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APRIL 1969

# APERTURE AVERAGING OF OPTICAL SCINTILLATION

H. T. Yura and R. F. Lutomirski

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PREFACE

This Memorandum is part of RAND's study for the Advanced Research Projects Agency of those phenomena which affect the performance of optical reconnaissance and guidance equipment. The objective of these studies is to provide sufficient understanding to permit the systems analyst to compute performance estimates under various operational conditions.

In optical communications and related devices, the random variations in the received signal due to atmospheric turbulence can represent a severe limitation to system performance. This Memorandum gives an improved expression for the effect of a finite diameter receiving aperture in reducing the variance of a fluctuating light signal, and should be of interest to those concerned with the use of lasers in the atmosphere.

SUMMARY

The aperture-averaging factor of a circular aperture is given. This factor gives the effect of a finite receiving aperture on spherical and plane waves in reducing the variance of a fluctuating light signal. Curves of the reduction factor and normalized signal standard deviation as a function of range and receiver aperture diameter are presented and are compared with those which were previously calculated by Fried.<sup>(1)</sup> It is shown that Fried's expression agrees with the results obtained here only for propagation distances much less than the propagation distance where the average field is down by a factor of the order  $e^{-1}$ .

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## I. INTRODUCTION

When a large receiving aperture is used to collect scintillating light, such as laser light that has propagated through a turbulent medium, the percentage fluctuation is not as large as would be obtained if a small aperture were used. The aperture performs an averaging process. This is explained by the fact that for  $D > \rho_0$ , where  $\rho_0$  is the characteristic length of the log-amplitude covariance, several field inhomogeneities with different signs can be found within the limits of the receiving aperture, and therefore they partially compensate one another. In the inertial subrange,  $\rho_0 \approx \sqrt{\lambda z}$ , where  $D$  is the diameter of the assumed circular receiving aperture,  $\lambda$  is the optical wavelength, and  $z$  is the propagation distance.

This process has been studied by Fried,<sup>(1)</sup> who calculates the aperture-averaging factor (defined below) for a large range of the relevant parameters. Central to his calculation is the covariance of irradiance, which he takes as that result that would be obtained from the Rytov approximation. As has been discussed elsewhere,<sup>(2)</sup> the Rytov approximation is invalid for propagation distance  $z$  exceeding a critical distance  $z_c$ . The range  $z_c$  is the range at which the average component of the field is down by a factor of the order  $e^{-1}$  and is typically  $\sim 1$  km for horizontal propagation paths near the earth's surface. The Rytov approximation predicts large dispersion of the intensity for  $z > z_c$ , whereas experimentally,<sup>(3)</sup> the dispersion of the intensity tends asymptotically to unity for  $z > z_c$ . Hence, calculations based on the Rytov approximation are invalid for  $z > z_c$ .

Here we derive the aperture-averaging factor based upon a functional



form of the dispersion of the intensity that is consistent with the experimental observations. Both plane and spherical waves are discussed. The effect of a finite transmitting aperture is not considered here. Only propagation paths with constant turbulence parameters are discussed (e.g., horizontal paths), since the dependence of the turbulence parameters on height is not known well and unduly complicates the problem mathematically.

## II. APERTURE-AVERAGING FACTOR

Consider a circular receiving aperture with diameter  $D$ . The signal variance  $\sigma_s^2$  is given by<sup>(1)</sup>

$$\sigma_s^2 = 2\pi \int_0^D d\rho \rho K(\rho, D) C_I(\rho) \quad (1)$$

where

$$K(\rho, D) = \begin{cases} \frac{D^2}{2} \left\{ \cos^{-1}\left(\frac{\rho}{D}\right) - \left(\frac{\rho}{D}\right) \left[1 - \left(\frac{\rho}{D}\right)^2\right]^{\frac{1}{2}} \right\} & \text{if } \rho \leq D \\ 0 & \text{if } \rho > D \end{cases} \quad (2)$$

$$C_I(\rho) = \overline{(I(\vec{x}) - I_0)(I(\vec{x}') - I_0)} \quad (3)$$

$I_0$  is the average irradiance in the absence of turbulence,  $\rho = |\vec{x} - \vec{x}'|$ , and the bar over a quantity represents the ensemble average of the quantity. The quantity  $C_I(\rho)$  is the covariance of the intensity in a plane perpendicular to the propagation direction at distance  $z$  from the source. The quantity  $K(\rho, D)$  is a geometrical factor which is seen to be just the area of overlap of two circles of diameter  $D$  whose centers are displaced a distance  $\rho$ .

For normalization purposes, it is convenient to divide  $\sigma_s^2$  by  $(\pi D^2/4)^2 C_I(0)$ , the signal variance that would be obtained if the irradiance fluctuations were perfectly correlated over the entire aperture. We call this ratio  $\theta$ , the aperture-averaging factor. We have

$$\theta = \frac{\sigma_s^2}{\left(\frac{\pi}{4} D^2\right)^2 C_I(0)} \quad (4)$$

$$= \frac{32}{\pi D^4} \int_0^D d\rho \rho \frac{C_I(\rho)}{C_I(0)} K(\rho, D)$$

from which, by using appropriate values of  $C_I(\rho)$ , the aperture-averaging factor may be obtained.

Fried<sup>(1)</sup> based his calculation on

$$C_I(\rho) = I_0^2 \left\{ \exp [4C_\ell(\rho)] - 1 \right\} \quad (5)$$

where  $C_\ell(\rho)$  is the covariance of the log-amplitude. Equation (5) is valid at all ranges provided the log-amplitude is normally distributed. Fried has computed the log-amplitude covariance using a modified Rytov approximation (modified to be consistent with energy conservation), which will be denoted by  $C_\ell^R(\rho)$ . As has been discussed elsewhere,<sup>(2)</sup> this approximation is invalid for  $z > z_c$ . The critical range  $z_c$  for  $\lambda = 0.6328\mu$  (He - Ne laser wavelength) is:

<u>Structure Constant</u>	<u>Plane Waves</u>	<u>Spherical Waves</u>
$C_n^2 = 0.3 \times 10^{-15} \text{ cm}^{-2/3}$	$z_c = 3.6 \text{ km}$	$z_c = 5.9 \text{ km}$
$= 3 \times 10^{-15} \text{ cm}^{-2/3}$	$= 1.0 \text{ km}$	$= 1.7 \text{ km}$
$= 30 \times 10^{-15} \text{ cm}^{-2/3}$	$= 0.3 \text{ km}$	$= 0.48 \text{ km}$

No theoretical expression for  $C_I(\rho)$  which is valid for arbitrary  $z$  exists. An expression which is a smooth interpolation between the

limits of small and large  $z$  and gives a value for the intensity variance which agrees to within 20 percent with the experimental values<sup>(3)</sup> has been given by Yura:<sup>(2)</sup>

$$C_I(\rho) = I_0^2 \left[ 1 - \exp(-2 \sqrt{C_\ell^R(\rho)}) \right]^2 \quad (6)$$

Equation (6) agrees with Eq. (5) for small  $z$  (where  $C_\ell^R(0) \ll 1$ ) and tends asymptotically to  $I_0^2$  for large  $z$  (where  $C_\ell^R(0) \gg 1$ ). Figure 1 is a graph of  $C_I(0)/I_0^2$  versus  $C_\ell^R(0)$  as given by Eqs. (5) and (6).

The aperture-averaging factor  $\theta$  is obtained here from  $C_I(\rho)$  as given by Eq. (6). Fried and Cloud<sup>(4)</sup> have given an analytic representation of  $C_\ell^R(\rho)$  for plane waves assuming horizontal propagation paths and that the Kolomogorov spectrum represents the turbulence-induced fluctuations of the index of refraction. Their result is given by Eq. 2.12 of Ref. 4. Similarly, Fried<sup>(5)</sup> has derived an analytic expression for  $C_\ell^R(\rho)$  for spherical waves. This result is given by Eq. 2.8 of Ref. 5.

We can now calculate the normalized signal standard deviation  $[\sigma_s^2/(\bar{S})^2]^{\frac{1}{2}}$ , where  $\bar{S}$ , the average signal, equals  $\pi D^2 I_0/4$ . From Eq. (4) we find

$$\sigma = \left( \frac{\sigma_s^2}{(\bar{S})^2} \right)^{\frac{1}{2}} = \left( \frac{\theta C_I(0)}{I_0^2} \right)^{\frac{1}{2}} \quad (7)$$

The values of  $C_\ell^R(0)$  have been derived by Fried and Cloud<sup>(4)</sup> for plane waves and by Fried<sup>(5)</sup> for spherical waves. They are given by

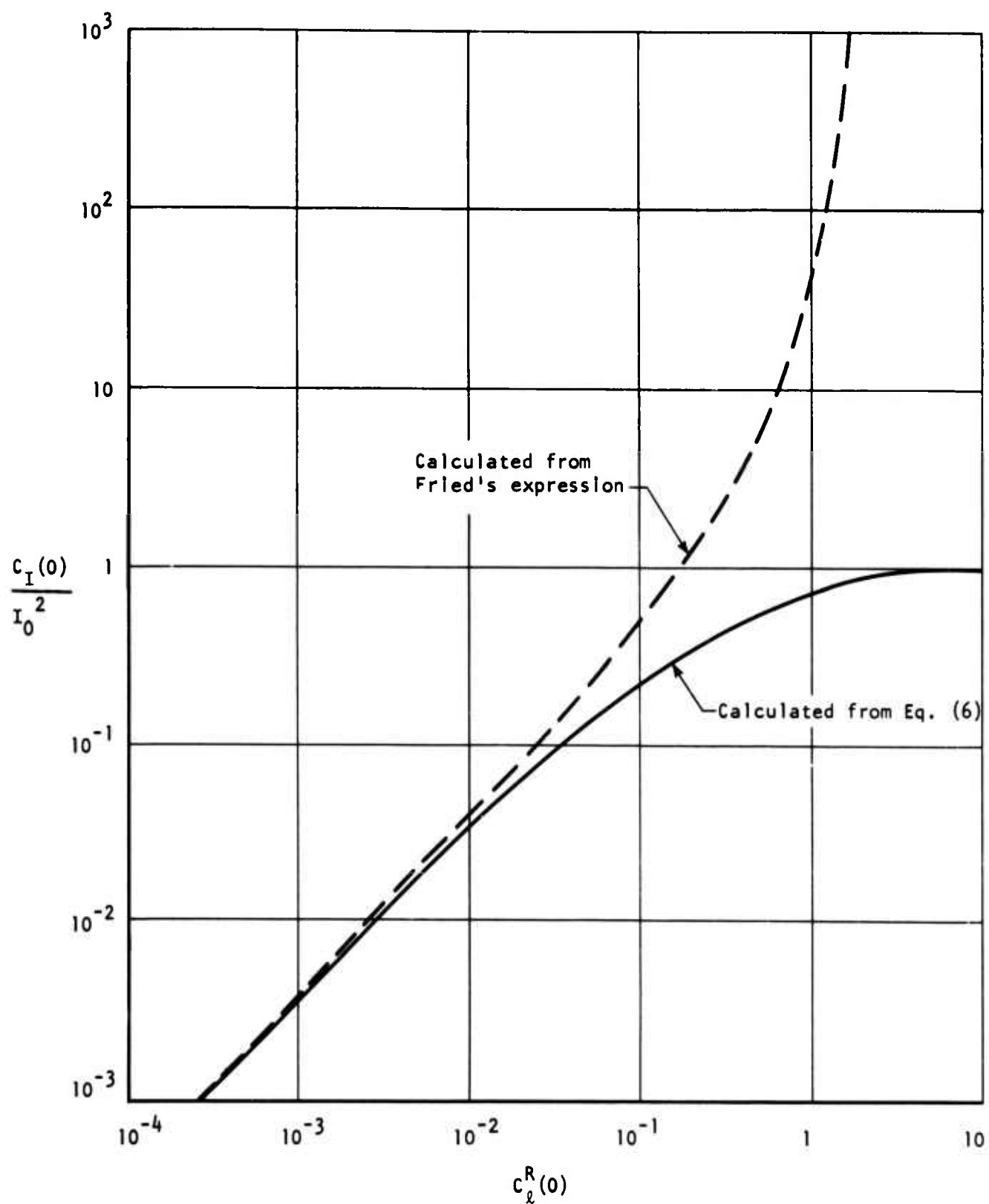


Fig. 1--The normalized variance of irradiance  $C_I(0)/I_0^2$  as a function of log-amplitude variance  $C_l^R(0)$

$$C_{\ell}^R(0) = 0.309k^{7/6} z^{11/6} C_n^2 \quad (\text{plane waves}) \quad (8)$$

and

$$C_{\ell}^R(0) = 0.124k^{7/6} z^{11/6} C_n^2 \quad (\text{spherical waves}) \quad (9)$$

Figure 2 gives the aperture-averaging factor for plane and spherical waves as a function of the normalized receiver aperture diameter  $D/\sqrt{4z/k}$  ( $k = 2\pi/\lambda$ ) for  $z = 1$  km,  $\lambda = 0.6328\mu$ , and  $C_n^2 = 3 \times 10^{-15} \text{ cm}^{-2/3}$ , where  $C_n^2$  denotes the index structure constant.\*

Figure 3 gives the aperture averaging factor for plane and spherical waves as a function of the normalized range  $4z/kD^2$  for  $D = 5$  cm,  $\lambda = 0.6328\mu$ , and  $C_n^2 = 3 \times 10^{-15} \text{ cm}^{-2/3}$ . For comparison purposes, the results based on Fried's expression are included in these graphs.

The differences in  $\theta$  for long propagation paths are readily discernible. This is because the aperture standard deviation as computed from Eq. (6) tends to a finite value for large  $z$ , while the same quantity computed from Eq. (5) increases without limit. Figures 4 and 5 give the aperture-averaging factor as a function of the normalized range  $4z/kD^2$  (for  $D = 5$  cm) and as a function of  $D/\sqrt{4z/k}$  ( $z = 1$  km) for  $\lambda = 0.6328\mu$ , and various values of  $C_n^2$  for plane and spherical waves, respectively.\*\* Figure 6 is a plot for plane waves

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\*The values of  $C_n^2$  typical of turbulence in the first few hundred meters of the atmosphere are:  $30 \times 10^{-15} \text{ cm}^{-2/3}$  (strong daytime turbulence within a few meters of the ground),  $3 \times 10^{-15} \text{ cm}^{-2/3}$  and  $7 \times 10^{-15} \text{ cm}^{-2/3}$  (moderate daytime turbulence and/or strong nighttime turbulence), and  $0.3 \times 10^{-15} \text{ cm}^{-2/3}$  (very weak turbulence which occurs in the near-neutral periods at dawn or dusk).

\*\*The specific expressions used here for the normalized range and receiver aperture are introduced for convenience and are not to be interpreted as scaling parameters.

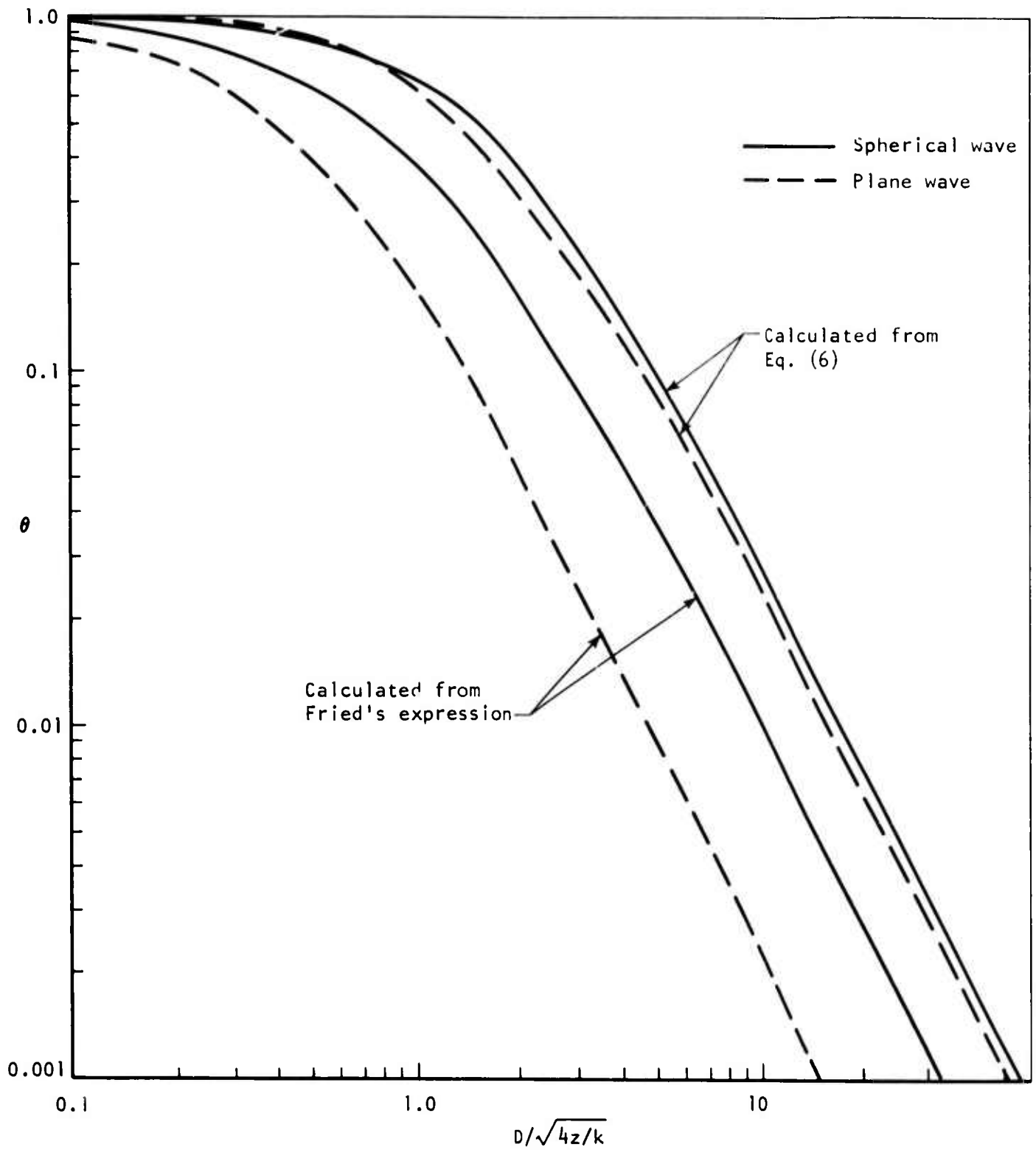


Fig. 2--The aperture-averaging factor  $\theta$  as a function of normalized receiver aperture diameter  $D/\sqrt{4z/k}$   
 ( $\lambda = 0.6328\mu$ ,  $z = 1$  km, and  $C_n^2 = 3 \times 10^{-15} \text{ cm}^{-2/3}$ )

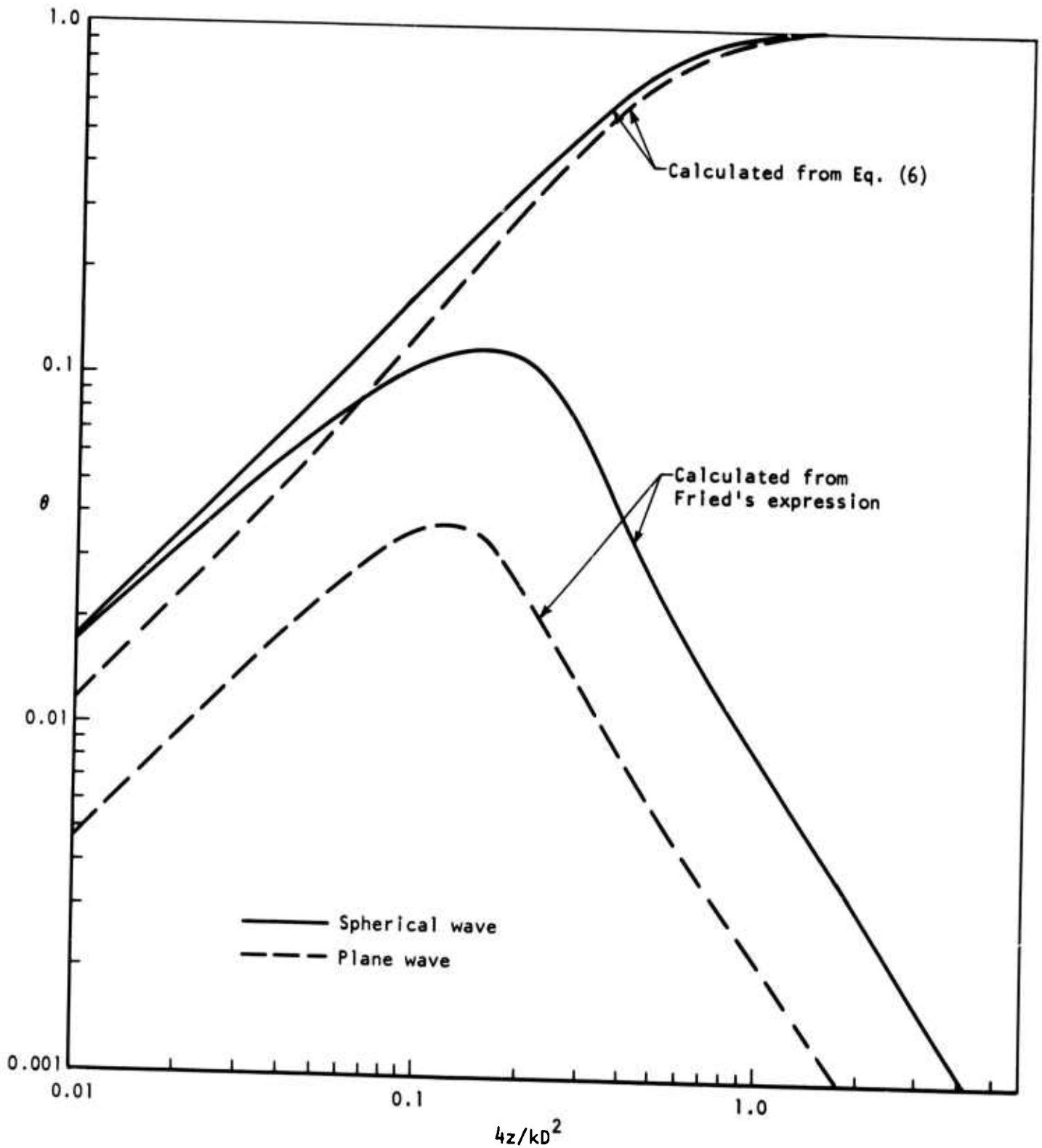


Fig. 3--The aperture-averaging factor  $\theta$  as a function of normalized range  $4z/kD^2$   
 $(\lambda = 0.6328\mu, D = 5 \text{ cm}, \text{ and } C_n^2 = 3 \times 10^{-15} \text{ cm}^{-2/3})$



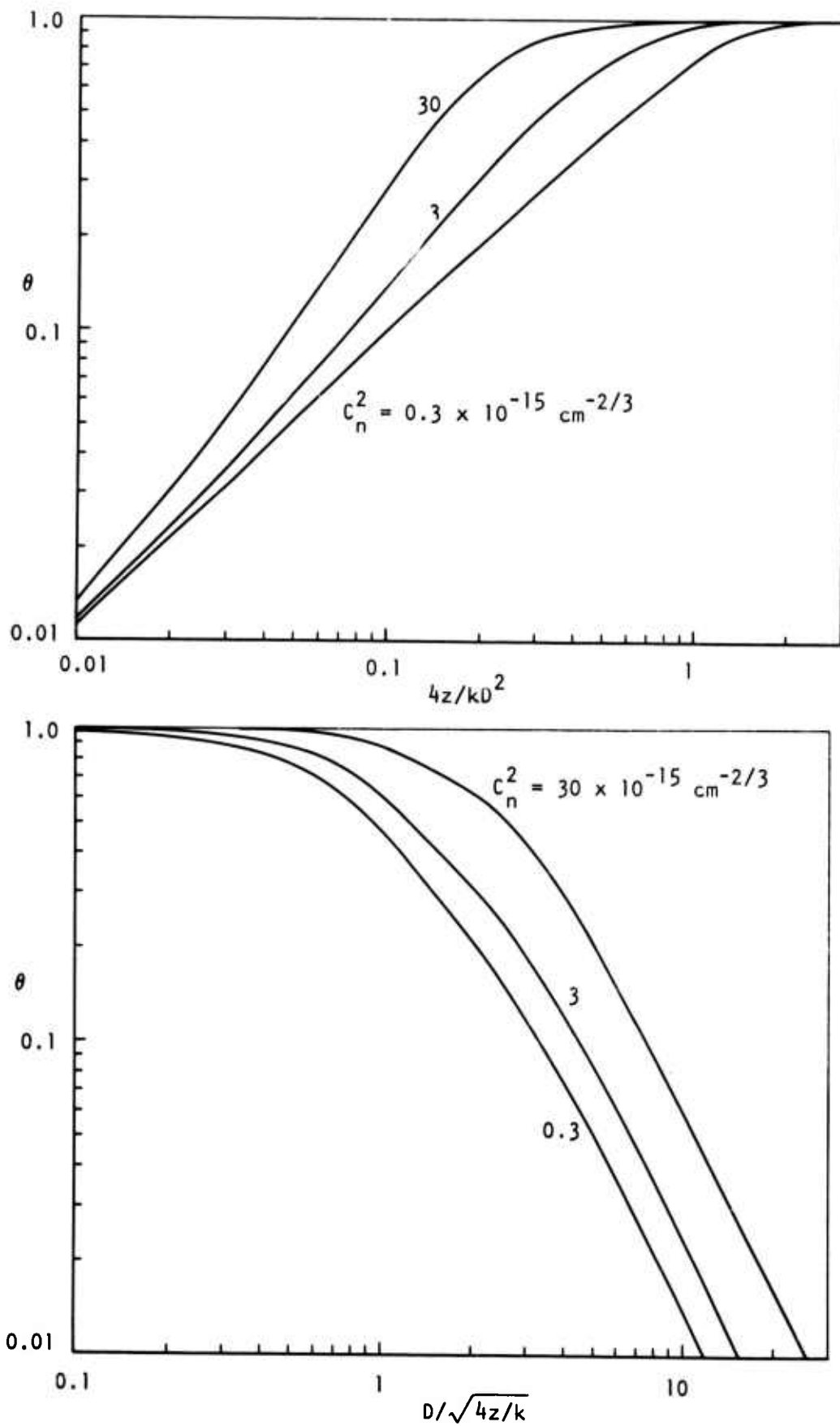


Fig. 4--The aperture-averaging factor  $\theta$  for plane waves as a function of normalized range  $4z/kD^2$  ( $\lambda = 0.6328\mu$ ,  $D = 5 \text{ cm}$ ) and normalized receiver diameter  $D/\sqrt{4z/k}$  ( $\lambda = 0.6328\mu$ ,  $z = 1 \text{ km}$ ), for various values of the index structure constant

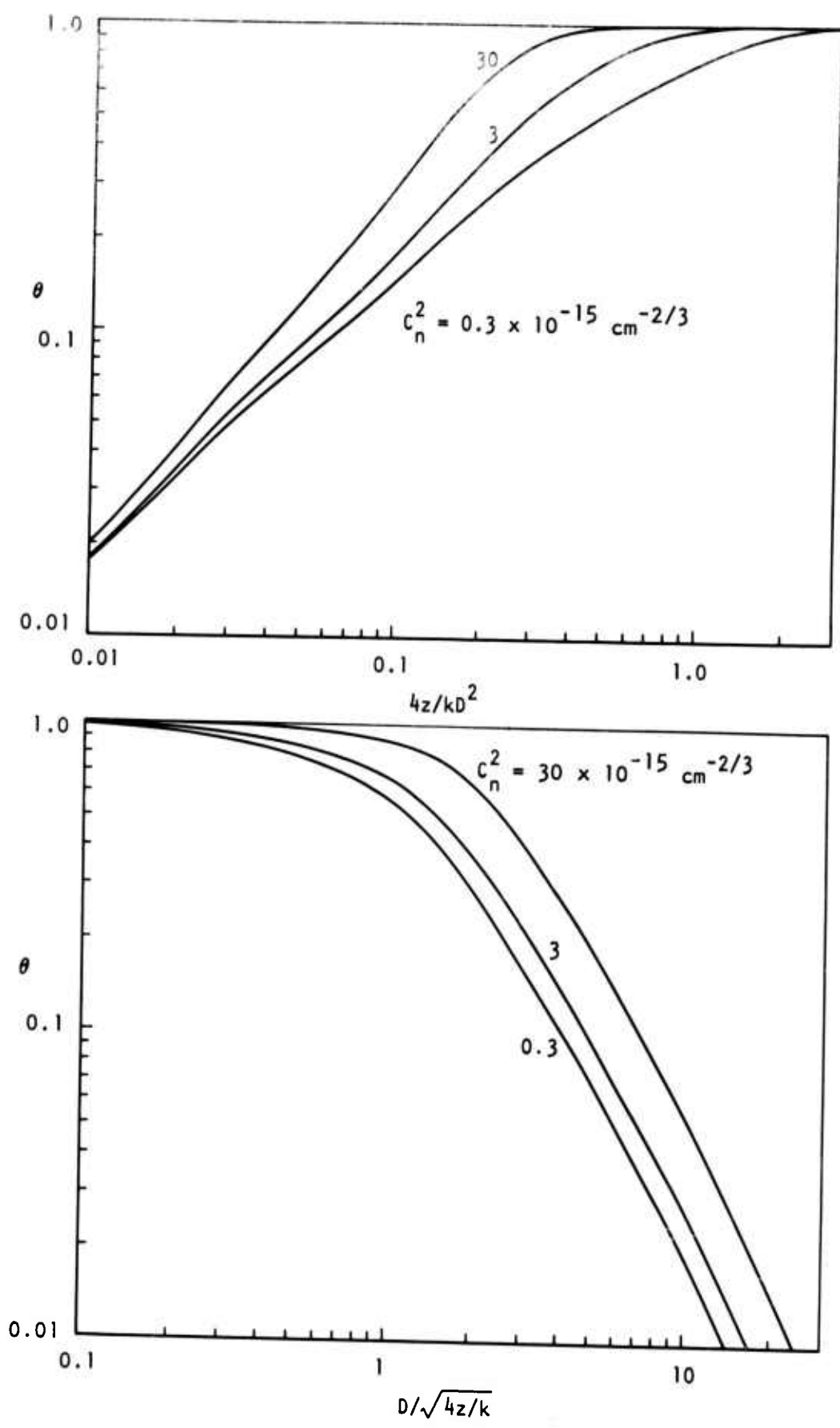


Fig. 5--The aperture-averaging factor  $\theta$  for spherical waves as a function of normalized range  $4z/kD^2$  ( $\lambda = 0.6328\mu$ ,  $D = 5 \text{ cm}$ ) and normalized receiver diameter  $D/\sqrt{4z/k}$  ( $\lambda = 0.6328\mu$ ,  $z = 1 \text{ km}$ ), for various values of the index structure constant

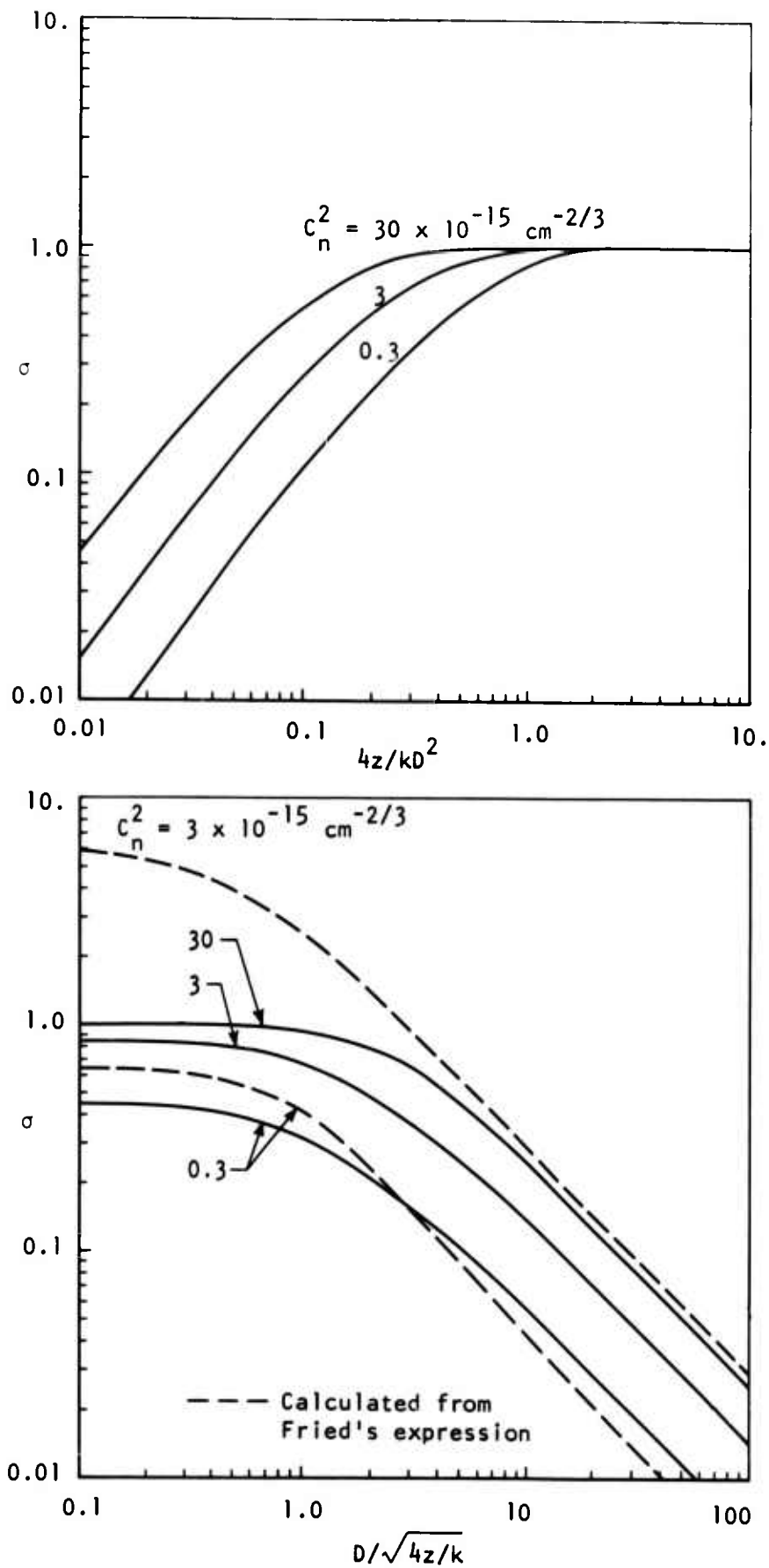


Fig. 6--The normalized signal standard deviation  $\sigma$  for plane waves as a function of normalized range  $4z/kD^2$  ( $\lambda = 0.6328\mu$ ,  $D = 5$  cm) and normalized receiver diameter  $D/\sqrt{4z/k}$  ( $\lambda = 0.6328\mu$ ,  $z = 1$  km), for various values of the index structure constant

of the normalized signal standard deviation  $\sigma$  as a function of  $4z/kD^2$  and as a function of  $D/\sqrt{4z/k}$  for the same values of the parameters, while Fig. 7 gives similar graphs for the normalized signal standard deviation for spherical waves. For comparison purposes, the results of using Fried's expression are included in Figs. 6 and 7. At small apertures the difference between the present analysis and that of Fried should be readily observable.

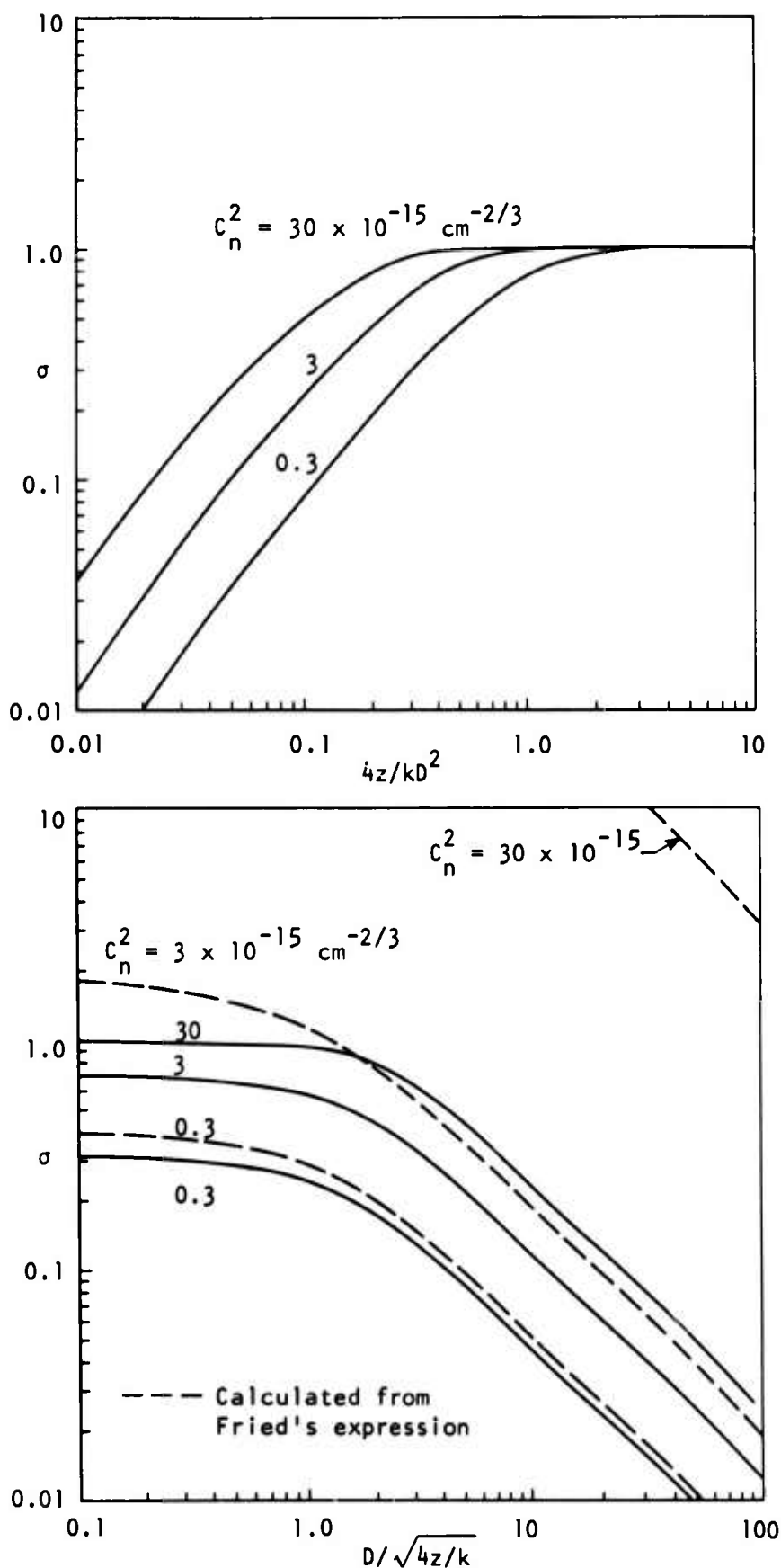


Fig. 7--The normalized signal standard deviation  $\sigma$  for spherical waves as a function of normalized range  $4z/kD^2$  ( $\lambda = 0.6328\mu$ ,  $D = 5$  cm) and normalized receiver diameter  $D/\sqrt{4z/k}$  ( $\lambda = 0.6328\mu$ ,  $z = 1$  km), for various values of the index structure constant

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