RANKING AND SELECTION METHODS FOR CAPITAL INVESTMENT
DECISIONS IN PRIVATE AND PUBLIC SECTORS

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I. INTRODUCTION

One of the most important areas for decisionmaking within our
socio-economic system is that of capital investment decisions. The
basic idea of capital investment decisions is that expenditures of
capital are made in exchange for expected benefits over future periods
of time. In its largest sense, capital investment would include all
expenditures other than current account or consumption-oriented expen-
ses. As such, it would encompass the decisionmaking process of all
individuals, business enterprises (firms), governmental agencies, and
other organizations within the economy.

The purpose of this paper is to survey and illustrate various ana-
lytical methods for assisting the capital investment decision process.
Attention is limited to capital investments made by firms--referred to
collectively as the private sector--and those made by governmental or-
ganizations within the public sector. Even though such a dichotomization

is not complete--as witnessed by the governmental regulation, and some-
times control, of private entities--it does serve as a usable framework
for the various methodologies discussed in this paper. The scope of
private investment decisions includes new fixed assets, replacement of
existing fixed assets, make or buy decisions buy or lease decisions,
new product lines, and changes in distribution systems. Alternatively,
governmental investment decisions could involve such public areas as
health, education, transportation, recreation, and even space.

The relative importance of capital investment decisions is best
seen by examining the components of total economic activity as por-
trayed by an aggregate gross national product (GNP) model

$$\text{GNP} = \text{C} + \text{I} + \text{G} \tag{1}$$
$$(843) \quad (524) \quad (121) \quad (198)$$

In equation (1), the private sector is represented by consumption C
and investment I, while the public sector is designated by G. The num-
bers in parenthesis represent the estimates for fiscal 1968 in billions
of dollars. Within the investment category, expected expenditures for
plant construction and equipment purchases (i.e., depreciating type
assets) together account for $88 billion, or over 10 percent of total
GNP. Within the governmental category, it is more difficult to assess
such relationships because depreciation accounting is not used. None-
theless, capital investment budgets for the Department of Transportation
($6 billion) and other governmental agencies would probably approach
$75-$100 billion. Clearly, the importance of capital investment

decisions within the private and public sectors of our economic system can be noted in just a short period of one year.[1]

A useful starting point in analyzing capital investments is to recognize that the decisionmaking process really consists of two impor-tant steps: (1) generating investment projects, and (2) ranking and selecting investment projects. The first step essentially one of analysis and data preparation, while the second step is one of deci-sion. Although it would seem clear that success in step (2) can be no better than the inputs which are prepared in step (1), still the majority of the published literature has dealt with the latter. Un-fortunately, this inequity is continued in this paper.

After a brief overview of the problem of generating investment projects in Section II, a review of suggested ranking method and their associated decision rules is presented in Section III. Section IV then focuses on methodology for use in the private sector. This necessitates a careful classification of problem types and a suggested solution for each. Example investment projects are evaluated to illustrate the sug-gested ranking and selection methods. This is followed in Section V with a discussion of methodology applicable to the public sector, and again an illustrative example is included. The important problem of uncertainty, and its relationship to time discounting, is the subject of Section VI, and alternative means of handling uncertainty are sug-gested. The final section is by no means a complete coverage of the problem of uncertainty, but rather is intended to summarize possibili-ties which warrant further research.

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After a brief overview of the problem of generating investment projects in Section II, a review of suggested ranking method and their associated decision rules is presented in Section III. Section IV then focuses on methodology for use in the private sector. This necessitates a careful classification of problem types and a suggested solution for each. Example investment projects are evaluated to illustrate the suggested ranking and selection methods. This is followed in Section V with a discussion of methodology applicable to the public sector, and again an illustrative example is included. The important problem of uncertainty, and its relationship to time discounting, is the subject of Section VI, and alternative means of handling uncertainty are suggested. The final section is by no means a complete coverage of the problem of uncertainty, but rather is intended to summarize possibilities which warrant further research.

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In order to expand upon these thoughts, it is necessary to consider private and public sectors individually. Consider first, the generating of investment projects by private firms. Ideas for cost reduction or income expansion can come from anywhere within the organization--from workers at the lowest level (perhaps from a suggestion system) to project engineers specifically paid to generate such ideas. For small projects with a relatively low cost, the investment may be approved locally and immediately implemented. For larger projects, however, review and evaluation may only be made at a higher (or even top) level of the organizational hierarchy. Clearly, one of the more important responsibilities of top management in most firms is the reviewing and evaluation of the final list of investment projects which survive lower level scrutiny in the process of being reviewed up the organizational hierarchy.[2]

Certain information is necessary for each investment project to be considered. First of all, the project life must be estimated. Then all relevant benefits and costs during the project life must be forecasted. Included would be all direct benefits and costs as well as all indirect effects on other projects. That is, a systems-type viewpoint should be utilized so as to gain full perspective about each proposed investment. The relevant unit for measuring costs is logically in dollars, while the unit for benefits is also typically in dollars. More specifically, benefits should be measured as net cash flows--after tax and including relevant depreciation charges. The reason for the latter

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[2] For further discussion of the organizational aspects involved in generating investment projects, see Norton [19] and Hill [15]. Illustrative manuals and forms to be used in collecting the necessary information for each investment project are included in Pessemier [20] and Bierman and Smidt [2].

is that the objective (theoretically at least) of the firm is to maxi-
mize the potential wealth of the common shareholders (i.e., the owners).
And although no attempt is made to specify the utility function for
wealth of the shareholder group other than that it is an increasing func-
tion at a decreasing rate, the usual assumption is that firm decisions
which maximize the available cash position (for dividend payments or
reinvestment within the firm) will tend to maximize shareholder wealth.

Whereas the generating process is relatively straightforward with-
in the private sector, undefined boundaries and obscure objectives ob-
viate analogous simplicity within the public sector. First of all, the
ideas for public projects can come from anywhere within the society.
The benefits of a given project may only focus on a local area (such as
a bridge) or alternatively, the project may have far-reaching impact
(such as a dam and irrigation system). Moreover, the impetus for some
public projects may be relatively small, while others are backed by
strong lobby groups or other political pressures.[3] This characteristic
of the public sector clearly precludes an impartial and equal evalua-
tion of all investment projects. And since numerous investment proj-
ects are typically in competition for limited capital resources, the
projects must be reviewed at increasingly higher levels of authority.

Between the conception of investment projects and the process of
evaluation lies the formidable task of identifying and measuring all
relevant benefits and costs. A major difficulty here is that the sphere
in which benefits accrue may not be well defined--and secondly, there

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[3]For a more detailed look at how a public project evolves from
idea to reality, see Eckstein [8].

may be no convenient unit for measurement. It is far more difficult
to justify the "dollarizing" of benefits in the public sector than it
is for private enterprise. In addition, benefits may pertain to differ-
ent entities such as individual citizens (as they use or confront the
investment project), private firms (which indirectly may become in-
volved with the project), and the public or society in an aggregate sense.
Even if he succeeds in measuring such benefits, the analyst must confront
the weighting problem across these areas of involvement. The implica-
tions of these observations should become more apparent in subsequent
sections.

For the remainder of this paper, these various problems of project
generation will be overlooked (or assumed solved), and attention will
shift to the second step of ranking and selecting. In so doing, it is
convenient to establish a common notation for discussing investment
projects. A useful scheme is to represent a given project in a time-
line diagram as follows:
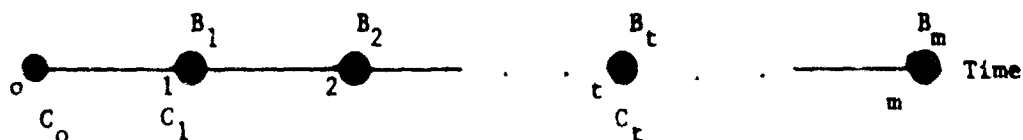
The large circles represent points, along a time continuum from left
to right, which divide the expected horizon of the investment project
into m periods. The periods need not be of equal length--although an-
nual periodicity is commonplace. For the general project illustrated,
B_t and C_t represent the benefit and cost, respectively, accruing at the
end of period t. If both benefits and costs can be measured in dollars,
then $R_t = B_t - C_t$ will represent the net cash benefit accruing at the

end of period t. Thus, in simplest form, an investment project can be described (for purposes of quantitative evaluation) by (1) its initial cost C_o, (2) the expected horizon length m in number of periods, and (3) a series of m net cash benefits R_t. If aggregation is not possible or desired, C_o, m, B_t, and C_t will describe the project.

A final caution should be made. The investment projects, so described, are, at best, estimates of the future. And because we live in a world of uncertainty, the B_t and C_t should be taken as "expected" values which summarize (explicitly or implicitly) the analyst's feelings about the future in a single measure. In Section VI, this rather limiting assumption is dropped and more exact methods for incorporating uncertainty are suggested.

III. A REVIEW OF RANKING METHODS

One might well argue that the foregoing is the critical part of
the capital investment decision process, and that if all benefits and
costs are forecasted in an appropriate manner, tne ranking and selec-
tion tasks could be largely routinized into a set of computational pro-
cedures and a series of decision rules. An examination of the academic
literature, however, in such fields as industrial engineer ng, mana-
gerial finance, operations research, and economics, suggests that no
clear consensus exists. One of the reasons for the lack of uniformity
is that certain methods are normatively developed, while others are
empirically suggested as how the real world behaves. Secondly, there
is a wide spectrum of different problems to which the ranking and se-
lecting methods have been applied. This has tended to confuse the de-
velopment of a consensus.

It is convenient, at thi point, to distinguish between the task
of ranking and the task of selecting investment projects. The selec-
tion task, which is really one of decision among alternatives, and
which is thereby closely related to the particul sector involved, is
considered in the next two sections. The subject of this section is
the ranking task which must be done first.

The ranking of capital investments is a further quantification,
given the time line representation of a series of investment projects.
Six distinct methods of ranking will be explained and discussed here.
All six are potentially useful in evaluating alternative investment
projects. If,for some reason, an investment project 's considered so
important to the welfare of the firm or society that it need not be

subject to the "competition" of alternative projects, then it should
be immediately accepted, regardless of how its benefits compare to its
costs. Such a situation of "urgency" or "priority" is recognized as
a real world occurrence, but will not be considered further in a com-
parison of ranking methods. In order to illustrate the different rank-
ing methods to be discussed, consider the nine capital investment proj-
ects whose characteristics are described in Table 1. All are relatively
short-lived and the benefits and costs have been simplified where
possible.

Table 1

EXAMPLE INVESTMENT PROJECTS

Project	Investment C_o	Net Cash Benefits			
		R_1	R_2	R_3	R_4
A	500	600			
B	1000	1200			
C	400	564			
D	240	2500	-2500		
E	200	120	165		
F	400	240	160	200	
G	300	240	240	60	
H	2000	2200	-300	1700	
I	1000	200	200	200	800

A "simple" project is one where an initial cash outlay is followed by
one or more positive net cash benefits (or inflows). Project D and H
are not of this type since negative net cash benefits occur in subse-
quent periods. An example of such a non-simple project would be the
need to perform major maintenance on a machine.

1. PAYBACK RECIPROCAL (PBR)

One of the commonly used ranking methods in the private sector is called "payback" and has to do with how quickly the capital outlay (or cost) of the project is recovered. That is, it determines the number of periods such that the total of the R_j is just equal to the initial C_o. Since other of the ranking methods are couched in a measure of return or profitability, it is convenient to work with the reciprocal of the payback measure. Hence, define

$$PBR = 1/n \text{ such that } \sum_{t=1}^{n} R_t = C_o \qquad (2)$$

Although the R_t are defined to accrue only at the end of a period, the usual practice is to assume continuity of benefit accrual evenly through the preceding period. For Project F, the initial outlay $C_o = 400$ is recovered during the second period; that is, $n_F = 2$ or $PBR_F = .500$. Values of PBR for all nine projects are presented in Table 2. Assuming that quickness of capital recovery is desirable,

Table 2

SUMMARY OF RANKING METHOD RESULTS
FOR EXAMPLE INVESTMENT PROJECTS

Project	Ranking Method					
	PBR	ARR	IRR	NPV	BCR	AEV
A	1.200	.400	.200	66.038	1.132	70.000
B	1.200	.400	.200	132.076	1.132	140.000
C	1.410	.820	.410	132.076	1.330	140.000
D	10.417	-1.000	(a)	-106.503	0.556	-58.091
E	.673	.425	.257	60.057	1.300	32.757
F	.500	.333	.244	136.739	1.342	51.155
G	.300	.533	.448	190.392	1.635	71.227
H	1.100	.513	.403	1182.429	1.591	442.358
I	.286	.200	.116	168.280	1.168	48.564

[a] Two positive real roots: .250 and 4.000.

projects would be ranked in order of decreasing PBR. It is immediately
noted that PBR is independent of project size--i.e., $PBR_A = PBR_B$. More-
over, the ranking by capital recovery does not agree with the other
ranking methods to be discussed. The highest (Project D) will even be
seen to be unacceptable for certain of the other methods.

The major advantage of payback reciprocal is its ease of computa-
tion. But in only measuring capital recovery, PBR does not consider
the "profitability" of the project since subsequent benefits beyond
capital recovery are ignored. Nevertheless, PBR (or payback) is prob-
ably the most popular ranking method used in the private sector. The
decision criterion for project acceptance using PBR is some minimum
cut-off which must be specified by the decision maker.

2. AVERAGE RATE OF RETURN (ARR)

Another popularly used ranking method within the private sector
is the average rate of return. Although there are various versions of
this method, it is usually defined as the ratio of average net income
to average investment cost. As such, the ranking method is closer to
an accounting concept which depends closely on the particular deprecia-
tion schedule used, and thus it is not a cash flow concept. The aver-
age rate of return can be expressed by

$$ARR = \frac{2\left[\sum_{t=1}^{n} R_t - C_o\right]}{mC_o} \tag{3}$$

If all the net cash benefits are equal ($R_t = R$) and assuming that straight-
line depreciation is used ($C_o = mD$), then it is easily shown that

$$ARR = \frac{2(R-D)}{C_o} = \frac{2R}{C_o} - \frac{2}{m} \tag{4}$$

and as the investment horizon gets increasingly larger

$$\text{limit ARR} = \frac{2R}{C_o} = 2 \cdot \text{PBR} \tag{5}$$
$$m \longrightarrow \infty$$

That is, it approaches twice the payback reciprocal.[4]

The average rate of return is preferable to payback reciprocal in that it comes closer to being a measure of profitability. The chief disadvantage of ARR is that there is no consideration given as to when the benefits accrue, and therefore the time value of money is ignored. If used, however, the decision maker is again forced to specify a minimum cut-off value and all projects whose ARR is above the cut-off are accepted. As seen from Table 2, Project C exhibits the highest value when the alternative investment projects are ranked using ARR. The ARR for Projects A and B are again the same, thus indicating the insensitivity of ARR to project size.

3. INTERNAL RATE OF RETURN (IRR)

The remaining four ranking methods are all superior to PBR and ARR in that they consider the relative timing of benefits and costs during the expected lifetime of the investment project. All four can be considered as different versions of a discounted cash flow approach which does consider timing. Current academic thought is in general agreement that there is a time value of money, and hence discounting should be reflected in some manner.[5]

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[4] If $R_t = R$, then from expression (2), PBR = R/C_o. See Gordon [12] for further comparisons of these ranking methods.

[5] Eiteman [9] has suggested another version of payback which does include discounting. The method does not appear to have gained wide acceptance, however.

One of the earliest of the discounted cash flow techniques is the internal rate of return. Although other names have been used, the meaning is clear. It is that rate of return (IRR) used for discounting such that the present value of all project benefits just equals the present value of costs. In equation form

$$\sum_{t=1}^{m} \frac{B_t}{(1 + IRR)}t = \sum_{t=o}^{m} \frac{C_t}{(1 + IRR)}t \qquad (6)$$

or if the only cost is an initial outlay

$$\sum_{t=1}^{m} \frac{B_t}{(1 + IRR)}t = C_o \qquad (6a)$$

It is an acceptable measure since timing is reflected and also because it is a measure of profitability.

An immediate disadvantage is that IRR appears more than once in equation (6) and thus cannot be solved for directly. In particular, the determination of IRR requires the solution of an m-degree polynomial equation. In addition to being computationally cumbersome (although approximating computer methods do exist), there may well be more than one real solution--provided there is a change in sign for the R_t values during the horizon. A necessary condition for multiple roots is that there is a change in signs of the net cash benefits during the project horizon.[6] Consider, for example, Project D which has such a change of sign and hence IRR_D = .25 or IRR_D = 4.00. Conversely, Project H has only one real root.

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[6] For a clearly written resolution of the multiple-root problem, see Teichroew, et al. [27].

A second disadvantage is that the relative size of the investment project is nowhere reflected in the IRR ranking method. For example, project A and B both have an IRR = .20 but differ by a factor of two in cash outlay requirement.

The third, and probably most damaging, disadvantage of IRR is that all intermediate cash inflows are assumed to be reinvested at the IRR. That is, the important reinvestment assumption depends on the particular project itself rather than alternative opportunities available to the firm (or government agency).[7] The extent of this poor assumption is related to the associated decision rule for IRR--which is to accept all projects with IRR larger than a specified cut-off rate. Much of the literature suggests that the appropriate cut-off rate (at least for a firm) is its cost of capital, k. More specifically, if the firm's investment alternatives are ranked by IRR, one obtains a marginal efficiency of capital (or demand for capital) schedule. If it is then superimposed on the firm's marginal cost of capital (or supply of capital) schedule, the optimal level of capital investment can be determined. Only near such a solution is the reinvestment assumption approximately valid. If capital rationing exists, the implied reinvestment rate (IRR) is likely to be unrealistically high.

Finally, it can be shown that in the limiting case, IRR approaches payback reciprocal. Again assuming equal benefits, one can show

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[7]The reinvestment disadvantage can be circumvented by specifying a precise opportunity rate available to the firm and compounding all intermediate flows forward to the end of the horizon. See Porterfield [21].

$$IRR = \frac{B}{C_0} - \frac{B}{C_0} \left(\frac{1}{1 + IRR}\right)^m \tag{7}$$

and hence

$$\lim_{m \to \infty} IRR = \frac{B}{C_0} = PBR \tag{8}$$

The IRR ranking method has been suggested for use in both private and public sectors.

4. NET PRESENT VALUE (NPV)

A closely related ranking method which fortunately does not share in the disadvantage of IRR is the net present value. It is also of the discounted cash flow type and differs mainly in that a reinvestment rate is assumed, a priori, but now as a discount rate rather than as the cut-off criterion. Again, the appropriate reinvestment rate is usually defined as the firm's cost of capital, k. The ranking method is given by

$$NPV = \sum_{t=1}^{\bullet} \frac{B_t}{(1 + k)^t} - \sum_{t=0}^{\bullet} \frac{C_t}{(1 + k)^t} \tag{9}$$

or alternatively using the R_t notation

$$NPV = \sum_{t=0}^{\bullet} \frac{R_t}{(1 + k)^t} \tag{9a}$$

where $R_0 = - C_0$.

This measure, unlike the three preceding methods, is not in per-centage units, but rather in dollars. An alternative definition,

therefore, of NPV is that it represents the number of current dollars which a decisionmaker should be willing to pay for the opportunity of investing in the given project. A decisionmaker should be willing to pay at least $66 for Project A, for example. This means, therefore, that for the NPV measure, size is reflected, computations are relatively easy (although present value tables must be used), and the reinvestment rate is made explicit. The single disadvantage of NPV is that it does not reflect the relative efficiency of the investment project. For example, Projects B and C both have the same NPV when a discount rate of k = 6% is used, but Project C clearly makes better use of the initial outlay which is required. The associated decision rule for this ranking method is simply to accept all investment projects having a positive NPV. Thus, all the projects in Table 1 except D would be accepted, but Project H would be the first choice. There is general agreement among academicians that, among the discounted value methods, NPV is preferable to IRR because of computational ease and the explicit reinvestment assumption.[8]

5. BENEFIT COST RATIO (BCR)

Another ranking method which has received widespread usage, particularly in the public sector is the benefit cost ratio. It is similar in many respects to both NPV and IRR. It is defined by the following expression

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[8] For additional comparisons of IRR versus NPV, see Bierman and Smidt [2], or Quirin [23].

$$BCR = \frac{\sum\limits_{t=1}^{m} \dfrac{B_t}{(1 + k)^t}}{\sum\limits_{t=o}^{m} \dfrac{C_t}{(1 + k)^t}} \qquad (10)$$

which is similar to equation (9) except that a ratio is used rather than the numerical difference. Again, the particular opportunity rate k is specified as the appropriate measure for discounting (and also reinvestment).

The associated decision rule is to accept all projects whose BCR is greater than unity. It is readily noted that if BCR = 1.0 for a given project, then equation (10) reduces to the same as equation (6) -- this means that the discount rate is identical to the IRR for the project. Conversely, the main difference between BCR and NPV is that the former measures the "relative efficiency" of the project while the latter focuses on the size of the net benefit available to the decisionmaker. Notice that in comparing Projects A and B, the BCR is similar--as were PBR, ARR, and IRR--and only NPV indicates the effect of project size. Conversely, Projects B and C have a similar NPV, but $BCR_C > BCR_B$ which indicate that Project C makes more efficient use of capital.

Another possible advantage of BCR is that if benefits cannot be measured in dollars, a ratio (such as BCR) can still be used, while a difference (such as NPV) would make no sense. This might explain the widespread use of BCR in the public sector--but a closer look will reveal that other problems also arise if benefits and costs cannot be measured in the same units.

6. ANNUAL EQUIVALENT VALUE (AEV)

The final ranking method to be discussed can be traced to the industrial engineering literature. The basic idea here is to reduce the "net benefits" of an investment project to a period (or annualized) basis. This is done as follows:

$$AEV = \frac{NPV}{DA(m,k)} \tag{11}$$

where $DA(m,k)$ is the present value of a \$1 annuity received for m periods when the discount rate is $k\%$ per period. In other words, the annual equivalent value simply puts the net present value method on an annualized basis. This, of course, shifts focus from the size of the project to the length of its expected horizon.

Alternatively, if all benefits are the same ($B_j = B$), then

$$AEV = B - \frac{C_o}{DA(m,k)} \tag{11a}$$

In this version, the second term reduces the initial outlay cost to an annual basis (using a capital recovery factor) and subtracts it from the annual benefit.

The associated decision rule is to accept the investment project if AEV is positive. Its major advantage is when comparisons are being made between projects having different expected lifetime. Comparing Projects F and I, for example, one observes that $NPV_I > NPV_F$, but $AEV_I < AEV_F$. The difference is that NPV does not make any assumption about what happens between the end of the short project (F) and the end of the long project (I). Conversely, AEV assumes a continuity of

similar quality projects. A similar answer is obtained from NPV by
determining the least common denominator of project lives. In comparing
a 3-period project versus a 4-period project, one would assume a series
of four 3-period projects--and compare with another series of three
4-period projects. That is, the common denominator is twelve periods.

This completes the review of six methods for ranking investment
projects. Although inter-method comparisons have been made and examples
discussed, no overall conclusions have been inferred--except that some
form of discounting is desirable. The reason for this is that the
appropriate choice and implementation of a particular ranking method(s)
will depend upon this particular problem that is faced by the decision-
maker. This is the subject of the next two sections--and it is appro-
priate to begin with the private sector since it is somewhat easier
to categorize the capital investment problems of a business enterprise.

IV. RANKING AND SELECTING IN THE PRIVATE SECTOR

All of the six ranking methods discussed in the preceding section have at least one significant disadvantage. Furthermore, there would not appear to be a single, unique choice of ranking method which would be preferable in all cases. The critical test for any ranking method-- together with its associated decision rule--is whether or not their usage leads to correct decisions. The correct decision, in turn, depends on the particular objective of the decisionmaker and also the particular problem of investment project selection. The objective of the financial manager (decisionmaker) has already been specified as an attempt to maximize the wealth position of the common shareholders.

As far as the different types of investment problems which may arise are concerned, it is useful to categorize them as falling along a spectrum of increasing inter-relationship among alternative invest- ment projects. This is illustrated in the following schematic diagram:

Mutually Exclusive Projects	Independent Projects	Partially Related Projects	Highly Related Projects

Furthermore, it is important to specify the nature of the rela- tionship for both partially and highly related projects. Such a rela- tionship may be technological--which is a phenomenon internal to the firm--or stochastic which means that the future prospects of the projects are somehow related. The stochastic relationship--which is external to the firm--will be discussed in a subsequent section. As far as the technological consideration is concerned, if the projects are quite

closely related, it may be preferable to treat them as a single project.
As for partially related projects, the best guideline is to be sure
and consider all factors when evaluating a given project.

In order to move toward more specific recommendations, it is use-
ful to consider a series of increasingly complex projects and/or objec-
tives--and for each case, to indicate which ranking method and selective
rule is most appropriate. In what follows, it may well be questionable
whether a given firm, at a given point in time, would ever be confronted
with such simple alternatives. The retort to this is that it is method-
ologically useful to begin with the simple situations and gradually re-
lax assumptions (or add constraints).

The first and simplest case is the evaluation of a single invest-
ment project. For example, should the firm purchase a given machine?
Whereas it should be noted that all decisions involve alternatives, the
alternative to the single project is simply "doing nothing." Assume
then that the single project is being evaluated, there is no limitation
of available capital funds to the firm, and finally that the firm has
a cost of capital, k. The appropriate decision rule, in such a situa-
tion, is to accept the project if its NPV, where discounting is done
at k, is positive.[9] Such an acceptance means that the firm is expected
to have a net dollar benefit, over the lifetime of the project but in
current dollars, equal to NPV. Of course they should accept the project,
ceteris paribus, because it is analogous to simply receiving a check
for that amount.

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[9]An equivalent decision rule, under these conditions, is to accept
the project if its BCR is greater than unity. Such an equivalence will
not always be true, however, as will be seen.

The second case is where the firm is evaluating a series of independent projects, but again there is no capital constraint. Because there is no relationship between projects, the correct solution is simply to accept all projects with a positive NPV. Such a decision, just as in the first case, is entirely appropriate in view of an overall objective of maximizing the wealth position of the common shareholders of the firm.

The third case is to choose between two alternative investment projects which are mutually exclusive--in the sense that only one of them can be accepted--and further assuming that the two projects have the same expected lifetime or horizon. The appropriate procedure here is to select the project with the largest, positive NPV. This is comparable to ranking the projects in order of their increasing dollar outlays and examining the merits of the "incremental" investment between them.

To illustrate, assume that Projects F and G, from before, are mutually exclusive, and a choice must be made. Clearly, $NPV_G > NPV_F > 0$, to that the optimal choice is Project G. But Project G, the smallest project, if taken alone would be acceptable. The incremental project F-G would be represented by

$$\underset{100}{\bullet\!\!-\!\!-\!\!-\!\!-}\bullet\underset{80}{-\!\!-\!\!-\!\!-\!\!-\!\!\bullet\!\!-\!\!-}\overset{140}{-\!\!-\!\!\bullet}$$

This incremental project has a net present value of -53.653, and thus it should not be accepted. So again, Project G is the optimal choice. If a single choice must be made among several mutually exclusive projects, the direct use of NPV, or using it indirectly in an incremental

manner, should lead to the appropriate selection decision.

One additional comment should be made about this third case. If two mutually exclusive projects have very similar measures of ''PV, it is recommended that their benefit cost ratios also be compared, and the project with the highest BCR > 1.0 should be selected because it makes more efficient use of capital. If a certain project has a higher BCR than its alternative, but simultaneously a slightly lower NPV, then the decisionmaker is forced to make a trade-off between wealth maximization (NPV) and capital efficiency (BCR). Such trade-offs can often lead to a confounding of a already complex problem. Thus, except when the alternative has an almost identical NPV, it is suggested that the NPV ranking and selection criterion be used.

The fourth case is a comparison of two (or more) projects which are mutually exclusive but have unequal project lives. Use of NPV is inappropriate because it ignores the interim period between the end of the shorter and of the larger projects. It is suggested that ideally the analyst (or decisionmaker) would make explicit assumptions about the interim period. If that is not particularly feasible, then use of the AEV method is recommended. It implicitly assumes that projects of a similar quality will be generated in the future in a continuing manner, and thus places the two alternatives on a comparable basis.

Returning to the earlier comparison between Projects F and I, it was mentioned that Project I would be selected if NPV were the ranking method used[10] and no attention were given to their uneven lines. What

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[10]Notice that this is another example where NPV and BCR give opposite rankings.

this means is that Project F is "penalized" in that "nothing happens"

between the third and fourth periods. More specifically, no reinvest-

ment opportunities are assumed to exist. In moving to the preferred

method, AEV, this poor assumption is circumvented when both projects

are placed on a periodic basis. As a result, Project F is considered

preferable to Project I and should be selected.

The fifth case relaxes the assumption of unlimited capital and

recognizes that, in many instances, a condition of capital rationing

may exist. Economic theory would suggest, that as long as projects

having positive NPV are available, they should be accepted so as to

maximize shareholder wealth.[11] Unfortunately, most firms impose a

budgetary ceiling on the extent to which investment projects can be

accepted in a given period. The question arises, therefore, as to

what ranking and selection method should be used.

A quick answer might be to continue to use NPV which has been the

major ranking method suggested for the foregoing cases. In some cases,

NPV will lead to the correct decision, but it is easy to find examples

where it does not. Assume that only the independent Projects C, F,

and I are being considered and a budget ceiling of 1000 is imposed.

Using NPV, Project I would be chosen and the ﹍ would be exhausted.

From Table 2, it is seen that the combination of Projects C and F would

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[11]As more and more investment projects are accepted, the firm
must generate additional capital funds from its various financial sources.
As a result, the firm's cost of capital tends to rise--thus necessitating
a re-appraisal of the ranking (viz. discounting) results. Solution of
the capital investment problem thus necessitates a simultaneous consid-
eration of both demand and supply conditions. See Weston and Brigham [29].

result in a total NPV equal to 268.815 (as opposed to $NPV_{I} = 168.280$) and also only 80 percent of the budget would have to be used.

A second possibility is to rank the projects using BCR and then accept the projects until the budget is exhausted. In the case of Projects C,F, and I, such a procedure does lead to the optimal selection of C and F. Although the BCR method will work for many capital ration- ing problems, one can still construct examples for which it will not lead to the optimal solution. Non-optimality generally results when the lumpiness of investment projects causes a subset of smaller projects to result in a better solution than one from certain of the larger projects.[12]

What is needed for the case of capital rationing is a method which will always give the correct solution--not just "most" of the time. Such a solution is obtainable if the capital investment decision is formulated into a programming context.[13] If n projects are being considered, the problem becomes

$$\text{Maximize} \quad \sum_{j=1}^{n} X_j \; NPV_j \tag{12}$$

$$\text{Subject to} \quad \sum_{j=1}^{n} X_j \; C_{jo} \leq \bar{C} \tag{13}$$

where X_j is a binary variable for project j taking on the values unity or zero depending upon whether project j is accepted or rejected, re- spectively. Furthermore, C_{jo} is the initial cash outlay for project j and $\bar{C}$ is the extent of the allowable capital budget. If it is not

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[12]See Van Horne [30] for such an example.

[13]For a comprehensive survey of the programming approach to capi- tal rationing, see Weingartner [28].

appropriate to "net out" benefits and costs and/or there are cash out-
lays in subsequent periods as well, the constraint as given by expres-
sion (13) must be enlarged to

$$\text{Subject to} \quad \sum_{j=1}^{n} X_j \, C_{jt} \leq \overline{C}_t \tag{13a}$$

where $t = 0, 1, \ldots, m$ refers to the various periods in the horizons of
the proposed investments where capital rationing will occur, and $\overline{C}_t$
is the maximum capital outlay allowed in period t. In other words,
the constraint set consists of m equations rather than just one.

Because of the binary nature of X_j, this formulation is in essence
an integer programming problem for which there are solution algorithms
but few operational programs. The general nature of such a programming
approach is exactly as required--to find the best (via aggregate NPV)
combination of investment projects within the budget constraint. A
difficulty of the expanded system, given by expressions (12) and (13a)
is that projects to be initiated at a later date must be "generated"
now. That is, the programming methodology does not really allow for
the evaluation of investment projects that may arise during the next
period(s).

This completes a survey of recommended ranking and selection methods
for business firms in the private sector. Although not all possibili-
ties have been exhausted, the five cases covered do include a wide
spectrum of possible decision situations. The net present value has
been seen to be the chief ranking method suggested--together with modi-
fications for particular problems.

V. RANKING AND SELECTING IN THE PUBLIC SECTOR

Economic theory would suggest, that given reasonably competitive

markets and resource mobility, capital resources s ould flow between

business enterprises until the marginal contribution of capital in

each firm is a constant. Such a distribution and flow of capital assets

is not observed, however, as many firms continue operations despite a

dearth of investment opportunities. It is not within the scope of this

paper to explore the several reasons for this less than ideal situation.

The important point is that the capital investment process of business

firms can typically be evaluated in isolation from that of other firms.

The realities of such a "shielding" greatly simplifies the nature of

the ranking and selection methods used by business firms. There are,

of course, conflicting demands for capital funds within a given firm,

but they can be resolved (using the previous recommendations) rather

easily since all are subsumed under the common objective of maximiz-

ing areholder wealth.

In moving to a discussion of the public sector, it is also well

to indicate how a decisionmaking unit--such as a governmental agency--

is related to other agencies, and how this will influence the ranking

and selection methods to be used. One quickly realizes that the scope

of the capital investment process within the public sector is far

more complex due to (1) the absence of the shielding phenomenon that

is pertinent to a business firm,(2) the lack of well-defined objectives

for the public sector, and (3) problems of measuring benefit and costs.

Quantitative analysis of capital investments with the public

sector is generally referred to as "cost benefit analysis" and is but

a part of an overall evaluation scheme known as the planning-programming-budgeting system. This system, which was first introduced into the Department of Defense and later into all federal agencies, has forced decisionmaking into an explicit quantitative evaluation of alternative systems. Furthermore it has attempted to integrate annual budget decisions with long-range planning.[14]

Whereas cost-benefit analysis refers to the overall process of identifying objectives, technological alternatives, and relevant benefits and costs, there are still different ranking methods which have been suggested. By far the most prevalent ranking method used within the public sector is the benefit cost ratio. A distinction should be made between benefit cost ratio which is a particular ranking method, and cost-benefit analysis which is an overall approach to decision-making. It is well to first review the use of benefit cost ratios and indicate potential pitfalls in their using. And in order to do so, it is convenient to focus on the decisionmaking process within a single government agency. Later in this section, the higher-order problem of capital allocation between government agencies will be considered.

Suppose, for example, that a transportation agency is considering alternative systems for meeting expected demand for transportation services over an extended horizon--say of 25 years. Assume, further, that four alternative systems, or system improvements, have been identified, some of which can be implemented in combination. Let the four alternatives be designated as Projects J,K,L, and M.

14.
    For further description of planning-programming-budgeting systems and cost-benefit analysis, see Due [7] . A theoretical discussion of cost-effectiveness is provided by Heuston and Ogawa [14] , and a survey of cost-benefit analysis is found in Pheat and Turvey [22].

For each project, there is both an investment cost C_I and an annual cost for operation and maintenance C_{OM}. Assume, for simplicity, that if projects are accepted in combination, these cost components will be additive. Table 3 summarizes the cost components of the four basic projects and also the feasible combinations. Note that a total of ten distinct combinations--each referred to as a system change--are identified. This includes five combinations of two projects and one combination of three projects.

Table 3

RELEVANT COSTS AND BENEFITS FOR ALTERNATIVE SYSTEM CHANGES

System Change	Project(s)	Investment Cost, C_I	Annual Cost, C_{OM}	Annual Benefit, B
1	J	2,800,000	40,000	1,510,000
2	K	3,000,000	42,000	1,400,000
3	L	3,200,000	40,000	1,600,000
4	M	12,000,000	85,000	2,620,000
5	JK	5,800,000	82,000	1,830,000
6	JL	6,000,000	80,000	2,500,000
7	JM	14,800,000	125,000	2,700,000
8	KL	6,200,000	82,000	1,920,000
9	KM	15,000,000	127,000	2,040,000
10	JKL	9,000,000	122,000	2,300,000

Also included in Table 3 are the annual benefits expected from each of the system changes. Although there is no particular reason to assume that such benefits should be the sa · in each of 25 years of the planning period, it is a simple matter to convert an unequal stream R_t into an equal stream R using an appropriate discount rate (k = 6%),

according to the following relationship

$$R = \frac{\sum\limits_{t=1}^{m} \frac{R_t}{(1+k)^t}}{DA\ (m,k)} \qquad (14)$$

In addition, there is no particular reason to assume that it is even

possible to estimate benefits in dollar terms. And this, in fact, is

one of the important problems confronting the design of a methodology

for the public sector. Assume for purposes of the illustration, how-

ever, that such estimation can be done. Finally, note that, ur   e the

costs, the annual benefits are not additive. Certain project combina-

tions such as JK and JL result in a total benefit less than the sum of the

individual components.

In order to compare among the ten alternative system changes, it

is first necessary to also convert costs to an annualized basis. This

is done, in a manner similar to that for benefits, using

$$C = C_{OM} + \frac{C_I}{DA\ (m,k)} \qquad (15)$$

where, as before DA(m,k) is an appropriate annuity factor which converts

C_I to an annual equivalent.[15] Table 4 lists the alternative system

changes in order of their increasing total annual cost, C. The ten

system changes are then evaluated using the net present value method

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[15] In this illustration, a discount rate k has been used without
specification. The determination of the appropriate discount rate for
both the private and public sectors is deferred until the next section
of the paper.

and also the benefit cost ratio method. Notice that, unlike the calcu-
lations used in Table 2, here the NPV and BCR are both computed on an
annualized basis. The BCR is unaffected by such a change, and the NPV
is only affected by a scale change--i.e., there is no change in the
relative rankings.

Table 4

EVALUATION OF ALTERNATIVE SYSTEM CHANGES

System Change	Annual O&M Cost[a]	Annual Inv. Cost[a]	Annual Cost[a]	Annual Benefit[a]	Annual NPV[a]	Annual BCR
1	40	219	259	1,510	1,251	5.83
2	42	2?5	277	1,400	1,123	5.05
3	40	250	290	1,600	1,310	5.51
5	82	454	536	1,830	1,294	3,41
6	80	469	549	2,500	1,951	4.55
8	82	485	567	1,920	1,353	3.39
10	122	704	826	2,300	1,474	2.78
4.	85	939	1,024	2,670	1,596	2.56
7	105	1,158	1,263	2,700	1,437	2.14
9	107	1,173	1,280	2,040	760	1,59

[a] In thousands.

NOTE: Calculations based on a 6% discount rate.

If there is no capital rationing involved, then the optimal system
change would appear to be the one which gives the largest excess of
"dollarized" benefits over dollar cost--either in the aggregate or on
an annualized basis. In this example, system change 6 which results
in an expected NPV equal to 1951 is the optimal choice. But notice that

this is clearly not the system change with the largest BCR.[16] Hence, system change 1 (Project J) gives the largest BCR, while system change 6 (combination of Projects J and L) yields the largest NPV. This serves to illustrate that benefit cost ratio--if used incorrectly--can lead to an incorrect solution.

It is possible, however, to salvage the benefit cost ratio. This can be accomplished if the benefit cost ratio is used in an incremental fashion involving only pair comparisons. Continuing the same example, the analysis is presented as Table 5. In this analysis, system change 0 refers to an unchanged base system which is postulated to be operational, without further investment, during the planning horizon. The analysis proceeds in order of increasing total annual cost--that is, in order of presentation in Table 4. System change 1 compares favorably with system change 0 and is accepted. System change 2 does not compare favorably with system change 1 and is rejected. The process continues for all alternatives--always comparing the "challenger" with the last acceptable system change. As noted in the last column of Table 5, the last acceptance--and hence the optimal selection--is system change 6. This agrees with the NPV ranking in Table 4 and this illustrates how the benefit cost ratio can be used correctly.

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[16]This illustrative example is based on a similar problem presented by Grant and Ireson [13].

Table 5

INCREMENTAL BENEFIT COST ANALYSIS
OF ALTERNATIVE SYSTEM CHANGES

System Comparison	Incremental Cost	Incremental Benefit	Incremental BCR	Decision
1 versus 0	259	1,510	5.83	Accept 1
2 versus 1	18	-110	-6.11	Reject 2
3 versus 1	31	90	2.90	Accept 3
5 versus 3	246	230	0.93	Reject 5
6 versus 3	259	900	3.47	Accept 6
8 versus 6	18	-580	-32.22	Reject 8
10 versus 6	277	-200	-0.72	Reject 10
4 versus 6	475	120	0.25	Reject 4
7 versus 6	714	200	0.28	Reject 7
9 versus 6	731	-460	-0.63	Reject 9

One of the advantages of a situation involving only a few alterna-
tives, such as is depicted in the sample of this section, is that all
combinations can be explored directly. For if complete enumeration is
not feasible, then a programming approach must be used. Moreover, it
is possible to illustrate the results graphically and indicate certain
features of the problem. Consider Fig. 1 which relates total benefit
and total cost and where the ten system changes of the example are
plotted.

It can be immediately noted that certain of the system changes--
in particular 2, 8, 9, and 10--are dominated by another. Dominance
means that a certain system has higher benefits at a lower cost.[17]

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[17]A more thorough discussion of dominance and other decisionmaking
criteria is found in MacCrimmon [17]. A graphical discussion of cost-
effectiveness (benefit) trade-off is presented by Fox [11] .

Benefit (in thousands)

Fig. Graphical Analysis of Alternative System Changes

Graphically, this means that any system change that lies in the south-
east quadrant of another system change is "dominated by it." A domin-
ated system change also shows up in Table 5 with a negative benefit cost
ratio.

Other system changes can be eliminated from contention because of
concavity--a condition having to do with the overall location of points
in the space. Note that system changes 0, 1, 6, and 7 form a locus
which is concave to the origin. Now consider system changes 3 and 5.
If the decision rule allowed either of them to be accepted (as in Table
5), then system change 6 would automatically be accepted because the
segment from 5 to 6 has a "higher" slope than from 1 to 3 or 3 to 5.
Hence system changes 3, 4, and 5 are eliminated because they fail to
be on the concave locus of feasible alternatives. Also note that these
alternatives have benefit cost ratios less than unity in Table 5. By
the same benefit cost ratio criterion, system change 7 would necessarily
be eliminated also. However, in Fig. 1, it is part of the concave locus.

The graphical analysis serves to emphasize an important aspect of
decisionmaking in the public sector which has already been mentioned.
And that is the difficulty in many instances to evaluate benefit in
dollar terms. In the private sector, this does not usually present a
problem because the wealth maximization objective lends itself to
evaluating all investment projects in terms of dollars of benefits over
the lifetime of the project. Within the public sector, however, objec-
tives are less clear (usually in terms of social welfare, etc.) and
dollar measurements are not always appropriate. In addition, the ana-
lyst may not be willing to place a "dollar equivalent" value on what-
ever unit of measurement is used.

Suppose, then, that the benefits in the example are in "gratiles" or some such non-monetary unit. The question is simply how would a decision be reached? First of all, the NPV ranking method becomes power-less if a common denominator for costs and benefits cannot be reached. This also explains in part why the benefit cost ratio has received wider acceptance in the public sector. But whereas the BCR may still be appropriate as a relative measure of efficiency for comparing among investment projects, the "unity" selection criterion is no longer appropriate. In such cases, the decisionmaker must provide his appropriate preference function between dollar costs and gratile benefits.

In the orientation of Fig. 1, this would probably take the form of a set of indifference curves. Such curves would be convex to the origin--thus indicating that an increasing increment of benefit is necessary to justify an additional dollar of capital investment. Finally, such indifference curves would represent higher levels of satisfaction as they lie further up in the space--the optimal solution, of course, would lie at the tangency of the concave "opportunity" locus and the highest attainable indifference curve.

The implication of this is that in the private sector, the deci-sionmaker is spared the responsibility of specifying preference func-tions.[18] But in the public sector, this is not usually possible--and some one must determine the appropriate terms for trade-off. At best, the analyst can indicate the full range of alternatives (such as in a benefit-cost plot like Fig. 1) and the implication of different trade-off terms.

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[18]Such a statement must be relaxed when uncertainty is brought into the analysis. See the next section.

The other important observation relates to the isolation of de-
cisionmaking in one government agency from that in all others. The
"shielding" effect which is prevalent in the private sector is less
likely to occur in the public sector as various government agencies
compete for centrally allocated funds. Economic theory would again
suggest an equal marginal contribution across all government agencies.
This means that the slope, at solution, along each concave opportunity
locus (for each agency) should be approximately the same. It is doubt-
ful that such a condition could ever exist in practice because of poli-
tical pressures, lobbies, and the real world tendency of treating cer-
tain investment projects as "so urgent" that no analysis is required.

Finally, if the practice is to impose a dollar ceiling on each
agency, then this ceiling is simply treated as a vertical constraint
in a benefit cost space--and no projects which lie beyond (to the
right) should be included in the analysis.

This completes a brief survey of the formidable ranking and se-
lecting problems confronting the public sector. Again, all possibilities
have not been considered. In essence, an attempt has been made to
clarify the appropriateness of using the benefit cost ratio method, to
suggest the implication of decision criteria when measurement problems
exist with respect to benefits, and finally to introduce the complexi-
ties of agency inter-relationships

VI. THE PROBLEM OF UNCERTAINTY

This final section is intended to introduce a very important aspect of the capital investment decision process--that of uncertainty. The section is by no means intended as a complete analysis of risk and uncertainty, but instead, as an introduction to the need for further research--since it is felt that treatment of uncertainty is the weakest aspect of the decision process for capital investments.

In discussing appropriate procedures for ranking and selecting investment projects within both the private and public sectors, two important dimensions have been considered--one explicitly and the other only implicitly. The first dimension, the need for discounting future benefits and costs in order to reflect the time value of money, has been explicitly handled through the use of a discount rate k. In the private sector, k is typically taken as the firm's cost of capital. In the public sector, k is vaguely thought of as some sort of opportunity rate-- the important point here is that discounting is involved in the recommended NPV and BCR methods.

The second dimension has to do with uncertainty--the fact that investment decisions are based on expectations which may or may not materialize. Uncertainty, therefore, is an unfortunate characteristic of the forecasting task which, in turn, is part of the first step of generating investment projects. Thus far in the paper, uncertainty has only been considered implicitly in that forecasted benefits and costs have been taken as their expected values. In reality, one would expect that both benefits and costs should better be thought of as probability distributions rather than just expected values. The intent here is to

suggest different ways in which the entire probability distribution
can be reflected in the recommended ranking and selection methods.[19]
And in the process, the relationship between the uncertainty dimension
and the time value dimension will hopefully be made clear.

The first method of handling uncertainty is to ignore it under the
guise of certainty--such as has been done thus far in this paper. This
is felt to be a poor procedure in that it assumes that all projects are
of the same quality--and can be evaluated entirely on the basis of their
respective estimates of benefits and costs. The point is that uncer-
tainty cannot really be ignored, and assuming a world of certainty may
lead to incorrect--and even disastrous--decisions. If uncertainty is
ignored, then discounting pertains only to the time value of money con-
cept--which itself is not clear in the case of public sector decisions.

The second method of handling uncertainty is to adjust the discount
rates which are used. In particular, the more uncertain a future bene-
fit is felt to be, the higher the discount rate which is employed. The
effect of this is to penalize an uncertain benefit (or cost) by decreasing
its present value. Since net present value has been the most recommended
ranking method in this paper, it is well to review its computation.

In defining NPV using expression (9) or (9a) the discount rate was
considered to be constant over time and also among all projects investi-
gated. It has already been suggested that different rates could be
applied to different firms in different risk classes. Then too, it may
be desirable to change the rates over time--viz. to reflect the fact
that uncertainty increases with time during the project lifetime. This

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[19]For a complete survey of possible adjustments for uncertainty,
see Canada [4].

suggests a modified formula which would only pertain to the jth project

$$NPV_j = \sum_{t=0}^{m} \frac{R_{jt}}{(1 + k_{jt})} \tag{9b}$$

and where $k_{j1} < k_{j2} < \ldots < k_{jm}$ reflects an increasing uncertainty over time.[20] This type of adjustment for uncertainty can be referred to as a "denominator" adjustment. Operationally, it is cumbersome because the analyst is forced to specify a whole family of discount rates over time and for each different investment project.

An additional difficulty concerning the denominator type adjustment is that the discount rate involves the time value of money as well as the problem of uncertainty. In particular, the discount rate k_{jt} can be enlarged as follows

$$k_{jt} = i + k_{jt}^{*} \tag{16}$$

where i is the risk-free interest rate which denotes the time value of money, and k_{jt}^{*} is the "risk premium" for project j and pertaining to period t.

The confounding of these two factors provides motivation for a third method of handling uncertainty. In this method, which may be termed a "numerator" adjustment, uncertainty is penalized by a direct adjustment of the net cash benefit itself. This type of adjustment may be specified by another expression for calculating net present values

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[20] Robichek and Myers [24] have demonstrated that use of a constant discount rate, such as in equations (9) and (9a), infers a particular growth pattern for uncertainty over time. Also, see English [10].

$$NPV = \sum_{t=o}^{\blacksquare} \frac{\alpha_t R_t}{(1 + i)^t} \qquad (9c)$$

where α_t is an adjustment factor for period t, and hence $\alpha_t \leq 1$. Note also that if uncertainty is handled in the numerator, the risk-free interest rate alone is used for discounting.[21]

The product $\alpha_t R_t$ is known as a certainty equivalent, which simply means that a condition of uncertainty is reduced to a condition of certainty using an appropriate α_t penalty. Although little empirical work has been done on the nature of such an adjustment or penalty, it might, for instance, take the form

$$\sigma_t = 1 - w \, \sigma_t \qquad (17)$$

where w is a pre-determined constant (depending on risk preferences) and σ_t is the standard deviation of the probability distribution on the net cash benefit--and for which R_t was previously taken to mean the expected value. An alternative formulation would be to use the coefficient of variation, rather than the standard deviation, because it is in standardized units.

Despite its operational difficulties, the certainty equivalent or numerator adjustment would appear to be more attractive than the

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[21] This still does not completely solve the discounting problem--although it does simplify it considerably. For arguments about the appropriate choice of discount rates in both the private and public sectors, see Berman [1], Eckstein [8], Musgrave [18], and Quinn [23].

previous methods because it separates the risk and uncertainty question
from the time value o money, or discounting, problem:

The _fourth_ and final method for handling uncertainty is especially
pertinent when one is concerned with "partially related" projects. As
defined earlier, partially related projects can have to do with techno-
logical (already discussed) or stochastic considerations. The latter
means that two investment projects are assumed to be somehow related
as far as the uncertainty to which they are subject. In particular, a
stochastic relationship means that the two projects are subject to a
joint probability distribution. And in such a case, it is necessary
to consider the interrelationships between all pairs of investment proj-
ects that are being evaluated as potential additions to a capital budget.

The suggested approach for the class of problems is, in essence, a
portfolio approach wherein the decisionmaker must attempt to select the
optimal portfolio of capital investments.[22] The relative contribution
of a given project within the portfolio is analogous to the certainty
equivalent approach except that the numerator adjustment includes a
project's inter-dependencies as well as its individual riskiness. The
solution requires a programming formulation--not unlike that suggested
earlier for capital rationing.

Although this portfolio method is relatively undeveloped and un-
tried at present, together with the certainty equivalent method, it
provides the potential for better handling of the problem of uncertainty--
and therefore better decisions for selecting investment projects.

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[22]For an elaborate development of this complex problem, see
Lintner [16] and Cramer and Smith [5]

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