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ASSESSMENT AND DEVELOPMENT OF METHODS OF ACOUSTIC PERFORMANCE PREDICTION FOR JET NOISE SUPPRESSORS

by

D. Middleton and P. J. F. Clark

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SUMMARY

The acoustical behaviour of certain nozzles designed to reduce the exhaust noise from turbo-jet engines has usually been established by direct measurement. The need for a simple yet adequate method of prediction of such behaviour is clear. The present paper reviews the quasi-empirical approaches which have been adopted in the past, and paying particular attention to methods suggested by Eldred to deal with the power spectral density and Lee for deriving directivity patterns, develops these for application to (axisymmetric) nozzles where the elements are not all of the same size. The measure of agreement between predicted levels and typical results quoted in the literature is generally reasonably good. Some implications of the theory are discussed. Additionally, a mathematical model is presented to calculate the noise reduction due to the interference of adjacent twin round jets.

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NOTATION

A	nozzle cross section area
a	speed of sound
b	distance between jets (Sec. 5.1.1)
D	equivalent nozzle diameter (Sect. 5.1.1)
d	nozzle or tube diameter
E	half the distance between the peripheries of similar jets (Sect. 4.2)
e	tube efficiency (Sect. 5.1.1)
f	frequency
K	Proportionality constant in Lighthill's relation (eqn.1)
k	$= K \rho a_c^{-5} U_e^8$ (Sect. 4.1)
$\hat{K}$	empirical constant (Sect. 3)
L	typical length scale
ℓ	eddy length scale
M	Mach number
N	number of tubes in a multi-tube nozzle
n	number of tubes of a given style (Sect. 5.1.1)
P	acoustic power
$\tilde{P}$	pressure ratio at nozzle exit
$\hat{p}$	spatial average r.m.s. sound pressure
R	nozzle radius
r	radius of flow at distance x (Sect. 4.2)
$\tilde{r}$	distance from noise source (Sect. 5.2)
T	absolute temperature
U	jet velocity
u	local time-mean velocity
$\tilde{u}$	turbulent velocity
x	distance downstream from nozzle efflux plane
α	separation parameter = $R/(R + E)$
γ	ratio of specific heats

θ	angle from jet axis
μ	angle defined by sketch near eqn. 5
ρ	density
ϕ	inner mixing angle of jet
ψ	outer mixing angle of jet

SUBSCRIPTS

c	convection
e	exit
i	octave number, or i^{th} nozzle tube, according to context
o	ambient
s	secondary

1. INTRODUCTION

The necessity for lessening the exhaust noise from the jet engines of modern commercial aircraft has meant the devotion of a considerable amount of man-power, time and money in Universities, Government Research Institutes and the aircraft and aero-engine industries to the problems of suppressor-nozzle behaviour. The theories of aerodynamic noise propounded by Lighthill[1,2], Ribner[3] and others, have gone a long way towards elucidating the principle parameters in the generation of jet noise, but the lack of knowledge, either from theoretical or practical approaches, of various quantities associated with the turbulent mixing of jets has made difficult the complete evaluation of the full expressions derived for the acoustic output of a jet. Although behavioural patterns are being steadily brought to light, e.g., the work of Davies, Fisher and Barrett[4] and of Chu[5] on turbulence convection velocities in round jets, only by extensive series of experiments has it been possible to produce quite detailed methods, necessarily semi-empirical, for estimating the noise at any position in the radiative field of an (unsuppressed) turbojet engine, stationary or in flight, e.g., Franken[6] and, more recently, Coles[7] and Kobrynski[8]. The methods of interpolation which their data permit naturally involve the parameters which Lighthill first elucidated in the dimensional analysis of his resultant expression for the far-field noise, though usually with possible modification of their exponents, plus allowance of course for any reflection and atmospheric absorption effects. (Modification of the exponents in Lighthill's expression has also been considered by both Ribner[9] and himself[10,11], in the light of subsequent data).

Although the sound field can therefore be estimated with considerable accuracy for a round convergent nozzle operating within the typical range of conditions for aircraft take-off and cruise, the situation with regard to other designs of exhaust unit is not nearly so satisfactory. Although relatively few tests are necessary to determine the general trend of behaviour for a given type of suppressor nozzle, it has not proved possible to <u>predict</u> the behaviour with too great an accuracy. Several attempts at such prediction have been made, e.g. Greatrex and Brown[12], and Lee and Semrau[13], with varying degrees of success, and it is the purpose of the present paper to introduce an improved method for forecasting the acoustic performance of certain types of nozzle. The method is applied to examples of both corrugated and multi-tubed nozzles which to date have been the form most favoured by aircraft/engine manufacturers. It is evident that any method which can successfully anticipate the structure of the noise field from a given design of nozzle operating under a given condition must prove a powerful tool in an attempt to optimize suppressor nozzle design. This in turn would imply a considerable saving in the ad hoc testing of nozzles and therefore in the time, man-power and money aforementioned, as well as achieving the principal object of the exercise - less noise annoyance to those who live or work close to the take-off routes of modern jet airliners.

2. NOISE PARAMETERS FOR STANDARD JETS

Lighthill's classical analyses[1,2], of the problem of aerodynamically-generated sound were produced at the time that first measurements were being made of the noise radiated from turbulent jets. He was able to derive an expression which formally represented the sound intensity at a point in the acoustic far-field. Assuming the correctness of his hypotheses, the problem would be solved were one able to evaluate this expression, but lack of knowledge of turbulence structure either from a mathematical or an experimental aspect has not made this possible until the more recent approaches of Chu[5] and Jones[14]. Several theoreticians, notably Lilley[15], Ribner[3] and Corcos[16] have tried to adapt or reformulate the equations in an endeavour to produce expressions which can be handled with greater facility from either experimental or theoretical viewpoints. Lighthill himself went on to consider a dimensional analysis of his formula, and deduced that for a jet of moderate Mach number, the total acoustic power output, obtained by integration over a large sphere centred on the jet as 'point source' was given by

$$P = K \frac{\rho^2}{\rho_o} \frac{L^2 U^8}{a_o^5} \qquad (1)$$

where U, L and ρ are respectively typical velocity, length scale and density associated with the flow. ρ_o is the density of the ambient fluid, the velocity of sound in which is a_o and K is the constant of proportionality. Some notation is covered in the list of notation.

The many experiments carried out on round subsonic jets, e.g., as reported by Westley and Lilley[17], Waterhouse and Berendt[18], Greatrex[19], etc., show that this relationship is essentially well-substantiated over a large range of conditions: The L^2 factor may be replaced by the efflux area of the convergent nozzle over a range of at least one thousand (Coles[7]). The correct density to use in the numerator has been the subject of some controversy, good correlation having been found in some cases with $\rho = \rho_o$ (e.g. Howes[20]). Lighthill [11] himself in discussing this point in Appendix A to his Wright Brothers Lecture, suggests an intermediate density associated with the centre of the mixing region, and a comprehensive investigation by Lee's noise research team[21] at the General Electric Company showed that best agreement was obtained using the density at the nozzle efflux. Since the density of the fluid in a jet varies only slowly with operating condition, its effect is usually very difficult to separate from that of velocity which is clearly the dominant parameter in the expression. As typical variations in the ambient speed of sound a_o are small, therefore little data exists on the a_o^{-5} factor effect. The value of K is of the order of 10^{-4}, being constant for any one series of experiments but depending on the amount of initial turbulence.

It is evident that the velocity, entering at some high power, is the quantity dominating the expression for the acoustic output. In the 1961 Bakeran lecture, Lighthill[10] gives consideration to experimental work on turbulence in jets which would indicate that his dimensional analysis should produce a dependence more akin to U^6 than U^8, but he is able to conclude that the correction factor is sufficient to restore the relationship to approximately an eighth power one. It should be observed that this proportionality is intended to refer to jets with moderate Mach number, say of the order of 0.5 to 1.0. Below the lower limit, as Ffowcs-Williams and Gordon[22] have pointed out, noise generated upstream of the nozzle could overwhelm the mixing noise. For the upper limit, when the jet flow becomes sonic, the possibility of shock noise exists as a superimposing phenomenon, and when the turbulent 'eddies' are being convected at a speed higher than the local velocity of sound (which itself requires a flow Mach number in excess of unity), the dependence can be shown to fall to more nearly the third power of the jet velocity (Ffowcs-Williams[23]). In practice, the acoustic output from the exhaust of typical turbojet engines, as used on commercial aircraft, tends to obey the relationship

$$p \sim \frac{\rho^2 A V^n}{\rho_o a_o^5} \qquad \text{where } n = 8.5$$

over the range 1000 ft/sec < V < 1800 ft/sec 2000 ft/sec, with a convergent nozzle being expected to choke at about 1600 ft/sec. Below about 1000 ft/sec velocity, other engine noise sources, e.g., compressor whine, tend to intrude, making measurements of the exhaust situation difficult. Since this range covers virtually all the velocities, both actual and relative, experienced by typical modern airlines during take-off, the crucial condition as far as jet noise suppression is concerned, there was no call for a variation in the value of the velocity index for the calculations reported herein.

The foregoing has dealt with the total acoustic output. Behaving in somewhat similar manner is the overall sound pressure level registered at a particular point in the far field. It remains to give consideration to the directivity of the noise and its spectral content. Lighthill's derivation[2] was adjusted by Ffowcs-Williams[24] to give a directional factor of $(1-M_c \cos\theta)^{-5}$, M_c being the Mach number of the eddy convection, together with a factor due to 'some preferred orientation of the quadrupoles'. Lighthill[10] gives examples of this in his Bakerian lecture. Meanwhile Ribner[3] has separated the output into 'self' noise (non-directional) and 'shear' noise, with directivity $(\cos^2\theta + \cos^4\theta)$, together with a convection factor of $\left\{(1 - M_c \cos\theta)^2 + \delta^2 M_c^2\right\}^{-5/2}$, δ being a 'fluctuation parameter', and an allowance for refraction. Numerical evaluation of the convected wave equation by Schubert[25] is giving good agreement with practical results. Lilley[15] in his analysis also produced some expressions for the directivity of the noise.

With regard to the frequency content, dimensional analysis by Powell[26] showed that the high frequency noise should fall off as f^{-2} whilst the noise in the lower frequencies should increase at least as quickly as f^{+2}. There is, of course, a large intermediate regime, and so that although these forecasts are broadly followed (e.g. Lighthill[10]), there is only limited application for any qualitative procedure. Most of the foregoing points have been considered in detail by Ribner[27] in his review article.

Further results are that about half the noise appears to come from the annular mixing region, (Lighthill [11]) though this has recently been the subject of some controversy[28], and by dimensional analysis, Ribner[29], Lilley[15], and Powell[26] were each able to show that the acoustic power output per unit length of this region was approximately constant, whereas in the fully developed region the output fell off approximately as x^{-7}. These latter results are considered in greater detail in Section 4.

3. REVIEW OF PREVIOUS 'THEORETICAL' ESTIMATIONS OF SUPPRESSOR NOZZLE BEHAVIOUR

Before proceeding to discuss various assessments of suppressor nozzle behaviour, it is of interest to discuss the work of Potter[30], Chu[5] and Jones[14], who appear to have made the only attempts to measure the noise-producing parameters directly. Potter measured the mean velocity, the longitudinal and radial intensities of the turbulence scales for the round and notched sides of a nozzle having a single corrugation. By directly substituting his experimental results into Lilley's expression, Potter was able to calculate the estimated acoustic power output per unit volume of the flow for various stations, and show that the decrease in shear and in turbulence intensity on the corrugated side outweighed the greater mixing volume produced, with the result that the noise output from a corrugated nozzle should indeed be lower than from the round nozzle of equivalent efflux area. He also estimated that the optimum suppressor might be one consisting of four tubes, three encircling the center one.

Chu[5], on the other hand, used the one-dimensional Fourier cosine transform of Lighthill's aerodynamic noise equation (as modified by Proudman[31]) to obtain a formulation more suitable, and then measured the mean velocity profiles and the two-point space-time correlation of the turbulence velocities and their square in a round jet. It was then possible to estimate the basic directivity, the intensity, and the spectrum of both the 'shear' and 'self' noise generated by unit volume of turbulence. The acoustic power estimated for the jet turned out to be about an order of magnitude greater than that obtained by extrapolation from results measured on round jets at velocities higher than the 150 ft/sec value employed by Chu.

Jones[14] employed a Fourier analysis in space and time to Lighthill's equations so that the problem of evaluating the acoustic radiation was reduced to determining the intensity of the fluctuating Reynolds stresses, their spectra, and the eddy volume of each Fourier component. The experimental work was lessened by appealing to the self-preserving nature of jet flow and various estimates. Comparison of the estimates and measurements of 'self'- and shear'-noise gave reasonable agreement, and work was continuing.

The 'philosophy' behind estimation of suppressor nozzle behaviour is expressed by quoting from the work of Lee et al[21]:

'The approach adopted ... pre-supposes that even in the absence of a true physical model there exists a unique and definite relationship between the radiated sound power direction and the distribution of mean flows in the jet field. This by-passes a number of difficulties, e.g. turbulence data, directivity effects, attenuation effects due to propagation of sound through a non-homogeneous fluid and Mach number convection effects'.

He continues, 'Where necessary quasi-empirical constants are used to develop the relationship between acoustics and aerodynamics into a wc .ing form. That this phenomenonological approach may represent a gross oversimplification is quite apparent'.

It is clear that any procedure which uses only the geometrical properties of the suppressor design can give not more than a single estimate for each of the factors such as total acoustic output, directivity, spectral content of the noise, etc. Only when appeal is made to aerodynamic data is it possible to produce results which are dependent on operating condition, and experimental results have shown that suppressor nozzle behaviour, either in absolute terms or relative to that of a standard nozzle is generally a function of engine speed. On the other hand, to be of practical use, the method of prediction must be relatively easy to perform. (Some of the methods to be outlined require a knowledge of mean flows basically rather more difficult to procure experimentally than the corresponding acoustic output).

One of the earliest attempts at correlating suppressor design and acoustic behaviour was by Greatrex and Brown[12], who considered nozzles mainly of the corrugated form. They hypothesized that 'the total acoustic power remained substantially constant irrespective of nozzle shape, but that due to interference some of the noise was re-directed or scattered'. The efflux was considered as made up of two parts, namely, the volume between the efflux plane and the plane at which the individual jets coalesce, and the remainder. This downstream volume is assumed to have precisely the properties of the corresponding circular jet, whereas the first region differs from the equivalent region of the circular jet by a factor of λ which is assumed to be a unique function of N, the number of corrugations'. In this paper N was subsequently replaced by an effectively equivalent parameter, the 'thickness ratio', as the diameter of the largest inscribable circle at the plane of the efflux divided by the diameter of the circular nozzle of equal efflux area. An allowance was also introduced to permit estimation of the effect of a central core of larger diameter than the breadth of a corrugation. The results of these calculations are compared with the peak-to-peak reduction in polar sound pressure level measured for a velocity of 1800 ft/sec on a full-scale turbo-jet engine. The agreement is not unreasonable, but the details given are too sparse to follow through the calculations or to permit any re-evaluation. However, it should be noted that although this method presents a reduction independent of operating condition, the graph of experimental results presented in their paper do not entirely support this.

A second paper by Greatrex[32] was presented some three years later.

The first part dealt mainly with ejector flows, with the hypothesis that

$$\text{noise} \propto \text{core length} \times (V_j - V_a)^n$$

this expression being corrected in the subsequent discussion by dividing by 'the core length for $V_a = 0$'. V_j is the efflux velocity of the primary discharge, V_a is the velocity of the induced flow, and the velocity exponent n is 'around 8 to 10'. This time the 'theoretical' attenuation is plotted against 'the reductions in peak overall noise heard by an observer walking nearly parallel to the jet axis'. The conclusion was that 'the test points are undoubtedly scattered but the agreement between calculation and measurement is very reasonable at the small ejector lengths which can be used in practice. We attribute the high attenuation at larger ejector lengths to a change in directivity of the noise due to multiple reflections inside the ejector'. Similar calculations were carried out for a seven-tube suppressor nozzle exhausting into an ejector, though once again it is not possible to check the details. One result of these calculations was that the 'theoretical' attenuation produced by an ejector decreased as its diameter increased, which is contrary to the outcome of the calculations in the previous paper. An estimate of the attenuation of an ejector in flight was also included.

Contemporary with the Greatrex and Brown[12] paper, Dyer, Franken and Westervelt[33] published an alternative approach to estimating suppressor nozzle behaviour. 'We present here a simple analysis of jet noise reduction due to a combination of a jet with an induced secondary airflow. When the secondary air combines with the primary air of the jet, it forms a new jet stream of larger area and lower velocity. The net results of this new jet of lower velocity may be noise reduction'. Using suffix 1 for the primary discharge, suffix 2 for the induced flow, and suffix 3 for the final (assumed fully-mixed) flow, the solution of the one-dimensional continuity, momentum and energy equations together with the equation of state gives

$$\frac{U_3}{U_1} = \frac{1}{2}\left\{\frac{A_1}{A_3}\left(1-\frac{T_2}{T_1}\right) + \frac{U_2}{U_1} + \left[\frac{A_1^2}{A_3^2}\left(1-\frac{T_2}{T_1}\right)^2 + \frac{U_2^2}{U_1^2} - 2\frac{A_1}{A_3}\left\{\frac{U_2}{U_1}\left(1+\frac{T_2}{T_1}\right) - 2\left(\frac{T_2}{T_1}\right)\right\}\right]^{\frac{1}{2}}\right\} \qquad (2)$$

Using $P \sim \dfrac{\rho_o A U^8}{a_o^5}$ they obtained a formal change in power level of

$$\left(10 \log_{10} \frac{A_3}{A_1} + 80 \log_{10} \frac{U_3}{U_1}\right) \text{ dB}$$

On Strouhal number considerations, they suggested that frequencies transform according to

$$f_3/f_1 = (U_3/U_1)\,(A_1/A_3)^{\frac{1}{2}}$$

They continued by arguing that the values of all the quantities are essentially known, except for area A_3. An upper bound for this can be estimated by taking 'the area of the circle, ellipse or rectangle that completely circumscribes the exit plane of the modified nozzle' as upper limit, which in turn leads to a prediction of the upper limit on noise reduction. They compare some measured power level changes at spectrum peak with these theoretically-derived maximum reductions, showing the estimate to be optimistic to a certain degree. Further comments are offered on the incompleteness of mixing near the nozzle which will both increase the noise level and tend to give a directivity broadening with the possibility of a secondary peak at high frequencies in the spectrum. They admit that 'the theory does not take into account the induction of secondary air that occurs even with a standard nozzle', and it would seem on their basis of estimation, the noise from an elliptical or rectangular nozzle would equal that of the equivalently-sized circular nozzle. Extension of the theory to flight conditions also was briefly considered.

Their approach was the subject of re-examination by Powell[34] whose main objective was to make an allowance, albeit somewhat empirical, for the non-uniformity of the jet parameters over the cross-sectional area A_3. In comparing 'observed' and 'theoretical' attenuation, both the ordinate and abscissa differ from the earlier paper. Again the data given are too sparse to permit re-appraisal.

These latter approaches would estimate an attenuation for ejectors which was independent of their length, whereas Greatrex's method[32] allows this attenuation to vary if the primary potential flow from the nozzle protrudes beyond the exit plane of the ejector. By an adaptation of Ribner's[29] method of analysis for the sound output per unit slice of jet, Middleton[35] was able to estimate the change in acoustic output produced by ejectors of various lengths and diameters modifying the flow from a round nozzle. Agreement with experimental results for the shorter and longest ejectors was quite good, but for those of 'medium' length, (ejector length/nozzle diam. = 10-20) an empirical correction factor improved the measure of agreement. In certain cases the analysis of the practical work was complicated by the presence of intense discrete frequencies generated by the flow in the ejector.

Greatrex and Brown[12] were also able to present an alternative approach. Arguing that 'the operation of a silencer consists of two independent effects:

(i) interception of generated noise
(ii) reduction of noise generated by interference of mixing regions,

they suggested that the attenuation due to (i) could be estimated for a nozzle consisting of equal tubes as $10 \log_{10} s$ dB, where s was the proportion of tubes which could be viewed at right angles to the face in that direction. This would in general produce some asymmetry in the noise field, as is indeed measured under such circumstances. Some further experimental results, discussed more fully in section 4, enabled the additional attenuation due to the 'interference' of (ii) to be estimated.

Lee and Wenzelberger[36] presented 'quasi-empirical equations' for correlating the acoustic and aerodynamic properties of a free subsonic jet. It subsequently proved possible to extend this to suppressor nozzles (Lee et al[21]). Starting effectively with the relationships Ribner[29] had used, that the amount of sound power δP emitted by an elemental slice of jet of thickness δx is given by

$$\delta P(x) \sim \rho_o a_o^{-5} \overline{u(x)^8} \{\ell(x)\}^{-1} A(x) \delta x$$

where $A(x)$ is the cross-sectional area of the mixing region, the following relationships were assumed

(i) $\overline{u(x)^8} = \frac{1}{A(x)} \int_A u^8 \, dA$

(ii) $\ell(x) = \text{const.}\ x$

(iii) the frequency of sound and the location of its source is given by the empirically established relationship $f_x = (\xi x)^{\eta}$

(iv) the power spectrum shapes of the noise sources in the jet stream are approximately the same with respect to dimensionless distance downstream.

Defining $G(f)$ as the sound power spectrum $\frac{dP}{df}$ with $x = \frac{1}{\xi} \cdot f^{-1/\eta}$

$$G(f) = \frac{dP}{dx}\frac{dx}{df} \sim \rho_o a_o^{-5} \, \xi f^{\frac{1}{\eta}} \, \frac{1}{\xi\eta} f^{-1-\frac{1}{\eta}} \int_A u^8 \, dA$$

$$= \hat{k} \frac{\rho_o a_o^{-5}}{\eta f} \int_A u^8 \, dA \qquad (3)$$

where $\hat{k}$ is the appropriate constant of proportionality, and 'obviously depends on geometry of design'.

The noise-producing region of the circular jet is split into two -- the 'high-frequency noise-generating zone' extending from the plane of the nozzle to the disappearance of the potential cone, and the 'low-frequency noise-generating zone' beyond that. Similarity of the velocity profiles within each zone is used, together with numerical value for the constants involved established from the data of Lawrence[37]. The contributions from the two zones are then added to give the emitted sound power spectrum. Choosing $\hat{k}$ to give the best fit, comparison with the measured results obtained on current tests with a round nozzle gave agreement appearing to be 'rather close'.

It was possible to transpose this method directly to the cases of single and two interfering rectangular jets and to an eight-lobed suppressor, with the additional assumption that shielding and dissipative effects are insignificant. The same numerical values for G and N as in the case of the standard nozzle, namely 1.25 and 1.22 respectively, were chosen, and the integral was evaluated numerically, from the results of many velocity traverses. The characteristic dimension of the rectangular jet was taken to be the diameter of the circular nozzle of equal area, and a similar definition was chosen for each lobe element of the eight-lobe nozzle. Reasonable agreement was again found between the power spectrum levels obtained from measurements in a reverberant room and those predicted by this method. However, for optimum agreement it is still necessary to determine a value for k by empirical comparison. Values obtained were as follows:

Conical nozzle	Two interfering rectangular jets	Eight-lobed nozzle
4.38×10^{-5}	6.9×10^{-5}	5.27×10^{-5}

This difficulty was effectively surmounted in a subsequent section of the same reference. The noise output was regarded as coming from two regions, one where the jet intermixing had scarcely begun, near the efflux plane of the suppressor nozzle, and the other where the individual flows had effectively coalesced. In each case, it was argued that the resulting power spectra could be obtained from the generalized power spectrum of a conical nozzle, with the characteristic dimension (and hence frequencies) suitably chosen. In consistent units, the peak frequencies from the two regions were deduced as $0.13\ U/\Delta$, where Δ is the maximum diameter of the nozzle or the width of an individual lobe as appropriate, U being the efflux velocity. The value of the power spectrum at the peak frequency for either region depended on a constant of proportionality which was empirically found to be represented by $0.00228/(\pi/\Delta)^2$ where π is the perimeter of the nozzle or 'petal' as appropriate. Again agreement with experimental results was reasonable.

It remains to consider the directivity of the jet noise. In Lee's[21] report the directivity indices are plotted (dB vs. angle) for each of the customary eight octaves for full-scale conical and eight-lobe nozzles. From these 'it becomes evident that the directional characteristics, for any given frequency band, are the same for (both) nozzles'. This 'has led to the hypothesis that the directivity characteristics of jet noise are functions only of frequency, and are essentially independent of nozzle .... configuration'. This of course assumes that the nozzle is axisymmetric, or virtually so. The somewhat bold hypothesis does, however, enable one to determine the directivity behaviour, once the sound power spectrum of the source and the total acoustic output is known. (The directivity index gives the shape of the spatial distribution of the octave in question, and the integral of this must give the appropriate octave power level. Summation of the octave levels at a fixed point yields the overall level from which the directivity of the sound pressure levels is established). Although quite reasonable agreement was established between the predictions of the method and experimental results on a variety of nozzles, it was conceded that no satisfactory method had yet been achieved for predicting the directivity indices themselves (which apparently depended on jet temperature amongst other things).

The approaches given by Lee have been extended by Semrau[38] of the same company, and 'experimental evidence is presented which indicates that jet noise power spectra can be adequately predicted for the various nozzle configurations at flow pressure ratios up to 3.0, and flow temperatures up to 1800°R. Comparison between measured and predicted overall sound power shows that the prediction method used provides calculated levels within ±2dB of acoustically measured levels for nearly all nozzle shapes and flow conditions tested. An attempt is then made to predict 'aerodynamic properties of jet flows from suppressor nozzles through the use of a computer program'. In the method suggested five empirical constants appear, and only partial completion of the programmes had been achieved by the time the report was presented.

A research team led by Eldred[39] has also given consideration to the prediction of far-field acoustic behaviour from aerodynamic details of the flow, with the comment 'for the purposes of predicting the power radiated from an arbitrary flow, it is necessary to examine the acoustic power generation as a function of axial distance and frequency in relation to the actual flow parameters'. Tolerable agreement was achieved between the measured and estimated figures. An innovation here was a discussion of the effect of ground reflection, which is not a straightforward task since the assumptions of a phase-coherent source is scarcely justified. A normalized power spectrum is presented for round jets, but no reference is given. Instead of the more customary Strouhal number $\frac{fd}{U}$ the quantity $\frac{fd}{U_e}\frac{a_e}{a_o}$ is used for the dimensionless frequency, where suffix e denotes exit condition and suffix o ambient condition, and the quantity d represents the effective nozzle diameter. The latter differs from the true nozzle diameter only for pressure ratios $\hat{p}$ above choking, Eldred actually quoting the relationship for a value of γ of 1.4, namely

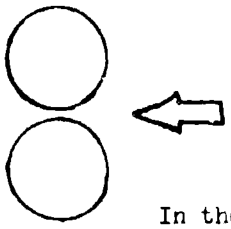
$$d = d_e\{1 + 1.71(0.53\hat{p} - 1)\}^{\frac{1}{2}} \qquad (\hat{p} > \frac{1}{0.53}) \tag{4}$$

In order to consider the output from a mixing nozzle of tubular form, Eldred suggested that the noise should consist of that 'radiated by the outer portion of the individual tube flows before they coalesce at some downstream station, plus the noise radiated by the combined flow further downstream'. It was claimed that the noise from the inner position of the flows from the tubes can be ignored because 'the details of this internal mixing near the nozzle appear to have little bearing on the noise generation, except when there is insufficient spacing between the outer nozzle elements to permit the necessary inflow of ambient air to the center of the jet...' This, it is suggested, is due in part to the lower internal turbulence levels.

The assumption of this approach have the appeal of simplicity, as well as being quite compatible with the implications of Lighthill's basic theory of jet noise[1,2]. With ease of computation a further consideration, these factors have commended this method for further examination and development in the present work (see especially Section 5).

4. THE NOISE FROM TWIN INTERFERING JETS

The first case for which it is of interest to make an estimation of the change in noise output is when two jets are sufficiently close that their



flows interact. The jets are assumed to issue from equal co-planar round nozzles with their centrelines parallel. If the jets are considered from a point perpendicular to the plane joining their centres, there can be no 'shielding' effect and any difference in the acoustic output from that due to two jets operating independently can be ascribed to 'interference'.

In the model to be constructed, it is assumed that the flow of each jet is unaffected by the other <u>except</u> where the jets physically intersect. Thus the growth of the outer mixing boundary and the decay of the inner potential core will be taken as varying linearly with distance downstream, and the inner and outer mixing angles Φ and ψ may be taken as equal. If then the jets are each of radius R at the nozzle efflux plane and their centre-lines are a distance 2R + 2E apart, the jets intersect at a distance $x = E\cot\psi$ downstream. Concerning the zone of interference itself, as Lawrence and Benninghoff[40] and Potter[30] have shown, the turbulence intensity and scale in such a region is much lower than when no interference occurs. The hypothesis is therefore made that the noise-generating parameters are so low in the common zone that its contribution to the acoustic output is negligible and may therefore be neglected. It is therefore necessary to establish the contribution of such a volume when no interference is occurring, and to do this the unmodified jet must be considered in further detail.

4.1 The Unmodified Round Jet

The basic relationship is, (cf. Ribner[29], Powell[26])

$$\delta P \sim \rho a_o^{-5} f^4 \tilde{u}^4 \ell^3 \delta V$$

where $\tilde{u}$ and f are turbulent velocities and frequencies and l is a scale of turbulence, and δP is the power emitted from a volume element $\delta V > \ell^3$.

Assuming that $\frac{f\ell}{\tilde{u}}$, a 'turbulence Strouhal number' is constant, and that the ratio $\frac{\tilde{u}}{U}$ is also constant, the relationship becomes

$$\delta P \sim \rho a_o^{-5} U^8 \ell^{-1}\, \delta V$$

In the annular mixing region, $U \sim U_e$, the jet exhaust velocity, the correlation length $\ell \sim x$, and

$$\delta V = \pi(\tan\psi + \tan\Phi)\,\{2Rx + x^2(\tan\psi - \tan\Phi)\}\delta x$$

$$= 4\pi R x \tan\psi\,\delta x, \text{ equating } \Phi \text{ and } \psi.$$

Therefore, incorporating the density, velocity of sound and proportionality constants into a single quantity k, the acoustic power output for the region up to the core end is

$$k\int_0^{R\cot\psi} \frac{4\pi R x \tan\psi}{x}\,dx \qquad \text{ie.}\quad 4\pi R^2 k \quad \text{where } k = k\, a_o^{-5} U_e^{8}$$

For any section downstream of the core, $\delta V \doteq \pi(R + x\tan\psi)^2 \delta x$. The correlation length ℓ by experiment appears to be fairly constant over an appreciable range[11], and hence is conveniently taken as its value at the end of the annular mixing region. As will be shown, due the rapid fall-off in sound output per unit slice of jet downstream it makes very little difference to the result whether ℓ in the fully developed region is taken to vary as $(x - R\cot\psi)$ or to be constant.

It remains to find a relationship with distance downstream for the velocity, the asymptotic form for which is known to be inversely proportional to x. A form which is found to agree well with experimental results is

$$\frac{U}{U_e} = 1 - \frac{1}{4}\left(\frac{x - R\cot\psi}{R\cot\psi}\right)^3$$

for the intermediate mixing region, taken as the range $R\cot\psi < x < 2R\cot\psi$ together with

$$\frac{U}{U_e} = \frac{3/4R\cot\psi}{x - R\cot\psi}$$

for the fully developed region $x > 2R\cot\psi$. It will be observed that both the velocity and its derivative are continous at the boundaries of the regions. The shapes together with some experimental points of Corcos[16], are shown in Figure 1.

The sound power output due to the adjustment region is therefore

$$k\int_{R\cot\psi}^{2R\cot\psi} \frac{\pi(x\tan\psi + R)^2}{R\cot\psi}\left\{1 - \frac{1}{4}\left(\frac{x - R\cot\psi}{R\cot\psi}\right)^3\right\}^8 dx$$

where the value of k is the same a previously.

Substituting $z = \dfrac{x - R\cot\psi}{R\cot\psi}$ the integral becomes $k\displaystyle\int_0^1 \pi(z+2)^2\left\{1 - \frac{z^3}{4}\right\}^8 dz$

which is easily evaluated in closed form with value $3.88\pi R^2 k$, correct to 2 decimal places.

For the fully-developed region, the sound power is

$$k\int_{2R\cot\psi}^{\infty} \frac{\pi(x\tan\psi + R)^2}{R\cot\psi}\left\{\frac{3/4R\cot\psi}{x - R\cot\psi}\right\}^8 dx \text{ or } k\int_{2R\cot\psi}^{\infty} \frac{\pi(x\tan\psi + R)^2}{x - R\cot\psi}\left\{\frac{3/4R\cot\psi}{x - R\cot\psi}\right\}^8 dx$$

according as $\ell \sim R\cot\psi$ or $\ell \sim (x - R\cot\psi)$ in this range. The integrations are again straightforward, and are respectively

$$\frac{151}{105}\left(\frac{3}{4}\right)^8 \pi R^2 k \quad \text{and} \quad \frac{130}{105}\left(\frac{3}{4}\right)^8 \pi R^2 k$$

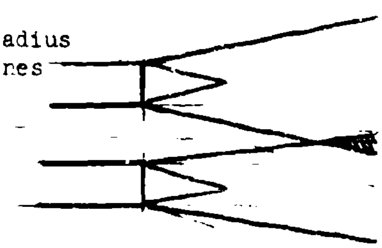
On this hypothesis the total output from these two regions is $(3.88 + .15 \text{ or } .13)\pi R^2 k$ i.e., about $4.02\ \pi R^2 k$ units, comparing with the $4\pi R^2 k$

units from the annular mixing region. Thus the condition that the noise outputs are approximately equal has been satisfied by the chosen empirical relationships.

It is of interest to plot the sound power output per unit slice of the jet. This has been done in Figure 1. The region up to the core end is Ribner's well-known 'x^0 - law' whilst in the downstream zone the x^{-7} relationship is sketched. In the adjustment zone it is seen that the acoustic output per slice of unit thickness actually increases for a while before falling off to the eventual x^{-7} law. This is because, on the present hypothesis, the increasing mixing volume which results after the eradication of the core more than counterbalances the initial fall-off in velocity. This means that the main noise-producing region in a jet should be more truly regarded as being of about one-and-a-half core lengths than just the core length as has been sometimes supposed. This matter has been recently considered by Ffowcs-Williams[28].

By way of comparison the results of Dyer[41] have been added to this graph. The flat portion of his curve has been made to agree with the line and since he plotted results against dimensionless nozzle distance downstream as opposed to the present 'core-lengths', the downstream scale has been chosen to give the same area under the graph (i.e., same acoustic output) as the contemporary results. Dyer's[41] reproduction of Sanders' calculations from the data of Lawrence are also plotted. These results add credence to the method of approach adopted.

4.2 Two Jets Interfering



Consider two identical coplanar jets of radius R at their efflux plane, and whose parallel centre-lines are at a distance 2R + 2E apart. The inner and outer mixing angles are assumed equal, and the flows are unmodified except where they physically intersect. It is convenient to introduce the dimensionless separation parameter $\alpha = \frac{R}{R+E}$ which has therefore the range (0,1], and which is the reciprocal of the parameter used by Greatrex and Brown[12]. Then the two jets will intersect at a distance downstream of $x = E \cot \psi$. This will be in the annular mixing region if $E < R$ and in the adjustment region if $R<E<2R$. Only the fully-developed region is affected if $E>2R$. We therefore have

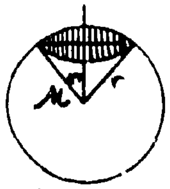
α: $0 \to \frac{1}{3}$	Fully developed region only affected
α: $\frac{1}{3} \to \frac{1}{2}$	Fully developed and adjustment regions affected
α: $\frac{1}{2} \to 1$	All three regions affected.

Now on the present basis, less than 2% of the acoustic output emanates from the fully-developed region. Thus if the whole of this region were ignored there would be a change of less than 0.1 dB in the estimations. For this reason the change in acoustic output due to a separation parameter value of less than one-third will be taken to be zero and the region will subsequently be neglected.

At a distance downstream x, the radius r of each jet will be

R + xtanψ, and the distance between the centre-line is 2r cos μ = 2(R+E)

Thus the overlapping area is $2\left\{\frac{1}{2} r^2(2\mu) - \frac{1}{2} r^2 \sin 2\mu\right\}$



and $\delta V = 2r^2(\mu - \sin\mu\cos\mu)\,\delta x$

$$= 2(R + x\tan\psi)^2 \left\{\sec^{-1}z - \frac{1}{z}\sin(\sec^{-1}z)\right\}\delta x \quad \text{where } z = \frac{R+x\tan\psi}{R+E}$$

$$= 2\frac{R^2}{\alpha^2}\left\{z^2\sec^{-1}z - (z^2-1)^{\frac{1}{2}}\right\}(R+E)\cot\psi\,\delta z \tag{5}$$

The Annular Mixing Region

Suppose now that $\frac{1}{2} < \alpha < 1$ i.e., the annular mixing region of the 2 jets intersect. Then assuming, as intimated, that no noise is generated within the common volume, the acoustic output from the region up to the core end will be deficient by an amount J_1

$$\text{where } J_1 = k\int_{E\cot\psi}^{R\cot\psi} \frac{2(R+x\tan\psi)^2}{x}\left\{\sec^{-1}z - \frac{1}{z}\sin(\sec^{-1}z)\right\} dx$$

$$= \frac{2R^2k}{\alpha^2}\int_1^{2\alpha}\frac{1}{z-\alpha}\left\{z^2\sec^{-1}z - (z^2-1)^{\frac{1}{2}}\right\}dz$$

$$= \frac{2R^2k}{\alpha^2}\left\{\left[\frac{z^2}{2} + \alpha z\right]_1^{2\alpha} + \alpha^2\int_1^{2\alpha}\frac{\sec^{-1}z}{z-\alpha}\,dz - \left[(z^2-1)^{\frac{1}{2}} + \alpha\cosh^{-1}z + (1-\alpha)^{\frac{1}{2}}\sin^{-1}\left(\frac{1-\alpha z}{z-\alpha}\right)\right]_1^{2\alpha}\right\}$$

$$= \frac{2R^2k}{\alpha^2}\left[\int_1^{2\alpha}\frac{\sec^{-1}z}{z-\alpha}\,dz + 4 - \frac{1}{\alpha}\left\{1 + \cosh^{-1}2\alpha - \left(\frac{1}{\alpha^2} - 1\right)^{\frac{1}{2}}\left\{\frac{\pi}{2} - \sin^{-1}\left(\frac{1-2\alpha^2}{\alpha}\right)\right\} + \left(4 - \frac{1}{\alpha^2}\right)^{\frac{1}{2}} + \frac{1}{2\alpha}\right\}\right] \tag{6}$$

The Transition Region

Consider now the situation where there is interference in the transition region, i.e. $1 > \alpha > \frac{1}{3}$. If α belongs to the range $(\frac{1}{3},\frac{1}{2})$ then intersection takes place at $x = E\cot\psi$. On the other hand if α is greater than one half, the jets first intersected upstream of this region, and the lower limit for integration is therefore $x = R\cot\psi$.

The amount J_2 by which the region is deficient in acoustic output is therefore

$$J_2 = k\int_{\max(R\cot\psi,\ \frac{R}{\alpha}\cot\psi)}^{2R\cot\psi} \frac{2(R+x\tan\psi)^2}{R\cot\psi}\left\{1-\frac{1}{4}\left(\frac{x-R\cot\psi}{R\cot\psi}\right)^3\right\}^8 \left\{\sec^{-1}z - \tfrac{1}{2}\sin(\sec^{-1}z)\right\} dx$$

$$= \frac{2R^2k}{\alpha^2}\int_{\max(2\alpha,1)}^{3\alpha}\left\{z^2\sec^{-1}z - (z^2-1)^{\frac{1}{2}}\right\}\left\{1-\frac{1}{4}\left(\frac{z}{\alpha}-2\right)^3\right\}^8 dz \qquad (7)$$

Now if $I_{n+1} = \int z^{n+1}\sec^{-1}z\,dz$, integration by parts yields

$$(n+2)I_{n+1} = \left\{z^{n+2}\sec^{-1}z - z^n(z^2-1)^{\frac{1}{2}}\right\} + n\int z^{n-1}(z^2-1)^{\frac{1}{2}}\,dz$$

Further, $\int z^{n-1}(z^2-1)^{\frac{1}{2}}\,dz = \int \cosh^{n-1} w\,\sinh^2 w\,dw$ putting $z = \cosh w$

$$= \int(\cosh^{n+1} u - \cosh^{n-1} u)\,du$$

and powers of the hyperbolic cosine are readily integrated, the form depending on whether the positive integer n is even or odd.

Thus the integral J_2 can be reduced to standard form and integrated, but due to the high powers involved the coefficients are somewhat tedious to evaluate.

The proportions of $8\pi R^2k$ that J_1 and J_2 are respectively, are shown in the table for an appropriate range of values of α. Ignoring any effects in the fully-developed region, and taking the total unmodified acoustic output as $8\pi R^2k$, due to the interference of the regions there is a decibel change of

$$10\log_{10}\left\{1-\frac{J_1+J_2}{8\pi R^2k}\right\}\ \text{dB}$$

These attenuations have been evaluated for the α-range and are shown in Figure 2. It is seen that interference effects appear negligible below an α-value of 0.5, but increase in a roughly linear manner to about 2 dB when the jets are (virtually) touching at their discharge plane. Slightly more attenuation is due to interference in the transition region than in the annular region

Added to the curves in the figure are the experimental results reported by Greatrex and Brown[12], and which appear to be the only ones available in the open literature. In the original paper the points are labelled attenuation ~ dB' without specifying precisely the noise reduction being quoted. Results appear to be given to the nearest quarter-decibel but it is stated[12] that these 'preliminary' results are 'of uncertain reliability'. (However these same results were in fact reproduced subsequently[32]).

In view of this it appears that the present analysis goes as far as is permissible at the moment. Certainly the general agreement of form between the measured and theoretical values is encouraging.

TABLE

$\alpha = \frac{R}{R+E}$	$\leq 1/3$	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$J_1/8\pi R^2 k$	0	0	0	.051	.076	.103	.128	.153
$J_2/8\pi R^2 k$	0	.008	.040	.092	.139	.183	.221	.238

5. SUPPRESSOR NOZZLE BEHAVIOUR - RE-EXAMINATION OF THE APPROACHES OF ELDRED AND OF LEE

5.1 The Power Spectrum

The simple approach adopted in the preceding section for estimating the interference effect due to two jets in proximity does not lend itself to ready application for the more complicated flow pattern from multi-lobe and other designs of suppressor nozzles. For these it is required to find the far-field directivity of the noise, both in octave bands and for the overall sound pressure level, and the power spectral density. In order to carry out the calculations completely it will be necessary to appeal to certain (normalised) experimental curves, namely those for power spectra, and directivity indices (both in octave bands and overall) for a standard nozzle.

The case of the directivity of noise is quite straightforward to deal with. From a wealth of experimental data, von Gierke[42], Eldred[39], Lee[21], Franken[6] and others have published curves both for overall noise and the level in octave bands. In most cases the precise details of the jets used for the measurements are not given, but agreement amongst the curves is generally very good. Three of these curves for overall noise are compared in Figure 3, the levels being plotted relative to the average sound pressure level recorded in the polar traverse. Lee's curve is somewhat higher than the other two for angles above 100° from the rearward-pointing jet axis, but in this region the levels are sufficiently low to be quite unimportant and tend to be swamped by other sources of engine noise.

Because of the completeness of his results for application to the cases under present investigation, the directivity curves of Lee both for the overall and octave bands noise levels (Figure 4) have been the ones used herein. Where comparison was possible between the predicted and experimental results, it was also found that his curves gave slightly better agreement than some of the others.

All the above authors except Franken also produce curves for the power spectrum, but comparison is not too easy in that each has used a different dimensionless parameter with which to normalize the results*. Thus two spectrum curves which agree well at one condition may differ rather more at another. For this reason only the curve of Eldred[39], which is the one used in the present investigation, is presented (Figure 5). Suffice it here to observe that when typical calculations were performed using any of the available data, the results appeared to differ only slightly.

Several authors, e.g. Powell[34], Greatrex and Brown[12], Ffowcs-Williams[28], [illegible] ted that the noise from a suppressor-type nozzle may conveniently be [illegible] coming from two regions, namely, that volume near the nozzle efflux [illegible] the flows from each tube or corrugation manifests its individuality, [illegible] Mixing Region (IMR), and the downstream region where these flows [illegible] to a fully-developed situation and the initial condition 'for- [illegible] the [illegible] Mixing Region (SMR). This is illustrated in [illegible] already been used by the teams of Lee[21] and of [illegible] the present method of determining power spec- [illegible]

[illegible]

[illegible] in Section [illegible], Greatrex and Brown[12] [illegible] noise from a multi-tube nozzle [illegible] and 'interference' concepts. [illegible] from the I.M.R., establish- [illegible] of the nozzle. Neither of [illegible] the constituent

[illegible]

[illegible] such a situation, the following [illegible] tube centers which are visible [illegible] along the radius vector direct- [illegible] (This has been done for two nozzles [illegible] typical of designs where the tubes [illegible] The suggestion is now made that within the [illegible] the hatched region in each figure, the induced velocities [illegible] the turbulence levels are reduced to an order making [illegible] generated in such a region negligible. Outside the hatched region [illegible] remaining behave as free flows, unmodified by interference. [illegible] Eldred's hypothesis, but the details have been modified for [illegible] generality. These contentions are supported by his analysis of avail- [illegible] dynamic data. We therefore establish an 'efficiency factor' which is [illegible] function of the geometry of the nozzle.

For a simple situation like that in Figure 7a, where there are n equal outer tubes regarded as generating the (I.M.R.) noise (the inner tubes being ineffective) each will have a relative efficiency of $(\frac{1}{2} + \frac{1}{n})$ this being

* This is due to the fact that the spectrum of jet noise does not appear to be completely correlated on the basis of simple Strouhal number considerations, and that therefore greater sophistication is required.

the ratio of the 'unhatched' angle to the full 2π radians. The efficiencies of equally-sized units may be added, and so their total output is equal to $(\frac{n}{2} + 1)$ 'complete' tubes of such size. An example of this form is the well-known Boeing 21-tube nozzle.

As the ensuing frequency spectrum is diameter-dependent, it is necessary to consider separately each size of tube. Thus if there are

n_1 tubes, each of diameter d_{n_1} with total efficiency e_{n_1}

----

n_k tubes, each of diameter d_{n_1} with total efficiency e_{n_k}

then the difference in dB levels between the noise of the unmodified jet and that due to the n_j tubes of diameter d_{n_j} is

$$10 \log_{10} \left\{ \frac{d_{n_j}^{\;2} e_{n_j}}{D^2} \right\} \quad \text{dB}$$

where $D^2 = \sum_{j=1}^{k} n_j d_{n_j}^{\;2} = \frac{4}{\pi}$ x area of the standard nozzle, since the effective efflux areas must be the same.

In the case where the power level of the unmodified jet is known explicitly for the condition in question, the corresponding estimated level due to the I.M.R. of each size of tube may be immediately determined. As an alternative, a satisfactory empirical relation based on Lighthill's expression has been found to be[39], in dB re 10^{-13} watt,

$$P = 146 + 20 \text{ Log}_{10}D + 80 \log_{10}\left(\frac{U}{1000}\right) \tag{8}$$

where D the effective nozzle diameter is measured in feet

U the jet velocity is measured in feet/second and the empirical constant of 146dB corresponds to a value of 2.86×10^{-5} for the constant K in equation (1). Again the corresponding power level from the I.M.R. can be written down. The frequency distribution of the power level is then established for each size of tube, from the normalised curve of Figure 5, and the power spectrum due to the I.M.R. is found by adding the contributions from the individual sizes of tube.

Noise from the Secondary Mixing Region

In order to consider the noise from the downstream portion of the flow, it is necessary to establish the position where the individual jets may be regarded as having coalesced into a large jet, thereby losing their identity. This fusing, being somewhat asymptotic in nature, is somewhat difficult to quantise.

A satisfactory approach was found to be obtained by assuming that the gross jet commences at the plane where the hydrodynamic flow fields of any two jets having non-zero efficiency parameters first encounter each other. Thus if the minimum distance between two such jets at the efflux plane is b, then each jet is assumed to have increased in radius by $\frac{1}{2}b$ at the commencement of the S.M.R. and an effective area A_s is defined for this plane by

$$A_s = \frac{\pi}{4}\sum_{i=1}^{N}(d_i + b)^2 \qquad \text{where N = no. of tubes} = \sum_{j=1}^{k} n_j$$

The effective diameter is consequently $\left\{\sum_{i=1}^{N}(d_i + b)^2\right\}^{\frac{1}{2}}$

The apposite flow conditions for the jet at this plane have been derived by Eldred[39] who found the ratio of the velocity there to the exit efflux velocity to be

$$\frac{U_s}{U_e} = \frac{1}{2}\left[\widetilde{\alpha\beta}(\tilde{E}-1) + \{[\widetilde{\alpha\beta}(\tilde{E}-1)]^2 + 4\widetilde{\alpha\beta}(1+\tilde{d})\}^{\frac{1}{2}}\right] \tag{9}$$

where $\tilde{\alpha} = \frac{A_2}{A_s}$ $\quad \tilde{\beta} = \frac{P_e}{P_o}$ $\quad \tilde{d} = \frac{P_e - P_o}{P_e U_e^2}$ $\quad \tilde{E} = \frac{T_e}{T_o}$

when the primary jet is choked, and the secondary jet is taken to be fully expanded. In the case when the jets at each plane are unchoked, d = 0 and $\tilde{E} = \frac{1}{\tilde{\beta}}$ so that the above expression simplifies to

$$\frac{U_s}{U_e} = \frac{1}{2}\left[\tilde{\alpha}(1-\tilde{\beta}) + \left\{[\tilde{\alpha}(1-\tilde{\beta})]^2 + 4\widetilde{\alpha\beta}\right\}^{\frac{1}{2}}\right] \tag{10}$$

The power level for the S.M.R. is then found by substituting the appropriate values for the effective diameter and velocity into equation (8), and the power spectrum is obtained from Figure 5 again. Since the new diameter will be considerably greater than that of any constituent tube, the spectrum from the S.M.R. will be of markedly lower frequency than that resulting from the I.M.R. The final power spectrum curve, which is the sum of the power spectra for the two regions, therefore tends to consist of two humps, respectively attributable to those regions. The overall power level is the sum of the power level due to each region, and hence the reduction in power level due to the particular nozzle configuration in use may be determined.

5.1.2 Multi-lobe Suppressor Nozzles

The Initial Mixing Region

The other common form of suppressor design has been the multi-lobed nozzle. The extension of the method used for multi-lobed nozzle to this situation is straightforward, since the extremities of the lobes are usually well-approximated to by arcs of circles. The centres of these circles can therefore be joined in the manner outlined for tubular nozzles, so that an 'inoperative' region is formed within the hatching. A typical example, that of an eight-lobed nozzle, is shown in Figure 7c. The diameter and relative efficiencies then follow through as before, and hence the calculations of the power spectral density may be carried out.

The Secondary Mixing Region

To deal with the noise from the S.M.R., there is again the problem of defining the area A_e satisfactorily. In this case it was found more appropriate to choose the interior of the circle circumscribing the efflux plane (see Figure 7c). This, it will be noted, is equivalent to Eldred's[39] definition of A_e, but is different in form from that selected for the multi-tube nozzle. (It was found that this definition if applied to the multi-tube situation led to low-frequency levels somewhat less than those reported in the literature). The calculation was then carried out as before, and the I.M.R. and S.M.R. contributions added.

5.2 The Angular Distribution of Octave and Overall Noise Levels

The fundamental hypothesis which enables progress to be made on the estimation of octave spectra at any point in the acoustic far-field of an (essentially axi-symmetric) suppressor nozzle is that 'the directivity characteristics of jet noise are functions only of frequency and are essentially independent of nozzle (or suppressor) configuration'. This was the conclusion of Lee after comparing the directivity patterns of an eight-lobed nozzle with those of a conical nozzle. Figure 8a plots from Lee's data the difference between these directivity patterns in each of the octaves

20	75	150	300	600	1200	2400	4800	
-	-	-	-	-	-	-	-	Hz
75	150	300	600	1200	2400	4800	10000	.

and these have been averaged in Figure 8b. It is seen that although differences as large as 5 decibels arose, the customary difference is only of the order of one decibel, at virtually all angles to the jet. The actual distributions for the standard nozzle are given in the two sheets which comprise Figure 4.

A qualitative argument advanced by Ribner[43] in support of the similarity of these directivity indices is essentially as follows: at a given operating condition the discharge from a suppressor nozzle will roughly match the round jet in average velocity in the mixing regions and hence convection effects will be similar. Additionally, for refractive purposes the effective volume of the suppressor jet will again resemble that of the standard jet. The overall refraction and convection effects being somewhat akin, the general result will be for similar directivity patterns to ensue for given frequency band. The overall noise directivity may however be markedly different due to differences in sound power spectra.

The procedure for calculating the octave band noise levels at any angle is now straightforward, since the sound power spectrum of the source has been determined by the methods of the preceding section. Ignoring absorption, in the acoustic far field the spatial average R.M.S. sound pressure $\hat{p}_i$ in the i th octave at distance r, is given by

$$\hat{p}_i = \left\{ \frac{P_i \rho_o a_o}{2\pi r^2} \right\} \qquad (11)$$

where P_i is the power level in the i th octave band, and hemispherical radiation is assumed, all data being obtained from static tests on engines mounted near the ground

Choosing a reference sound pressure of 0.0002 microbar, and a radius of 200 feet, substituting standard atmospheric values for ρ_o and a_o into the expression,

noise level in i th octave band at angle θ and 200 ft. radius

$$= P_i + D.I.(i,\theta) - 53.5 \text{ dB} \qquad (12)$$

where P_i is the i th octave band power level now expressed in dB re 10^{-13} watt, and $D.I._i$ is the corresponding directivity index.

As the contribution from frequencies beyond the eight under consideration is small, the distribution of the overall noise against angle may then be found by summing the contributions of the constituent octaves. A typical result for the overall noise directivity from a suppressor nozzle is shown in Figure 10.

6. RESULTS AND DISCUSSION

A whole range of nozzles of multi-tube and multi-lobe form were examined. They were both full-scale, i.e., operating with a turbo-jet engine of the 10,000 lb. thrust class, and scale-versions which used either hot or cold flows. The example given in this report (Figures 9 and 10) are typical of the results, and are actually for the full size 12-lobe nozzle with centerbody reported in the investigation of Ciepluch, North, Coles and Antl[46]. Some calculations were found to give slightly better agreement than the ones shown, and some worse ones also ensued. This may be due in part to the necessity of estimating the appropriate operating conditions in certain cases.

On the whole the agreement between the predicted and measured data in the high-frequency region was found to be good. As can be seen from Figure 9, this noise is virtually entirely due to the I.M.R., on the present hypotheses. Whether there is one peak in the region or more depends on the number of differing diameters of tube or lobe which contribute to this noise. For the low-frequency region variable agreement was found, there being a tendency to underestimate this contribution, especially for the multi-tube nozzles. Errors cannot be ascribed to merely mis-estimating the exit flow conditions as adjustment in these figures would similarly affect the high-frequency estimate. It would therefore seem that the source of error may be in the choice of A_8 and any future analyses of this type should pay careful attention to how this might better be defined.

In all the cases investigated, the agreement between the computed and measured directivities was good. This gives further credence to Lee's hypothesis that the directivity characteristic of a jet are primarily functions of frequency, and that nozzle configuration is a secondary consideration.

The methods described in this paper for the estimation of nozzle behaviour may be used in an attempt to predict an optimum design of suppressor, but it would be well advisable to check some additional implications as further tests of their validity. An example of this is the result that other things being equal it is better in a tubular design to have all inner tubes mounted along the radii joining the centers of the circumferential tubes to the nozzle center, thereby reducing the 'effective' number of tubes. A specific experiment along these lines would be straightforward to perform given the requisite apparatus.

Another result concerns the 'area ratio', using the terminology of Greatrex and Brown[12]. Spacing tubes further apart by increasing the radial scale will, on the present hypothesis, have no effect on the I.M.R. noise. It will, however, place the plane of coalescence further downstream, with a resulting larger A_s and lower 'mixed' velocity. The noise from the S.M.R. should consequently be less than formerly, and with the typical frequencies therefrom lower, the subjective annoyance from the spectrum decreases further. This effect may not be large, however, if the noise is dominated by that from the initial mixing region. On the other hand this movement of the S.M.R. downstream is contrary to the concept of Large[45], who argues that increasing the area ratio leads to an increase in the rate of secondary air mixing, promoting the development of the self-preserving jet closer to the nozzle exit.

A third investigation could concern the distribution of elements within the periphery, when these do not apparently contribute to the I.M.R. noise as determined by the 'hatching principle' and shown illustratively in Figure 7a. Any changes measured in the noise output where tubes within a hatched area are replaced by an alternative array of equivalent efflux area, would provide a positive indication of their acoustic importance.

These, and similar simple experiments, for example, more precise knowledge of the source strength distribution along a jet and of the noise field from two interfering parallel jets, as studied in Section 4, would go a considerable way in assessing the accuracy of the hypotheses discussed herein, and might also indicate new approaches to be investigated in order to predict, more completely than ever before, the design of the optimum noise-suppressing array.

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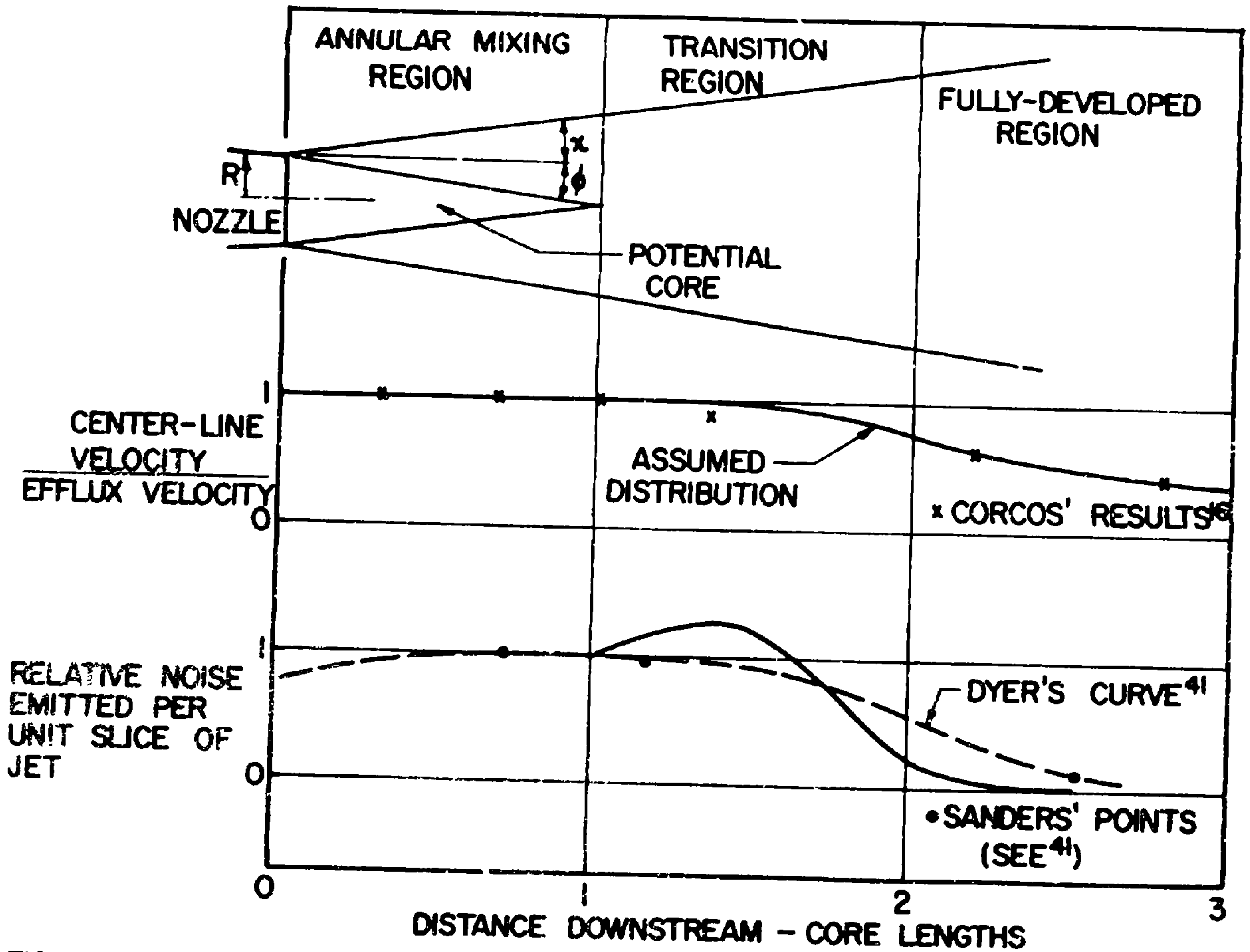

FIG. 1 ASSUMED STRUCTURE OF CONICAL JET AND INFERRED NOISE DISTRIBUTION

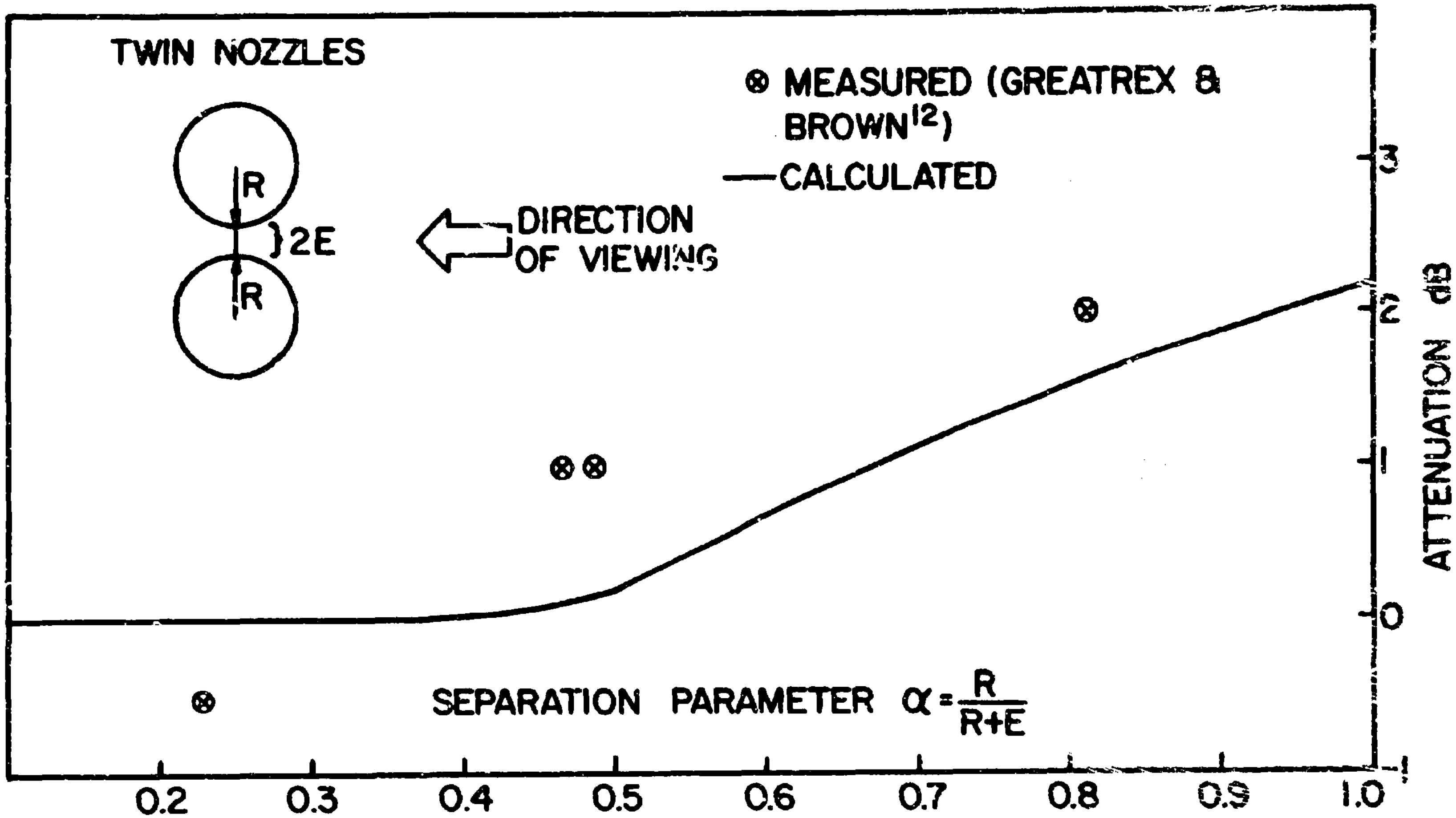

FIG. 2 COMPARISON OF MEASURED AND PREDICTED ATTENUATION DUE TO THE INTERFERENCE EFFECT OF TWO ROUND JET IN PROXIMITY.

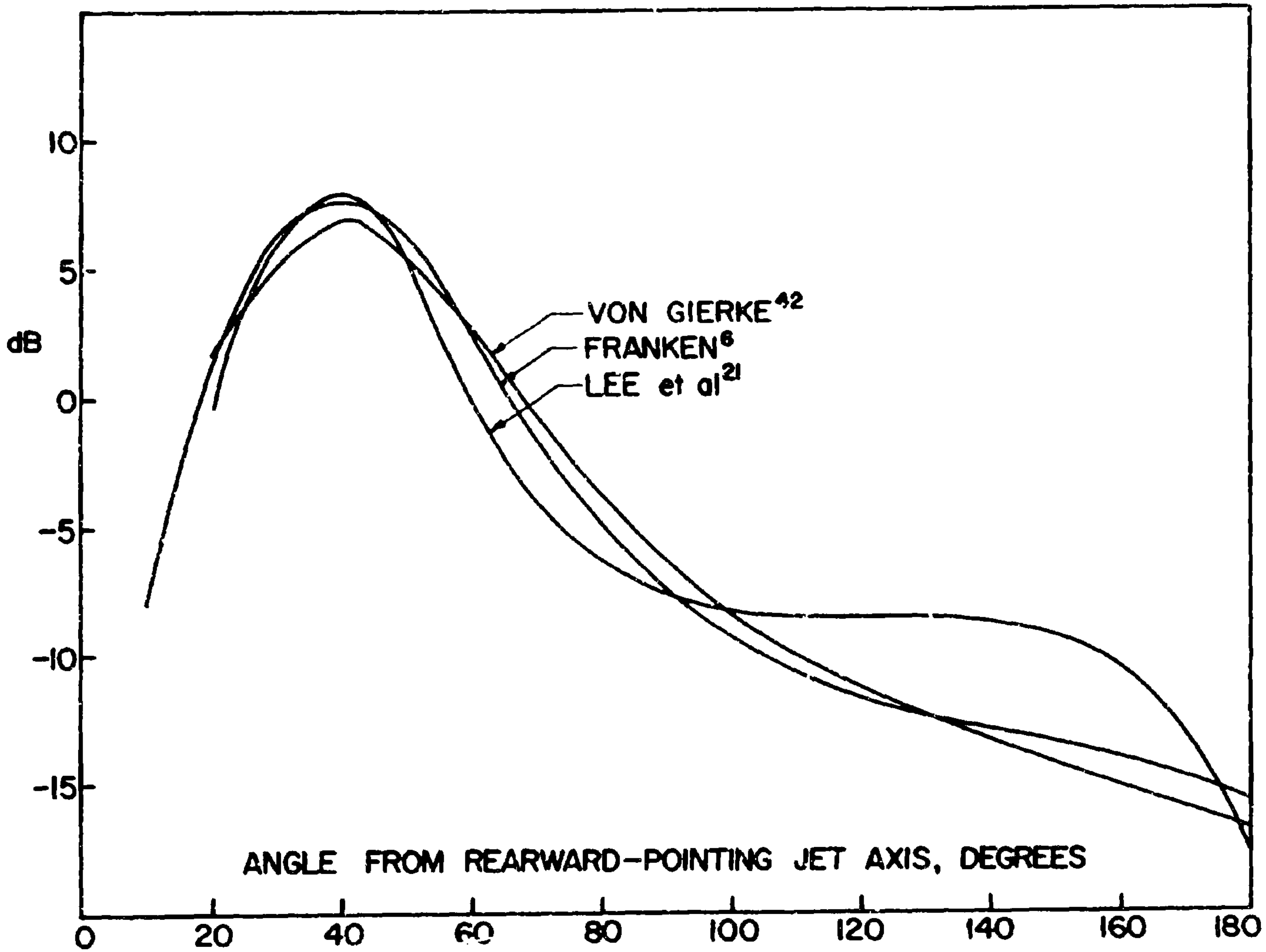

FIG.3 DIRECTIVITY INDICES, OVERALL NOISE FROM A STANDARD NOZZLE, VARIOUS WORKERS

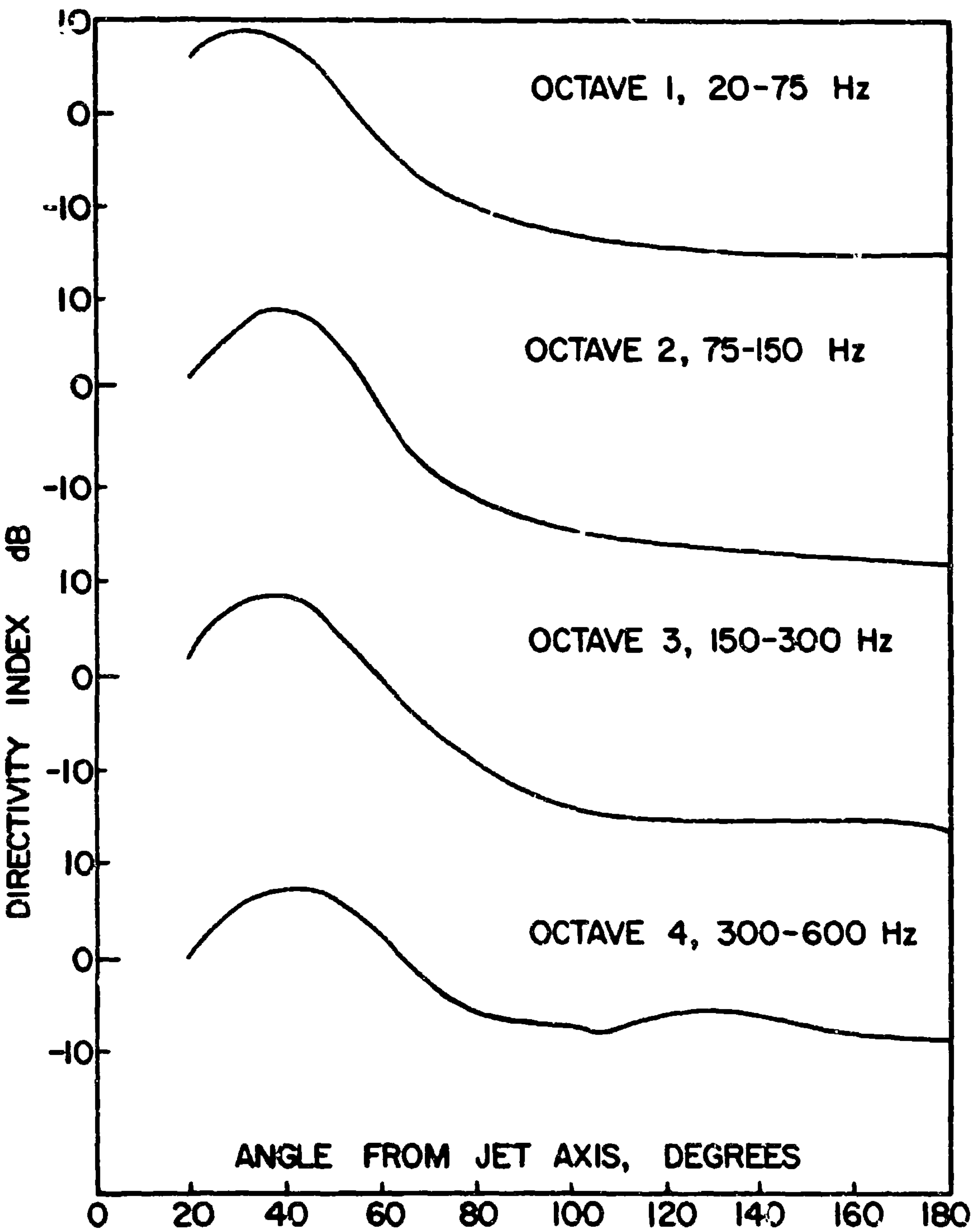

FIG. 4a DIRECTIVITY INDICES IN OCTAVE BANDS FOR CONICAL NOZZLE, (Lee et al[21])

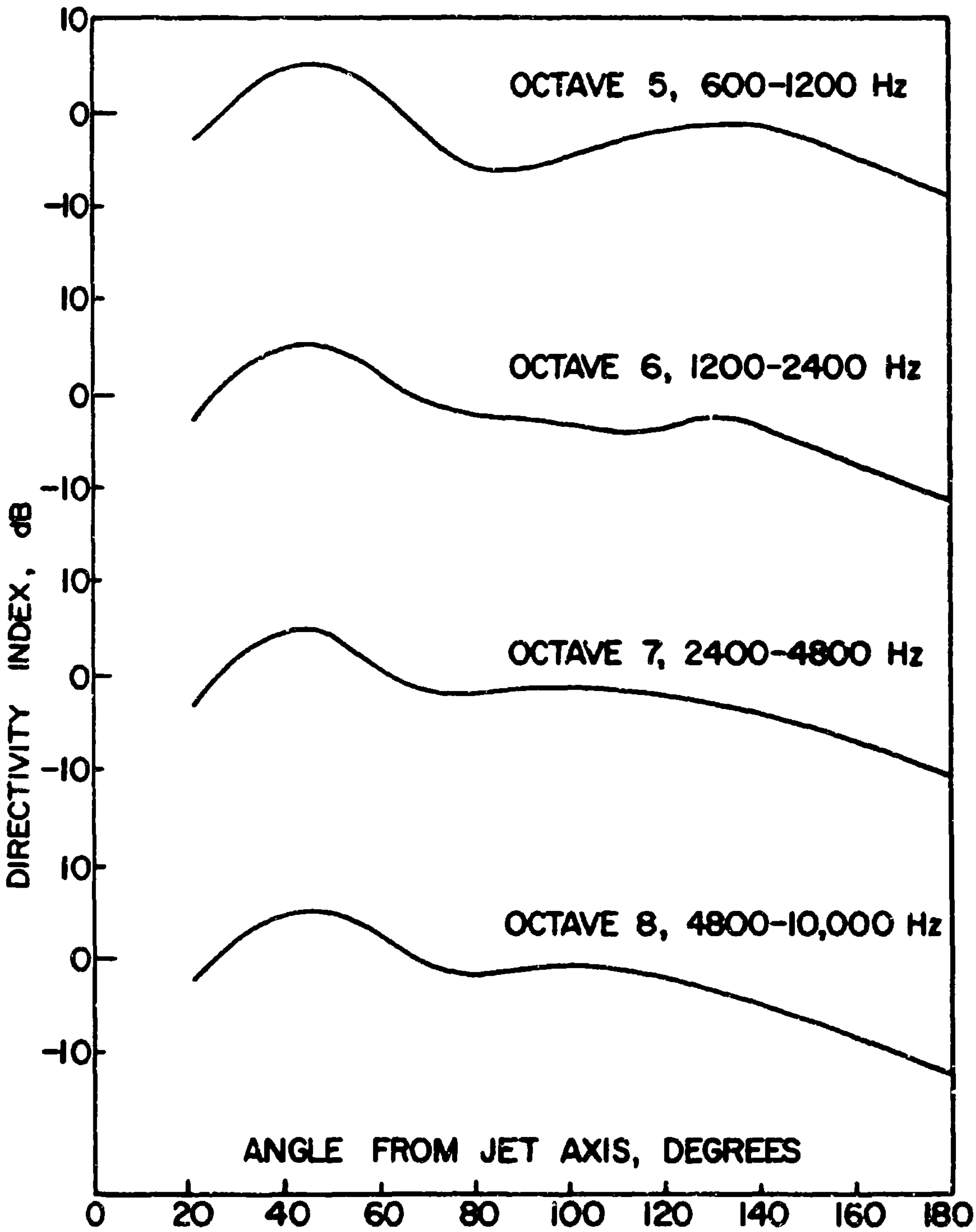

FIG. 4b DIRECTIVITY INDICES IN OCTAVE BANDS FOR CONICAL NOZZLE (Lee et al[21])

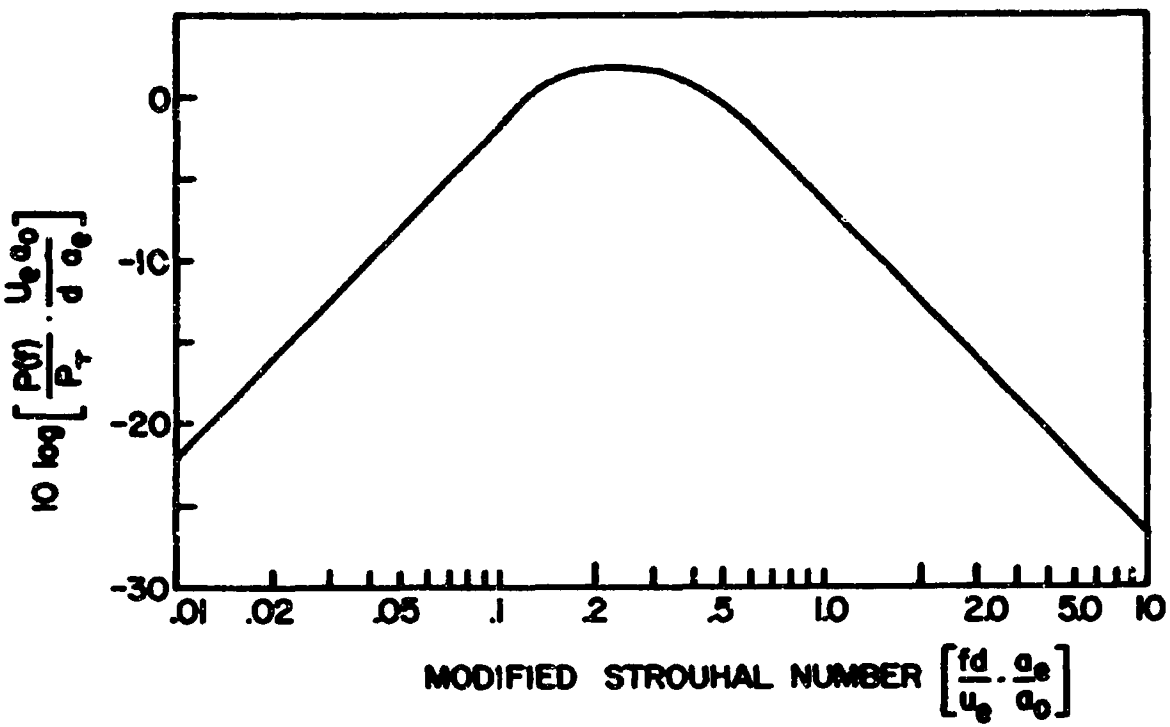

FIG. 5 NORMALIZED POWER SPECTRUM FOR AXISYMMETRIC JETS ISSUING FROM CONVERGENT NOZZLES (AFTER ELDRED[39])

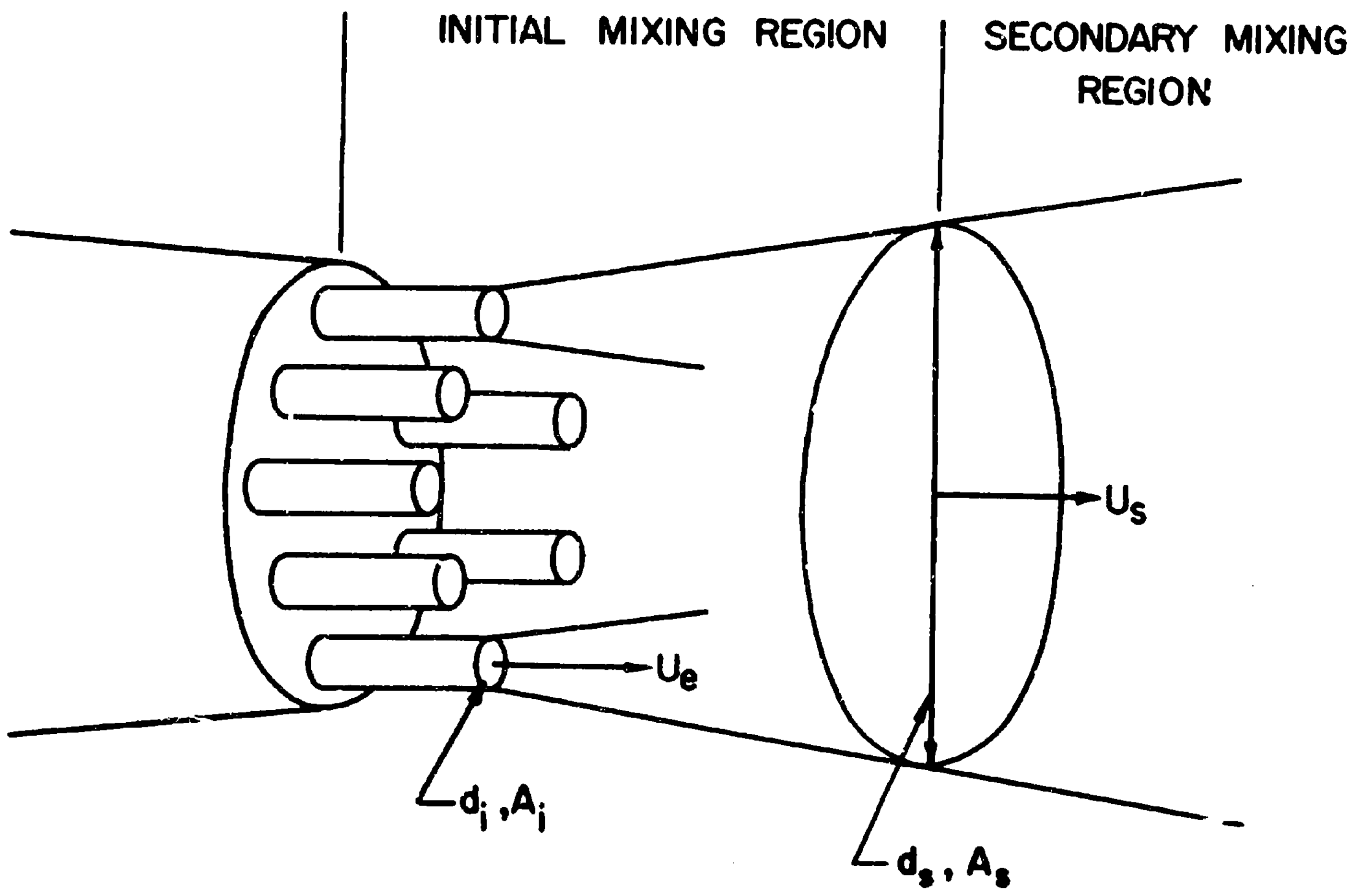

FIG. 6 FORMATION OF 'INITIAL' AND 'SECONDARY' MIXING REGIONS FOR SUPPRESSOR NOZZLE

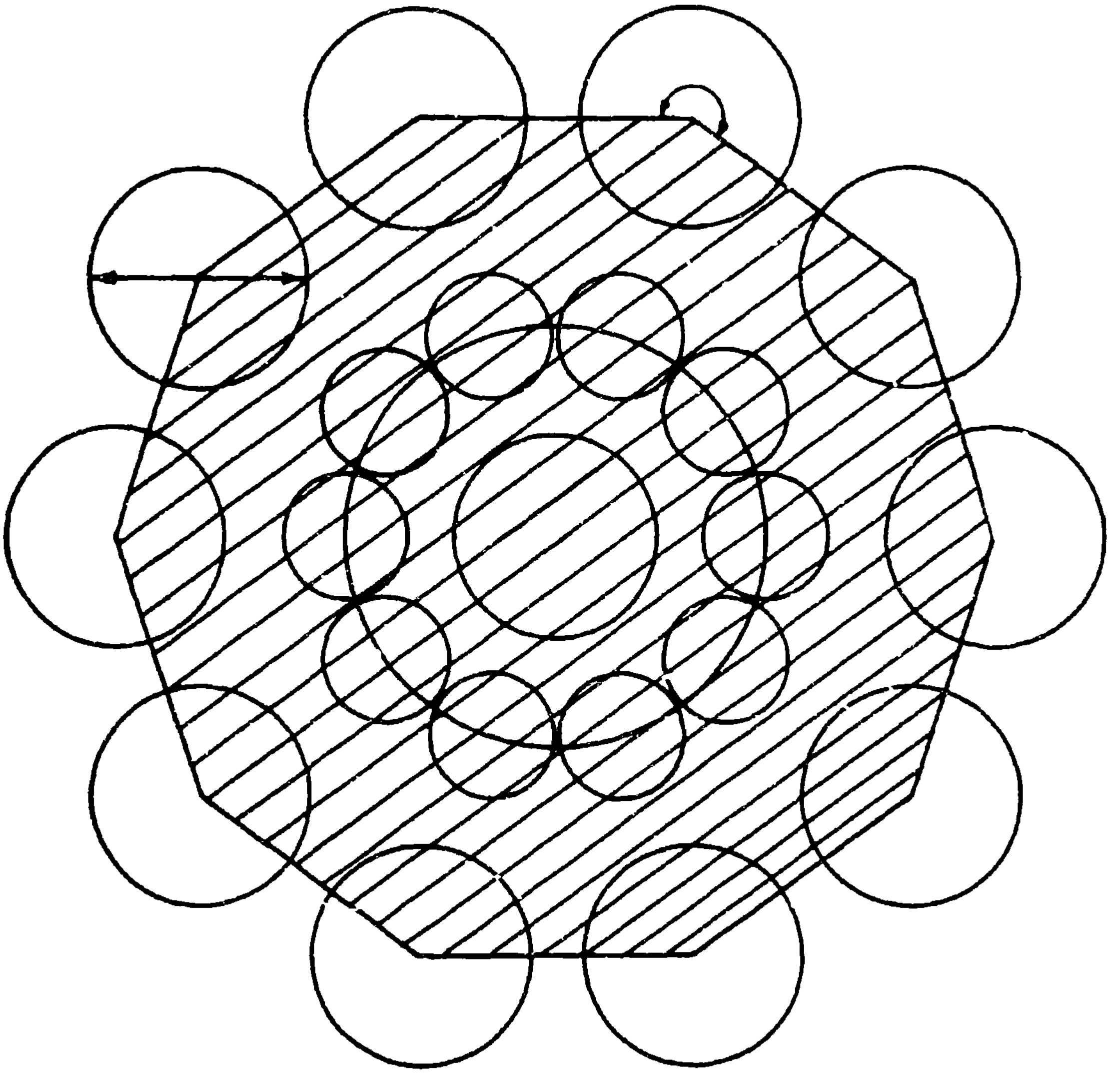

FIG. 7a END VIEW OF TYPICAL RADIALLY-ALIGNED MULTI-TUBE SUPPRESSOR NOZZLE (NOISE GENERATED WITHIN SHADED ZONE CONSIDERED NEGLIGIBLE)

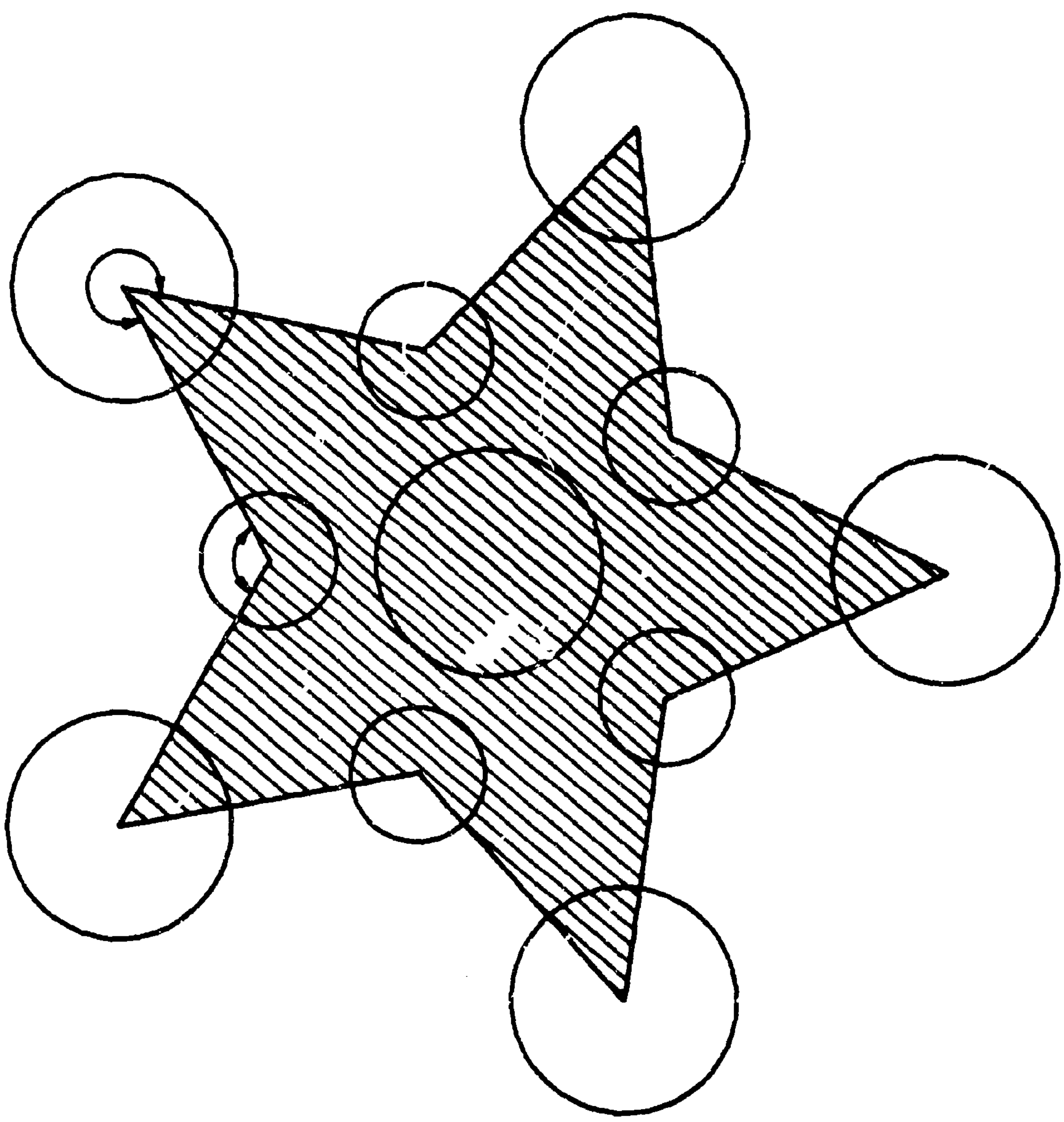

FIG. 7b END VIEW OF TYPICAL NON-RADIALLY-ALIGNED MULTI-TUBE SUPPRESSOR NOZZLE

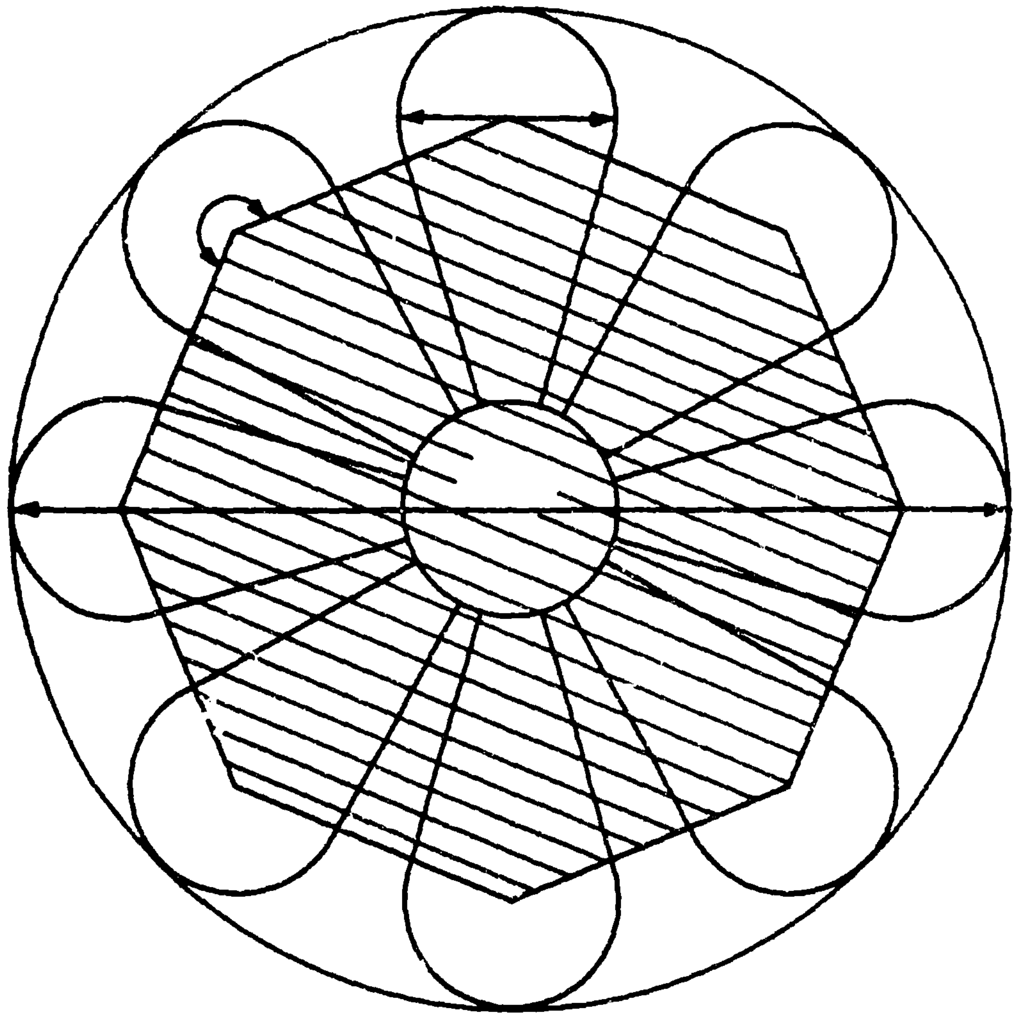

FIG. 7c END VIEW OF TYPICAL MULTI-LOBED NOZZLE (WITH CENTER-BODY)

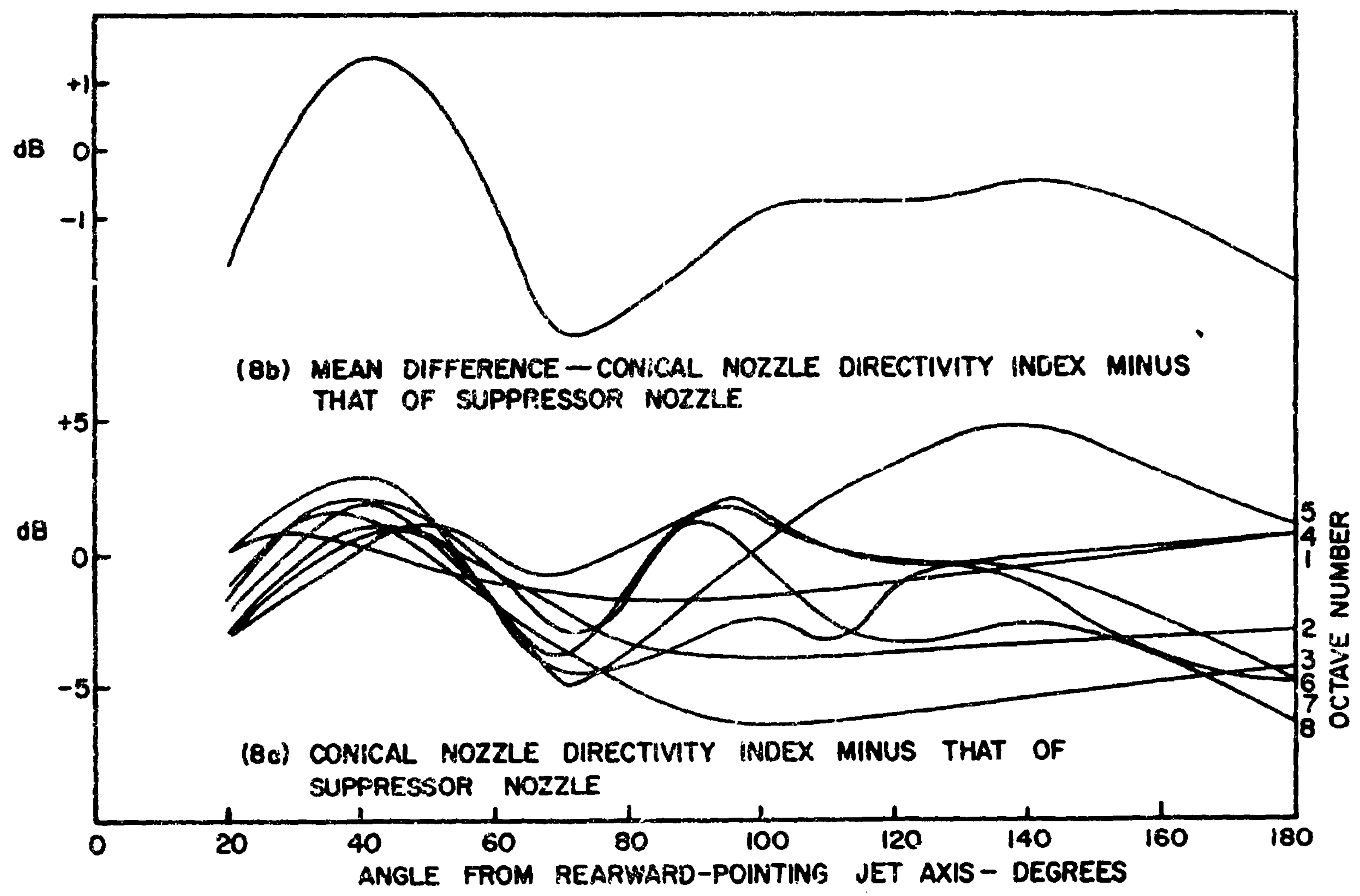

FIG. 8 DIFFERENCES IN DIRECTIVITIES — CONICAL AND EIGHT-LOBE NOZZLES

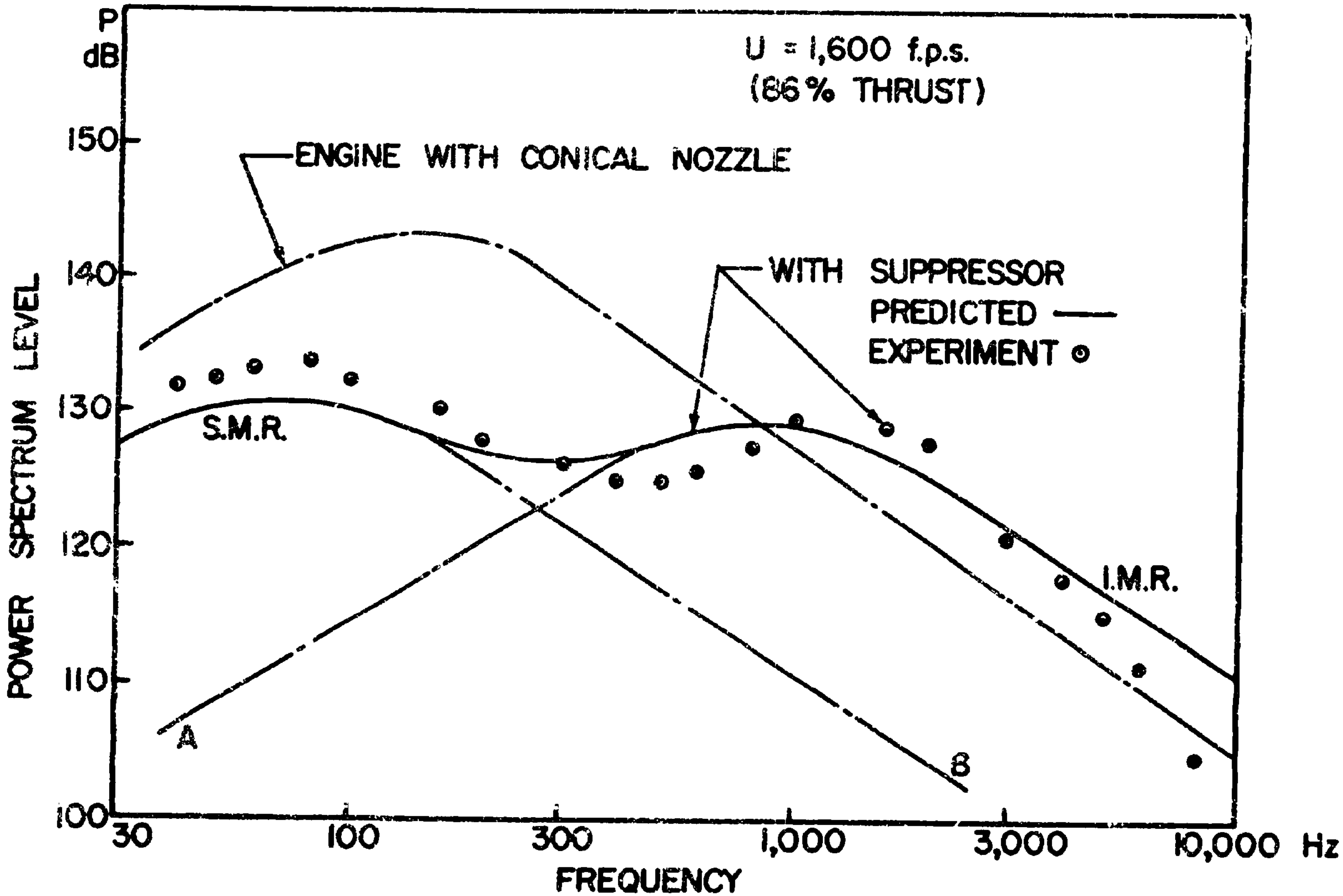

FIG. 9 COMPARISON OF MEASURED AND PREDICTED POWER SPECTRUM LEVELS, (TWELVE-LOBE SUPPRESSOR NOZZLE WITH CENTER-BODY)

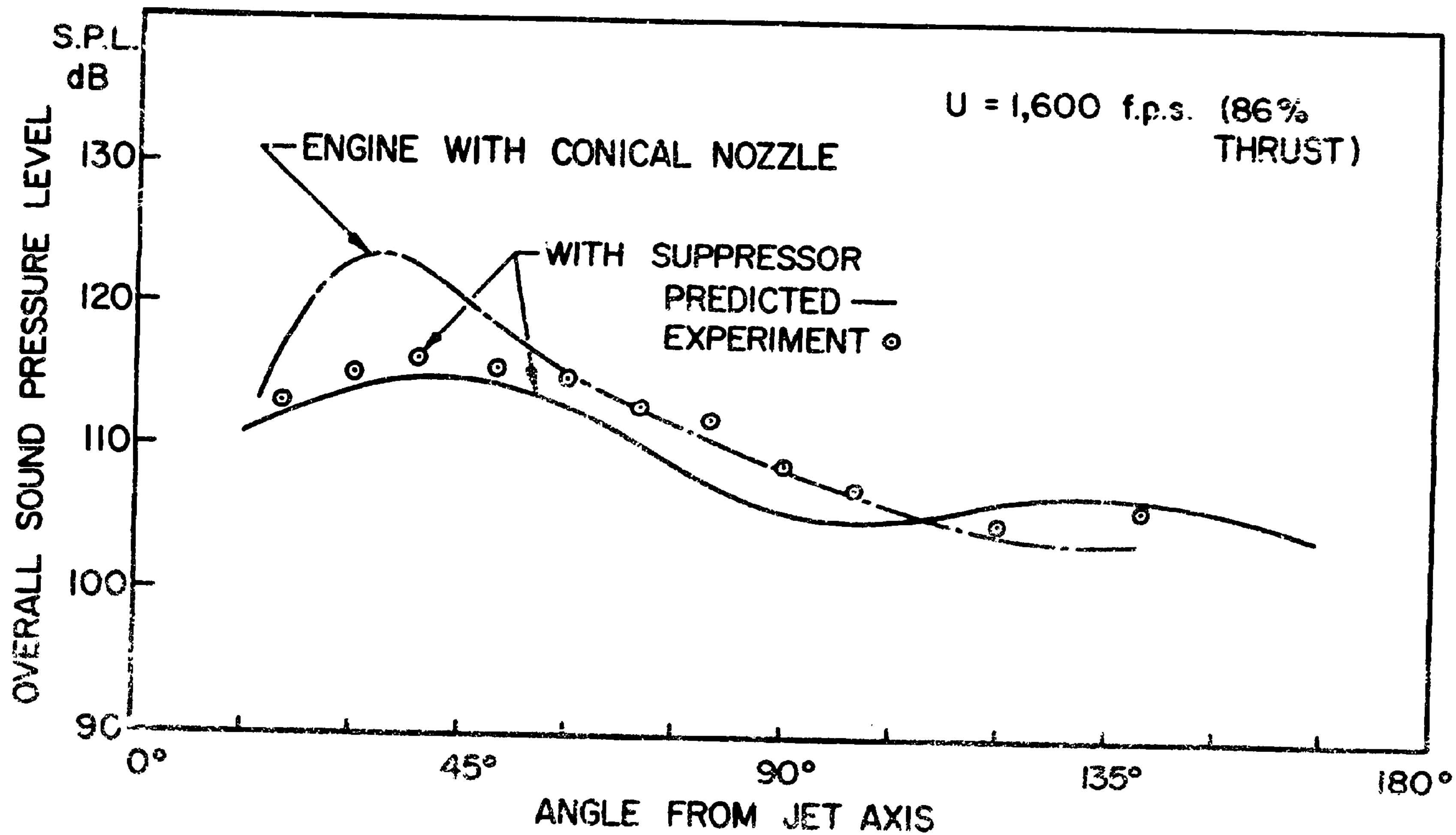

FIG. 10 COMPARISON OF MEASURED AND PREDICTED POLAR NOISE DISTRIBUTIONS (TWELVE-LOBE SUPPRESSOR NOZZLE WITH CENTER-BODY)

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13. ABSTRACT

The acoustical behaviour of certain nozzles designed to reduce the exhaust noise from turbo-jet engines has usually been established by direct measurement. The need for a simple yet adequate method of prediction of such behaviour is clear. The present paper revie s the quasi-empirical approaches which have been adopted in the past, and p ying particular attention to methods suggested by Eldred to deal with the power spectral density and Lee for deriving directivity patterns, develops these for application to (axisymmetric) nozzles where the elements are not all of the same si e. The measure of agreement between predicted levels and typical results quoted in the literature is generally reasonably good. Some implications of the theory are discussed. Additionally, a mathematical model is presented to calculate the noise reductio due to the interference of adjacent in round jets.

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