Technical Note 1969-17

Pointing Accuracy of Lincoln Laboratory 28-Foot Millimeter-Wave Antenna

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14 February 1969

Prepared under Electronic Systems Division Contract AF 19(628)-5167 by

Lincoln Laboratory
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Lexington, Massachusetts

The work reported in this document was performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology, with the support of the U.S. Air Force under Contract AF 19(628)-5167.

ABSTRACT

Three experiments have provided a rough evaluation of the pointing errors of the 28-foot paraboloid and its mount. One examined the errors introduced by the pedestal, including the azimuth bearing, the second evaluated the pointing of an optical telescope attached to the backup structure, and the third showed that the departure between the telescope boresight and the millimeter-wave boresight is not a large fraction of a beamwidth. The experiments yielded estimates of base tilt, structural sag, elevation-axis skew, and collimation error. When allowance is made analytically for these defects, static pointing of the optical telescope at night has RMS errors of about 0.01 deg on each axis; this quantity is one-seventh of a 35-GHz beamwidth.

Accepted for the Air Force
Franklin C. Hudson
Chief, Lincoln Laboratory Office

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POINTING ACCURACY OF LINCOLN LABORATORY 28-FOOT MILLIMETER-WAVE ANTENNA

I. INTRODUCTION

The 28-foot antenna on Building D has been used chiefly for lunar radar, along with some radio astronomy, but in the future it may be considered for use in space communications. For this application, or in radio astronomy that will be in the front rank in the coming years, pointing accuracy may be a limitation. Because the mount was initially inexpensive and has been in use for five years or more, its pointing capability has been in doubt.

A study of the antenna pointing was made in 1964 by J. R. Cogdell, but it was inconclusive; moreover, the pointing control and readout devices have since been greatly improved. During recent months, therefore, we have been evaluating the pointing control and sensing systems, to determine the present pointing performance of the antenna.

The antenna itself is well suited for astrometrical measurements in the millimeter range; its high gain and high angular resolution mark it as one of the best antennas in the non-Communist world. High angular resolution makes crucial the pointing control of the antenna, and it is here that problems arise. For first-class work, the fluctuations due to antenna pointing should be negligible. If we know the position within one-tenth beamwidth, then the fluctuations will be less than 3 percent. Specifically, since the 3-dB beamwidth is 4.2 minutes of arc (0.070 deg), we would like to know the antenna direction within a limit of ±0.007 deg. At present the pointing is certainly not known to one-tenth beamwidth, but with further analysis and some system improvement, that level of accuracy may be attainable.

This report comprises a description of the pointing controls, a statement of possible errors, descriptions of the experiments performed by the authors, evaluations of the elevation and azimuth errors, and suggestions for improving the pointing. One appendix gives the coordinates of the antenna and tells how to set the pointing indicators. Other appendixes analyze the consequences of defects of the mount, give a program for computing pointing data for stars, discuss the focusing adjustment of the antenna, and make some recommendations for improving the telescope.

II. POINTING OF ANTENNA

A. Pointing Systems

The antenna is on an elevation-over-azimuth mount. To protect the azimuth encoder from being hit by a cable clamp, azimuths between 49 and 92 deg are forbidden by stops. Pointing is accomplished by one of three methods:

(1) A slewing motor and gear train, which turn the antenna at a rate of approximately 60 deg per minute; this system is useful for gross angular displacements.

(2) An amplidyne, motor, and gear train allowing variable speeds from 0 to 6 deg per minute; this system is used both for fine positioning and for tracking.

(3) An entirely mechanical handwheel system, previously used for fine positioning but now used only for emergencies such as power failure.

The position of the antenna beam is monitored by three independent methods:

(1) A closed-circuit-television camera behind a telescope boresighted parallel to the antenna beam.

(2) Digital encoders directly measuring rotation of the azimuth table and elevation trunnions.

(3) Synchro systems measuring rotation about the azimuth and elevation axes. Until the summer of 1967, these were the only indicators of position. The data boxes of these two-speed synchros are separated from the azimuth table and the elevation trunnions by 3 and 6 meshes of gears, respectively; in addition, there are 2 meshes in each data box. This method of indication is therefore crude by comparison with the encoder method recently installed.

The telescope-television monitor allows the antenna to be positioned relative to some visible object on earth or in the heavens. Optical sensitivity of this monitor is sufficient to display blue third-magnitude stars; with the aid of a reference circle inscribed on a reticle, a practiced observer can estimate positions to ±0.002 deg.

The digital position monitor in each coordinate consists of an optical binary encoder with stator affixed to the mount, and rotor affixed directly to the elevation trunnions or to the azimuth table. Since the encoders avoid involvement with the gear trains and synchros previously used to measure the position, uncertainties in readout are much reduced.

The encoders give 17 bits, and the RMS accuracy claimed for them by the manufacturer is $\pm\frac{1}{2}$ of the least significant bit, namely ±0.00137 deg. The encoder reading is a binary fraction of 360 deg; the binary fraction is multiplied by 36,000 in an operation that ignores all bits after the twentieth, thereby introducing a truncation error that lies between zero and −0.0025 deg. Then the sixteen most significant bits are converted to binary-coded decimal, and the result is rounded off to the nearest hundredth of a degree, so there is a roundoff error as large as ±0.005 deg. The worst-case totals are +0.00637 deg and −0.00887 deg. These are about a tenth of a 35-GHz beamwidth.

Because of the many meshes of gears, the servo monitor system is subject to backlash and other problems; it is known that the servo monitors can disagree with the encoders by as much as 0.3 deg. At present the servo position monitor is used only for antenna pattern measurements and the position command system.

The antenna is positioned by first using the slew system for crude pointing and then using one of two modes that control the amplidyne system.

(1) The rate mode is used by an operator to control tracking. It uses a closed-loop velocity servo, with feedback from a tachometer. Approximate tracking rates are found by turning two wirewound potentiometers until the image of the tracked body is motionless on the telescope-television screen. A control stick mounted on the console is used for perturbing the rates manually to compensate for accumulated error.

Although the rate mode can be used for positioning as well as for tracking, it is not the proper mode to use for positioning, because leakage currents cause the antenna to drift from the desired position.

(2) The position command mode uses a closed-loop servo system that is governed by the two-speed-synchro data gear boxes and a command servo system whose azimuth and elevation readouts can be changed by handwheels. The operator dials a position, and the amplidyne system moves the antenna toward that position at a speed of 6 deg per minute in both coordinates. When the new antenna position nearly agrees with the command, the amplidynes slowly decrease the speeds until the antenna is positioned in accord with the command.

Radio astrometric measurements are usually made in one of two ways. In the "drift-scan" method, the antenna beam is positioned ahead of the source to be observed and then the earth's motion causes the source to drift across the antenna beam. The second method is "on-off tracking," usually used for sources small compared to the beamwidth. The source is tracked for a specified time, and then the antenna beam tracks a region of the sky several beamwidths away.

The antenna can be used for either of these methods of observation. The quality of an antenna mount can best be evaluated by drift-scanning, which is independent of any judgment (or controls) involving rates.

The position monitors have been aligned with the antenna beam by boresighting the antenna on a known location near the horizon. For this purpose we use an antenna range whose transmitter is located on the Billerica water tower, six miles distant. Antenna patterns are taken in azimuth and elevation to establish the direction of the main lobe of the antenna; then the peak of the main lobe is set on the transmitter and the telescope-television monitor is sighted on a light mounted adjacent to the transmitter dish, in such a place as to avoid parallax. The digital and synchro readouts are then set to the azimuth and elevation of the transmitter dish, which are discussed in Appendix A. Ideally, we should thereafter be able to point the antenna anywhere in the visible hemisphere and the monitors would indicate truly the direction of the antenna beam; departures from that ideal constitute the subject of this study.

B. Possible Errors in Pointing

There are many effects that could contribute to pointing error. To simplify our discussion, the kinds of error are grouped into four categories:

(1) Mechanical play and flexure
(2) Faulty fabrication or installation
(3) Errors in locating the antenna with respect to the sources
(4) Refraction in the atmosphere.

Under mechanical play and flexure, we classify any errors caused by changes in the physical structure of the antenna or its mount. The following are considered possible mechanical causes of error.

(1) The antenna rests on a foundation composed of four concrete pillars extending from below ground to the roof of Building D. Settling, or expansion and contraction, may cause variations in base position. Building vibrations may couple through these columns and cause small errors.

(2) In an X configuration, four metal legs extend from the four pillars upward and inward to the pedestal that supports the azimuth bearing assembly. If the antenna is not balanced about the elevation axis, flexing of the legs may occur as the antenna is moved from position to position in the celestial sphere.

(3) Each leg rests on a jack, which in turn is attached to a concrete pillar. The jacks are used for adjustment of the azimuth table to zero tilt. Although the jacks do not compress appreciably, there is play of 0.015 inch in an upward direction, and this freedom may allow rocking from one leg to the opposite leg. Error from this cause cannot be very important, because the play in the jacks permits the azimuth table to tilt less than 10 seconds of arc.

(4) The azimuth bearing assembly has not been oiled since installation, because the bearing is inaccessible. Weathering and loading have probably caused wear; there is evidence that damage exists and causes some wobble of the azimuth table.

(5) The antenna dish with its counterweight is known to be heavier than the load for which the antenna support structure was designed. Flexing under gravitation must be taken into consideration as a possible cause of error.

(6) The waveguide that supports the feed may flex, so that millimeter-wave pointing will not bear a constant relationship with the boresight of the telescope. This effect would make the antenna point higher than the position monitors indicate; dependence on the cosine of the elevation angle is expected.

(7) The telescope-television monitor must flex in its mounting on the back-up structure; the calculated flexure is very small.

(8) There is play in the gears that position the reticle, which is in the front focal plane of the television camera. Furthermore, the reticle mounting supports an infrared filter assembly, which – as it is rotated in and out of the field of view of the camera – may either move the reticle or introduce a refractive offset that moves the optical boresight.

(9) The antenna dish itself will change shape under thermal gradient and also under gravity loading; these problems have not been investigated in this study.

Of the many faults of fabrication or installation from which the mount may suffer, four are particularly worthy of note. The consequences of the first three are examined in Appendix B.

(1) The azimuth table (assumed plane) may not be horizontal.

(2) The elevation axis may not lie in a plane that is parallel to the azimuth table.

(3) The boresight of the antenna may not be perpendicular to the elevation axis; this shortcoming is of importance to surveyors, who call it collimation error.

(4) Imperfect construction or faulty mounting can cause error in the behavior of the position indicators, the shaft encoders.

The location of the antenna is, in our case, judged by a latitude-longitude-altitude fix provided by a surveyor. Details on the assumed location are given in Appendix A.

The only atmospheric cause of pointing error is refraction, but it has two aspects that we can consider separately. One is the apparent displacement of stars, planets, or the Billerica transmitter as seen through the telescope; the other is the departure of the millimeter-wave propagation from the line indicated by the telescope.

For refraction of the optical rays, the Nautical Almanac has tables that give sufficiently accurate corrections for celestial objects that are well above the horizon – perhaps 20 deg or more. Terrestrial refraction, which causes displacement in the apparent elevation of the Billerica transmitter as seen by the telescope, can be estimated in first approximation by consulting a handbook of surveying. Fortunately, what is most important about this effect is only the difference between it and the refraction of millimeter waves.

Refractions at optical and millimeter wavelengths have kindly been calculated for us by R. K. Crane (Group 61), using one atmospheric profile measured on a cold dry day in February and another measured in August on a day that was warm and humid. For celestial objects at any elevation that we can use – say, more than 5 deg – the refractions of millimeter and optical waves differ by too little for us to be concerned about (less than a hundredth of a beamwidth). For pointing on Billerica, the difference in refraction was 0.03 milliradian for the February atmosphere and 0.3 milliradian – a quarter of a beamwidth – for the August atmosphere. The refraction was, in both cases, larger for the millimeter waves than for the visible light.

In warm humid weather, therefore, aligning the telescope with the Billerica light when the millimeter beam is peaked on the Billerica transmitter can introduce some divergence of the optical and millimeter beams, in such a direction that the millimeter beam will point somewhat above the line of vision of the telescope. On one occasion, at about 5 a.m. on 12 April 1968, we found the millimeter beam from Billerica to be raised by one milliradian. A few hours earlier (10 p.m.) and a few hours later (about 3 and 9 p.m.) it was at its usual elevation. Similar behavior has been observed a few times since then; we are fairly sure that the apparent displacement of the beam was caused by anomalously large differences in the refractions of the millimeter and optical beams.

The direction of the millimeter beam can, in principle, be perturbed by reflections from the ground and from the tops of buildings that lie along the range. The range was originally satisfactory;* however, since then the Laboratory has been extended by Building 1, which has a flat roof of corrugated iron covered with a few inches of concrete and a two-inch layer of mineral fibers. This new reflector lies, in part, along the range. Moreover, it lies squarely in front of the probe tower that was used to verify the uniformity of illumination of the dish, so that the probe tower samples a field that can, and presumably does, differ from the field at the dish. It would seem that this tower should be abandoned, but the portable access tower, if wheeled up as close as it will go to the antenna mount, should make a suitable support for a movable receiver to sample the field. In the present work, it has been assumed that reflections have not significantly affected the apparent position of the Billerica transmitter. The assumption is supported by the radiometer experiment described below; it is also supported by the lack of any pronounced scalloping on the elevation patterns obtained by means of the Billerica transmitter.

III. THE EXPERIMENTS

In an effort to isolate the sources of pointing error, three separate experiments were conducted. The first examined the tilt of the plane on which the azimuthal motion occurs. The second compared, by optical means, the indicated and actual positions of certain stars and Venus. The third experiment verified that the optical system does properly indicate the direction in which the millimeter-wave antenna is pointing (except for possible refraction effects described above, which can be significant only at elevations of a degree or less).

A. Azimuth-Table Experiment

The first experiment was designed to evaluate the performance of the base and the azimuth bearing. A bubble level (Brunson Model 65210) was used to measure slope; on the scale used, it was capable of an accuracy of one second.

When the level was placed below the azimuth bearing and the antenna was turned in azimuth, changes in reading were expected from two mechanical effects: rocking of the antenna from one leg to another, and flexing of the legs. The ideal base would show no change in tilt as the antenna moves about the azimuth and elevation axes. Thus on a polar graph for that ideal base, we should expect to see a circle centered at the origin, with radius equal to the slope of the level. Antenna rocking from one leg to another would cause a change in the radius at angles

*V. L. Lynn, E. A. Crocker and J. W. Meyer, "Performance Evaluation Techniques for a Large-Aperture Millimeter System," Proceedings of the Symposium on Electromagnetic Windows, Ohio State University (4-6 June 1962).

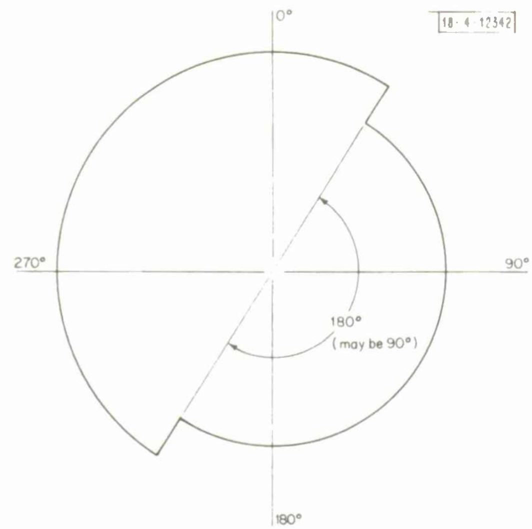


Fig. 1(a). Idealized behavior of four-legged base if rocking were dominant fault. Reading of level would change only when base rocked from one leg to another.

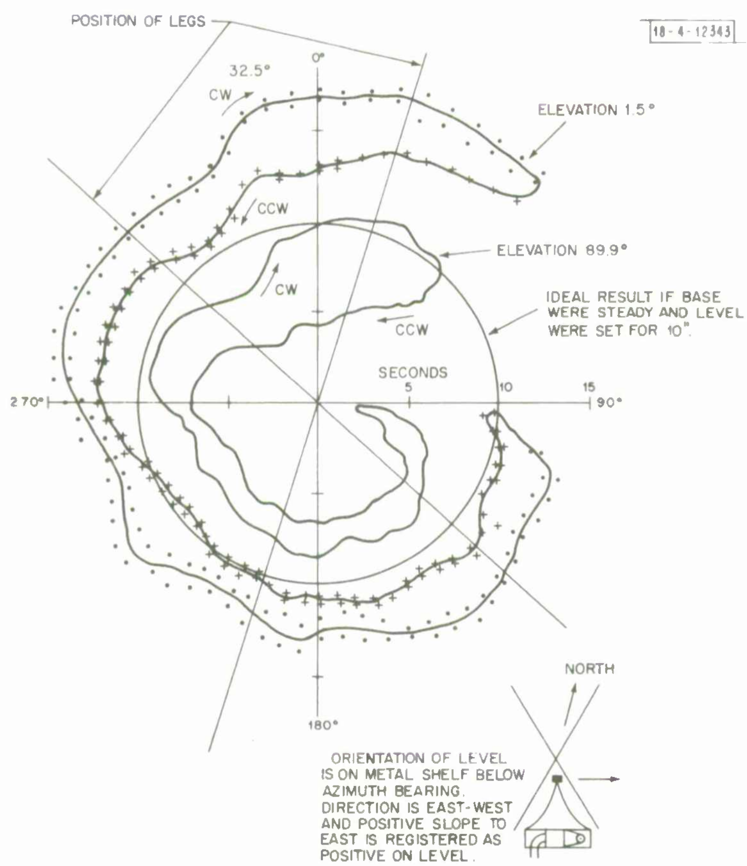


Fig. 1(b). Actual behavior of four-legged base as antenna is moved in azimuth. Level was on ledge below azimuth bearing and was oriented approximately east-west.

180 deg apart in azimuth [Fig. 1(a)]. Flexing of legs may occur if the antenna is not balanced about the elevation axis.* The flexing would be describable by a superposition of two sine-like functions dependent upon the azimuth and having amplitudes proportional to the cosine of the antenna elevation. Two sinusoids would be needed because the feet form an 18- by 24-foot rectangle; their phase difference would be 2 arctan (18/28).

Figure 1(b), for the actual base, shows neither rocking nor flexing. Instead, two other features appear:

(1) There is hysteresis, a difference between curves indicating clockwise and counterclockwise rotations; it has an amplitude of 5 seconds.

(2) The average slope seems to be dependent upon the antenna elevation angle. However, the data were taken on a hot sunny afternoon and the change in slope seems very likely to have been caused by thermal changes in the legs as the experiment progressed.

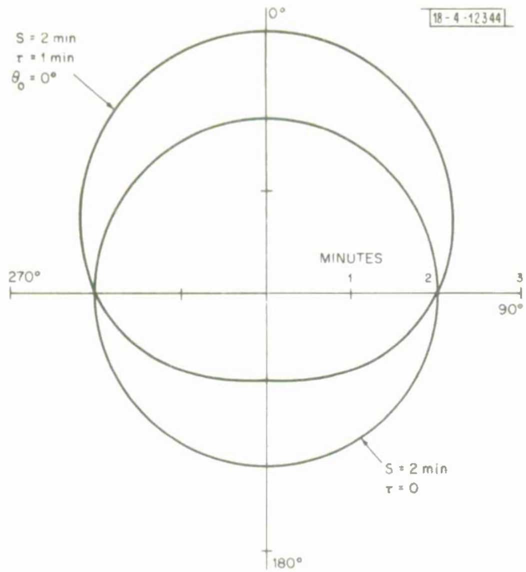
Error from thermal change can be avoided by observing at night. The error observed, whatever its cause, is less than 10 seconds in amplitude and this is less than 4 percent of a beamwidth. Therefore, whether the error is thermal or not, this experiment indicates that the base is sufficiently stable.

The second phase of the azimuth-table experiment was evaluating the azimuth bearing assembly. The level was carefully placed above the azimuth bearing, and the electrical leads from the level were so attached that, when the level rotated with the antenna, little or no force would be exerted on the level by reason of twisting or pulling of leads. The leads ran from the antenna mount to the operator's console, where the meter was located.† Taking data then became the job of one man, who controlled the pointing of the dish and also recorded the data. The technique of data taking was to turn the antenna at maximum amplidyne rate, 6 deg per minute, while the elevation control was held fixed. As the level experiences an inward radial acceleration, a small error would be expected because of the rotation. However, the radial acceleration is practically constant and corresponds to constant slope of the level; moreover, calculation of the centripetal acceleration shows that the error introduced by the rotation is less than 0.1 second of arc — entirely negligible. As an experimental check, several sets of data were taken by stopping the antenna at each data position and then recording the slope. There were no significant changes in the experimental results.

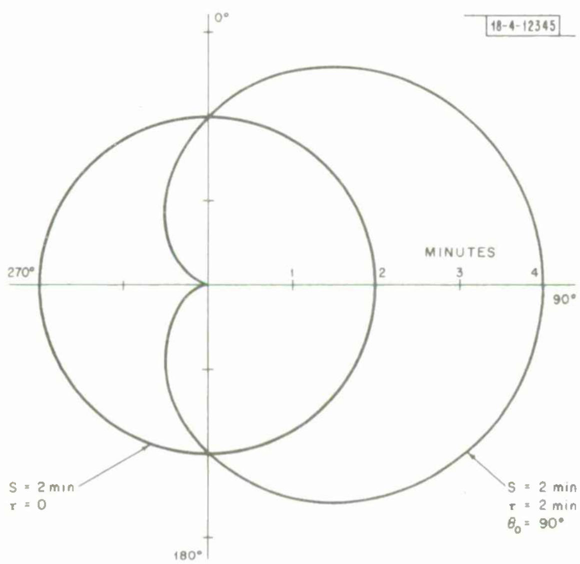
On a level installed above the azimuth bearing, we would expect to see the effects already observed on the base below the bearing; in addition, we should see effects of shortcomings of the bearing assembly. A measure of the azimuth-table tilt can be easily made by plotting the level reading (slope) as a function of the azimuth, on a polar graph. Given that the level itself has a constant slope S with relation to the bearing assembly and that the azimuth table is tilted at a small angle τ toward the azimuth direction Θ_o, then on a polar graph the slope will be $A = S + \tau \cos(\Theta - \Theta_o)$. Figure 2(a-c) shows the resulting curves for different values of the azimuth plane tilt τ and the level slope S. Additional variations would be due to base movements, thermal effects, and the mechanical response of the bearing assembly. Each individual roller

* When determined in 1961, probably with little weight in the cab, the unbalanced torque on the elevation axis was 740 inch-pounds, tail heavy; for a 20,000-pound structure, this is very good balance.

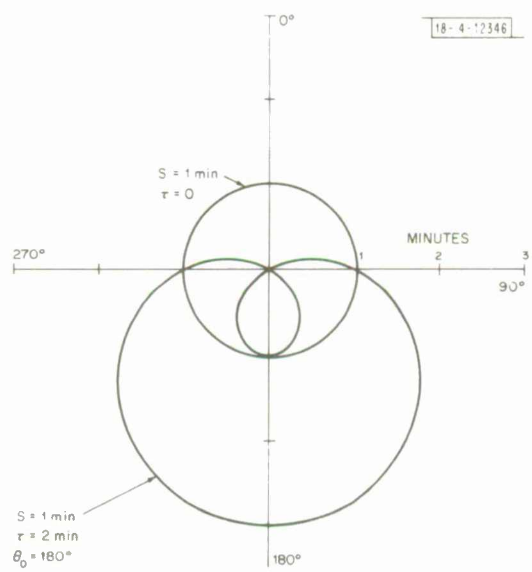
† The level readings had been checked against a calibrated sine bar to make sure that the long cable needed for this remote monitoring does not affect the operation of the instrument.



(a) Tilt smaller than offset.



(b) Tilt equal to offset.



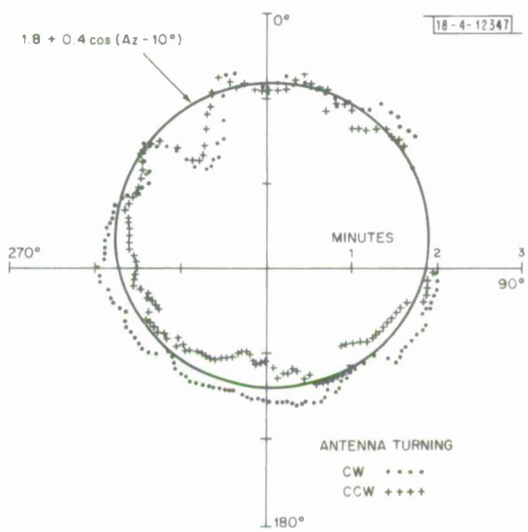
(c) Tilt larger than offset.

Fig. 2. Ideal polar diagrams of level reading vs azimuth of antenna, when level is above azimuth bearing and rotates with antenna. S is offset of level and τ is tilt of base. Curves have form $S + \tau \cos(\Theta - \Theta_0)$.

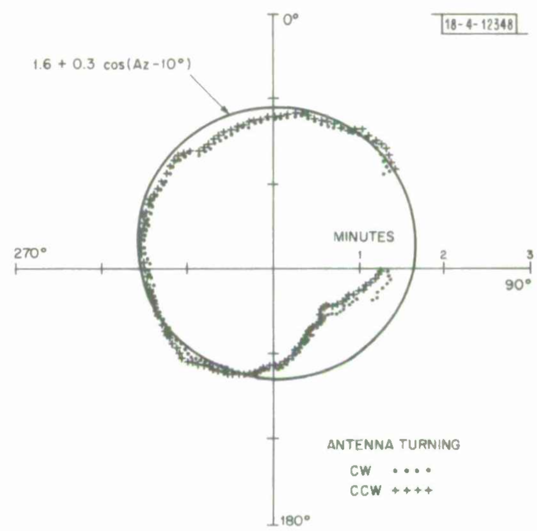
within the assembly will have some play in positioning; when the direction of motion is reversed, each bearing will move to take up that play. Thus we would expect to see hysteresis in the azimuth-table tilt above the bearing assembly, but no such effect below the bearing assembly. The fact that there seems to be a small hysteresis effect below the bearing assembly is, at present, difficult to explain. Motion of the base has already been shown to be small, and thermal change has been neglected in this analysis because the antenna was tested mostly at night, when the thermal disturbances would be very small.

Two very different responses are seen with the actual system, depending upon whether the level is placed parallel to the azimuth of the antenna boresight or parallel to the elevation axis. When the level is parallel to the antenna-boresight azimuth and is above the azimuth bearing, its slope adheres pretty well to a pattern $S + \tau \cos(Az - 10°)$, with τ, the tilt of the azimuth table, equal to about 20 seconds of arc. As Fig. 3(a-b) shows, however, the data depart from the calculated curves by as much as 30 seconds of arc. The irregularities are believed to be due to damage to the azimuth bearing, which has no provision for lubrication. On one occasion a few winters ago, during a radiometry experiment, the antenna refused to turn until brute force was applied to the handcranks; at the time, it was believed that there was ice in the bearing. Measurements similar to those under discussion here had been made shortly before this incident, and another set was made about four months afterward; the measurements in the second set were markedly less regular and less repeatable. A new bearing was procured at that time, but it has not been installed.

When the level was parallel to the axis defined by the elevation trunnions, greater variations in level readings occurred. Moreover, the azimuth-bearing behavior seems dependent on the direction the antenna turns in and has recently turned in (Figs. 4, 5 and 6).

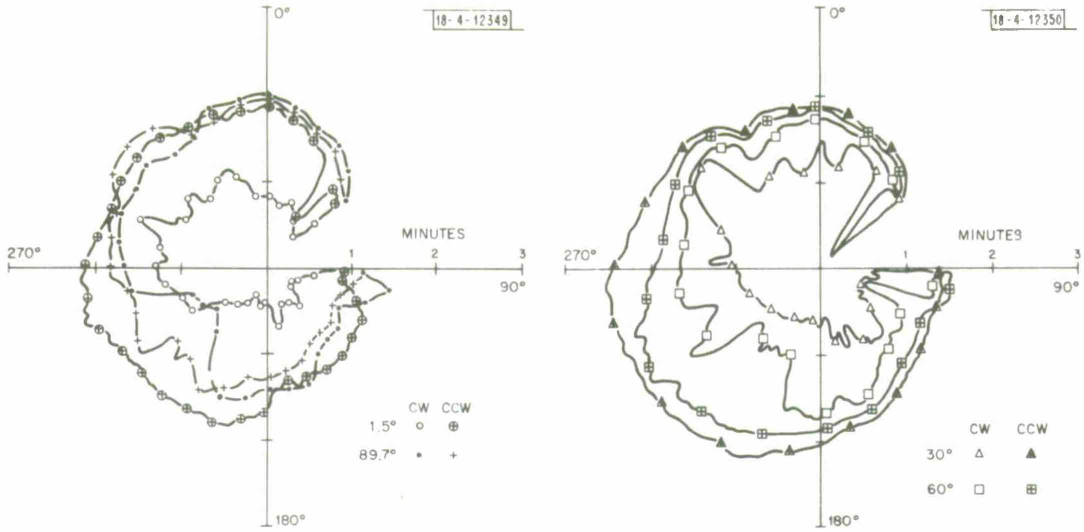


(a) Antenna elevation 1.5 deg.



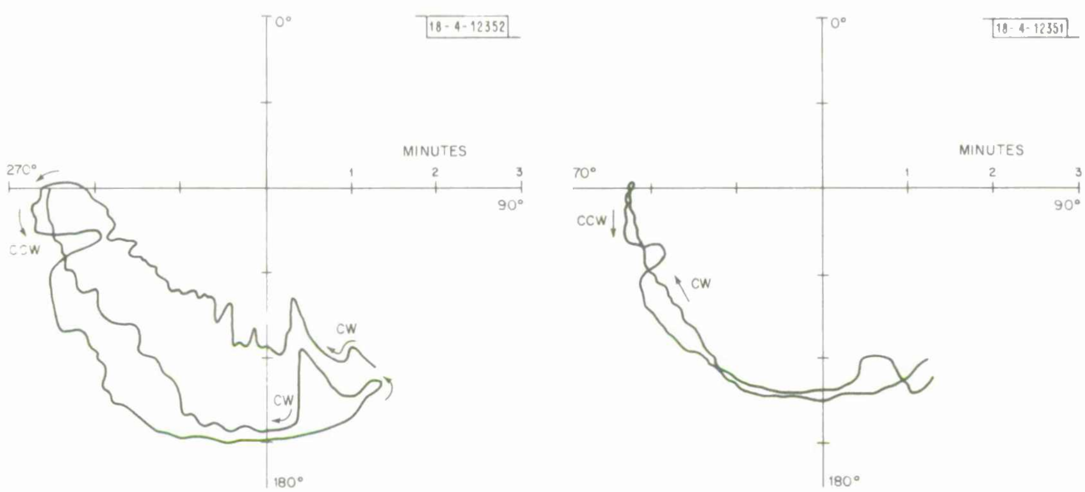
(b) Antenna elevation 89.9 deg.

Fig. 3. Readings of level on azimuth table, above azimuth bearing and aligned with antenna azimuth, as a function of antenna azimuth.



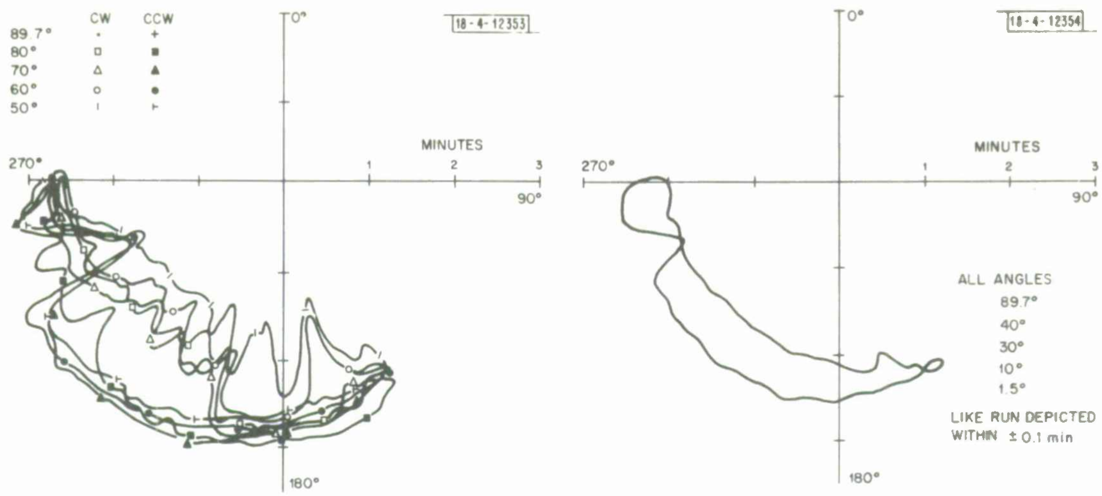
(a) Antenna elevations 1.5 and 89.9 deg. (b) Antenna elevations 30 and 60 deg.

Fig. 4. Readings of level on azimuth table, above azimuth bearing and parallel to axis of trunnions, as a function of antenna azimuth.



(a) 10 July 1967, breeze up to 10 knots. (b) 11 July 1967, hardly any breeze.

Fig. 5. Readings like those of Fig. 4, on two successive days. Antenna elevation 89.9 deg.



(a) 10 July 1967, breeze up to 10 knots. (b) 11 July 1967, hardly any breeze.

Fig. 6. Readings like those of Fig. 4, on two successive days. Antenna at a variety of elevations from 1.5 to 89.9 deg.

The data show that:

(1) Hysteresis, the difference between curves indicating clockwise (CW) and counter (CCW) rotations, tends to decrease as the elevation angle increases.

(2) There is a larger number of unpredictable changes in the slope when the antenna turns in a CW direction. Amplitude of these changes varies from 0 to 1 minute of arc.

(3) Backlash occurs about 20 deg in azimuth after reversal of direction. Although the backlash seems to have been independent of elevation and azimuth, it varied from day to day, sometimes amounting to as much as 1.5 minutes of arc.

(4) A 10-knot breeze probably causes noticeable disturbance of the azimuth table.

B. Optical Drift-Scan Experiment

The television-telescope sighting system was used to compare the indicated positions of some stars and Venus with their calculated positions. Azimuth and elevation angles for specific local times were calculated by means of programs described in Appendix C. The precision was 0.001 degree; however, there is a possibility of systematic error due, for example, to error in site latitude or longitude.

The observations were made by the drift-scan method, because the object was to check not tracking accuracy, but the relation between the readings of the encoders and the static azimuth and elevation of the antenna beam. To minimize errors due to defects in the azimuth bearing assembly, the antenna was always (with an exception noted below) moved from east to west. By successive approximations, the pointing was adjusted so that the star or planet passed very close to the dot that marked the center of the reticle. The setting of the reticle was such that the dot coincided with the lamp at Billerica when the encoders were set for the estimated azimuth and elevation of a light ray from the lamp. Making the star pass very close to the dot made the experiment independent of distortion by nonlinearities in the television system.

Errors in azimuth ranged from about +0.005 to −0.030 deg, varying with elevation in the way indicated in Fig. 7. The reason for plotting the curves that appear there will be given in Sec. IV-B.

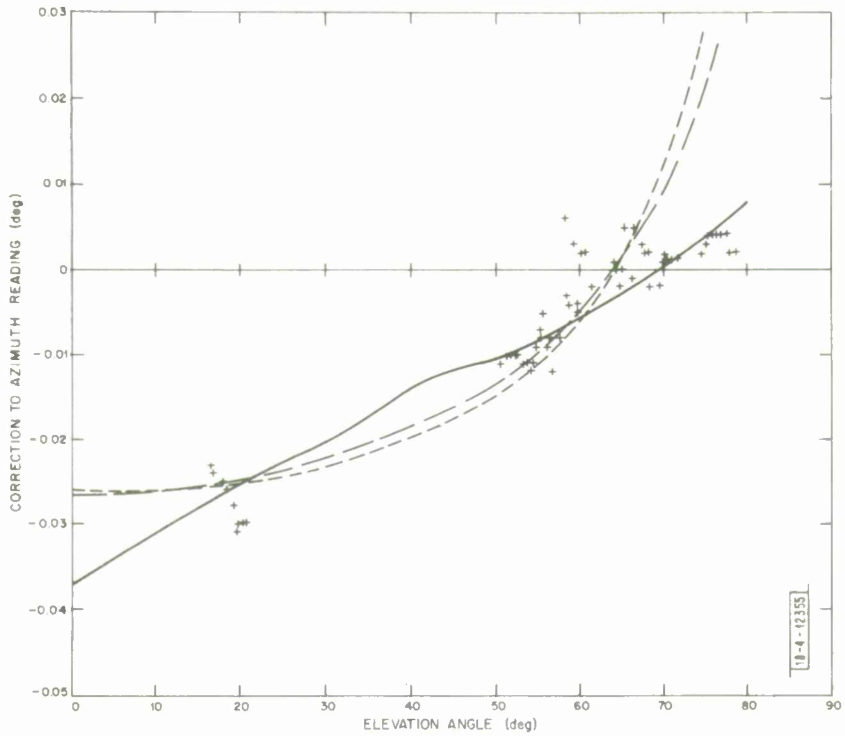


Fig. 7. Corrections needed to convert azimuth encoder readings to azimuth as indicated by star positions. Broken curves are attempts to fit data by assuming that elevation axis is tilted and boresight is perpendicular to it (short dashes) or that axis is level and boresight is not perpendicular to it (long dashes). Solid curve is fitted by assuming that both defects are present, in amounts stated in Sec. IV-B. For all curves, it is assumed that azimuth table is horizontal; tilt actually measured was small enough so that this assumption does not seriously distort picture.

The elevation error was dependent on elevation in a highly systematic way. Figure 8 is a plot of the needed correction in elevation, as a function of elevation. Here, as elsewhere, the correction is defined as the amount that has to be added to the encoder reading to make it agree with the calculated position of the star.* The calculated position used for Fig. 8 takes no cognizance of refraction; the data are replotted in Fig. 9 to show how much must be added to the encoder reading to make it fit the calculated position of the star when the calculation takes into account the effect of atmospheric refraction, which was computed from the tables in the Nautical Almanac, using temperature and pressure readings recorded by the Air Force at Hanscom Field.

Figure 10 is similar to Fig. 9, except that the data are for Venus, and they have been adjusted for movement of the reticle in a way described in Appendix E and Fig. E-1.

The curve drawn in on Figs. 9 and 10 is the same on both plots. It has the form 0.23 (1 − cos El), the unit being the degree, and it is adjusted in ordinate for good fit of the data of Fig. 9.

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* By "error," we mean a quantity opposite in sign, and equal in magnitude, to the correction, for elevation or azimuth as the case may be.

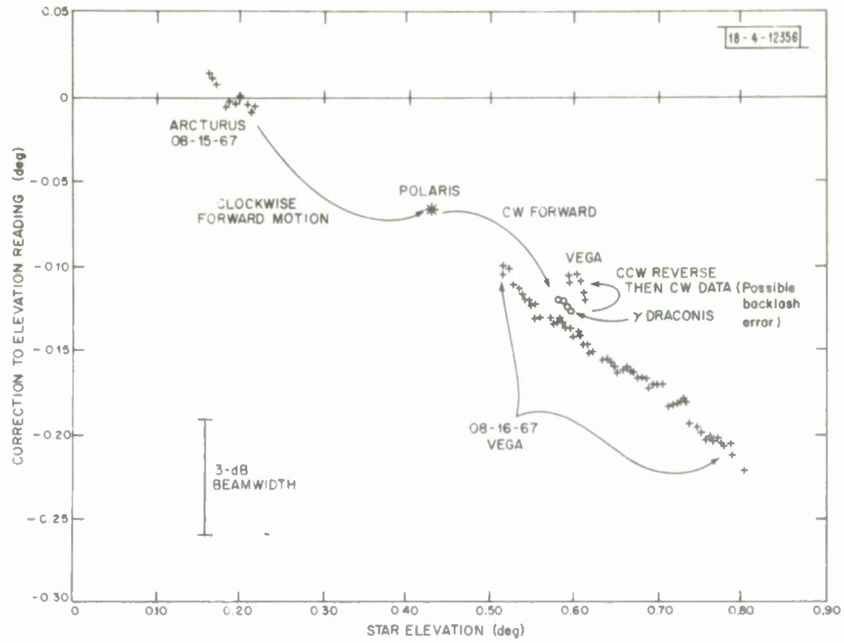


Fig. 8. Correction needed to make elevation encoder reading agree with calculated elevations of some stars, on two nights. Calculated elevations included no allowance for refraction.

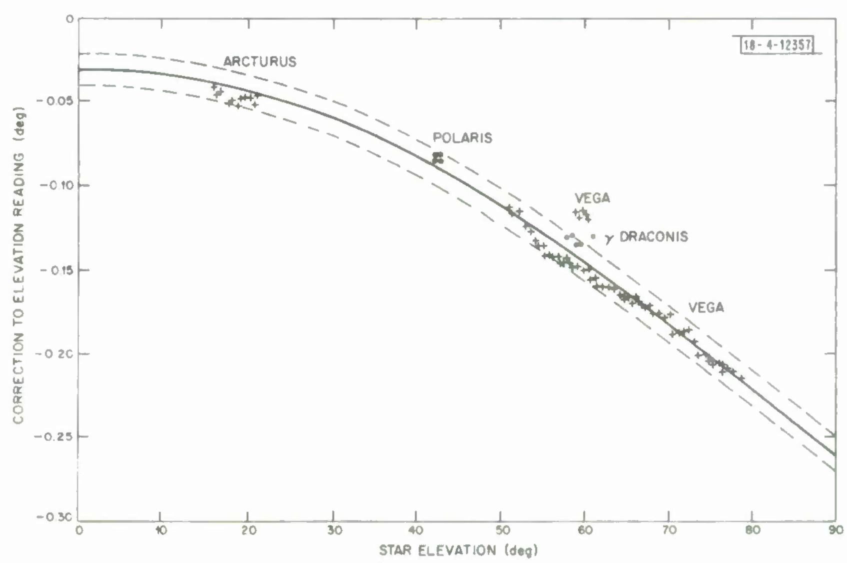


Fig. 9. Correction needed to make elevation encoder reading agree with calculated elevations of some stars, when calculated elevations included allowance for refraction. Observations are same as those in Fig. 8. Of the correction at zero elevation, about 0.02 deg was caused by using traditional and incorrect value for elevation of Billerica transmitter used as reference. Solid curve is constructed analytically from $-0.23 \cos El$. Dashed lines are ±0.01 deg from solid one.

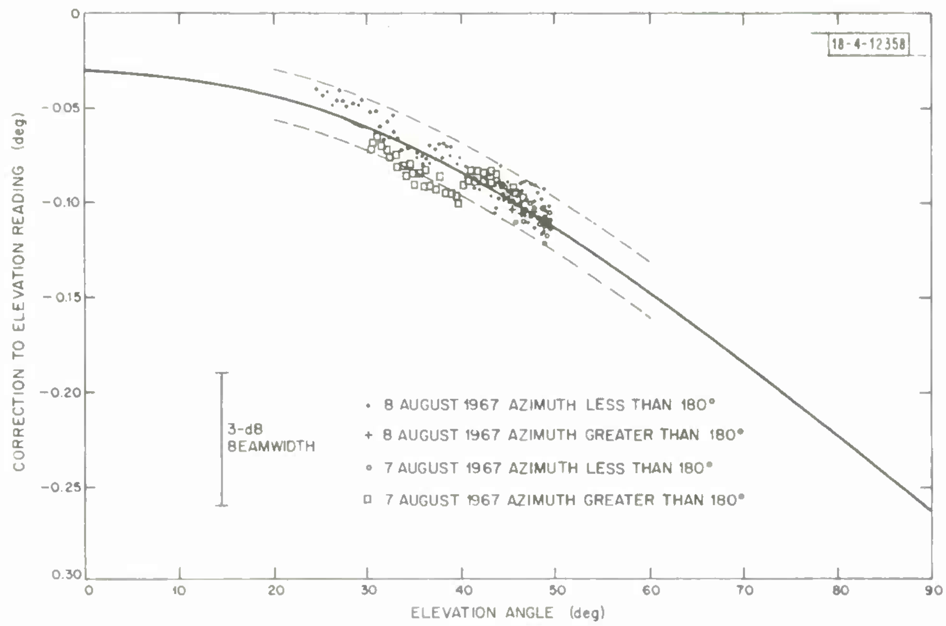


Fig. 10. Corrections needed to make elevation encoder readings agree with calculated elevations of Venus. Calculated positions include allowance for refraction. Solid line is that derived to fit star data in Fig. 9, and dashed lines are ±0.01 deg from it. These data, taken in daytime on two days, show more scatter than star data, but are well fitted by same curve. Venus trailed sun by 3 hours.

The implication of this curve is important. It says that most of the elevation error is a simple smooth function of the elevation angle; the form of the function strongly suggests that this part of the error is caused by mere sagging of the structure under its own weight. If the TV boresight is aligned on the Billerica lamp and the encoder is set for the elevation angle of that direction (with due allowance for atmospheric refraction), then near the zenith, the boresight elevation will be higher than the encoder reading, by 0.23 deg, which is 3 one-way beamwidths.

More extensive data would have been desirable for this experiment, but the work was limited by cloudy nights and hazy days. The amount of data gathered was, however, sufficient to provide reasonably firm conclusions.

C. Radiometer Experiment

Having found a sag in the pointed structure, we wanted to know if the telescope-TV assembly was flexing or if the entire antenna was sagging. (Sag of the feed alone was ruled out, because that would give an error having the wrong sign.) The question was decided by testing the boresight of the telescope against the direction of the radiometer beam, using the moon as a source.

After parallelism of the millimeter-wave and optical boresights had been checked on the antenna range when the elevation was only a fraction of a degree, the antenna was pointed at the full moon at various times on a night in September when the moon was as high as 50 deg (Fig. 11),

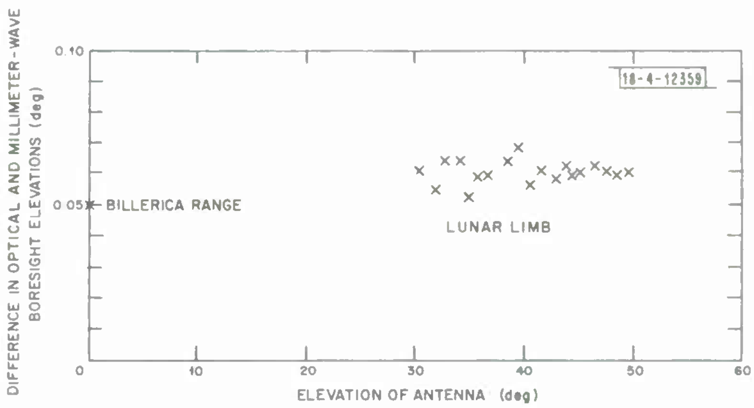


Fig. 11. Elevation of millimeter boresight minus elevation of optical boresight, as a function of elevation encoder reading. Constancy of the difference (which on a later night was verified out to 65 degrees) shows that sag inferred from Fig. 9 is principally or entirely sag of dish, not merely sag of telescope.

and again in February at an hour when the moon's elevation exceeded 65 deg. When the 35-GHz radiometer reading on an area near the lower limb of the moon had been established, the beam was lowered until the radiometer output was halved, showing that half the beam was missing the moon, and that the limb was therefore on the nose of the beam. A photograph of the TV screen recorded the relation between the limb and the reticle at that moment. After several repetitions, similar observations were made on the upper limb, and then on the left and right limbs. Throughout the experiment, including the initial boresighting at near-zero elevation, it was assumed that the atmospheric refraction was the same for the millimeter waves as for the optical ones.

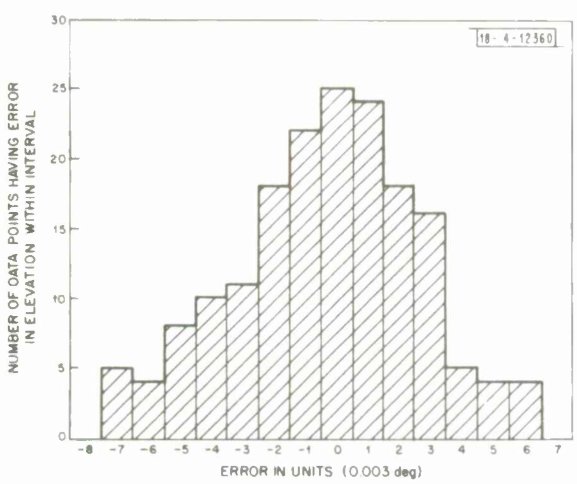
The experimental uncertainty in the settings was about 0.02 deg, a quarter of a beamwidth. To this accuracy, the optical boresight lay on the limb of the moon when the radiometer signal was at half maximum and the elevation of the moon was 65 deg. We conclude that alignment of the telescope-TV axis and the axis of the antenna pattern is not appreciably dependent on the elevation setting. The sag disclosed by the drift-scan experiment, which at 65-deg elevation causes an error of $0.23 \cos 65° = 0.10$ deg, is not caused by flexing of the telescope in relation to the dish. It must be ascribed to a droop of the dish and backup structure with respect to the elevation axis.

IV. ANALYSIS OF EXPERIMENTS

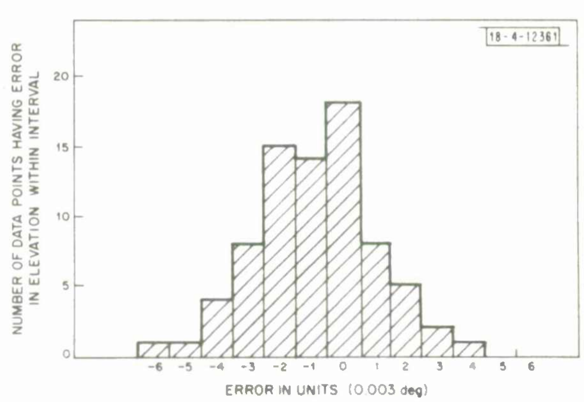
A. Analysis of Elevation Error

The azimuth-table experiment (Sec. III-A and Fig. 3) showed that elevation error due to the azimuth bearing assembly is about ±0.008 deg. From Fig. 9, except for some data on Vega and α Draconis, we find that the optical boresight follows the sag curve to within ±0.010 deg. Only about 80 percent of the Venus data, corrected for the reticle displacement mentioned in Appendix E, lie within these limits (Fig. 11). Greater error in the Venus data implies that other effects than sag are present during daytime observations. With the azimuth bearing and the tilt of the azimuth table in their present condition, limiting the use of the antenna to night observations and using only the sag correction, we can predict the elevation of the optical boresight to within ±0.010 deg, or about one-seventh of a beamwidth (Fig. 9).

A better, more conclusive measure of elevation error is found in Fig. 12(a-b). Data from Figs. 11 and 9, respectively, were used to estimate the distribution of error about the sag curve. Both in daytime and nighttime runs, we find that the distribution appears to be at least approximately Gaussian. With a confidence level of fifty percent, we can state that if backlash is avoided, the static elevation of the optical boresight, after correction for the sag, is in error



(a) Daytime observations on Venus. Square root of second moment of distribution about mean is 0.009 deg.



(b) Nighttime observations on stars. Square root of second moment of distribution about mean is 0.006 deg.

Fig. 12. Distributions of errors in elevation. Error is defined as departure of data from solid curve in Figs. 9 and 10.

by not more than ±0.01 deg during nighttime runs in calm weather and not more than ±0.01 deg during the particular daytime runs that were made.

From Appendix B, we find that alignment errors call for the elevation reading to be corrected by an amount that is the slope of the azimuth table in the direction of the boresight. Figure 3 plots that slope as a function of azimuth. Though perhaps dependent on elevation, the correction was approximately 0.4 minute of arc, or 0.007 deg, times cos (Az − 10°). This amounts, at most, to a tenth of a beamwidth. It could probably be reduced by adjusting the leveling jacks. However, even with the jacks as they were, the error in elevation calculated from the azimuth-table tilt was at most equal to one division on the decimal readout of the elevation encoder (0.01 deg). It was therefore below the threshold of detectability.

It is intuitively apparent, and Appendix B demonstrates, that collimation error and tilt of the elevation axis do not cause any first-order error in the elevation readings.

B. Analysis of Azimuth Error

Appendix B shows that the azimuth correction needed because of misalignments is

$$Az - Az' = [b' + p\cos\delta - q\sin\delta]\tan El' + c\sec El'$$

where $\delta = b'\tan El' + c\sec El'$; the other quantities are defined in the appendix. Because b' and c are not likely to exceed 10^{-3} radian, the approximations $\cos\delta = 1$, $\sin\delta = 0$ are adequate when $El' < 80°$. With that restriction,

$$Az - Az' = [b' + p]\tan El' + c\sec El' \quad .$$

According to Fig. 3, the highest point on the rim of the azimuth table is at azimuth 10 deg. Since $p = \tau\cos a'_o$ and a'_o is the azimuth reading for the left (when looking out along the boresight) end of the trunnion axis, measured from the lowest point of the rim rather than from north, we need a'_o as a function of Az'. These two quantities would differ by 270° if Az' were measured from the lowest point on the rim, and it actually is measured from 0°, which is 190° from the lowest point. Hence

$$Az' + 270° = a'_o + 190°$$

$$\tau\cos a'_o = \tau\cos(Az' + 80°) \quad .$$

Because weather restricted the amount of data on star positions that was collected, nearly all the data happen to have azimuths in the 255- to 281-deg range, so that cos (Az' + 80°) ranged from 0.90 to 1.00. The azimuth-table experiment (which was performed a few weeks before the star positions were observed) gave the result $\tau \doteq 0.4$ min $\doteq 0.007°$. For most of the star observations on azimuth error, it is an acceptable approximation to say $p = 0.007°$.

The pointing errors found in the drift-scan experiment are plotted in Fig. 7 to show their dependence on elevation. On that plot, the broken curves are attempts to fit the data with a tangent function only, or with a secant function only. It is clear that curves with these forms cannot be fitted to the data. This negative finding demonstrates that the azimuth error is not caused predominantly by misalignment of the trunnion axis with respect to the azimuth table (measured by b'), nor is it caused predominantly by collimation error (measured by c). The solid curve is constructed by assuming values

$$p = 0.007° \qquad b' = 0.032° \qquad c = -0.037° \quad .$$

These values for b' and c (about 2 minutes and −2 minutes) that are needed to produce a fit are physically plausible. Note that they are appreciably larger than τ, so that taking account of the individual azimuths of the stars, instead of simply setting $p = \tau$, would make little difference in the values obtained for b' and c.*

Though the estimated values of b' and c are large fractions of a beamwidth, they are such that b' tan El' and c sec El' have a strong tendency to cancel one another at elevations in the range from 20 to 80 deg. At elevations that are likely to be of practical interest, therefore, corrections for the combined effect of collimation error and skew of the elevation axis are not large and can be worked into the program for computing the pointing data for any celestial object †

If the antenna is to be pointed by use of the encoders only, without the visual check provided by the TV system, the drift-scan experiment should be done again, in more detail, to improve on the estimates given here. The range of elevations 20 to 80 deg should be covered, and the value of p should be monitored during the observations. To get good measurements of b' and c, it is, of course, helpful to make τ small by adjusting the leveling jacks. Imprecise mounting of the encoders could perturb the estimates of b' and c. At the time of installation, however, the encoders were mounted with a precision that satisfied all the specifications set by the manufacturer, and under these conditions the encoder error introduced by the coupling does not exceed 1 second of arc, which is 0.0003 deg.

What can be said at present is that the azimuth error at elevations below 80° can approximate half a beamwidth (Fig. 7), but that the greater part of this error can be accounted for by assuming certain amounts of skew in the elevation axis and error in collimation, and that error from these causes can readily be taken into account when computing the pointing data.

It is important to notice that at high elevations, the angle between the boresight and the desired direction may be appreciably smaller than the difference between the desired azimuth and the azimuth of boresight. In fact, if we simplify by postulating zero error in elevation, their ratio will equal the cosine of the elevation; at elevation 80°, an error of 0.12 deg in azimuth would cause an error of only 0.02 deg along a great circle to the desired point on the celestial sphere. Even if the estimates of b' and c are not very accurate, therefore, the multiplication of b' and c by tan El' and sec El' does not imply a rapid worsening, at high elevations, of the pointing error that is of interest to the user of the antenna.

A question that would bear investigation is whether the errors for millimeter-wave pointing are the same — within tolerable limits — as the errors in optical pointing such as used in our drift-scan experiment. We have demonstrated that the visual boresight maintains rather well its alignment with the millimeter-wave boresight, but that experiment was investigating the nature of the observed sag, which amounts to a few beamwidths (0.23 deg); a closer investigation would be needed before assuming that the collimation error for the antenna can be evaluated well by observing the pointing error of the TV system. In particular, it seems possible that ground or near-field effects when aligning the two boresights on the antenna-range tower in Billerica may make the boresights depart from parallelism when they are directed at the sky. If a better

* The two points at elevation 59 deg that lie well above the calculated curve were observed at azimuth 305 deg, and adjusting them for this fact would bring them closer to the curve by about 0.003 deg.

† There is no cancellation when boresighting on the antenna range, because at zero elevation, the effect of the skew vanishes (tan El' = 0), whereas the effect of collimation error does not (sec El' = 1).

receiver becomes available, it may be practical to measure the various errors by observing radio sources instead of stars, as has been done for the Haystack antenna.*

Because one works with point sources and because the boresighting on Billerica is not perturbed in azimuth – by, for example, the horizontal metal-lined roof of Building I, which borders on one side of the antenna range – using optical observations is an advantageous way of finding errors that arise from the shortcomings of the mount.

The whole range of azimuth errors that were observed, over an elevation range from 20 to 80 deg, was about half a beamwidth (+0.003 to −0.030 deg). The observations covered only a limited range of azimuths (mainly 255 to 280 deg). These azimuths were such that the azimuth error caused by the tilt of the base was near its maximum, if we are safe in assuming that the tilt did not change its nighttime orientation during the few weeks during which the work was done. However, at certain azimuths one might find errors caused by rocking of the azimuth table on the azimuth bearing, which is in need of replacement. As noted earlier, a replacement has since been procured, but not installed.

Given a new azimuth bearing and a slightly more detailed investigation of the kind described here, it seems likely that by evaluating b', c, and τ or p, one can establish a correction curve that will permit pointing to within 0.01 deg in azimuth, which is about a seventh of a beamwidth. Except at low altitudes, such accuracy in azimuth is superfluous, because the effect of azimuth error on the angle between boresight and target is diminished by the cosine of the elevation.

V. SUGGESTIONS FOR IMPROVEMENT

For visual pointing, there is no necessity to change the mount or its appurtenances, but the improvements in the telescope that are outlined in Appendix E would be easy and inexpensive.

Focusing, discussed briefly in Appendix D, would be aided by making the feed waveguide move more readily. At present, it gets stuck when not moved frequently. The cause of the sticking is not known, but it may well be chemical action between the copper waveguide and the magnesium-alloy diaphragms inside the mast. This reaction could probably be prevented by varnishing the waveguide. Putting silicone grease on it has been tried and does not cure the trouble.

The waveguide now runs through a 2-foot section of stainless steel tubing that keeps the feed from wobbling. The clearance between the tubing and the waveguide is small, and this situation compounds the difficulty of moving the waveguide. The next time the waveguide is removed from the mast, the tight-fitting stainless steel tube should be replaced by a loose-fitting one that touches the waveguide only at brass plugs shrunk into the ends of the tube. Two such tubes have been fabricated; they are stored in the antenna cab.

The command system uses the old synchro readouts as reference. The command is set in by use of handwheel-driven dials. These are separated from the dish by numerous meshes of gears. A better command system would use the encoders for reference, thereby taking the gears out of the loop. It would match the digits of the encoder reading to a set of command digits, which could be selected on a small array of keys such as those on a desk calculator. That system would be an improvement over the present one in a number of ways:

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* M. L. Meeks, J. A. Ball and A. B. Hull, "The Pointing Calibration of the Haystack Antenna," IEEE Trans. Antennas Propag. AP-16, 746-751 (November 1968).

(1) The readings would be closer to the true values of azimuth and elevation

(2) The corrections for systematic errors would be the same as with other methods of pointing

(3) Operator time would be saved

(4) Operator error would be nearly eliminated.

(5) With a moderate amount of additional equipment, pointing or tracking could be done by computer in real time.

The digitized command system would have no disadvantages over the handwheel command system. It would be useful for day-to-day operations like those of the past three years, highly useful for the drift-scan mode of observing sources, and essential for control by computer.

For pointing at invisible sources or targets, several changes seem to offer benefits. Before making them, the work described in this report should be extended and refined in some ways.

As a first step, one should repeat the azimuth-table experiment (Sec. III-A) to see what state the azimuth bearing is in, and should decide whether to replace it. Then the drift-scan experiment should be repeated, on a slightly larger scale, using a spread of azimuths, in order to find out whether correction curves such as those in Figs. 7 and 9 are indeed valid, and to refine the estimates of the parameters in the correction equations. It would be useful to have a pair of remote-reading levels underneath the azimuth table while the optical drift scans are being observed (Sec. III-B), so that the parameters p and q can be recorded. The sag, collimation error, and skew of the elevation axis can then be evaluated more closely than was possible with the data reported here.

Making the remote-reading levels a permanent part of the installation would make good pointing independent of hysteresis and backlash and of thermal or other changes in the legs of the mount and in the height of the piers that support it. During the day, the effects of these changes are probably small compared with the shift in millimeter-wave boresight caused by thermal distortion of the dish. At night, the changes probably have negligible effect; if so, the levels would only offer knowledge that all is well with the base and with the azimuth bearing.

VI. CONCLUSIONS

For visual pointing at night, the mount and the drive-and-control systems are highly satisfactory as they are now. The telescope would benefit from the simple revisions proposed in Appendix E.

For visual pointing in the daytime, e.g., at Venus, solar heating may cause the millimeter-wave boresight to shift with respect to the mount. The probable magnitudes of such shifts are not known; the present study was concerned with the mount itself.

When the pointing is to be inferred from the encoder readings, the only large correction is that for sag. It can be as much as three 35-GHz beamwidths (0.23 deg), but seems highly systematic. After this correction was applied to the elevation readings, they were accurate at night to about 0.01 degree, which is one-seventh of a 35-GHz beamwidth.

The chief causes of error in azimuth readings were collimation error and misalignment of the trunnion axis. Each of these defects amounts to about half a beamwidth, but they have opposite sign. Consequently, the azimuth corrections that were measured did not exceed half a 35-GHz beamwidth. When corrected analytically for the estimated systematic defects of the mount, but with no correction for the shortcomings of the azimuth bearing, the azimuth pointing errors were mostly less than 0.01 deg.

The observations on the azimuth table (Sec. III-A and Figs. 4, 5, and 6) show hysteresis and backlash in the component of tilt that is parallel to the axis of trunnions. Except at very high elevations, these effects – which alter the quantity p (Appendix B) – cause no appreciable error in elevation, and at zero elevation they cause no error in azimuth. At higher elevations, their effect on azimuth is multiplied by tan El, but the hysteresis itself diminishes so rapidly with elevation that its effect is perhaps never very serious. Moreover, the deleterious effects of a given error in azimuth diminish as the cosine of the elevation. The data are not extensive enough to show whether the backlash changes with elevation.

The stellar and planetary observations on which these conclusions were based had to be done during one summer-staff appointment, and the observations did not exhaust all possible orientations of the dish. In particular, at some azimuths the pointing errors could conceivably increase because of the damaged portion of the azimuth bearing. It did not seem proper to attempt any detailed study of the effects of the bad bearing, if only for the reason that the bearing is not likely to remain in a steady state of badness. On the other hand, there was no attempt to choose azimuths at which the defects of the bearing would have minimum influence.

We conclude that by applying some easily manageable corrections to compensate for the defects of the mount, the root mean square of the pointing errors (per axis) can be reduced to about 0.01 deg for the telescope boresight at night, and that the pointing of the millimeter beam is of comparable accuracy. The observations made in the daytime show that the pointing of the telescope is then not as good as it is at night, but their scope was too limited to yield any quantitative estimate of the degradation caused by the range of conditions that prevail in daylight.

APPENDIX A
POSITION OF 28-FOOT DISH AND BEARINGS OF TRANSMITTER AT BILLERICA

A letter from Robert E. Cameron of Harry R. Feldman, Incorporated, of Boston, to Paul Gaudette of Lincoln Laboratory, 1 May 1964, gives the position of the center line of the lunar radar mount as latitude 42°27'37.100'' north, longitude 71°16'0.01.008'' west. It says that the bearing from that center line to "an antenna at the Billerica Water Tower" is 4°35'36'' east of true north. A technician who has been continuously involved with the 28-foot dish attests that the dish now at Billerica is in the same position as the one that was there in 1964.

It is disquieting that the same survey, plus some data and calculations (Lincoln Laboratory Notebook 3240), gives the elevation of the center of the present (two-polarization) Billerica dish as −0.428 deg. This is ridiculous. It would put the top of the Billerica water tower close to 235 feet below the center of our dish. But that center is at altitude 258 feet, so the survey says that Billerica draws its water from a tower whose lower part is below sea level!

In a paper by Lynn, Crocker and Meyer,* there is a graphical profile of the range, obtained presumably from maps. It shows the center of the 28-foot dish to be at altitude 258 feet, and the Billerica dish to be at 360 feet; it puts the Billerica end of the range 102 feet higher than the Lexington end. This is more plausible, but it turns out to be inaccurate.

A search for data on the water tower was complicated by the fact that Billerica has at least two of them, and initially the town officers gave us data on the wrong one. The one we are using was erected by Camp, Dresser and McKee, of Boston, who say that the grade of the concrete base of "our" tower is 275.50 feet above mean sea level, and that the height to the top of the cylindrical section is 120 feet. The center of our dish is 39 inches higher, hence at 399 feet. This is about 40 feet higher than on the Lynn, Crocker and Meyer profile; the implied error somewhat exceeds one beamwidth of the lunar radar at 35 GHz. We conclude that the dish at Billerica is centered 141 feet higher than the center of the 28-foot dish.

The slant distance between the dishes is 31,555.60 feet. (This figure comes from the Feldman survey that is demonstrably wrong about the elevation angle; the distance was measured directly with a geodimeter, and is probably not in error by any amount that is relevant here.) Combining this distance with the 141-foot difference in altitude gives +0.256 deg as the angle of elevation if one uses a flat earth. Allowing for the radius of the earth reduces the figure to +0.212 degree, but the usual allowance for refraction [B. Breed, Surveying, 7th ed. (Wiley, New York, 1942), p. 96] raises this to +0.219 deg. This is the elevation at which presumably reliable surveys say the center of the Billerica dish will appear when observed visually through our TV telescope on a day when refraction along the range is of the handbook amount.

Paul Gaudette of Lincoln Laboratory measured the angle one morning late in August, at about 9:30 a.m., daylight saving time. When offset to apply to the centers of our dishes, his observed value is +0.219 deg.

We have assumed that the Feldman report is correct except for a mistake in elevation. The good agreement between our measured and calculated values of elevation suggests that the

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*V. L. Lynn, E. A. Crocker and J. W. Meyer, "Performance Evaluation Techniques for a Large-Aperture Millimeter System," Proceedings of the Symposium on Electromagnetic Windows, Ohio State University (4-6 June 1962).

Feldman figure for the distance is not far wrong. The possibility exists that the bearing to Billerica or the latitude or longitude of the dish may be in error by an amount that has significance when one is determining the pointing corrections. It appears from the experiments reported in Sec. III-B that such errors are not huge, but their possible existence should be kept in mind if more elaborate experiments of the same kind are ever conducted.

Pending verification or revision of the bearing to Billerica, the antenna should be pointed to receive maximum power over the range, the reticle should be set so that it is lined up with the light at Billerica, and the encoders should be set for elevation +0.22 deg, azimuth 4.59 deg. This should be done on a night when the humidity seems favorable for having the same refraction for the millimeter waves and for visible light. Whether the humidity has indeed been favorable can be checked by observing the reticle position with respect to the moon when the radiometer says that the millimeter beam is pointed at the upper or lower limb of the moon (Sec. III-C). Similar observations at the left or right limb will show whether the millimeter beam on the range has been displaced sideward by reflection from the existing buildings.

APPENDIX B
SOME EFFECTS OF MISALIGNMENTS IN MOUNT

This appendix examines the effects of three departures from mechanical perfection in an azimuth-elevation mount: (1) the azimuth table is not perfectly horizontal; (2) the elevation shaft and azimuth table are not perfectly orthogonal; (3) the boresight is not perpendicular to the elevation shaft. The subject is discussed by K. Stumpff in Geographische Ortsbestimmungen (VEB Deutscher Verlag der Wissenschaften, Berlin, 1955) pages 35-42, and what follows has drawn on his analysis. There is a more condensed treatment in M. L. Meeks, J. A. Ball and A. B. Hull, "The Pointing Calibration of the Haystack Antenna," IEEE Trans. Antennas Propag. AP-16, 746-751 (November 1968); an advantage of the present discussion is that it provides a basis for estimates of the elevations at which it breaks down.

Let τ = tilt of azimuth table

b = tilt of trunnion axis with respect to the horizontal; $b > 0$ if left trunnion is higher than right one

b' = tilt of the trunnion axis with respect to the azimuth table; $b' > 0$ if the left trunnion is farther from azimuth table than the right one

c = collimation error of antenna (the angle between the antenna boresight and a plane normal to the trunnion axis, taken positive if the boresight lies to the right of the plane).

Assume that these four quantities are small (<1°) and constant.

Primed quantities pertain to the coordinate system determined by the antenna structure and its shaft encoders, whereas the angles in the true azimuth and elevation coordinates are not primed. Thus, in Fig. B-1, Z is the zenith, and Z' is the position (on the celestial sphere) of the azimuth axis; z and z' are the actual and instrumental zenith distances of a star at S. The axis of trunnions projected meets the celestial sphere at K.

In the spherical triangle ZZ'K, the law of cosines gives

$$\cos(90° - b) = \cos(90° - b')\cos\tau + \sin(90° - b')\sin\tau\cos(\pi - a_o')$$

$$\sin b = \sin b' \cos\tau + \cos b' \sin\tau \cos a_o' \quad .$$

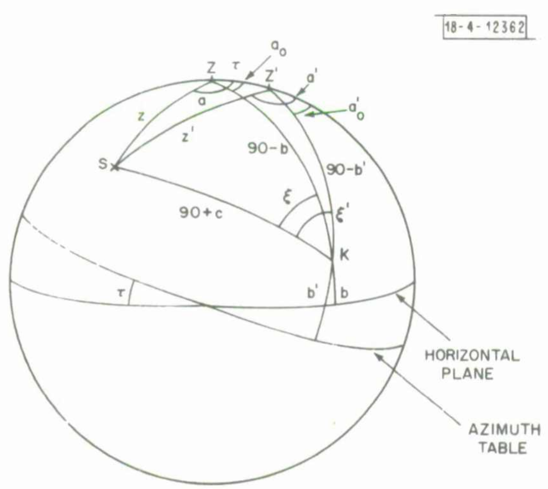

Fig. B-1. Relation of mount to celestial sphere.

Since τ, b and b' are small, this reduces to

$$b \doteq b' + \tau \cos a'_o \quad .$$

Likewise, in the triangle SZ'K

$$\cos(90° + c) = \cos(90° - b') \cos z' + \sin(90° - b') \sin z' \cos(a' - a'_o)$$

$$-c = b' \cos z' - \sin(a'_o - a' + 90°) \sin z' \quad .$$

Since K is the projection of the trunnion axis and S is on the boresight, $a' - a'_o$ is near 90 deg, and we can say

$$c = -b' \cos z' - (a'_o - a' + 90°) \sin z'$$

$$a' - a'_o - 90° = b' \cot z' + c \csc z' \quad .$$

In the triangle ZZ'S, the law of sines gives

$$\frac{\sin a}{\sin z'} = \frac{\sin(\pi - a')}{\sin z}$$

and the law $\sin\alpha \cos B = \cos\beta \sin\gamma - \sin\beta \cos\gamma \cos A$, in which the small letters are sides and the capitals are angles, gives

$$\sin z \cos a - \cos z' \sin\tau - \sin z' \cos\tau \cos(\pi - a') \quad .$$

Since τ is small, these relations take the forms

$$\sin a \sin z = \sin a' \sin z'$$

$$\cos a \sin z \doteq \cos a' \sin z' + \tau \cos z' \quad .$$

Multiplying the first of these by $\cos a'$ and the second by $\sin a'$, and subtracting, eliminates $\sin z'$

$$\sin(a - a') \sin z = -\tau \sin a' \cos z' \quad .$$

This relation shows that $a - a'$ must be small when τ is small, so we can say

$$a \doteq a' - \tau \sin a' \cot z'$$

where the approximation $\sin z \doteq \sin z'$ has been introduced into the last term, which is a small one.

We have already found that

$$a' = 90° + a'_o + b' \cot z' + c \csc z'$$

and making this substitution gives

$$a \doteq 90° + a'_o + \cot z' \left[b' - \tau \sin(90° + a'_o + b' \cot z' + c \csc z')\right] + c \csc z'$$

$$a \doteq 90° + a'_o + \left[b' + \tau \cos(a'_o + b' \cot z' + c \csc z')\right] \cot z' + c \csc z' \quad .$$

Here a is an actual azimuth, a'_o is the azimuth <u>reading</u> of the left end of the trunnion axis, and z' is the <u>reading</u> of the zenith distance of the star S. The equation can usefully be put into simpler form by introducing

$$\delta = b' \cot z' + c \csc z'$$

$$p = \tau \cos a'_o$$

$$q = \tau \sin a'_o$$

where (because τ is a very small angle, which can be thought of as having orthogonal components) p is the azimuth table's inclination parallel to the axis of trunnions, and q is the inclination in a plane perpendicular to that axis.* We then have

$$a \doteq 90° + a'_o + [b' + p \cos\delta - q \sin\delta] \cot z' + c \csc z' \quad .$$

The angles a and a'_o are referred to the line ZZ'; they differ from azimuths referred to north by just the angle from north to ZZ'. (The same angle converts a to true azimuth Az, and a'_o to instrumental azimuth Az', because for the azimuth circle, $z' = \pi/2$ and the corrections for the misalignments vanish.) Also, the instrumental elevation El' is the complement of z'. Consequently, the true azimuth is to be found from the instrumental readings Az' and El' by

$$Az = Az' + [b' + p \cos\delta - q \sin\delta] \tan El' + c \sec El'$$

where b', c, p and q are defined above and

$$\delta = b' \tan El' + c \sec El' \quad .$$

Now seek a comparable relation for El, the true elevation; this means solving for $z' - z$. In the triangle SZ'K,

$$\cos z' = \cos(90° + c) \cos(90° - b') + \cos c \cos b' \cos\xi'$$

$$= -\sin c \sin b' + \left(1 - \frac{c^2 + b'^2}{2}\right) \cos\xi'$$

$$\cos\xi' - \cos z' = cb' + \frac{c^2 + b'^2}{2} \cos\xi'$$

$$2 \sin\frac{z' - \xi'}{2} \sin\frac{z' + \xi'}{2} = cb' + \frac{c^2 + b'^2}{2} \cos\xi' \quad .$$

Because b' and c are both small, the left side must be small, which means $z' - \xi'$ is small. (The geometrical argument is that $z' \doteq \xi'$ because the sides adjoining z' are nearly quadrants.) Consequently, the last equation is well approximated by

$$z' - \xi' = cb' \csc\xi' + \frac{c^2 + b'^2}{2} \cot\xi' \quad .$$

Using triangle SZK in the same way gives

$$z - \xi = cb \csc\xi + \frac{c^2 + b^2}{2} \cot\xi \quad .$$

In this pair of equations, the right sides are of second order unless ξ and ξ' are small, which happens only for large elevations. For elevations not too large, $z' \doteq \xi'$ and $z \doteq \xi$. It is of interest to find out how large an elevation is too large.

---

*Actually, this interpretation is a trifle loose, because it ignores the effects of the alignment errors, but for any usable mount it is a valid approximation.

Suppose that $c = b = b' = 1$ deg of arc; which is 0.0174 radian. Then z and ξ differ by 5×10^{-4} radian, which is half a beamwidth of the 28-foot dish at 35 GHz when ξ is about one radian. This means that equating z' to ξ' and z to ξ would cause appreciable error in pointing at elevations higher than about 30 deg. However, values of c, b, and b' as large as one degree should not be encountered in practice. It is to be hoped that in a mount like ours, the assumption that $c = b = b' = 1$ minute of arc is more realistic. For these values of the alignment errors, one finds that $z - \xi$ = half a beamwidth when $\xi = 1$ minute of arc. If the alignment errors have magnitudes as expected, therefore, we can use with confidence, at elevations from 0 to almost 90 deg, the approximations

$$z' \doteq \xi' \quad \text{and} \quad z \doteq \xi$$

and we can find $z' - z$, the correction to the elevation reading, by finding $\xi' - \xi$. For the triangle Z'ZK, the law of sines gives

$$\frac{\sin(\xi' - \xi)}{\sin \tau} = \frac{\sin(180° - a'_o)}{\sin(90° - b)}$$

$$\sin(\xi' - \xi) = \sin\tau \sin a'_o \sec b \quad .$$

Replacing the sines by the small angles and sec b by 1, we have

$$z' - z = \tau \sin a'_o \quad .$$

In terms of notation already introduced,

$$z' - z = q \quad .$$

Consequently,

$$El = El' + q$$

where q is the inclination of the azimuth table along a line perpendicular to the axis of trunnions. For low elevations, this relation is obvious, and its failure near the zenith is also obvious; the analysis shows that for plausible values of the alignment errors, the relation fails only for elevations very near the vertical.

Supplementary Note:– In the calculation for the azimuth correction, sin z was replaced by sin z'. To see that this substitution is legitimate, consider again a pair of equations that preceded it,

$$\sin a \sin z = \sin a' \sin z'$$

$$\cos a \sin z \doteq \cos a' \sin z' + \tau \cos z' \quad .$$

Multiply these respectively by sin a' and cos a' and add. The result is

$$\sin z \doteq \sin z' + \tau \cos a' \cos z'$$

because $\cos(a - a') \doteq 1$. Therefore, sin z' differs from sin z by an amount smaller than τ. Since the substitution of sin z' for sin z was made in a term that was already multiplied by the small quantity τ, the substitution introduces no appreciable error.

APPENDIX C
COMPUTER PROGRAMS

Planetary altazimuth positions were obtained using the Mason emphemeris program as modified and used by Drs. I. I. Shapiro and M. Ash of Group 63. The program is used to calculate planetary altazimuth positions for many radioastronomy observatories including Haystack and Arecibo, and probably will continue to be available in the future. Only the antenna site position and elevation need be altered to adapt the radiometry mode control program for the 28-foot antenna.

Star positions were calculated in another program written by one of us. Essentially the program takes the local hour angle and declination of the star and transforms the position into azimuth and elevation components. Star coordinates in right ascension and declination and the Julian day of observation are used to calculate the local hour angle. Then, the azimuth and elevation are calculated using the formulas

$$\sin(\text{elevation}) = \cos(\text{declination}) \cdot \cos(\text{local hour angle}) \cdot \cos(\text{latitude}) + \sin(\text{dec}) \cdot \sin(\text{lat})$$

$$\tan(\text{azimuth}) = \frac{\cos(\text{dec}) \cdot \sin(\text{lha})}{[\cos(\text{dec}) \cdot \cos(\text{lha}) \cdot \sin(\text{lat}) - \sin(\text{dec}) \cdot \cos(\text{lat})]}$$

Necessary input data cards are as follows:

(1) Minimum elevation of star card having format (2X, F5.2). The angle must be in degrees.

(2) Observation period card having a format of (2(2I3, 3X), 15X, I2). It contains the following information; the bars indicate blank columns: _XX_XX_TO_YY_YY,_at_intervals_of_ZZ_minutes. _XX_XX and _YY_YY are the beginning and ending times on the 24-hour clock denoting local time and _YY_YY may be smaller than _XX_XX. _ZZ is in minutes and defines the time interval between observations.

(3) Star data cards having a format of (5A4, 2(6X, 3I3)). From one to two hundred star data cards may be inserted here. Star cards used are listed following the program listing at the end of this appendix. The first twenty spaces contain information about the star and the next thirty spaces contain the right ascension and declination. Notice when preparing the cards that the right ascension is in hours, minutes, and seconds while the declination is in degrees, minutes, and seconds.

(4) A blank card which tells the computer that all star data cards have been read.

(5) Observation day cards. Each card has two dates punched in the form:

_MM_DD_YYYY,_JJJJJJJ.J,_

where MM is the month, DD is the day and YYYY is the year. JJJJJJJ.J is the corresponding Julian day number. The first date on each card must always be there, otherwise the computer reads a blank card and thinks that computations are complete. The second date, if omitted, does not end a run, but rather tells the computer to compute positions for only the observation interval for the first date.

(6) A blank card which tells the computer that all position calculations are completed.

```
      DIMENSION N(12),HOLL(200,5),NRA(200,3),NOECL(200,3),RA(200,3),
     2DECL(200,3),ALT(6),AZI(6),TIM(6),EN(12),LTIM(6),JTIM(6),SHA(200)
      IMPLICIT REAL*8 (A-H,O-Z)
      PI=3.1415927
      TWOPI=2.0*PI
      PI2=PI/2.0
      PI32=3.0*PI2
C   GREENWICH HOUR ANGLE FOR 0 HRS 1 JAN, 1968
      GHA068=(2.0*PI*((6.0+34.0/60.0+56.143/3600.0)/24.0))
C   JULIAN DAY NUMBER
      DAY068=2439855.0
      DEGREE=(2.0*PI)/360.0
      TIMON=(2.0*PI)/24.0
C   GEOCENTRIC POSITION OF OISH
      SLONG=71.267*OEGREE
      SLAT=42.462*DEGREE
      SINLAT=DSIN(SLAT)
      COSLAT=DCOS(SLAT)
      PLAT=PI/2.0-SLAT
      SPLAT=-PLAT+2.0*PI/36.0
C   NUMBER OF STARS=JSTAR
C  REAO IN TIME TO BEGIN AND TIME TO ENO ANO TIME INTERVAL
      REAO (5,115) ELVMIN
  115 FORMAT (2X,F5.2)
      ELVMIN=ELVMIN*OEGREE
      SINMIN=DSIN(ELVMIN)
      READ (5,100) (N(I),I=7,11)
      IF (N(11).GT.0) GO TO 40
      WRITE 112
      GO TO 25
C   FINOING NUMBER OF TIME INTERVALS
   40 OO 8 IIII=1,12
    8 EN(IIII)=N(IIII)
      TIMELO=(EN(7)+EN(8)/60.0)
      TIMEL1=(TIMELO+5.0)*TIMON
      TIMEL2=((EN(9)+EN(10)/60.0)+5.0)*TIMON
      TILT=EN(11)/60.0
      IF (TIMEL2.GT.TIMEL1) GO TO 19
      TIMEL2=TIMEL2+2.0*PI
   19 OELTAT=TILT*TIMON
      DIF=(TIMEL2-TIMEL1)/DELTAT
      KK=DIF+1.0
      KJ=(KK+5)/6
C   INPUT OF STAR RA AND DECL POSITIONS
      WRITE (6,62)
   62 FORMAT (1H1,60X,'STAR POSITIONS'/
     250X,' STAR      RIGHT ASCENSION      DECLINATION')
      DO 28 I=1,200
      READ (5,102) (HOLL(I,IB),IB=1,5),(NRA(I,K),K=1,3),(NOECL(I,M),M=
     21,3)
      WRITE(6,1020) (HOLL(I,IB),IB=1,5),(NRA(I,K),K=1,3),(NOECL(I,M),M=
     21,3)
      NRAR=NRA(I,1)+NRA(I,2)+NRA(I,3)
      IF (NRAR.GT.0) GO TO 28
      JSTAR=I-1
      GO TO 14
   28 CONTINUE
C MAKING NEGATIVE OECLINATIONS COMPLETELY NEGATIVE
   14 OO 1 II=1,JSTAR
      IF (NDECL(II,1).GT.0) GO TO 22
      NDECL(II,2)=-NDECL(II,2)
      NDECL(II,3)=-NOECL(II,3)
```

```
   22 CONTINUE
C  CHANGING DEGREES,MINUTES,SECONDS TO RADIANS
      DO 2 III=1,3
      RA(II,III)=MRA(II,III)
    2 DECL(II,III)=NDECL(II,III)
      RA(II,1)=(2.0*PI*(RA(II,1)+RA(II,2)/60.0+RA(II,3)/3600.0))/24.0
      SHA(II)=2.0*PI-RA(II,1)
    1 DECL(II,1)=(DECL(II,1)+DECL(II,2)/60.0+DECL(II,3)/3600.0)*DEGREE
      WRITE (6,111)
      WRITE (6,64)
   64 FORMAT (' '/////' '50X,' STAR POSITIONS IN DEGREES')
      DO 67 IA=1,JSTAR
   67 WRITE (6,66) (HOLL(IA,IK),IK=1,5),RA(IA,1),DECL(IA,1)
   66 FORMAT (40X,5A4,10X,2(F9.5,10X))
C  READ IN DAY POSITIONS
   24 CONTINUE
      READ (5,101) (N(I),I=1,3),DAYNOW,(N(I),I=4,6),DAYFUT
  101 FORMAT (2(2I3,I5,XX,F9.1,XXX))
      IF (N(1).EQ.0) GO TO 25
      NDAY=DAYFUT
      NOWDAY=DAYNOW
      DO 99 MODAY=NOWDAY,NDAY
      DAYNOW=MODAY
      WRITE (6,111)
  111 FORMAT ('1')
C  CORRECTION BY JULIAN DAY
      GHAVEQ=GHA068+0.002737909*(DAYNOW-DAY068)*2.0*PI
C  PREPARING TO CALCULATE STAR POSITIONS FOR GIVEN DAY
      DO 3 I=1,JSTAR
      JM=0
      EJM=0.0
      SPRINT=0.0
      PPRINT=0.0
      D=DECL(I,1)
      SINDEC=DSIN(D)
      COSDEC=DCOS(D)
C  PRINT OUT INFO
      WRITE (6,103) (HOLL(I,IJI),IJI=1,5),(N(IJJ),IJJ=1,3),DAYNOW,(N(IQ
     2),IQ=7,11)
      WRITE (6,1021) (NRA(I,K),K=1,3),(NDECL(I,M),M=1,3)
      WRITE (6,104)
      DO 4 JJ=1,KJ
      IF (DECL(I,1).GT.SPLAT) GO TO 16
      WRITE (6,108)
      GO TO 3
   16 CONTINUE
C  CALCULATION OF LOCAL HOUR ANGLE
      DO 5 JI=1,6
      TIM(JI)=TIMEL1+(EJM*DELTAT)
      IF (JM.GT.KK) GO TO 6
      GHA=GHAVEQ+TIM(JI)*(1.002737909)
      HOAN=GHA+SHA(I)-SLONG
C  CALCULATION OF AZIMUTH AND ELEVATION
      SINHOA=DSIN(HOAN)
      COSHOA=DCOS(HOAN)
      CALC1=SINDEC*SINLAT+COSDEC*COSHOA*COSLAT
      ALTO=DARCOS(CALC1)
      ALT(JI)=PI2-ALTO
      IF (ALT(JI).LT.ELVMIN) GO TO 33
   36 CALC2=(COSDEC*SINHOA)/(COSDEC*COSHOA*SINLAT-SINDEC*COSLAT)
      ALT(JI)=ALT(JI)/DEGREE
      AZI(JI)=DATAN(CALC2)/DEGREE
      IF (ALT(JI).LT.0.0) GO TO 6
```

```
      GO TO 7
    6 LTIM(JI)=0
      JTIM(JI)=0
      ALT(JI)=0.0
      AZI(JI)=0.0
      GO TO 11
   33 CONTINUE
      DO 34 IX=JM,KK
       HOAN=HOAN+DELTAT
      COSHOA=DCOS(HOAN)
      CALC1=SINDEC*SINLAT+COSDEC*COSHOA*COSLAT
      IF (CALC1.GT.SINMIN) GO TO 35
   34 IXO=IX+1
      IF (HOAN.GE.PI32) GO TO 41
      SHOAN=HOAN-5.0*DELTAT
      IF (SHOAN.LE.PI2) GO TO 41
      WRITE (6,107)
      GO TO 3
   41 DO 37 IX=JI,6
      LTIM(IX)=0.0
      JTIM(IX)=0
      ALT(IX)=0.0
   37 AZI(IX)=0.0
      DO 39 LL=1,6
   39 ALLT=ALT(LL)+ALLT
      IF (ALLT.LT.1.0) GO TO 42
      WRITE(6,105) (LTIM(MO),JTIM(MO),AZI(MO),ALT(MO),MO=1,6)
   42 WRITE (6,109)
      GO TO 3
   35 JM=IXO
      EJM=JM
      TIM(JI)=TIMEL1+EJM*DELTAT
      JJ=(JM+5)/6
      GO TO 36
    7 CONTINUE
      TIME=TIMELO+EJM*TILT
      LTIM(JI)=TIME
      ELLTIM=LTIM(JI)
      JTIM(JI)=(TIME-ELLTIM)*60.0
      IF (JM.LT.1) GO TO 31
      JTIM(JI)=JTIM(JI)+1
      IF (JTIM(JI).LT.60) GO TO 31
      JTIM(JI)=JTIM(JI)-60
      LTIM(JI)=LTIM(JI)+1
   31 CONTINUE
C    CALCULATION TO PLACE THE AZIMUTH ANGLE IN THE PROPER QUADRANT
C   IF TRANSFER TO 9, MEANS THAT ANGLE IN FIRST OR SECOND QUADRANT
C IF TRANSFER TO 10, MEANS THAT IN THIRD OR FOURTH QUADRANT
C THIS IS ACCOMPLISHED BY USING THE LOCAL HOUR ANGLE FOR REFERENCE
C   LOGIC IS THAT ARCSIN(X)>0 IN LST AND 3RD QUADRANTS AND <0. IN 2ND AND
C   AND ALSO 0<LOCAL HOUR ANGLE<180. AZIMUTH ANDLE IN 3RD AND 4TH QUADRAN
C   AND IF LHA BETWEEN 180.0 AND 360.0, THEN AZIMUTH ANGLE IS IN 1ST OR 2
      LHOAN=HOAN/(2.0*PI)
      HLHOAN=LHOAN
      HOAN=HOAN-HLHOAN*2.0*PI
      MHOAN=HOAN/PI
      IF (MHOAN.EQ.1) GO TO 9
      GO TO 10
    9 IF (AZI(JI).GT.0.0) GO TO 11
   12 AZI(JI)=AZI(JI)+180.000
      GO TO 11
   10 IF (AZI(JI).GT.0.0) GO TO 12
      AZI(JI)=AZI(JI)+360.000
   11 CONTINUE
      JM=JM+1
      EJM=JM
```

```
    5 CONTINUE
      DO 21 LI=1,6
      IF (LTIM(LI).LT.24) GO TO 21
      LLTIM=LTIM(LI)/24
      LTIM(LI)=LTIM(LI)-24*LLTIM
      IF (SPRINT.GT.0.0) GO TO 21
      DAYNEW=DAYNOW+1.0
      NN=N(2)+1
C  ROUTINE TO CHANGE DATE PROPERLY
      IF (NN.LT.29) GO TO 1001
      IF (NN.EQ.29) GO TO 290
      IF (NN.EQ.30) GO TO 1001
      IF (NN.EQ.31) GO TO 310
      IF (N(1).NE.12) GO TO 1000
      N(3)=N(3)+1
      NN=1
      N(1)=1
      GO TO 1001
  290 IF (N(1).NE.2) GO TO 1001
      NEN=N(3)/4
      LEAP=4*NEN
      IF (N(3).NE.LEAP) GO TO 1000
      GO TO 1001
  310 IF (N(1).EQ.4) GO TO 1000
      IF (N(1).EQ.6) GO TO 1000
      IF (N(1).EQ.9) GO TO 1000
      IF (N(1).EQ.11) GO TO 1000
      GO TO 1001
 1000 NN=1
      N(1)=N(1)+1
 1001 CONTINUE
      WRITE (6,110) N(1),NN,N(3),DAYNEW
  110 FORMAT (' DAY IS NOW',3I5,' JULIAN DAY',F11.1)
      SPRINT=1.0
   21 CONTINUE
   15 WRITE(6,105) (LTIM(MO),JTIM(MO),AZI(MO),ALT(MO),MO=1,6)
      PPRINT=1.0
      IF (JJ.EQ.KJ) GO TO 32
    4 CONTINUE
    3 CONTINUE
   99 N(2)=N(2)+1
      GO TO 24
   32 IF (JM.GT.KK) GO TO 3
      JJ=JJ-1
      GO TO 4
   25 CONTINUE
      WRITE (6,106)
  100 FORMAT (2(2I3,3X),15X,I2)
  102 FORMAT (5A4,2(6X,3I3))
 1020 FORMAT (40X,5A4,2(6X,3I3))
 1021 FORMAT ('+',50X,'RA:',3I3,', DECL: ',3I3)
  103 FORMAT(////' AZIMUTH-ELEVATION COORDINATES FOR ',5A4/' GMT DATE',
     22I3,I5,' JULIAN DAY ',F10.1/' START',2I3,' END',2I3,'  INCREMENT'
     3,I3,' MINUTES')
  104 FORMAT (' ',6('  LT AZIMUTH ALTITUDE')/
     2' ',6(' HR MIN   DEG    DEG '))
  105 FORMAT (' ',6(I3,I3,X,F7.3,X,F6.3))
  106 FORMAT (' END OF DATA')
  107 FORMAT (' RA OF STAR NOT ABOVE HORIZON DURING TIME OF OBSERVATI
     6ON.')
  108 FORMAT (' DECLINATION OF STAR IS TOO FAR SOUTH TO BE SEEN ')
  109 FORMAT (' ALTHOUGH RIGHT ASCENSION OF STAR IS OBSERVABLE, STAR IS
     2 TOO FAR SOUTH TO BE SEEN NOW')
  112 FORMAT (' TIME INCREMENT MISSING AND CAUSES INFINITE LOOP.
     2 EXECUTION DELETED ')
      STOP
      END

T=0.40/4.84 10.58.16
```

STAR	STAR POSITIONS RIGHT ASCENSION	DECLINATION
ALPHA AND (2.1) A3P	0 6 41	28 54 30
BETA CASS ((2.4) F5	0 7 24	58 58 4
GAM PEG (2.9) B2	0 11 32	15 0 1
ALPHA CASS (2.3) K0	0 38 37	56 21 24
BETA CENT (2.2) K0	0 41 56	18 10 3
BETA AND (2.4) M0	1 7 52	35 26 46
DELTA CASS(2.8) A5	1 23 38	60 3 52
BETA ARE (2.7) A5	1 52 49	20 38 51
GAM1 AND (2.3) K0	2 1 52	42 10 20
A U MI (2.1) K2	2 0 57	89 6 43
ALPHS ARI (2.2) K2	2 5 19	23 18 26
ALPHA CET (2.8) M0	3 0 56	3 57 41
BETA PER (2-3) B8	3 6 1	40 49 47
A PER (1.9) F5	3 21 57	49 44 43
ETA TAU (3.0) B5P	3 45 31	24 0 16
CSI PER (2.9) B1	3 52 3	31 47 13
ALDEBARAN(ATAU)1.1	4 34 2	16 26 40
IOTA AUR (2.9) K2	4 54 51	33 6 56
BETA EUR (2.9) A3	5 6 14	-5 7 40
RIGEL (BORI) 0.3	5 12 57	-8 14 19
CAPELLA (A AUR)0.2	5 14 15	45 57 59
GAM ORI (1.7) B2	5 23 22	6 19 16
BETA TAU (1.8) B8	5 24 12	28 34 53
DELTA /RI (2.5) B0	5 30 19	0 19 20
IOTA ORI (2.9) OE5	5 33 49	-5 55 49
EPS ORI (1.7) B0	5 34 32	-1 13 18
KSI TAU (3.0) B3P	5 39 6	-1 57 31
KAPPA ORI (2.2) V0	5 46 11	-9 40 48
BETELGEUSE(AORI)0-1	5 53 23	7 24 8
BETA AUR (2.1) A0P	5 57 28	44 56 46
THETA AUR (2.7) K0	5 57 28	37 12 43
GAMMA GEM (1.9 A0	6 35 48	16 25 44
SIRIUS(A CMA) -1.6	6 43 42	16 40 10
EPS C MA (1.6) B1	6 57 20	28 55 34
CASTOR(A GEM) 1.6	7 32 30	31 57 43

STAR	STAR POSITIONS RIGHT ASCENSION	DECLINATION
PROCYRON (A CMI)0.5	7 37 35	5 18 39
POLLUX (B GEM) 1.2	7 43 18	28 6 27
REGULUS (A LE/) 1.3	10 6 37	12 7 45
A U MA 1.9	11 1 42	61 55 46
SPICA (A VIR) 1.2	13 23 27	10 59 23
ETA 1 MA (1.9) B3	13 46 15	43 28 39
ETA BOO (2.8) G0	13 53 7	18 33 45
ARCTURUS(A BOO) 0.2	14 14 9	19 21 12
GAMMA BOO (3.0) F0	14 30 45	38 27 6
EPS BOO (2.7) K0	14 43 33	27 12 45
ALPHA2 LIB (2.9) A3	14 49 3	15 54 21
BETA U MI (2.2) K5	14 50 47	74 17 25
BETA LIB (2.7) B8	15 15 14	-9 15 45
ALPHA CR B (2.3) A0	15 33 17	26 49 29
ALPHA SER (2.7) K0	15 42 38	6 31 41
DELTA OPH (3.0) M0	16 12 37	-3 36 39
ETA DRA (2.9) G5	16 23 32	61 35 37
ANTARES (A SCO) 1.2	16 27 22	26 21 38
B HER (2.8) K0	16 28 48	21 33 37
KSI OPH (2.7) B0	16 35 20	10 30 6
KSI HER (3.0) G0	16 40 3	31 39 41
BETA DRA (3.0) G0	17 23 41	52 19 30
A OPH (2.1) A5	17 33 24	12 34 58
BETA OPH (2.9) K0	17 41 50	4 34 46
GAMMA DRA (2.4) K5	17 55 50	51 29 31
VEGA (A LYR) 0.1	18 35 49	38 45 7
ALTAIR (A AQ 6) 0.9	19 49 10	8 46 48
GAM CYG (2.3) F8P	20 21 3	40 9 1
DENEB (A CYG) 1.3	20 40 18	45 9 42
E CYG (2.6) K0	20 44 53	33 50 45
FORMALHAUT(APSA)1.3	22 55 50	21 47 51
A CEP (2.6) A5	21 17 48	62 26 43
E PEG 2.5 K0	21 42 34	9 43 22
B PEG (2.6) M0	23 2 10	27 54 12
A PEG (2.6 A0	23 3 7	15 1 38

APPENDIX D
ANTENNA FOCUSING

There are two nearly separate adjustments for focusing the antenna, i.e., for placing correctly the image of the waveguide aperture that is formed by reflection in the secondary reflector. One adjustment puts the axis of the waveguide aperture on the axis of the dish, and the other locates the image at the right point along the axis. Both adjustments use the Billerica transmitter as a source.

Motion perpendicular to the axis of the dish is controlled by turnbuckles. When these are set properly, the sidelobes close to the main lobe are approximately symmetrical about the main lobe. If this adjustment has to be disturbed, one should put into the waveguide aperture a plastic plug with a little ball on the end, and sight on the ball through telescopes clamped to the pads provided on the rim of the dish for that purpose. When the adjustment is to be restored, the telescopes will show when the end of the plug is in its former position.

The axial adjustment is made (at least in first approximation) by maximizing the gain of the antenna. The six miles of our antenna range do not suffice to put the range transmitter in the far field. In other words, when the antenna is focused so that the gain on the range is maximized, the gain for signals from a greater distance is not maximized.

Probably the final adjustment, for maximum gain at infinity, could be made by maximizing the radiation received from the moon and detected with the radiometer. However, the resulting pattern and gain could not be measured with the moon as a source. It is possible to focus and measure gain and pattern on the range, and then to adjust for the far field by computation. The problem has been treated by Cheng and Moseley, Trans. IRE, PGAP AP-4, No. 4, 214-216 (October 1955) and by Karachun, Kuzmin and Salomonovich, Radioteknika i Electronika 6, 430-436 (March 1961); the following way of viewing the problem may be helpful.

In Fig. D-1, the x-axis is the axis of the paraboloid of revolution. By definition of a parabola, the focus is at (f, 0) such that $r = f + x$. In consequence, $y^2 = 4fx$. The transmitter is in the

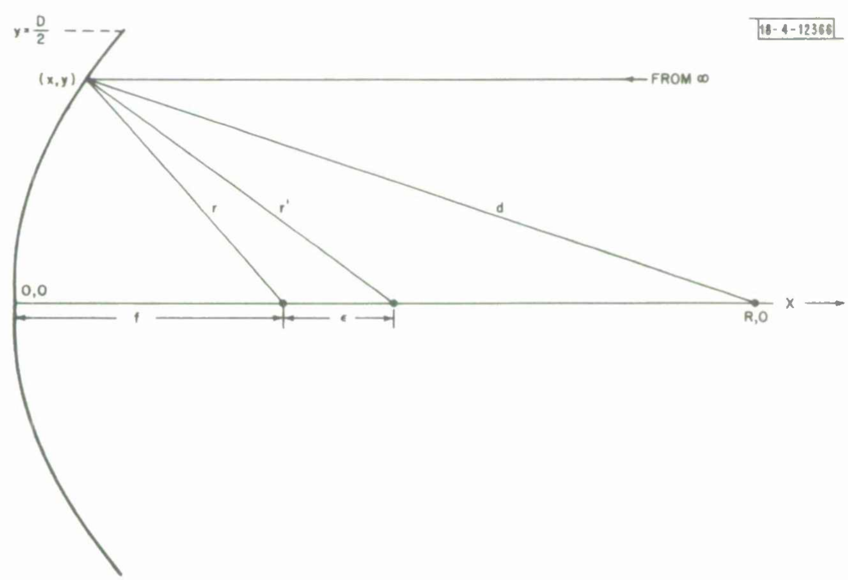

Fig. D-1. Defocusing of ray originating in near field.

near field at (R, 0). Let a ray from there strike the mirror, in the vertical plane through the axis, at (x, y) and cross the axis, after reflection, at $(f + \epsilon, 0)$. If we neglect ϵ^2,

$$r'^2 = (f + x)^2 + 2\epsilon(f - x)$$

and

$$r' = (f + x)\sqrt{1 + 2\epsilon(f - x)/(f + x)^2}$$

$$\doteq f + x + \epsilon\,\frac{f - x}{f + x} \quad .$$

Likewise,

$$d^2 = y^2 + (R - x)^2$$

$$= (R - x)^2\left[1 + \frac{y^2}{(R - x)^2}\right]$$

so that

$$d \doteq (R - x)\left[1 + \frac{2fx}{R^2}\right] \quad .$$

In our situation, the term in x^2/R^2 does not exceed 10^{-6}, and the other terms are much larger, so it can be dropped. The total path, then, has length

$$d + r' = R + f + (2fx/R) + \epsilon(f - x)/(f + x) \quad .$$

The ray reflected at (x, y) will interfere constructively with the axial ray if their path lengths are the same, i.e., if

$$R + f + \epsilon = R + f + (2fx/R) + \epsilon(f - x)/(f + x) \quad .$$

We see at once that this happens for all x if R is very large and $\epsilon = 0$, but that when $2fx/R$ is an appreciable fraction of a wavelength, the interference is perfectly constructive for only a single nonzero x. In other words, a parabola does not focus perfectly unless the source is at infinity. For a source at a finite distance R, there is perfectly constructive interference at $(f + \epsilon, 0)$ between the axial ray and the ray for which

$$\epsilon\left[1 - \frac{f - x}{f + x}\right] = \frac{2fx}{R}$$

which means

$$\epsilon = \frac{f^2}{R}\left[1 + \frac{y^2}{4f^2}\right] \quad .$$

If we knew the value of y that gives perfectly constructive interference when the overall gain is a maximum, then we could focus for maximum gain on Billerica and then displace the feed toward the dish by an amount ϵ calculated from the last equation; then the antenna would be focused on infinity. However, we have had no exact knowledge of what value of y to use for this purpose.

Karachun, et al., and Cheng and Moseley (who use a more elaborate approach based on the diffraction integral) both evaluate ϵ, in effect, by selecting $y = D/2$, so that the phase is matched for the axial and the peripheral rays. However, this seems like not the best choice, because if curvature of the wave front is compensated correctly at the rim of the dish, it is overcompensated everywhere else.

It is seen from above that the path difference between the axial ray and any other ray is

$$\Delta = 2fx/R + \epsilon(f - x)/(f + x) - \epsilon \quad .$$

Manipulation gives

$$\Delta = \frac{2fx}{R} - \frac{2\epsilon x}{f + x} \quad .$$

The phase error resulting from this path difference is plotted in Fig. D-2. The abscissa is the square of the distance of the reflecting point from the axis, because the contribution of each

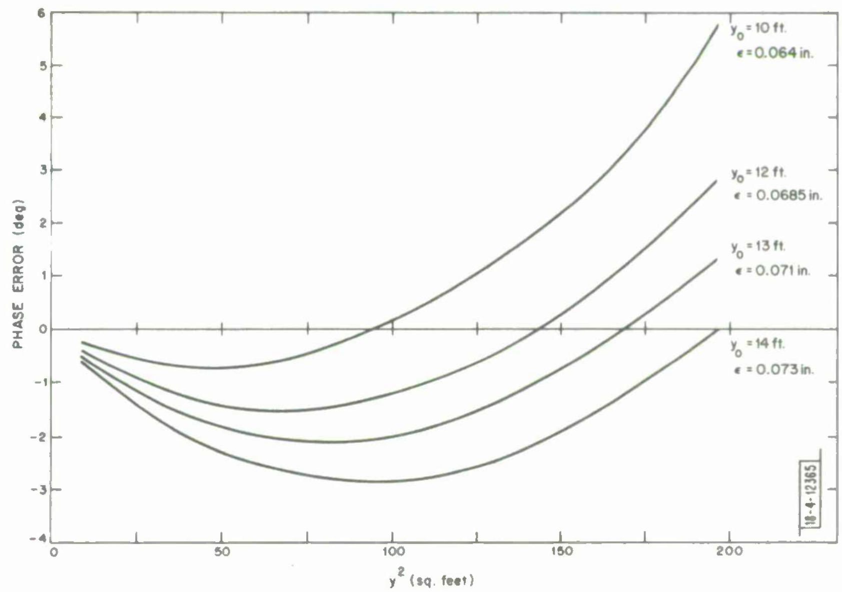

Fig. D-2. Phase error as a function of distance y from axis of paraboloid, for various positions of secondary reflector, which are chosen to make the phase error vanish at different radii y_o.

annulus is proportional to its area. Each curve relates to a particular ϵ, chosen so that y_o, the radius for which the phase difference is zero, has a preassigned value. Positive phase differences represent undercompensation for curvature of the wave front. The curves show that the tightest bound on the phase error is achieved when ϵ = 0.070 inch.

We have assumed that when the antenna is focused for maximum gain on Billerica, the image of the feed aperture is 0.070 inch from the focal point of the dish. Therefore, after focusing for maximum gain, we have moved the feed 0.070 inch closer to the dish and called it focused for infinity.

The criterion of Cheng and Moseley and of Karachun, et al., namely, that $y_o = D/2$, leads to the same result for our analysis as it does for theirs. As Fig. D-2 shows, their criterion implies that ϵ = 0.073 inch. This is so close to our value that the difference (0.01 λ) is not important, since the gain changes only slowly as the feed is moved near the point of maximum gain.

In fact, analysis* shows that for our range, if we focused for maximum gain on Billerica and made no further adjustment at all, the gain for a 35-GHz source at infinity would not differ from the maximum by more than 0.3 dB. We have therefore been unnecessarily detailed in our analysis, which is presented here because it clarifies some points that were cloudy for us after we had read the articles cited.

The foregoing says nothing about taper of the feed pattern. Figure D-3 shows that taper has very little effect. The reason is that even with illumination at the rim 12 dB less intense than at the center, the field near the rim dominates, because most of the area of the dish is near the rim.

A focusing adjustment of 0.070 inch can confidently be expected to give a far-field pattern similar to that measured on the range with the antenna adjusted for maximum gain on the range.

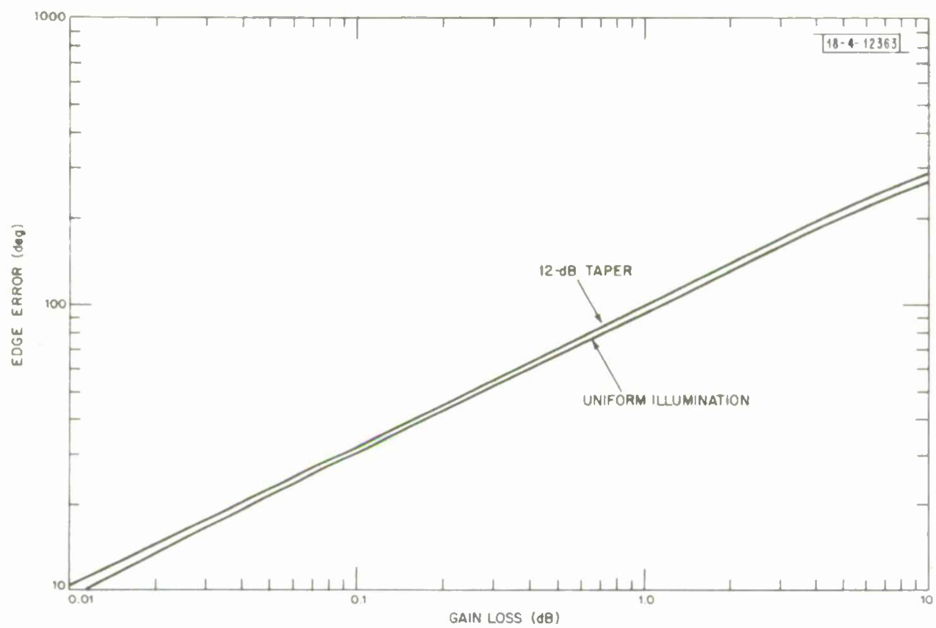

Fig. D-3. Loss in gain of circular aperture because of defocusing, for uniform and tapered illuminations. Assumed illumination varies as $\exp[-\alpha y^2 - j\beta y^2]$. (This figure was supplied by Dr. John Ruze).

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*S. Silver, Microwave Antenna Theory and Design, Radiation Laboratory Series, Vol. 12 (McGraw-Hill, New York, 1949) Chapter 6, especially p. 199.

APPENDIX E
THE TELESCOPE

The telescope (Lincoln Laboratory Drawing No. S-23510) has several weaknesses that should be recorded here, so that they may be avoided or remedied if the installation is used again.

The object lens, which was purchased from A. Jaeger, Lynbrook, New York, is an achromat of good quality. Its focal length is 24.5 inches. The relay lens that puts the image on the Vidicon of the TV camera is what limits the resolution of the system. For this function, one needs a lens that is designed to work at small object distance. The one that was purchased was believed to be of this character, because it is mounted for use in an enlarger. However, a test by J. A. Daley of Group 21 showed that this 50 mm lens is designed for object distances that are large; it is merely a camera lens mounted to fit into an enlarger. Its quality, even as a camera lens, is poor when it is wide open (f2.8) but much better at f4. When it is stopped down more, the image deteriorates because of diffraction.

The resolution of this lens is less than the resolution of the Vidicon. Replacement of it with a good copying lens of the same focal length would improve the system.

The reticle that carries the reference circle is positioned by means of a staging made for a microscope. The mechanism is a cheap one, and the stability that is needed in a holder of microscope slides is less than one would like to have in a telescope reticle mount. The stability of the boresighting is not known to be limited by unwanted motion of the staging, which has been reduced by installing a spring, but in any overhaul of the telescope, mounting the reticle on a better pair of cross slides should be given some consideration.

We can give no quantitative statement about the performance of the microscope staging because during the investigation reported here, the shortcomings of the slide were masked by the motions caused by the infrared filter and its mount. The filter is intended to protect the Vidicon when there is a possibility that the sun will come into the field of view. It was supposed to offer complete protection, but when the telescope was used for a short time to view the sun, with no other protection, an image of the reticle was "burned" onto the Vidicon screen. Actually, of course, the reticle image protected the part of the screen on which it fell, it was the rest of the Vidicon screen that changed. We have not changed the tube because its performance against the moon has been satisfactory, but for working with stars, a new Vidicon might give a useful improvement in sensitivity.

For looking at the sun, the effective aperture of the objective lens should be greatly reduced by capping it with a metallized filter or with a brass cover in which there is a $\frac{1}{8}$-inch hole covered with several layers of exposed film. When this is done, the infrared filter is not needed for looking at the sun.

The infrared filter can be swung in and out of the beam. The original intent was that it be left in the beam unless the degradations in sensitivity and in resolution that it introduces are objectionable. In practice, one degradation is objectionable for looking at the stars, and the other is objectionable when looking at the moon, so the only useful aspect of the filter is that, if it is in place, accidental passage of the sun across the field of view will not ruin the Vidicon.

The design of the filter holder is fundamentally unsound, because the stop that limits its motion is rigidly attached to the reticle holder; bumping the filter against its stop can jar the reticle. If the filter is to be used at all, this situation should be remedied.

Another trouble with the filter is that its front and rear surfaces are not parallel; it is, in effect, a prism. After some of the drift scans on Venus (Sec. III-B) had been made, there was an attempt to use Jupiter, which was only 6 deg from the sun. The infrared filter was called into use as a precaution against accidentally pointing at the sun. Subsequent drift scans on Venus showed a shift in the pointing error, as in Fig. E-1. Though this finding was at first ascribed

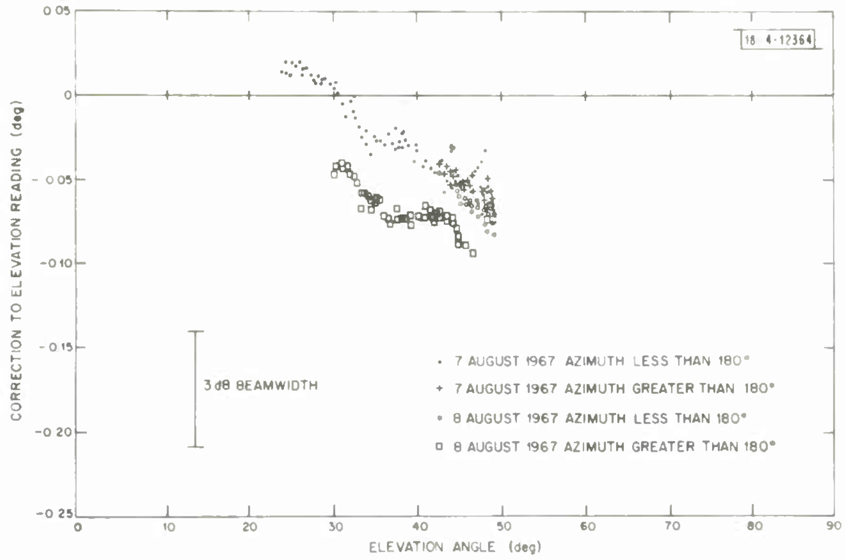

Fig. E-1. Raw data on Venus. Jump was caused by infrared filter, which was moved into beam after observations designated by circles were made. Some of jump may have come from displacement of reticle because of bumping by filter.

wholly to a displacement of the reticle, it prompted inspection of the filter. This was placed on a first-surface mirror that was normal to the axis of an autocollimator; as the filter was rotated, the cross hairs moved. The image formed by rays that had passed through the filter and those that were merely reflected from the front surface could easily be distinguished, because the glass colored them differently. Displacement of the returned beam showed that two-way transmission through the filter, at practically normal incidence, changed the direction of the light by about 4 minutes of arc. Even if the filter holder is modified so that it causes no motion of the reticle holder, this filter will still cause an optical displacement of the reticle by about half a 35-GHz beamwidth.

Another difficulty with the telescope is that when the screws for adjusting the reticle position are turned, nothing happens for a while, and then the reticle takes a leap. The flexible cables are double spirals, so torsion in them is probably not causing trouble. It is likely that they are twisting, and that if they were anchored near their middles, the motion would be smoother.

When the telescope was designed, there was no provision for illuminating the reticle. For the star-pointing experiments described in Sec. III-B, an illuminated reticle would have been a useful aid. As a step toward getting one, the reticle pattern was etched into a suitable glass disk, which is now in the reticle holder. The next step (not yet taken because it would have delayed the work on lunar reflectivity) would be to polish the edge of the dish at two ends of a diameter, and then some small lamps (cylinders about 0.1 inch in diameter) could be set in holes in the reticle holder, in the plane of the reticle. The voltage on the lamps should be controllable from the console in the penthouse.

DOCUMENT CONTROL DATA – R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Lincoln Laboratory, M.I.T.	2a. REPORT SECURITY CLASSIFICATION Unclassified	2b. GROUP None

3. REPORT TITLE

Pointing Accuracy of Lincoln Laboratory 28-Foot Millimeter-Wave Antenna

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
Technical Note

5. AUTHOR(S) *(Last name, first name, initial)*

Gull, Thomas R. and McCue, J.J. Gerald

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
14 February 1969	46	6

8a. CONTRACT OR GRANT NO. AF 19 (628)-5167	9a. ORIGINATOR'S REPORT NUMBER(S) Technical Note 1969-17
b. PROJECT NO. 649L c. d.	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)* ESD-TR-69-11

10. AVAILABILITY/LIMITATION NOTICES

This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
None	Air Force Systems Command, USAF

13. ABSTRACT

Three experiments have provided a rough evaluation of the pointing errors of the 28-foot paraboloid and its mount. One examined the errors introduced by the pedestal, including the azimuth bearing, the second evaluated the pointing of an optical telescope attached to the backup structure, and the third showed that the departure between the telescope boresight and the millimeter-wave boresight is not a large fraction of a beamwidth. The experiments yielded estimates of base tilt, structural sag, elevation-axis skew, and collimation error. When allowance is made analytically for these defects, static pointing of the optical telescope at night has RMS errors of about 0.01 deg on each axis; this quantity is one-seventh of a 35-GHz beamwidth. Appendixes give data on the location of the antenna and the procedure for focusing.

14. KEY WORDS

space communications
radar astronomy
radioastronomy
millimeter waves
microwave antennas