Report 3010

NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER
Washington, D.C. 20007

A BULK CAVITATION THEORY WITH A SIMPLE EXACT SOLUTION

STRUCTURAL MECHANICS LABORATORY
RESEARCH AND DEVELOPMENT LABORATORY

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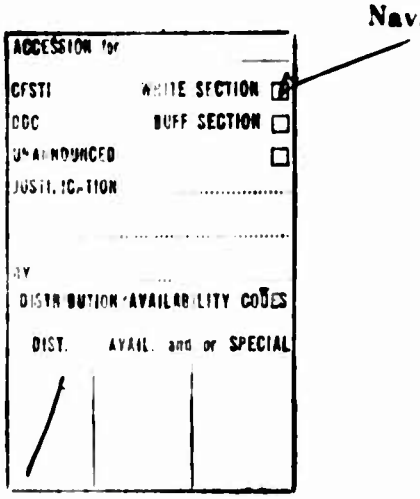

DEPARTMENT OF THE NAVY
NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER
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A BULK CAVITATION THEORY WITH A SIMPLE EXACT SOLUTION

by

G. V. Waldo, Jr.

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NOTATION

A	Small cross-sectional area of a water column
c	Shock wave velocity (≈ 5000 fps for sea water)
e	Base of natural logarithms (≈ 2.718 28)
g	Deceleration due to gravity (≈ 32.2 ft/sec^2)
g^*	Nondimensional deceleration due to gravity
h	Depth below surface
h'	Depth below surface ($h \geq h' \geq 0$)
k	Exponential depth decay constant
P	Pressure of incident shock wave at a given time after arrival
P_L	Pressure in the slab just above the cavitated zone
P_a	Air pressure (≈ 14.7 psi)
P_0	Peak pressure of shock wave
T	Time after arrival of the incident shock wave
t	Time at which the surface slab has grown to a given thickness
t'	Time after kickoff ($t \geq t' \geq 0$)
t_c	Closure time
t_∞	Time at which $y = \infty$
t^*	Nondimensional time at which the surface slab has grown to a given thickness
t_c^*	Nondimensional closure time
U	Kickoff velocity of the water at a given depth
U_0	Kickoff velocity of the water surface
V	Velocity of the water surface at a given time
V_g	Velocity of the water surface at a given time without the effect of gravity
V^*	Nondimensional velocity of the water surface at a given time
V_b^*	Nondimensional velocity of the water surface at a given time without the effect of gravity
y	Thickness of the surface slab at a given time
y'	Depth below surface ($y \geq y' \geq 0$)
y_b	Depth of water particle which falls back to its initial position before joining the surface slab

y_c	The closure depth
y^*	Nondimensional thickness of the surface slab at a given time
y_c^*	Nondimensional closure depth
y_1	Depth of upper bound of the below surface slab
y_2	Depth of lower bound of the below surface slab
z	Depth below surface
β	Angle which a plane wave makes with the surface
δ	Surface displacement
δ_g	Surface displacement without the effect of gravity
δ^*	Nondimensional surface displacement
δ_g^*	Nondimensional surface displacement without the effect of gravity
ρ	Density (~ 2 slugs/ft^3 for sea water)
τ	Surface cutoff time at a given depth
θ	Exponential time delay constant

ABSTRACT

A theory of bulk cavitation is presented in which an equation governing the motion of the water surface is derived. This equation is found to have a simple exact solution with which calculations can be performed without a computer. This solution not only yields the results of Walker and Gordon for the closure depth and time, but also predicts the complete surface velocity history rather than just a straight line model of the surface velocity history. Simple exact equations for the surface slab thickness history and the surface displacement history are also derived. The theory is found to be physically plausible and is in reasonable agreement with the experimental data of Walker and Gordon.

ADMINISTRATIVE INFORMATION

The work of this report was funded under Defense Atomic Support Agency Subtask NB004(3089).

INTRODUCTION

When a shock wave caused by an underwater explosion reaches the surface, it is reflected as a negative pressure wave which is superposed on the incident positive pressure wave. At a certain depth below the surface, the total pressure may become negative. At about this depth, the water cavitates since under the usual test conditions, water seems unable to withstand substantial negative pressures.[1] The cavitation may persist down to significant depths and cause what is known as "bulk cavitation." A knowledge of this phenomenon is necessary to determine the motion of the water and thus the effect of underwater explosions on surface ships and submarines.

This report presents an incompressible theory of bulk cavitation in which an equation governing the motion of the water surface is derived. This equation is found to have a simple exact solution with which calculations can be performed without a computer. This solution not only yields the results of Walker and Gordon[1] for closure depth and time, but also predicts the complete surface velocity history rather than just a straight line model of the surface velocity history. Other theoretical details have been presented in reports by Cushing et al.[2, 3] A more involved theory which includes the compressibility of the water is being studied and will be the subject of later work. It is expected that the results of the compressible theory can also be applied without the aid of a computer.

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[1]References are listed on page 11.

THEORY OF BULK CAVITATION

After a shock wave reaches the surface, it is reflected as a negative pressure wave. At a given depth h, the time interval $\tau(h)$ between the passage of the front of the shock wave and the front of the reflected wave is called the "cutoff time." If for all depths h the total pressure of the water immediately after cutoff is positive, then no cavitation occurs. However, if the total pressure is negative immediately after cutoff, then the water cavitates. In this case, it is assumed that cavitation is present at all depths h just after cutoff. When the water cavitates, the water particles at a depth h are "kicked off" with a vertical velocity $U(h)$ which is called the "kickoff velocity." Using the results presented in this report, it can be shown that the cutoff times of interest are much smaller than the times characterizing the development of bulk cavitation. Therefore, it is assumed that the time of kickoff is the same for all particles regardless of depth. A more detailed model will be considered in later work on the compressible theory.

In the theory of the present report, the water is assumed to be incompressible. It is also assumed that all the water has zero pressure just after cutoff; this assumption is justified because the pressure in the cavitated zone is approximately the water vapor pressure which is negligible for cases of practical interest. Immediately after kickoff, air pressure decelerates the water particles at the surface. The particles just below the surface then join up with the surface particles. This is the beginning of the formation of the "surface slab." Air pressure continues to decelerate the surface slab so that even more particles below the surface join the surface slab. In this manner, the thickness of the surface slab grows from zero initial thickness. Figure 1 shows a surface slab which has grown to a thickness y at the time t.

As explained later, there is a "bottom zone" (see Figure 1) in which the water motionless. At time t, this zone will have the depth y_b. Since y_b decreases as a function of time, this bottom zone moves upward. Because the thickness of the surface slab increases as a function of time, there will be a certain time, called the "closure time," at which the surface slab meets the bottom zone. After closure, the water is assumed to have no further movement.

KICKOFF VELOCITY

The impulse imparted by a shock wave to a column of water of small cross-sectional area A above a depth h is $\int_0^{\tau(h)} AP(T)\,dT$, where $P(T)$ is the pressure of the incident shock wave at a time T after arrival. This impulse must be equal to the momentum of the column of water which is $\int_0^h \rho A U(h')\,dh'$, where ρ is the density of the water. Equating these two expressions, differentiating both sides with respect to h, and dividing both sides by ρA gives

$$U(\lambda) = \frac{P(\tau(\lambda))}{\rho} \frac{d\tau(\lambda)}{d\lambda} \qquad [1]$$

SURFACE DISPLACEMENT

Since the particles in the cavitated zone are all at zero pressure, the only deceleration they experience is gravity. Thus, at any time t' (where $t \geq t' \geq 0$), the displacement of a particle which was kicked off at a depth λ is

$$U(\lambda)\, t' - \frac{g}{2}\, t'^2$$

where g is the deceleration due to gravity. That is, "free fall" displacement will occur until the particle joins the surface slab. At the instant this particle joins the surface slab, it will have the displacement

$$U(\lambda)\, t - \frac{g}{2}\, t^2$$

Since the water is incompressible, the thickness of the surface slab y must be equal to λ. Thus the displacement of the surface is

$$\begin{aligned} \delta &= U(\lambda)\, t - \frac{g}{2}\, t^2 \\ &= U(y)\, t - \frac{g}{2}\, t^2 \end{aligned} \qquad [2]$$

Differentiation of Equation [2] with respect to t yields the velocity of the surface

$$V = \frac{d\delta}{dt}$$

$$= \frac{d[U(y)\, t]}{dt} - gt \qquad [3]$$

$$= U(y) + t \frac{dU(y)}{dy} \frac{dy}{dt} - gt \qquad [4]$$

MOMENTUM EQUATION

Now consider the water in the surface slab at time t having a thickness y. At kickoff, all of this water was undisplaced and had the kickoff velocity distribution $U(y')$ for $y \geq y' \geq 0$. The momentum of a column of small cross-sectional area A of this water was

$$\int_0^y \rho A U(y')\, dy'$$

However, in the time interval from kickoff to the time t, the momentum of the column of water of cross sectional area A has been reduced due to air pressure P_a and† gravity by the amount

$$P_a At + \rho Agyt$$

If there is no momentum flux into (or out of) the sides of this water column, the momentum of the column at time t is

$$\int_0^y \rho AU(y')\, dy' - P_a At - \rho Agyt$$

which must be equal to ρAVy. That is,

$$\rho AVy = \int_0^y \rho AU(y')\, dy' - P_a At - \rho Agyt \qquad [5]$$

which is the momentum equation. The effect of momentum flux into (or out of) the sides of the water column will be considered in later work on the compressible theory.

SURFACE SLAB THICKNESS

Substituting Equation [3] into Equation [5], dividing both sides by A, and adding ρgyt to both sides gives

$$\rho y \frac{d[U(y)\,t]}{dt} = \rho \int_0^y U(y')\, dy' - P_a t \qquad [6]$$

It should be noted that y is independent of y in Equation [6]. This is expected since the rate at which the surface slab is being decelerated by gravity is the same as the rate at which the particles in the cavitated zone are being decelerated by gravity.

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†Actually the value of P_a should be the air pressure minus the water vapor pressure. However, the water vapor pressure is negligible for cases of practical interest. In these cases, the small error produced by neglecting the water vapor pressure tends to be counteracted by the small error caused by neglecting the increase in air pressure due to the sudden upward movement of the water surface.

Equation [6] can be written as

$$yt \frac{dU(y)}{dy} \frac{dy}{dt} = \int_0^y U(y')\,dy' - yU(y) - (P_a'/\rho)\,t \qquad [7]$$

or

$$\frac{dy}{dt} = \frac{\int_0^y U(y')\,dy' - yU(y) - (P_a/\rho)\,t}{yt \frac{dU(y)}{dy}} \qquad [8]$$

It will be shown later that for all cases of interest y is a monotonic increasing function of t. Therefore, we can let t become the dependent variable and y become the independent variable. So, reciprocation of both sides of Equation [8] gives

$$\frac{dt}{dy} = \frac{yt \frac{dU(y)}{dy}}{\int_0^y U(y')\,dy' - yU(y) - (P_a/\rho)\,t} \qquad [9]$$

Equation [9] relates the time to the surface slab thickness and consequently governs the motion of the water surface. It can be easily verified by direct substitution that the solution of Equation [9] for which $t = 0$ at $y = 0$ is

$$t = 2 \frac{\rho}{P_a} \left[\int_0^y U(y')\,dy' - yU(y)\right] \qquad [10]$$

Differentiating Equation [10] with respect to y and substituting the resulting expression into Equation [4] yields

$$V = U(y) - \frac{P_a}{2\rho} \frac{t}{y} - gt \qquad [11]$$

PHYSICAL CONSIDERATIONS

Equation [1] demonstrates that for most cases of interest $U(y)$ is a monotonic decreasing function of y. Thus, using Equation [10], it can be shown that $dt/dy \geq 0$. That is, y is a monotonic increasing function of t. If $U(y)$ were not a monotonic decreasing function of y, several slabs would form (see Appendix). Also, at $y = 0$, $dt/dy = 0$, i.e., initially the surface grows infinitely fast. This can be seen physically by remembering that initially the

surface slab is of zero thickness and is being infinitely decelerated by the air pressure. Thus, the particles of water just under the surface will join the surface slab immediately.

Equation [1] also shows that as $y \to \infty$, $U(y) \to 0$ and $\int_0^y U(y')\,dy' \to$ constant for all cases of interest. Therefore, Equation [10] implies that as $y \to \infty$, $t \to t_\infty$ assymptotically, i.e., as $t \to t_\infty$, the slab grows infinitely fast and gets infinitely thick.

It is now convenient to define the surface displacement without the effect of gravity as (see Equation [2])

$$\begin{aligned} \delta_g &= \delta + \frac{g}{2} t^2 \\ &= U(y)\, t \end{aligned} \qquad [12]$$

and the surface velocity without the effect of gravity as (see Equation [11])

$$\begin{aligned} V_g &= \frac{d\delta_g}{dt} \\ &= V + gt \\ &= U(y) - \frac{P_a}{2\rho} \frac{t}{y} \end{aligned} \qquad [13]$$

It is seen from the above discussion that as $y \to \infty$, $\delta_g \to 0$. So it is concluded (Equation [12]) that initially δ_g rises from zero and eventually falls back to zero at $t = t_\infty$. Physically, the air pressure eventually reverses the upward motion of the surface and forces it back down to its original level. The fact that the final surface level is the same as the initial surface level is consistent with the assumption of incompressibility. This implies (see Equation [13]) that V_g must decrease from its initial value $U(0)$ to zero and become negative. Finally, at $t = t_\infty$, Equation [11] shows that V_g must be zero. This means that the acceleration of the surface must be zero some time after the velocity becomes negative but before $t = t_\infty$.

Because the surface slab is assumed to be incompressible and of uniform velocity, the pressure in the slab must vary linearly with depth from P_a at the surface to P_L just above the cavitated zone. Thus, the total force per unit area on the slab is $P_L - P_a - \rho y g$ which must be equal to $\rho y\, dV/dt$, i.e.,

$$\rho y \frac{dV}{dt} = P_L - P_a - \rho g y \qquad [14]$$

which becomes

$$P_L = -\frac{P_a^2}{4\rho}\,\frac{t}{y^2\,\dfrac{dU(y)}{dy}} \qquad [15]$$

by using Equations [10] and [11].†

Expanding $U(y)$ in a Taylor series about $y=0$ and substituting Equation [10] into Equation [15], it can be shown that as $y \to 0$, $P_L \to P_a/4$. It can also be shown that $P_L \to \infty$ as $y \to \infty$. Equation [15] demonstrates that P_L is always positive. If this were not the case, the surface slab itself would be a cavitated zone which is physically meaningless.

BOTTOM ZONE

It must also be assumed that if a particle which was kicked off at a depth y_b falls back to the same depth *before* it joins the surface slab, then it has no further movement. That is, it remains stationary after it returns to its initial position rather than continuing to fall. If this were not the case, the final water level would be different than the initial water level. This assumption is consistent with the assumption of incompressibility. The time t at which this particle returns to its initial depth is

$$t = \frac{2U(y_b)}{g} \qquad [16]$$

From this equation and the fact that $U(y_b)$ is a monotonic decreasing function of y_b and that $U(y_b) \to 0$ as $y_b \to \infty$ for all cases of interest, it is realized that at any time t there is a depth below which the water is motionless. This deep zone of motionless water is called "the bottom zone" (see Figure 1). Initially, at $t=0$, $y_b = \infty$. Later, as t increases, y_b decreases.

CLOSURE

Eventually, there will be a time t_c at which $y_b = y \equiv y_c$. That is, t_c is the time at which the surface slab meets the bottom zone. After this time, all the water is assumed to be motionless. The time t_c is called "the closure time," and the depth y_c is called "the closure depth." Substitution of Equation [16] into Equation [2], when $y_b = y \equiv y_c$, shows that the displacement of the surface at the closure time is zero. This is consistent with the

†Alternatively, P_L can be found by realizing that it is the momentum with respect to the surface slab which is imparted to a unit area in a unit time, i.e., $P_L = \rho\, dy/dt\,(U(y) - V_g)$, which can be shown to be the same as Equation [15].

assumption of incompressibility. Equating Equations [10] and [16], setting $y_b = y - y_c$, and multiplying both sides by $y'2$ gives

$$\frac{g\rho}{P_a}\left[\int_0^{y_c} U(y')\,dy' - y_c U(y_c)\right] = U(y_c) \qquad [17]$$

which can be used for determining y_c and thus t_c. Equation [17] gives the same values of y_c (and t_c) as the model in Reference 1.

Since $U(y_c) = gt_c'2$, it is seen by using Equation [11] that the velocity of the surface slab at the closure time is $-(g + P_a'(\rho y_c))t_c'2$ which is also the same as that predicted by the model in Reference 1.† This completes the general exact solution of the bulk cavitation problem for this theory.

EXPONENTIAL SHOCK WAVE

The most important special case of a shock wave exponentially decreasing with time will now be examined. In this case, the pressure history has the form

$$P(T) = P_0 e^{-T/\theta} \qquad [18]$$

where P_0 is the peak pressure and θ is the exponential time decay constant. For a plane wave making an angle β with the surface, the surface cutoff time is

$$\tau(y) = \frac{2}{c}\, y \cos\beta \qquad [19]$$

where c is the velocity of the shock wave. Substituting Equations [18] and [19] into Equation [1] yields

$$U(y) = \frac{2P_0}{\rho c}\cos\beta\, e^{-\left(\frac{2}{c}\,\frac{y\cos\beta}{\theta}\right)} \qquad [20]$$

$$= U_0\, e^{-y/k} \qquad [21]$$

where

$$U_0 = \frac{2P_0}{\rho c}\cos\beta \qquad [22]$$

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†A rough estimate of the closure pulse can be obtained by assuming that the problem is strictly one-dimensional. The closure pulse is then a square wave which arrives at a point of depth z at the time $t_c + |z - y_c|/c$, where c is the velocity of sound in water. The duration of the pulse is $2z/c$ for $0 \leq z \leq y_c$ and $2y_c/c$ for $z > y_c$. In this approximation, z should not be too deep. Perhaps it should be less than about $3y_c$. The pressure of the pulse is $\rho c/2$ times the absolute value of the velocity of the surface slab at the closure time.

is the kickoff velocity of the water surface and

$$k \equiv \frac{c\theta}{2 \cos \beta} \qquad [23]$$

is the exponential depth decay constant.

NONDIMENSIONAL VARIABLES

The nondimensional time t^* is defined as

$$t^* \equiv t/\left(\frac{\rho}{P_a}\ U_0\, k\right) \qquad [24]$$

and the nondimensional thickness y^* is defined as

$$y^* \equiv y/k \qquad [25]$$

Using these definitions Equation [10] becomes

$$t^* = 2\,[1 - (1 + y^*)e^{-y^*}] \qquad [26]$$

for the exponential wave. Similiarly, defining the nondimensional velocity V^* as

$$V^* \equiv V/U_0 \qquad [27]$$

and the nondimensional gravitational deceleration as

$$g^* \equiv g\rho k/P_a \qquad [28]$$

Equation [11] becomes

$$V^* = e^{-y^*} - \frac{t^*}{2y^*} - g^*\, t^* \qquad [29]$$

for the exponential wave. Finally, Equation [2] determines the nondimensional displacement

$$\begin{aligned} \delta^* &\equiv \delta/\left(\frac{\rho}{P_a}\ U_0^2\, k\right) \\ &= e^{-y^*} t^* - \frac{g^*}{2}\ t^{*2} \end{aligned} \qquad [30]$$

for the exponential wave.

CLOSURE DEPTH AND TIME

Using Equation [17], it is found in the case of the exponential wave

$$e^{y_c^*} = 1 + y_c^* + 1/g^* \qquad [31]$$

where $y_c^* \equiv y_c/k$ is the nondimensional closure depth.
Also, the nondimensional closure time is

$$t_c^* \equiv t(y_c)/\left(\frac{\rho}{P_a}\ U_0 k\right)$$

$$= t^*(y_c^*) \qquad [32]$$

$$= \frac{2/g^*}{1 + y_c^* + 1/g^*}$$

Equations [31] and [32] for the closure depth and time are the same as the results of of the model presented in Reference 1 as expected. This means that the simple model of Reference 1 is just as good as this more detailed theory in finding the closure depth and time. However, the surface slab thickness, velocity, and displacement *histories* are found more accurately by using Equations [24] through [30] of this report. The experimental velocity and displacement histories in Reference 1 seem to verify this more detailed theory. Unfortunately the accuracy of these histories is not good enough to conclude that this theory is significantly better for predictions than the simplified model in Reference 1.

CALCULATIONAL AIDS

As an aid to calculations, Equation [26] relating t^* and y^* has been plotted in Figure 2. As explained before, y^* grows infinitely fast at t^* 0. At t^* = 0.5285, the curve has an inflection point. At t^* = 2, the surface slab is again growing infinitely fast and is of infinite thickness.

Figure 3 shows the nondimensional velocity and displacement without the effect of gravity (i.e., $V_g^* = V^* + g^* t^*$ and δ_g^* $\delta^* + (g^* 2)t^{*2}$, plotted using Equations [29] and [30]) as functions of t^*. At t^* = 0, the deceleration of the surface is infinite. Later, at t^* = 0.715, V_g^* is zero and δ_g^* has its maximum value, 0.204. Still later, at t^* = 1.661, V_g^* has its minimum value, −0.218. Finally, at t^* = 2, V_g^* and δ_g^* are zero and the acceleration is infinite. Figure 4 shows an enlarged view of the early part of Figure 3.

An example of the use of these graphs is illustrated in Figure 5. In this figure, the velocity history resulting from the detonation of a 10,000-lb charge of HBX-1 at a depth of 100 ft and horizontal standoff of 400 ft is presented according to the theory of this report. Also shown are the predictions in accordance with the theory of Walker and Gordon[1] and the experimentally observed velocity history.[1] The curves are terminated at the time of arrival of the very complicated velocity associated with the expansion of the explosion bubble in this particular experiment. It can be seen that the theory of this report gives a more realistic characterization of the velocity history than does the theory of Walker and Gordon.[1] However, there are other experimental velocity histories in Reference 1 to which the theory of Walker and Gordon appears to give a better fit than the theory of this report at later times. The accuracy of the experimental data is not sufficient to determine that the theory of this report is significantly better.

SUMMARY AND CONCLUSIONS

A bulk cavitation theory which has a simple exact solution has been presented. The solution is physically plausible and is in reasonable agreement with the experimental data of Walker and Gordon.[1] Calculations can now be made for the complete surface slab thickness, velocity, and displacement histories using Equations [10], [11], and [2] for any monotonic decreasing kickoff velocity distribution. The most important special case of the exponential shock wave can be treated with the aid of Figures 2, 3, and 4. All calculations can be performed without the aid of a computer.

REFERENCES

1. Walker, R. R. and Gordon, J. D., "A Study of the Bulk Cavitation Caused by Underwater Explosions," David Taylor Model Basin Report 1896 (Sep 1966).

2. Cushing, Vincent, "Study of Bulk Cavitation and Consequent Water Hammer," Engineering-Physics Company Final Report on Contract NONR-3389(00) EPCO Project 106 (Oct 31, 1961).

3. Cushing, Vincent et al., "Three-Dimensional Analysis of Bulk Cavitation," Engineering-Physics Company Interim Report on Contract NONR-3709(00), EPCO Project 106 (Sep 24, 1962).

APPENDIX

$U(y)$ FUNCTIONS WHICH ARE NOT MONOTONIC DECREASING

If $U(y)$ were not monotonic decreasing, a slab other than the surface slab would instantly form starting at a depth y_1 shallower than the point at which $U(y)$ begins to increase and extending to some depth y_2 deeper than the point at which $U(y)$ is again decreasing (see Figure 6). Since the water is incompressible, this slab must be of uniform velocity and of zero pressure. Because it must have the same momentum as the water in it had initially, the velocity of this slab is $\frac{1}{y_2 - y_1} \int_{y_1}^{y_2} U(y')\,dy'$ which must be equal to $U(y_1)$ and to $U(y_2)$. The bulk cavitation problem in this case can now be treated using the procedure in the main text by replacing $U(y)$ by $\frac{1}{y_2 - y_1} \int_{y_1}^{y_2} U(y')\,dy' = U(y_1) = U(y_2)$ for $y_1 < y \leq y_2$. When the depth of the surface slab is y_1, it will instantly grow to the depth y_2. At this time, there will be a sudden increase in the vertical velocity of the surface. If there is more than one such increase in the $U(y)$ function, these additional increases can be treated in a similar fashion.

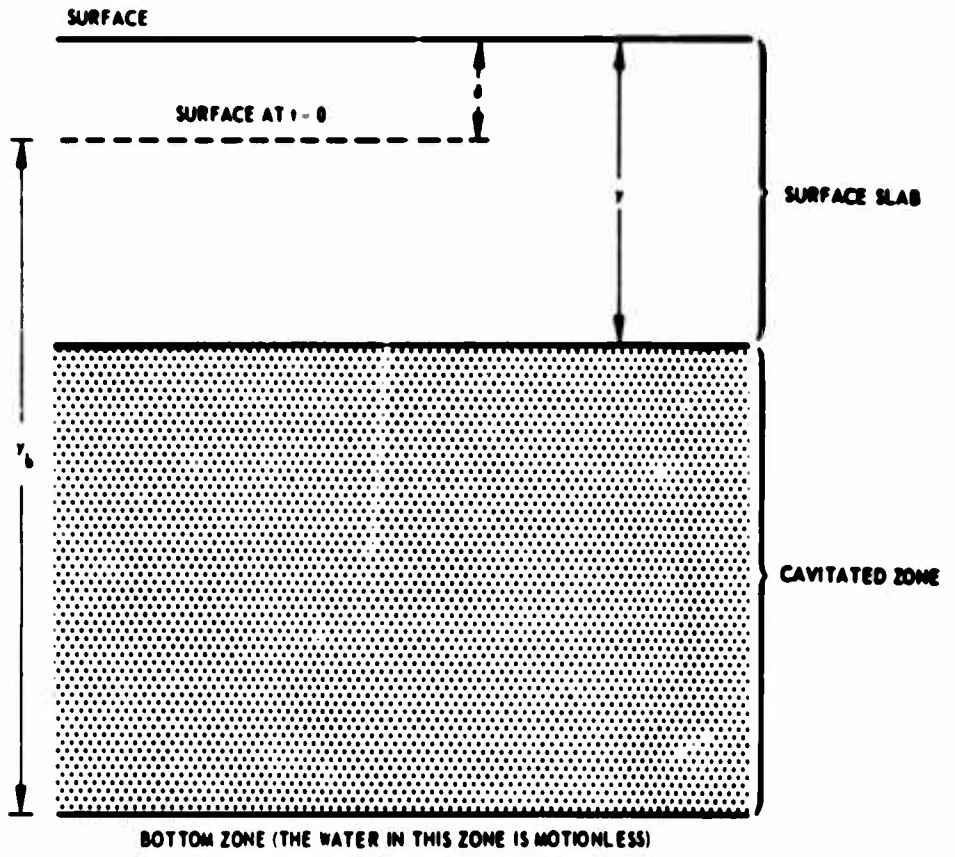

Figure 1 – Bulk Cavitation at Time t

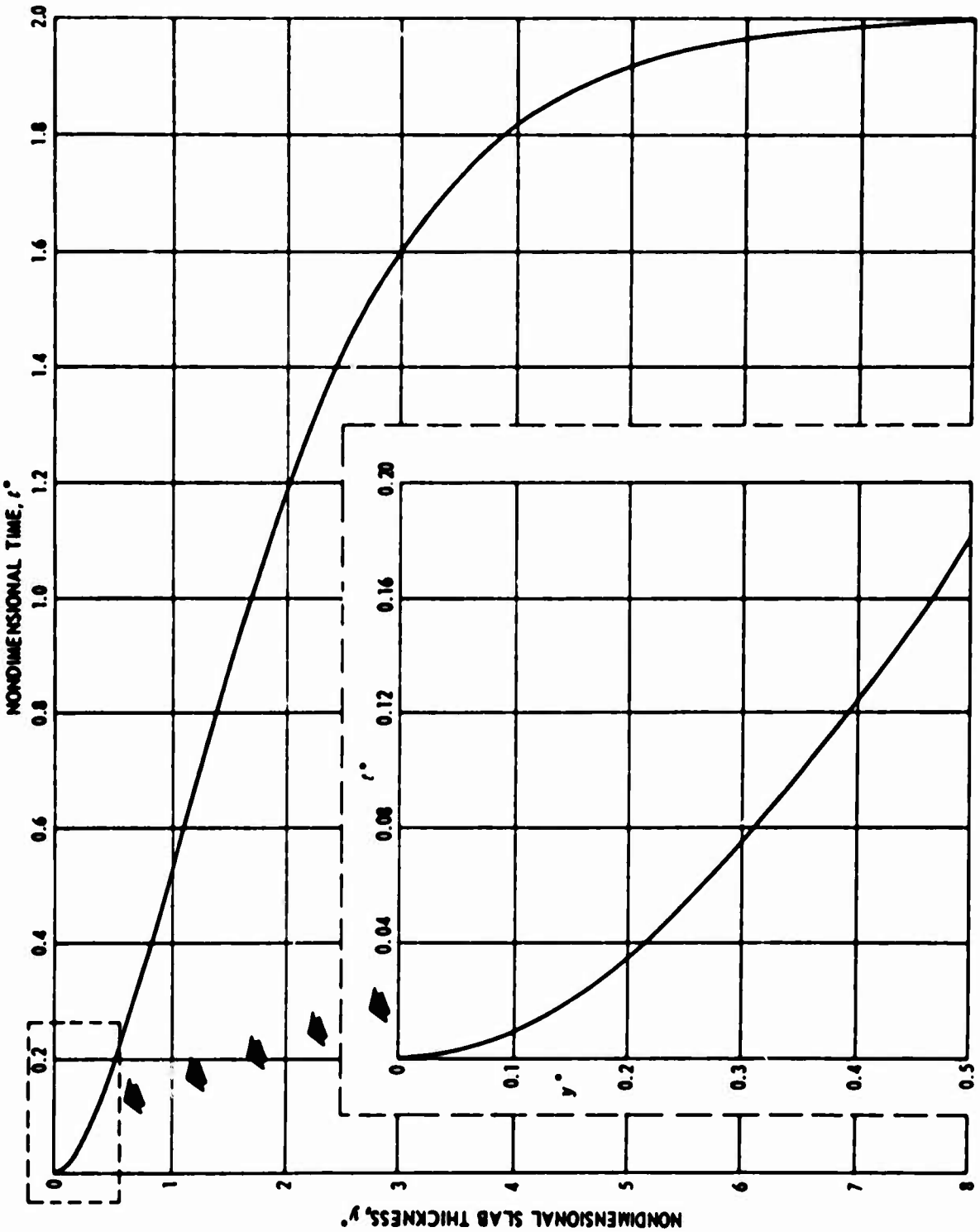

Figure 2 – Nondimensional Slab Thickness versus Nondimensional Time

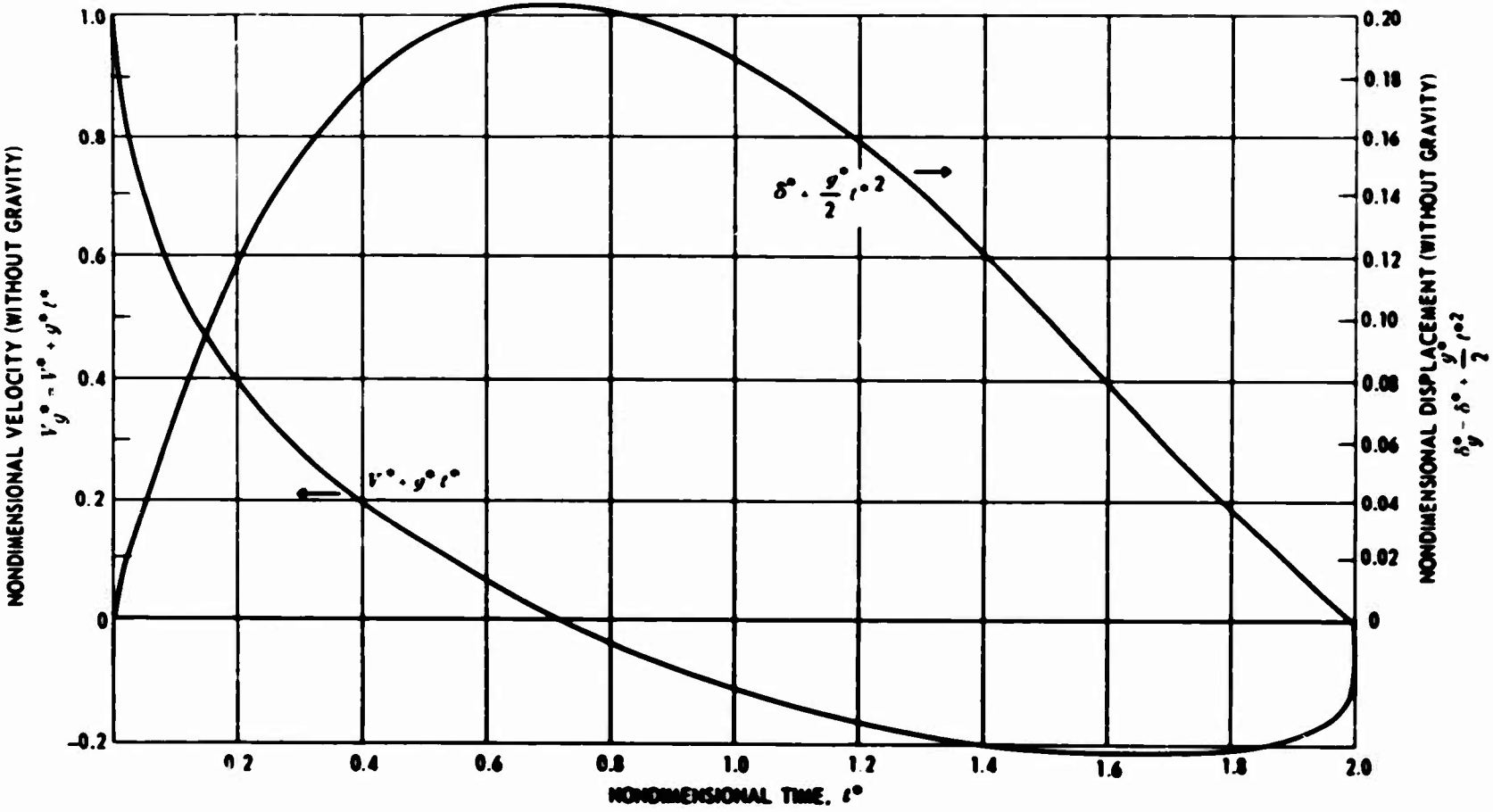

Figure 3 – Nondimensional Velocity and Displacement without the Effect of Gravity versus the Nondimensional Time

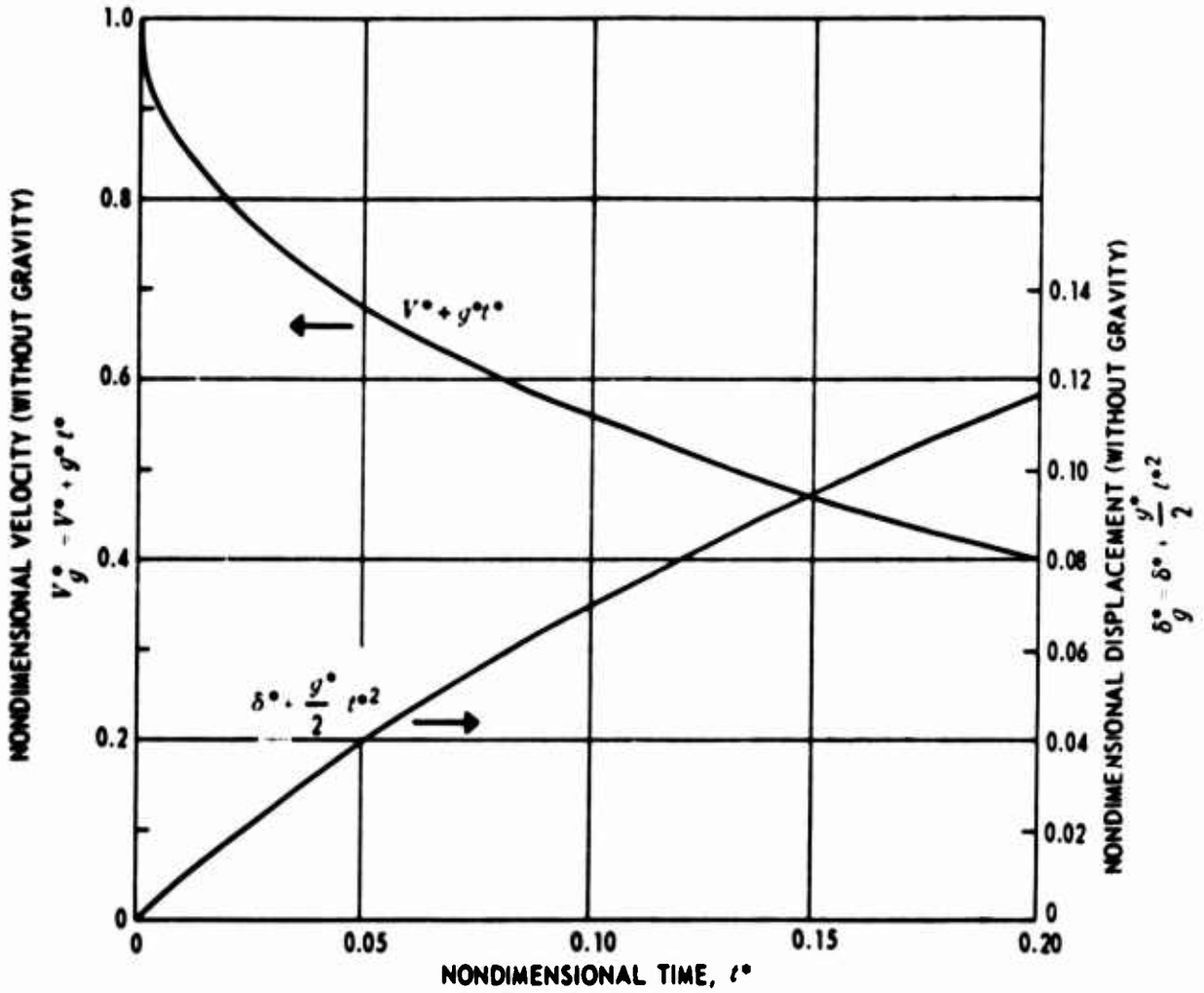

Figure 4 – Early Nondimensional Velocity and Displacement without the Effect of Gravity versus the Nondimensional Time

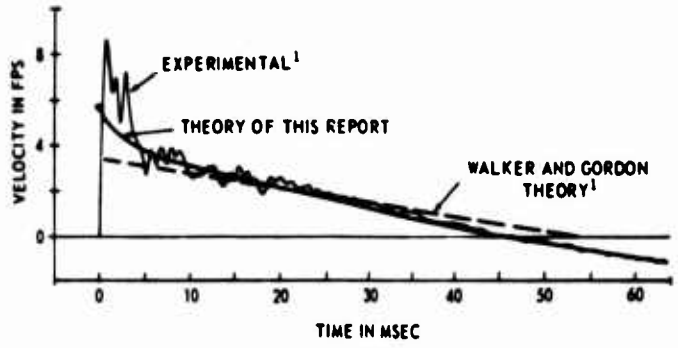

Figure 5 – Comparison of Experimental and Theoretical Velocity Histories

Charge: 10,000 lb HBX-1, burst depth: 100 ft, horizontal standoff: 400 ft.

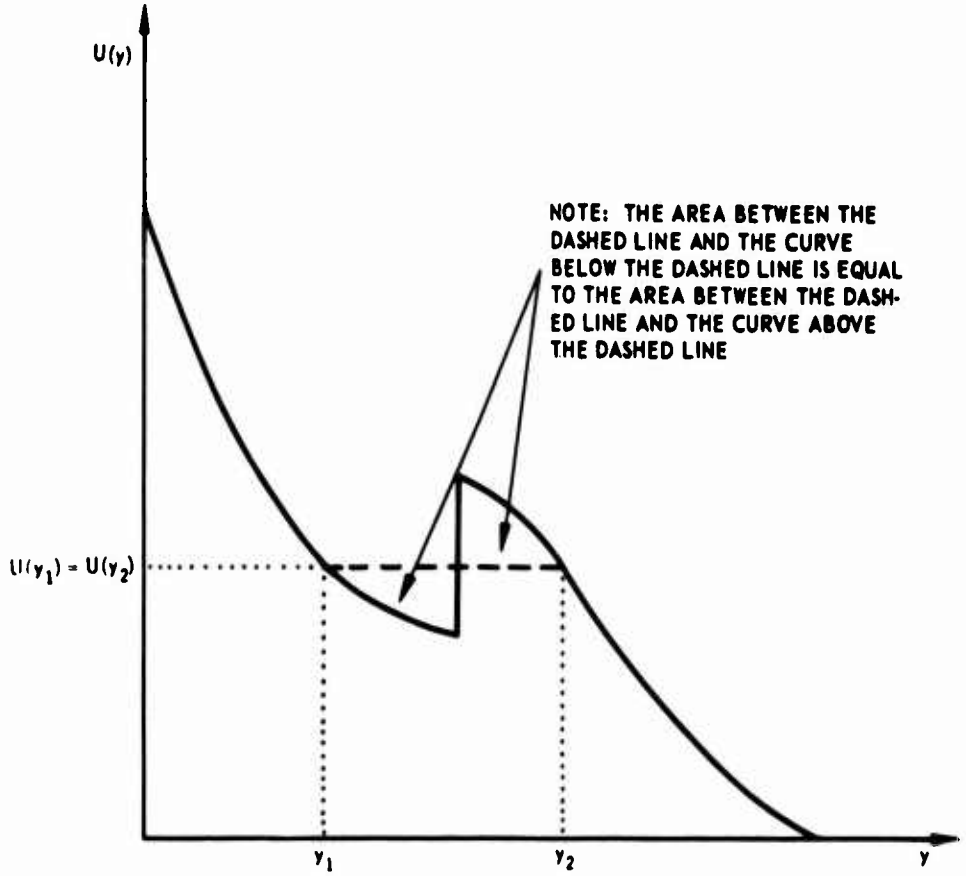

Figure 6 – A $U(y)$ Function Which Is Not Monotonic Decreasing

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G. V. Waldo, Jr.

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13. ABSTRACT

A theory of bulk cavitation is presented in which an equation governing the motion of the water surface is derived. This equation is found to have a simple exact solution with which calculations can be performed without a computer. This solution not only yields the results of Walker and Gordon for the closure depth and time, but also predicts the complete surface velocity history rather than just a straight line model of the surface velocity history. Simple exact equations for the surface slab thickness history and the surface displacement history are also derived. The theory is found to be physically plausible and is in reasonable agreement with the experimental data of Walker and Gordon.

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