

AD 688225

AFOSR Scientific Report

AFOSR 69-0878 TR

Air Force Office of Scientific Research

Office of Aerospace Research

Grant AFOSR 1424-68

THE STABILITY OF A CLAMPED SHALLOW ARCH
SUBJECTED TO IMPULSIVE LOADING

Maxwell C. Cheung and Charles D. Babcock, Jr.



Graduate Aeronautical Laboratories

California Institute of Technology

Pasadena, California

March 1969

1. This document has been approved for public
release and sale; its distribution is unlimited.

Reproduced by the
CLEARINGHOUSE
for Federal Scientific & Technical
Information Springfield Va 22151

AD 68825

AFOSR Acientific Report

AFOSR 69-0878 TR

THE STABILITY OF A CLAMPED SHALLOW ARCH
SUBJECTED TO IMPULSIVE LOADING

by

Maxwell C. Cheung and Charles D. Babcock. Jr.

March 1969

This work was supported by the Air Force Office of Scientific Research,
Office of Aerospace Research, United States Air Force
under Grant No. AFOSR 68-1424

Graduate Aeronautical Laboratories

California Institute of Technology

Pasadena, California

TABLE OF CONTENTS

	Page
I. INTRODUCTION	1
II. EXPERIMENT	3
1. Test Specimens	3
2. Impulsive Load	4
3. Initial Imperfection Measurement	5
4. Camera Setup	6
5. Response Measurement	6
6. Test Procedure	7
III. TEST RESULTS	9
IV. CONCLUSION	11
REFERENCES	12
TABLES	14
FIGURES	16

ACKNOWLEDGMENT

The authors wish to express their sincere appreciation to Dr. E. E. Sechler for his valuable suggestions and encouragement received during the course of this investigation. The comment and support from Dr. W. G. Knauss is also appreciated. They also want to thank Mr. George Carlson of the Aeronautics Shop and Mr. Marvin Jessey of the Electronics Laboratory. Very much appreciated was the help of Miss Helen Burrus for the typing and Mrs. Betty Wood for the graphs and figures.

This study was supported by the Air Force Office of Scientific Research, Office of Aerospace Research, United States Air Force under Grant No. AFOSR 68-1424.

ABSTRACT

Clamped circular aluminum arches have been explosively loaded using a spray deposited light ignited explosive (Silver Acetylide-Silver Nitrate). Using high speed photography and computerized data reduction, a detailed time history of the arch deflection has been obtained. This deflection was fitted with mode shapes in order to display the motion graphically. The experimental data shows that the clamped arch does not have a jump in response with increasing load level, but a transition from small displacement response to a large displacement response over a sizable range of load level.

NOMENCLATURE

A	Surface area of the arch (in ²)
E	Young's Modulus (psi)
h	Arch thickness (in)
L	Arch length (in)
I	Specific impulse (lb-sec/in ²)
\bar{I}	Nondimensional impulse = $\frac{4I^2 R^2}{h^4 E \rho}$
q_1, q_2, q_3	Generalized displacement coordinates
R	Arch radius (in)
t	Time (sec)
w	Arch displacement (in)
W_{\max}^a	Maximum nondimensional average displacement
W_e	Weight of explosive (gm)
α	Independent variable (radian)
β	Arch half angle (radian)
γ	Geometric parameter = $\frac{L^2}{4Rh}$
ρ	Mass density (lb sec ² /in ⁴)

I. INTRODUCTION

The problem of dynamic buckling or nonlinear response of shell like structures has been widely discussed in the past few years. The question of dynamic buckling is of concern since this phenomenon is associated with a large change in the structural response with a small change in the loading conditions. This change of response character with load is similar to the stability problem of static loading and hence the name dynamic buckling has often been adopted. However, since the loading and response are functions of time, the analogy is not quite complete. This has led to a variety of definitions of dynamic buckling.

As a result of the interest in this problem, much work has been done to determine simplified methods of finding the critical loading conditions without resorting to a direct solution of the time dependent nonlinear partial differential equations. This has led to methods of determining the relation between the static and dynamic buckling load for step loading (Refs. 1 and 2). Methods depending upon energy balances have also been developed (Refs. 3 and 4).

The shallow arch has been chosen as a model for many of these analytical studies. This rather simple structure has all of the nonlinear characteristics of much more complicated shell structures. A wide variety of this nonlinear behavior can be exhibited by changing the end conditions, loading or geometry of the arch.

From an experimental point of view, the clamped arch is the most practical problem when dynamic loading is used. Other boundary conditions are very difficult to duplicate in the laboratory. The clamped

arch was the subject of a study by Humphreys (Ref. 5). His analysis used a Galerkin procedure and an analogue computer. In addition, he performed experiments using a clamped arch and sheet explosive.

The clamped arch was also considered by Simitzes (Ref. 4) who used an energy analysis. However, Vahidi (Ref. 6) recently pointed out that the clamped arch under impulse loading does not have any equilibrium positions, stable or unstable, except the undeformed position. This would indicate that Simitzes analysis is in error. In fact, the nonexistence of other equilibrium positions suggest that the clamped arch under impulse may not exhibit a large increase in response at some critical impulse level as does the simply supported arch.

In order to clarify the behavior of the clamped arch under impulsive loading, the problem is reexamined in the report. Carefully controlled experiments have been carried out to obtain a time history of the arch motion after being explosively loaded. This data is then examined in detail and presented in a manner showing the difference in the behavior for this type of structure as opposed to the impulsively loaded simply supported arch.

II. EXPERIMENT

The experimental part of this work consisted of loading a clamped circular arch with a very short duration uniform pressure load. The resulting motion of the arch was recorded using high speed photography. A detailed description of the experimental work is presented in the following sections.

1. Test Specimens

The equations that describe the motion of a shallow arch show that the circular arch can be characterized by one nondimensional parameter called γ in this report (See Figure 1).

$$\gamma = \frac{L^2}{4Rh} = \frac{\beta^2 R}{h}$$

For all of the arches tested, the thickness was held constant and the same nominal radius of curvature was used.

The specimens were cut from 1/16 inch thick 2024 aluminum sheet, and were trimmed by the milling machine to 3/4 inch wide. The strips were then rolled to approximately 30 inches radius in a three-roll roller. The radius was rough checked using a template. After rolling, the arches were heat treated. Tensile tests on curved specimens cut from the arches gave a Young's Modulus of 10.5×10^6 lb/in² and a proportional limit above 45,000 lb/in².

The arches are mounted into a heavy steel frame for testing. The ends are secured using Devcon B. An arch mounted in the frame is shown in Fig. 2.

2. Impulsive Load

Silver Acetylide-Silver Nitrate was used to apply a very short duration pressure loading on the arch. The advantage of this chemical is the low level of impulse possible, which is approximately one-tenth of commercially available sheet explosives. The general properties are introduced in references 7 and 8. More details are given in references 9 and 10. The explosive is produced in the form of a fine grained powder and is applied to the test specimen by spraying using acetone as a thinner.

This explosive can be detonated by an electric spark or an intense light. It has been shown that a fairly large area can be detonated within a few microseconds if expendable Xenon flash tubes are used (Ref. 8). It was found that if one nonexpendable Xenon flash tube was used that complete detonation over the longest arch could be accomplished in less than 150 microseconds. This duration of loading on the whole arch is short enough that it can be considered impulsive.

A GE-522 Xenon flash tube, mounted in a parabolic reflector, was used as the detonator. The tube was driven by a 250 microfarad capacitor charged to 5,000 volts. The rise time of the tube was about 20 microseconds and the duration of the light pulse was of the order of 300 microseconds.

The calibration of the impulse generated by this explosive was carried out on a ballistic pendulum. The explosive was deposited evenly on a 2 inch square steel plate and attached to the end of the pivoted arm of the pendulum. The calibration curve of impulse vs. weight is shown in Fig. 3. The offset from the origin is due to the

friction at the pivot point. From this data an impulse level of $0.152 \frac{\text{lb-sec/in}^2}{\text{gm/in}^2} (6.75 \times 10^4 \frac{\text{dyne-sec/cm}^2}{\text{gm/cm}^2})$ was calculated. This is about 3.6 per cent lower than previously found in reference 7.

3. Initial Imperfection Measurement

A pendulum like apparatus was built to measure imperfections of the arch. It consisted of a fixed center and rotatable arm which can be adjusted in length. A dial indicator with a working range of 0.060 inches was installed at the tip of the arm (See Figure 4).

The measurement was made by first adjusting the arm to the appropriate radius then pressing the concave side of the arch gently against the dial gauge. Starting from one end of the arch the dial gauge readings were taken at half inch interval along the arch until the dial gauge could not advance another full step. This gave the deviation of the arch shape from the preset radius. Applying the "Least Square Method" to the measured data the "best fit radius" and the "best fit imperfection" can be found.

Before the arches were mounted on the supports the imperfections were measured. Only those arches with imperfection amplitude less than five thousandth of an inch were used. Also, they were cut from longer ones so that the selected sections had a minimum amount of asymmetric imperfection. When the arch was mounted and the Devcon was hardened, the arch shape was measured again to make sure that it had not been excessively deformed due to the process of mounting.

4. Camera Setup

A 16 mm, high speed motion picture camera (HyCam, Red Lake Laboratories) was used to record the response of the arch. The axis of the camera was aligned perpendicular to the base line of the arch. To eliminate as much distortion as possible, the camera was placed at least 7 feet away.

The light was provided by four 1000 watt quartz-iodine flood lamps. They were placed on the side of the arch opposite the camera and aimed directly at the camera. In order to collect more light at the camera, a large plastic Fresnel lens of focal length of 30 inches was placed between the arch and the lamps. A sheet of clear Mylar was placed on the flat side of the Fresnel lens to help diffuse the intense and unevenly concentrated light. A spot light meter at the camera position was used to check the evenness of the lighting of the Fresnel lens. The camera setup is shown in Fig. 5.

A 400 foot roll of Kodak Tri-X negative (ASA rating of 400) of standard thickness (0.006 inches) was used for each test shot. However, the desired framing rate of 10,000 pictures per second was only obtained on the last 100 feet of film. The exact framing rate was determined by a timing light which exposed a small dot of light on the edge of the film every 1/1000 of a second.

5. Response Measurement

A detailed deflection history of the arch motion was obtained by reading the film frame by frame. The film was projected using a 500 watt slide projector on a mirror at a distance of about 15 feet. The image was reflected to a screen ruled with 21 equally spaced lines

(Fig. 6). The distance to the mirror was adjusted until the arch image from support to support fit exactly on the twenty equal division on the screen. The lines then serve as the X coordinate for the arch. The Y distance was measured using a stretched wire perpendicular to the X lines. This wire was attached to a moving slide whose position was measured by a linear potentiometer. The wire was carefully moved to each intersection of the arch image and the lines ruled on the screen. The potentiometer reading was automatically recorded on punched cards for digital computer reduction. Approximately 50 frames were read for each test covering about 10 milliseconds of motion.

6. Test Procedure

When a particular value of γ has been selected the arch was cut to the appropriate length. The surface of the arch was then cleaned and the weight determined to the nearest 1/2000 of a gram. An estimation of the weight of the explosive was made and then it was sprayed on the convex side of the arch. The explosive was cured for one hour at 100 degrees Fahrenheit. The arch and the explosive are then weighed and the amount of deposited explosive determined. If the correct amount is not obtained the process was repeated until the weight differs from that desired by a few hundredths of a gram. Next the arch was secured in the steel frame with Devcon B.

After the Devcon has hardened, the arch and support are aligned with the camera. The Xenon flash tube was placed about 10 inches above the surface of the arch and the evenness of the light source was checked. The camera was started and an Event Synchronizer

built into the camera discharges the Xenon tube when 325 feet of film has been exposed. The deformation of the arch that is of interest takes about 3 feet of film. The last 100 feet of exposed film was developed and the part of interest retained.

III. TEST RESULTS

A total of 11 tests were recorded by the high speed camera. These tests were divided into two groups. Group B had a geometric parameter close to 10 and for group C, γ was close to 20. The dimensions are listed in Table I. The arches used for testing were selected so as to minimize the size of initial imperfections. In all cases the deviation from the perfect arch was less than 5 per cent of the thickness.

The response of the arches to impulsive loading was examined in several manners. First the average displacement was calculated for the time of response of interest. This displacement is defined as the area between the deformed and undeformed arch. It is normalized to the area enclosed by the undeformed arch and the base line. The values of the maximum average displacement are listed in Table II and are shown in Fig. 7 as a function of the nondimensional impulse \bar{I} . This figure shows that the arch exhibits a large increase in maximum response over a rather small range of impulse level. The maximum rate of change of response (the inflection points for the faired curves in Fig. 7) are at an impulse level \bar{I} of 9 and 22 for γ equal to 10 and 20 respectively. It is interesting to note that these points are close to the point of average displacement equal to one, which was used by Humphreys (Ref. 5) to define the critical load. However, they are about 6-7 times as high as Humphreys' experimental results. The reason for this is not known at this time.

In order to get a more quantitative picture of the arch deformation, a three mode approximation to the deformed shape was calculated.

The representation used is as follows:

$$w = \left(1 - \frac{\alpha^2}{\beta^2}\right)^2 \left[q_1(t) + \frac{\alpha}{\beta} q_2(t) + \frac{\alpha^2}{\beta^2} q_3(t) \right] h$$

The coefficients were determined by using a "Least Square Fit" of the experimental data from the high speed pictures. The time history of the three modes is shown in Fig. 8 for a typical test. The trajectories in a q_1, q_3 space can also be displayed. Figure 9 shows a supercritical and subcritical response for each group of arches. The line of average displacement equal to one is also shown in the figures.

In addition to the dynamic test results described above, an attempt was made to determine the static equilibrium positions of the clamped arch. This is of interest since the existence of these positions is a requirement in an energy approach to determining the dynamic buckling loads. These tests were carried out by pushing the arch through to a large displacement configuration by hand and attempting to find a position where it would stay. This was unsuccessful for the 4 arches used with $\gamma \approx 10$ and 20. In addition, the existence of an unstable equilibrium position could not be detected. This is somewhat more difficult to determine experimentally since it is like trying to balance a ball on top of a hill. However, there did not seem to be any equilibrium points other than the undeformed position for the arch tested. This is in agreement with Vahidi's calculation (Ref. 6).

IV. CONCLUSION

The experimental work on the impulsively loaded clamped arch shows that the maximum response has a significant increase in value for a small increase in load at some value of impulse. It would appear for the data available that this increase is a smooth transition from a small response to a large response at some critical impulse level. Therefore, the use of a definition of dynamic buckling which requires a finite change in response for an infinitesimal increase in load would not consider this problem as a dynamic buckling problem. However, from a practical point of view the increase in deflection is of the order of three for a small increase in impulse level. This increase is about the same amount as one obtains for step loading on a simply supported arch in the range of geometric parameter where direct snapping occurs (Ref. 11).

It is of interest to note that this problem is like the one of direct snapping as categorized by Lock (Ref. 12). In other words the structure reaches its maximum displacement on the first oscillation of the fundamental mode. This can be seen from the response plot (Fig. 8) combined with the trajectories (Fig. 9). An examination of the nonsymmetric response shows that it does not appear to be parametrically excited by the symmetric mode as is characteristic of indirect snapping.

In summary, while no evidence has been found that the clamped circular arch under impulse loading can be rigorously categorized as a dynamic buckling problem, it is clear that over a small range

in impulse the arch undergoes a significant increase in response. It is therefore of practical significance to determine this range of impulse. In addition, it was experimentally determined that no stable equilibrium position exists for the clamped arch free from lateral load other than the undeformed position. Also, the existence of an unstable equilibrium position was not detected.

References

1. Budiansky, B.: "Dynamic Buckling of Elastic Structures: Criteria and Estimates", Dynamic Stability of Structures. Edited by Herrmann, G., Pergamon Press, 1967, pp. 83-106.
2. Hutchinson, J. W. and Budiansky, B.: "Dynamic Buckling Estimates", AIAA Journal, Vol. 4, No. 3, March 1966, pp. 525-530.
3. Hsu, C. S.: "On Dynamic Stability of Elastic Bodies with Prescribed Initial Conditions", Int. J. Engineering Sci., Vol. 4, March 1966, pp. 1-21.
4. Simitzes, G. J.: "Dynamic Snap-Through Buckling of Low Arches and Shallow Spherical Caps", Ph. D. Dissertation, Dept. of Aeronautics and Astronautics, Stanford University, June 1965.
5. Humphreys, J. S.: "On Dynamic Snap-Buckling of Shallow Arches", AIAA Journal, Vol. 4, No. 5, May 1966, pp. 878-886.

6. Vahidi, B.: "Non-Existence of Snap-Through for Clamped Shallow Elastic Arches Subjected to Impulsive Load", TR No. 8, University of California, San Diego, March 1969.
7. Nevill, G. E. and Hoese, F. O.: "Impulsive Loading Using Sprayed Silver Acetylide-Silver Nitrate", Experimental Mechanics, Sept. 1965, pp. 294-298.
8. Hoese, F. O.; Langner, C. G. and Baker, W. E.: "Simultaneous Initiation over Large Areas of a Spray Deposited Explosive", Experimental Mechanics, Sept. 1968, pp. 392-397.
9. Clift, G. D. and Fedoroff, B. T.: "A Manual for Explosives Laboratory", Vol. 1 and 2, Lefax Soc. Inc., 1943.
10. Nieuwland, J. A. and Vogt, R. R.: "The Chemistry of Acetylene", Reinhold, New York, 1945.
11. Lock, M. H.: "The Snapping of a Shallow Sinusoidal Arch under a Step Pressure Load", AIAA Journal, Vol. 4, No. 7, July 1966, pp. 1249-1256.
12. Lock, M. H.: "A Study of Buckling and Snapping under Dynamic Loads", TR-0158(3240-30)-3, Aerospace Corp., Dec. 1967.

TABLE I

Geometric Description of Tested Arches

Arch	Thickness (in)	Radius (in)	Length (in)	γ
B9	0.0623	31.76	8.64	9.43
B6	0.0623	30.22	8.43	9.44
B7	0.0622	25.45	8.13	10.44
B8	0.0623	32.33	8.57	9.12
C3	0.0605	31.31	12.08	19.25
C2	0.0605	31.57	12.20	19.48
C5	0.0605	30.55	12.28	20.40
C4	0.0605	28.10	12.12	21.60
C8	0.0604	29.47	12.03	20.33
C7	0.0604	26.64	12.11	22.79
C9	0.0603	31.84	12.31	19.73

TABLE II

Summary of Impulse Tests

Arch	$W_e/A(\text{gm/in}^2)$	\bar{I}	W_{max}^a
B9	0.0626	8.43	0.5596
B6	0.0749	10.45	1.3905
B7	0.0964	12.19	1.5830
B8	0.0847	14.82	1.7342
C3	0.0742	12.60	0.2281
C2	0.0799	15.08	0.3616
C5	0.0826	16.66	0.3969
C4	0.0854	16.95	0.4669
C8	0.0962	20.33	0.6239
C7	0.1098	21.65	0.9185
C9	0.1037	27.75	1.6227

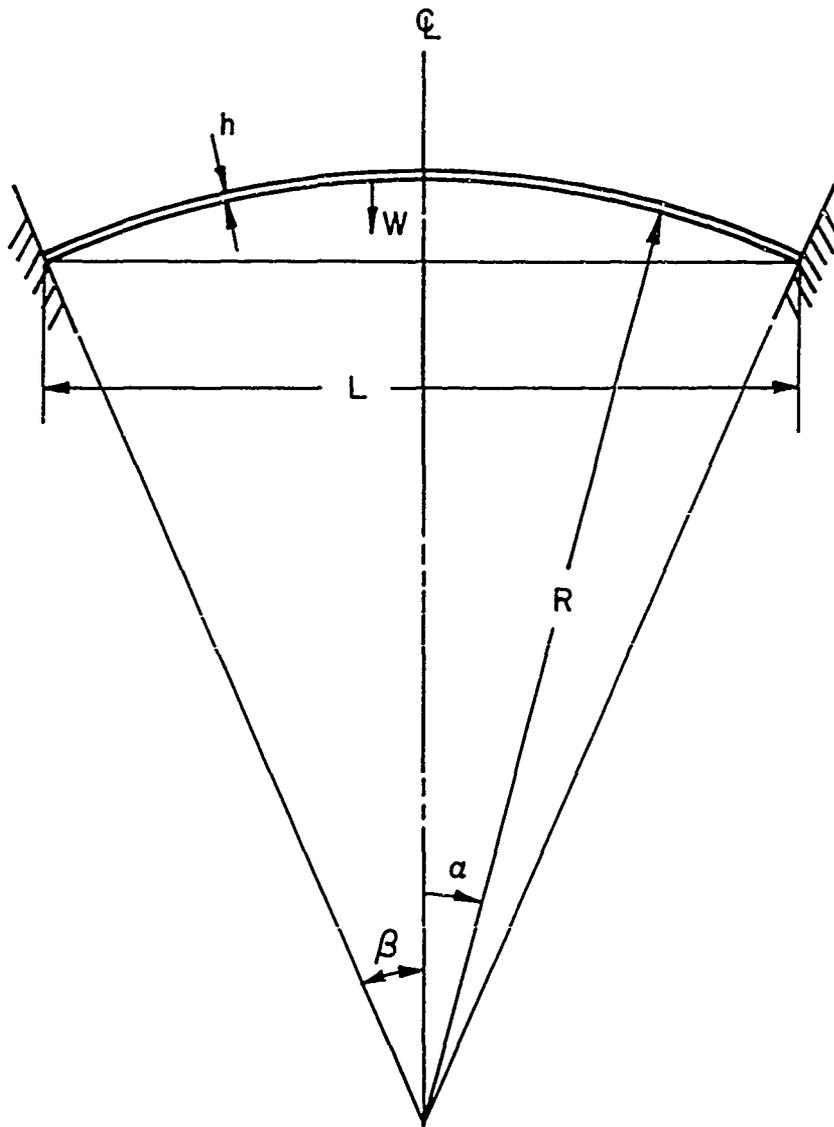


FIG. 1 COORDINATE SYSTEM OF CIRCULAR ARCH

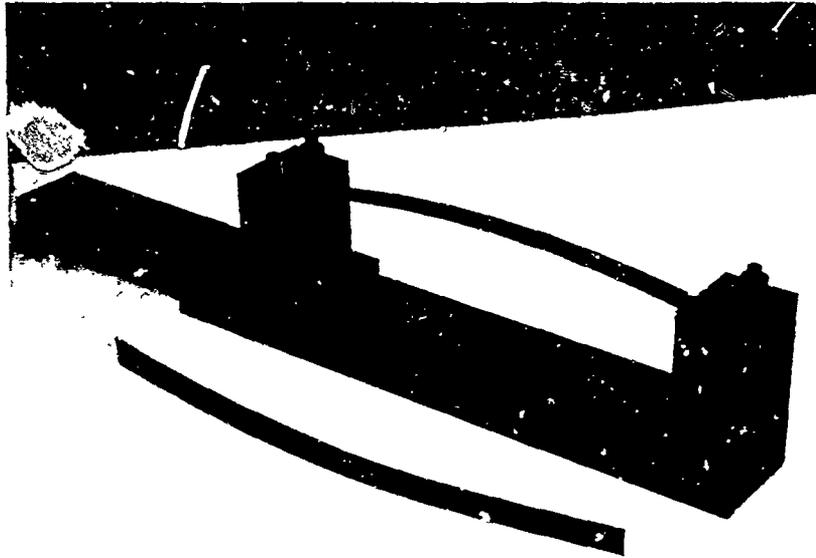


Fig. 2. Circular Arch Before and After Installation in Steel Frame.

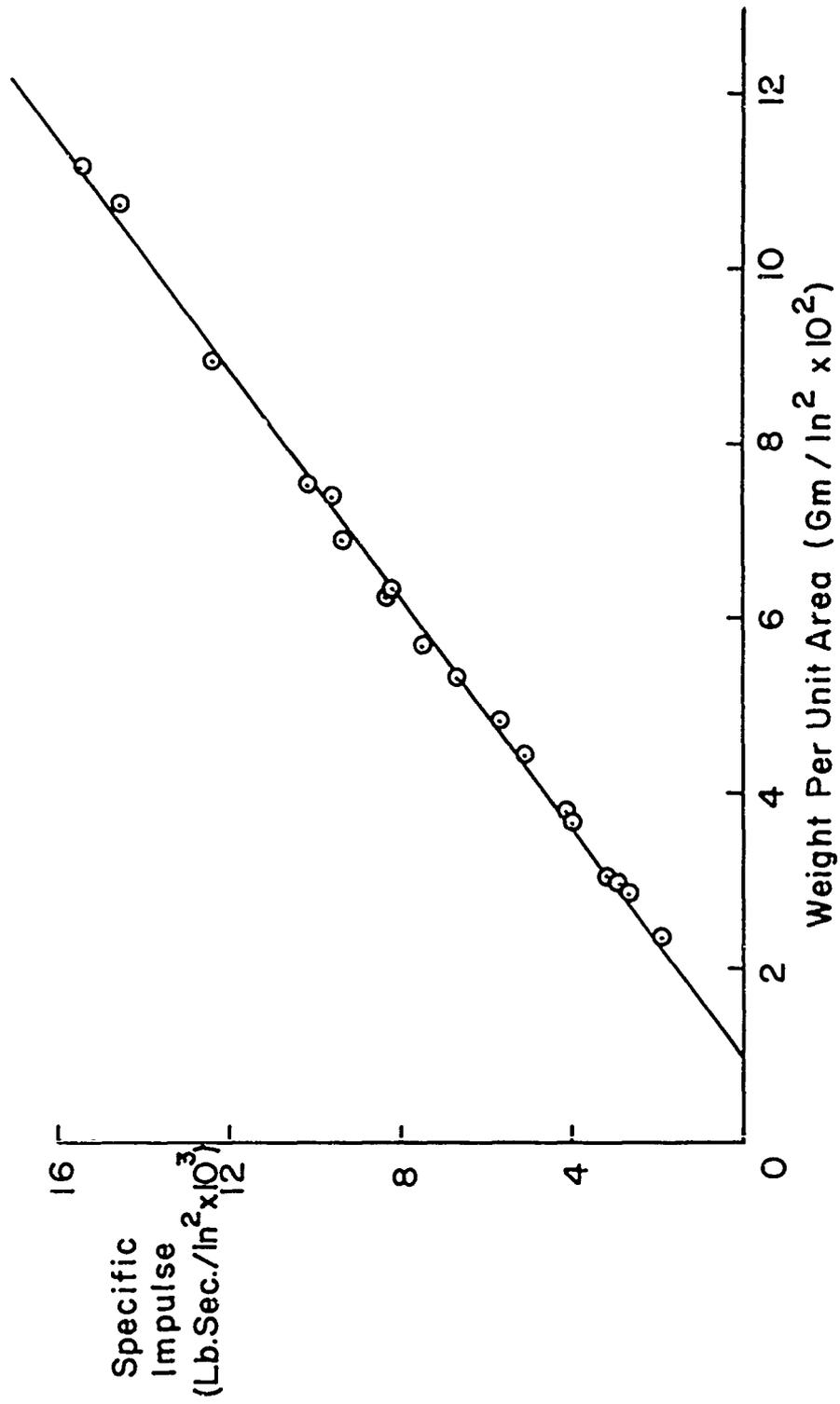


FIG. 3 CALIBRATION OF EXPLOSIVE (SILVER ACETYLIDE-SILVER NITRATE)

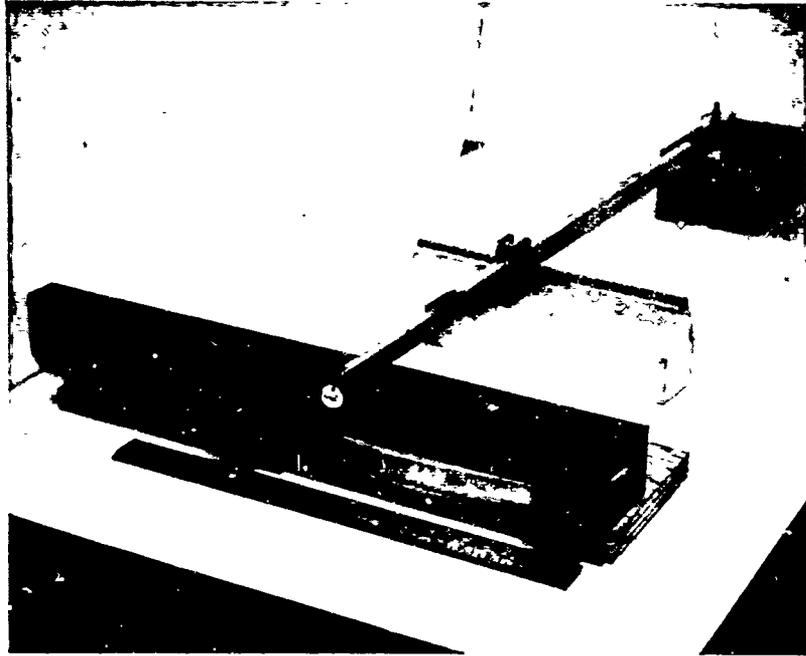


Fig. 4. Measuring of Initial Imperfection of Circular Arch.

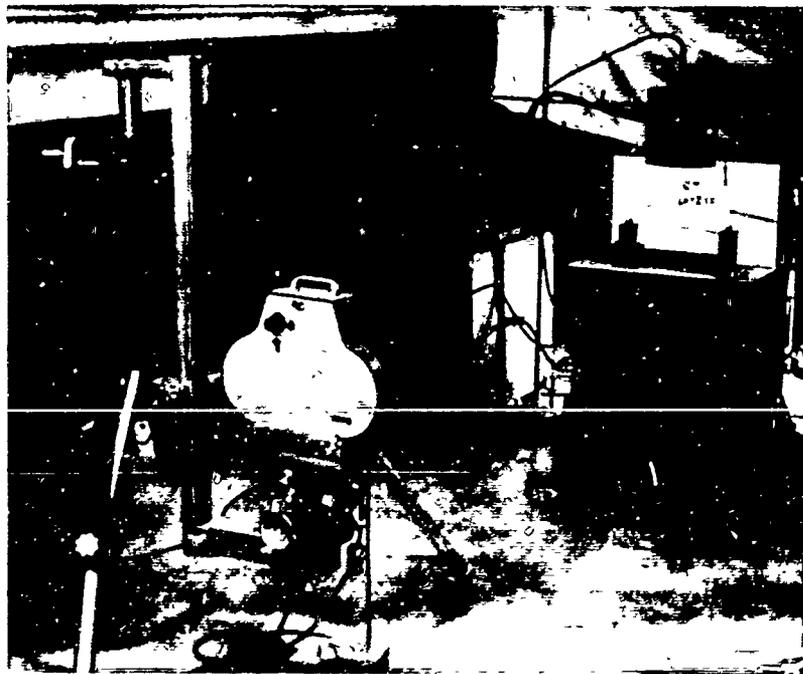


Fig. 5. Camera Setup



Fig. 6. Film Reader to Measure Deformed Arch Shape.

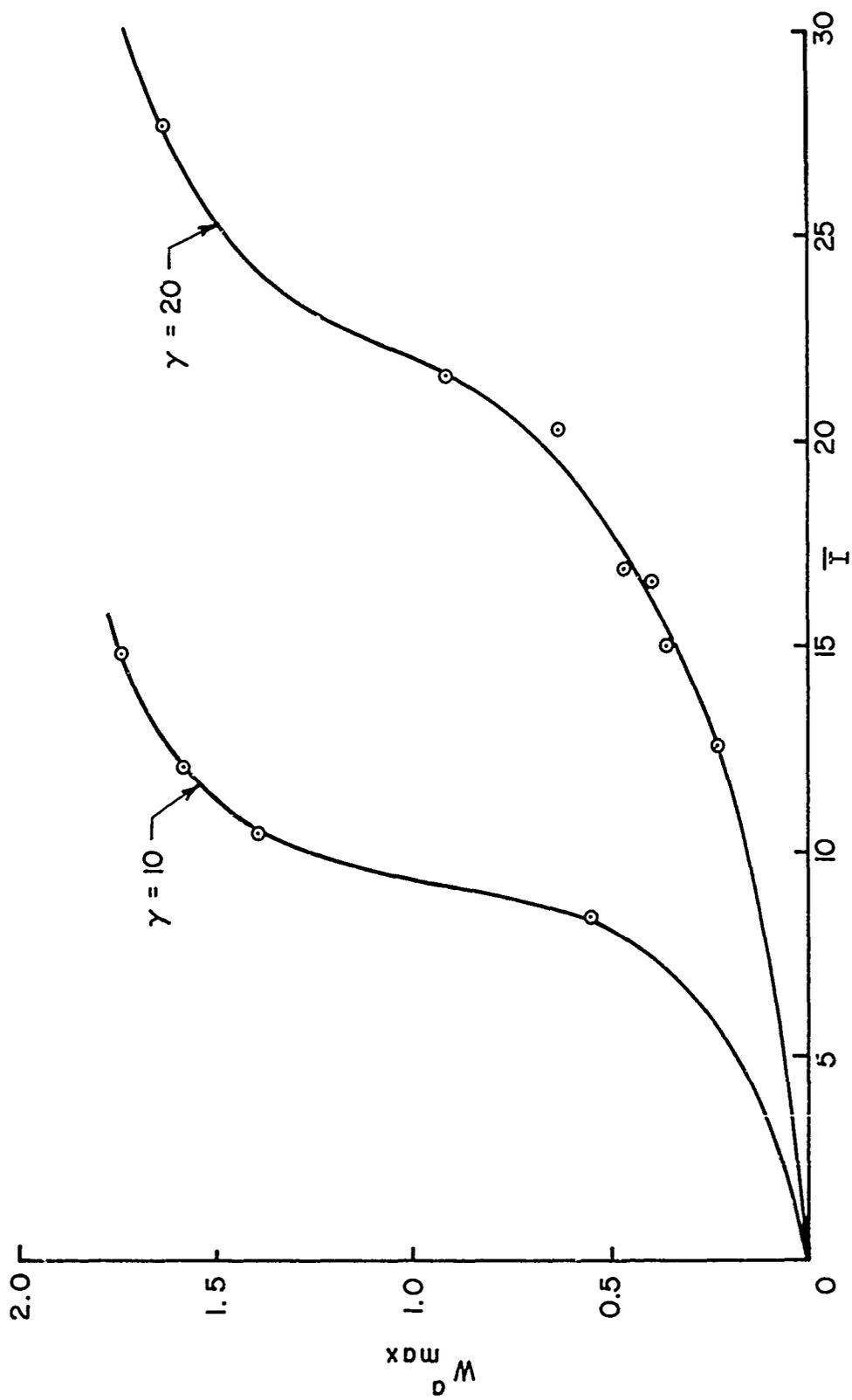


FIG 7 IMPULSE VS. MAXIMUM AVERAGE DISPLACEMENT FOR CLAMPED ARCH

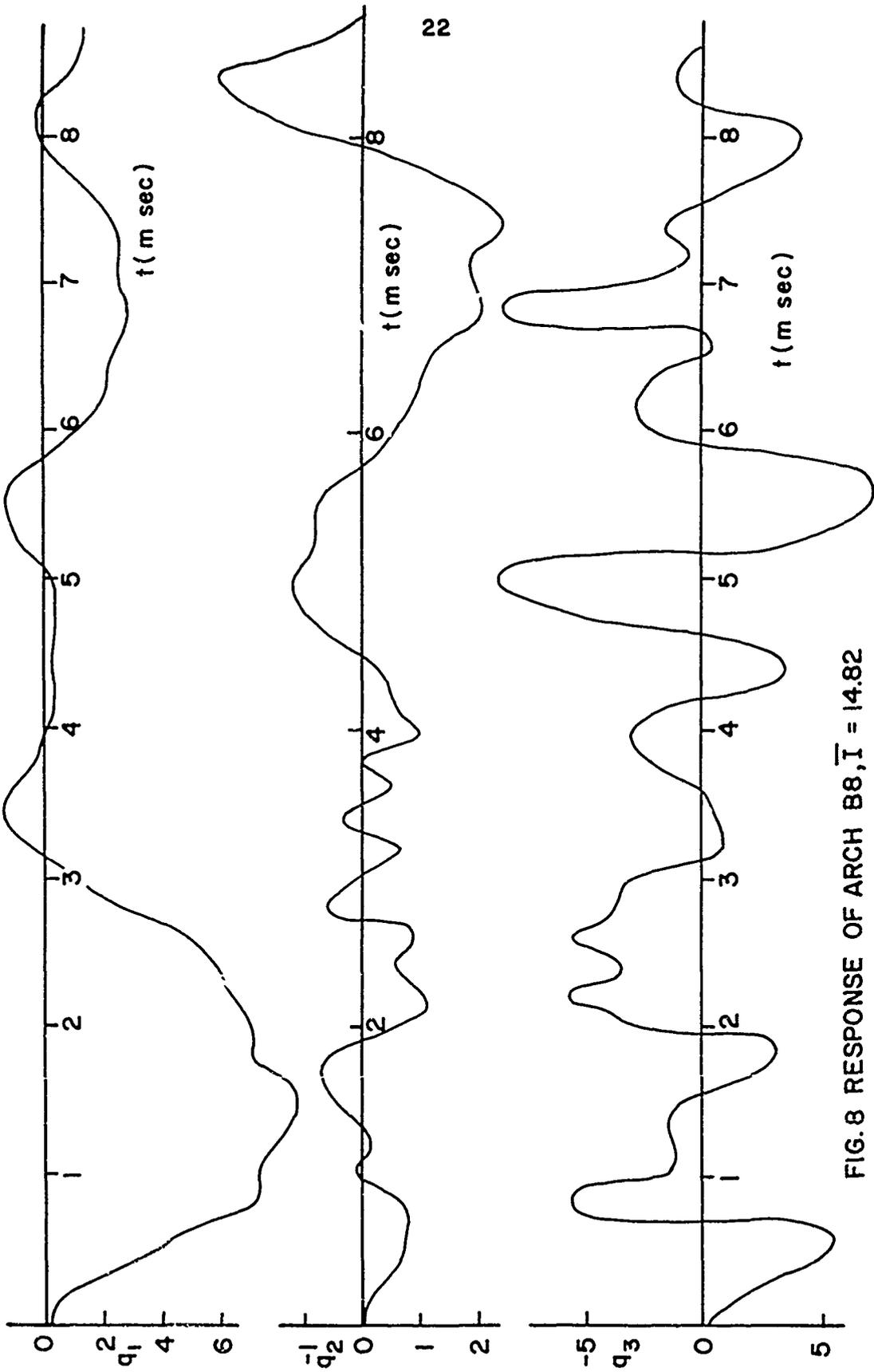


FIG. 8 RESPONSE OF ARCH B8, $\bar{I} = 14.82$

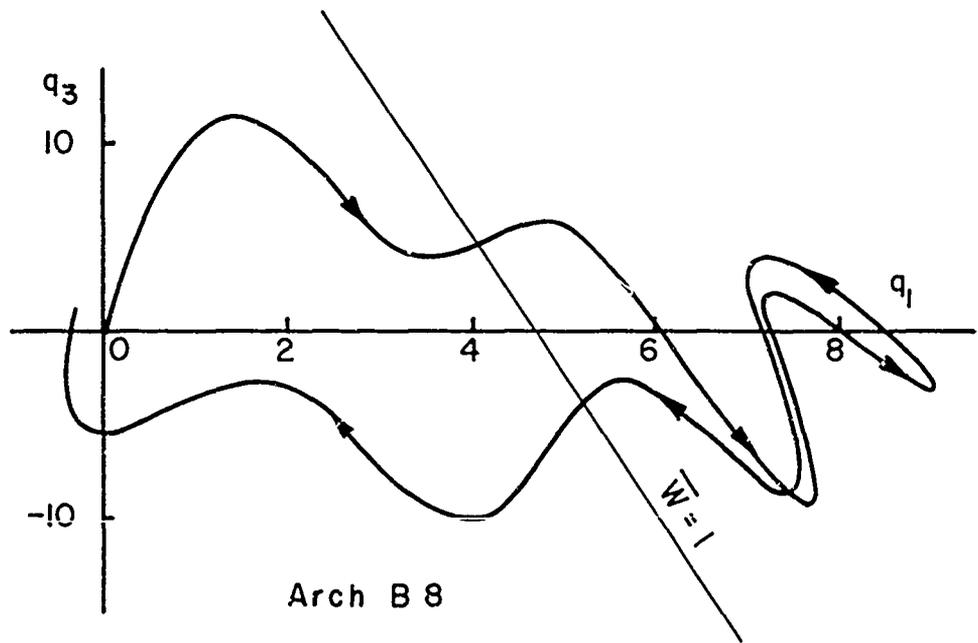
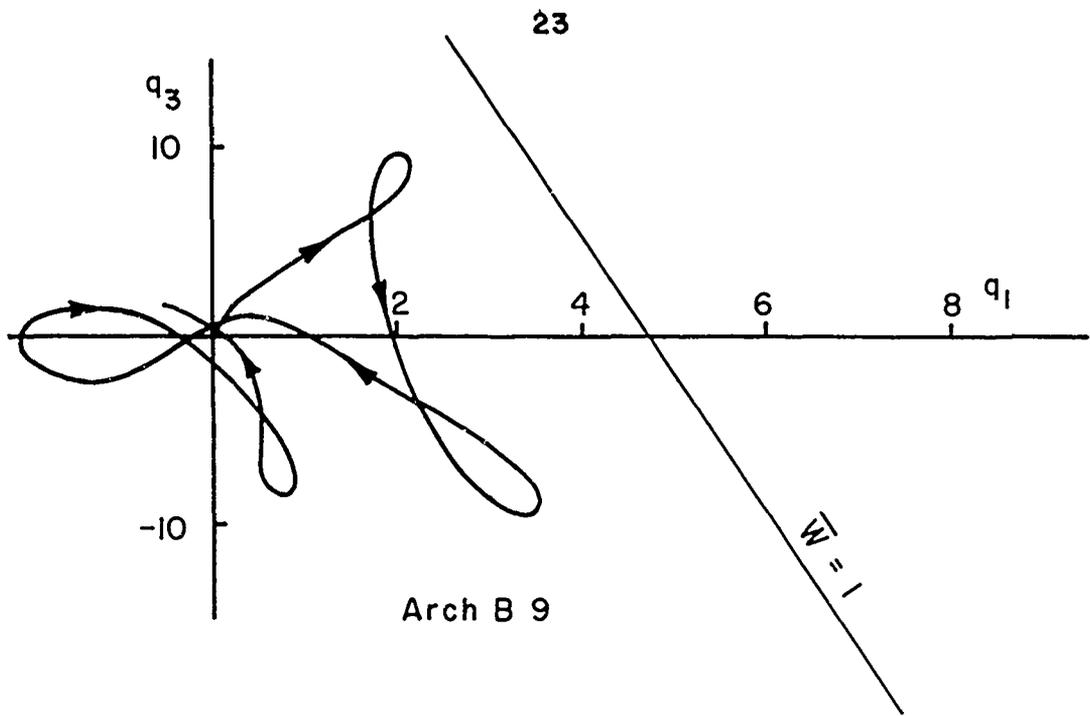


FIG.9a TRAJECTORIES OF ARCHES FOR $\gamma \approx 10$

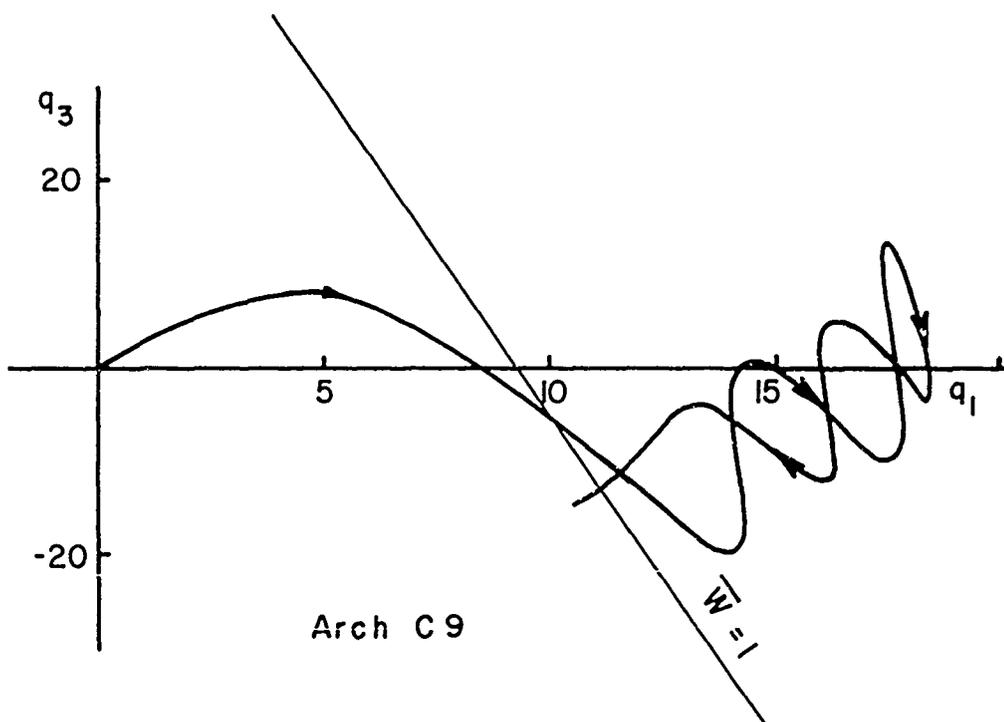
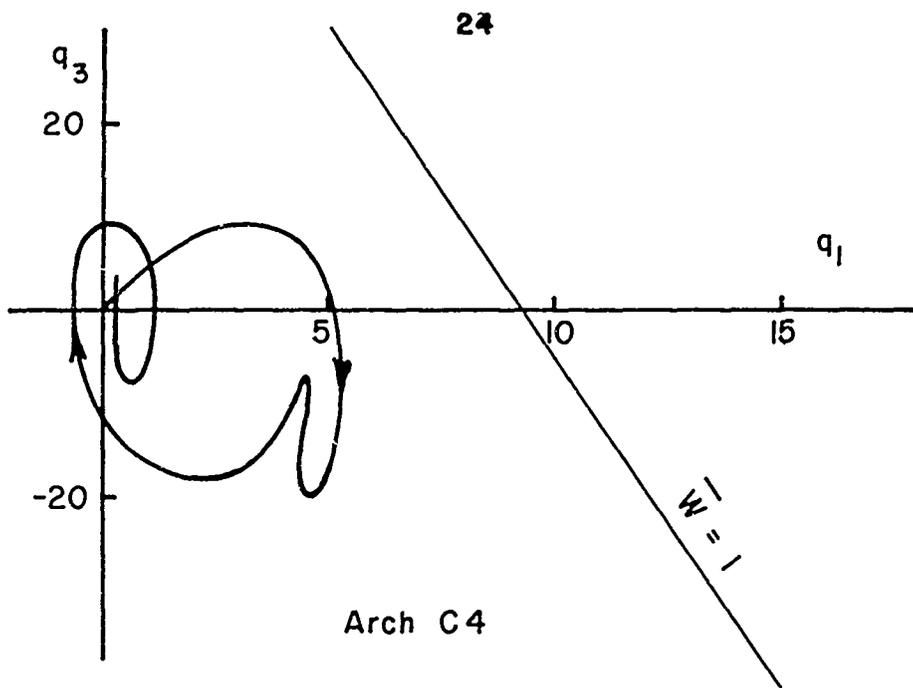


FIG. 9b TRAJECTORIES OF ARCHES FOR $\gamma \approx 20$

UNCLASSIFIED
Security Classification

AD 688225

DOCUMENT CONTROL DATA - R & D		
<i>Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified</i>		
1. ORIGINATING ACTIVITY (Corporate author) California Institute of Technology Graduate Aeronautical Laboratories Pasadena, California 91109		2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED
		2b. GROUP
3. REPORT TITLE THE STABILITY OF A CLAMPED SHALLOW ARCH SUBJECTED TO IMPULSIVE LOADING		
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Scientific Interim		
5. AUTHOR(S) (First name, middle initial, last name) Maxwell C. Cheung Charles D. Babcock, Jr.		
6. REPORT DATE March 1969	7a. TOTAL NO. OF PAGES 24	7b. NO. OF REFS 12
8a. CONTRACT OR GRANT NO AFOSR-1424-68	9a. ORIGINATOR'S REPORT NUMBER(S)	
b. PROJECT NO 9782-01		
c. 6144501F	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d. 681307	AFOSR 69-0878 TR	
10. DISTRIBUTION STATEMENT 1. This document has been approved for public release and sale; its distribution is unlimited.		
11. SUPPLEMENTARY NOTES TECH, OTHER	12. SPONSORING MILITARY ACTIVITY AF Office of Scientific Research (SREM) 1400 Wilson Boulevard Arlington, Virginia 22209	
13. ABSTRACT <p>Clamped circular aluminum arches have been explosively loaded using a spray deposited light ignited explosive (Silver Acetylide-Silver Nitrate). Using high speed photography and computerized data reduction, a detailed time history of the arch deflection has been obtained. This deflection was fitted with mode shapes in order to display the motion graphically. The experimental data shows that the clamped arch does not have a jump in response with increasing load level, but a transition from small displacement response to a large displacement response over a sizable range of load level.</p>		

DD FORM 1473

UNCLASSIFIED
Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Dynamic Buckling Clamped Arches Impulse Load						