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PROCEEDINGS OF THE TECHNICAL WORKSHOP ON RADAR SCATTERING FROM RANDOM MEDIA

WORKSHOP HELD AT
THE UNIVERSITY OF CALIFORNIA
LAJOLLA, CALIFORNIA

5-16 AUGUST 1968

ADVANCED RESEARCH PROJECTS AGENCY
STRATEGIC TECHNOLOGY OFFICE
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PREFACE

This volume is a summary of presentations and discussions of a technical workshop on Radar Scattering from Random Media, held at the Institute for Pure and Applied Sciences, University of California (San Diego), La Jolla, California, on 5 - 16 August 1968, and sponsored by the Advanced Research Projects Agency. The Workshop was divided into Theoretical and Experimental Panels. Summaries of the reports of these Panels are the result of collaboration among several Workshop participants. Special thanks are given to Dr. S. Rand (IPAS) and to Dr. S. C. Lin (IPAS) for preparing and editing the panel reports. A complete transcript of the proceedings of the Workshop was prepared by Dr. R. Ruquist (MIT/LL), and, although not included herein, is available upon request. Special thanks is given to Dr. T. O. Philips (BTL), general editor of this report, who skillfully molded the diverse sections of the report into a coherent document.

K. Kresa

K. Kresa (ARPA)

Workshop Chairman

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Chapter 1

INTRODUCTION

Section 1. A The Scope of the Workshop and of This Report

The understanding of radar returns from turbulent plasmas has been the focus of much interest for many years. This subject is important because radar is extensively utilized in both the field and the laboratory as a remote sensor to provide basic information on turbulent flows and many aspects of the physics of reentry.

The use of radar scattering in this diagnostic capacity is difficult, since radar scattering provides only an indirect method of probing the fluid dynamics and turbulent structure of the medium. In a weakly-ionized turbulent plasma (such as a turbulent wake) microwave radiation is scattered by the free electrons and one must know or guess the relationship between the electronic phenomena observed and the underlying fluid dynamics and turbulent structure. Moreover, in order even to interpret radar scattering measurements in terms of electron densities and electron motions, it is usually necessary to have some theoretical understanding of the electromagnetic scattering process.

In certain limiting cases (for example, in the case of low mean electron densities and small electron density fluctuations) simple and reliable theories are available. But it is seldom easy to determine the conditions under which these simple theories should be valid or the nature of the deviations from those theories. Other theories which may be more widely applicable are frequently so complex that they are not useful when detailed quantitative results are required.

This Workshop was motivated by ARPA's desire to assess the present state of knowledge of scattering theory as applied primarily to radar scattering from turbulent wakes. Both theoretical and laboratory experimental research were reviewed and pertinent correlations between theory and experiment were examined. In addition, the results obtained from radar measurements were presented and the relevance of present theories to the understanding of these data were discussed.

The goal of the Workshop, and the purpose of this report, was to prepare a critical review of the present research in this area with respect to the understanding of radar scattering from ionized turbulent media and, in addition, to recommend areas for future emphasis and support.

The activities of the Workshop were highly informal, as is this volume of reports emanating from the Workshop. Although several participants were invited to prepare lectures on relevant topics, most of the Workshop sessions consisted of informal discussion and debate. There was no attempt to record these discussions in detail herein, and this present volume is not intended to be a thorough documentation of the material considered by the Workshop. Instead it should be viewed as a collection of written reports, prepared by individuals or groups present at the Workshop, which may be of some interest or use to a wider audience.

Section B of this introductory Chapter 1 summarizes some particularly significant conclusions and recommendations of the Workshop. A list of the Workshop participants and a summary of the program are found in Chapter 2.

Chapter 3 reviews the present status of the theory of incoherent electromagnetic scattering from random media. The theoretical models reviewed are those which were discussed at the Workshop; those which appear most useful for the understanding of wake scattering received the most attention both in the Workshop discussions and in this report. Table I (in Chapter 3) presents a useful summary and comparison of these various theories. Chapter 4 contains a brief description of some of the laboratory facilities in which scattering experiments are being performed. Results from these scattering experiments were presented to the Workshop but, since they are well documented in the technical literature, these results are not included here.

During the last three days the Workshop was divided into a Theoretical Panel and an Experimental Panel, according to the research interests of the participants. Each panel reviewed the Workshop discussions and prepared a set of conclusions and recommendations which are to be found in Chapter 5.

Three appendices are attached to this report. Appendix A contains a bibliography of papers on the theory of scattering from random media, as well as papers documenting the results of laboratory scattering experiments. It is based upon a bibliography of theoretical papers prepared for the Workshop by A. Hochstim; additional entries were supplied by T. O. Philips. Appendix B is a paper by S. C. Lin on the spectrum of electron density fluctuations in hypersonic wakes. It is an expanded version of a lecture given at the Workshop. Appendix C represents an attempt to determine the applicability of various theories of scattering from random media to the interpretation of field data. This paper is the output of a "working session" of the Workshop, which was organized by S. Edelberg. Some of the conclusions reported in Section B of Chapter 5 are based upon this study. Appendix C is the only classified section of this Workshop Report and is therefore being distributed separately.

Section 1. B Major Conclusions

Chapter 5 contains conclusions and recommendations which were identified by the Theoretical and Experimental Panels of the Workshop. A few of these conclusions appear to be especially significant for future investigations of scattering from reentry wakes.

Validity of the first Born approximation. The simplest and most widely used theory of electromagnetic scattering from random media is the first Born approximation; the conditions for the validity of this theory have been investigated by Salpeter and Treiman. Several different sources reviewed at the Workshop suggest that these Salpeter-Trieman conditions may be unnecessarily restrictive for the purposes of scattering studies where agreement of theory and experiment within a factor of two is adequate. For example, the computer experiments of Hochstim and the laboratory experiments of Guthart and his colleagues suggest that the first Born approximation may be valid (to within a factor of two) for electron density fluctuation levels which approach the critical electron density. (On the other hand, the experiments of Granatstein suggest that significant deviations from the first Born approximation may occur at somewhat lower levels of electron density fluctuations.)

A brief study performed at the Workshop, using radar wake scattering data from reentry experiments, and using a theoretical approach due to Shkarofsky and to Feinstein, also indicates that the first Born approximation may be valid under a wider range of reentry conditions than had previously been realized. It will require further theoretical and experimental studies to verify that the first Born approximation is indeed so widely applicable to the wake scattering problem.

Use of the distorted wave Born approximations. There are several generalizations of the first Born approximation which have appeared in the scattering theory literature. Most of these generalizations are equivalent to one of the five "distorted wave Born approximations" which are discussed in Chapter 3, where they are given the names DWBA-1, -2a, -2b, -2c, and the "Kraichnan model". Each of these theories is a special case of the theories which follow it in the list. It was concluded by the Workshop participants that there may be little point in applying the theories DWBA-1, -2a, or -2b to the wake scattering problem, since it appears that the model DWBA-2c is only slightly more complicated and would be expected to yield significantly better quantitative results. Other theories, such as the Kraichnan model and the Information

Theory Formulation (see Section 2.D) may in principle be more general, but do not appear to yield practical results for the study of wake scattering.

Definition of the scattering medium. Even if one has a collection of theoretical models for scattering from a random medium, accompanied by conditions for the validity of these models, the choice of the most useful model depends upon the nature of the scattering medium. Is the medium, for example, adequately characterized by a statistical description in terms of correlation functions of random variables? Is it necessary to specify in detail the nature of the boundary of the medium? Are the gradients of mean electron density sufficiently small that diffraction effects may be neglected?

It is not clear that the currently used statistical descriptions of a turbulent medium are always adequate; under some conditions it may be necessary to have a more complete description of a single realization of a random medium. The development of alternative characterizations of the scattering medium should be a major goal of future studies, both theoretical and experimental, of the fluid dynamics, the turbulence properties, and the chemistry of wakes.

Chapter 2

WORKSHOP PARTICIPANTS AND PROGRAM

The following persons participated in one or more of the sessions of the Workshop:

KEITH A. BRUECKNER, University of California, San Diego
ANTHONY DEMETRIADES, Aeronutronic Division, Philco Corporation
SEYMOUR EDELBERG, Lincoln Laboratory, Massachusetts Institute of Technology
LEOPOLD B. FELSEN, Polytechnic Institute of Brooklyn
CARL H. GIBSON, University of California, San Diego
VICTOR L. GRANATSTEIN, Bell Telephone Laboratories
HAROLD GUTHART, Stanford Research Institute
ADOLF HOCHSTIM, Institute for Defense Analyses
JOHN JAREM, Drexel Institute of Technology
TUDOR W. JOHNSTON, RCA Victor Research Laboratory (Montreal)
KENT KRESA, Advanced Research Projects Agency
S. C. LIN, University of California, San Diego
L. R. MARTIN, Lincoln Laboratory, Massachusetts Institute of Technology
RAYMOND F. MISSERT, Cornell Aeronautical Laboratory
MARVIN H. MITTLEMAN, University of California, Berkeley
S. S. PENNER, University of California, San Diego
THOMAS O. PHILIPS, Bell Telephone Laboratories
ROBIN I. PRIMICH, A.C. Electronics Defense Research Laboratories
ANDREW PROUDIAN, Heliodyne Corporation
S. RAND, University of California, San Diego
JACQUES RENAU, Aerospace Corporation
RICHARD S. RUFFINE, Advanced Ballistic Missile Defense Agency
RICHARD RUQUIST, Lincoln Laboratory, Massachusetts Institute of Technology
I. P. SHKAROFSKY, RCA Victor Research Laboratory (Montreal)
BURTON STROM, Riverside Research Institute
K. SULZMANN, University of California, San Diego
EMMETT A. SUTTON, Avco-Everett Research Laboratory
KENNETH M. WATSON, University of California, San Diego
S. ZIVANOVIC, A.C. Electronics Defense Research Laboratories

The Workshop sessions included formal lectures, spontaneous presentations, informal discussions, and working sessions (which sometimes involved the entire Workshop and sometimes only a subcommittee of the attendees). In the following outline we attempt to refer to all of the significant discussions (and not only to the more formal lectures). We have assigned titles to the informal and spontaneous discussions; the titles will, we hope, characterize the nature of the subjects discussed.

Monday, August 5, 1968

Registration.
Goals for the Workshop, K. Kresa.
Definitions, posing of the problem, and review of scattering theory literature, A. Hochstim.
Review of scattering from random media, J. Keller (paper presented by J. Jarem).
Discussions:
- Comments on a paper by Frisch, J. Jarem.
- The Tatarskii approach to scattering theory, L. Felsen.
- Identification of physical parameters pertinent for scattering phenomena, R. Ruffine and others.
- Comments on the conditions for various scattering theories, I. Shkarofsky.

Tuesday, August 6

Derivation of the transport equation for scattering from a random medium, K. Watson.
A one-dimensional scattering experiment on the computer, A. Hochstim.

Wednesday, August 7

A hueristic multiple scattering theory, I. Shkarofsky.
The distorted wave Born approximation, M. Mittleman.
A solution for scattering from a one-dimensional slab, J. Jarem.

Working session: Numerical values of physical parameters pertinent for scattering phenomena (R. Ruffine and others).

Discussions:

The applicability of scattering theories to scattering from reentry vehicle wakes.

Summary of available theoretical methods, A. Hochstim.

A theorem of Lax, K. Watson.

Distorted wave Born approximations, J. Jarem, L. Felsen.

Limitations to transport theory, K. Watson.

Thursday, August 8

Distorted wave Born approximations, L. Felsen.
Statistical methods in scattering theory, A. Proudian.
Properties of wakes of hypersonic projectiles, E. Sutton. (with comments by A. Demetriades).
Discussions: Necessary extensions to current theory.
Working session: Preparation of reports on the status of theories of scattering from turbulent plasmas.

Friday, August 9

Working session: Preparation of reports on the status of theories of scattering from turbulent plasmas.

Monday, August 12

Summary of theories of scattering from turbulent plasmas, A. Proudian.
Discussions: The current status of scattering theory.
Experiments on scattering from a turbulent plasma jet, H. Guthart.
Characteristics of turbulent wakes, A. Demetriades.
Diagnostic study of a turbulent plasma jet, T. Johnston.
Scattering experiment with a flowing turbulent plasma, V. Granatstein.

Tuesday, August 13

The spectrum of wake turbulence - Comparisons of field data with theory, E. Sutton.
Recent TRADEX wake scattering observations, L. Martin.
Recent wake scattering observations at L-band and C-band, R. Missert.

Discussions:

A multiple scattering criterion from ionospheric physics, J. Renau.

The limits of the Born approximation based upon cross-polarized scattering, R. Ruffine.

Wednesday, August 14

Working session (Experimental panel): Comparison oi theoretical scattering models with recent field data, S. Edelberg and others.

Thursday, August 15

The spectrum of electron density fluctuations in turbulent reentry wakes, S. Lin.

Working session (Theoretical and experimental panels): Formulation of the conclusions of the Workshop and recommendations for future research.

Chapter 3

THEORETICAL STUDIES OF SCATTERING FROM RANDOM MEDIA

Section 3.A Introduction

During the first four days (August 5 through 8) the Workshop heard lectures on a number of different approaches to the theory of scattering from random media. The principal goals during these four days were (i) to achieve an understanding of the relationships among the various approaches, (ii) to estimate the range of validity of each approach, and (iii) to assess the likelihood that any given approach could be usefully applied to the specific problem of wake scattering. Much of the information assembled concerning these questions is presented in Table I. A summary of the general conclusions and recommendations concerning theoretical studies is given in Chapter 5.

In most, but not all, cases, the various approaches to scattering calculations were discussed in the context of a scalar wave equation. Little detailed study was given to the possibility of extending the techniques to electromagnetic scattering problems in which polarization effects are important; however, some assessment of the possibility of such an extension was included in Table I.

TABLE I. SCATTERING THEORY STATUS

	Range of Validity	Application to Slab	Finite Geometry
First Born	$\frac{\delta\varepsilon}{\varepsilon} \ll 1$ $\sigma_B D \ll 1$	yes	yes
DWBA-1	$\frac{\delta\varepsilon}{\varepsilon} \ll 1$ $\sigma_B L \ll 1$ $\sigma_B D \ll 1$	done 1-D Jarem 3D in progress	hard for exact Green's function*
Heuristic Model (DWBA-2a)	$\frac{\delta\varepsilon}{\varepsilon} \ll 1$	yes	yes - Shkarofsky
Simplified Model (DWBA-2b)	$R \ll 1$ †	half-plane in process - Felsen	probably with <u>much</u> work
Full Model (DWBA-2c)	$R \ll 1$ †	1-D possibly Mittleman	very <u>hard</u>*
Transport Theory	$kD \gg 1$ $R \ll 1$ $\nabla \ln n \ll k$	has been done in radiative transfer	→
Kraichnan	R unbounded	can be done - hard*	hard*
Information Theory Formulation	R unbounded	?	?

<u>Definitions:</u>

$R = \left(\frac{\delta n}{n} k L\right)^2$ L = correlation length $k = 2\pi/\lambda$

D = physical dimension along beam

σ_B = total Born cross section per unit volume

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* Ray tracing ($\nabla \ln n \ll k$) makes problem more tractable.

† Watson claims models give correct backscatter power to a factor of 2 provided $R \ll 1$ for simplified model and $\sqrt{R} < 1$ for full model.

TABLE I. SCATTERING THEORY STATUS (Cont'd)

	Status of Research	Verification	Polarization Effects
First Born	complete	with experiment	yes - none for first Born; δ^4 effect 2nd Born
DWBA-1	slab with 3-D formulated (Jarem)	1-D Hochstim 3-D none	hasn't been done, but possible
Heuristic Model (DWBA-2a)	extend to include density profiles, anisotropy	pending	yes
Simplified Model (DWBA-2b)	?	?	from curved paths only or 2nd Born
Full Model (DWBA-2c)	formulated	1-D underway Felsen, Mittleman	from curved paths only or 2nd Born
Transport Theory	diffusion limit done - ready for add'l applications	probably in other problems - diffusion limit will be checked during workshop	yes
Kraichnan	formulated	verified on other models	yes
Information Theory Formulation	formulated	?	should give correct value

It was decided to defer to some other occasion any discussion of the use of the various scattering theories to calculate the frequency spectrum of waves scattered from random media. A discussion of frequency effects is important and should be attempted as a sequel to this Workshop.

In order to attempt to understand the ensemble-averaged microwave scattering from a turbulent wake, four general approaches were considered: (i) perturbation techniques, (ii) Watson's transport equation, (iii) the so-called information theory formulation, and (iv) computer experiments.

The approach which received the most attention involved a variety of modifications of the perturbation scheme, whereby an expansion is performed in terms of a small parameter. The prototype method, beyond which generalizations were proposed, is the well-known "first Born approximation," an attractive starting point because of its simplicity as well as its applicability to a sufficiently underdense wake. Attempts to generalize the first Born approximation included a number of techniques which are collectively referred to as "distorted wave Born approximations"; in these techniques the expansion parameter is the electron density fluctuation, rather than the total electron density as in the first Born approximation. These methods are a useful improvement when there are regions of the wake where the mean electron density approaches, or possibly even exceeds, the critical value corresponding to the incident microwave frequency, while the statistical fluctuations from the mean are much less than the critical density. These perturbation methods will be discussed in Section 3.B.

A second approach to the scattering problem was outlined in a lecture by K. M. Watson in which he discussed the derivation of a vector transport equation for the incoherently scattered energy. The derivation starts from the multiple scattering equations for scattering from a collection of N individual electrons. This approach has the merit that it could take advantage of the many existing solutions to the transport equation. Watson's equation is discussed in Section 3.C.

Limited attention was given to a completely different approach, which has been called the "Information Theory Formulation," a technique which is of the nature of a variational method. The theory as presented to the Workshop is in a primitive stage, but is outlined in Section 3.D.because it has a slim possibility of being the only true theory which may eventually be used to describe scattering from a wake with overdense fluctuations. The discussion was brief because, when applied to the wave equation with a random variable, the formulation has not reached the point where much can be surmised about its applicability.

A fourth class of techniques was proposed which included either numerical methods, that is, "computer experiments," or actual scattering experiments to be performed on models. None of the models would simulate the wake, but rather they would be simple mocked-up situations, with statistics which are completely prescribed, in order to test the various scattering theories. The advantage is clear in that the complicated wake structure is ignored, so that the testing is limited entirely to the scattering theories. The various methods for extending some simple one-dimensional computer experiments, which have already been performed, will be described in Section 3.E.

An obvious omission in the discussions at this Workshop, by consensus of the participants, involved theories of scattering from overdense random surfaces. It is hoped that these theories, which are expected to be appropriate to wake scattering under some reentry conditions, will be the subject of another meeting.

Section 3.B Perturbation Techniques

Five proposals were considered for extending the first Born approximation.* The first Born approximation assumes that the coherent wave propagates in the random medium with the free space wave number k_o. (The coherent wave is then exactly the wave incident upon the medium.) In all of the proposed extensions to the theory the coherent wave propagates with an effective wave number $k(\vec{x})$ which differs from the free space value k_o. These extensions are all referred to as "distorted wave Born approximations."

In the first such model, which will be referred to as the "simple model" (or, in the language used by the Workshop, DWBA-1), the effective wave number $k(\vec{x})$ differs from k_o because of the mean electron density. The effects of electron density fluctuations on the coherent field are ignored.

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* Some calculations have been performed in the second Born approximation (that is, second order terms of the Born perturbation series are retained). However, even these calculations are considerably more difficult than the first Born approximation and, except in very special cases, it seems hopeless to expect to be able to calculate higher order terms in the Born series.

The other four "distorted wave Born approximations" include some of the effects of electron density fluctuations on the coherent wave. The "heuristic model" (DWBA-2a) includes in an ad hoc manner an attenuation of the coherent wave; this attenuation is the result of the scattering of energy out of the coherent beam by electron density fluctuations. The fluctuations therefore modify the imaginary part of the effective wave number $k(\vec{x})$. In the "expanded model" (DWBA-2b) the real part, as well as the imaginary part, of the effective wave number is modified by fluctuations to first order in the scattering coefficient. The "full model" (DWBA-2c) brings in the effects of higher order scatters in the equation for the coherent field, but ignores them in the equation by which the incoherent field is derived from the coherent field. Although there appears to be an inconsistency in this model, in that terms are ignored which seem to be as large as terms which are retained, this model is taken seriously not only because of the reasonableness of the formulation, but also because it has been shown to result in improvement over the simplified theories when applied to nuclear scattering problems. Finally, the "Kraichnan model" includes additional multiple scatters beyond the full model, and appears to be somewhat of an improvement. However, it involves a nonlinear integral equation, which in most applications would be extremely difficult to solve.

The five perturbation schemes described above involve increasing complications, and each includes the previous models as special cases. A rather direct derivation of the "full model" (DWBA-2c) is available, and this will be outlined here.

Consider the equation

$$(L_o+L_1)\psi = 0 \ , \qquad (1)$$

where L_o and L_1 are linear operators, L_o being sure and L_1 being stochastic. The electron density fluctuations are considered to be a random process and are included in L_1. The mean electron density may appear either in L_1 (as in the Born approximation) or in L_o (as in the various distorted wave Born approximations). Because of L_1 the total field will contain a fluctuating (stochastic) part $\delta\psi$ as well as a coherent part ϕ,

$$\psi = \phi + \delta\psi \ . \qquad (2)$$

By substituting equation (2) into equation (1), and averaging over the random variables, one obtains

$$L_o\phi + \langle L_1\delta\psi\rangle = 0 \qquad (3)$$

where it will be assumed that L_1 is centered, that is, $\langle L_1\rangle = 0$, and therefore also $\langle\delta\psi\rangle = 0$. By subtracting equation (3) from equation (1), we have the equation for $\delta\psi$ associated with a given realization,

$$L_o\delta\psi + L_1\phi + L_1\delta\psi - \langle L_1\delta\psi\rangle = 0. \qquad (4)$$

The fundamental assumption of DWBA-2c is that the quantity $L_1\delta\psi - \langle L_1\delta\psi\rangle$ is much smaller than the other terms of equation (4), and may therefore be discarded. This is an ad hoc assumption, with no theoretical justification offered by any of the participants at the Workshop. It is accepted therefore, that the "full model" DWBA-2c may constitute no improvement over the "simple model" DWBA-1 in which additional terms, which appear to be of the same order of magnitude, are neglected. Although the a priori expansion parameter of both DWBA-1 and DWBA-2c (as well as of the "expanded model" DWBA-2b, of intermediate complexity) would appear to be the same, namely the electron density fluctuations, the model DWBA-2c is expected to be an improvement because it has given results which compare better with experiment in the corresponding nuclear scattering problem. Thus for DWBA-2c, equation (4) is replaced by

$$L_o \delta\psi + L_1 \phi = 0 \tag{5}$$

with the formal solution,

$$\delta\psi = -L_o^{-1} L_1 \phi . \tag{6}$$

When equation (6) is combined with equation (3), we have a single equation for the coherent field,

$$\left[L_o - \langle L_1 L_o^{-1} L_1 \rangle \right] \phi = 0 . \tag{7}$$

The quantity $\langle L_1 L_o^{-1} L_1 \rangle$ is an integral operator which involves the statistical properties of the electron density fluctuations. This term is ignored in both the first Born approximation and the "simple" distorted wave Born approximation (DWBA-1).

In the first Born approximation the sure operator L_o is taken to be

$$L_o = \nabla^2 + k_o^2 \tag{8a}$$

where k_o is the free space wave number ($k_o = 2\pi f_o/c$) of the monochromatic plane wave incident upon the random medium;

the wave equation for the coherent field ϕ is

$$L_o\phi = \left(\nabla^2+k_o^2\right)\phi = 0. \qquad (8b)$$

In the model DWBA-1, the incident free space wave number k_o is replaced by an effective wave number $k(\vec{x})$, where

$$k(\vec{x})^2 = k_o^2\left[1 - \frac{\langle n(\vec{x})\rangle}{n_c}\right] \qquad (9a)$$

which includes the effects on the coherent wave of the mean electron density $\langle n(\vec{x})\rangle$. Here n_c is the critical electron density corresponding to the incident frequency f_o. Then the wave equation for the coherent field is

$$L_o\phi = \left[\nabla^2 + k(\vec{x})^2\right]\phi = 0. \qquad (9b)$$

It is known that equation (9b) can be solved for the coherent field in a number of simple geometries, even when ϕ is a vector field. The problem of an infinite cylinder, with $k(\vec{x})$ a step function, is exactly solvable.

A major problem of applying the full model, given by equation (7), is associated with the determination of L_o^{-1}, that is, of obtaining the Green's function for a real problem. For example, if one writes

$$L_1 = k(\vec{x})^2\mu(\vec{x}) \tag{10}$$

where $\mu(\vec{x})$ is a stochastic variable which is proportional to the electron density fluctuation, then equation (7) may be written

$$0 = \left[\nabla^2 + k(\vec{x})^2\right]\phi(\vec{x}) - \int k(\vec{x})^2 k(\vec{x}')^2 G^{(o)}(\vec{x},\vec{x}')\langle\mu(\vec{x})\mu(\vec{x}')\rangle\phi(\vec{x}')d^3x' \tag{11}$$

where $L_o^{-1} = G^{(o)}(\vec{x},\vec{x}')$ is the Green's function. Even if $k(\vec{x})$ is constant throughout the wake, the Green's function is extremely complicated (although available for the case of a cylinder) because of the finite geometry. It was proposed at the Workshop therefore, to use the Green's function associated with an infinite space, but with wave number k which corresponds to some mean electron density inside the wake. The assumption is that the correlation length and the dimension of the wake are both large compared to the radiation wavelength inside the wake. Using the proposed assumption, we have

$$G^{(o)}(\vec{x},\vec{x}') = \frac{\exp(ik|\vec{x}-\vec{x}'|)}{4\pi|\vec{x}-\vec{x}'|} . \tag{12}$$

We also define the two-point correlation function

$$\langle \mu(\vec{x})\mu(\vec{x}')\rangle = R(\vec{x},\vec{x}') \qquad (13)$$

which is assumed to depend only on $\vec{x}-\vec{x}'$. With the additional assumption of isotropic turbulence, namely $R = R(|\vec{x}-\vec{x}'|)$, the Fourier transform of equation (11) may be written in the form

$$0 = (K^2-k^2)\phi(\vec{K})$$

$$+ k^4 \int G^{(o)}(|\vec{x}-\vec{x}'|)R(|\vec{x}-\vec{x}'|)\phi(\vec{x}')\, e^{i\vec{K}\cdot\vec{k}}\, d^3x d^3x' \qquad (14)$$

where

$$\phi(\vec{K}) = \int \phi(\vec{x})\, e^{i\vec{K}\cdot\vec{x}}\, d^3x.$$

Finally, it is assumed that the correlation length is small compared to the dimensions of the medium. Then after making the transformation, $\vec{y} = \vec{x} - \vec{x}'$, the integrals of equation (14) are separable, and the resulting dispersion relation is found to be

$$0 = K^2 - k^2 + \frac{k^4}{4\pi} \int \frac{\exp(ik|\vec{y}|+i\vec{K}\cdot\vec{y})}{|\vec{y}|} R(|\vec{y}|)d^3y \qquad (15)$$

where Green's function (12) has been used. Although equation (15) has been derived by assuming that k is constant throughout the wake, it is reasonable to generalize this equation, for a spatially dependent k, so that

$$0 = K(\vec{x})^2 - k(\vec{x})^2 + \frac{k(\vec{x})^4}{4\pi} \int \frac{\exp[ik(\vec{x})|\vec{y}| + iK(\vec{x})\cdot\vec{y}]}{|\vec{y}|} R(|\vec{y}|)d^3y \tag{16}$$

under the condition that $k(\vec{x})$ does not change significantly in a distance $1/k$. The final version of the full model for the coherent field, as proposed at the Workshop, involves the replacement of equation (11) by

$$\left[\nabla^2 + K(\vec{x})^2\right]\phi = 0 \tag{17}$$

where $K(\vec{x})$ is the solution of the integral equation (16). Note that $K(\vec{x})$ depends upon the mean electron density through the quantity $k(\vec{x})$ and upon the statistics of the electron density fluctuations through the correlation function $R(|\vec{x}-\vec{x}'|)$.

The infinite space Green's function of equation (12) appears in equation (16) as the factor $\exp(iky)/4\pi y$. An obvious generalization would involve the replacement of this factor, in equation (16), by a Green's function appropriate to a bounded medium. This proposal was considered at the Workshop, but it was felt that such a calculation would be very difficult and should not be attempted now.

Equations (16) and (17) constitute an extreme simplification of the stochastic problem, and it is important to study the conditions of validity. In passing from equation (4) to equation (5) one omits the term $L_1\delta\psi - \langle L_1\delta\psi\rangle$, which is of order $k^2\mu\delta\psi$, while retaining the terms $(\nabla^2+k^2)\delta\psi$ and $k^2\mu\phi$. The operator ∇^2, when applied to $\delta\psi$, is expected to be of order $1/L^2$, where L is the correlation length, so that the condition

$$\mu k^2 L^2 << 1$$

implies that the omitted term is indeed small compared to $\nabla^2\delta\psi$. There is, however, no a priori reason to believe that the term $L_1\delta\psi - \langle L_1\delta\psi\rangle = k^2[\mu\delta\psi - \langle\mu\delta\psi\rangle]$, which is omitted, is much smaller than the term $k^2\mu\phi$, other than the fact that the full model DWBA-2c is an improvement over simpler models when applied to nuclear scattering problems. The derivation of equation (16) from equation (11) regarded as heuristic, and no corresponding conditions were proposed. The condition that the correlation length be small compared to the dimensions of the wake is obvious.

The "expanded model" (DWBA-2b) is a minor specialization of the full model. The assumption is that the solution of equation (16) for K is not very different from k, so that an expansion is possible. The solution of equation (16) is approximately by

$$K = k - \frac{k^3}{8\pi} \int \frac{\exp(iky + i\vec{k}\cdot\vec{y})}{y} R(y) d^3y. \qquad (19)$$

This value of K may be no worse than the value obtained from the full model. The validity conditions are clearly the same, since it has already been assumed above that the fluctuations, by means of which K differs from k, may be treated as a perturbation.

The heuristic model is somewhat of an inconsistency, but was introduced in order to obtain answers very quickly. Essentially it is the Born approximation, but with the beam allowed to attenuate exponentially with a damping coefficient given by the imaginary part of expression (19), that is, one writes for the field

$$E = E_0 \exp(ikx - \alpha x)$$

where

$$\alpha = \frac{k^2}{2} \int_0^\infty y \sin^2(ky) R(y) dy.$$

This expression for α is obtained from equation (19) after some elementary angular integrations. The heuristic model includes scattering out of the beam to lowest order in the fluctuations, but then it requires the assumption that this same value of α can be used to all orders in an exponential attenuation. It ignores scattering into the beam (and may therefore, be applicable, in some sense, to a thin beam), as well as modifications due to fluctuations of the real part of the index of refraction.

The Kraichnan model goes beyond the full model, and is the most general perturbation technique which was considered. According to the full model of equation (7), one can write a generalized averaged Green's function for the problem in the form

$$\langle G \rangle = G^{(o)} + G^{(o)} \langle L_1 G^{(o)} L_1 \rangle G^{(o)} \qquad (20)$$

whereby the statistics are included directly in the Green's function. According to the Kraichnan model, equation (20) is replaced by the nonlinear integral equation,

$$\langle G \rangle = G^{(o)} + G^{(o)} \langle L_1 \langle G \rangle L_1 \rangle \langle G \rangle \qquad (21)$$

No discussion was given as to the manner in which equation (21) is an improvement over equation (20) and possibly the answer is unknown. At the present time, however, this may be an academic question, unless a relatively simple method of solving equation (21) is proposed.

In all of these methods the result is a solution for the coherent field ϕ. For a given realization of the random medium, that is, for a given choice of the function $L_1 = k(\vec{x})^2\mu(\vec{x})$, the corresponding incoherent field would in principle be calculated from equation (6)

$$\begin{aligned} \delta\psi(x) &= -L_o^{-1}L_1\phi \\ &= \int d^3x' G^{(o)}(\vec{x},\vec{x}')k(\vec{x}')^2\mu(\vec{x}')\phi(\vec{x}') \end{aligned} \tag{22}$$

The scattered energy, however, is proportional to the quantity

$$\langle|\delta\psi(\vec{x})|^2\rangle = \int d^3x'd^3x''k(\vec{x}')^2k(\vec{x}'')^2G^{(o)}(\vec{x},\vec{x}')G^{(o)}(\vec{x},\vec{x}'')^* \cdot\langle\mu(\vec{x}')\mu(\vec{x}'')\rangle\phi(\vec{x}')\phi(\vec{x}'')^* \tag{23}$$

Given a coherent field ϕ, the quantity $\langle|\delta\psi|^2\rangle$ can be calculated since the correlation function $\langle\mu(\vec{x}')\mu(\vec{x}'')\rangle = R(|\vec{x}'-\vec{x}''|)$ is assumed known.

Section 3.C Watson's Transport Equation

K. M. Watson has studied multiple scattering of electromagnetic waves in a random medium, starting from the problem of scattering by individual electrons. Scattering from a collection of N scattering centers (the individual electrons) can be exactly described by a collection of coupled multiple scattering equations. When appropriate statistical averages are taken over the positions of individual electrons, it becomes possible to separate the scattered waves into coherent and incoherent parts. The coherent wave may be assumed to propagate in a medium of varying effective wave number which, as in the perturbation theories discussed earlier, is expressed in terms of the electron density and the correlation function of the electron density fluctuations.

Under suitable conditions, the most important being the validity of the eikonal approximation, Watson derives a transport equation for the intensity I_{ij} of the incoherently scattered wave:

$$\frac{dI_{ij}(\vec{x},\hat{p})}{ds} = -\left\{\frac{1}{2}\sum_{i,t}\int\Big(ii\,|M(\hat{p},\hat{p}')|\,tt\Big)d\Omega_{\hat{p}'}\right\}I_{ij}(\vec{x},\hat{p}) \qquad (24)$$

$$+\sum_{st}\int\Big(ij\,|M(\hat{p},\hat{p}')|\,st\Big)I_{st}(\vec{x},\hat{p}')\,d\Omega_{\hat{p}'}\ ,$$

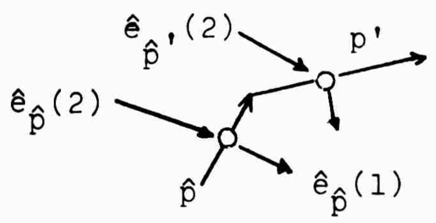
$$i = 1,2,\ j = 1,2$$

Here $I_{ij}(x,p)$ is the four-component intensity function, corresponding to polarization directions $i,j = 1,2$, for the incoherent waves propagating in direction $\hat{p}$ at position $\vec{x}$. The scattering function is a 4×4 matrix, which is given by

$$(ij|M(\hat{p},\hat{p}')|st) = \frac{3}{8\pi}\,\sigma_T\rho^2(x)\left[\hat{e}_{\hat{p}}(i)\cdot\hat{e}_{\hat{p}'}(s)\right]\left[\hat{e}_{\hat{p}}(j)\cdot\hat{e}_{\hat{p}'}(t)\right]$$

$$\cdot\int g(z,R)\;e^{in_1(z)k(\hat{p}'-\hat{p})\cdot\vec{R}}\;d^3R \qquad (25)$$

where σ_T is the Thompson scattering cross section, $\rho(x)$ is the local electron density, $g(z,R) \equiv g(z,|\vec{z}'-\vec{z}\;|)$ is the pair correlation function (assumed to be spherically symmetric), and the polarization directions are defined by:



If a solution to equation (24) for I_{ij} is obtained, then the energy density at a point $\vec{z}$ with polarization $\hat{e}$ is given by

$$U(\vec{z};\hat{\epsilon}) \equiv \frac{1}{8\pi}\langle|\hat{e}\cdot\vec{E}(\vec{z})|^2\rangle$$

$$= \frac{1}{c}\int d\Omega_{\hat{p}} \sum_{i,j=1}^{2} \left[\hat{e}\cdot\hat{e}_{\hat{p}}(i)\right]\left[\hat{e}\cdot\hat{e}_{\hat{p}}(j)\right] I_{ij}(\vec{z},\hat{p}) \tag{26}$$

It is clear from inspection of equation (22) that the eikonal approximation has been made in its derivation, except that the wave properties associated with polarization have been retained (and therefore I_{ij} has four components). The principle value of Watson's derivation lies in the explicit evaluation of the scattering matrix, equation (25), in terms of the pair correlation function. In the derivation of this expression it is necessary to make the same assumptions required in the distorted wave Born approximations.

According to equation (24) the scattering length $\ell(\vec{x})$ is given by

$$\frac{1}{\ell(\vec{x})} \equiv \frac{1}{2}\sum_{i,t}\int \left(ii|M(\hat{p},\hat{p}')|tt\right) d\Omega_{\hat{p}'}$$

$$= \rho^2(\vec{x})\int \sigma_T(\hat{p}\cdot\hat{p}')g(z,R)\, e^{in_1(\vec{x})(\hat{p}'-\hat{p})\cdot\vec{R}}\, d^3R\, d\Omega_{\hat{p}'} \tag{27}$$

where $\sigma_T(\hat{p}\cdot\hat{p}')$ is the differential Thompson scattering cross-section. In order of magnitude, one has

$$\frac{1}{\ell} \approx r_o^2 R_c^3 \delta\rho^2, \tag{28}$$

where $r_o = (e^2/mc^2)$, the classical electron radius, R_c is the correlation length, and $\delta\rho$ is the electron density fluctuation.

The most important assumptions required in Watson's analysis are

(i) Scattering occurs in the wave zone, namely $\ell >> \lambda$, where λ is the radiation wave length;

(ii) Only one scatter occurs in each correlated volume, namely $\ell >> R_c$;

(iii) Geometrical optics, namely $\lambda|\nabla(\ell n\ n)| << 1$ where n is the index of refraction;

(iv) All assumptions required by the distorted wave Born approximations, that is, sufficiently underdense electron density fluctuations;

(v) In order to avoid bending of the ray paths (note that s of equation (22) is measured along a ray path) as well as to avoid rotation of polarization during propagation one requires $|n-1| << 1$, namely an underdense plasma.

Section 3.D Information Theory Formulation

The informational entropy maximization principle represents an attempt to obtain closure of the stochastically nonlinear random equation of wave propagation, without either using a perturbation expansion or invoking a priori closure assumptions. It provides closure at any specified level of the hierarchy of moment equations, by requiring that the included unknown moments lead to a maximum informational entropy, and therefore provides an unbiased estimate of the included moments of the field, the higher moments remaining unspecified.

Consider the wave equation

$$\nabla^2\psi + k^2(1+\mu)\psi = 0 \tag{29}$$

where μ is stochastic, and not necessarily small. The first two equations of the moment hierarchy are

$$(\nabla^2+k^2)\langle\psi(x)\rangle + k^2\langle\mu(x)\psi(x)\rangle = 0$$

$$\left(\nabla_x^2+k^2\right)\langle\mu(x')\psi(x)\rangle + k^2\langle\mu(x)\mu(x')\psi(x)\rangle = 0 \tag{30}$$

The last terms of both of the equations (30) are the stochastically nonlinear terms which lead to the closure problem.

The maximum informational entropy principle proceeds conceptually as follows.

For any given choice of $\langle\mu(x)\mu(x')\psi(x)\rangle$, the equations (30) are a closed system and can in principle be solved in order to obtain, in addition to $\langle\mu(x)\mu(x')\psi(x)\rangle$, the ensemble averages $\langle\mu(x')\psi(x)\rangle$ and $\langle\psi(x)\rangle$. But by definition

$$\langle\mu(x)\mu(x')\psi(x)\rangle$$

$$= \int P\{\mu(x) = \bar{\mu},\mu(x') = \bar{\mu}'\}P_c\{\psi(x) = \bar{\psi}|\bar{\mu},\bar{\mu}'\}\bar{\mu}\bar{\mu}'\bar{\psi}d\bar{\mu}d\bar{\mu}'d\bar{\psi}, \tag{31}$$

where $P\{\mu(x) = \bar{\mu},\mu(x') = \bar{\mu}'\}$ is the joint probability that $\mu(x)$ and $\mu(x')$ will assume the values $\bar{\mu}$ and $\bar{\mu}'$ respectively and $P_c\{\psi(x) = \bar{\psi}|\bar{\mu},\bar{\mu}'\}$ is the conditional probability that $\psi(x)$ will assume the value $\bar{\psi}$, given that the values of $\mu(x)$ and $\mu(x')$ are $\bar{\mu}$ and $\bar{\mu}'$. The other two ensemble averages of equations (30) are

$$\langle\mu(x')\psi(x)\rangle =$$

$$= \int P\{\mu(x) = \bar{\mu},\mu(x') = \bar{\mu}'\}P_c\{\psi(x) = \bar{\psi}|\bar{\mu},\bar{\mu}'\}\bar{\mu}'\bar{\psi}d\bar{\mu}d\bar{\mu}'d\bar{\psi} \tag{32}$$

and

$$\langle\psi(x)\rangle = \int P\{\mu(x) = \bar{\mu}, \mu(x') = \bar{\mu}'\} P_c\{\psi(x) = \bar{\psi}|\bar{\mu},\bar{\mu}'\}\bar{\psi}d\bar{\mu}d\bar{\mu}'d\bar{\psi} \tag{33}$$

Now the probability $P\{\mu(x) = \bar{\mu}, \mu(x') = \bar{\mu}'\}$ is a prescribed function which is determined by the stochastic properties of the medium. Therefore by specifying the quantity $\langle\mu(x)\mu(x')\psi(x)\rangle$, equation (31) is a constraint on the conditional probability $P_c\{\psi(x) = \bar{\psi}|\bar{\mu},\bar{\mu}'\}$. Furthermore, the wave equations (30) could then be solved uniquely for $\langle\mu(x')\psi(x)\rangle$ and $\langle\psi(x)\rangle$, so that equations (32) and (33) constitute two additional constraints on P_c. Given these three constraints, there exists a unique conditional probability function $P_c\{\psi(x) = \bar{\psi}|\bar{\mu},\mu'\}$ which maximizes the corresponding informational entropy,

$$S_c(x,x') = \int P_c\{\psi(x) = \bar{\psi}|\bar{\mu},\bar{\mu}'\}\ell n\, P_c\{\psi(x) = \bar{\psi}|\bar{\mu},\bar{\mu}'\}d\bar{\psi} \tag{34}$$

The stationary value of S_c, say $\hat{S}_c(x,x')$, is determined by the condition

$$\left.\frac{\delta S_c}{\delta P_c}\right]_{S_c=\hat{S}_c} = 0. \tag{35}$$

The resulting value of $\hat{S}_c$ depends on the original choice of $\phi(x,x') = \langle\mu(x)\mu(x')\psi(x)\rangle$; in fact $\hat{S}_c$ is a functional of ϕ. Therefore maximization of $\hat{S}_c$ with respect to ϕ,

$$\frac{\delta\hat{S}_c[\phi]}{\delta\phi} = 0, \tag{36}$$

provides one with the best choice of $\phi(x,x')$, consistent with the retention of two moment equations (30). Clearly the method can be extended to an increased number of moment equations, with a great deal of increased difficulty.

In summary, for a given choice of $\phi(x,x') = \langle\mu(x)\mu(x')\psi(x)\rangle$, equations (30) are solved for $\langle\mu(x')\psi(x)\rangle$ and $\langle\psi(x)\rangle$. Then S_c of equation (34) is maximized with respect to P_c, subject to the constraints (31), (32), and (33). Finally, the result is maximized with respect to ϕ. An increased number of moment equations improves the accuracy by providing more constraints on P_c.

The procedure described above, while well defined, has not been significantly explored at this time. It is discussed here because it appears to have potential as a systematic and practical variational procedure, and because it has been found to be useful in studies of equilibrium and nonequilibrium statistical mechanics.

Section 3.E Computer Experiments

Hochstim has performed a computer "experiment" which is intended to simulate scattering from a one-dimensional random medium. He solves the problem of propagation through a stack of a finite number of slabs, each of which may be assigned a thickness and a dielectric constant. The transmitted and reflected intensities are calculated for one given set of slabs; this is a well-defined mathematical problem which reduces to matrix inversion. The calculation is then repeated many times with the thickness and/or the dielectric constant of each slab varying in a random fashion from one realization to the next. These random variations can be described statistically, and the statistical properties of an ensemble of such realizations are hence known. The outputs of such an "experiment" for one ensemble of dielectric constant profiles are the ensemble averages of the transmitted and reflected intensities.

The "experimental" results can be compared with theoretical calculations for propagation in a one-dimensional random medium. The theoretical results consist of approximate solutions to the one-dimensional wave equation for a medium with a randomly varying dielectric constant; these solutions can be derived using various techniques which are analogous to the approaches to the three-dimensional scattering problem (the first Born approximation, the distorted wave Born approximations, etc.). Comparisons of the one-dimensional theoretical and "experimental" results give some indication of the ranges of validity of the various theoretical approaches.

It was proposed that there might be several types of extensions to Hochstim's approach.

Extensions of the one-dimensional model. A number of additional features of scattering phenomena might be studied within the context of Hochstim's recent one-dimensional studies:

a. Thus far Hochstim has assumed a spatially uniform mean electron density upon which statistical fluctuations are superimposed. Similar calculations could be performed using a more realistic spatially non-uniform (possibly parabolic) mean electron density distribution.

b. Average phase shifts for the reflected and transmitted waves could be calculated.

c. Time correlations between the various statistical realization could be specified.

d. The statistics of the random medium could be made more realistic. For example, Hochstim assumes that the thickness of each slab and its electron density are statistically independent, whereas these parameters should be correlated.

e. Hochstim's results for the reflected and transmitted intensities are presented as a function of the parameter $(\delta n/n)^2$, where δn is the RMS electron fluctuation and n is the mean electron density. The computer experiments might suggest meaningful alternative parameters.

f. Attenuation of the beam due to side scattering could be included even in the context of a one-dimensional calculation.

g. The incident wave could be allowed to impinge upon the stack of slabs at an angle, in order to study aspect angle dependence.

Solutions of the multiple scattering equation of Watson. If there are N scatterers, which are in the wave zone relative to each other (scatterer separations large compared to the wave length), then Maxwell's equations are exactly replacable by the system of algebraic equations,

$$\vec{E}(\vec{z}_\alpha) = \vec{E}_I(\vec{z}_\alpha) + \sum_{\beta\neq\alpha=1}^{N} \sum_{j=1}^{2} e_{\alpha\beta}(j) F_{\alpha\beta}(\vec{z}_\alpha j)$$

and

$$F_{\alpha\beta}(\vec{z}_\alpha, i) = G^{o}_{\alpha\beta} f_{i1}(\alpha\beta, \beta 0) E_I(\vec{z}_\alpha) + \sum_{\sigma \neq \beta = 1}^{N} \sum_{j=1}^{2} G^{o}_{\alpha\beta} f_{ij}(\alpha\beta, \beta\sigma) F_{\beta\sigma}(\vec{z}_{\beta}, j) \tag{37}$$

where $\vec{E}_I(\vec{z}_\alpha)$ is the incident field at position $\vec{z}_\alpha$; $\hat{e}_{\alpha\beta}(j)$ describes the polarization of the field while travelling from points $\vec{z}_\beta$ to $\vec{z}_\alpha$; $f_{ij}(\alpha\beta, \beta\sigma) = -r_o \hat{e}_{\alpha\beta}(i) \cdot \hat{e}_{\beta\sigma}(j)$ is the Thompson scattering amplitude; $f_{i1}(\alpha\beta, \beta 0)$ is the scattering amplitude for the incident field; and

$$G^{o}_{\alpha\beta} = \frac{e^{ik|\vec{z}_\alpha - \vec{z}_\beta|}}{|\vec{z}_\alpha - \vec{z}_\beta|}$$

is the free-space Green's function in the wave zone. The solution of equations (37) involves diagonalization of an N×N matrix. This might be done fairly easily with N as large as 100, in order to serve as a check on scattering theories, and if computer costs permit, it might be possible to include a statistical distribution of scatterer realizations.

Laboratory experiments on a simple physical model. A statistical distribution of metal spheres might be constructed, and either microwaves or acoustic waves scattered from the distribution. With the statistics completely prescribed, this is an additional check on scattering theories. There was some disagreement among members of the Workshop concerning the feasibility of this experiment.

One-dimensional doppler spread. Hochstim's one-dimensional calculations could be redone with the boundaries of the slabs being given random velocities. Because of multiple scattering, the transmitted and reflected waves would reflect a sequence of Doppler shifts from the individual boundaries. A question to be answered by such a study would be the effect upon the frequency spectrum of significant multiple scattering.

Chapter 4

LABORATORY FACILITIES FOR SCATTERING EXPERIMENTS

There have been several laboratory investigations of the scattering of microwaves from turbulent plasmas. The scattering media in these experiments have been turbulent plasma jets and ionized turbulent pipe flows. These facilities have made possible direct comparisons with scattering theories. Published results of the BTL (Granatstein) pipe flow experiment and the SRI (Guthart et al) plasma jet are included among the references listed in Appendix A.

In the SRI and BTL experiments the mean electron density can be varied from a level of at least two orders of magnitude below critical electron density, up to at least critical electron density. The ratio of electron density fluctuations to mean density is about 0.5 for both experiments. In both facilities it is possible to study the cross-polarized return and the frequency spectrum of the scattered fields.

Recently the RCA (Montreal) laboratory (Johnston et al) has begun to study microwave scattering from a turbulent plasma jet. In their present configuration the scattering is from regions in which the electron density exceeds the critical electron density. (More recent results, obtained since this Workshop, have given information about scattering at lower electron densities.)

In all of these facilities there has been a careful diagnostic study of the properties of the turbulent medium. In addition an extensive diagnostic study of a plasma jet has been performed by Demetriades at the Philco Corporation.

The properties of the scattering media and the results of the scattering experiments are summarized in the following table.

Table II

Diagnostics and Scattering from Laboratory Plasmas

Investigator	Granatstein (BTL)	Guthart (SRI)	Johnston (RCA)	Demetriades (Philco)
Flow Type	Pipe	Jet	Jet	Jet
Flow Diameter	2.2 cm	10-15 cm	10 cm	10-30 cm
Similarity?	?	Yes	No	Yes
Measured Properties	Scattered Intensity, n, Δn, Spectra, Correlations	Scattered Intensity n, Δn, Spectra, Corr. Fn, Scales	Scattered Intensity Long. and Lat. Correlations	All mean, turbulent, and intermittent properties
Reynolds no.	8000 (based on diameter)	3000	2000	5000
Ambient Pressure	45 mm (Ar+1% N_2)	6 mm	2 mm	10 mm
Gas Temperature	400°K	--	600°K	400-1400°K
Electron Temperature	12,000°K	--	10,000°K	4000-6000°K

Table II (continued)

Investigator	Granatstein (BTL)	Guthart (SRI)	Johnston (RCA)	Demetriades (Philco)
Electron Density on Axis (per cm^3)	7×10" - 10^{14}	10^{11} - 10^{13}	10^{13} - 10^{14}	10^{9} - 10^{13}
Velocity Fluctuation $\Delta u/u$	3%	~20%	~20%	22%
Temperature Fluctuation T/T	--	--	--	15%
Electron Fluctuation $\Delta n/n$ (on Axis)	~0.5	~0.5	0.5 - 1	0.2 - 5
Large ("Spiky") Electron Fluctuation	No	?	Yes	Yes
Temperature Spectra	--	--	--	-5/3 range observed
Electron Spectra	No -5/3 range	--	-5/3 range observed	No -5/3 range
Longitudinal Correlation Scale	~1/5 of flow diameter	~1/6 of flow diam	~1/4 of flow diameter	~1/6 of flow diameter
Lateral Correlation Scale	~1/5 of long. scale	~1/2 of long scale	~1/3 of long. scale	--

Table II (continued)

Investigator	Granatstein (BTL)	Guthart (SRI)	Johnston (RCA)	Demetriades (Philco)
Microwave Wavelength	0.86 cm 0.43 cm	3.2 cm	0.86 cm 1.9 cm	--
Ratio of wave-length to Correlation Scale	~2	~1	0.3 0.45	--
Ratio of ν to ω	0.1	0.2	--	--
Conclusions regarding scattering	*	†	‡	—

* Departure from first Born theory is small for $(\Delta n)_a^2 / \left(n_{cr}^2 \sin \alpha\right) \lesssim 0.05$ where $(\Delta n)_a$ is the RMS electron density fluctuation on the axis, n_{cr} is the critical electron density, and α is the aspect angle. For values of $(\Delta n)_a$ larger than that given by the above inequality the scattering cross section quickly saturates. When a core of plasma of diameter equal to the microwave wavelength becomes critically dense, the scattering cross section decreases with any further increase in electron density.

† Measured backscattered cross section agrees with first Born theory to within 3 dB for electron density (on the axis) up to 10^{12} cm^{-3}. Cross polarized backscattering cross sections appear to vary as the fourth power of the electron density.

‡ Scattering is apparently from overdense regions.

Chapter 5

CONCLUSIONS AND RECOMMENDATIONS

Section 5.A Conclusions and Recommendations of the Theoretical Panel

Scattering theory. (Small electron density fluctuations.) In Chapter 3 we reviewed a number of scattering theories which are currently felt to be fruitful. In the limit of small electron density fluctuations the first Born approximation is frequently applicable, as well as extensions to it such as the various distorted wave Born approximations. The panel recommends that extensions of the first Born approximation should continue to receive extensive study, especially with regard to applications to the wake scattering problem; it is felt that the "full model" (DWBA-2c) should receive the greatest attention. The full model is chosen over some of the simpler extensions discussed in Chapter 3 (namely DWBA-1, -2a, and -2b) because it is only slightly more complicated and is expected to yield significantly better results.

It appears (on the basis of the studies presented at the Workshop) that the first Born approximation has a greater range of validity than had been previously acknowledged. The limits of validity of the Born approximation, including the Salpeter-Treiman condition, should be reconsidered.

The role of Watson's transport theory as a supplement to the various perturbation approximations should be studied further.

Scattering theory. (Partially or totally overdense media.) Studies of scattering from overdense random surfaces should be continued and the theories modified, if necessary, in order to make them applicable to scattering from an overdense turbulent wake.

As a preliminary to the study of scattering from a wake with overdense blobs, scattering and absorption by single overdense geometrical structures should be studied. A considerable literature on this subject is available in the context of ionospheric physics. It is known that considerable absorption occurs when an electron density profile becomes overdense with a gentle slope; therefore, it is conceivable that the inclusion of absorption may simplify the multiple scattering problem by reducing the number of scatterers required for the investigation.

Also a preliminary to the study of overdense scattering, it may be useful to develop techniques for ray tracing with representative electron density profiles.

The panel is not optimistic about the chances of success for these approaches to scattering from partially or totally overdense media.

Inputs to theory. (Representation of the scattering medium.) It is very important to develop a mathematical description of a random scattering medium which is clearer than is presently available in the statistical descriptions. Thus if a wake could be characterized as an irregular tube with sharply defined boundaries within which the electron density is reasonably uniform and outside of which the electron density is zero the proper electromagnetic theory to use would most likely be one in which surface scattering was equally as important as volume scattering. On the other hand, the statistical description of electron density for a wake could imply a theory such as a distorted wave Born approximation. Therefore, it is recommended that fluid dynamicists make an effort to provide a description of a single realization of a turbulent wake in sufficient detail to enable scattering theorists to choose the correct model of scattering.

Inputs to theory. (Scattering experiments.) A major problem in the development of scattering theory is the determination of the effective index of refraction (or equivalently, the effective wave number) in a turbulent medium. The importance of this information is clear from the review of scattering theories in Chapter 3. It is therefore recommended that microwave scattering experiments be designed to measure the forward scattered wave, as a function of scattering angle, as well as the associated phase shifts, and that these experiments be analyzed to determine the propagation properties of the turbulent medium.

Studies of frequency spectra. Studies of the frequency spectra of electromagnetic waves scattered from turbulent media have appeared in the literature, almost exclusively in the context of the first Born approximation. However, the effect of multiple scattering upon the frequency spectrum is an extremely important problem upon which there is presently little theoretical information. Therefore, the current effort should be extended to calculations of frequency spectra using the distorted wave Born approximations.

Numerical experiments. The feasibility of extending Hochstim's computer experiment to three dimensions should be investigated. For each realization of an ensemble of random distributions of scatterers this "experiment" would involve the diagonalization of an N×N matrix, where N is the number of scatterers in the system. The study should begin by calculating the scattering from a single realization, thereby modeling the scattering problem without statistics. If the cost is reasonable, then many realizations, perhaps with specified statistics (or at least known statistics), should be studied. It is recommended that research programs along these lines have relatively low priority.

Other useful extensions of Hochstim's work, all in the context of the one-dimensional problem, have been discussed in Section D of Chapter 3.

Some consideration should be given to the validity of extrapolating to three dimensions the results of one- and two-dimensional numerical experiments.

Section 5.B Conclusions and Recommendations of the Experimental Panel

Scattering mechanisms. The first Born approximation for scattering from a turbulent plasma appears to be valid in the far wake of many of the ballistic range projectiles and full-scale reentry vehicles which have been studied. The range of applicability of the first Born approximation appears to be greater than that given by the Salpeter-Treiman conditions.

Departures from the predictions of the first Born approximation may be caused by one or more of the following physical processes:

a) The presence of locally overdense "blobs" in an otherwise underdense plasma.

b) Attenuation due to absorption and/or scattering.

c) Variation of the effective wave number of the coherent wave propagating in the turbulent medium.

The mechanism (a) was not considered in any detail by the Workshop; mechanisms (b) and (c) are taken into account, at least to first order in the electron density fluctuations in the various distorted wave Born approximations. The relative importance of these various mechanisms has not been specifically studied. Only volume scattering effects were considered by the Workshop; no attention was given to surface scattering.

Comparisons of theory and experimental data. For sufficiently small electron density fluctuations scattering from turbulent laboratory plasmas appears to agree with the first Born approximation. Departures from the first Born approximation (such as those seen in the experiments of Granatstein) may be satisfactorily interpreted in terms of attenuation effects, as in the theories of Shkarofsky and of Feinstein. (These theories fall into the category DWBA-2a of Chapter 3.)

Preliminary attempts were made (during the Workshop) to interpret a limited amount of wake scattering field data from non-ablating, non-seeded spheres at high altitudes. It appears possible to achieve a self-consistent interpretation of this data using (i) the first Born approximation, (ii) attenuation corrections to the first Born approximation using the heuristic models (DWBA-2a) of Shkarofsky and of Feinstein, (iii) extrapolation to reentry conditions to turbulence theories and experiments on low speed wakes and plasma jets, assuming local isotropy and uniform molecular transport properties. These interpretations must be considered preliminary until more reliable values of electron densities, electron density fluctuations, and turbulent scale lengths become available.

Similar attempts were made to interpret wake scattering field data for heavily ablating reentry vehicles. These attempts were not successful. It appears that for many ablating reentry vehicles the electron densities are sufficiently large that strong attentuation and/or multiple scattering effects must be considered.

Studies of this sort should be continued, with an effort being made to use the best available estimates of turbulent spectra. Using presently available theoretical models (one of the distorted wave Born approximations, for example) field data should be tested for a consistent interpretation in terms of these models.

Studies of the nature of the wake. The attempts to obtain a consistent interpretation of field data have suggested areas where further theoretical and experimental studies of the properties of wake turbulence are necessary.

a) For any given reentry vehicle there are significant variations in the turbulence spectrum (including the turbulent scale sizes) as a function of Reynolds number and of downstream distance. More information about these variations should be obtained from theory and experiment.

b) It has been assumed during the course of the studies of the field data that the spectrum of the electron density fluctuations is identical to the spectrum of the fluid velocity fluctuations. In the near future this assumption will be tested experimentally for various plasma conditions.

c) It has been assumed that the spectrum of electron density fluctuations is isotropic. Further studies of departures from isotropy in reentry vehicle wakes is required.

Many of these studies of wake turbulence can be furthered by laboratory experiments in plasma jets and in ballistic ranges and shock tubes. Specific proposals for useful studies in such facilities are given below.

<u>Laboratory experiments</u>. To gain better understanding of the properties of turbulent shear flows in a compressible fluid with variable molecular transport coefficients, finite reaction rates, etc., experiments should be designed to determine such relevant quantities as:

a) Lateral and transverse correlation functions (that is, the full three-dimensional wave number spectrum) of velocity, temperature, and electron density fluctuations and the dependence of these correlation functions on all of the various parameters of interest: Reynolds number, temperature fluctuation amplitude, Debye length to wavelength ratio, etc.

b) Spatial distributions of mean electron density and of electron density fluctuation amplitudes in turbulent shear flows, measured either directly or through inference from distributions of other convected scalar quantities.

c) The relationship between the spectra of electron density fluctuations and other scalar quantities and the velocity spectrum, over the range of parameters of interest.

To aid the study and testing of scattering theories the range of parameters under which scattering experiments are performed should be extended. In all of these scattering experiments it is important to adequately map the turbulence field in the scattering medium.

All of these laboratory investigations should be attempted in ballistic ranges or shock tunnels so that the differences between low velocity and hypersonically generated wakes can be studied.

Ballistic range and shock tunnel experiments should study the variations in electron density fluctuations for a fixed flow field profile in order to determine quantitatively whether departures from the first Born approximation are to be expected. Scattering experiments on wakes in such facilities should be performed to demonstrate quantitatively agreement or disagreement with various theoretical models for scattering.

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APPENDIX A

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APPENDIX B

Small-Scale Structure and Viscous Cutoff in Scalar Spectrum of Hypersonic Wake Turbulence

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Appendix B - 2

1. INTRODUCTION

Interpretation of radar returns from turbulent plasmas is often complicated by such important effects as multiple scattering[1,2] and three dimensional geometry. However, under certain restrictive conditions regarding the plasma density and the overall dimension of the scattering volume,[1] it is generally agreed that the averaged scattering intensity can be reliably calculated according to the Booker formula,[3] which was a single-scattering theory based on the first-order Born approximation[4] and requires only relatively simple specification of the statistical properties of the scattering medium. The Booker formula gives, for the time-averaged scattering intensity from a turbulent plasma of sufficiently slowly varying spatial configuration,[1,3]

$$\sigma(\theta,\lambda_o) = \frac{k^4 \sin^2\psi}{4\pi} \int_V \overline{\Delta\epsilon^2(\underset{\sim}{r})} F(\underset{\sim}{r},\underset{\sim}{q}) d^3r \qquad (1)$$

Here $\sigma(\theta,\lambda_o)$ is defined, as usual, as 4π times the power scattered per unit solid angle per unit incident power density along the direction of scattering $\underset{\sim}{k}$, which makes an angle θ with respect to the incident wave propagation vector $\underset{\sim}{k}_o$; $k \equiv |\underset{\sim}{k}| = |\underset{\sim}{k}_o| \equiv 2\pi/\lambda_o$ is the incident wave number; ψ is the angle between $\underset{\sim}{k}$ and the electric field vector $\underset{\sim}{E}_o$ of the incident wave; $\overline{\Delta\varepsilon^2(\mathbf{r})}$ is the local mean-square fluctuation of the dielectric constant about its mean value $\overline{\varepsilon(\mathbf{r})}$ at point $\mathbf{r}$; and

$$F(\mathbf{r},\mathbf{q}) \equiv \int_V S(\mathbf{r},\mathbf{r}')\, e^{i\mathbf{q}\cdot\mathbf{r}'}\, d^3r' \qquad (2)$$

defines the local spectrum function for $\overline{\Delta\varepsilon^2(\mathbf{r})}$ as a spatial Fourier transform of the normalized two-point correlation function

$$S(\mathbf{r},\mathbf{r}') \equiv \overline{\Delta\varepsilon(\mathbf{r})\ \Delta\varepsilon(\mathbf{r}+\mathbf{r}')} \Big/ \overline{\Delta\varepsilon^2(\mathbf{r})} \qquad (3)$$

along the direction of the vector $\mathbf{q} \equiv \mathbf{k} - \mathbf{k}_0$. The volume of integration V in equations (1) and (2) is understood to be extended over the effective scattering volume as defined by the plasma boundaries and/or the range cell of the radar under consideration. The frequency- or wavelength-dependence of the scattering intensity is, of course, implicitly contained in the Fourier parameter $\mathbf{q}$, which has a magnitude $q = (4\pi/\lambda_0)\sin(\theta/2)$.

In the study of radar return from the turbulent wake of hypersonic objects, there has been a strong temptation on the part of some early investigators to compare the frequency-dependence of the scattering intensity $\sigma(\theta,\lambda_0)$ with what could be inferred from simple turbulence theories.

In particular, the temptation to draw conclusions from a limited number of frequency samplings about whether a "universal spectrum" of the classical Kolmogorov[5] or Oboukhov-Corrsin[6,7] form does exist or not in the turbulent hypersonic wake seemed irresistable. Such an exercise was bound to be unfruitful and misleading since it not only overlooked the fact that the statistical properties of the turbulent hypersonic wake plasma within the scattering volume V as sampled by the finite range cell of the radar might not at all be isotropic nor spatially homogeneous, but also the fact that even for a homogeneous, isotropic turbulence field of finite Reynolds number in a constant-density fluid of constant transport properties (i.e., kinematic viscosity and molecular diffusivity), a "universal spectrum" can exist only over a limited range of wave number which, in turn, depends on a number of scaling parameters.[8,9,10] Furthermore, the effects of variable density and rapid chemical reactions within the hypersonic wake plasma may greatly complicate the form of the scalar fluctuation spectrum.[11] We shall review these complications briefly as follows.

2. ENERGY SPECTRUM FOR ISOTROPIC TURBULENCE IN AN INCOMPRESSIBLE FLUID OF CONSTANT KINEMATIC VISCOSITY

As discussed at length in most standard texts,[12-1] the three-dimensional energy spectrum E(q,t) for a decaying homogeneous isotropic turbulence field in an incompressible fluid of constant kinematic viscosity ν is generally of the form illustrated in Fig. 1. As usual, the function $E(q,t)$ is defined in such a way that the ensemble-averaged turbulence kinetic energy per unit mass lying within the wavenumber range between q and $q + dq$ at time t is given by $E(q,t)dq$, so that the mean-square velocity fluctuation along any one of the three orthogonal rectilinear coordinates is given by the integral

$$\frac{3}{2}\,\overline{u'^2(t)} = \int_0^\infty E(q,t)\,dq \,. \tag{2}$$

The entire energy spectrum is roughly divisible into three main ranges in wavenumber space. Referring to Fig. 1, these are:

(i) The Low Wavenumber Range, consisting of the largest eddies* with longest persistency, and those large eddies which are mostly responsible for the macroscopic diffusion property (i.e., eddy diffusivity D_e) of the turbulence field.[13]

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* It should be noted that "eddies" in turbulence is actually a loose term referring to certain Fourier components in configuration space. Eddies of different sizes are not really separable entities since they may share the same part of the fluid at any given time.

The shape for this part of the spectrum is slowly time-varying, and depends strongly on the initial condition of formation.

(ii) <u>The Energy-Containing Wavenumber Range</u>, which forms the hump of the spectrum in the vicinity of the wavenumber denoted by q_E in Fig. 1.

(iii) <u>The Universal Equilibrium Range</u> at high wavenumber beyond the hump of the spectrum. The shape of this part of the spectrum becomes "universal" and independent of the initial condition of formation only in the sense that it can be collapsed into a single curve at all time t when the wavenumber scale is normalized with respect to the Kolmogorov wavenumber[5]

$$q_K \equiv (\varepsilon/\nu^3)^{1/4} \qquad (3)$$

Here ε denotes the instantaneous rate of dissipation of turbulence energy per unit mass, which is, of course, a function of time t in a decaying turbulence field.

It is important to note that the famous Kolmogorov law which predicted a $q^{-5/3}$ dependence for the energy spectrum, is only applicable to the "inertial subrange" which lies somewhere within the lower wavenumber portion of the universal equilibrium range as illustrated in Fig. 1. In the high wavenumber end near q_K, the energy spectrum decreases much more rapidly than $q^{-5/3}$ due to the action of viscosity. The extent of the inertial subrange thus depends on the instantaneous separation between the two wavenumbers q_E, q_K in a decaying turbulence field.

The inertial subrange, as well as the universal equilibrium range of the energy spectrum, has been found to exist not only in low speed grid turbulence,[12] but also in a large number of shear flows ranging from tidal channels[15] to low speed wakes in water tunnels.[16] The normalized energy spectrum as deduced from these latter experiments[15,16] showing the $q^{-5/3}$ inertial subrange and the subsequent viscous cutoff near q_k is reproduced here in Fig. 2.

As mentioned earlier, the lower wavenumber portion of the energy spectrum (i.e., ranges (i) and (ii) cited above) is not universal even in low speed grid-generated turbulence of constant mean flow velocity U. This latter type of flow is, of course, a close laboratory simulation of a decaying homogeneous isotropic turbulence field in which the time after formation t is replaced by the averaged flow time past the grid x/U at any given axial station of distance x downstream of the grid. A typical time evolution of the normalized energy spectrum in grid turbulence is illustrated in Fig. 3. The lack of universality for the lower wavenumber portion of the spectrum is quite evident. In the case of turbulent shear flow, one may expect not only similar continuous evolution of the spectral shape, but also pronounced anisotropy associated with the lower wavenumber portion of the energy spectrum as well.

3. RELATIONSHIP BETWEEN ENERGY SPECTRUM AND SCALAR FLUCTUATION SPECTRUM

From the Booker formula (1), it is quite clear that the averaged scattering intensity (or radar cross-section) does not depend directly on the energy spectrum $E(\underset{\sim}{r},\underset{\sim}{q})$ of the turbulence field, but rather on the spectrum function $F(\underset{\sim}{r},\underset{\sim}{q})$ for mean-square fluctuation of the dielectric constant as defined in equations (2) and (3). Even though in subsonic turbulence, the fluctuation of any scalar quantity, such as the dielectric constant ε, is mostly driven by the convective motion of the turbulence, there exists no simple relationship between $E(\underset{\sim}{r},\underset{\sim}{q})$ and $F(\underset{\sim}{r},\underset{\sim}{q})$ in turbulent shear flow.

In the case of homogeneous isotropic turbulence of sufficiently high Reynolds number in a fluid of constant mass density ρ, kinematic viscosity ν, and molecular diffusivity D for a particular conserved, passive scalar quantity θ, however, it has been shown by Oboukhov,[6] and independently by Corrsin,[7] that the fluctuation spectrum for θ in the "convection subrange" should be of the form

$$\Gamma(q,t) \propto \chi \varepsilon^{-1/3} q^{-5/3} \qquad (4)$$

in which χ denotes the instantaneous rate of diffusive dissipation of mean-square fluctuation of θ per unit volume; and ε denotes the instantaneous rate of viscous dissipation of turbulence kinetic energy per unit mass, as before. The fluctuation spectrum $\Gamma(q,t)$ is defined in such a way that the Fourier component for mean-square fluctuation of θ lying within the wavenumber range between q and q + dq at time t is given by $\Gamma(q,t)dq$, so that the total mean-square (i.e., ensemble-averaged) fluctuation of θ at time t is given by

$$\overline{\Delta\theta^2(t)} = \int_0^\infty \Gamma(q,t)dq \ . \qquad (5)$$

In a way analogous to the definition of the "inertial sub-range" for the energy spectrum (Fig. 1), the "convection subrange" here refers to a range of wavenumber lying somewhere between q_θ where the peak of $\Gamma(q,t)$ is located, and a certain cut-off wavenumber which we shall call the Oboukhov-Corrsin wavenumber,

$$q_{oc} \equiv \left(\varepsilon/D^3\right)^{1/4} . \qquad (6)$$

While Oboukhov[6] and Corrsin[7] both predicted that the wavenumber dependence of $\Gamma(q,t)$ within the convection subrange should be identical to that of the energy spectrum within the inertial subrange, and that the respective diffusive and viscous cut-off wavenumbers for the two subranges should bear the ratio

$$\frac{q_{oc}}{q_K} = \left(\frac{\nu}{D}\right)^{3/4} \qquad (7)$$

Batchelor[8] pointed out that such would be the case only when D and ν are of the same order of magnitude. When $D << \nu$, such as diffusion of electrolytes (e.g., salt) in water, the $q^{-5/3}$ dependence for $\Gamma(q,t)$ should extend only to the neighborhood of the Kolmogorov wavenumber q_K. In the range of wavenumber between q_K and a new diffusive cut-off wavenumber which we shall call the Batchelor wavenumber

$$q_B \equiv \left(\varepsilon/\nu D^2\right)^{1/4} \tag{8}$$

the wavenumber dependence of $\Gamma(q,t)$ should become q^{-1}.

The existence of the viscous-convective subrange with q^{-1} dependence for the fluctuation spectrum of a weakly diffusive passive scalar as predicted by Batchelor[8] has indeed been confirmed in the grid experiment of Gibson and Schwarz,[17] and also more recently in the wake experiments of Gibson, Lyons, and Hirschsohn.[18] It is worth noting, however, that the transition point from the $q^{-5/3}$ spectrum to the q^{-1} spectrum observed in the experiments[17,18] turned out to lie in the neighborhood of 0.04 q_K instead of q_K.

When the molecular diffusivity for the passive scalar of interest is much greater than the kinematic viscosity, i.e., $D \gg \nu$, Batchelor, Howells, and Townsend[9] predicted that the diffusion cut-off to the scalar fluctuation spectrum should begin at the Oboukhov-Corrsin wavenumber q_{oc}, and that in the "inertial-diffusive subrange" between q_{oc} and q_K, the spectrum should be of the form

$$\Gamma(q) = \frac{1}{3}\, C\chi\varepsilon^{2/3} D^{-3} q^{-17/3} \quad . \tag{9}$$

In other words, the spectrum should fall off rapidly with increasing wavenumber at a rate approaching q^{-6}.

In a series of recent papers, Gibson[10] has proposed a unified spectral theory for the fine structure of scalar fields mixed by turbulence in which the inertial subrange of Oboukhov and Corrsin $\left(\Gamma \propto q^{-5/3}\right)$ and the viscous-convective subrange of Batchelor $\left(\Gamma \propto q^{-1}\right)$ were reproduced. However, for strongly diffusive scalars (i.e., $D \gg \nu$), Gibson predicted a new inertial-diffusive subrange with $\Gamma \propto q^{-3}$ and upper cut-off at the Batchelor wavenumber q_B. This latter result by Gibson thus contradicted sharply with the inertial-diffusive subrange obtained earlier by Batchelor, Howells, and Townsend.[9] As of this time, there yet exists no definitive experimental measurement on the spectral structure of strongly diffusive scalars in isotropic turbulence from which the shape of the inertial-diffusive subrange can be clearly established. In view of the fact that the physical models employed by Batchelor et al. and by Gibson in arriving at their respective theoretical results were quite different , and they were believed to be respectively correct with equal conviction, the question of spectral shape for strongly diffusive scalars in the inertial-diffusive subrange must be regarded as an unsettled problem in basic turbulent scalar mixing theory.

At this point, it may be noted that the scalar fluctuation spectrum $\Gamma(q)$ discussed above is still quite distinct from the scattering spectrum $F(\underset{\to}{r},\underset{\to}{q})$ defined earlier in Section 1. However, it is easy to show that the two are simply related as follows.

Assuming that the correlation length scale for the fluctuating scalar quantity of interest (e.g., the dielectric constant of the wake plasma) is generally much smaller than the dimensions of the scattering volume V under consideration one may extend the finite range of volume integration in equation (2) to infinity without making appreciable error. Then, from the well known properties of Fourier transforms,[19] one obtains from equation (2),

$$S(\underset{\to}{r},\underset{\to}{r}') = \frac{1}{8\pi^3} \iiint_{-\infty}^{\infty} F(\underset{\to}{r},\underset{\to}{q})\, e^{-i\underset{\to}{q}\cdot\underset{\to}{r}'}\, d^3q \ . \qquad (10)$$

Upon multiplying both sides of the above equation $\overline{\Delta\varepsilon^2(\underset{\to}{r})}$ and subsequently letting $\underset{\to}{r}' = 0$, one obtains, with the help of equation (3),

$$\overline{\Delta\varepsilon^2(\underset{\to}{r})} = \frac{1}{8\pi^3} \iiint_{-\infty}^{\infty} \overline{\Delta\varepsilon^2(\underset{\to}{r})} R(\underset{\to}{r},\underset{\to}{q}) d^3q \ . \qquad (11)$$

If the scalar field were locally isotropic about $\vec{r}$, the function F varies only with the magnitude of the vector $\vec{q}$ in accordance with equation (2), so that equation (11) reduces to

$$\overline{\Delta\varepsilon^2(\vec{r})} = \frac{1}{2\pi^2}\int_0^\infty \overline{\Delta\varepsilon^2(\vec{r})}F(\vec{r},q)q^2dq \ . \qquad (12)$$

Comparing this with the definition of the local scalar fluctuation spectrum Γ defined earlier in equation (5), with θ presently replaced by ε, and t by $\vec{r}$, we obtain the relationship,

$$F(\vec{r},q) = 2\pi^2\ \Gamma(\vec{r},q)/q^2\ \overline{\Delta\varepsilon^2(\vec{r})} \qquad (13)$$

4. DIELECTRIC CONSTANT AND ELECTRON DIFFUSIVITY IN HYPERSONIC WAKE PLASMAS

As was discussed at considerable length in Ref. 11, the local dielectric constant $\varepsilon(\vec{r})$ in a hypersonic wake plasma is generally given by a linear combination of terms involving the local number density of free electrons $n_e(\vec{r})$, the number densities of molecular ions of both positive and negative charges $n_\zeta(\underset{\rightarrow}{r})$, and the mass density $\rho(\underset{\rightarrow}{r})$. Accordingly, the two-point auto-correlation function for dielectric constant fluctuation $S(\underset{\rightarrow}{r},\underset{\rightarrow}{r}')$ is generally made up of a large number of terms corresponding to auto-correlation of, and cross-correlation among, the various scalar quantities just mentioned. The relative importance of the various correlation terms in contributing to the resultant dielectric constant fluctuation spectrum has been shown[11] to depend strongly on the electron chemistry within the wake,[20] and also on the incident electromagnetic wave frequency under consideration.

In the present discussion, we shall limit ourselves to the simpler type of situation in which the dielectric constant fluctuation is dominated by the fluctuation of a single scalar quantity, namely, the electron number density n_e. Such would be the situation, for example, in a "recombination-controlled" wake where the rate of disappearance of free electrons is governed by the charge-neutralization process rather than by the electron-attachment (negative-ion formation) process.[11,20,21] If one further assumes that the characteristic time for depletion of free electrons due to chemistry is relatively long in comparison with the local characteristic time scale for turbulent mixing, then one may treat n_e as a passive scalar quantity in the application of existing turbulent mixing theories.

As is well known in plasma physics, the effective molecular diffusivity for electrons in a weakly-ionized plasma is a strong function of the ratio between the local Debye length[22]

$$h \equiv \left(kT/4\pi n_e\ e^2\right)^{\frac{1}{2}} \qquad (14)$$

and the local electron density gradient length scale ℓ of interest. When $h \ll \ell$, the diffusion process is essentially ambipolar,[23] and the effective diffusivity D for the electron-ion pair through the gas can be expected to be roughly the same as the local kinematic viscosity ν in a single temperature plasma. When $h \gg \ell$, the diffusion of electrons becomes "free" from the impeding effect of the heavier ions, and the effective diffusivity for these "free" electrons will become much greater than the local kinematic viscosity (by a factor of the order of $(m_M/m_e)^{\frac{1}{2}}$, i.e., the square-root of the molecule/electron mass ratio, or a factor of about 200 in air). Thus, according to equation (14), and the physical constants $k = 1.3804\times10^{-16}$ erg/°K (Boltzmann constant), $e = 4.803\times10^{-10}$ e.s.u. (electron charge), the criteria for ambipolar- and for free-diffusion of electrons are respectively given by (with T in K and ℓ in cm), For ambipolar-diffusion:

$$n_e >> 48T/\ell^2 \quad \text{electrons/cm}^3 \qquad (15a)$$

For free-diffusion:

$$n_e << 48T/\ell^2 \quad \text{electrons/cm}^3 \qquad (15b)$$

From the above criteria, it is seen that for a gradient length scale of the order of 1 cm in a wake plasma up to a few thousand °K temperature, ambipolar diffusion will prevail at electron number density much greater than $10^5/\text{cm}^3$.

5. APPLICATION TO HYPERSONIC FAR-WAKE

The near-wake of hypersonic objects is generally dominated by a high Mach number (relative to the ambient air), anisotropic compressible flow field with strong temperature and density gradients, as well as rapid chemical reactions.[20,21] To such a flow field, classical turbulence theories are clearly not applicable.

At sufficiently high flight Reynolds number $Re_\infty \equiv U_\infty r_n/\nu_\infty$ (i.e., far above that required for laminar-turbulent transition in the wake) and at sufficiently large distances downstream of the object (typically a few hundred times the characteristic dimension r_n of the object), however, ballistic range experiments[24] showed that the gas density fluctuation as inferred from the film contrast of schlieren photographs tended to become statistically homogeneous and isotropic within the turbulent wake volume.* Thus, the possibility exists that classical isotropic turbulence theories remain applicable to at least some part of the hypersonic far-wake. The range of applicability for the classical theories will then be limited only by the restriction that the fluctuation amplitudes for gas temperature, density, kinematic viscosity, and scalar diffusivity all be very small in comparison with their respective mean values, as assumed in all classical turbulence theories.

In turbulent shear flows, the fluctuation amplitude of scalar quantities is generally of the same order of magnitude as the averaged "defect" or "excess" of such quantities within the turbulent zone from the ambient condition.

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* As indicated in the paper of Herrmann et al,[24] the film contrast correlation actually sampled the transverse curvature of the refractive index distribution along the averaged ray path within the turbulent wake volume, and hence tended to be heavily biased toward the smaller eddies as long as the density fluctuation associated with these eddies remained strong enough to be seen by the schlieren system. Thus, the observed homogeneity and isotropy in density fluctuation could well be restricted only to the smaller scale structure of the wake turbulence, as one would expect.

In the case of hypersonic wakes, the temperature excess $\Delta T \equiv \overline{T} - T_\infty$ is expected to remain greater than or comparable to the ambient air temperature T_∞ for a very long distance behind the object. As an example, in the case of quasi-equilibrium flow behind a hypersonic sphere of 22,000 ft/sec velocity,[21] the mean wake temperature at a thousand sphere radii behind the sphere ($x/r_n = 1{,}000$) is expected to be approximately 1,500°K, so that the temperature excess would be 4 or 5 times the ambient air temperature (depending on altitude). In the case of non-equilibrium flow (frozen dissociation), the temperature excess would be smaller, but it generally would still take a distance of a few thousand sphere radii for ΔT to subside to values smaller than T_∞.

Since the gas density ρ is roughly inversely proportional to the gas temperature (for constant-pressure wake flow) while the kinematic viscosity ν is roughly proportional to T^2, the condition of small fluctuation amplitudes for T, ρ, and ν does not appear to be an easy condition to satisfy except at the "very-far-wake". On the other hand, in the very-far-wake where these fluctuation amplitudes become small, one may very well run into the difficulty of transition from ambipolar-diffusion to free-diffusion for the electrons as n_e decreases [equations (15a) and (15b)]. When such transition occurs, the electron diffusivity D becomes a rapidly varying function of the local electron density $n_e(\underset{\rightarrow}{r})$.

From the preceding discussion, it is quite clear that existing turbulent scalar mixing theories may find very limited range of applicable conditions in hypersonic wake plasmas on account of the many conflicting restrictions. Nevertheless, in the event that ambipolar-diffusion does prevail in a recombination-controlled far-wake (or, alternatively, in a very-far-wake where the dielectric constant fluctuation is dominated by molecular ion density fluctuations[11]), then one shall not be too surprised to find a universal equilibrium range of plasma dielectric constant fluctuation with a spectral shape identical to that observed in low speed turbulence. As illustrated in Fig. 4, the shape of such a spectrum for any scalar quantity with nearly unity Prandtl number (i.e., $\nu/D \simeq 1$, such as diffusion of heat in perfect gases under the condition of small temperature fluctuation amplitudes or ambipolar-diffusion of electrons) is indeed very similar to the turbulence energy spectrum, as Oboukhov,[6] Corrsin,[7] and Batchelor[8] all predicted. (See Section III above, and compare Figs. 2 and 4.)

If one divides the scalar fluctuation spectrum shown in Fig. 4 for Pr = 0.7 by the square of the wavenumber q in accordance with equation (13), one then obtains the universal equilibrium form of the scattering spectrum $F(\underset{\rightarrow}{r},q)$. If the spectral separation between the energy-containing wavenumber q_E and the Kolmogorov wavenumber q_K (which is roughly the same as the Oboukhov-Corrsin wave-number q_{oc} in this case) is sufficiently large, such scattering spectrum would be characterized by a power-law segment $F \propto q^{-11/3}$ corresponding to the inertial-convection subrange ($\Gamma \propto q^{-5/3}$) of the scalar fluctuation spectrum up to a wavenumber of approximately $0.2q_K$, and dissipative region beyond $q/q_K \approx 0.2$ where the spectrum falls off rapidly with increasing value of q/q_K at an ever accelerating pace. As indicated symbolically in Fig. 1, the existence of the inertial subrange in the energy spectrum is conditioned on a sufficiently high value of the turbulence Reynolds number (in comparison with unity), and so would be the inertial-convection subrange in the scalar fluctuation spectrum and the $q^{-11/3}$ segment of the scattering spectrum.

The preceding discussion on the spectral form of $F(\underset{\rightarrow}{r},q)$ in the universal equilibrium range applies only to a limited spatial region about the point $\underset{\rightarrow}{r}$ within the wake. Since the radar cross-section $\sigma(\theta,\lambda_o)$ as defined in equation (1) is given by an integral over the scattering volume V, the wavelength-dependence (or frequency-dependence) of the radar cross-section cannot be used directly to infer the spectral form of $F(\underset{\rightarrow}{r},q)$ unless the spatial variations of both $\overline{\Delta\varepsilon^2(\underset{\rightarrow}{r})}$ and $F(\underset{\rightarrow}{r},q)$ happened to be negligibly small ove the scattering volume.

VI. ESTIMATE OF THE KOLMOGOROV WAVENUMBER

Within the universal equilibrium range discussed in the preceding section, the form of the scattering spectrum $F(\underset{\rightarrow}{r},q)$ depends only on the normalized wavenumber q/q_K. It also follows that the range of radar wavelengths over which the scattering intensity would experience the effect of diffusive cutoff to the scattering spectrum is completely fixed by the averaged value of the Kolmogorov or Oboukhov-Corrsin wavenumber $q_K = (\mathcal{E}/\nu^3)^{1/4} \simeq (\mathcal{E}/D^3)^{1/4}$ within the scattering volume of interest at any given scattering angle θ (assuming that the spatial variation of F is sufficiently mild throughout the scattering volume).

In classical turbulence theory, the local rate of dissipation of turbulence kinetic energy $\mathcal{E}$ is assumed to be a given quantity characteristic of the macroscopic properties of the turbulence field. It is also a directly measurable quantity in low-speed turbulence experiments.[16] In the case of hypersonic wake flow, the dissipation rate $\mathcal{E}$ has never been directly measured, and has only been crudely estimated on occasion. One such estimate, for the case of hypersonic spheres, was reported in Ref. 26. This estimate, which was based on a simple extrapolation of low-speed turbulence data and on experimentally observed wake growth law of hypersonic spheres in ballistic ranges, gave for the dissipation rate in the far wake

$$\varepsilon \simeq 0.06(x/r_n)^{-2} U_\infty^3 / r_n \qquad (16)$$

where x is the distance behind the sphere along the wake axis, r_n is the radius of the sphere, and U_∞ is the velocity of the sphere relative to the ambient air. It is interesting to note that the dissipation rate so estimated turned out to be not very different from that measured by Gibson et al[16] in a low speed wake behind a sphere in a water tunnel, which gave, for the limited range of value of x/r_n covered by this latter experiment,

$$\varepsilon \simeq 0.27(x/r_n)^{-2.35} U^3 / r_n \; . \qquad (17)$$

By substituting (16) into equation (3), and defining a "flight Reynolds number" for the sphere $Re_\infty = U_\infty r_n / \nu_\infty$, one obtains the following estimate for the Kolmogorov wavenumber in the wake of a hypersonic sphere,

$$q_K \simeq 0.5(x/r_n)^{-\frac{1}{2}} (\nu_\infty/\nu)^{\frac{3}{4}} r_n^{-1} Re^{\frac{3}{4}} \qquad (18)$$

In the hypersonic wake, the kinematic viscosity ν varies roughly with the square of the gas temperature T, so that the above estimate of the Kolmogorov wavenumber is meaningful only if the temperature fluctuation amplitude is indeed small in comparison with the local mean temperature $\overline{T}$ as discussed earlier, and in such case, one may let $(\nu_\infty/\nu)^{\frac{3}{4}} = (T_\infty/\overline{T})^{\frac{3}{2}}$. The mean gas temperature in a reacting turbulent flow field is generally a very difficult quantity to calculate. It depends on the turbulent transport and mixing rates as well as on the chemical reaction model for the turbulent flow field. At any rate, such mean temperature, and subsequently the Kolmogorov wavenumber, have been calculated according to the reacting turbulent flow model proposed in Ref. 26 for some typical hypersonic flight conditions. The result is reproduced here in Fig. 5. It is interesting to note that the Kolmogorov wavenumber so calculated (dotted curves) turned out to be a somewhat wavy function of distance behind the sphere instead of a monotonically decreasing function. This was due to the competing effect between the rate of decrease of the estimated turbulence energy dissipation rate, and the rate of decrease of kinematic viscosity accompanying the continuously decreasing mean wake temperature.

Ignoring the detailed variation of the dotted curves with distance, the results illustrated in Fig. 5 indicate that the Kolmogorov wavenumber in the far wake of the sphere is of the order of $5\times10^{-3}\ r_n^{-1}\ Re_\infty^{3/4}$. Thus, for a sphere of 1 ft radius (r_n = 30.5 cm) at 22,000 ft/sec velocity (∿7 km/sec) and 150,000 ft altitude (∿50 km), one obtains $q_K \simeq 0.9\ cm^{-1}$. For radar back scattering, $\theta = \pi$, and $q = (4\pi/\lambda_o)\sin(\pi/2) = 4\pi/\lambda_o$. Since the dissipative range of the scalar spectrum begins at about $q/q_K = 0.2$ (see Figs. 2 and 4), one may conclude that diffusive cutoff to the scattering spectrum will be felt at radar wavelength λ_o shorter than about 50 cm according to this estimate.

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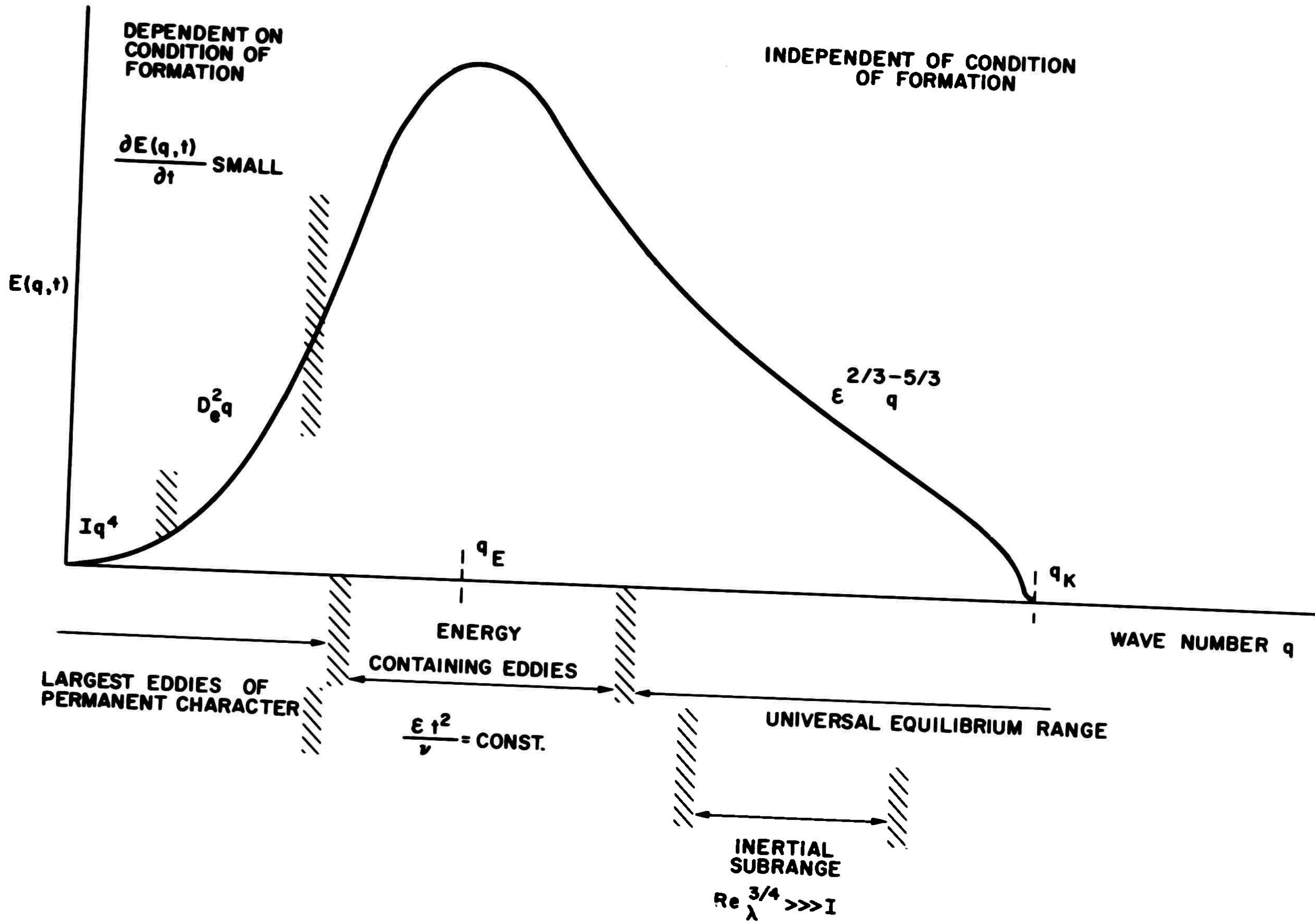

FIG. I FORM OF E (q,t) IN VARIOUS WAVENUMBER RANGES (PARTIALLY REPRODUCED FROM FIG. 3-11 OF REF. 13)

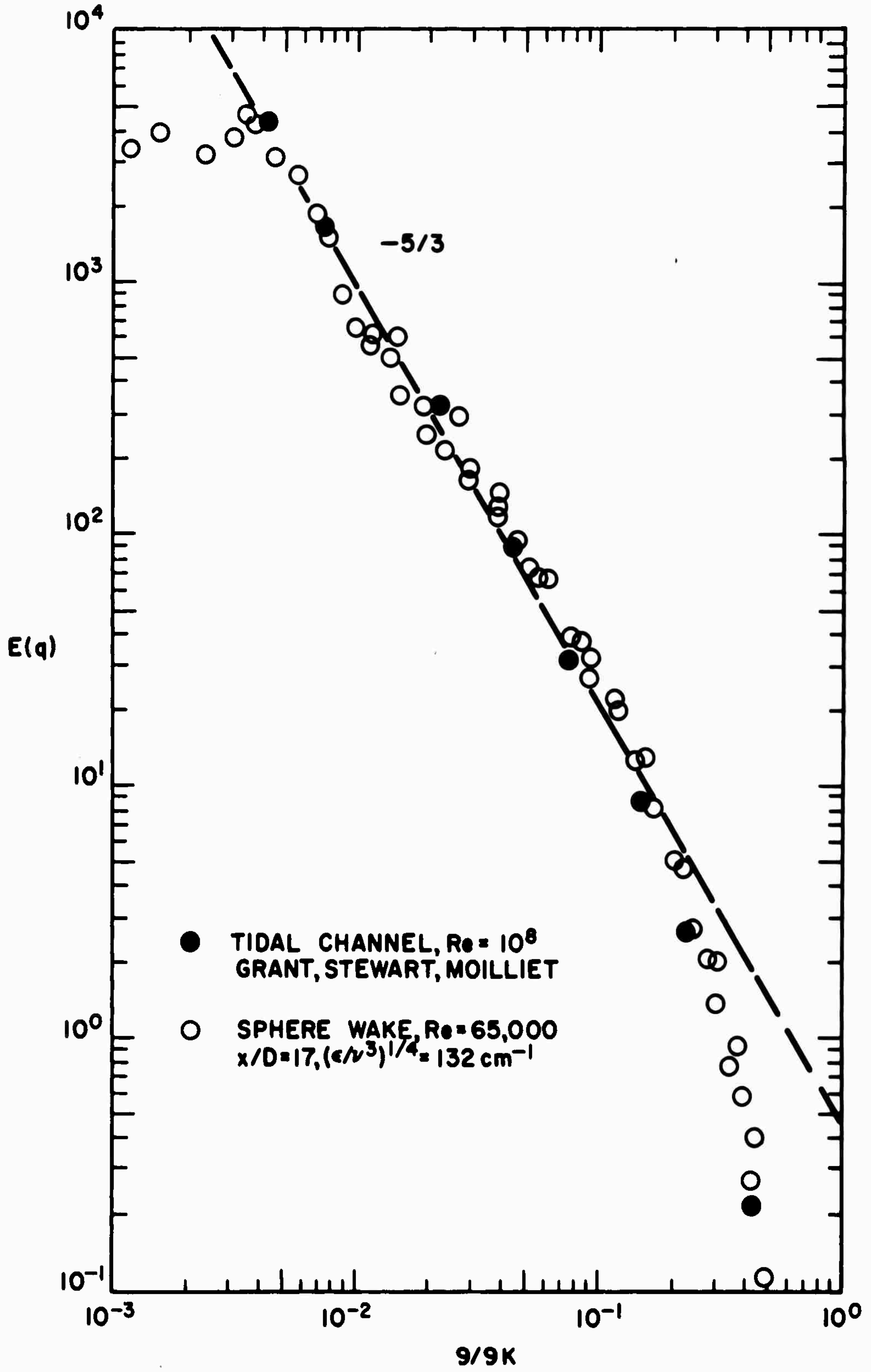

FIG. 2 EXPERIMENTALLY OBSERVED TURBULENCE ENERGY SPECTRUM IN UNIVERSAL EQUILIBRIUM RANGE. (REPRODUCED FROM FIG. 10 OF REF. 16)

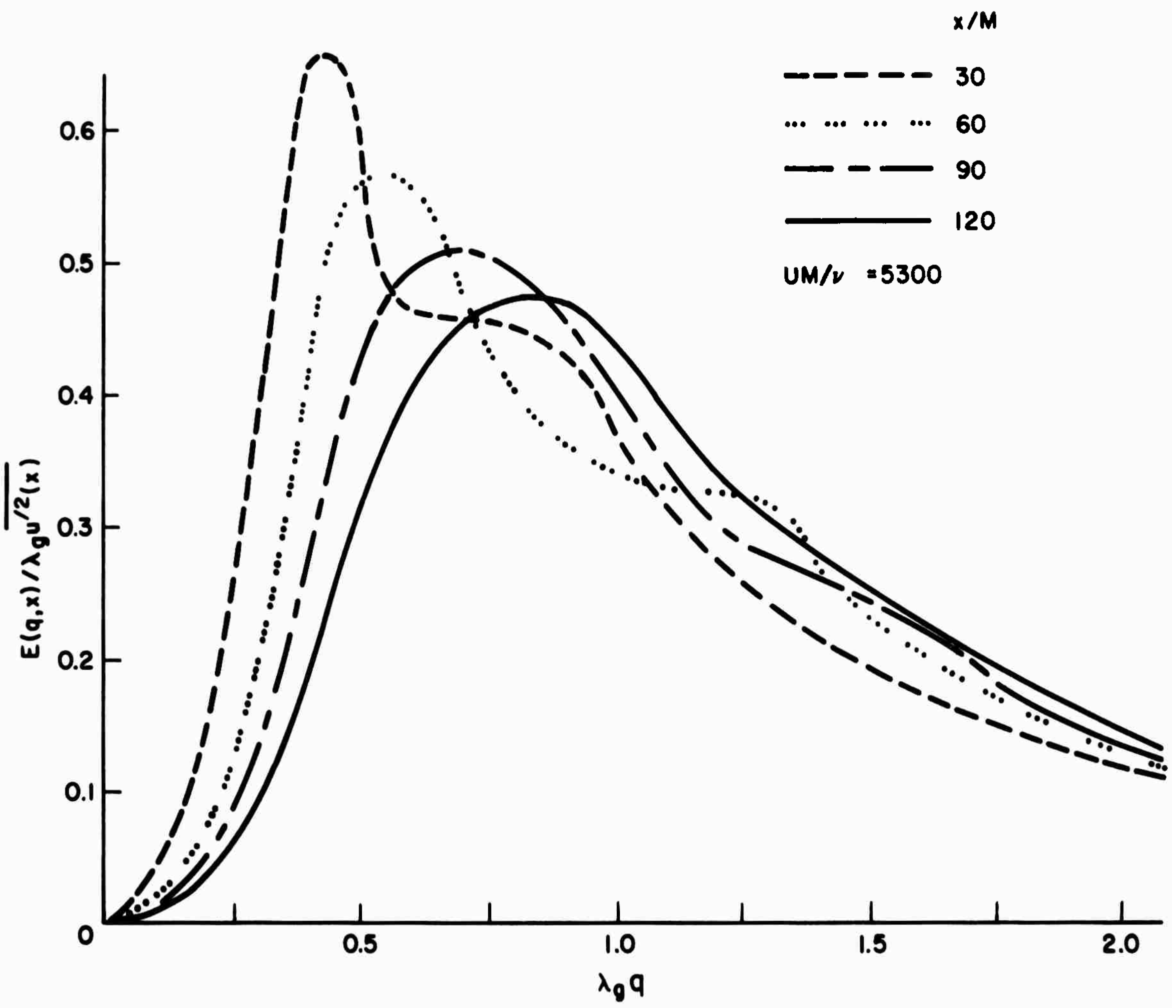

FIG. 3 ENERGY SPECTRUM AT DIFFERENT STAGES OF DECAY OBSERVED IN GRID-TURBULENCE EXPERIMENT OF STEWARD AND TOWNSEND. x/M IS THE DISTANCE DOWNSTREAM OF THE GRID IN UNITS OF THE GRID SPACING M. (REPRODUCED FROM FIG. 7.6 OF REF. 12.)

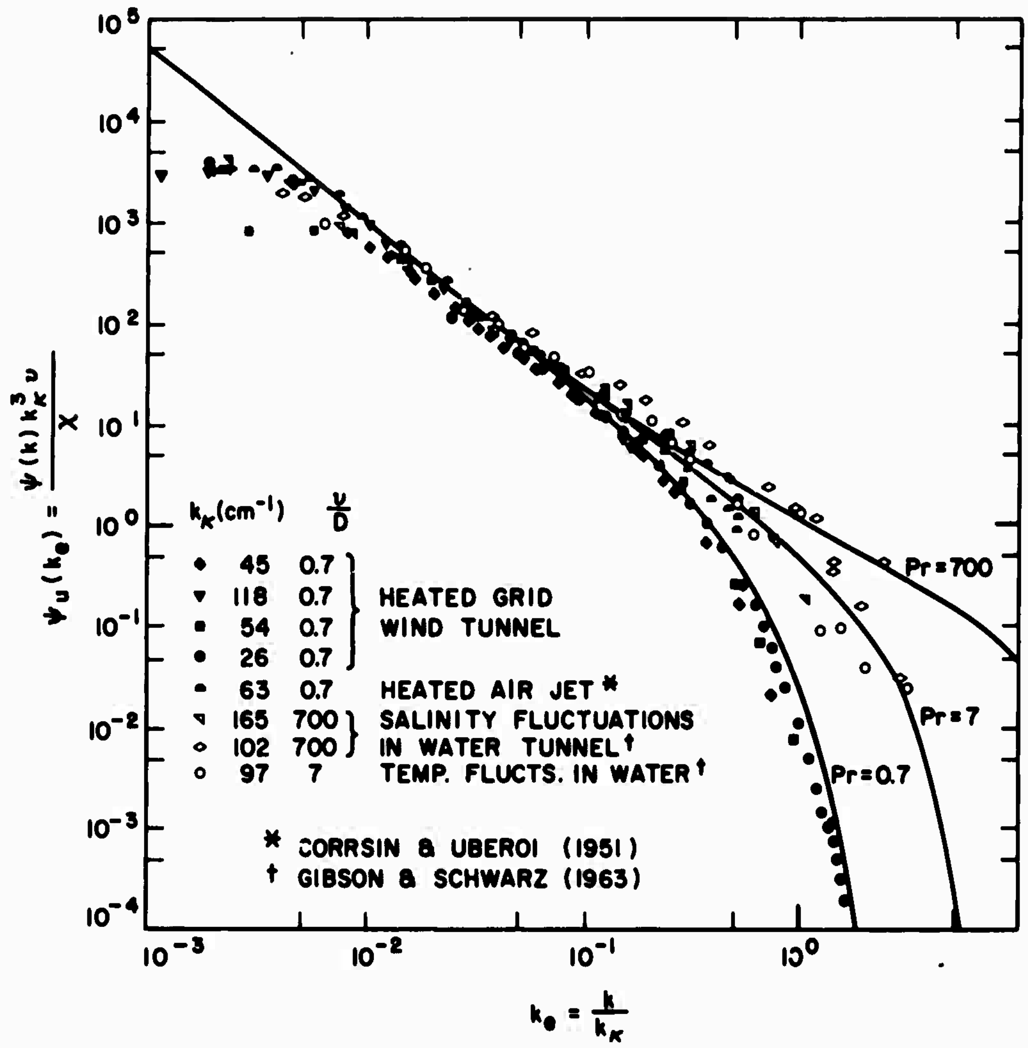

FIG. 4 COMPARISON OF SCALAR FLUCTUATION SPECTRA IN UNIVERSAL EQUILIBRIUM RANGE OBSERVED IN WIND TUNNEL AND IN WATER TUNNEL FOR SCALARS OF DIFFERENT PRANDTL NUMBER Pr ≡ ν/D. (REPRODUCED FROM FIG. 2 OF REF. 25).

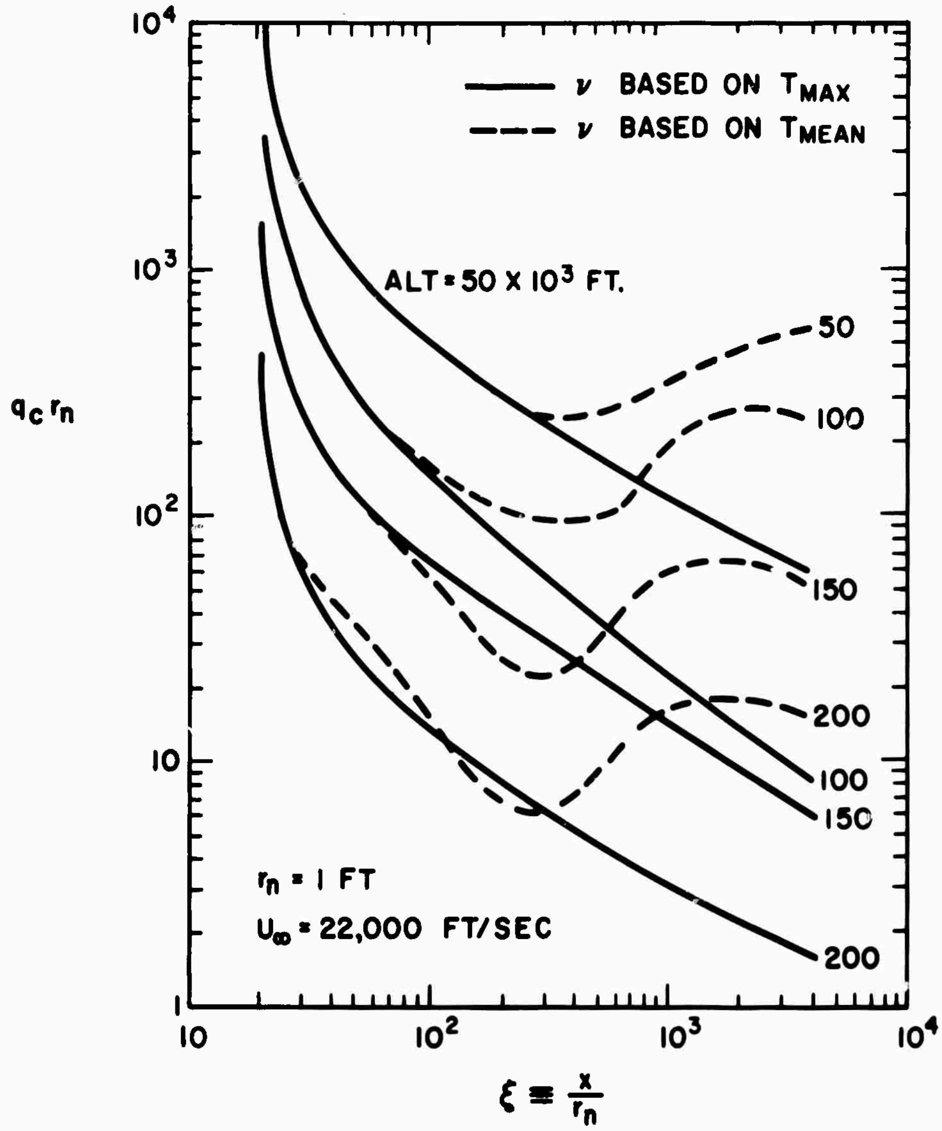

FIG. 5. ESTIMATED KOLMOGOROV WAVENUMBER IN THE HYPERSONIC WAKE OF A SPHERE OF 1 FT. RADIUS AS A FUNCTION OF ALTITUDE AND NORMALIZED AXIAL DISTANCE. (REPRODUCED FROM FIG. 7 OF REF. 26.)

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13. ABSTRACT

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