

AD

AD691109

REPORT NO. RE-TR-69-15

**THE WEIBULL DISTRIBUTION
APPLIED TO THE GROUND CLUTTER
BACKSCATTER COEFFICIENT**

by

Robert R. Boothe

June 1969

D D C
RECEIVED
AUG 5 1969
REGITTEL
B

Distribution of this document is unlimited.



U.S. ARMY MISSILE COMMAND

Redstone Arsenal, Alabama

Reproduced by the
CLEARINGHOUSE
for Federal Scientific & Technical
Information Springfield Va. 22151

This document has been approved
for public release and sale; its
distribution is unlimited

12 June 1969

Report No. RE-TR-69-15

**THE WEIBULL DISTRIBUTION
APPLIED TO THE GROUND CLUTTER
BACKSCATTER COEFFICIENT**

by

Robert R. Boothe

DA Project No. 1M262301D209
AMC Management Structure Code No. 5221.12.178

Distribution of this document is unlimited.

Action Group II
Research and Engineering Directorate (Provisional)
U. S. Army Missile Command
Redstone Arsenal, Alabama 35809

ABSTRACT

Spatial distributions of the ground clutter backscatter coefficient σ^0 for various types of terrain have been found to fit a Weibull distribution quite well. This report discusses the Weibull probability distribution function and its characteristics, and presents some clutter measurements that demonstrate the Weibull distribution. The data presented include measurements taken at L-, S-, and X-band frequencies, at several depression angles, and for various resolution cell sizes. A method for generation of random variables that fit a selected Weibull statistical population for use in radar simulations is also given.

CONTENTS

	Page
1. Introduction	1
2. The Weibull Distribution Applied to σ^0	1
3. Generation of Weibull Distributions in Simulations	4
4. Spatial Backscatter Distributions of Measured Land Clutter	5
REFERENCES	13
Appendix A. DETERMINATION OF THE VARIANCE OF THE WEIBULL FUNCTION	15
Appendix B. DETERMINATION OF SLOPE PARAMETER "a"	17

ILLUSTRATIONS

Table		Page
I	Weibull Parameters for Clutter Distributions	7

Figure		
1	Ground Clutter Spatial Distribution for Rocky Mountains at S-Band	8
2	Ground Clutter Spatial Distribution for Wooded Hills and Grassland near Huntsville, Alabama, at L-Band	9
3	Ground Clutter Spatial Distributions for Forest at Different Times of Year at X-Band	10
4	Ground Clutter Spatial Distributions for Cultivated Land in April at Several Depression Angles	11
5	Ground Clutter Spatial Distributions for Cultivated Land in May at Several Depression Angles	12
B-1	Example Slopes for Weibull Distributions Having Different Slope Parameters "a"	19

SYMBOLS

a	slope parameter (equal to $1/b$)
b, α	Weibull distribution parameters
P	probability that $\sigma_c^0 \leq \sigma^0$
r_i	the i^{th} random number of a uniform distribution
σ^0	clutter backscatter coefficient
$\overline{\sigma^0}$	mean value of σ^0
σ_c^0	σ^0 for any particular clutter cell
σ_m^0	median value of σ^0
σ_i^0	the i^{th} value of σ^0 from a Weibull population
σ^2	variance of the Weibull function

1. Introduction

Procedures for estimating the performance of a radar in a clutter environment vary widely. Probably the most common approach is to treat ground clutter as though it were homogeneous and characterized by a constant backscatter coefficient, σ^0 . Depending on the particular value chosen for σ^0 , the detection range can vary widely since the signal-to-clutter ratio is generally considered to vary inversely with range to the first power.

Other theoretical models utilize a statistical clutter model to describe the spatial distribution of land clutter. The most common probability distributions used to model the spatial distribution of σ^0 are the log-normal and exponential (or Rayleigh amplitude) probability density functions. At low depression angles, the land clutter density generally varies over a wider dynamic range than would be predicted by the Rayleigh amplitude distribution; and for this reason, the log-normal distribution is more widely used for low depression angles to model land clutter spatially.

Airborne land clutter measurements made by the Naval Research Laboratory [1] were found to fit approximately the Rayleigh amplitude distribution in a number of cases. These measurements, however, were made at depression angles on the order of 5 degrees or more, which may have accounted for this type of distribution.

Another probability function, the Weibull distribution function, has been observed to fit some types of land clutter spatial distributions of σ^0 quite well. The Rayleigh amplitude probability distribution function is a special case of the Weibull function, and preliminary results appear to indicate that the clutter distributions approach the Rayleigh case for larger depression angles.

This report discusses the Weibull probability distribution function and its characteristics, and presents some clutter measurements that demonstrate the Weibull distribution.

2. The Weibull Distribution Applied to σ^0

The Weibull probability density function is a single variate function having two parameters, α and b , and is given by

$$p(\sigma^0) = \frac{b\sigma^{0b-1}}{\alpha} \exp\left(-\sigma^{0b}/\alpha\right) , \quad (1)$$

where σ^0 is the variate in terms of the clutter backscatter coefficient. For the case where $b = 1$, Equation (1) becomes the exponential (or Rayleigh amplitude) density function

$$p(\sigma^0) = \frac{1}{\alpha} \exp(-\sigma^0/\alpha) , \quad (2)$$

where $\alpha = \overline{\sigma^0}$, the average value of σ^0 .

Integration of Equation (1) yields the probability that the actual clutter cross section per unit area will not exceed some value σ_c^0 , resulting in

$$P(\sigma_c^0 \leq \sigma^0) = 1 - \exp(-\sigma_c^{0b}/\alpha) . \quad (3)$$

The mean of the Weibull distribution function may be determined from the integral

$$\overline{\sigma^0} = \int_0^{\infty} \sigma^0 p(\sigma^0) d\sigma^0 . \quad (4)$$

A change of variable $x = \sigma^{0b}/\alpha$ (and defining a change of parameter $a = 1/b$) yields the integral

$$\overline{\sigma^0} = \alpha^{1/b} \int_0^{\infty} x^{1/b} e^{-x} dx , \quad (5)$$

which is the gamma function integral. The resulting mean value of σ^0 is then

$$\overline{\sigma^0} = \alpha^{1/b} \Gamma(1 + a) . \quad (6)$$

Equation (3) can be solved for σ^0 to obtain

$$\sigma^0 = \left[\alpha \ln\left(\frac{1}{1-P}\right) \right]^{1/b} = \left[\alpha \ln\left(\frac{1}{1-P}\right) \right]^a . \quad (7)$$

The median of the distribution is then determined by setting $P = 1/2$ in Equation (7) which reduces to

$$\sigma_m^0 = [\alpha \ln 2]^a . \quad (8)$$

The parameter α can now be expressed in terms of the median clutter backscatter coefficient σ_m^0 .

$$\alpha = \sigma_m^0 1/a / \ln 2 = \frac{\sigma_m^0 b}{\ln 2} . \quad (9)$$

Substitution of Equation (9) in Equation (3) results in the Weibull distribution function expressed as

$$P(\sigma_c^0 \leq \sigma^0) = 1 - \exp[-\ln 2 (\sigma^0/\sigma_m^0)^b] . \quad (10)$$

Analytical techniques are available for determining how well a measured distribution of the clutter backscatter coefficient approaches a Weibull distribution and for estimating the parameters "b" and σ_m^0 (or α if desired). A graphical approach, however, is useful and more easily applied to experimental data, and this method rather than the analytical approach will be described here.

Substituting the results of Equation (9) into Equation (7), one has

$$\sigma^0 = \frac{\sigma_m^0}{(\ln 2)^a} \ln^a \left(\frac{1}{1-P} \right) . \quad (11)$$

Taking the logarithm of σ^0 and multiplying by ten results in

$$10 \log \sigma^0 = 10 \log \sigma_m^0 - 10 a \log (\ln 2) + 10 a \log \left[\ln \left(\frac{1}{1-P} \right) \right] , \quad (12)$$

which is

$$\sigma^0(\text{dB}) = \sigma_m^0(\text{dB}) + 1.6 a + 10 a \log \left[\ln \left(\frac{1}{1-P} \right) \right] . \quad (13)$$

Now it can be seen that if the Weibull probability distribution is plotted on graph paper having the probability scale proportional to $\log \left[\ln \left(\frac{1}{1-P} \right) \right]$ and σ^0 in decibels on a linear scale, the result is a straight line having a slope determined by the parameter "a." The value of the median fixes the relative position of the distribution line. The graph paper having these special scales described above is sometimes referred to as Rayleigh paper, since the Rayleigh amplitude distribution is a special case of the Weibull type distribution for which $a = 1$.

Measured distributions of σ^0 can be plotted on Weibull paper to determine graphically if the distribution is of the Weibull type, and if so, to determine the value of "a" (or b). If the points lie in a reasonably straight line, the Weibull distribution can be considered to be a reasonable approximation to the actual distribution, especially if the points fit well within the 10- to 90-percent range. Values of σ^0 corresponding to the lower probabilities are generally well below a system's noise level and present virtually no problem to the radar, while strong values of σ^0 corresponding to the higher probabilities are generally too large to be suppressed effectively by moving target indicator (MTI) systems, anyway. The capability of a radar to operate satisfactorily in a land clutter environment is strongly determined by the manner in which the clutter is distributed spatially over the range of σ^0 which might be described as moderate. The use of any one particular value of σ^0 by itself tends to be misleading in describing a radar's performance from a system standpoint.

3. Generation of Weibull Distributions in Simulations

The Weibull distribution function is readily adaptable for use in digital computer simulations that utilize a Monte Carlo approach to simulate the performance of radar systems in a ground clutter environment. As a target moves over the ground, it competes with different clutter cells or with different combinations of clutter. If it can be assumed that there is little or no correlation between successive looks of the radar at a particular target, Equation (11) can be used with a random number generator to produce random variates σ_i^0 's whose density function is Weibull. The above assumption is generally valid for search radars with good spatial resolution and frame times on the order of several seconds against tactical aircraft.

Equation (11) is simply an expression which will transform or map a set of uniformly distributed random numbers which cover the unit interval between 0 and 1 into the domain of σ^0 . To generate random σ^0 's from a statistical population whose distribution is Weibull, uniformly distributed random numbers between 0 and 1 are first generated. Each random number r_i is then transformed into a corresponding σ_i^0 by

$$\sigma_i^0 = \frac{\sigma_m^0}{(\ln 2)^a} \left[\ln \left(\frac{1}{1 - r_i} \right) \right]^a, \quad (14)$$

where the slope parameter "a" and the median σ_m^0 have been previously defined for the particular type of clutter to be simulated.

The Weibull distribution function has been used to simulate spatial ground clutter distributions in the U. S. Army Missile Command Air Defense Simulation (MADS) digital computer model. This program utilizes digital terrain data at each 500-meter interval over the area simulated for use in determining masking effects. A code number is used to identify the type of ground clutter (forest, grassland, etc.) at each 500-meter grid coordinate. The code number selects the desired Weibull distribution having slope parameter "a" and median σ_m^0 for a particular clutter type, and a value of σ^0 is drawn from the selected Weibull population.

4. Spatial Backscatter Distributions of Measured Land Clutter

Several sources of measured distributions of clutter backscatter coefficient have been obtained which exhibit the Weibull distribution. The data points have been plotted on Weibull paper as described in Part 2 such that a distribution which is Weibull will result in a straight line. These sources represent measurements from both high and low resolution radars, relatively long and short clutter cell averaging times, and for various frequencies. The greatest deviations from the straight line appear at the very small values of σ^0 which might have been a result of the receiver noise level.

Figure 1 shows a distribution of Rocky Mountain [2-4] ground clutter taken at S-band with a resolution cell defined by a 2.0-microsecond pulse and a 1.5-degree beamwidth. The data were taken for ranges less than 20 miles from an unmasked location. No information is given concerning the depression angles or vegetation. Nevertheless, the distribution fits a straight line quite well on Weibull paper over a wide portion of the probability scale. The circled points indicate values taken from the distribution curve previously given [2].

The distribution of the clutter backscatter coefficient for data taken near Huntsville, Alabama, [5] is shown in Figure 2. These measurements were made at L-band out to a maximum range of approximately 40 kilometers with a resolution cell defined by a 3.0-microsecond pulse length and a 1.7-degree beamwidth. The terrain included grassland, low rolling wooded hills, wooded mountains, and some man-made structures. The data, taken with a non-scanning antenna and an averaging time of 3 seconds at each position, represent the strongest 241 clutter cells out of several thousand cells examined and therefore exhibit a higher median value when compared to other measurements. The radar site was located on one of the hills such that the depression angle from the antenna, as determined from maps of the area, was generally less than 0.5 degree. Even though the smaller clutter cells were neglected, the Weibull distribution still represents a good fit to the distribution of the measured clutter data.

Data taken at X-band by Linell [6] at the Research Institute of National Defense in Sweden is presented in Figures 3 through 5. The resolution cell is defined by a 0.17-microsecond pulse length and a 1.4-degree beamwidth. The data were taken with a rotating antenna atop a water tower. Figure 3 shows two spatial distributions of the clutter backscatter coefficient for a Swedish forest area at different times of year. These particular data are for a depression angle of 0.7 degree. Seasonal deviations appeared only during the month of November, resulting in an apparent shift in the distribution of about 6 decibels. The slope changed only slightly.

Figures 4 and 5 show spatial distributions for cultivated land taken in the months of April and May at varying depression angles of 1.25, 2.5, and 5.0 degrees. As the depression angle (and consequently the median σ_m^0) increases, the slope of the distribution approaches that of the exponential (or Rayleigh amplitude) distribution. This trend was also evident in measurements of the same terrain at other seasons of the year. Naval Research Laboratory airborne measurements [1] at several frequencies where depression angles were on the order of 5 degrees or greater also approached the Rayleigh amplitude distribution. This effect is probably attributed to increased shadowing effects at smaller depression angles. Hopefully, satisfactory relationships between the slope parameter "a," the median σ_m^0 , the frequency, the polarization, and the depression angle (or grazing angle) for various terrain types can be verified for use with Equation (14) in a spatial clutter model for air defense simulations.

The parameters for the distributions of this section are given in Table I. The median σ_m^0 is obtained by simply reading it from the distribution at P = 50 percent. The slope parameter "a" is obtained by the method described in Appendix B.

TABLE I. WEIBULL PARAMETERS FOR CLUTTER DISTRIBUTIONS

Figure No.	Terrain Type	Frequency Band	Depression Angle (deg)	Resolution ($\mu\text{sec} \times \text{deg}$)	Median σ_m^0 (dB)	Slope Parameter "a"
1	Rocky mountains	S	---	2.0×1.5	-46.25	3.9
2	Low rolling wooded hills, grassland	L	≈ 0.5	3.0×1.7	-27.0*	3.19
3	Forest (March-May, August)	X	0.7	0.17×1.4	-42.5	3.95
3	Forest (November)	X	0.7	0.17×1.4	-36.4	3.76
4	Cultivated land (April)	X	1.25	0.17×1.4	-47.75	3.3
4	Cultivated land (April)	X	2.5	0.17×1.4	-38.0	1.75
4	Cultivated land (April)	X	5.0	0.17×1.4	-29.8	1.1
5	Cultivated land (May)	X	1.25	0.17×1.4	-46.5	2.84
5	Cultivated land (May)	X	2.5	0.17×1.4	-37.8	2.33
5	Cultivated land (May)	X	5.0	0.17×1.4	-25.3	1.0

*This distribution neglected many cells of low clutter return, and the median is not considered typical of this type of terrain at low depression angles.

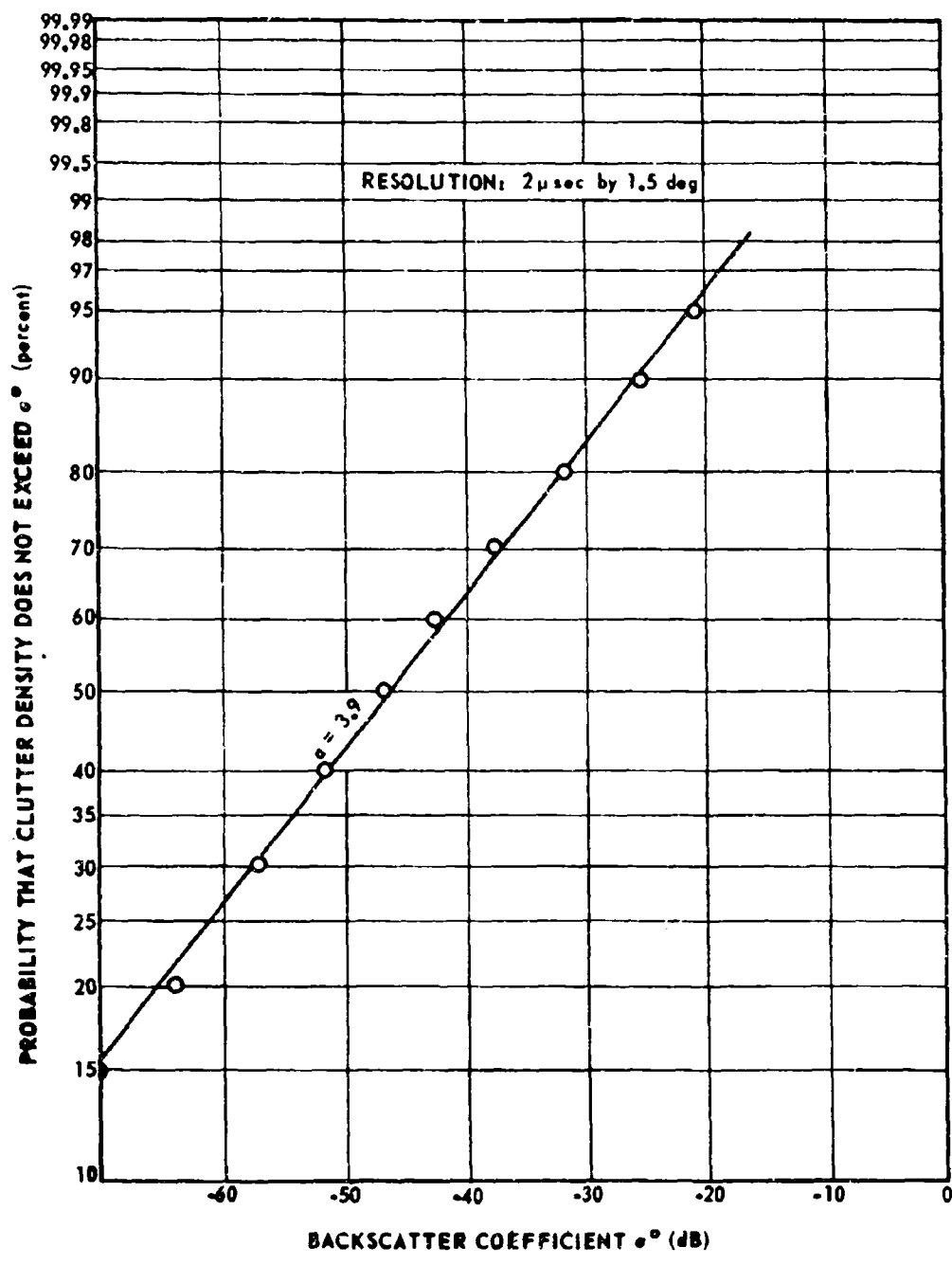


FIGURE 1. GROUND CLUTTER SPATIAL DISTRIBUTION FOR ROCKY MOUNTAINS AT S-BAND

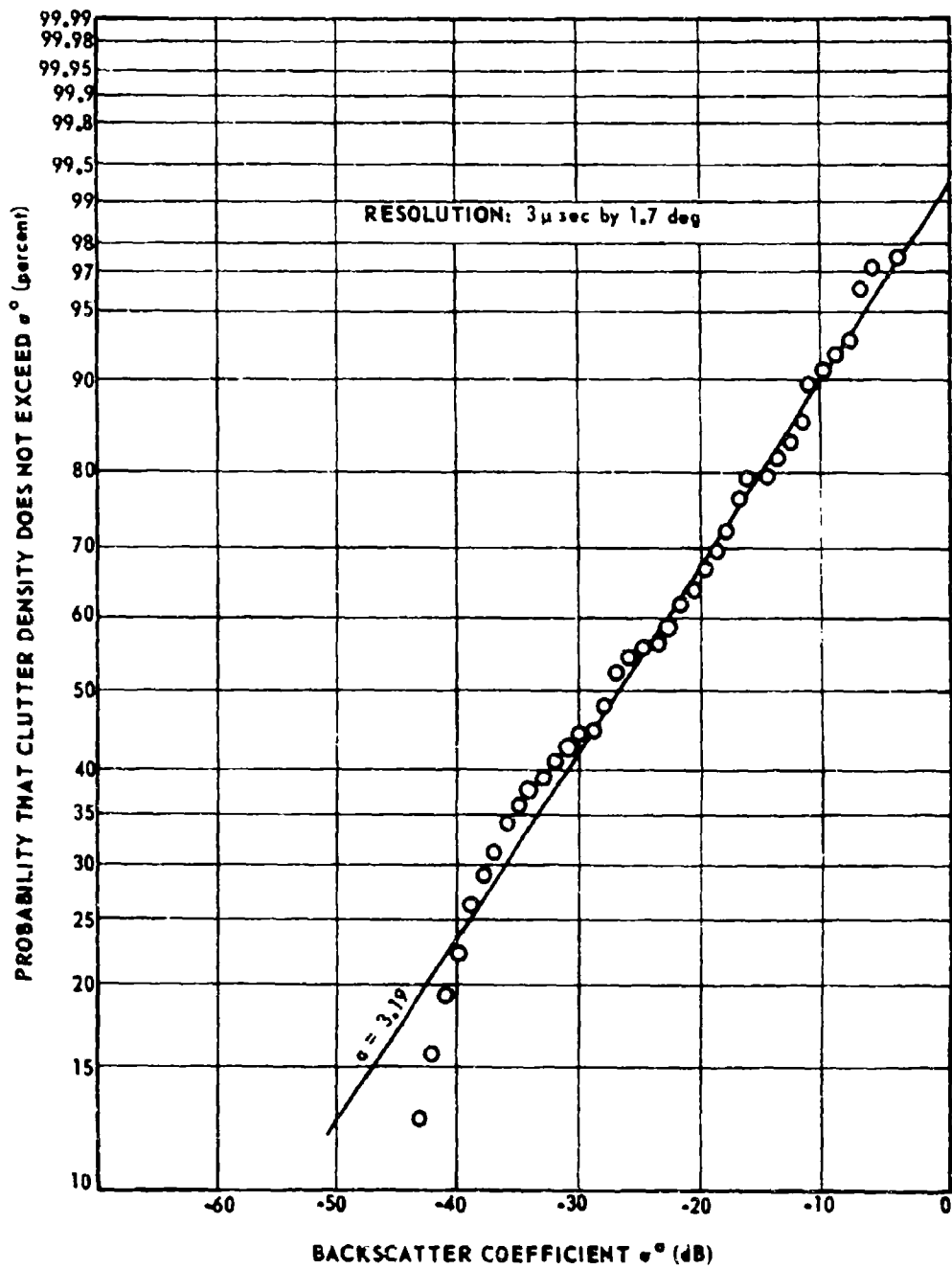


FIGURE 2. GROUND CLUTTER SPATIAL DISTRIBUTION FOR WOODED HILLS AND GRASSLAND NEAR HUNTSVILLE, ALABAMA, AT L-BAND

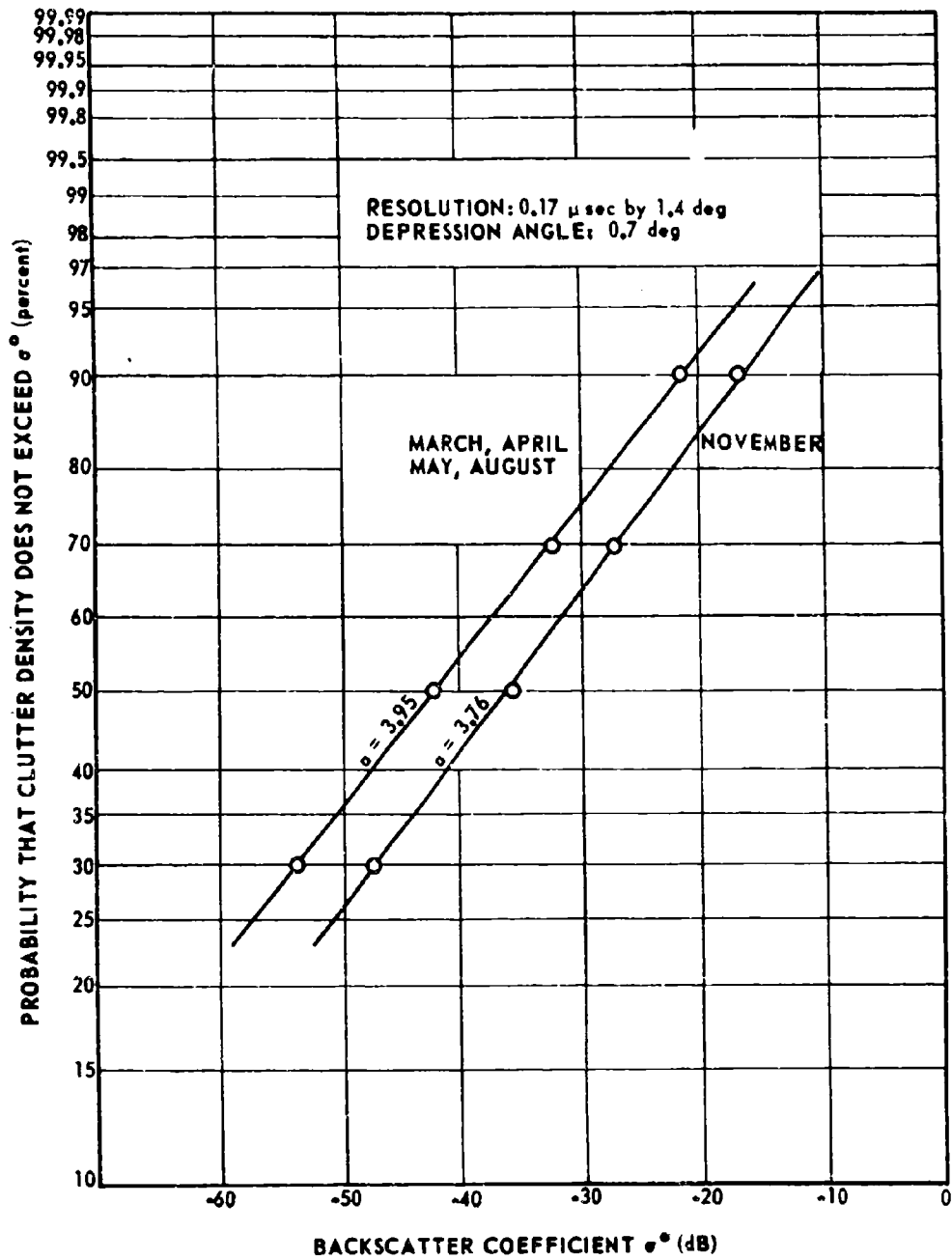


FIGURE 3. GROUND CLUTTER SPATIAL DISTRIBUTIONS FOR FOREST AT DIFFERENT TIMES OF YEAR AT X-BAND

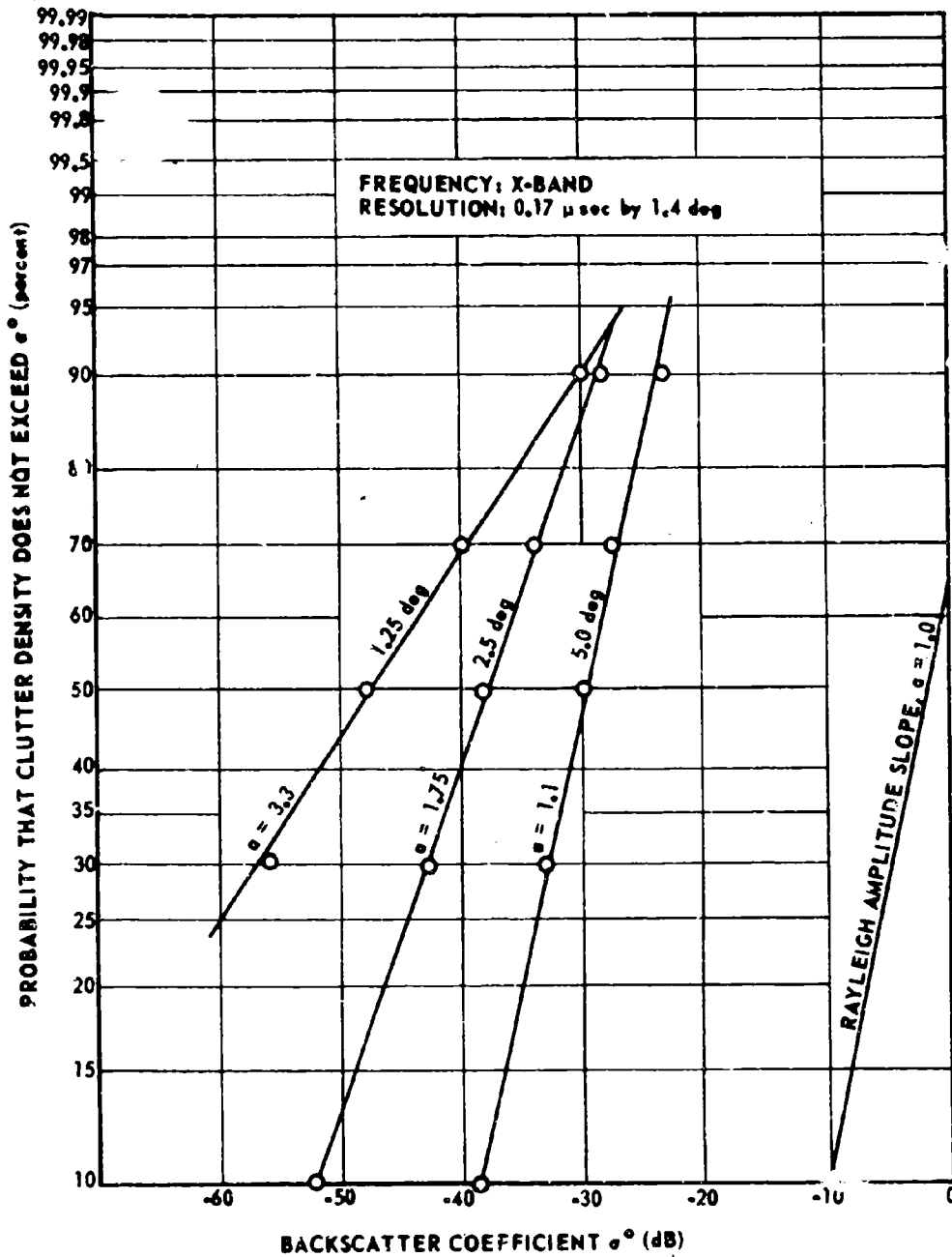


FIGURE 4. GROUND CLUTTER SPATIAL DISTRIBUTIONS FOR CULTIVATED LAND IN APRIL AT SEVERAL DEPRESSION ANGLES

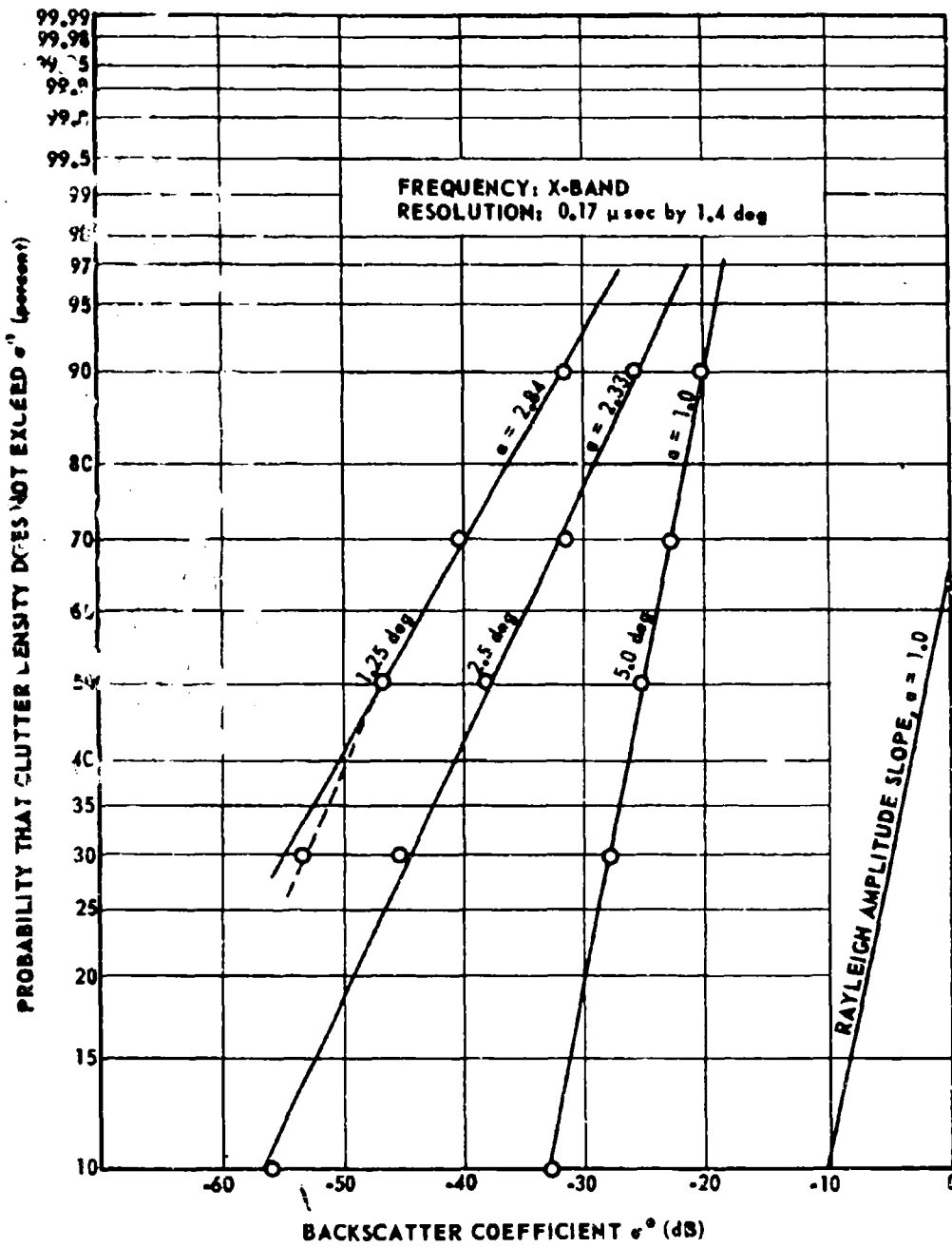


FIGURE 5. GROUND CLUTTER SPATIAL DISTRIBUTIONS FOR CULTIVATED LAND IN MAY AT SEVERAL DEPRESSION ANGLES

REFERENCES

1. Guinard, N. W., Ransone, J. T., Laing, M. B., and Hearton, L. E., NRL Terrain Clutter Study Phase I, Naval Research Laboratory Report No. 6487, 10 May 1967.
2. Barton, D. K., "Target Detection in Land Clutter," Raytheon Company, Wayland, Massachusetts, Internal Memorandum, 30 November 1966.
3. Expected Performance of SAM-D Against SRAM Type Targets in Realistic Clutter Environments, " Raytheon Company, Report No. BR-4356, 21 August 1967.
4. Nathason, F. E., Radar Training Program Notes, Chapter 7: "Sea and Land Backscatter," The Johns Hopkins University Applied Physics Laboratory.
5. Holliday, E. M., Wood, W. E., Powell, D. E., and Basham, C. E., L-Band Clutter Measurements, U. S. Army Missile Command Report No. RE-TR-65-1, 3 November 1964.
6. Linell, T., An Experimental Investigation of the Amplitude Distribution of Radar Terrain Return, Institute of National Defense, Stockholm, Sweden, Report No. D 3135-62, October 1966.

Appendix A
**DETERMINATION OF THE VARIANCE OF THE
 WEIBULL FUNCTION**

In determining the variance, the mean square value of σ^0 is first calculated:

$$\overline{\sigma^{0^2}} = \int_0^{\infty} \sigma^{0^2} p(\sigma^0) d\sigma^0 = \int_0^{\infty} \sigma^{0^2} \frac{b\sigma^{0^{b-1}}}{\alpha} \exp\left(-\sigma^0/\alpha\right) d\sigma^0 . \quad (\text{A-1})$$

A change of variable $x = \sigma^0/\alpha$ (and defining a change of parameter $a = 1/b$) yields the integral

$$\overline{\sigma^{0^2}} = \int_0^{\infty} \alpha^{2a} x^{2a} e^{-x} dx \quad (\text{A-2})$$

$$\overline{\sigma^{0^2}} = \alpha^{2a} \Gamma(1 + 2a) . \quad (\text{A-3})$$

From equation (6), the mean is shown to be

$$\overline{\sigma^0} = \alpha^a \Gamma(1 + a) ; \quad (\text{A-4})$$

the variance is found from

$$\sigma^2 = \overline{\sigma^{0^2}} - (\overline{\sigma^0})^2 . \quad (\text{A-5})$$

The variance is then

$$\sigma^2 = \alpha^{2a} [\Gamma(1 + 2a) - \Gamma^2(1 + a)] . \quad (\text{A-6})$$

Appendix B
DETERMINATION OF SLOPE PARAMETER "a"

The slope parameter "a" of a Weibull distribution may be determined directly from the plot of the distribution on Weibull paper. By recalling that the probability scale is proportional to $\log [\ln(1/1 - P)]$, a variable is introduced $x = \log [\ln(1/1 - P)]$, such that the probability scale is transformed into linear units of x. Substituting x into Equation (13) gives

$$\sigma^0(\text{dB}) = \sigma_m^0(\text{dB}) + 1.6 a + 10 ax \quad (\text{B-1})$$

The desired slope is the derivative $\frac{d\sigma^0(\text{dB})}{dx}$:

$$\frac{d\sigma^0(\text{dB})}{dx} = 10 a = \frac{\Delta\sigma^0(\text{dB})}{\Delta x} \quad (\text{B-2})$$

The slope parameter "a" is then found from

$$a = \frac{\Delta\sigma^0(\text{dB})}{10 \Delta x} \quad (\text{B-3})$$

It is now necessary to determine what interval of probability corresponds to a Δx of unity. The lowest probability shown on the Weibull paper used in this report is $P = 0.10$. Thus, for $P = 0.10$,

$$\begin{aligned} x_{P=0.1} &= \log \left[\ln \frac{1}{0.9} \right] \\ &= \log 0.1053 \\ x_{P=0.1} &= -0.9775 \end{aligned}$$

Then, for unity Δx ,

$$\Delta x = 1 = x' - x_{P=0.1}$$

and

$$x' = 1 + x_{P=0.1} = 0.0225$$

Solving the expression

$$\log \{ \ln (1/1 - P') \} = 0.0225$$

for P' , one obtains the value $P' = 0.651$. Thus, a Δx of unity corresponds to the interval on the probability scale between 10 and 65.1 percent. From the above result, it can be seen that the slope parameter "a" is found by the expression

$$a = \frac{\sigma^0(\text{dB})_{P=0.651} - \sigma^0(\text{dB})_{P=0.1}}{10} \quad (\text{B-4})$$

Graphically, this is easily determined by translating the slope of the distribution over to the probability scale such that it passes through $P = 0.651$ on the right-hand probability axis. The interval of $\sigma^0(\text{dB})$ between the right-hand probability axis and the line's intersection on the $P = 0.10$ axis is $(\sigma^0_{P=0.651} - \sigma^0_{P=0.1})$ above. This graphical technique is illustrated for several slopes in Figure B-1.

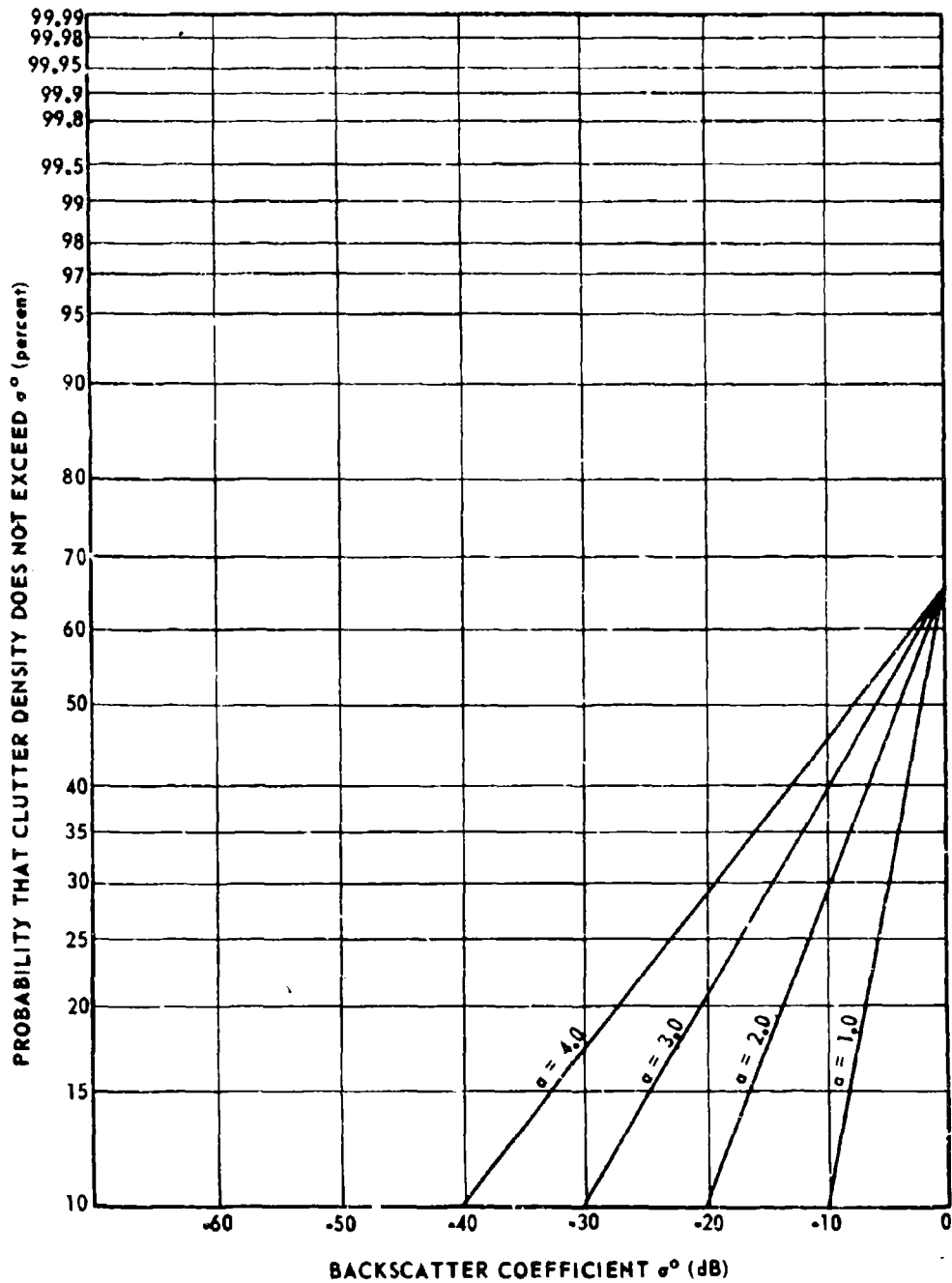


FIGURE B-1. EXAMPLE SLOPES FOR WEIBULL DISTRIBUTIONS HAVING DIFFERENT SLOPE PARAMETERS "a"

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D		
<i>(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)</i>		
1. ORIGINATING ACTIVITY (Corporate author) Action Group II Research and Engineering Directorate (Provisional) U. S. Army Missile Command Redstone Arsenal, Alabama 35809		2a. REPORT SECURITY CLASSIFICATION Unclassified
		2b. GROUP NA
2. REPORT TITLE THE WEIBULL DISTRIBUTION APPLIED TO THE GROUND CLUTTER BACKSCATTER COEFFICIENT		
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Technical Report		
3. AUTHOR(S) (First name, middle initial, last name) Robert R. Boothe		
6. REPORT DATE 12 June 1969	7a. TOTAL NO. OF PAGES 25	7b. NO. OF REFS 6
8a. CONTRACT OR GRANT NO.	8b. ORIGINATOR'S REPORT NUMBER(S) RE-TR-69-15	8c. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) AD _____
8. PROJECT NO. (DA) 1M262301D209 AMC Management Structure Code No. c. 5221.12.178 d.		
10. DISTRIBUTION STATEMENT Distribution of this document is unlimited.		
11. SUPPLEMENTARY NOTES None	12. SPONSORING MILITARY ACTIVITY Same as No. 1	
13. ABSTRACT <p>Spatial distributions of the ground clutter backscatter coefficient σ^0 for various types of terrain have been found to fit a Weibull distribution quite well. This report discusses the Weibull probability distribution function and its characteristics, and presents some clutter measurements that demonstrate the Weibull distribution. The data presented include measurements taken at L-, S-, and X-band frequencies, at several depression angles, and for various resolution cell sizes. A method for generation of random variables that fit a selected Weibull statistical population for use in radar simulations is also given.</p>		

DD FORM 1473

REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

UNCLASSIFIED
Security Classification

21

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Ground clutter backscatter coefficient Weibull distribution Spatial distribution Generation of variables (Weibull)						