ($K_{fbU} = 1$) feedbacks constitutes an "ideal" voltage amplifier, similar to an electronic amplifier, that requires practically no current, and hence also no power from the signal source. Actual amplifiers of this type can have an input resistance of several hundred kΩ, i. e., by several thousand times higher than the actual resistance of the control winding. The stability of the amplifier characteristic is increased in the same proportion. The nonlinearity of the characteristics does not exceed 0.1% in this case. The time constant is also minimal for the particular supply frequency, i. e., it does not exceed a supply period.

From a dynamic point of view, such an amplifier is a pure delay network, like an amplifier with a high resistance R_C, i.e., it is a high-speed amplifier (see the amplifier depicted in Fig. 7.22).

B. Parallel Electric Negative Feedback

In contrast to series feedback, parallel feedback (Figs. 7.27c and d) does not alter the voltage gain (the input voltage remains equal to the control-winding voltage), but diminishes the current gain by a factor of $(1+K_{I0}K_{fbI})$. Here K_{I0} is the current gain in the absence of feedback, and K_{fbI} is the parallel feedback factor.

Thus,

$$I_l = K_{I0} I_c,$$

where

$$I_c = I_{in} - I_{fb};$$
$$I_{fb} = K_{fbI} I_l.$$

Hence

$$I_l = \frac{K_{I0}}{1+K_{I0}K_{fbI}} I_c,$$

or

$$K_I = \frac{I_l}{I_c} = \frac{K_{I0}}{1+K_{I0}K_{fbI}}. \tag{7.43}$$

A $(1+K_{I0}K_{fbI})$-fold increase in the input current signifies a proportional decrease in the input resistance.

If one-hundred percent positive magnetic ($K_{fb} \approx 1$) and parallel negative ($K_{fbI} = 1$) feedback is used, we obtain an ideal current amplifier, with an input resistance close to zero (since the input voltage is zero at $K_{U0} \approx \infty$). This means that the amplifier does not need power from the input current source, and that its insertion into the circuit of this current has no effect whatsoever on the mode of operation of this circuit.

Parallel negative feedback, just as series feedback, increases the stability and linearity of the static amplifier characteristic and diminishes the time constant without changing the figure of merit. Since in the case of parallel feedback the input variable to be compared is the input current, such a type of feedback stabilizes the dependence of the output signal on the input current, and not on the input voltage, as in the case of series feedback. Hence the characteristic stabilized in the circuit of Fig. 7.27c is the static characteristic $U_l = f(I_{in})$, whereas the characteristic stabilized in Fig. 7.27d is $I_l = f(I_{in})$.

The compensation-type voltage and current magnetic amplifiers considered above are utilized in measuring devices for the amplification of signals generated by various sources, as operational amplifiers in computing devices, and as low-lag high-stability amplifiers in automatic controllers and servosystems.

Negative feedback is widely used in multistage magnetic amplifiers in the form of overall coupling of the amplifier. In contrast to a one-stage amplifier, such an overall negative feedback in a multi-stage amplifier not only diminishes the time constant, but increases also the figure of merit of the amplifier (see [23], chap. 10).

C. Magnetic Negative Feedback

Magnetic feedback (Fig. 7.2) has been already considered in detail in the present chapter.

Just as electric feedback, negative magnetic feedback can be used for increasing the stability and linearity of the amplifier characteristic. It has no effect, however, on the input resistance (the feedback signal is not applied to the control circuit). Magnetic feedback differs also from electric feedback by the fact that according to formula (7.6) it changes the figure of merit of the amplifier by a factor of $(1 \mp K_{fb})^{-1}$, where the minus sign corresponds to positive feedback, and the plus-sign to negative feedback, i.e., negative magnetic feedback diminishes the figure of merit.

For this reason negative feedback is normally applied in the form of electric feedback. Negative magnetic feedback is used only if it is undesirable that the amplifier output be resistance coupled to its input, and in the case of summing amplifiers, which contain several control windings. By using negative magnetic feedback in such an amplifier it is possible to effect dc signal summation with an error of less than 0.01%. Such amplifiers are utilized as operational amplifiers in computing devices of continuous service.

D. Dynamic Feedback

The lag of magnetic amplifiers can be reduced not only by means of lagless (electrical) rigid feedback, but also by means of negative rigid lag (delay) feedback and positive flexible feedback. By using such feedbacks it is possible to reduce the amplifier lag by five- to tenfold. These feedbacks operate according to the same principles as the corresponding compensating feedbacks in automatic control systems.

The lag of negative rigid feedback speeds up the transient process of variation of the amplifier output signal when the input signal varies, this being due to the fact that the delay of the negative feedback signal is equivalent to an increase in the input signal at the beginning of the transient process, i.e., it produces forcing.

The lag of the feedback is achieved by inserting an inductance (RL lag network) in series in the feedback circuit, or by shunting this circuit by a capacitor (RC lag network), as shown by the dashed line in Fig. 7.27a. Flexible feedback is produced with the aid of a differentiating RC network or of a differentiating transformer, which are connected in series in the feedback loop. Positive flexible feedback involves the supplementing of the input signal by a signal which is proportional to the rate of change of the amplifier output signal, so that this rate of change is increased.

Whereas rigid electric lagless feedback, while reducing the amplifier's time constant, does not affect its figure of merit, since it involves also a proportional decrease in K_p, on the other hand the introduction of lag into such a feedback loop or the use of positive flexible feedback diminishes the time constant without affecting K_p, i.e., it increases the figure of merit of the amplifier.

In multistage amplifiers the transient process is often oscillating too strongly; in this case one uses negative flexible feedback to dampen these oscillations. As a result, the figure of merit of the amplifier is diminished, but in exchange it is possible to obtain a transient process of the type desired.

§ 7.15. Review of Methods Used for Reducing the Lag of Magnetic Amplifiers

The basic shortcoming of magnetic amplifiers, that must be dealt with when these amplifiers are used in automatic systems, is lag. Below we list in systematic order the methods used for reducing the lag of magnetic amplifiers, most of which were already examined by us above.

A. Increasing the Supply Frequency

This is the most powerful method, whereby the figure of merit of the amplifier can be increased without limits. All the other methods of reducing the lag have a limit, which is determined by precisely the supply frequency. The figure of merit increases as the supply frequency, whereas the absolute value of the amplifier delay is bounded from below by the magnitude of the supply period. Yet to increase the supply frequency is also the most complicated method. The main difficulty in this respect consists in the need to possess a special supply source of a frequency higher than the line frequency. For high-power amplifiers or for groups consisting of a large number of amplifiers one uses as such a source electromechanical supply units (motor-generators) and various types of frequency multipliers, the most reliable and convenient of which are static magnetic frequency multipliers (§ 13.2). Low-power magnetic amplifiers are supplied by transistor or tube frequency generators.

B. The Use of Positive Magnetic Feedback

The maximum attainable figure of merit (assuming the supply frequency to be fixed) is determined in this case by the quality of the rectifiers and cores (see §§7.9 and 7.13A).

C. The Use of Dynamic Feedback and of Additional Signals at the Amplifier Input, Affecting the Rate of Change of the Input Signal

In this section we examine the methods, normally used in automatic systems for improving their dynamic performance. Among the types of feedback, we considered only dynamic feedbacks which permit the figure of merit of amplifiers to be increased — rigid negative lag-type feedback and flexible positive feedback (see § 7.14).

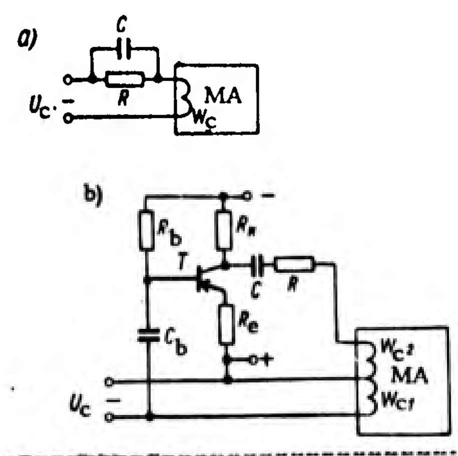

Fig. 7.28. Connecting a differentiating RC network at the input of a magnetic amplifier for the purpose of reducing its lag.

By applying an additional signal which is related to the rate of change of the input signal it is possible to force the transient process, without affecting the steady-state gain, i.e., it has the same result as the use of positive flexible feedback. This can be achieved with the aid of the same differentiating networks (a differentiating RC network or a differentiating transformer) as are used in a flexible feedback loop, though they are connected at the amplifier input in parallel with the channel over which the input signal itself is fed to the amplifier. As an example we show in Fig. 7.28 the circuit diagram of a differentiating RC network. Figure 7.28a depicts the use of the simplest passive RC network, whose shortcoming is a considerable loss in input signal, and hence also in gain, due to the resistance R.

The circuit of Fig. 7.28b uses a transistor amplifier in a differentiating network that permits complete compensation of the time constant T_C of the control circuit without reducing the amplifier gain. Here C is a differentiating capacitor, and C_b is a blocking capacitor that prevents, on the one hand, the steady-state input signal from reaching the transistor input, and on the other hand the transistor supply voltage from reaching the control winding. W_{C1}. The W_{C1} and W_{C2} windings may also coincide. Owing to the fact that in the steady state the transistor amplifier does not affect the magnetic amplifier, being separated from the latter by the capacitors C_b and C, the instability of the transistor parameters and of its supply voltage do not reduce the stability of the static characteristic of the magnetic amplifier as a whole.

Such a transistor amplifier can be also inserted in the flexible feedback loop in order to increase the transfer function of the latter.

D. The Use of Multistage Systems

In the case of series (cascade) connection of magnetic amplifiers, the figure of merit of the multistage amplifier obtained in this way will exceed by many times the figure of merit of a single-stage amplifier. This is due to the fact that the power gains of the individual stages are multiplied, whereas the lags are added, so that their ratio, which specifies the figure of merit, will increase together with the number of stages. Thus by going over from a single-stage amplifier to a two-stage amplifier the figure of merit increases by ten- and hundredfold.

This will be examined in more detail in Chapter 9 below, devoted to multistage amplifiers. Let us only note here that one can use also mixed amplifiers, in which the magnetic amplifier operates in series (before or after) with another type of amplifier, such as a transistor or electron-tube amplifier. In particular, if the amplifier of other type can be regarded as lagless in comparison to the magnetic amplifier, then the overall figure of merit of the mixed amplifier will exceed the figure of merit of the magnetic amplifier by the power gain of the other amplifier.

Footnotes

p. 193. [1] In the foreign literature this amplifier is also known as an <u>amplistat</u> (USA) or <u>magnestat</u> (Great Britian).

p. 212. [2] One can show that irrespective of the actual lag of the particular half-cycle relative to the instant of variation of U_C, the curve representing the variation of the mean value of the load voltage described by equation (7.30) always lags the instant variation of U_C by a quarter of the supply period. This is due to the fact that if the actual lag of the beginning of a half-cycle relative to the instant at which U_C jumps becomes smaller than T/2, then the jump in α_s during the next half-cycle will decreases accordingly, whereas if this lags becomes larger, the jump in α_s will also be larger.