A MIX-OF-MODES EVALUATION MODEL FOR TRANSPORTATION SYSTEMS

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I. INTRODUCTION

One of the more important problems associated with any systems or effectiveness analysis is that of synthesizing the various sub-studies which are made such that continuity of purpose and method are maintained and meaningful results are insured. Within the broad area of transportation planning and evaluation, the problem is particularly acute because the appropriate planning horizon may span many years, and the benefits of alternative transportation systems accrue to a host of different concerns within both the private and the public sectors. One approach toward such complex problems is to begin at the lowest level of consideration and develop sub-models which eventually will feed into higher-level models. The alternative approach--and one which is reflected in this paper--is to begin at the highest level of decision and work downward toward the lower levels of detail.

An important focus of a meaningful evaluation model for transportation systems is to evaluate alternatives within the context of the entire package or mix of transportation services. More specifically,

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it is the evaluation of the incremental benefits and costs from adding a modified or new transportation mode to the existing mix. If the transportation planning horizon covers the 1970-1995 period, for example, new or improved modes must be evaluated relative to the total mix-of-modes that is expected to be operational during that period.

The purpose of this paper is to develop a methodology which conceptually may be useful in evaluating alternative transportation systems within such a mix-of-modes context. An important characteristic of the suggested methodology is that alternatives are evaluated along several important dimensions so as to reflect users of transportation systems, business firms that are involved in providing transportation services, and also the general public. In order to concentrate on the more important aspects of the problem, it is convenient to confine attention to intercity transportation along a single link between two metropolitan areas. The suggested model is adaptable, however, to the larger problem of transportation within a network of large cities.

In Section II, a generalized model for evaluating the incremental effectiveness of alternative transportation improvements is developed. Section III explores the relative sensitivity of the model to changes in the important variables and parameters. The model is extended in Section IV to include cost considerations, and an appropriate decision rule is explained. The benefit-cost model and its associated sensitivity analysis are then illustrated in Section V, using an hypothetical example which involves two alternative changes to a basic system consisting of seven modes over a planning horizon of five years. The final section explores both the limitations and implications of the mix-of-modes evaluation model.

II. GENERALIZED EFFECTIVENESS MODEL

This section presents a generalized model for assessing the total effectiveness of a transportation system consisting of several distinct modes of travel. The fundamental unit of consideration is one-way trips along a single link between two nodes. Each trip within such a simplified network is assumed to consist of intra-city travel on one or more of M_1 feeder modes within each node and inter-city travel on exactly one of M_2 feeder modes along the single link. The basic system, therefore, consists of a total mix of $M = M_1 + M_2$ transportation modes.*

By making certain specifications concerning the number of allowable feeder modes within each node, it is possible to identify a unique number of possible mode combinations that could be used by a traveler for a single trip. For example, if exactly one feeder mode is used at each end of the trip, a total of $M_1^2 M_2$ possible mode combinations could be identified. If two or more feeder modes are allowed, the total number increases sharply since various combinations are possible. Conversely, many particular combinations could be ruled out on logical grounds. A useful way of describing a particular mode combination for a single trip is with the vector

$$\overline{X} = \left[X_1, X_2, \ldots, X_M\right] \tag{1}$$

where X_j is a binary variable with value <u>one</u> if mode j is used or value <u>zero</u> if the mode is not used on the trip.

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*The importance of considering the total package of transportation services has been emphasized in M. L. Manheim, "Principles of Transport Systems Analysis," <u>Highway Research Record: Number 180</u> (Highway Research Board, 1967), pp. 11-20.

Corresponding to the particular technological characteristics of individual modes, as well as overall economic conditions, there will be a demand pattern for transportation services from the basic system over the planning horizon. The demand pattern for a particular period can be described by the triplet $(W_t, \overline{X}_{wt}, \overline{D})$ which highlights an important distinction between primary demand and derived demand. Primary demand is given by W_t, the total number of one-way trips demanded between the two nodes during period t. Corresponding to each trip will be its usage vector $\overline{X}_{wt} = [X_{mwt}]$ where X_{mwt} is a binary variable, as defined above, and $1 \leq w \leq W_t$ trips, $1 \leq t \leq T$ periods, and $1 \leq m \leq I$ possible modes. Total demand for trips during the entire horizon is simply

$$W = \sum_{t=1}^{T} W_t \qquad (2)$$

Derived demand, on the other hand, refers to the demand for particular modes resulting from primary demand. Derived demand is denoted by the matrix $\overline{D} = [D_{mt}]$ where the demand for any given mode during a given period is specified by

$$D_{mt} = \sum_{w=1}^{W_t} X_{mwt} \qquad (3)$$

and where total horizon demand for mode m is given by

$$D_m = \sum_{t=1}^{T} D_{mt} = \sum_{t=1}^{T} \sum_{w=1}^{W_t} X_{mwt} \qquad (4)$$

The binary variable for describing a particular mode combination is thus seen to be a convenient means of distinguishing between primary demand and derived demand--the latter being central to the purposes of this study.*

The next step is to indicate the relative benefits to all those groups involved with the basic transportation system. One useful classification scheme is to consider benefits (or disbenefits) accruing to (1) users of the transportation system, (2) all those business firms involved in design, implementation, operation, or maintenance of the many parts of the transportation system, and (3) the general public.** Attributes are particular dimensions of involvement, for each of these categories, along which absolute or relative measurements can be made. Possible attributes for users might include travel time, travel cost, convenience, and safety. For operator firms, some measure of profitability could be used as an attribute with sales and market share as possible alternatives. Attributes for the general public are admittedly more vague--possibilities here might be noise level and air pollution. Assume, notationally, that a total of I attributes are considered.

Suppose further that it is possible to assign values of a matrix $\overline{B} = [B_{mi}]$. These values may be thought of as measures of relative utility or benefit on a unit-trip basis. Thus, B_{mi} would represent the

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*No attempt is made in this paper to establish a particular method of forecasting demand. Considerable literature exists on this important but lower level problem. See, for example, R. E. Quandt and W. J. Baumol, "The Demand for Abstract Transport Modes: Theory and Measurement," Journal of Regional Science, 6:13-26, 1966.

**An analogous breakdown was employed by M. Hill, "A Method for the Evaluation of Transportation Plans," Highway Research Record: Number 180 (Highway Research Board, 1967), pp. 21-34.

particular value along attribute dimension i from a single usage of mode m. Furthermore, entries in the matrix $\overline{B}$ would consist of ordinal type numbers--useful in comparing among the various modes--where numerical assignments are made on a "points" basis between designated upper and lower limits.* Such limits would refer to the best and worst benefits along a single attribute. For example, in comparing modes along the societal attribute of air pollution, modes such as automobile and bus would receive fewer points than subway.*

Although the two important matrices, $\overline{D}$ and $\overline{B}$, comprise the basic variables in the mix-of-modes methodology, two additional input parameters are also needed. The first of these is the vector $\overline{R} = [R_i]$ of relative weightings across the I attribute dimensions which are identified. That is, R_i represents the weighting assigned to attribute i.** The second parameter is given by the matrix $\overline{S} = [S_{it}]$ of relative weightings across the T time periods. The representation used for intertemporal trade-offs is deliberately general, but it can be made more explicit if so desired. For example, if such a weighting scheme is to follow the popular method of discounting all values back to the present time using a schedule of rates $\overline{K} = [K_i]$, then

*Although utility or effectiveness values such as the B_{mi} can be questioned on several grounds, no further attempt is made here to justify their usage. For a similar approach to this problem, see W. Jessiman, et al, "A Rational Decision-Making Technique for Transportation Planning," Highway Research Record: Number 180 (Highway Research Board, 1967), pp. 71-80.

**Not unlike the case for $\overline{B}$, specification of the values for $\overline{R}$ poses a formidable task. A promising methodology for rationally determining both R and B, see J. R. Miller, "The Assessment of Worth: A Systematic Procedure and its Experimental Validation," unpublished Ph.D. dissertation, Massachusetts Institute of Technology, June 1966.

$$S_{it} = \left[\frac{1}{1+K_i}\right]^t \tag{5}$$

A two-dimensional schedule is used because the weighting may well vary over time to reflect increasing risk and, secondly, because they may vary among different attributes such as in the case of private versus public sectors.

The suggested model for evaluating alternative transportation systems within a mix-of-modes context consists of the logical aggregation of the input information contained in $\overline{D}$, $\overline{B}$, $\overline{R}$, and $\overline{S}$. Letting F represent total system benefit or effectiveness, the model would be written as

$$F = \sum_{m=1}^{M} \sum_{i=1}^{I} \sum_{t=1}^{T} R_i S_{it} D_{mt} B_{mi} \tag{6}$$

Operationally, the aggregating procedure would proceed as follows. First, the utility or benefit values B_{mi} are weighted by the demand forecasts D_{mt}. This interim step yields a three-dimensional array of demand-weighted values $\overline{V} = [V_{tim}]$ where

$$V_{tim} = D_{mt} B_{mi} \tag{7}$$

Successive steps in the procedure aggregate the V_{tim} across time, attributes, and modes--in that order. The weightings S_{it} are used in the aggregation over time, and the weightings R_i are used in the aggregation across attributes. The final measure F represents the total system benefit--over the entire planning horizon--from the demand which is expected for the package of transportation services that will be available. Dimensionality of F is not important in itself since it depends on the range of allowable value for $\tilde{B}$. In addition, the model is not intended as a means of measuring just one system, but rather as a means of comparing on an incremental basis alternative changes in the transportation mix. Before proceeding to develop a benefit-cost model, together with an associated decision rule, for evaluating incremental changes, it is well to further explore the basic effectiveness model as summarized by expression (6).

III. SENSITIVITY ANALYSIS

In a decisionmaking area as complex as transportation, it is not enough simply to evaluate the relative benefits and costs of alternative systems. Because of the highly complex interaction of a variety of different considerations, it is also useful to examine the sensitivity of resulting evaluations to changes in the input variables and parameters. A procedure such as sensitivity analysis thus affords the decisionmaker additional information which may be useful, not only in the actual decision which is made, but in the degree of confidence that it is the optimal decision.

Within the context of this paper, it is thus useful to examine the sensitivity of the total effectiveness measure to different demand patterns, utility assessments, and also changes in the attribute and time weighting parameters. One approach toward this end is to examine the incremental change in effectiveness which is found by taking the total differential of expression (6) as follows:

$$dF = \left(\frac{\partial F}{\partial D_{mt}}\right) dD_{mt} + \left(\frac{\partial F}{\partial B_{mi}}\right) dB_{mi} + \left(\frac{\partial F}{\partial S_{it}}\right) dS_{it} + \left(\frac{\partial F}{\partial R_i}\right) dR_i \qquad (8)$$

In this total differential, the partial derivatives represent the responsiveness of the model to changes in its component parts, while the other terms are the absolute changes expected to result from a changing transportation system.

Although the merits of a sensitivity analysis can really not be appreciated until applied to a numerical example, it is of interest to briefly explore the expected directions of change when the inputs are altered. This is readily accomplished by examining further the partial derivatives of expression (8) as follows:

$$\frac{\partial F}{\partial D_{mt}} = \sum_{i=1}^{I} R_i S_{it} B_{mi} > 0 \qquad (9)$$

$$\frac{\partial F}{\partial B_{mi}} = \sum_{t=1}^{T} R_i S_{it} D_{mt} > 0 \qquad (10)$$

$$\frac{\partial F}{\partial R_i} = \sum_{m=1}^{M} \sum_{t=1}^{T} S_{it} D_{mt} B_{mi} > 0 \qquad (11)$$

$$\frac{\partial F}{\partial S_{it}} = \sum_{m=1}^{M} R_i D_{mt} B_{mi} > 0 \qquad (12)$$

Because all the input values are positive, all of the individual changes in F are also positive. For the special case where discount rates are used as time weightings, substitution of expression (5) into expression (6) would lead to

$$\frac{\partial F}{\partial K_i} = \sum_{m=1}^{M} \sum_{t=1}^{T} \frac{- t R_i D_{mt} B_{mi}}{(1+K_i)^{t+1}} < 0 \qquad (12a)$$

Hence, if a discount rate is increased, benefits from a future period are penalized more, and total effectiveness is reduced.

Since the suggested effectiveness model focuses on a mix-of-modes context, it is also useful to examine sensitivity from that particular viewpoint. Recalling that the "last" summation in expression (6) is across the various modes of the transportation system, it is possible to compute the relative contribution of each mode to the total transportation system. The appropriate calculation to do this would be

$$Y_m = \frac{\sum_{i=1}^{I} \sum_{t=1}^{T} R_i S_{it} D_{mt} B_{mi}}{F} \tag{13}$$

where Y_m represents the percentage contribution of mode m to the total transportation system.

IV. BENEFIT-COST DECISION RULE

Thus far, only the relative benefits of the basic transportation system have been reflected. In order to choose among alternative transportation proposals, it is necessary to bring cost considerations into the analysis. This section explains an appropriate benefit-cost decision rule which can be used in connection with the generalized effectiveness model which has been developed.

The suggested procedure is to utilize the benefit cost ratio which is but one of a set of "discounted-present-value" techniques which have been popularized in the academic literature as appropriate for ranking and selecting investment projects. The common theme of these techniques is to penalize all future benefits and costs for each project into today's value scheme--and choose accordingly. As opposed to the internal rate of return and net present value methods which are commonly suggested for decisionmaking within the business enterprise, the benefit-cost ratio has become more popular in decision problems involving the public sector.* A major reason for this is that use of the benefit-cost ratio does not require that benefits and costs be measured in the same units. And this is a typical characteristic of problems within the public sector--the transportation problem being no exception.

Recall that total effectiveness of the basic transportation system expected to be operational over the T-period horizon is denoted by F. If the total cost of that system in present dollars is C, the benefit-

*For an excellent comparison of these three evaluation methods, see G. D. Quirin, The Capital Expenditure Decision (Richard D. Irwin, 1967).

cost ratio would simply be F/C. It turns out that for the purposes of evaluating alternative changes or improvements in the transportation system, neither the ratio F/C nor the cost itself C has direct bearing on the appropriate decision.

Instead, the relevant framework is to consider incremental benefits and incremental costs as measured from the basic proposed system. Let J represent the total number of distinct changes (or combination of changes) that are being evaluated. Each particular change $1 \leq j \leq J$ could be an improvement to a single mode, or it might be a scheduling or other network innovation which might have impact across several modes. In any event, the suggested procedure is to treat each possible change j individually and compute its total system benefit F_j and system cost C_j. The incremental benefit and cost for that change would be given by $\Delta F_j = F_j - F$ and $\Delta C_j = C_j - C$, respectively. The appropriate benefit-cost ratio thus becomes $\Delta F_j/\Delta C_j$ and the suggested decision rule is to choose that system change which maximizes this ratio. More explicitly, the decision rule

$$\underset{j}{\text{Maximize}} \left[\frac{\Delta F_j}{\Delta C_j}\right] \qquad (14)$$

is suggested as the relevant criterion. Finally, the entire analysis is deliberately built around transportation between only two nodes. As such, the decision rule may lead to solutions which are sub-optimal relative to the entire network. Nonetheless, expression (14) does serve to tie the generalized effectiveness model to the point of decision. Some of the qualifications which have been mentioned will be considered further in the final section of the paper.

V. ILLUSTRATIVE EXAMPLE

In order to add further understanding to the generalized effectiveness model, the sensitivity analysis of the model, and the associated benefit-cost decision rule, a simplified example is presented in this section. As before, attention focuses on the easiest case of one-way travel along a single link--for example, between Philadelphia and New York City. The planning horizon consists of T = 5 periods (1971-1975).

The basic transportation system that is expected to be operational during that period consists of M = 7 travel modes. In particular, the system consists of the following $M_1 = 4$ intra-city feeder modes: Automobile (X_1), bus (X_2), subway (X_3), and taxicab (X_4). The $M_2 = 3$ intercity major modes are airplane (X_5), freeway (X_6), and railroad (X_7). Any one-way trip can be described by the vector $\bar{X}$ of binary-usage variables. For example, $\bar{X} = [1, 0, 1, 0, 0, 0, 1]$ could represent a modal combination such as automobile-railroad-subway. The case of an individual traveler making the entire trip by personal automobile would be described verbally as automobile-freeway-automobile or by the usage vector $\bar{X} = [1, 0, 0, 0, 0, 1, 0]$.

Suppose further that a demand analysis is made for the five-year period and for the postulated system of seven total modes. Results indicate a total primary demand of 35,000 one-way trips according to the schedule

$$W_1(1971) = 5000 \text{ trips}$$

$$W_2(1972) = 6000 \text{ trips}$$

$$W_3(1973) = 7000 \text{ trips}$$

$$W_4(1974) = 8000 \text{ trips}$$

$$W_5(1975) = \underline{9000} \text{ trips}$$

$$W = 35{,}000 \text{ trips}$$

Furthermore, by specifying a usage vector for each trip (in some aggregate sense) and by summation using equation (3), the following modal-time matrix is obtained as a measure of derived demand:

$$\begin{array}{rl} & \begin{array}{ccccc} (1971) & (1972) & (1973) & (1974) & (1975) \end{array} \\ \bar{D} = [D_{mt}] = & \left[\begin{array}{ccccc} 7000 & 8000 & 9000 & 13000 & 12000 \\ 2000 & 2000 & 2500 & 3000 & 3000 \\ 2000 & 3000 & 3000 & 4000 & 3000 \\ 2000 & 3000 & 3500 & 4000 & 3000 \\ \hdashline 1500 & 2000 & 2000 & 2500 & 3000 \\ 2500 & 3000 & 3500 & 4000 & 4500 \\ 1000 & 1000 & 1500 & 1500 & 1500 \end{array}\right] \begin{array}{l} \text{(Automobile)} \\ \text{(Bus)} \\ \text{(Subway)} \\ \text{(Taxicab)} \\ \text{(Airplane)} \\ \text{(Freeway)} \\ \text{(Railroad)} \end{array} \end{array}$$

Although these are only illustrative numbers, certain features of the matrix are worth noting. First, derived demand increase over time for all modes--the single exception being 1975 where the overall demand for feeder modes decreases. Secondly, since a single major mode is used for each one-way trip, a total of the values below the dashed line for each column in the matrix simply gives the demand for trips in that particular year W_t. In addition, the demand for automobiles is always at least twice as great as for the freeway, since the assumption is that only personal autos are driven on the freeway, but autos can also be used as a means of intra-city transportation to the railroad, airplane, or another feeder mode. Finally, for the entire planning horizon, an average of 2.62 feeder modes per trip are used.

The next step is to specify the different attributes which are to be measured. For this simplified example, the following $I = 4$ attributes will be considered: travel time, travel cost, return on investment, and air pollution. That is, users of the transportation system are only interested in travel time and travel cost, operator firms consider only their return on investment, and the public is primarily concerned with air pollution.

In order to assess the utility or effectiveness of this particular transportation system, it is first necessary to specify the unit-trip measures B_{im} as defined earlier. Hypothetically, these values might be as follows:

$$\bar{B} = [B_{mi}] = \begin{array}{cl} \begin{array}{cccc} \begin{pmatrix}\text{Travel}\\ \text{Time}\end{pmatrix} & \begin{pmatrix}\text{Travel}\\ \text{Cost}\end{pmatrix} & \begin{pmatrix}\text{Return}\\ \text{on Inv}\end{pmatrix} & \begin{pmatrix}\text{Air}\\ \text{Pollution}\end{pmatrix} \end{array} & \\ \left[\begin{array}{cccc} 8 & 6 & 10 & 3 \\ 4 & 9 & 7 & 3 \\ 6 & 10 & 5 & 10 \\ 10 & 4 & 7 & 3 \\ \hline 10 & 3 & 7 & 10 \\ 7 & 10 & 10 & 5 \\ 5 & 5 & 4 & 10 \end{array}\right] & \begin{array}{l} \text{(Automobile)} \\ \text{(Bus)} \\ \text{(Subway)} \\ \text{(Taxicab)} \\ \text{(Airplane)} \\ \text{(Freeway)} \\ \text{(Railroad)} \end{array} \end{array}$$

Assessments are made on a relative basis within each attribute category with a maximum of 10 points for the "best" feeder mode and also the "best" major mode.

Despite the arbitrary nature of these rankings it is possible to indicate obvious preferences. For example, along the travel time attribute, taxicab ranks highest within the city (node). Personal automobile is rated lower because of parking problems. Subway certainly moves "faster" in an absolute sense but is penalized because the traveler must somehow get to and from his home or other connecting modes. For the major mode category, airplane is clearly quicker as indicated. Again, these value assignments are independent of demand (at this point) and represent the relative contribution of each mode within the given mix of modes.

The other important inputs are the two sets of weighting parameters. The relative weighting among attributes R_i can be described by the vector.

$$\bar{R} = [R_i] = \begin{bmatrix} .25 \\ .25 \\ .25 \\ .25 \end{bmatrix} \begin{array}{l} \text{(Time)} \\ \text{(Cost)} \\ \text{(Return on investment)} \\ \text{(Air pollution)} \end{array}$$

which simply means that the four attributes of this example are weighted equally. The weighting across time will be made with the special case of discount rates. The discounting schedule can be represented by

$$\bar{K} = [K_i] = \begin{bmatrix} .10 \\ .10 \\ .10 \\ .05 \end{bmatrix} \begin{array}{l} \text{(Time)} \\ \text{(Cost)} \\ \text{(Return on investment)} \\ \text{(Air pollution)} \end{array}$$

which effectively assigns a higher rate to the private sector than to the public sector. The correspondence between $\bar{K}$ and the generalized weightings $\bar{S}$ is given by expression (5).

These inputs can now be used to compute the total effectiveness of the basic system. As previously mentioned, the first step in the aggregation is to compute the three-dimensional array $V = [V_{tim}]$ of demand-weighted benefit values. For example, if the first column of $\overline{D}$ is systematically multiplied times successive columns of $\overline{B}$, one obtains the aggregate 1971 utilities for each mode and attribute combination. In other words, this takes into account the expected demand pattern for the first year of the planning horizon. The end result is a series of five matrices--each representing one year of the horizon. For 1971, the matrix would be as follows:

$$\overline{V}_{t=1971} = [V_{1971,i,m}] = \begin{array}{cl} \begin{array}{cccc} \text{(Time)} & \text{(Cost)} & \text{(Return on Inv.)} & \text{(Air Pollution)} \end{array} & \\ \left[\begin{array}{cccc} 56000 & 420004 & 70000 & 21000 \\ 8000 & 18000 & 14000 & 6000 \\ 12000 & 20000 & 10000 & 20000 \\ 20000 & 8000 & 14000 & 6000 \\ \hdashline 15000 & 4500 & 10500 & 15000 \\ 17500 & 25000 & 25000 & 12500 \\ 5000 & 5000 & 4000 & 10000 \end{array}\right] & \begin{array}{l} \text{(Automobile)} \\ \text{(Bus)} \\ \text{(Subway)} \\ \text{(Taxicab)} \\ \text{(Airplane)} \\ \text{(Freeway)} \\ \text{(Railroad} \end{array} \end{array}$$

This process of weighting by demand results in an entirely different picture. Although taxicab was ranked higher than personal automobile on the basis of travel time, the higher demand for using one's own automobile reverses the ranking in the V_{itm} values. Other similar changes in the 1971 $\overline{V}$ matrix are also noted.

The series of five $\overline{V}$ matrices (one for each year of the horizon), together with $\overline{R}$ and $\ddot{K}$ (or $\overline{S}$), become the input values needed for the effectiveness model. Using expression (6), the total effectiveness of the proposed transportation system is found to be 663,675. This is a "dimensionless" quantity which reflects all considerations felt to be important--at least for this simplified example. The total cost of this basic transportation system is estimated to be C = \$10 billion, but as mentioned before, this particular value is not essential to the problem of choosing among prospective changes.

Consider now alternative changes in the transportation mix. Assume for continued simplicity that only $J = 2$ distinct alternatives are to be evaluated in addition to the base system already described. The first is simply an improvement of an existing major mode, while the second is the addition of a new major mode to the three already operational.

The first proposed change in the mix of modes available is to implement a technological improvement in railroad locomotives. The improved trains will be available at the beginning of the third period. The effect of this improvement, which can be realized at an added present-value cost of \$200 million over the planning horizon, is to reduce travel time between the two cities.

Although the primary demand for trips remains the same at $W = 35{,}000$, estimates of derived demand are altered to reflect the improved status of railroad viz-a-viz other major modes. The derived demand matrix for the first alternative change becomes

$$
\begin{array}{c}
\begin{array}{ccccc} (1971) & (1972) & (1973) & (1974) & (1975) \end{array} \\
\bar{D}^1 = [D^1_{mt}] = \left[\begin{array}{cc:ccc}
7000 & 8000 & 9000 & 12500 & 11500 \\
2000 & 2000 & 2500 & 3000 & 3500 \\
2000 & 3000 & 3500 & 4500 & 4000 \\
2000 & 3000 & 4000 & 5000 & 4000 \\ \hdashline
1500 & 2000 & 1700 & 2000 & 2300 \\
2500 & 3000 & 3300 & 3800 & 4200 \\
1000 & 1000 & 2000 & 2200 & 2500
\end{array}\right]
\begin{array}{l}
\text{(Automobile} \\ \text{(Bus)} \\ \text{(Subway)} \\ \text{(Taxicab} \\ \text{(Airplane)} \\ \text{(Freeway)} \\ \text{(Railroad)}
\end{array}
\end{array}
$$

where the superscript simply indicates the first proposed change in the mix of modes. The first two columns of $\bar{D}^1$ are similar to $\bar{D}$, while the last three columns reflect (1) the increased demand for railroad, (2) the reduced demand for airplane and freeway, and (3) the increased demand for feeder modes (particularly subway and taxicab) which is postulated. The $\bar{B}$, $\bar{R}$, and $\bar{K}$ input data are left unchanged for this case. Ideally, the B_{71} value for travel time using railroad would be adjusted upward--but is held constant for simplicity, and also because the improvement does not take effect until midway through the planning horizon.

The second proposed change is the addition of VTOL as a major mode for inter-city travel. It can be developed and implemented by the beginning of the fourth period at an added present-value cost of \$1.5 billion over the planning horizon.

The availability of this additional mode, together with its added flexibility for rapid air travel, is expected to increase primary demand (W = 38,000 trips) as well as derived demand for transportation services. The derived demand over time is given by

$$\bar{D}^2 = [D^2_{mt}] = \begin{array}{cl}
\begin{array}{ccc|cc} (1971) & (1972) & (1973) & (1974) & (1975) \end{array} & \\
\left[\begin{array}{ccc|cc}
7000 & 8000 & 9000 & 14000 & 14000 \\
2000 & 2000 & 2500 & 3500 & 4000 \\
2000 & 2000 & 2500 & 3500 & 4000 \\
2000 & 3000 & 3500 & 5500 & 5500 \\
\hline
1500 & 2000 & 2000 & 3000 & 3000 \\
2500 & 3000 & 3500 & 4000 & 4500 \\
1000 & 1000 & 1500 & 1500 & 1000 \\
0 & 0 & 0 & 500 & 2500
\end{array}\right] &
\begin{array}{l}
\text{(Automobile)} \\
\text{(Bus)} \\
\text{(Subway)} \\
\text{(Taxicab} \\
\text{(Airplane)} \\
\text{(Freeway)} \\
\text{(Railroad)} \\
\text{(VTOL)}
\end{array}
\end{array}$$

Note that, although this matrix now includes an eighth row, values of the first three columns are identical to those of $\bar{D}$. During 1974, a small demand for this "revolutionary" demand results, but also an added demand for airplane--perhaps because service on the latter is improved in the advent of a close competitor. During 1975, however, travelers begin to utilize the VTOL mode at the expense of the other modes. Again, the feeder mode demand has been adjusted accordingly.

The matrix of per-trip utility values is also augmented with an eighth row as follows:

$$\bar{B}^2 = [B^2_{mi}] = \begin{array}{c} \begin{array}{cccc} \text{(Travel Time)} & \text{(Travel Cost)} & \text{(Return on Inv.)} & \text{(Air Pollution)} \end{array} \\ \left[\begin{array}{cccc} 8 & 6 & 10 & 3 \\ 4 & 9 & 7 & 3 \\ 6 & 10 & 5 & 10 \\ 10 & 4 & 7 & 3 \\ \hdashline 10 & 3 & 7 & 10 \\ 7 & 10 & 10 & 5 \\ 5 & 5 & 4 & 10 \\ 9 & 4 & 6 & 10 \end{array}\right] \end{array} \begin{array}{l} \\ \text{(Automobile)} \\ \text{(Bus)} \\ \text{(Subway)} \\ \text{(Taxicab)} \\ \text{(Airplane)} \\ \text{(Freeway)} \\ \text{(Railroad)} \\ \text{(VTOL)} \end{array}$$

The other inputs, $\bar{K}$ and $\bar{R}$, are again held constant for this proposed change.

A benefit-cost comparison of these two alternatives can now be made along the lines suggested earlier. For each of the proposed changes, the total system effectiveness is calculated, and the incremental benefit and incremental cost--both measured relative to the basic transportation system--are used to calculate the incremental benefit-cost ratio. Results of such calculations for the two alternatives are presented in Table 1. It is immediately noted that the incremental benefit-cost ratio for an improved railroad mode is higher than that for the alternative of introducing a new VTOL mode. Appealing to the decision rule of

Table 1

BENEFIT-COST ANALYSIS OF

ALTERNATIVE TRANSPORTATION SYSTEMS 1971-75

	Basic System	Improved Railroad Mode (j=1)	New VTOL Mode (j=2)
System benefit	663,675	680,120	727,709
Incremental Benefit	--	16,445	64,034
System Cost ($000)	10,000	10,200	11,500
Incremental Cost ($000)	--	200	1,500
Benefit-Cost Ratio $\Delta F_j/\Delta C_j$	--	82.23	42.69

expression (14), one concludes that the optimal strategy is to improve the existing railroad major mode by making it faster, rather than the more ambitious project of developing and implementing VTOL as another major mode.

Clearly, the inputs dictated by this simplified example are arbitrary, incomplete, and perhaps even inconsistent. Nonetheless, together with the generalized effectiveness model described in the preceding section, they serve as an illustrative example of how one might evaluate alternative transportation systems in a mix-of-modes context--at least along a single link between two nodes over a finite time horizon. To further illustrate the use of the suggested sensitivity analysis, and also to emphasize the importance of the mix-of-modes context, it was convenient to continue the example. A total of seven distinct changes were investigated on the basis of how they affected the total system effectiveness.* The seven changes were incorporated via the $\overline{D}$, $\overline{B}$, $\overline{K}$,

*In order to focus on the sensitivity of the total effectiveness model as given by expression (6), costs were not considered for the remainder of the illustrative example.

and $\bar{K}$ inputs. An explanation of each of the seven changes follows:

(a) Change of attribute weightings to

$$\bar{R} = \begin{bmatrix} .40 \\ .40 \\ .10 \\ .10 \end{bmatrix} \begin{matrix} \text{(Travel Time)} \\ \text{(Travel Cost)} \\ \text{(Return on Investment)} \\ \text{(Air Pollution)} \end{matrix}$$

which places heavier emphasis on users than on operators or society.

(b) Change of attribute weightings to

$$\bar{R} = \begin{bmatrix} .20 \\ .20 \\ .20 \\ .40 \end{bmatrix}$$

which places heavier emphasis on the public attribute of air pollution.

(c) Change of the discounting schedule to

$$\bar{K} = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}$$

which effectively bypasses discounting, and thus all years are treated equally.

(d) Change of the discounting schedule to

$$\bar{K} = \begin{bmatrix} .05 \\ .05 \\ .05 \\ .05 \end{bmatrix}$$

which means that all attribute accounts are discounted using the 5 percent risk free interest rate--i.e., no distinction is made between private and public sectors.

(e) Change of demand for major modes to

$$\bar{D} = \begin{array}{l} \left[\begin{array}{ccccc} \vdots & \vdots & \vdots & \vdots & \vdots \\ 1500 & 1500 & 1500 & 2000 & 2000 \\ 2500 & 3500 & 5000 & 5500 & 6500 \\ 1000 & 1000 & 500 & 500 & 500 \end{array}\right] \begin{array}{l} \\ \text{(Airplane)} \\ \text{(Freeway)} \\ \text{(Railroad)} \end{array} \end{array}$$

where the big increase over time is for freeway. Meanwhile, railroad decreases over time. Total demand for one-way trips is held constant.

(f) Change of demand for feeder modes to

$$\bar{D} = \left[\begin{array}{ccccc} 6000 & 6500 & 7000 & 9000 & 8000 \\ 2000 & 2000 & 2500 & 3000 & 3000 \\ 3000 & 4500 & 5000 & 8000 & 7000 \\ 2000 & 3000 & 3500 & 4000 & 3000 \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{array}\right] \begin{array}{l} \text{(Automobile)} \\ \text{(Bus} \\ \text{(Subway)} \\ \text{(Taxicab)} \\ \\ \end{array}$$

where the significant increase is for taxicab at the expense of automobile. Total demand for feeder modes is held constant, as is the ratio of feeder to major modes.

(g) Change of per-trip utility values. For the $\bar{B}$ matrix used earlier, all values of 10 were increased to 15, while other values were unchanged. The effect here is to make "the best even better."

Results of these seven individual sensitivity changes are presented in Table 2. Total system effectiveness for each is calculated--again using expression ([illegible]) Results of the individual changes were all in relative directions as predicted by expressions (9) through (12a) in Section III. Several are worth mentioning. For example, weighting heavier toward user attributes increased the aggregate system benefit, but decreased when the weighting was shifted toward the air pollution. This resulted because there was only a single public attribute as against two user attributes. When time was ignored (i.e., no discounting), the system benefit increased substantially. Conversely, only slight increases in total system benefit resulted when major mode demand and feeder mode demand were shifted (not increased), respectively. Finally, a significant change was observed when the relative per-trip utility values were increased. The percentage change in total system benefit, for this illustrative example and also for the hypothetical changes which were made, ranged from -5.5 percent for change (b) to +31.0 percent for change (c).

Table 2

SENSITIVITY ANALYSIS FOR ILLUSTRATIVE EXAMPLE

System or Change	Effectiveness
Basic System	663,675
(a) Attribute weighting toward user	666,799
(b) Attribute weighting toward public	627,311
(c) No discounting	869,625
(d) Equal discounting	743,195
(e) Major mode demand	667,731
(f) Feeder mode demand	675,339
(g) Utility values	814,227

Also of interest, as part of a sensitivity analysis for the example, is the relative contribution of different mode to total system effectiveness. This calculation was made using expression (13). Results are presented in Table 3 for the basic system, the two alternative (proposed) transportation systems, and the seven changes which comprised the sensitivity analysis. The total for each column in this table is, of course, 100 percent.

An immediate observation is that the percentage contributions of different modes are relatively constant across the different transportation improvements.and sensitivity changes which were postulated. Major shifts occurred mainly when demand patterns for major or feeder modes were changed substantially. Conversely, for changes (a) through (d) which involved parameters for weighting and discounting, the mix changed very little. The value of this type of presentation will doubtless be enhanced when more realistic demand and utility values are incorporated into the effectiveness model.

Table 3

PERCENTAGE CONTRIBUTIONS OF DIFFERENT MODES TO TOTAL TRANSPORTATION SYSTEM BENEFIT

Mode	Basic System	Railroad (j=1)	VTOL (j=2)	Sensitivity Analysis Change (a)	(b)	(c)	(d)	(e)	(f)	(g)
1. Automobile	37.4	35.7	35.8	37.5	35.5	37.9	38.0	37.1	27.6	36.1
2. Bus	8.2	8.3	8.3	8.7	8.0	8.3	8.3	8.1	8.0	6.7
3. Subway	13.7	15.0	14.3	13.5	15.7	13.4	13.4	13.6	24.2	14.8
4. Taxicab	10.6	11.9	11.9	11.5	10.3	10.7	10.7	10.6	10.4	10.4
5. Airplane	9.6	8.2	9.2	8.6	12.5	9.1	9.0	7.5	9.5	10.5
6. Freeway	15.9	15.0	14.5	16.2	13.5	16.1	16.1	20.5	15.7	16.9
7. Railroad	4.6	5.9	3.9	4.0	4.5	4.5	4.5	2.6	4.6	4.6
8. VTOL	0.0	0.0	2.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

VI. IMPLICATIONS

This paper has presented and illustrated a generalized effectiveness model for evaluating alternative changes to a total transportation system. Both a sensitivity analysis and a benefit-cost decision rule have been suggested as appropriate adjuncts to the effectiveness model. The overall methodology is felt to have two important characteristics. The first is that benefit-cost relationships are measured within the context of the entire mix of transportation services. The second is that the methodology serves as a useful vehicle for drawing together or synthesizing lower-level considerations of demand, utility measurement, and comparisons across both time and attributes.

Certain limitations of the methodology should be mentioned. The first is simply a reminder that no single model is ever likely to provide a unique answer to a problem as complex as transportation planning. Nevertheless, if one believes in the possibility of somehow aggregating over the many aspects of the transportation problem, then the mix-of-modes evaluation model may prove useful in providing at least ballpark answers. More realistic examples should prove to be useful in further assessment of the mix-of-modes model.

A second limitation is that the model assumes a homogeneous user for the transportation system. This was necessary in order to focus on the important variables and parameters of the model. Because of its straightforward additive feature, the effectiveness model could be extended to consider several distinct classes of users--e.g., businessmen, shoppers, and vacationers. The cost of this added precsion is simply that demand and benefits must be estimated separately for each class that is identified. Moreover, additional weighting across user classes would be required.

A final limitation of the mix-of-modes model is that it focuses only on a single transportation link. It becomes of interest, therefore, to consider extending the model to include the relative effectiveness and costs over an entire network of several nodes and links. One possibility is to treat the network as a collection of individual links such as in the basic model. The difficulty with this is that the important relationship between inter-city and intra-city travel is likely to be blurred--particularly in the case of multi-link trips. It may also be possible to formulate the full network analysis into a programming context. This would not only allow the interfaces between modes and links to be made explicit, but it would serve as a framework for allocating a total systems budget across the various network links as well as the several modes which comprise the transportation system.

These limitations, plus others which could be mentioned, would appear not so much to refute the usefulness of the suggested methodology as to indicate the need for further extensions and enrichment. That is, the mix-of-modes evaluation model should be considered only the first step in making explicit the myriad of considerations and relationships that characterize the complex--and highly important--problem of selecting future transportation systems.