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by
J. C. Rotta

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SECOND ORDER BOUNDARY LAYER THEORY FOR TWO-DIMENSIONAL AND AXIALLY SYMMETRIC FLOW

GRENZSCHICHTTHEORIE ZWEITER ORDNUNG FÜR EBENE UND ACHSENSYMMETRISCHE HYPERSCHALLSTRÖMUNG

by

J. C. Rotta

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Translated by

Barbara Dickinson

Translation edited by

K. G. Winter

TRANSLATION AUTHOR'S SUMMARY

It is shown that there is an interaction between pressure gradients normal and parallel to the surface in supersonic flows and that the effect of longitudinal curvature increases strongly with Mach number. The second order boundary layer equations and the matching conditions with the inviscid flow are discussed. As well as longitudinal curvature, transverse curvature and variations in stagnation pressure and enthalpy, are considered as second order effects. Integral relationships of the boundary layer equations (momentum, energy equation, etc.) are given. The effects are demonstrated by a numerical example of a laminar boundary layer profile and possibilities for an approximate solution of the equations are discussed.

CONTENTS

1 INTRODUCTION

Methods of calculation for boundary layers have attained a high degree of perfection. The main questions still unsolved are those in which simple boundary layer theory is not sufficient.

Prandtl's boundary layer theory is based on the assumption of constant pressure across the boundary layer. In the case of boundary layers on curved surfaces this assumption is often not justified. In recent times attempts have been made to determine the curvature effect using second order boundary layer theory (see, for example, Murphy[1], Yen and Toba[2], Hayasi[3], van Dyke[4,5] and Massey and Clayton[6]). Most of the work is limited to the case of incompressible flow. It is shown that significant effects may be expected when the thickness of the boundary layer is comparable with the radius of curvature of the surface. Much stronger effects arise, however, in flows with high Mach numbers.

A feature of compressible flow is the fact that pressure gradients normal to the surface produced by centrifugal forces result in variations of the density distribution. These directly affect the force equilibrium and thus influence the development of the boundary layer. In the case of turbulent boundary layers the turbulence mechanism is also influenced in a complicated manner so that, even with moderate Mach numbers, strong induced effects are observed, as has been shown by boundary layer studies on a waisted body of revolution which were carried out jointly by R.A.E. Bedford and AVA Göttingen[7].

Further flow problems exist which are outside the scope of boundary layer theory. These occur primarily in flows with separation and near the trailing edge of bodies (see the work of Küchemann[11]). For this type of problem a theory of order higher than second is necessary which leads to differential equations of the elliptic type. For the second-order boundary layer theory used in the present investigation, the equations form a problem which can be solved by 'marching' downstream as in the case of the simple boundary layer theory. The details are limited to laminar boundary layers, although many are also valid for turbulent boundary layers.

2 THE EFFECT OF SURFACE CURVATURE

In order to emphasize the significance of the surface curvature, Fig.1 recalls the fact that for supersonic flow around a body there is a causal relationship between the pressure gradients normal and parallel to the wall. The pressure variations are propagated along Mach lines. According to simple theory the following relationship exists between the pressure gradients

$$\frac{\partial p}{\partial y} = -\frac{\partial p}{\partial s}\sqrt{Ma^2 - 1} \quad . \qquad (1)$$

For high Mach numbers Ma this is reduced to

$$\frac{\partial p}{\partial y} \approx -\frac{\partial p}{\partial s} Ma \quad . \qquad (2)$$

The pressure gradient normal to the surface increases proportionately to the pressure gradient in the flow direction, and increases strongly with Mach number reaching a multiple of $\partial p/\partial s$.

It is also shown that the relative pressure change in the boundary layer can be quite considerable. The approximation for the pressure gradient normal to the surface gives

$$\frac{\partial p}{\partial y} = \frac{\rho u^2}{R} \quad . \qquad (3)$$

In the case of an ideal gas, the integration of equation (3) gives, for the ratio of the local pressure to the wall pressure,

$$\ln \frac{p}{p_w} = \gamma \int_0^y \frac{Ma^2}{R} dy \qquad (4)$$

where γ is the ratio of specific heats. It can be estimated from this equation that, for example, for $Ma = 10$ the pressure at the boundary of a flow layer of thickness only 1% of R_w is approximately 4 times the surface pressure.

The distribution of pressure in the boundary layer on a convex surface is shown in Fig.2. Since the actual velocity u is different from the value u^* of the inviscid flow, different pressures are obtained for the two flows. For convex surfaces the actual pressure is greater than for inviscid flow. The difference can be considerable. Consequently a boundary layer theory, which neglects the influence of curvature can give, under some circumstances, erroneous results at high Mach numbers. In the following the longitudinal curvature, the transverse curvature, the variation of the total pressure (entropy) and total enthalpy are considered as second-order effects. Slip and temperature jump at the surface are not considered, although inclusion of these effects does not present any fundamental difficulty.

3 BOUNDARY LAYER EQUATIONS

The general formulation of the second order boundary layer theory corresponds exactly to Prandtl's concept (see Schlichting[8]): friction stresses and heat conduction are only considered in a relatively thin boundary layer near the surface of the body. Inside this boundary layer certain terms of the Navier-Stokes differential equations and the energy equation may be neglected. In the remaining flow field friction stresses and heat conduction are neglected. We shall discuss the boundary layer equations for steady axial flow past a body of revolution.

An orthogonal axis system s, y is introduced on a curved, axisymmetric surface (Fig.3). The generator of the surface is given by $r_w(x)$. The geometrical relationships are specified on the figure. The boundary layer simplifications derive from the assumptions

$$v \ll u, \quad \partial/\partial s \ll \partial/\partial y \qquad . \tag{5}$$

Deviating from the usual boundary layer theory, we do not retain the restrictions

$$1/R_w \ll \partial/\partial s, \quad \delta \ll r_w \tag{6}$$

where δ is the boundary layer thickness. The boundary layer equations for this case have already been treated by van Dyke[5]. We use the continuity, momentum and energy equations in the following form:

Continuity:

$$R_w \frac{\partial(\rho u r)}{\partial s} + \frac{\partial(\rho v r R)}{\partial y} = 0 \qquad . \tag{7}$$

Equations of motion:
meridional:

$$R R_w \frac{\partial(\rho u^2 r)}{\partial s} + \frac{\partial(\rho u v r R^2)}{\partial y} = - R R_w r \frac{\partial p}{\partial s} + \frac{\partial(\tau r R^2)}{\partial y} \tag{8}$$

normal to the surface

$$- \frac{\rho u^2}{R} = - \frac{\partial p}{\partial y} \qquad . \tag{9}$$

Energy:

$$R_w \frac{\partial(\rho u h_o r)}{\partial s} + \frac{\partial(\rho v h_o r R)}{\partial y} = \frac{\partial(q r R)}{\partial y} + \frac{\partial(\tau u r R)}{\partial y} \quad . \tag{10}$$

The total enthalpy h_o is defined by

$$h_o = h + \frac{u^2}{2} \quad . \tag{11}$$

The shear stress in the laminar case is

$$\tau = \mu\left(\frac{\partial u}{\partial y} - \frac{u}{R}\right) \tag{12}$$

and the heat flux is calculated from

$$q = \frac{\lambda}{C_p}\frac{\partial h}{\partial y} \tag{13}$$

where μ is the viscosity, λ the thermal conductivity, C_p the specific heat at constant pressure and h the enthalpy.

Apart from the occurrence of radii of curvature, the equations differ from those of simple boundary layer theory mainly by the addition of the equation for the change of pressure normal to the surface, equation (9). This raises by one the order of the differential equation system. The mathematical consequences for the behaviour of the equation system have not yet been completely investigated, but it appears that the fundamental properties remain unchanged. This suggests the use of similar methods of solution as in usual boundary layer theory.

Up to now no restrictions have been made on the behaviour of the gas, apart from the fact that it is taken as a continuum. At best the expression for q according to equation (13) is only valid in the case of thermodynamic and chemical equilibrium. The behaviour of the gas must be established in order to solve the system of equations and for the sake of simplicity, thermodynamic and chemical equilibrium will be assumed here. Pressure, density and enthalpy are then related by an equation of state of the usual form. The simplest case is to take the equation of state of an ideal gas; however, it is not necessary to be limited to this assumption. The addition of further differential equations to take into account real gas effects makes the equation system quite complicated but the quoted relationships remain valid.

4 BOUNDARY CONDITIONS

Under the assumption of an impermeable surface, the velocity components must fulfil the conditions $u_w = 0$ and $v_w = 0$ for $y = 0$. Also alternatively $h = h_w$ or $q = q_w$ can be given*.

The formulation of the boundary conditions is very simple with first order boundary layer theory. Since the pressure is constant through the boundary layer, the pressure distribution of the potential flow at the wall is used as a boundary condition, which is independent of y. In the case of the second-order theory the matching to the inviscid flow is more complicated since the properties are dependent upon y. We define a thickness δ_r such that friction and heat conduction terms need be considered only for $0 \leq y \leq \delta_r$ and are negligible for $y \geq \delta_r$ (Fig.4). Further, we define a boundary layer thickness $\delta > \delta_r$ so that for $0 \leq y \leq \delta$ the boundary layer approximations are permissible. For $y \geq d$ the complete differential equations of the inviscid flow are used. Thus, in the overlapping region $\delta_r \leq y \leq \delta$, boundary layer approximations are permissible and the friction forces and heat conduction are negligible. The boundary conditions for $\delta_r \leq y \leq \delta$ then read:

$$u = u^*, \quad v = v^*, \quad h = h^*, \quad p = p^* \quad . \tag{14}$$

The requirement $\delta_r \leq \delta$ defines the limit for the applicability of the stated theory, i.e. the viscosity must be low enough or the Reynolds number high enough for this condition to be fulfilled.

The inviscid flow may be considered as continuing up to the surface of the body. In the region $0 \leq y \leq \delta$ it obeys the boundary layer equations in which the friction and heat conduction terms are omitted. It is advantageous to describe the inviscid flow by the tangential velocity u_w^* or the pressure p_w^* or h_w^* at the surface. These conditions, however, are not adequate to specify the flow field. The equations of inviscid flow indicate that total pressure p_0^* and total enthalpy h_0^* are constant along streamlines. Thus p_0^* and h_0^* must be known as functions of the stream function. Only in special cases (e.g. irrotational flow) are these parameters constant.

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* Taking into account slip and temperature jump at the surface, u_w and v_w are given from a linear relationship with the gradients $(\partial u/\partial y)_w$, $(\partial h/\partial y)_w$ and $(\partial h/\partial s)_w$ (see Wuest[9]).

It may be further noted that the requirement $v^* = v$ for $\delta_r \leq y \leq \delta$ is not compatible with the condition $v_w^* = 0$ for $y = 0$. The values of v_w^* can only be determined in the case of a known boundary layer. In many instances the displacement effect of the boundary layer on the external flow is neglected. Possibilities exist of introducing the displacement effect into the calculation, i.e. in an iterative correction or in the introduction of pressure variations in the boundary layer equations (interaction theory); this will not be discussed further here.

3 INTEGRAL EQUATIONS OF THE BOUNDARY LAYER THEORY

Among the mathematical methods for the solution of boundary layer equations, approximation methods, which operate with integral theorems for momentum, energy etc., are of particular interest. Such methods will now be discussed.

The principle of the integral approach is that we consider the equilibrium of the momentum and energy losses produced by friction forces and heat conduction with respect to the inviscid flow considered as extended to the wall. In the derivation of the integral conditions we assume the inviscid flow to be known and require that in the region $0 \leq y \leq \delta$, as already mentioned, the boundary layer equations are adequate without the friction and heat conduction terms. The formal derivation is mathematically simple and need not be explained further here. It may be noted that in the boundary layer not only velocity, density and enthalpy but also static pressure values are assumed which differ from those of the inviscid flow. After rearranging the terms, the equations can be brought into forms which are largely similar to those of usual boundary layer theory, and which reduce to these if the radius of curvature relative to the boundary layer thickness tends to infinity.

By multiplication of the equation of motion by u^m, a differential equation for a loss thickness δ_{2+m} can be established for each value of m. For $m = 0$ this gives the momentum equation in the usual way; for $m = 1$ we have the integral equation for mechanical energy, while for higher values of m there is no obvious physical explanation.

The integral form of the boundary layer equation is:

$$\left.\begin{aligned} &\frac{d\delta_{2+m}}{ds} + \delta_{2+m}\left\{\left[2 + m + (1+m)\,\frac{\delta_1 - \delta_{m,s}}{\delta_{2+m}}\right] \times \left(\frac{du/ds}{u}\right)^*_w + \left(\frac{d\rho/ds}{\rho}\right)^*_w + \frac{dr_w/ds}{r_w}\right\} - \\ &\quad - (1+m)\left[\delta_{2+m,r}\,\frac{dr_w/ds}{r_w} + \delta_{2+m,R}\,\frac{dR_w/ds}{R_w}\right] = \\ &\quad = (1+m)\left[\left(\frac{u_w}{u^*_w}\right)^m \frac{\tau_w}{(\rho\ u^2)^*_w} + m\int_0^\delta \left(\frac{u_w}{u^*_w}\right)^{m-1} \times \frac{\tau}{(\rho\, u^2)^*_w}\,\frac{\partial(u^*R)/\partial y}{u^*_w\,R_w}\left(\frac{R}{R_w}\right)^{1+m}\frac{r}{r_w}\,dy\right] \end{aligned}\right\}. \qquad \ldots (15)$$

This contains the displacement thickness:

$$\delta_1 = \int_0^\delta \frac{(\rho\ u)^* - \rho\ u}{(\rho\ u)^*_w}\,\frac{r}{r_w}\,dy \qquad (16)$$

and the remaining boundary layer loss thicknesses are defined as follows:

$$\left.\begin{aligned} \delta_{2+m} = \int_0^\delta &\left\{\frac{\rho\,u}{(\rho\ u)^*_w}\left[1 - \left(\frac{u\,R}{u^*_w\,R_w}\right)^{1+m}\right] - \frac{(\rho\ u)^*}{(\rho\ u)^*_w}\left[1 - \left(\frac{u^*R}{u^*_w\,R_w}\right)^{1+m}\right] + \right.\\ &\left. + (1+m)\left(\frac{u}{u^*_w}\right)^m \frac{p^* - p}{(\rho\ u^2)^*_w}\left(\frac{R}{R_w}\right)^{1+m}\right\}\frac{r}{r_w}\,dy \end{aligned}\right\} \qquad \ldots (17)$$

$$\left.\begin{aligned} \delta_{m,s} = \int_0^\delta &\left\{\frac{u^{*m} - u^m}{u^{*m}_w} + m\left(\frac{u}{u^*_w}\right)^{m-1} \frac{\partial u/\partial s}{du^*_w/ds}\,\frac{p^* - p}{(\rho\ u^2)^*_w} - \right.\\ &\left. - \left[1 + \frac{\partial p^*/\partial s}{(\rho\ u\ du/ds)^*_w}\right]\frac{u^{*m} - u^m}{u^{*m}_w}\right\}\left(\frac{R}{R_w}\right)^{1+m}\frac{r}{r_w}\,dy \end{aligned}\right\} \qquad \ldots (18)$$

$$\delta_{2+m,r} = \int_0^\delta \left(\frac{u}{u^*_w}\right)^m \frac{p^* - p}{(\rho\ u^2)^*_w}\left(\frac{R}{R_w}\right)^{2+m} dy \qquad (19)$$

$$\delta_{2+m,R} = \int_0^\delta \left[\frac{(\rho\ u^{2+m})^* - \rho\ u^{2+m}}{(\rho\ u^{2+m})^*_w} + (1+m)\left(\frac{u}{u^*_w}\right)^m \frac{p^* - p}{(\rho\ u^2)^*_w}\right] \times \left(\frac{R}{R_w} - 1\right)\left(\frac{R}{R_w}\right)^m \frac{r}{r_w}\,dy\,. \qquad (20)$$

The integral parameters are distinguished from those of simple boundary layer theory (see Walz[10]) only by the loss thicknesses $\delta_{2+m,r}$ and $\delta_{2+m,R}$. The remaining second order effects are also included in the definitions of the loss thickness. Due to the curvature normal to the streamlines, the radii ratio r/r_w appear in the integrands with the exception of $\delta_{2+m,r}$. In the case of δ_{2+m} the first term corresponds to that of simple theory. In the second term only properties of the inviscid flow occur. This term disappears for isentropic flow for which u^*R = const. In the third term the difference between the static pressures appears. The expression for $\delta_{m,s}$, in the integrands of which, partial differential coefficients also arise, is written with δ_1 in a common term like that of usual boundary layer theory. For $m = 0$, $\delta_{m,s} = 0$.

On the right side of the integral equation are the friction terms. The first term contains the wall shear stress and, since $u_w = 0$, has a finite value only when $m = 0$ (momentum equation); in this case the second term also disappears. The latter corresponds to the dissipation of mechanical energy for $m = 1$.

The integral equation of total energy has the form:

$$\frac{1}{r_w}\frac{d}{ds}\left[(\rho\, u\, h_o)_w^*\, r_w\, \delta h\right] + (\rho\, u)_w^*\, \frac{dh_{ow}^*}{ds}\,\delta_1 = q_w \tag{21}$$

where q_w is the heat flow per unit area from the body*. For the enthalpy loss thickness

$$\delta h = \int_0^{\delta}\left[\frac{\rho u}{(\rho\, u)_w^*}\left(1 - \frac{h_o}{h_{ow}^*}\right) - \frac{(\rho\, u)^*}{(\rho\, u)_w^*}\left(1 - \frac{h_o^*}{h_{ow}^*}\right)\right]\frac{r}{r_w}\, dy \tag{22}$$

the first term in the square brackets again corresponds to simple boundary layer theory. The second term contains only properties of the inviscid flow. It disappears in the case of isoenergetic flow, since then h^* = const.

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* If we relinquish the no-slip condition at the surface then the term $+ \tau_w u_w$ is added to the right hand side of equation (21) and the first term of the right hand side of equation (15) has a finite value for $m > 0$. The equations are then valid even in the case of a temperature jump at the surface.

The loss thicknesses can be calculated for given velocity and enthalpy profiles. As an example we will show calculated results for two-dimensional flow over convex, plane and concave surfaces. The Pohlhausen profile assumed is easily modified to match the inviscid flow

$$\frac{u}{u_w^*} = \frac{2(y/\delta) - 2(y/\delta)^3 + (y/\delta)^4}{1 + (y/\delta)(\delta/R_w)} \quad \text{for} \quad 0 \leq y \leq \delta \quad . \qquad (23)$$

The equations of state of an ideal gas are taken. The inviscid flow is taken as irrotational with h_o = const. The velocity profiles deviate only slightly from each other for a relative boundary layer thickness of $\delta/R_w = 0.01$ (Fig.5). On the other hand the static pressure at $Ma_w^* = 8$ undergoes considerable variation. The marked difference between actual pressure and the pressure of the inviscid flow at $y = 0$ is particularly noteworthy. This difference is shown in Fig.6 as a function of Mach number.

Fig.7 shows the integrands of the momentum loss; I_{21} is the momentum loss thickness of the velocity, I_{22} is the contribution of the static pressure. This fraction has a large value in the example shown.

Fig.8 shows the thickness ratio δ_1/δ_2 and the local coefficient of friction c_f (nondimensional wall shear stress $= 2\,\tau_w\delta_2/(\mu u)_w^*$) on a logarithmic scale as a function of the Mach number. With large Mach numbers, Ma_w^* appreciable variations due to curvature of the wall are shown.

This suggests that an approximate method on the basis of a single-parameter velocity profile should be sought, where the Pohlhausen polynomial equation can be used. A combination of the momentum ($m = 0$) and mechanical energy ($m = 1$) equations appears most promising.

Another important relationship which is often used in boundary layer calculation is the so-called condition of compatibility at the wall. In the case of second order boundary layer theory this reads

$$\mu_w\left(\frac{\partial^2 u}{\partial y^2}\right)_w = \frac{dp_w^*}{ds} + \frac{d}{ds}\int_0^\delta \frac{(\rho\, u^2)^* - \rho\, u^2}{R}\, dy - \frac{\tau_w}{r_w}\left[\sqrt{1-\left(\frac{dr_w}{ds}\right)^2} + \frac{r_w}{R_w}\right] - \tau_w\left(\frac{\partial\mu/\partial s}{\mu}\right)_w \quad . \qquad \ldots(24)$$

Here the differential coefficient of an integrated boundary layer parameter appears, so that, in the extension of the Pohlhausen method to second order boundary layer theory, a first order differential equation arises instead of an algebraic expression for the shape parameter. Thus in using the wall compatibility condition and dispensing with the equation for mechanical energy, a system of two simultaneous differential equations is given.

With reference to the temperature profile, the assumption that the same distribution of total enthalpy occurs in the boundary layer as in the inviscid flow may be an acceptable approximation in the case of insulated surfaces (i.e. h_o = constant for isoenergetic inviscid flow). In the case of heat transfer the integral condition for the energy conservation, equation (21), must be taken as third differential equation and a suitable expression for the enthalpy profile introduced.

6 CONCLUSION

The importance of second order boundary layer theory in hypersonic flow has been indicated and an example given. The longitudinal and transverse curvature of the surface and the variations of the static pressure and static enthalpy of the inviscid solution have been considered as second order effects. The effect of the longitudinal curvature has been specially considered. The mathematical problem has been formulated and possibilities of approximate calculation discussed. In order to have a basis of comparison for the calculated results, more accurate methods are necessary based either on a larger number of integral conditions and therefore making possible a larger variation of the boundary layer profile or on difference methods for the partial differential equations. The quoted integral conditions are also valid for turbulent boundary layers; however, further data on the velocity profile, friction coefficient and shear stress distribution and also heat transfer coefficients are necessary for their application.

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LIST OF ILLUSTRATIONS

Fig.1 The relationship between the pressure gradients $\partial p/\partial y$ and $\partial p/\partial s$

Machsche Linien — Mach lines

Fig.2 Velocity and pressure distributions in the boundary layer (diagrammatic) u^*, p^* are the values for inviscid flow

Fig.3 Coordinate systems for an axially symmetric surface with double curvature

Fig.4 Boundary conditions for boundary layer equations

u^*, v^*, h^*, p^* are values for inviscid flow

Vollständige ---- Strömung	complete differential equations of the inviscid flow
Grenzschicht ---- zulässig	permissible boundary layer simplification
Reibungskräfte ---- vernachlässigbar	friction forces and heat conduction negligible
Reibungskräfte ---- wesentlich	friction forces and heat conduction considerable

Fig.5 Velocity and pressure distributions for the Pohlhausen profile

$Ma_w^* = 8$

Konvex	convex
Konkav	concave
Pohlhausen Profil	Pohlhausen profile

LIST OF ILLUSTRATIONS (Contd)

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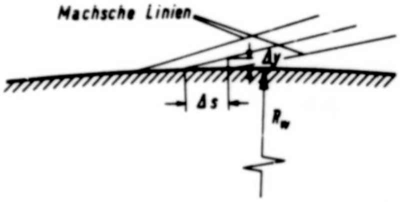


Fig.1

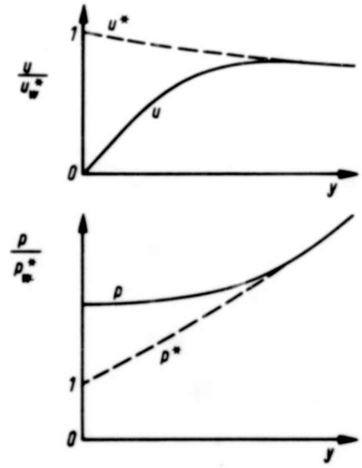


Fig.2

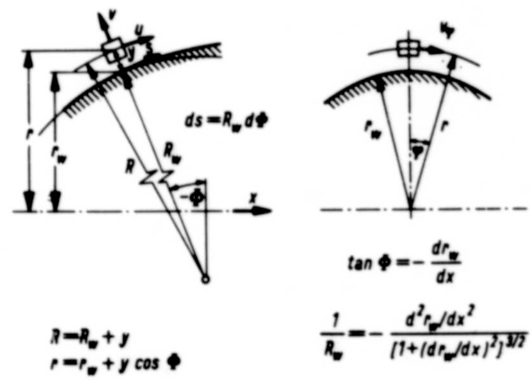
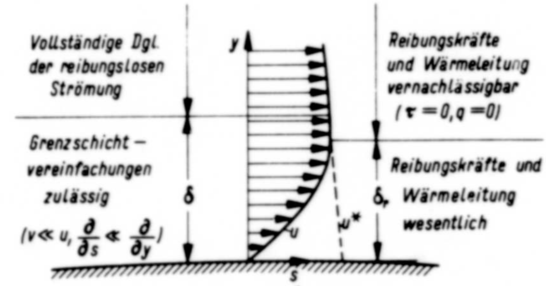


Fig.3



y=0	$\delta_r \le y \le \delta$
u=0	u=u*
v=0	v=v*
h=hw	h=h*
(q=qw)	p=p*

Fig.4

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Fig.5-8

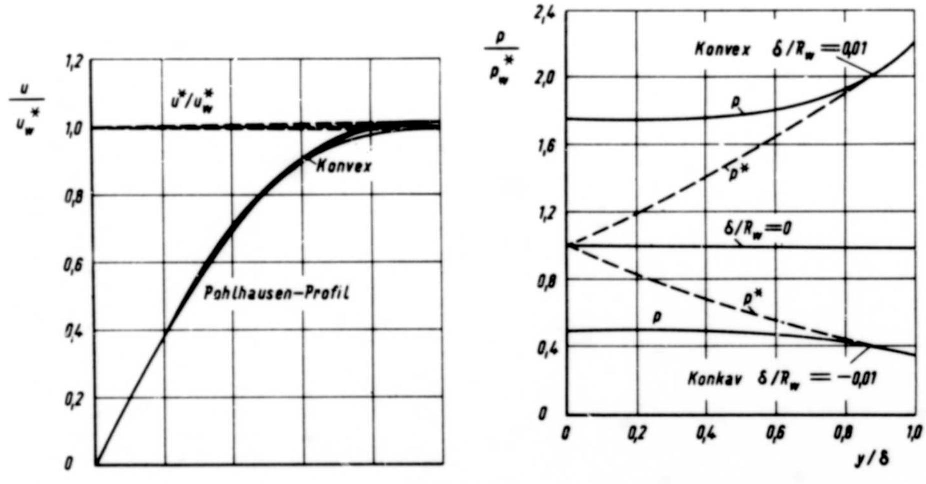

Fig.5

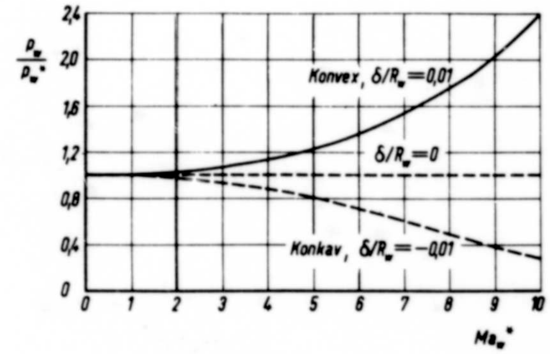

Fig.6

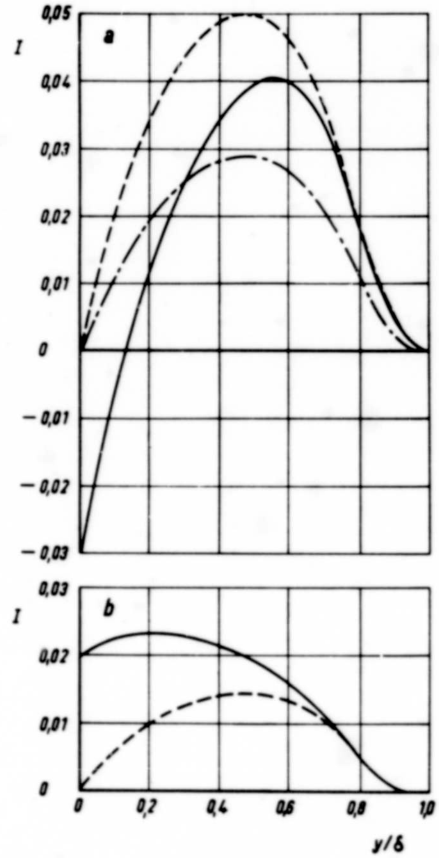

Fig.7

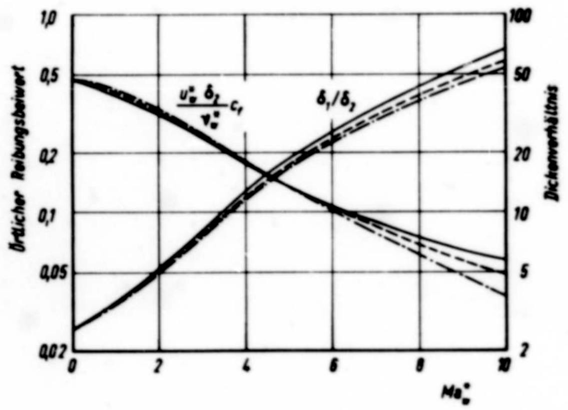

Fig.8