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Continuously Visible Satellite Constellations

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ABSTRACT

The results of this study show that a minimum of six satellites is required in a controlled-orbit constellation to have at least one satellite continuously visible from all points on the earth's surface. The study also examined the number of required satellites for different heights and various minimum elevation angles.

To have two satellites visible at all times it is sufficient to double the number required for a single-satellite condition; to have three satellites simultaneously visible it is sufficient to triple the number required for a single-satellite condition.

A constellation that is to have at least one visible satellite every place on the earth results in approximately twice the coverage of the earth's surface, but to date no easy method has been found for assuming that this duplication can be used to reduce the number required for having two, or three, mutually visible satellites at one time.

PROBLEM STATUS

This is a final report on this phase of the problem; work is continuing on other phases.

AUTHORIZATION

NRL Problem R04-16
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CONTINUOUSLY VISIBLE SATELLITE CONSTELLATIONS

INTRODUCTION

For many purposes (e.g., communication and navigation) it is necessary to have one or more satellites visible at all times. It is therefore important to know the minimum number of satellites required to meet the specific need.

BACKGROUND DISCUSSION

It is immediately evident that if the satellites were at least as large in diameter as the earth, two of them would be sufficient if they are kept 180 degrees apart from the earth's center.

Since a configuration of such enormous satellites is obviously impractical, the next possible constellation would appear to be four satellites spaced geometrically at the apices of a regular tetrahedron. These four satellites could illuminate the entire earth. Because the planes of these apices do not align with the center of the earth, the constellation is not fixed. It is therefore impossible to keep four satellites at points corresponding to the four apices of a regular tetrahedron.

A minimum of three satellites is required in one plane for complete coverage below that plane; Fig. 1 shows such a plane. To have complete coverage it would be necessary to have an additional plane of satellites. This second plane could be situated 90 degrees from the first plane, and providing that the satellite coverages in the two planes overlap sufficiently, the entire surface could be seen from these six satellites. These two satellite planes could be spaced either equatorial and polar or both polar, spaced at 90 degrees.

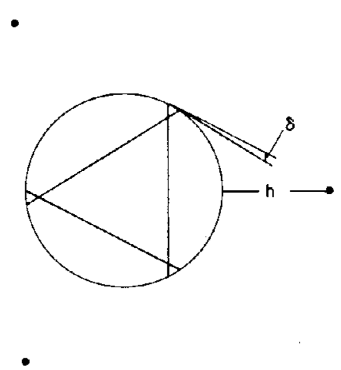

Fig. 1 - Complete coverage with three satellites in each of two planes

In such a six-satellite constellation three of the satellites are 120 degrees apart within a single reference plane, and the planes are 90 degrees apart. If one placed a set of four satellites in each of two planes, both the planes and the satellites within the planes would be equally spaced. Figure 2 shows the geometry and altitude(s) involved. In this second condition the total number of satellites is increased from six to eight, but their required altitudes are decreased by a factor greater than 2.

It is possible to use many other constellations for coverage at lower altitudes. Figure 3 shows some of the possibilities in which all have an equal number of satellites in each plane. From Fig. 3 it is obvious that the number of satellites required increases greatly as the altitude is decreased in order to have a complete earth coverage.

If a minimum elevation angles is required from each observation point, the altitudes needed for this angle are shown for a few elevation angles in Table 1.

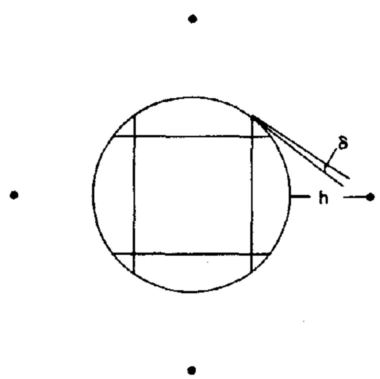

Fig. 2 - Complete coverage with four satellites in each of two planes

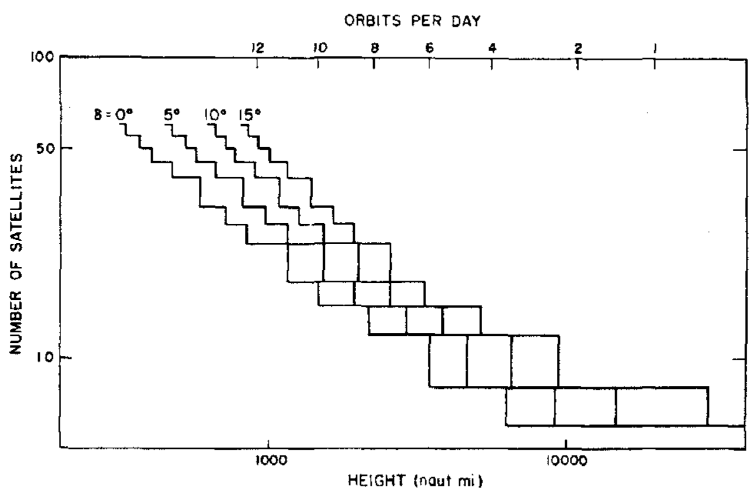

Fig. 3 - Number and height of polar circular satellites needed to insure at least one having an elevation angle greater than δ

For all of the cases described in the foregoing discussions the earth's surface is covered approximately an average of twice to assure that all parts will be covered at least once.

Table 1
Six-Satellite Configuration

δ (degrees)	Height (naut mi)	Period (hours)
0	6320	6.70
5	9200	9.91
10	14830	17.17
*15	30000	42.5

*The height for the δ of 15 degrees is greater than the requirement for a synchronous satellite condition.

To have two satellites visible at all times, it is sufficient to double the number of satellites required for a single-coverage condition. This doubling may be done by doubling the number per plane or by doubling the number of planes. For instance, in the case of the six-satellite configuration, a two-simultaneous-satellite-visibility constellation requires a total of twelve properly spaced satellites.

ANALYSIS

We can start with a constellation of six objects with three satellites rotating in two planes, which have a dihederal angle of 90 degrees. A "worst-case" analysis is used to determine the number and altitude of the satellite required for a given coverage.

Six-Satellite Configuration

To determine the altitude of the satellite in a six-satellite constellation, the spot on the surface most remote from any subsatellite point must be found. If it is assumed that one such satellite in each plane is at the equator with the others spaced 120 degrees apart in the plane and if these positions are spotted on a sphere, we are able to find the most, remote spot by the processes of spherical trigonometry (Fig. 4). For the position P to be a maximum distance from s_1 and s_3, P must be equidistant from them. This situation occurs when P is on the equator and a spherical triangle is thus formed with one side at 90 degrees and the angles between the unknown sides and the known side are each at 60 degrees. Using the cosine law, the side of the triangle representing distance x is calculated to be 63.43 degrees. This is a maximum but not necessarily the extreme angle. In Fig. 5 the remote point is at the center of a spherical rectangle. The extreme value of distance y is solved by one of the right-triangle equations and found to be 69.3 degrees. This is the extreme value of the great circle arc for the most remote point from any subsatellite point. From Fig. 6, given the earth central angle of 69.3 degrees and a desired minimum elevation angle δ, the height of the satellite necessary to yield such coverage can be calculated. For the six-satellite case the height for each value of δ is calculated as indicated by Table 1.

Eight-Satellite Configuration

If eight satellites are used, four in each plane and the planes at right angles, the required altitude is reduced as indicated by Table 2.

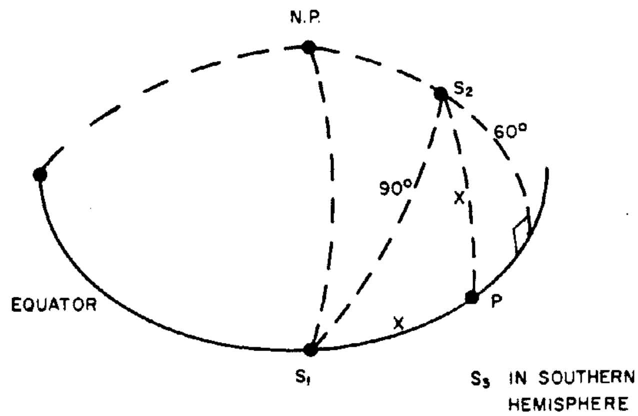

Fig. 4 - Location of a point most remote from the subsatellite points - case 1

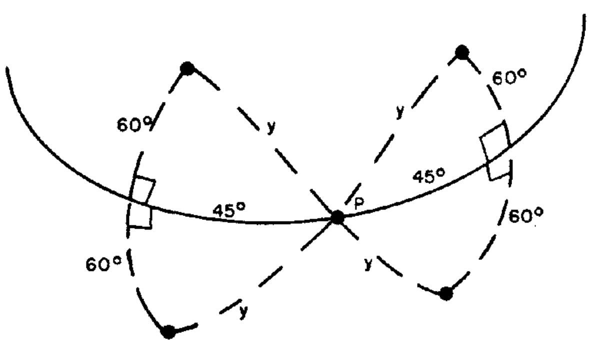

Fig. 5 - Location of a point most remote from the subsatellite points - case 2

Fig. 6 - Relation between earth central angle, minimum elevation angle, and height of the satellite

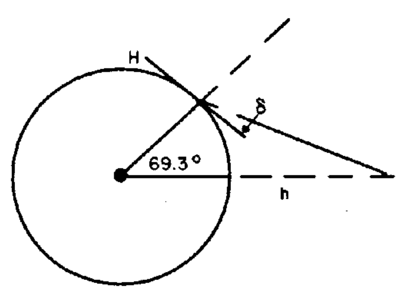

Table 2
Eight-Satellite Configuration

δ (degrees)	Height (naut mi)	Period (hours)
0	3450	3.96
5	4660	5.11
10	6480	6.87
15	9420	10.56

Twelve-Satellite Configuration

For a twelve-satellite system the height is reduced even more as indicated by Table 3.

Table 3
Twelve-Satellite Configuration

δ (degrees)	Height (naut mi)	Period (hours)
0	2180	2.95
5	2900	3.51
10	3870	4.35
15	1580	5.6

In the case for multiple satellite coverage a single satellite serves a segment of a sphere, whose area, as a fraction of the total sphere area, is (1 - cos a)/2, where a is the arc radius of the small circle. Some examples are indicated in Table 4.

Table 4
Multiple Satellite Coverage

No. of Satellites	Radius Coverage	Earth Fraction	Total Covered
6	69.3	0.324	1.944
8	60.0	0.250	2.00
12	52.2	0.194	2.33

The above figures show that a satellite arrangement that insures at least one body with an elevation angle greater than a specified value will also give multiple object coverage for most of the earth's surface. Figures 7 through 10 show the coverages for the above cases. Figure 11 goes a step further and shows the horizon coverage available from a system that insures at least one object above a 15-degree elevation.

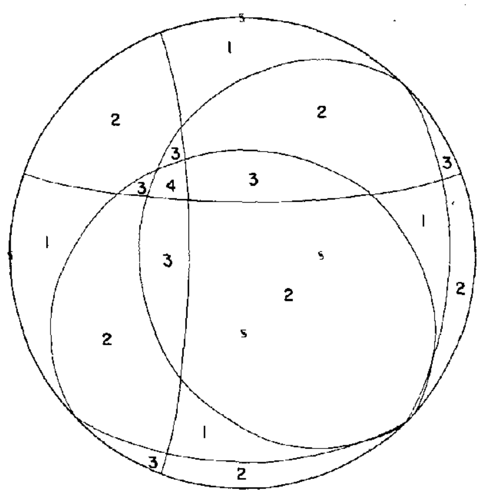

Fig. 7 - Six-station coverage with δ equal to 15 degrees

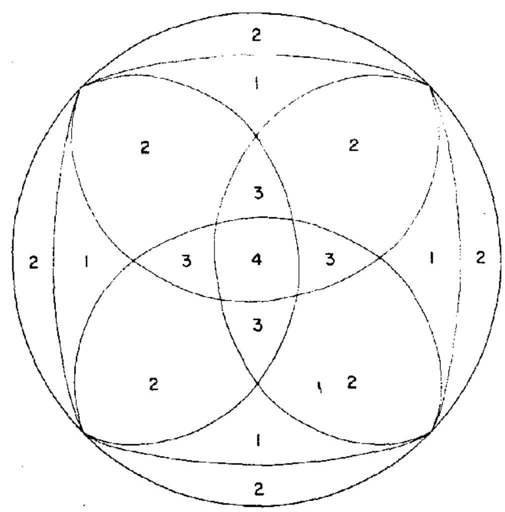

Fig. 8 - Eight-station coverage with δ equal to 15 degrees

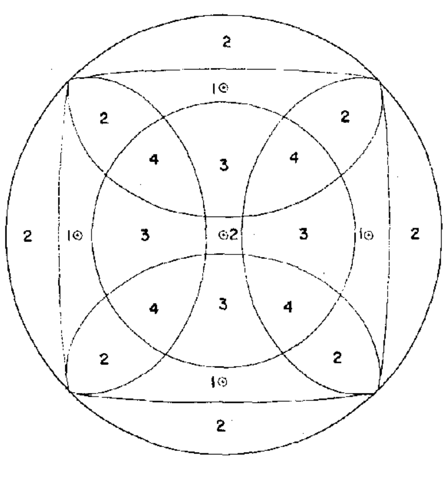

Fig. 9 - Twelve-satellite (in two planes) coverage with δ equal to 15 degrees

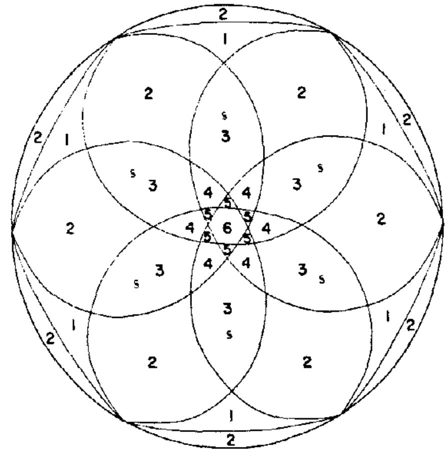

Fig. 10 - Twelve-satellite (in three planes) coverage with δ equal to 15 degrees

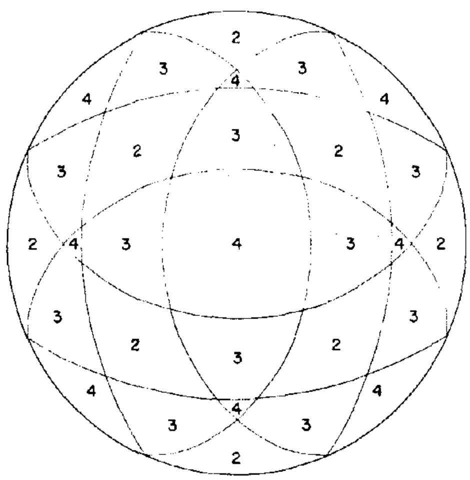

Fig. 11 - Eight-satellite horizon coverage

SUMMARY

The results obtained from the analysis indicate that a minimum of six satellites is required to obtain complete earth coverage, and by increasing this number to eight the required altitude is decreased by a factor greater than two. To assure that one satellite is visible everywhere approximately two will be visible on the average. To have two satellites visible everywhere it is sufficient to double the number required for the single-satellite case with an average now of four satellites being visible.

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