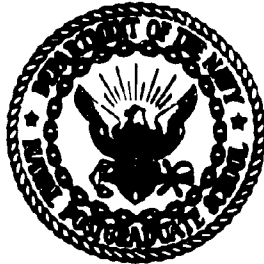# UNITED STATES
NAVAL POSTGRADUATE SCHOOL

THESIS

AN ECONOMIC MODEL OF THE

NAVAL POSTGRADUATE SCHOOL

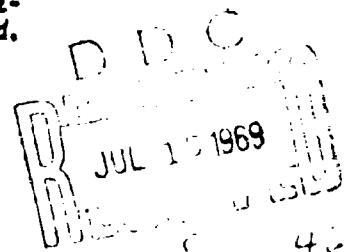by

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AN ECONOMIC MODEL OF THE

NAVAL POSTGRADUATE SCHOOL

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ABSTRACT

The increased size and complexity of educational institutions has created a need for an economic model of such institutions as an aid to planning and resource allocation. It is shown that economic theory can be applied to an analysis of the Naval Postgraduate School. A modified activity analysis technique is used to model the school's operation. The mathematical programming structure of the model emphasizes the objective of efficient operation. The necessary conditions of the programming problem are used to characterize decision rules for efficient operations. The decision rules stress the complexity of the problems facing school administrations. Application of the comparative statics technique to the model provides relationships which demonstrate that the total marginal effect of parameter variation can be decomposed. The components of these effects are interpreted as income, substitution, and revaluation effects.

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CHAPTER I

INTRODUCTION

In recent years there has been an increasing interest in the economics of higher education. This interest is chiefly due to the rapid growth of schools and the difficulties this growth has created. An increasing demand for the resources required by education has accompanied the growth of schools. This has raised many questions concerning the returns of education to society. Also, the expansion of many schools has made school administration a more complex task. Currently, most school administrations do not have available the formal economic tools with which to approach such problems as long range planning or the allocation of resources within the school. At this time there is considerable interest in the application of economic techniques to higher education to develop useful tools.

The purpose of this thesis is to develop an economic model of the Naval Postgraduate School. The model indicates the relative benefits of alternative decisions and exhibits the complexity of the overall decision problem. By demonstrating the complete effect of a decision, the model may assist the school's administration with the problems of planning and resource allocation.

The next chapter of the thesis presents a justification for the use of economic techniques. This is important, for some portions of economic theory are not always applicable

to public enterprises such as the Naval Postgraduate School.
Chapter III covers a description of the school's operation.
The operation is then related to a mathematical structure in
the form of a modified activity analysis. Chapter IV is con-
cerned with formulating the mathematical structure as a prob-
lem of efficient operation. Administrative decision rules
are analyzed and the feasibility of reformulating the struc-
ture as an implicit function is discussed. In Chapter V,
the comparative statics of the model are investigated.

CHAPTER II

THE SCHOOL IN ECONOMIC THEORY

A school is an institution which provides educational services to students. The educational process involves the combining of students with faculty, classrooms, libraries, and other resources. Culminating the process is the graduation of individuals appropriately credentialed in some field. Since a school can thus be considered as a producer of graduates, it seems reasonable to assume the traditional economic theory of the producing institution, the firm, is applicable. Unfortunately there are subtle differences between school and firm with respect to their underlying technologies and environments. These differences will be examined in the succeeding paragraphs.

The differences that exist between school and firm may be more easily understood when both institutions are considered as conflict systems. A conflict system is defined by two conditions. First, the system must be composed of basic elements, each of which has a preference ordering over all the possible states of the system. Second, there must be conflict among the preference ordering of the basic elements such that the system is unable to simultaneously satisfy the preferences of all the basic elements.[1]

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[1] James G. March, "The Business Firm as a Political Coalition," _Journal_ _of_ _Politics_ (October, 1962), pp. 663-664.

The firm is classified as a conflict system by consider-
ing the basic elements to be such subdivisions of a firm as
the sales and production departments. Conflict can arise
between the preferences of the two departmental organiza-
tions. For example, if the sales department organization
maximizes sales while the production department organization
attempts to improve product quality, conflict can arise with
respect to internal resource allocation. In the context of
the school, the various curriculum are considered as the
basic elements when classifying the school as a conflict
system. For example, each curriculum organization might de-
sire to increase the quantity of graduates it produces. In
turn, this increases the requirements for such resources as
students and faculty. If the school has insufficient re-
sources to satisfy the requirements of all the curriculum
organizations, conflict will arise among the curriculum
organizations for available resources.

The resolution of conflict within a system is classi-
cally described using two different techniques. The first
technique assumes that conflicts are resolved by cumulative
decisions within the basic elements. The preference ordering
of each basic element must be completely described and a de-
cision process modeled which will at least describe the equi-
librium conditions for the system. The second technique de-
scribes conflict resolution in terms of a superordinate goal.
This requires that the system behave as if there existed a
joint preference ordering for all the basic elements. All

conflicts among basic elements are resolved by selection of
the more preferred state of the system in terms of this joint
ordering.[2] In this thesis the latter technique will be used
as it has a simpler mathematical structure.

Determination of a superordinate goal for a school has
not been discussed in the literature. In seeking such a goal
it is helpful to examine the rationale for the goal of the
firm. In the economic theory of the firm, maximization of
profit is the goal most commonly used. This goal is chosen
because the firm is one of many decision units used to de-
scribe a market economy. The general economic equilibrium
model of a market economy is designed to investigate the con-
flict resolution process in the economic system as a whole.
As such the model abstracts from the internal operations of
the firm. That is, only the interaction of the firm with
its environment is analyzed. Hence a superordinate goal is
imputed to the firm which resolves the internal conflicts
and permits an analysis of the interactions among firms and
other decision units in a market economy. If the internal
conflicts of a firm are resolved in a manner not in agree-
ment with the superorindate goal, that firm will not exist
in the economy at equilibrium.

Unfortunately, it is not clear that a school is similarly
responsive to the market. There does exist a demand for
graduates but the exact nature of this demand has not been

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[2]Ibid., pp. 666-667.

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extensively investigated.[3] There is certainly competition between schools for resources, funds and students, but the effect of this competition again has not been extensively studied.[4] An even more important factor is that a large proportion of the schools of higher education are public enterprises. As such they are beyond the scope of the ordinary theory of the market. However, because of the nature of public enterprises, it is possible to assign to them a superordinate goal analogous to the firm's goal of maximization. Congressional and legislative discussions indicate that in order for a public enterprise to remain in existence it must at least exhibit behavior which minimizes waste. This minimizing waste behavior can also be described in terms of efficiency. That is, the output of a public enterprise should be maximized subject to the resource limitations on the enterprise. When a public enterprise produces more than one output, the single output maximization is replaced by the total value of the outputs. However, one of the difficulties encountered in a public enterprise is assigning values to the outputs. When such values are not available the criterion can only be the maximization of the vector of the outputs subject to resource constraints.

The specific school under examination in this thesis, the Naval Postgraduate School (NPGS), is clearly a public

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[3] Selma J. Mushkin (ed.), _Economics of Higher Education_ (Washington: Government Printing Office, 1962), p. 347.

[4] _Ibid._, p. 349.

enterprise. In response to pressures from the Congress, the NPGS must adhere to the superordinate goal common to all public enterprise, that is, maximization of the vector of outputs subject to the resource constraints. In addition, as an integral part of the Department of Defense, the NPGS is necessarily responsive to the requirements of the military services. Consequently, many of the analytical tools of the market economy, such as the generalized notion of supply and demand, may be applied to the operation of the NPGS.

CHAPTER III

OPERATION OF THE NAVAL POSTGRADUATE SCHOOL

In order to develop a model of a school, it is necessary to first understand the operation of the school. The various curriculum are the basic elements of the school. Table I lists the curricula at the Naval Postgraduate School. Each receives an input of students, usually twice a year. The size of this input is determined by the needs of the military services and can be modified only slightly by the school's administration. It is the responsibility of each curriculum organization to educate the students in the discipline of that curriculum. The actual proportion of students that obtain a graduate credential is determined by such factors as the individual student's ability, the educational technology in use and the level of resources employed. In addition to its teaching responsibility, the curriculum organization is required to conduct scholarly research.

The curriculum organizations are supplied with the resources needed to produce graduates and research by the school's administration. The demand for resources is dictated by two factors; the number of students in process and the state of educational technology. The school's available resources are constrained by the annual budget. In addition there are short term limitations such as the construction time of additional classrooms, the market situation for faculty in certain disciplines, and contractual obligations to present faculty and staff.

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TABLE I

CURRICULA AT THE NAVAL POSTGRADUATE SCHOOL*

Curriculum	Curriculum Number
Advanced Science	380
Aeronautical Engineering	610
Baccalaureate	461
Communications Engineering	600
Engineering Electronics	590
Staff Communications	620
Engineering Science	460
Meteorology	371
Advanced Meteorology	372
Oceanography	440
Mathematics	430
Naval Engineering	570
Computer Science	368
Computer Systems Management	367
Management	814 and 817
Operations Research/Systems Analysis	360
Nuclear Engineering (Effects)	521
Underwater Physics Systems	535
Ordnance Systems Engineering	530

*Extracted from the Naval Postgraduate School Catalogue for 1968-1970.

The role of the school's administration is to choose the educational technology which will best meet the supe - ordinate goal of the school subject to the various resource constraints. There are four broad methods in which the administration can affect the educational technology. First, academic standards, that is the quality of the graduates, can be raised or lowered. Sufficiently high standards are required to maintain the school's official and unofficial accredited status. On the other hand, higher standards will decrease the proportion of master credentialed graduates when a fixed level of resources is available to the school. In the second method, the average class size may be varied. As class size is increased, the number of faculty required will decrease but the proportion of master credentialed graduates might also be decreased. The third method varies the compactness of class scheduling. The compactness of a schedule is a measure of the uninterrupted study time available to a student. As a schedule is made more compact, classroom hours are assigned to adjacent periods and free periods between classes are eliminated. More compact scheduling would require more classrooms, but by making more study time available, could also increase the proportion of master credentialed graduates.[1]

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[1] "A Preliminary Cost-Benefit Analysis of Class Schedules" (Berkeley: University of California, Office of Analytical Stur es, February 13, 1968), pp. 2-4 (Mimeographed).

The fourth method is determination of the proportion of faculty effort to be devoted to teaching or research. If the size of the faculty is fixed, an increase in time spent on research will make fewer faculty available for teaching. This could decrease the number of students that may be enrolled in a curriculum if the other decision variables remain unchanged.

The mathematical technique selected to describe the school's operation is a modification of activity analysis. This consists of the "application of linear programming methods to general equilibrium theory."[2] The operation of the school can be described in terms of several processes such as output production and resource allocation. These processes are assumed to be linear. Each process is modeled by a vector equation. The first such process is the production of the school's output of graduates. These graduates are categorized by curriculum and by the credential awarded, such as M.S., B.S. or certificate of completion. This process is described by the vector equation[3]

$$\bar{q} = A\bar{s} \ . \tag{1}$$

The vector $\bar{q}$ defines the set of all possible outputs over time, and is partitioned by time periods. For this study the

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[2]William J. Baumol, Economic Theory and Operations Analysis (Englewood Cliffs, N.J., Prentice-Hall, Inc., 1965),p.491.

[3]Vectors are denoted by a lower case letter covered by a bar such as $\bar{q}$. All vectors are column vectors. A transpose is indicated by an apostrophe. For example, the transpose of $\bar{q}$ is $\bar{q}'$. Matrices are denoted by upper case letters such as A.

basic time period is one academic quarter. Thus $\bar{q}$ is partitioned into

$$(\bar{q}^1, \bar{q}^2, \ldots, \bar{q}^t, \ldots, \bar{q}^T)' \; ,$$

where the vector $\bar{q}^t$ denotes the set of outputs produced by the school in the $t\underline{th}$ quarter. Notice that all output from the present to the end cf the planning horizon (denoted by T) is included in $\bar{q}$. The vectors $\bar{q}^t$ are in turn partitioned by curriculum yieldin

$$\bar{q}^t = (\bar{q}^t_1, \bar{q}^t_2, \ldots, \bar{q}^t_i, \ldots, \bar{q}^t_I)' \; ,$$

where the vector $\bar{q}^t_i$ defines the set of outputs produced by the $i\underline{th}$ curriculum in the $t\underline{th}$ quarter. Each element q^t_{ij} represents a distinct type of output. For example, it might represent the number of graduates receiving M.S. degrees from the Operations Analysis curriculum in quarter t.

The vector $\bar{s}$ defines the set of student inputs over time and is also partitioned by quarters into the vectors $\bar{s}^t$ (t=1,2,...,T). The elements of each vector $\bar{s}^t$ are the number of students assigned as inputs to the $i\underline{th}$ curriculum in quarter t.

The matrix A describes the effect of the educational technology on the transformation of students into graudates. Partitioning of the matrix by quarters yields a column vector whose elements are the matrices A_t. Further partitioning of each matrix A_t by curriculum and by each curriculum's outputs results in a column vector whose elements are the row vectors

$$(\bar{a}^t_{11}, \bar{a}^t_{12}, \ldots, \bar{a}^t_{1J}, \bar{a}^t_{21}, \ldots, \bar{a}^t_{ij}, \ldots \bar{a}^t_{IJ})' \; .$$

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Each row vector $\bar{a}_{ij}^{t}$ corresponds to an element of the output vector $\bar{q}$ and identifies a particular production process.[4] The elements a_{ijk}^{t} of each such linear process indicate the proportion of the appropriate student input elements which will advance to graduate level and contribute to the graduating population represented by the element q_{ij}^{t} of the graduate output vector. Consequently each element is called a student advancement ratio and the matrix A is called the student advancement matrix. The values of the elements a_{ijk}^{t} depend upon the employed educational technology. Thus, each of these elements is a function of the four types of administrative decisions concerning academic standards, class sizes, class scheduling and distribution of faculty effort.

The second process to be modeled is the distribution of resources to the various curriculum. This process is described by the equation

$$\bar{k} = B\bar{s} \quad . \tag{2}$$

The resource vector $\bar{k}$ is partitioned into vectors whose elements k_{i}^{t} represent the requirement for the i$\underline{th}$ type of resource in quarter t. The vector $\bar{s}$ is the same student input vector discussed previously. The matrix B is partitioned by quarters into a column vector whose elements are the matrices B_{t}. Each row of B_{t} is denoted $\bar{b}_{i}^{t}$ which again represents a

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[4]Traditional activity analysis would require that equation (1) be written $\bar{A}q = \bar{s}$. Each column vector from the matrix A represents a linear process, and each process describes the effect of one output on the demand for all the inputs.

linear process generating the demand for the ith type resource. The elements of each row, b_{ij}^t, indicate the marginal increase in the ith type resource required for each additional student enrolled in a curriculum in quarter t. The values of these elements depend upon the educational technology employed. As previously discussed, the employed technology is a function of the administrative decisions.

The third important process to be described is the production of research. Research can be separated into student conducted research and faculty conducted research. Student research consists almost entirely of theses completed as a part of the educational process. Because student research and the number of graduates are related closely, they need not be considered as separate outputs. In succeeding chapters thesis research will be incorporated as part of the graduate output for appropriate types of graduates.

The production of faculty conducted research is related to the number of faculty available for research work and to the demand for such research. Analysis of the demand for research is beyond the scope of this thesis since it is primarily generated by factors external to the school's operation. It is important to note that there is also internally generated demand for faculty research because of the effect of this type of work upon the quality of instruction and student thesis research. For the purpose of this model it is sufficient to assume that a demand for research does exist. The amount of research conducted is described by the equation

$$\bar{f} = D\bar{k} \tag{3}$$

where $\bar{f}$ is the faculty research output vector, $\bar{k}$ is the re-
source vector previously described and the elements of the
matrix D indicate the marginal change in research production
for a change in resource levels. Recall that for a fixed
student input, resource requirements depend on the administra-
tive decisions ffecting the educational technology. The
fourth type of administrative decision concerning the distribu-
tion of faculty effort is extremely important in determining
the amount of research conducted by the faculty.

The processes previously described are carried out sub-
ject to three different constraints. The first of these con-
straints concerns the student advancement ratios, the elements
of the matrix A. Each element indicates the proportion of a
particular student input which will contribute to a specific
type of graduate output. Since no more than one hundred per
cent of any student input can contribute to the total grad-
uate population, the proportions for each input indicated by
the elements in each column of A must sum to one. This rela-
tionship is denoted by the equation

$$\sum_i a_{ij} - 1 = 0 \qquad (4)$$

for each of the j columns of A. There is also the property
that the value of each element be between zero and one.

The second constraint upon the school's operation is the
availability of resources. Operations in the current quarter
cannot utilize more resources than are already on hand. This
constraint is denoted by the equation

$$B_1 \bar{s} - \bar{k}^0 \le 0 \qquad (5)$$

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where $\bar{k}^0$ is the vector of resources initially on hand and $B_1\bar{s}$ is the vector of resources required in the first quarter. In succeeding quarters, acquisition of additional resources is limited by factors such as the market for new faculty and the construction time for new buildings. The vector $\bar{h}^t$ denotes the maximum increase in resources possible in quarter t due to these limitations. The resource constraint for quarter t is represented by the equation

$$B_t\bar{s} - B_{t-1}\bar{s} - \bar{h}^t \le 0 \tag{6}$$

which requires that the vectors of resources demanded in two adjacent quarters must not differ by more than the vector of maximum increase in resources.

The annual budget is the third operational constraint. Although determined annually, the budget is allotted to the school on a quarterly basis. For the purpose of this model it is considered as a quarterly budget, denoted by β_t. Total expenditures for resources must not exceed the budget in any quarter. Using the vector $\bar{p}_t$ to denote the market prices for resources in quarter t, the budget constraint for that quarter is represented by the equation

$$\bar{p}_t{}'B_t\bar{s} - \beta_t \le 0 \ . \tag{7}$$

This completes the development of the mathematical structure required to describe the operation of the school. The next chapter will be concerned with the formulation and solution of the efficiency problem in the context of this mathematical structure.

CHAPTER IV

THE EFFICIENCY PROBLEM

Recall that the superordinate goal of the NPGS is to
have an efficient operation such that outputs are maximized
subject to the constraints on the operation. The equations
developed in Chapter III can be used to characterize the
efficiency problem. In mathematical terms the criteria, the
object function, is stated as: Maximize $(\bar{g}, \bar{s})'$. Recall
from equation (1) that $\bar{q}$ is equal to $A\bar{s}$ and from equations
(2) and (3) that $\bar{f}$ is equal to $DB\bar{s}$. The constraints are:

$$\sum_i a_{ij} - 1 = 0, \quad j=1,2,\ldots,J; \tag{4}$$

$$B_1\bar{s} - \bar{k}^0 \leq \bar{0}; \tag{5}$$

$$B_t\bar{s} - B_{t-1}\bar{s} - \bar{h}^t \leq \bar{0}, \quad t=2,3,\ldots,T; \tag{6}$$

$$\bar{p}_t' B_t \bar{s} - \beta_t \leq 0, \quad t=1,2,\ldots,T . \tag{7}$$

Note that the elements a_{ij} and the elements of the matrices
B_t are functions of the four administrative decision variables:
academic standards, class size, schedule compactness and dis-
tribution of faculty effort. The decision variables are de-
noted by the vector $\bar{x}$.

The model is now formulated as a mathematical programming
problem and is amenable to solution by any one of several pro-
gramming techniques. Such a solution will indicate the appro-
priate administrative decisions to optimize the school's opera-
tion. It is also possible to characterize the optimal deci-
sions by examining the necessary conditions.

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For a problem with inequality constraints such as equation (5), the accepted method for examining the necessary conditions for optimality is the generalized Lagrange multiplier technique. This technique requires that the function to be maximized, in this case the vector of the outputs, be multiplied by a vector of constants.[1] This vector defines a supporting hyperplane to the surface of the objective function in the space spanned by the output vector. In economic theory this vector is interpreted as a vector of efficiency prices[2] which represent the relative values, the marginal rates of transformation, of the different types of outputs. The efficiency prices are denoted by the vector $\bar{\psi}$. The vector of criteria can be replaced by a scalar criterion by use of the efficiency prices. The new criterion is to maximize the total output value. This is denoted as:

$$\underset{(\bar{x})}{\text{maximize}}\ \bar{\psi}'\begin{bmatrix}\bar{q}\\ \bar{f}\end{bmatrix} \qquad . \qquad (8)$$

The next step in the Lagrange technique is to form the function $L(\bar{x},\bar{c})$ where $\bar{x}$ is the vector of decision variables and $\bar{c}$ represents the vectors of multipliers for the constraints.

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[1]G. Hadley, Nonlinear and Dynamic Programming, (Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1964), p. 209.

[2]Tjalling C. Koopmans, Three Essays on the State of Economic Science (New York: McGraw-Hill Book Company, Inc., 1957), p. 89.

The function is

$$L(\bar{x},\bar{c}) = \bar{\psi}' \begin{bmatrix} \bar{q} \\ \bar{f} \end{bmatrix} \qquad (9)$$

$$-\bar{\lambda}_1[B_1\bar{s}-\bar{k}^0]-\ldots-\bar{\lambda}_t[B_t\bar{s}-B_{t-1}\bar{s}-\bar{h}^t]-\ldots-\bar{\lambda}_T[B_T\bar{s}-B_{T-1}\bar{s}-\bar{h}^t]$$

$$-\mu_1[\bar{p}_1'B_1\bar{s}-\beta_1]-\ldots-\mu_t[\bar{p}_t'B_t\bar{s}-\beta_t]-\ldots-\mu_T[\bar{p}_T'B_T\bar{s}-\beta_T]$$

$$-\sigma_1[\sum_i a_{i1}-1]-\ldots-\sigma_j[\sum_i a_{ij}-1]-\ldots-\sigma_J[\sum_i a_{iJ}-1] \quad .$$

A saddle point for this function at $(\bar{x},\bar{c})$ is at least a local optimum for the original criterion. To locate such a saddle point, $(\bar{x},\bar{c})$ must satisfy the necessary conditions placed upon the partial derivatives of $L(\bar{x},\bar{c})$ with respect to $\bar{x}$ and $\bar{c}$. Note that $\bar{c}$ represents the vectors of multipliers $\bar{\sigma}$, $\bar{\mu}$ and $\bar{\lambda}_t$. Because of sign restrictions on $\bar{x}$ and $\bar{c}$ the following necessary conditions are applicable.[3]

$$\frac{\partial L}{\partial \sigma_i} = \sum_i a_{ij} - 1 = 0; \quad \sigma_i \cdot \frac{\partial L}{\partial \sigma_i} = 0; \quad \sigma_i \text{ unrestricted}$$

$$(i=1,2,\ldots,I). \qquad (10)$$

$$\frac{\partial L}{\partial \mu_t} = \bar{p}_t'B_t\bar{s} - \beta_t \geq 0; \quad \mu_t \cdot \frac{\partial L}{\partial \mu_t} = 0; \quad \mu_t \geq 0$$

$$(t = 1,2,\ldots,T). \qquad (11)$$

$$\frac{\partial L}{\partial \bar{\lambda}_1} = B_1\bar{s} - \bar{k}^0 \geq 0; \quad \bar{\lambda}_1'I\begin{bmatrix} \frac{\partial L}{\partial \bar{\lambda}_1} \end{bmatrix} = \bar{0}; \quad \bar{\lambda}_1 \geq 0. \qquad (12)*$$

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*Note that vector differential calculus is applied to constraints (12), (13) and (14). The identity matrix is denoted by I in these constraints.

[3]Hadley, op. cit., p. 188.

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$$\frac{\partial L}{\partial \bar{\lambda}_t} = B_t \bar{s} - B_{t-1} \bar{s} - \bar{h}^t \geq 0 \; ; \quad \bar{\lambda}_t I \left[\frac{\partial L}{\partial \bar{\lambda}_t} \right] = \bar{0} \; ; \quad \bar{\lambda}_t \geq \bar{0}$$

$$(t=2,3,\ldots,T). \quad (13)*$$

$$\frac{\partial L(\bar{x}, \bar{\psi}, \hat{c})}{\partial x_k} = \bar{\psi}' \begin{bmatrix} \frac{\partial A}{\partial x_k} \bar{s} \\ D \frac{\partial \bar{B}}{\partial x_k} \bar{s} \end{bmatrix} \quad (14)*$$

$$- \bar{\lambda}_1' \frac{\partial B_1}{\partial x_k} \bar{s} - \ldots - \bar{\lambda}_t' \frac{\partial B_t}{\partial x_k} \bar{s} + \bar{\lambda}_t' \frac{\partial B_{t-1}}{\partial x_k} \bar{s} - \ldots$$

$$- \bar{\lambda}_T' \frac{\partial B_T}{\partial x_k} \bar{s} + \bar{\lambda}_T' \frac{\partial B_{T-1}}{\partial x_k} \bar{s}$$

$$- \mu_1 \bar{p}_1' \frac{\partial B_1}{\partial x_k} \bar{s} - \ldots - \mu_t \bar{p}_t' \frac{\partial B_t}{\partial x_k} \bar{s} - \ldots - \mu_T \bar{p}_T' \frac{\partial B_T}{\partial x_k} \bar{s}$$

$$- \sigma_1 \sum_i \frac{\partial a_{i1}}{\partial x_k} - \ldots - \sigma_j \sum_i \frac{\partial a_{ij}}{\partial x_k} - \ldots - \sigma_J \sum_i \frac{\partial a_{iv}}{\partial x_k} \leq 0 \; ;$$

$$x_k \left(\frac{\partial L}{\partial x_k} \right) = 0 \; ; \quad x_k \geq 0 \quad (k=1,2,3,4) \quad .$$

The Lagrange multipliers are customarily interpreted as the marginal rate of change of the optimal value of the Lagrange function with respect to a small change of the appropriate constraint.[4] Thus λ_{ti} corresponds to the change in value with respect to a change of the resource constraint h_{tk} while μ_t corresponds to a change with respect to the budget constraint β_t. Also σ_j corresponds to the change in value

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*Note that vector differential calculus is applied to constraints (12), (13) and (14). The identity matrix is denoted by I in these constraints.

[4]Kelvin Lancaster, _Mathematical Economics_ (New York: Macmillan Company, 1968), p. 142.

with respect to a relaxation of the constraint that no more than one hundred per cent of the student input can graduate. Of course the latter constraint is a physical one which cannot be relaxed, but the hypothetical interpretation is still of interest. Note that, due to the inequality relationships in the necessary conditions, when the constraints are satisfied so that equations (4), (5), (6), and (7) are strict inequalities the Lagrange multipliers are at zero level. Therefore the corresponding terms in equations (14) vanish, and the equations are greatly simplified.

For further interpretation it is convenient to assume that equations (14) are strict equalities.[5] Then transposing all terms except the partial derivative of the objective function to the right hand side, the equations are

$$\bar{\psi}' \begin{array}{c} \frac{\partial A}{\partial x_k} \bar{s} \\ D\frac{\partial B}{\partial x_k}\bar{x} \end{array} = (\ldots \bar{\lambda}_t' \frac{\partial B_t}{\partial x_k} \bar{s} - \bar{\lambda}_t' \frac{\partial B_{t-1}}{\partial x_k} \bar{s} + \ldots) +$$

$$(\ldots + \mu_t \bar{p}_t' \frac{\partial B_t}{\partial x_k} \bar{s} + \ldots) + (\ldots + \sigma_j \sum_i \frac{\partial a_{ij}}{\partial x_k} + \ldots) .$$

$$(15)$$

Grouping the terms on the right by quarters creates a term

$$(\bar{\lambda}_t - \bar{\lambda}_{t+1} + \mu_t \bar{p}_t)' \frac{\partial B_t}{\partial x_k} \bar{s}$$

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[5] For the case when inequalities are present see R. P. King, "Necessary and Sufficient Conditions for Inequality Constrained Extreme Values," _Industrial and Engineering Chemistry - Fundamental_, V (1966), pp. 484-489.

for each quarter. This vector term represents the marginal opportunity costs of additional resources in quarter t. The partial derivative of the objective function on the left side of the equation represents the total marginal benefit accrued by the school's outputs from the present to the planning horizon. Consequently equations (15) represent the relationship: total marginal benefit equals total marginal opportunity cost. This relationship provides the school's administration with the following decision rule for efficient operation: select the values of the decision variables $\bar{x}$ which will cause marginal benefit to equal marginal cost. It should be noted that a change in any decision variable will effect every term in the equation. Therefore in most cases the decision rule is too complex to be applied directly. However, it does serve to summarize the difficult decision problems faced by the school's administrators.

The original problem formulation as a mathematical program has some of the structural requirements needed to apply the decomposition principle.[6] This technique could provide subordinate decision rules needed by the curriculum organizations. Other solution techniques for decentralized organizations might also be used.[7]

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[6]George B. Dantzig, Linear Programming and Extensions (Princeton: Princeton University Press, 1963) pp. 448-470.

[7]E. Malinvaud and M. O. L. Bacharach (eds.), Activity Analysis in the Theory of Growth and Planning (New York: St. Martins Press, 1967) pp. 170-210.

The model developed so far is in an explicit form. It is often useful to have available an implicit form for analysis. The remainder of this chapter is devoted to a conceptual demonstration of the development of an implicit form of the model. The previous formulation of the efficiency problem represented the vector of outputs as a vector function of the decision variables. Using $\bar{F}_1$ to denote the function, the mathematical relationship was

$$(\bar{q},\bar{f})' = \bar{F}_1(\bar{x}) \quad . \tag{16}$$

The equality necessary conditions for the efficiency problem can be solved by applying the implicit function theorem to obtain a function relating the decision variables to the operational parameters.[8] With the function denoted by $\bar{F}_2$ this relationship is

$$\bar{x} = \bar{F}_2(\bar{\psi},\bar{k}^0,\bar{h},\bar{\beta},\bar{s}) \quad .$$

Substituting this expression for $\bar{x}$ into equation (16) yields

$$(\bar{q},\bar{f})' = \bar{F}_3(\bar{\psi},\bar{k}^0,\bar{h},\bar{\beta},\bar{s}) \quad . \tag{17}$$

This equation describes the output surface in terms of the operational parameters. By applying the implicit function theorem to equation (17), the vector of efficiency prices can be obtained as a vector function, $\bar{F}_4$, of the other parameters and the output vector. Since the elements of $\bar{\psi}$ must sum to one, only N-1 of the N elements of $\bar{\psi}$ are needed to completely specify the efficiency prices. Therefore one element of the output vector is not needed for this application

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[8]King, op. cit., pp. 484-489.

of the implicit function theorem.[9]   The absence of this element from the output vector is denoted by $\bar{q}^{(-)}$.   The expression for $\bar{\psi}$ is

$$\bar{\psi} = \bar{F}_4(\bar{q}^{(-)}, \bar{f}, \bar{k}^0, \bar{h}, \bar{\beta}, \bar{s}) \ . \qquad (18)$$

The remaining element of the output vector can be found by substituting equation (18) into equation (17).   This yields the equation

$$q_i = F_5(\bar{q}^{(-)}, \bar{f}, \bar{\psi}, \bar{k}^0, \bar{h}, \bar{\beta}, \bar{s}) \ ,$$

which is easily transformed into the implicit form

$$F_6(\bar{q}, \bar{f}, \bar{\psi}, \bar{k}^0, \bar{h}, \bar{\beta}, \bar{s}) = 0 \ .$$

It is important to realize that the transformations required by this development may be rather difficult to perform.

If the implicit form can be obtained, it is quite useful, since this form is a summary of all the relationships among outputs and parameters.   More specifically the summary relates the outputs to parameters such as student input, faculty and budget levels when the school's operation satisfies the efficiency criterion with the employed technology.   Appropriate partial derivatives of the implicit function can be interpreted as the marginal rates of transformation between outputs, the marginal rates of substitution between inputs and the marginal effect on outputs of a parameter variation.

[9]S. Karlin, _Mathematical Methods and Theory in Games, Programming and Economics_ (Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1959) pp. 216-218.

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In the next chapter, partial derivative techniques are applied to the explicit formulation of the efficiency problem. This will indicate the marginal effect of a parameter variation on a decision variable.

CHAPTER V

COMPARATIVE STATICS

The necessary conditions previously developed indicated some properties of the model at a saddle point solution. The development depended on an implied assumption that the parameters of the school's operation were fixed at specified values. It is also important to understand the effect of a variation of the parameters upon the decision variables and Lagrange multipliers.[1] As an example of the analysis required, the effect of the variation of $p_{\alpha i}$, the market price of the $i\underline{\text{th}}$ type of resource in quarter α, is examined in this chapter.

When all of the necessary conditions are assumed to be strict equalities,[2] differentiation with respect to $p_{\alpha i}$ with all other parameters held constant yields equation set I as shown in Table II. In vector notation these equations can be represented by

$$E\bar{r}_0 = \bar{y}_0 . \tag{19}$$

The vector $\bar{r}_0$ is the vector of partial derivatives of $\bar{x}$ and $\bar{c}$ with respect to $p_{\alpha i}$. The matrix E is square and can be

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[1]C.R. Jones, "On the Theory of Cost-Benefit Analysis" (paper presented at the Annual Western Economic Association Meeting, Corvallis, Oregon, August, 1968), pp. 21-33.

[2]R.P. King, "Necessary and Sufficient Conditions for Inequality Constrained Extreme Values," _Industrial and Engineering Chemistry-Fundamental_, V (1966), pp. 484-489.

TABLE II

TYPICAL ELEMENTS OF EQUATION SET I

$$
\begin{array}{|cccc|}
0 & 0 & -\bar{p}_t'\,\dfrac{\partial B_t}{\partial x_k}\text{'s} & \bar{p}_t'\,\dfrac{\partial B_t}{\partial x_k}\text{'s} \\[2ex]
0 & 0 & \left(\dfrac{\partial \bar{b}_i^t}{\partial x_k} - \dfrac{\partial \bar{b}_i^{t-1}}{\partial x_k}\right)\text{'s} & 0 \\[2ex]
0 & 0 & \displaystyle\sum_i \dfrac{\partial a_{ij}}{\partial x_k} & 0 \\[2ex]
-\displaystyle\sum_i \dfrac{\partial a_{ij}}{\partial x_k} &
\begin{array}{l}
\bar{\psi}'\left[\dfrac{\partial^2 A}{\partial x_k^2}\text{'s}\quad + (\ldots - \mu_t' \bar{p}_t\,\dfrac{\partial^2 B_t}{\partial x_k^2}\text{'s}-\ldots) \right. \\[2ex]
\left. D\,\dfrac{\partial^2 B}{\partial x_k^2}\text{'s}\quad + (\ldots -\sigma_j \displaystyle\sum_{j,i} \dfrac{\partial^2 a_{ij}}{\partial x_k^2}-\ldots) \right] \\[2ex]
+ \left(\ldots -\bar{\lambda}_t\left(\dfrac{\partial^2 B_t}{\partial x_k^2} - \dfrac{\partial^2 B_{t-1}}{\partial x_k^2}\right)\text{'s}-\ldots\right)
\end{array}
& \dfrac{\partial u_t}{\partial p_{\alpha i}} & -\bar{b}_i^\alpha\text{'s} \\[6ex]
& & \dfrac{\partial \lambda_{ti}}{\partial p_{\alpha i}} & 0 \\[2ex]
& & \dfrac{\partial \sigma_i}{\partial p_{\alpha i}} & 0 \\[2ex]
-\bar{p}_t'\,\dfrac{\partial B_t}{\partial x_k}\text{'s} & -\left(\dfrac{\partial \bar{b}_i^t}{\partial x_k} - \dfrac{\partial \bar{b}_i^{t-1}}{\partial x_k}\right)\text{'s} & \dfrac{\partial x_k}{\partial p_{\alpha i}} & \mu_\alpha\,\dfrac{\partial \bar{b}_i^\alpha}{\partial x_k}\text{'s} \\
\end{array}
$$

partitioned into a two by two matrix where the submatrix E_{11} is the null matrix. The partitioned matrix E is represented by

$$E = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} \quad .$$

It can be shown that the inverse of E exists and can be formed by a computational formula.[3] The inverse, denoted by E^{-1}, is

$$E^{-1} = \begin{bmatrix} (-E_{12}E_{22}^{-1}E_{21}) & ((E_{12}E_{22}^{-1}E_{21})^{-1}E_{12}E_{22}^{-1}) \\ (E_{22}^{-1}E_{21}(E_{12}E_{22}^{-1}E_{21})^{-1}) & (E_{22}^{-1}-E_{22}^{-1}E_{21}(E_{12}E_{22}^{-1}E_{21})E_{12}E_{22}^{-1}) \end{bmatrix} \quad .$$

A convenient notation for the elements of the partitioned inverse is

$$E^{-1} = \begin{bmatrix} (E^{-1})_{11} & (E^{-1})_{12} \\ (E^{-1})_{21} & (E^{-1})_{22} \end{bmatrix} \quad .$$

Now, by using E^{-1}, equation (19) can be solved for the vector $\bar{r}_0$ yielding

$$\bar{r}_0 = E^{-1}\bar{y}_0 \quad . \tag{20}$$

Two elements of the vector $\bar{r}_0$ are of interest. These elements, $\frac{\partial x_k}{\partial P_{\alpha i}}$ and $\frac{\partial \mu_\alpha}{\partial P_{\alpha i}}$, are given by the following equations[4]

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[3] G. Hadley, Linear Algebra, (Reading Massachusetts: Addison-Wesley Publishing Company, Inc., 1961), pp. 108-109.

[4] The partial derivatives of interest in these equations are indicated by the typical element in the vectors on the left side of the equations.

$$\left[\frac{\partial \mu_\alpha}{\partial p_{\alpha i}}\right] = (E^{-1})_{11} \left[\begin{array}{c} -\bar{b}_i^{\alpha}\bar{s} \\ \bar{0} \end{array}\right] + (E^{-1})_{12} \left[\mu_\alpha \frac{\partial \bar{b}_i^{\alpha}}{\partial x_k} \bar{s}\right] \qquad (21)$$

$$\left[\frac{\partial x_k}{\partial p_{\alpha i}}\right] = (E^{01})_{21} \left[\begin{array}{c} -\bar{b}_i^{\alpha}\bar{s} \\ \bar{0} \end{array}\right] + E_{22}^{-1} \left[\mu_\alpha \frac{\partial \bar{b}_i^{\alpha}}{\partial x_k} \bar{s}\right] - E_{22}^{-1}E_{21}(E^{-1})_{12}\left[\mu_\alpha \frac{\partial \bar{b}_i^{\alpha}}{\partial x_k} \bar{s}\right]$$

$$(22)$$

The last term of equation (22) is a part of the total change
of μ_α with a change in $p_{\alpha i}$. Denoting this partial change as

$\dfrac{\partial \mu_\alpha^{(-)}}{\partial p_{\alpha i}}$, equation (22) is written as

$$\left[\frac{\partial x_k}{\partial p_{\alpha i}}\right] = (E^{-1})_{21} \left[\begin{array}{c} -\bar{b}_i^{\alpha}\bar{s} \\ \bar{0} \end{array}\right] + E_{22}^{-1} \left[\mu_\alpha \frac{\partial \bar{b}_i^{\alpha}\bar{s}}{\partial x_k}\right] - E_{22}^{-1}E_{21} \frac{\partial \mu_\alpha^{(-)}}{\partial p_{\alpha i}} \ .$$

$$(23)$$

The meaning of the terms on the right side of equation (23)
is not obvious. To examine them more closely, first con-
sider the partial derivatives of the necessary conditions
with respect to the parameter β_α, the budget in quarter α,
with all other parameters including $p_{\alpha i}$ held constant. These
derivatives are represented by

$$E\bar{r}_1 = \bar{y}_1 \quad .$$

The matrix E is the same one previously developed in equation
set I. Typical elements of the vectors $\bar{r}_1$ and $\bar{y}_1$ are

$$\bar{r}_1 = \left(\begin{array}{cccc} \dfrac{\partial \mu_t}{\partial \beta_\alpha} & \dfrac{\partial \lambda_{ti}}{\partial \beta_\alpha} & \dfrac{\partial \sigma_j}{\partial \beta_\alpha} & \dfrac{\partial x_k}{\partial \beta_\alpha} \end{array}\right)' \quad ,$$

$$\bar{y}_1 = (\begin{array}{cccc} 0 & 0 & 1 & 0 \end{array})' \quad .$$

Consequently, solving for the elements $\frac{\partial x_k}{\partial \beta_\alpha}$ and $\frac{\partial \mu_\alpha}{\partial \beta_\alpha}$ in the

vector $\bar{r}_1$ yields

$$\left[\frac{\partial x_k}{\partial \beta_\alpha}\right] = (E^{-1})_{21}\begin{bmatrix} \bar{0} \\ \cdots \\ 1 \end{bmatrix} \quad \text{and} \quad \left[\frac{\partial \mu_\alpha}{\partial \beta_\alpha}\right] = (E^{-1})_{11}\begin{bmatrix} \bar{0} \\ \cdots \\ 1 \end{bmatrix} .$$

Substituting the latter relationship into equation (21)

yields

$$\left[\frac{\partial \mu_\alpha}{\partial P_{\alpha i}}\right] = -\bar{b}_i^{\alpha\bar{s}}\left[\frac{\partial \mu_\alpha}{\partial \beta_\alpha}\right]_{P_{\alpha i}=\text{constant}} + (E^{-1})_{12}\left[\mu_\alpha \frac{\partial \bar{b}_i^\alpha}{\partial x_k^{\bar{s}}}\right] .$$

Using the usual interpretations from the economic theory of

consumer choice the last term on the right side of this equa-

tion may be interpreted as the residual variability in the

value of the budget constraint in quarter α. The residual

variability appears in equation (23) denoted by $\frac{\partial \mu_\alpha^{(-)}}{\partial p_{\alpha i}}$.

Substituting the just derived equation for $\frac{\partial x_k}{\partial \beta_\alpha}$ into equation

(23) yields equation (24) as follows:

$$\left[\frac{\partial x_k}{\partial P_{\alpha i}}\right] = -\bar{b}_i^{\alpha\bar{s}}\frac{\partial x_k}{\partial \beta_\alpha}\Bigg|_{P_{\alpha i}=\text{constant}} + E_{22}^{-1}\left[\mu_\alpha \frac{\partial \bar{b}_i^\alpha}{\partial x_k^{\bar{s}}}\right] - E_{22}^{-1}E_{21}\frac{\partial \mu_\alpha^{(-)}}{\partial p_{\alpha i}} .$$

To examine the remaining term it is necessary to consider

the economic dual of the efficiency problem. The economic

dual problem is to minimize cost subject to a fixed minimum

output constraint. Stated mathematically, the problem is

$$\text{Minimize:} \quad \gamma_0 \bar{p}_\alpha' B_\alpha \bar{s}$$

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subject to: $\quad \sum\limits_{i} a_{ij} - 1 = 0,$

$$B_1 \bar{s} - \bar{k}^0 \leq \bar{0},$$

$$B_t \bar{s} - B_{t-1} \bar{s} - \bar{h}^t \leq \bar{0},$$

$$\begin{bmatrix} \bar{q}^\star \\ \bar{f}^\star \end{bmatrix} - \begin{bmatrix} A\bar{s} \\ DB\bar{s} \end{bmatrix} \leq \bar{0},$$

$$\bar{p}_t' B_t \bar{s} - \beta_t \leq 0, \quad t \neq \alpha.$$

The Lagrange function for this problem is

$$\hat{L} = \gamma_0 \bar{p}_\alpha' B_\alpha \bar{s} + (\ldots - \delta_j(\sum\limits_{i} a_{ij} - 1) - \ldots)$$

$$+ (\ldots - \bar{\theta}_t (B_t \bar{s} - B_{t-1} \bar{s} - \bar{h}^t) - \ldots)$$

$$+ (\ldots - \gamma_t (\bar{p}_t' B_t \bar{s} - \beta_t) - \ldots)$$

$$- \bar{\phi}\left(\begin{bmatrix} \bar{q}^\star \\ \bar{f}^\star \end{bmatrix} - \begin{bmatrix} A\bar{s} \\ DB\bar{s} \end{bmatrix} \right) \quad .$$

The partial derivatives with respect to the Lagrange multipliers yield the following necessary conditions:[5]

$$\frac{\partial \hat{L}}{\partial \hat{\phi}} = \begin{bmatrix} \bar{q}^\star \\ \bar{f}^\star \end{bmatrix} - \begin{bmatrix} A\bar{s} \\ DB\bar{s} \end{bmatrix} \geq 0,$$

$$\frac{\partial \hat{L}}{\partial \delta_j} = \sum\limits_{i} a_{ij} - 1 \geq 0,$$

$$\frac{\partial \hat{L}}{\partial \bar{\theta}_t} = B_t \bar{s} - B_{t-1} \bar{s} - \bar{h}^t \geq 0$$

$$\frac{\partial \hat{L}}{\partial \gamma_t} = \bar{p}_t' B_t \bar{s} - \beta_t \geq 0, \quad t \neq \alpha,$$

---

[5] It should be noted that these conditions are incomplete. Only the relevant portions have been presented.

$$\frac{\partial \hat{L}}{\partial x_k} = \gamma_0 \bar{P}_\alpha' \left[\frac{\partial B_\alpha}{\partial x_k} \right] \bar{s} + (\ldots - \delta_j (\sum_i \frac{\partial a_{ij}}{\partial x_k}) - \ldots)$$

$$+ (\ldots - \bar{\theta}_t \frac{\partial B_t}{\partial x_k} - \frac{\partial B_{t-1}}{\partial x_k} \bar{s} - \ldots)$$

$$+ (\ldots - \gamma_t \bar{P}_t' \frac{\partial B_t}{\partial x_k} \bar{s} - \ldots)$$

$$+ \bar{\phi} \begin{bmatrix} \frac{\partial A}{\partial x_k} \bar{s} \\ D\frac{\partial B}{\partial x_k} \bar{s} \end{bmatrix} .$$

These economic dual necessary conditions are quite similar
to the necessary conditions of the original efficiency prob-
lem. It can be shown that there is an exact correspondence
between the dual Lagrange multipliers and the original ones
such that

$$\gamma_t = \mu_t, \quad t \neq \alpha,$$

$$\bar{\theta}_t = \bar{\lambda}_t ,$$

$$\delta_j = \sigma_j .$$

Also, the vector of efficiency prices correspond to the La-
grange multipliers in the output constraint so that

$$\bar{\phi} = \bar{\psi} .$$

Since the dual objective function is a scalar, it is multi-
plied by a generalized scalar Lagrange multiplier.[6] This

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[6]G. Hadley, Nonlinear and Dynamic Programming, (Reading,
Massachusetts: Addison-Wesley Publishing Company, Inc., 1964),
pp. 66-68.

generalized multiplier corresponds to the remaining original Lagrange multiplier

$$\gamma_0 = \mu_\alpha \ .$$

Again assuming equality necessary conditions, differentiation with respect to $p_{\alpha i}$ yields equation set II as shown in Table III. These equations are denoted by

$$G\bar{r}_2 = \bar{y}_2 \ .$$

The elements of the vector $\bar{r}_2$ are the partial derivatives with respect to $p_{\alpha i}$ with output fixed. This is denoted as

$$\left. \frac{\partial x_k}{\partial p_{\alpha i}} \right|_{(\bar{q},\bar{f})=\text{constant}} \ .$$

Notice that the matrix G can be partitioned so that

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & E_{22} \end{bmatrix} \ .$$

The inverse of the matrix G then is

$$G^{-1} = \begin{bmatrix} (G^{-1})_{11} & (G^{-1})_{12} \\ (G^{-1})_{21} & E_{22}^{-1} - E_{22}^{-1}G_{21}(G^{-1})_{12} \end{bmatrix} \ .$$

Solving for the element

$$\left. \frac{\partial x_k}{\partial p_{\alpha i}} \right|_{(\bar{q},\bar{f})=\text{constant}}$$

as before yields

$$\left[\frac{\partial x_k}{\partial p_{\alpha i}} \right]_{(\bar{q},\bar{f})=\text{constant}} = E_{22}^{-1}\left[\mu_\alpha \frac{\partial \bar{b}_{i}^{\alpha}}{\partial x_k} s \right] - E_{22}^{-1}G_{21}\frac{\partial \bar{\phi}}{\partial p_{\alpha i}} \right|_{(\bar{q},\bar{f})=\text{constant}} \ ,$$

TABLE III

TYPICAL ELEMENTS OF EQUATION SET II

$$
\begin{bmatrix}
-\left(\dfrac{\partial A}{\partial x_k}\bar{s}, D\dfrac{\partial B}{\partial x_k}\bar{s}\right)' & 0 & 0 & 0 & 0 \\[2ex]
-\bar{p}_t' \dfrac{\partial B_t}{\partial x_k}\bar{s} & 0 & 0 & 0 & 0 \\[2ex]
\left(\dfrac{\partial \bar{b}_i^t}{\partial x_k} - \dfrac{\partial \bar{b}_i^{t-1}}{\partial x_k}\right)\bar{s} & 0 & 0 & 0 & 0 \\[2ex]
\sum_i \dfrac{\partial a_{ij}}{\partial x_k} & 0 & 0 & 0 & 0 \\[2ex]
E_{22} & -\sum_i \dfrac{\partial a_{ij}}{\partial x_k} & \left(\dfrac{\partial \bar{b}_i^t}{\partial x_k} - \dfrac{\partial \bar{b}_i^{t-1}}{\partial x_k}\right)\bar{s} & -\bar{p}_t \dfrac{\partial B_t}{\partial x_k}\bar{s} & \left\{\dfrac{\partial A}{\partial x_k}\bar{s} \atop D\dfrac{\partial B}{\partial x_k}\bar{s}\right\}
\end{bmatrix}
\begin{bmatrix}
\dfrac{\partial \bar{\psi}}{\partial p_{\alpha i}} \\[2ex]
\dfrac{\partial u_t}{\partial p_{\alpha i}} \\[2ex]
\dfrac{\partial \lambda_{ti}}{\partial p_{\alpha i}} \\[2ex]
\dfrac{\partial \sigma_i}{\partial p_{\alpha i}} \\[2ex]
\dfrac{\partial x_k}{\partial p_{\alpha i}}
\end{bmatrix}
=
\begin{bmatrix}
0 \\[2ex]
0 \\[2ex]
0 \\[2ex]
0 \\[2ex]
\dfrac{\partial \bar{b}_i^\alpha}{\partial x_k}\bar{s} \; n_\alpha
\end{bmatrix}
$$

or

$$E_{22}^{-1}\left[\mu_\alpha \frac{\partial \bar{b}_{i\bar{s}}^\alpha}{\partial x_k}\right] = \left[\frac{\partial x_k}{\partial p_{\alpha i}}\right]_{(\bar{q},\bar{f})=constant} + E_{22}^{-1}G_{21}\frac{\partial \bar{\phi}}{\partial p_{\alpha i}}\bigg|_{(\bar{q},\bar{f})=constant}.$$

Substituting this equation into equation (24) yields

$$\left[\frac{\partial x_k}{\partial p_{\alpha i}}\right] = -\bar{b}_{i\bar{s}}^\alpha \left[\frac{\partial x_k}{\partial \beta_\alpha}\right]_{p_{\alpha i}=constant} + \left[\frac{\partial x_k}{\partial p_{\alpha i}}\right]_{(\bar{q},\bar{f})=constant}$$

$$+ E_{22}^{-1}G_{21}\frac{\partial \bar{\phi}}{\partial p_{\alpha i}}\bigg|_{(\bar{q},\bar{f})=constant} - E_{22}^{-1}E_{21}\left[\frac{\partial \mu_\alpha^{(-)}}{\partial p_{\alpha i}}\right]. \qquad (25)$$

Equation (25) shows that the marginal effect of a change in market prices on the decision variables can be decomposed into separate component effects. The terms on the right side of equation (25) represent these component effects. The components may be given the usual interpretations from the economic theory of consumer choice. The first such term,

$$-\bar{b}_{i\bar{s}}^\alpha \frac{\partial x_k}{\partial \beta_\alpha}\bigg|_{p_{\alpha i}=constant} \qquad ,$$

is that portion of the total effect which is associated with a change in income with constant market prices. This is called the income effect. The second term,

$$\frac{\partial x_k}{\partial p_{\alpha i}}\bigg|_{(\bar{q},\bar{f})=constant} \qquad ,$$

is the effect due to a market price change when the outputs are held constant. This is called the substitution effect. The third term represents the imputed revaluation of the efficiency prices, and the fourth term represents the revaluation of the residual variability in the value of the quarter α budget constraint.

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In many cases in economic theory it is possible to determine the sign of each of these effects. Since the exact nature of the underlying technology has not been determined, inferences as to signs cannot be made without further research. However, it is important to realize that the total marginal effect of a parameter variation can be decomposed into the sum of the income, substitution and revaluation effects. Relationships such as equation (25) can be used in conjunction with statistical techniques to test the model's relevance to empirical phenomenon.

CHAPTER VI

CONCLUSIONS

In summary, it has been shown that economic theory can be applied to the Naval Postgraduate School. Because of characteristics in common with the firm and public enterprise, the school has been described as a conflict system. The superordinate goal of efficient operation was used for conflict resolution.

A modified activity analysis technique was used to develop a model of the school. The decision rules produced by this model serve to emphasize the complexity of the decision problems facing the school administrators. The analysis of the comparative statics of the model has produced relationships which could be used to test the model's relevance to empirical phenomenon. These relationships demonstrate that the total marginal effect of parameter variation can be decomposed. The components of these effects are interpreted as income, substitution, and revaluation effects.

There are several areas which seem suitable for further study. First, statistical estimation procedures could be developed for the relationships in the model. Examples of these are the nature of the dependence of student advancement and resource demands upon the decision variables.

A second area for study is the actual testing of the model. Using statistical techniques and probably emphasizing the simultaneous determination of many of the variables, it should be possible to determine if the equations developed

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with the comparative statics technique actually hold. A third research area is the development of accounting techniques and a management data system which would reveal the marginal rates of return of various resources. This might involve the development and implementation of decentralized planning procedures.

Finally, the model could be extended to other institutions of higher education. This would involve further investigation into the position of such institutions in the economy. Supply and demand functions for graduates from such institutions is also a worthwhile area for further study.

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13. ABSTRACT

The increased size and complexity of educational
institutions has created a need for an economic model of
such institutions as an aid to planning and resource alloca-
tion. It is shown that economic theory can be applied to an
analysis of the Naval Postgraduate School. A modified activ-
ity analysis technique is used to model the school's opera-
tion. The mathematical programming structure of the model
emphasizes the objective of efficient operation. The neces-
sary conditions of the programming problem are used to
characterize decision rules for efficient operations. The
decision rules stress the complexity of the problems facing
school administrations. Application of the comparative
statics technique to the model provides relationships which
demonstrate that the total marginal effect of parameter
variation can be decomposed. The components of these effects
are interpreted as income, substitution, and revaluation
effects.

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14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
ECONOMIC MODEL						
SCHOOL MODEL						
ECONOMICS OF SCHOOL						