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HIGH INTENSITY LASER PROPAGATION
IN THE ATMOSPHERE

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Prepared by: Approved by:

L.M. Frantz   S. Altshuler (Project Manager)
Theoretical Physics Department   Director
                                      Quantum Physics Laboratory

Theoretical Physics Department
QUANTUM PHYSICS LABORATORY
Physical Research Center

TRW SYSTEMS
One Space Park
Redondo Beach, California 90278

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I.   INTRODUCTION

The self-defocusing effect is a phenomenon in which energy is absorbed by the air from an intense laser beam traversing the atmosphere, with a resultant decrease in the local refractive index. The region of decreased refractive index is, in effect, a lens, and may cause the beam to defocus.

In a previous report[1] a study of the self-defocusing effect was initiated. It was pointed out that there exists the possibility of shaping the power profile of the laser beam in such a way that, at least initially, the beam self-focuses instead of self-defocusing. One of the goals of the study, then, was to determine in detail how the shape of the initial power profile affects the development of the beam as it progresses through the atmosphere.

The problem was formulated mathematically in terms of a nonlinear, partial differential equation, whose solution describes the trajectories of the rays comprising the beam. In this report we present an approximate solution to the equation, and obtain from it some qualitative and semi-quantitative features of the ray trajectories.

The complete nonlinear, partial differential equation has the horizontal displacement of any given ray as its dependent variable, and contains three independent variables: time, height, and one horizontal position coordinate (assuming cylindrical or planar symmetry). The analysis to be discussed here is in terms of an approximation to the true equation in which the time appears as a trivial parameter, rather than as an independent variable. Physically, the approximation is as follows. The change in the refractive index at any point in space is proportional to the amount of heat deposited,

which in turn is proportional to the energy flux at that point integrated over time. We have approximated the time integral by the product of the instantaneous flux and the time. The model is now equivalent to a steady state situation in which the refractive index has an incremental term proportional to the flux; the coefficient of the incremental term contains the time as a parameter. The model equation is somewhat simpler than the original, but still contains all the features of mutual feedback between the beam and the refractive index.

In reference 1 it was pointed out that the beam's initial power profile may be shaped as in Fig. 1 so that the rays of the inner region, r < R, initially converge, because the gradient of the refractive index generated by the beam itself will be inward; however, the rays of the outer region, r > R, will initially diverge because the gradient of the refractive index then will be outward. As the rays of the outer region diverge farther with increasing height, the power density decreases, so that the region of outward gradient can be expected to eat its way into the interior region, and eventually cause the entire beam to spread.

In this report we shall be concerned with evaluating the rate at which the divergent outer region of the beam eats its way into the interior. To study the phenomenon we have considered an initial flux distribution having the shape indicated in Fig. 2. Here, since we are basically interested only in the behavior of the edge of the beam, we have chosen a planar geometry (for which the equations have a somewhat simpler form) rather than the more realistic cylindrically symmetrical geometry. The initial power profile is taken to be uniform along the negative axis, and to fall off exponentially

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along the positive axis. Since it follows from geometric optics that the rays deflect in the direction of the gradient of the refractive index, we expect that initially the rays along the positive axis will deflect to the right, but the rays along the negative axis, which represents the interior of the beam, will not be deflected at all. In this case the degradation of the beam will be due solely to the outer edge's eating its way in.

We have obtained an approximate solution to the model equation, which shows that the interior rays (the rays originating along the negative axis) go up undeflected to a certain height, then begin to deflect outwards (towards the positive axis) parabolically with height. This behavior is indicated in Fig. 3. The solution is valid only in certain regions of space, but for any ray it is quite accurate up to and including the point at which the ray starts to deflect. Therefore, it provides a first order description of the desired rate at which the beam edge eats into the interior. Specifically, the height at which any ray begins to deflect is at any time proportional to its original distance in from the edge, and is inversely proportional to the square root of the energy per unit volume deposited up to that time along the ray path. Furthermore, once the ray begins to deflect, the sharpness with which it does so depends both upon the energy per unit volume deposited and upon the steepness with which the original (exponential) outer edge descends: the greater the deposited energy density is, and the steeper the outer edge is, the more sharply the ray deflects.

2. THE MODEL EQUATION

In this section we shall derive the model equation, making use of some of the relationships developed in reference 1. We assume a planar geometry with z denoting the height coordinate and x the horizontal coordinate, and all quantities being independent of the third coordinate. The eikonal equation, which describes the ray trajectories in a medium of refractive index n, may be written in the form

$$\left(\frac{\partial n\hat{a}}{\partial \sigma}\right)_s = \nabla n , \qquad (2.1)$$

where the unit vector $\hat{a}$ is tangent to the ray path at any point, σ is the path length along a given ray, and s is the value of the horizontal coordinate of any ray at z = 0, that is, $x(s,z = 0) = s$. The projection of Eq.(2.1) along the horizontal axis is

$$\left(\frac{\partial n \sin\theta}{\partial \sigma}\right)_s = \left(\frac{\partial n}{\partial x}\right)_z , \qquad (2.2)$$

where θ is the angle between $\hat{a}$ and the vertical axis. It will be convenient to use, instead of the (x,z) coordinate system, one defined by the planes of constant z and the surfaces generated by the ray paths (surfaces of constant s), which are the solutions to the problem. When transformed to the (s,z) coordinate system (2.2) takes the form

$$\left(\frac{\partial n \sin\theta}{\partial z}\right)_s \cos\theta = \left(\frac{\partial n}{\partial s}\right)_z \left(\frac{\partial x}{\partial s}\right)^{-1} ; \qquad (2.3)$$

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we have used the relation $(\partial\sigma/\partial z)_s = \cos\theta$, which can be seen readily from Fig. 4.

The requirement of conservation of flux (assuming negligible absorption losses in the atmosphere) is given by

$$\int_A F\hat{a}\cdot d\vec{A} = 0 , \tag{2.4}$$

where $d\vec{A}$ is a surface element, the integration is over any closed surface A, and F denotes the magnitude of the flux. Taking a surface of the form illustrated in Fig. 5, in which the sides are surfaces of constant s (since these are everywhere tangent to the ray paths, no flux escapes through them, i.e., $\hat{a}\cdot d\vec{A} = 0$), and the top and bottom are line elements dx and ds, we have from (2.4),

$$F(s,z,t)\cos\theta \, dx = F_0(s,t)ds , \tag{2.5}$$

where $F_0(s,t) \equiv F(s,\theta,t)$. Now using the relation $dx = (\partial x/\partial s)_z ds$, we obtain from (2.5) the flux conservation equation,

$$F(s,z,t) = \frac{F_0(s,t)}{(\partial x/\partial s)_z \cos\theta} \tag{2.6}$$

Next we need the relation between the flux and the refractive index. The rate of change of refractive index is proportional to the rate of change of the density, which is proportional to the rate at which heat is deposited in the atmosphere; the latter in turn is proportional to the flux. The relation is given explicitly in Ref. 1 as

$$\dot{n}(s,z,t) = -\frac{(\gamma-1)(n^2-1)k \, F(s,z,t)}{2\gamma nP} . \tag{2.7}$$

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Here γ is the ratio of specific heats, k is the effective absorption coefficient for heating, and P is the ambient pressure. $\dot{n}$ is the time rate of change of the index at a fixed point in space, i.e., at fixed x,z,

$$\dot{n} = \frac{\partial}{\partial t} n[s(x,z,t),z,t]_{x,z} . \tag{2.8}$$

Therefore, the time integral of Eq. (2.7) is

$$n(s,z,t) - n_a = - \frac{(\gamma-1)(n_a^2-1)k}{2\gamma n_a P} \int_{x,z} F[s(x,z,t),z,t]dt , \tag{2.9}$$

where n_a is the ambient index, and the integral is to be evaluated at fixed x,z. Since, as noted in Ref. 1, the fractional change in refractive index is much less than unity, we have replaced n in (2.7) by the constant value n_a, and removed the factor $(n^2-1)/n \equiv (n_a^2-1)/n_a$ from the integral.

Now we make the principal approximation which characterizes the model equation, viz., we replace the time integral of the flux by the product of the instantaneous flux and the time,

$$\int_{x,z} F[s(x,z,t),z,t] \, dt \rightarrow F(s,z,t)t . \tag{2.10}$$

Strictly speaking, as the beam contracts or expands with time, the flux F increases or decreases with time accordingly. Therefore, even if we assume that the flux leaving the laser face $F_o(s,t)$ is constant in time (as we shall, in fact, do below), F at any other location is not time independent, and the replacement (2.10) is not strictly valid. However, it does significantly simplify the mathematical system by decreasing the number of independent variables from three to two. Furthermore, the resulting simplified relation

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between the index and the flux[*],

$$n(s,z) - n_a = - \frac{(\gamma-1)(n_a^2-1)kt\, F(s,z)}{2\gamma n_a P} , \qquad (2.11)$$

still has the same gross time dependence as (2.10), and still maintains the basic linear relationship between n and F.

We now have the three fundamental equations: the ray Eq. (2.3), the flux conservation Eq. (2.6), and the equation of coupling between the index and the flux (2.11). The flux may immediately be eliminated from (2.6) and (2.11) to provide an expression for the index at any point in space in terms of the (known) index at the laser face[**],

$$n(s,z) - n_a = \frac{n(s,0)-n_a}{(\partial x/\partial s)\,\cos\theta} , \qquad (2.12)$$

where

$$n(s,0)-n_a = - \frac{(\gamma-1)(n_a^2-1)kt\, F_0(s)}{2\gamma n_a P} . \qquad (2.13)$$

$F_0(s)$ is the flux emerging from the laser face, which we now take to be constant in time for $t \geqslant 0$, and zero for $t < 0$. Since $F_0(s)$ is assumed to be known, $n(s,0)$ is also known.

It is observed in Ref. 1 that the deviation of any ray from the vertical can be expected to be quite small, i.e., $\theta \ll 2\pi$. Therefore, we shall use the ray Eq. (2.3) in the small angle approximation,

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[*] In all expressions below the explicit t-dependence will be suppressed, since t now appears only as a trivial parameter.

[**] Hereafter only the s,z coordinates will be used, with x being a dependent variable. Therefore, it is no longer necessary to indicate explicitly which coordinates are being held fixed in the partial derivatives.

$$\frac{\partial x}{\partial s}\left(n\frac{\partial \theta}{\partial z} + \theta\frac{\partial n}{\partial z}\right) = \frac{\partial n}{\partial s} \quad . \tag{2.14}$$

Since the variation in index is very small compared to n itself, the term $\theta\partial n/\partial z$ is of second order in small quantities, and we drop it. Similarly, because of the small variation in index, we replace the n in $n\partial\theta/\partial z$ by n_a. Finally, noting that in the small angle approximation θ is given by $\theta = \partial x/\partial z$, we have

$$\frac{\partial x}{\partial s}\frac{\partial^2 x}{\partial z^2} = \frac{\partial}{\partial s}\left(\frac{n}{n_a}\right) = -\frac{\partial}{\partial s}\left(\frac{n_a-n}{n_a}\right) \quad . \tag{2.15}$$

We now define the dimensionless and positive function

$$f(s) \equiv \frac{n_a-n(s,0)}{n_a} \quad , \tag{2.16}$$

so that from (2.12) we have ($\theta \ll 1$)

$$\frac{n_a-n}{n_a} = \frac{f(s)}{\partial x/\partial s} \quad , \tag{2.17}$$

and from (2.13) we have

$$f(s) = \frac{(\gamma-1)(n_a^2-1)ktF_0(s)}{2\gamma n_a^2 P} \quad . \tag{2.18}$$

Inserting (2.16) into (2.15) we finally obtain the model equation,

$$\frac{\partial x}{\partial s}\frac{\partial^2 x}{\partial z^2} = -\frac{\partial}{\partial s}\left[\frac{f(s)}{\partial x/\partial s}\right] \quad . \tag{2.19}$$

The boundary conditions are

$$x(s,0) = s,$$ (2.20a)

$$\theta(s,0) = \frac{\partial x}{\partial z}(s,0) = 0.$$ (2.20b)

Equation (2.20b) states that all rays emerge vertically from the laser face. The function $f(s)$ is proportional to the flux at the laser face, and therefore has the same shape as the initial flux distribution $F_o(s)$. It may be regarded as a reduced initial flux distribution.

3. THE METHOD OF SOLUTION

Performing the derivative operation in (2.19) and reassembling terms leads to

$$\left(\frac{\partial x}{\partial s}\right)^3 \frac{\partial^2 x}{\partial z^2} - f \frac{\partial^2 x}{\partial s^2} + \frac{df}{ds} \frac{\partial x}{\partial s} = 0.$$ (3.1)

We now define

$$x \equiv s + x_1,$$ (3.2)

where x_1 is the horizontal displacement of any ray from its original location s (at $z = 0$). The boundary conditions are now

$$x_1(s,0) = 0$$ (3.3a)

$$\frac{\partial x_1}{\partial z}(s,0) = 0,$$ (3.3b)

and the Eq. (3.1) clearly becomes

$$\left(1 + \frac{\partial x_1}{\partial s}\right)^3 \frac{\partial^2 x_1}{\partial z^2} - f \frac{\partial^2 x_1}{\partial s^2} + \left(1 + \frac{\partial x_1}{\partial s}\right)\frac{df}{ds} = 0.$$ (3.4)

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Shortly, we shall drop $\partial x_1/\partial s$ relative to unity. First, we transform from the independent variable s to the new independent variable p by

$$\frac{dp}{ds} = \frac{1}{\sqrt{f(s)}} \; . \qquad (3.5)$$

To avoid notational confusion caused by the transformation we define the functions S and P,

$$p = \int_{b}^{S(p)} \frac{ds'}{\sqrt{f(s')}} \; , \qquad (3.6a)$$

$$P(s) = \int_{b}^{s} \frac{ds'}{\sqrt{f(s')}} \qquad (3.6b)$$

where b is arbitrary. The transformation and its inverse are then, respectively,

$$s = S(p), \quad p = P(s) \; . \qquad (3.7)$$

In addition we define

$$X_1(p,z) \quad x_1(s,z) \; . \qquad (3.8)$$

Now, by transforming selected terms in (3.4) we may convert it to the form

$$\left(1 + \frac{\partial x_1}{\partial s}\right)^3 \frac{\partial^2 x_1}{\partial z^2} - \frac{\partial^2 x_1}{\partial p^2} + \left(1 + \frac{3}{2} \frac{\partial x_1}{\partial s}\right) \frac{df}{ds} = 0 \qquad (3.9)$$

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then we assume

$$\left|\frac{\partial x_1}{\partial s}\right| \ll 1 \;, \tag{3.10}$$

so that

$$\frac{\partial^2 x_1}{\partial z^2} - \frac{\partial^2 x_1}{\partial p^2} + \frac{df}{ds} = 0 \;. \tag{3.11}$$

It will be seen that useful semiquantitative information concerning the ray trajectories can be obtained in the region for which (3.10) is satisfied. Next we transform the last term in (3.11) as follows, using (3.5),

$$\frac{df}{ds} = \frac{dp}{ds} \frac{d}{dp} f\left[S(p)\right] = \frac{1}{\sqrt{f}} \frac{df}{dp} = 2 \frac{d\sqrt{f}}{dp}$$

$$= 2 \frac{d}{dp} \frac{ds}{dp} = 2 \frac{d^2 S(p)}{dp^2} \;, \tag{3.12}$$

so that

$$\frac{\partial^2 x_1}{\partial z^2} - \frac{\partial^2 x_1}{\partial p^2} + 2 \frac{d^2 S}{dp^2} = 0 \;. \tag{3.13}$$

Finally, we define

$$y \equiv x_1 - 2S \tag{3.14}$$

to obtain

$$\frac{\partial^2 y}{\partial z^2} - \frac{\partial^2 y}{\partial p^2} = 0 \;. \tag{3.15}$$

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The solution to (3.15) is well known to be

$$y(p,z) = A(p + z) + B(p - z) , \tag{3.16}$$

where A and B are arbitrary functions. Consequently, we have

$$X_1(p,z) = 2S(p) + A(p + z) + B(p - z) . \tag{3.17}$$

A and B are readily determined from the boundary conditions (3.3) with the result

$$X_1(p,z) = 2S(p) - S(p + z) - S(p - z) . \tag{3.18}$$

In terms of the original independent variables, then,

$$x_1(s,z) = 2s - S\left[P(s) + z\right] - S\left[P(s) - z\right] . \tag{3.19}$$

To cast the S functions into a more convenient form we note that from the definition (3.6b) of $P(s)$ we have

$$P(s) + z = \int_b^s \frac{ds'}{\sqrt{f(s')}} + z , \tag{3.20}$$

and from the definition (3.6a) of $S(p)$,

$$P(s) + z = \int_b^{S\left[P(s) + z\right]} \frac{ds'}{\sqrt{f(s')}} . \tag{3.21}$$

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Subtracting (3.20) from (3.21) yields

$$z = \int_{s}^{S[P(s)+z]} \frac{ds'}{\sqrt{f(s')}} \cdot \tag{3.22}$$

Finally, defining

$$g(s,z) \equiv S[P(s)+z], \tag{3.23}$$

we have the solution

$$x_1(s,z) = 2s - g(s,z) - g(s,-z), \tag{3.24}$$

where $g(s,z)$ is defined by

$$z = \int_{s}^{g(s,z)} \frac{ds'}{\sqrt{f(s')}} \cdot \tag{3.25}$$

Because of the boundary condition (3.3a) we have

$$\frac{\partial x_1}{\partial s}(s,0) = 0. \tag{3.26}$$

Therefore [see (3.10)] (3.24) satisfies the model Eq. (3.1) and its boundary conditions exactly at the $z = 0$ boundary plane.

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4. SPECIAL CASE

We consider now the special case for which $f(s)$, the reduced initial flux distribution, is given by

$$f(s) = \begin{cases} a^2, & s \leq 0 \\ a^2 e^{-2s/\lambda}, & s > 0 \end{cases} \tag{4.1}$$

This is illustrated in Fig. 2, which shows $F_o(s)$, to which $f(s)$ is proportional. The dimensionless constant a^2 can be expressed in terms of the relevant physical parameters directly from Eq. (2.18).

The region of negative s represents the interior of the beam, which in reality of course, will have cylindrical rather than planar symmetry. In this region the initial flux distribution is uniform, so that the refractive index will initially be constant. According to the ray Eq. (2.1), the rays deflect towards the direction of the gradient of the index. Here the gradient vanishes, so we can expect that, at least at first, the rays will go up vertically, undeflected.

The edge of the beam begins at $s = 0$, and the intensity falls off exponentially for $s > 0$. The heating rate, then, will be greatest along the positive axis near $s = 0$. Consequently the index will be smallest near $s = 0$, and the gradient will be in the positive direction for $s > 0$. Thus all the rays in the beam's edge can be expected to deflect outwards (in the positive direction). We are interested in seeing when and how the rays of the interior region ($s \leq 0$) first begin to deflect.

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The function $g(s,z)$ can be calculated straightforwardly from (3.25) and we find for the ray trajectories,

$$x_1(s,z) = \begin{cases} 0 \;,\; s \leq -az \\ s+az- \lambda \log_e(1+s/\lambda+az/\lambda) \;,\; -az < s \leq 0 \\ 2s+\lambda+az-\lambda e^{s/\lambda}- \lambda \log_e(az/\lambda+e^{s/\lambda}) \;,\; 0 < s < \lambda \log_e(1+az/\lambda) \\ 2s- \lambda \log_e(e^{2s/\lambda}-a^2z^2/\lambda^2) \;,\; \lambda \log_e(1+az/\lambda) < s \end{cases} \quad (4.2)$$

$$\frac{\partial x_1}{\partial s}(s,z) = \begin{cases} 0 \;,\; s \leq -az \\ \dfrac{az+s}{\lambda+az+s} \;,\; -az < s \leq 0 \\ 2-e^{s/\lambda}\left(1 + \dfrac{1}{1+az/\lambda+e^{s/\lambda}}\right),\; 0 < s < \lambda \log_e(1+az/\lambda) \\ 2\left[1 - \dfrac{1}{1-(az/\lambda)^2 e^{-2s/\lambda}}\right] \;,\; \lambda \log_e(1+az/\lambda) < s \;. \end{cases} \quad (4.3)$$

It can be seen immediately that in the region $s \leq -az$, the solution is exact, since here we have $\partial x_1/\partial s = 0$. Furthermore, in this region the rays are undeflected. For $s \geq -az$, but $az -|s| \ll 1$, we see that $\partial x_1/\partial s$ is very small, and the solution is again quite accurate. From (4.2) we see that under these conditions the ray trajectories are

$$x_1(s,z) \approx \frac{1}{2\lambda}(az+s)^2 = \frac{1}{2\lambda}(az - |s|)^2 \;. \quad (4.4)$$

Therefore, we find that the outer edge of the beam (the region of diverging rays) eats into the interior region (the region of initially uniform flux distribution) along the curve $s = -az$. That is, for any negative s, the corresponding ray goes up undeflected until it reaches a height $z = -s/a$;

then it begins to diverge parabolically according to (4.4). This behavior is illustrated in Fig. 3. It should be noted that the height at which an interior ray first begins to deflect, $z = -s/a$, does not depend on the steepness of the beam's original edge shape. However, once the ray begins to deflect, it does so more rapidly for a more steeply descending beam edge; more precisely, we see from (4.4) that the ray displacement x_1 is proportional to $1/\lambda$, where λ is the scale length for the exponential fall-off of the beam edge.

The dimensionless constant a is immediately found from (2.18) to be

$$a = \left[\frac{(\gamma-1)(n_a^2-1)ktF_o}{2\gamma n_a^2 P} \right]^{\frac{1}{2}} . \tag{4.5}$$

Consequently, the height at which an interior ray first begins to deflect, $z = -s/a$, is seen to be inversely proportional to the square root of ktF_o, the amount of heat per unit volume thus far deposited along the ray path. In addition, we see from the appearance of a in (4.4) that the more heat per unit volume deposited the more sharply the ray begins to deflect.

Now, if we denote by z_o the target height, and by s_o the maximum distance which we can tolerate having the beam's edge eat into its interior, then from

$$z_o = s_o/a \tag{4.6}$$

and (4.5) we can solve immediately for the maximum tolerable absorption coefficient k. Taking $s_o = R/2$, where R is the initial radius of an equivalent

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cylindrically shaped laser beam, we find

$$k = \frac{\gamma n_a^2 \, PR^2}{2(\gamma-1)(n_a^2-1)F_o t z_o^2} \quad . \tag{4.7}$$

This expression, except for a factor of two, is the same as that found by Brueckner, et al[2]. It should be noted, however, that qualitatively the pictures are considerably different. In the latter analysis no specific consideration is given to the initial shape of the power profile, so that all rays are treated as equivalent and are implicitly assumed to begin deflecting as they leave the laser face; the criterion for determining k is that the beam may not be permitted to more than double its initial radius when it reaches target height. By contrast, in the present analysis we find that for the specific power profile chosen the interior rays do not deflect at all until they reach a certain height, and the criterion for determining k is that rays originating in from the edge by more than half the initial beam radius R may not begin to deflect until target altitude is reached. Thus, for conditions under which the effective absorption coefficient for heating is given by Eq. (4.7), the present study would indicate that the beam power is somewhat less diluted at target altitude than is indicated by the results in Ref. 2.

5.   SUMMARY

A solution has been found to a model equation, which approximates the true equation describing the ray trajectories. The approximation occurs in an expression for the amount of energy per unit volume deposited at any point along the ray path by the laser beam; it consists of replacing a time integral over the laser flux by the product of the instantaneous flux and the time. As a result, the time appears only as a trivial parameter in the model equation, rather than as an independent variable, as in the true equation.

The model equation has been solved for the special case of the initial flux distribution illustrated in Fig. 2; here the geometry is planar and the flux distribution is uniform in the region representing the interior of the beam (the negative axis) and descends exponentially in the region representing the edge of the beam (the positive axis). It is found that the rays of the interior region behave as illustrated in Fig. 3. That is, they go up undeflected to a height $z = |s/a|$, where $|s|$ is the original distance of the ray in from the edge,[*] and the dimensionless constant a is given by[**]

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[*] Stated conversely, at any height z the exterior region of diverging rays has eaten its way in from the original edge by a distance $|s| = az$.

[**] In this expression γ is the ratio of specific heats, $\gamma = c_p/c_v$, n_a is the ambient refractive index, P is the ambient pressure, k is the effective absorption coefficient for heating of the atmosphere by the laser light, t is the time, and F_o is the energy flux leaving the laser face in the interior, or uniform region.

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$$a = \left[\frac{(\gamma-1)(n_a^2-1)ktF_0}{2\gamma n_a^2 P} \right]^{\frac{1}{2}}$$

then they begin to deflect outwards (toward the positive axis) according to

$$x_1(s,z) \cong \frac{a^2}{2\lambda}(z-|s/a|)^2 \, ,$$

where x_1 is the displacement at height z of any ray from its original horizontal coordinates.

Thus it is seen that at any time the height at which an interior ray first begins to deflect outwards is proportional to its original distance in from the edge $|s|$, and is inversely proportional to the square root of ktF_0, the amount of heat per unit volume deposited along the ray path up to that time. Furthermore, once the ray begins to deflect outwards, the horizontal displacement per unit height is proportional to ktF_0. It is also inversely proportional to λ, the scale length for the exponential fall-off of the beam intensity at its outer edge. Thus, the more sharply the beam edge is shaped, the more sharply the interior rays will deflect, once they begin to do so.

The solutions from which these conclusions are drawn are not valid in all regions of space. They are, however, exact solutions to the model equation in the region in which the interior rays are undeflected, and are quite accurate in a very small region about which they first begin to deflect. They are not accurate elsewhere, so that it is not possible to ascertain from them the actual extent to which the interior rays a e displaced as a function of height.
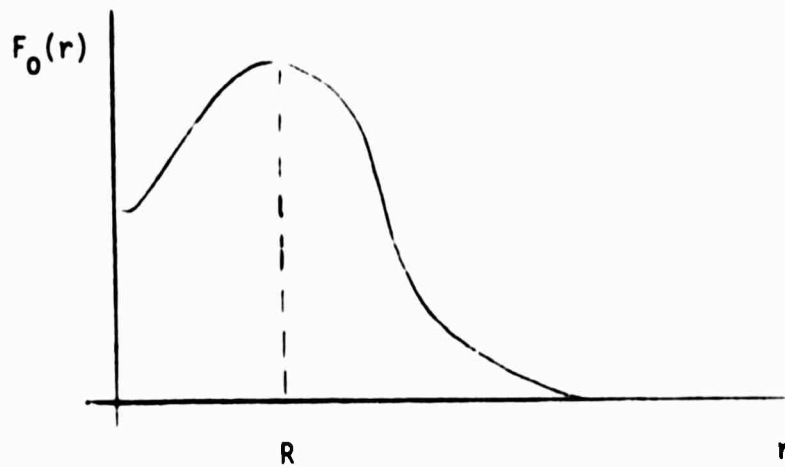
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Fig. 1. Radial distribution of flux $F_0(r)$ at the laser face for which the gradient of the refractive index points inward for r < R and outward for r > R. In this case the rays in the outer region r > R will diverge, but the rays in the inner region r < R will converge initially.
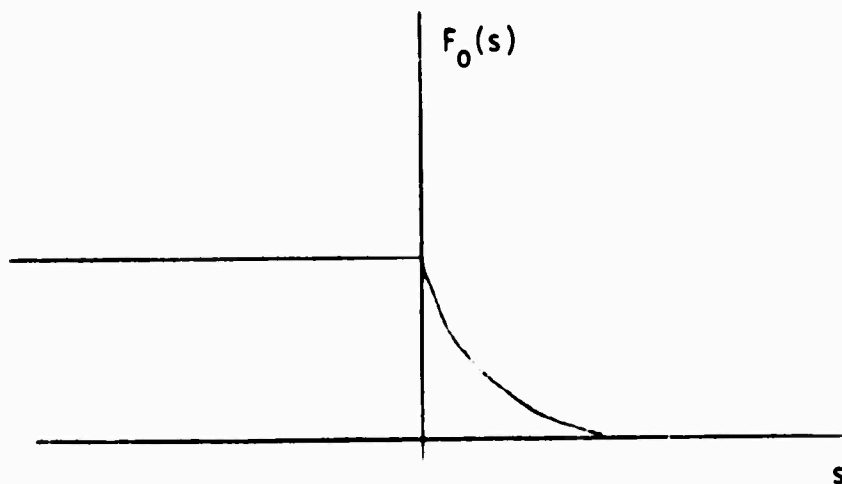

Fig. 2. Uniform distribution of flux $F_0(s)$ emanating from a semi-infinite laser face (s < 0), with an exponential fall-off at the edge. The rays originating in the uniform region along the negative s-axis are initially undeflected. Those originating along the positive s-axis are immediately deflected to the right.
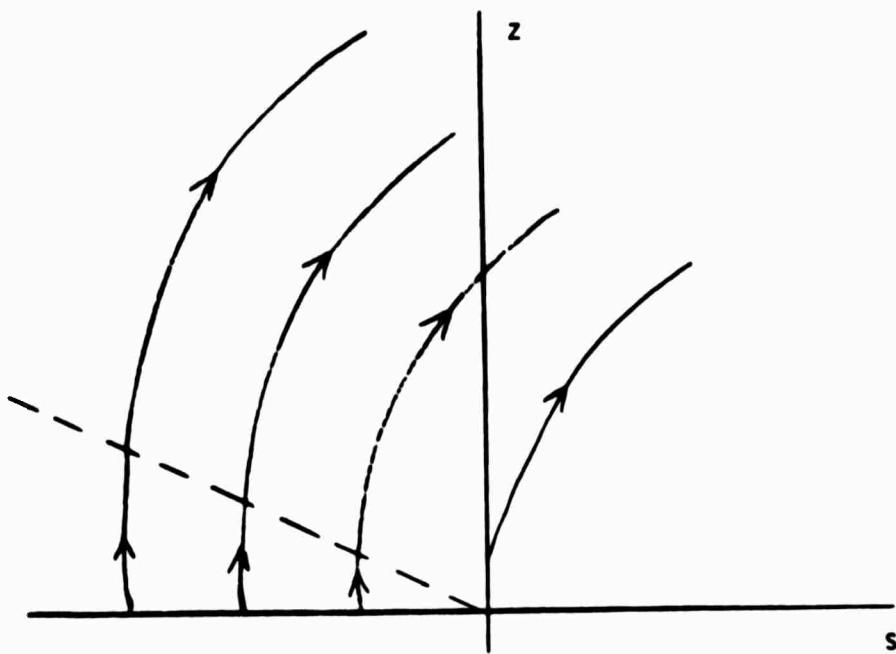
Fig. 3. Trajectories of rays emanating from the region of uniform
flux distribution (s ⪝ 0) depicted in Fig. 2. The rays
go up vertically until they intersect the dashed line,
then they begin deflecting to the right parabolically
with height z. The slope of the dashed line is inversely
proportional to the square root of the energy per unit
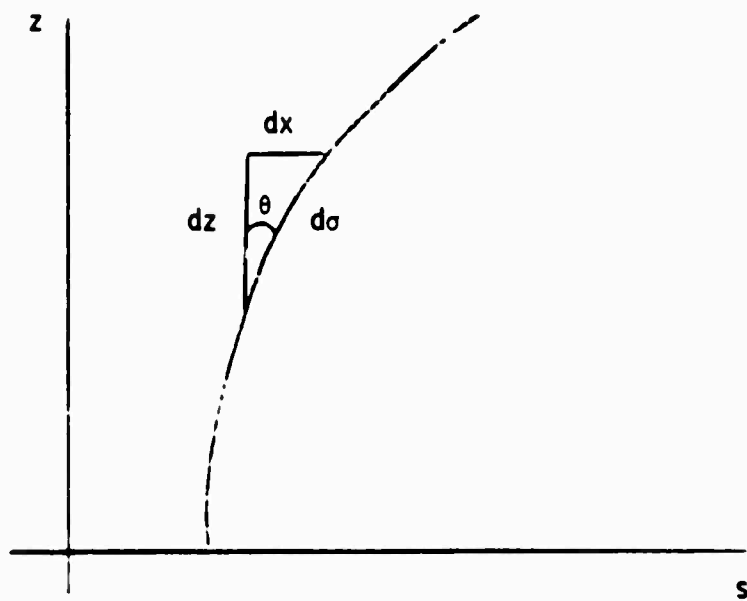volume deposited along the ray path.

Fig. 4. Typical ray trajectory, showing the angle θ and the incremental quantities dx, dz, and dσ at fixed s.
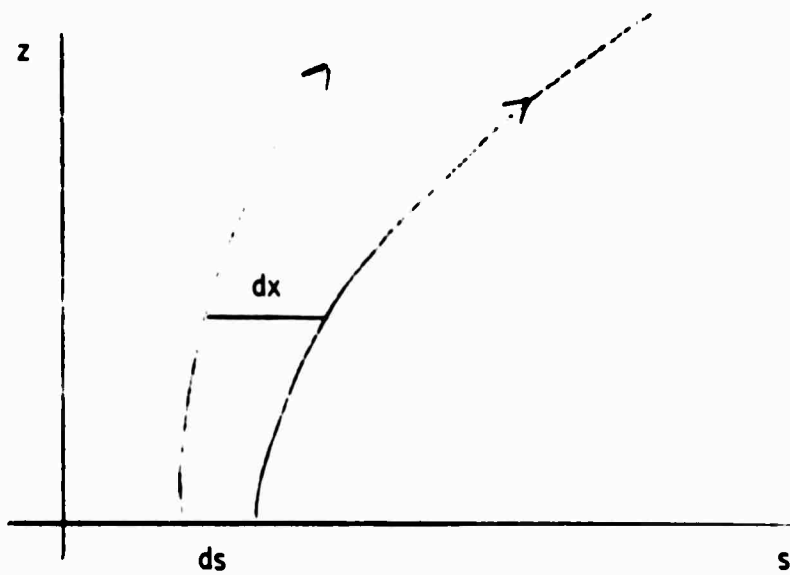

Fig. 5. Neighboring ray trajectories separated by an infinitesimal radial distance ds at the laser face, and by a corresponding radial distance dx at an arbitrary height z.

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2. Brueckner, et al., JMDR $\underline{2}$, 11 (1964).

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13 ABSTRACT

An approximate solution is found to the nonlinear, partial differential equation describing the thermal self-defocusing effect. The solution is for the special case of a flux distribution leaving the laser face, which has planar symmetry, is uniform over a semi-infinite region (simulating the interior of the beam) and falls off exponentially at the edge. The rays originating in the interior region are found to go up undeflected until they reach a height proportional to their original distance in from the edge and inversely proportional to the square root of the amount of heat per unit volume thus far deposited along the ray path. Then they begin to deflect outwards parabolically, more sharply for a larger heat density so far deposited, and also more sharply for a more steeply descending edge of the initial flux distribution.

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14. KEY WORDS	LINK A		LINK B		LINK C	
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Nonlinear propagation effect						
Shaped power profile						
Initial beam convergence						