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Dipoles with Lossy Stub Matching Networks

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DIPOLES WITH LOSSY STUB MATCHING NETWORKS

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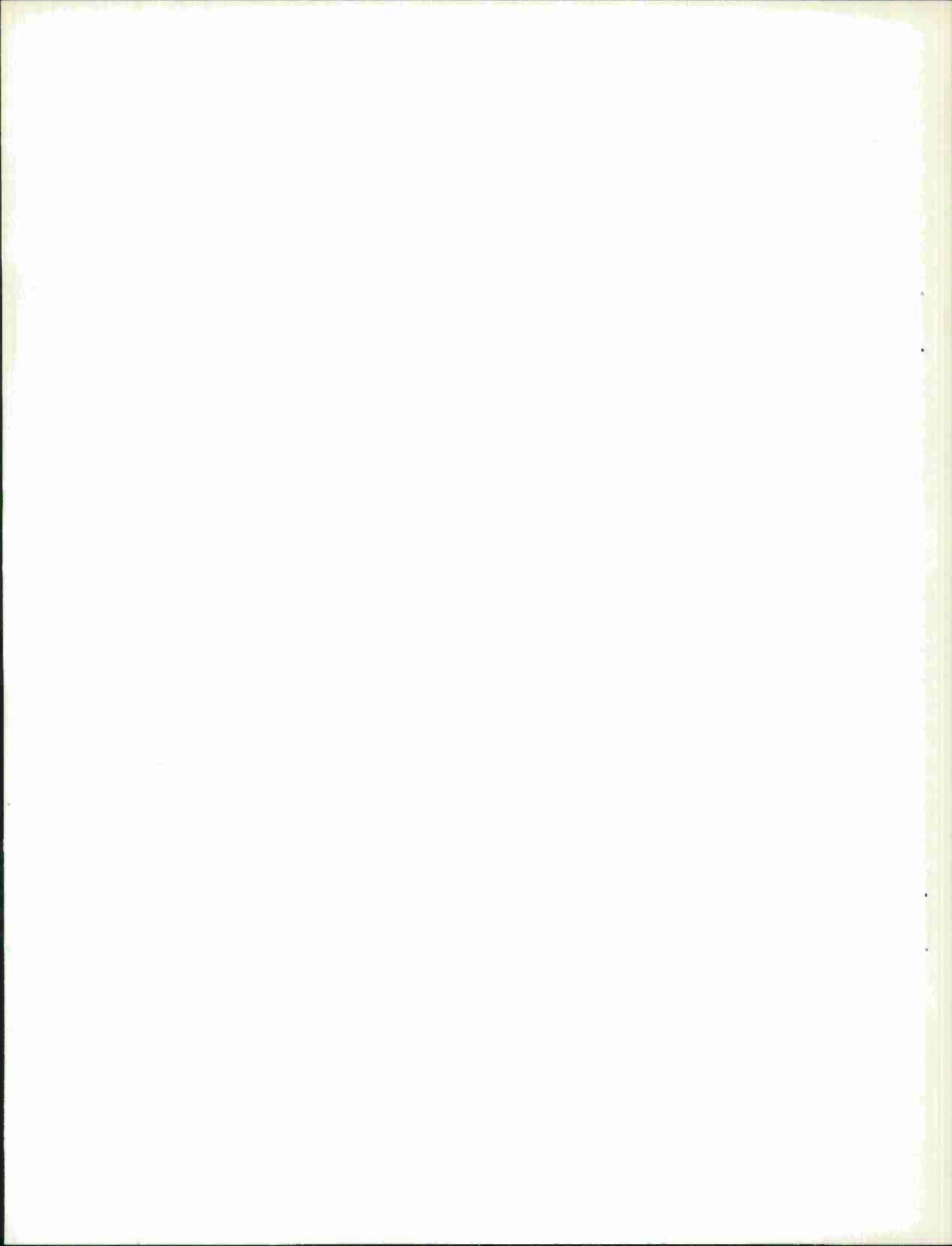
ABSTRACT

The problem of obtaining an acceptable impedance match of a thin, short dipole with simple stub matching networks has been investigated. A technique is described showing how the susceptance slopes of a dipole and a lossy stub matching network are combined to yield a result that would be more amenable to broad-band matching. The application of this technique is not restricted to dipoles and stubs and may prove useful in other matching problems in which lossy transmission lines must be used.

I.   INTRODUCTION
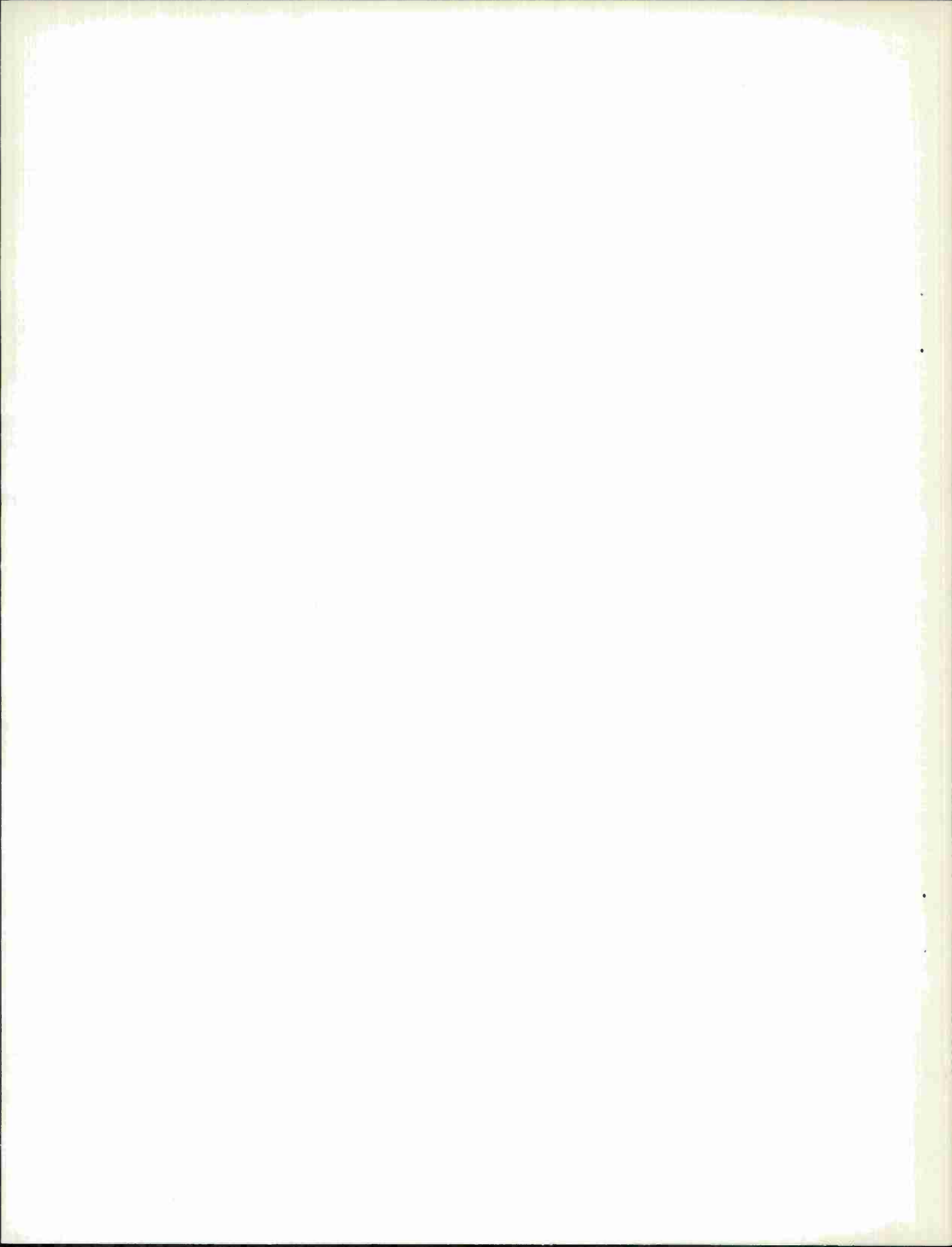
The problem of obtaining an acceptable input impedance match of a thin, short dipole with simple stub matching networks has been investigated. The approach used was to combine the positive slope of the dipole suscep- tance vs. electrical length $\beta \ell$ ($\frac{2\pi}{\lambda} \ell$) with the negative slope susceptance of either an open or shorted section of lossy transmission line. The slope re- ferred to in this discussion is the change of susceptance with respect to fre- quency. Because the slopes are equal and of opposite sign, the resultant admittance of the structure will have a constant input susceptance as a func- tion of electrical length or of frequency and only the input conductance will change. This condition is then more amenable to further broad-band match- ing using lumped elements or other techniques.

The general procedure is as follows. A point on the dipole input susceptance vs. electrical length curve at the center of the frequency band of interest is chosen. Next, the slope is determined, and this information is then used to find the attenuation constant and admittance of the required matching network. The effects of these parameters upon the matching sus- ceptance slope will also be shown.

II.   DIPOLE AND MATCHING STUB ADMITTANCE PROPERTIES

The input admittance of a dipole[1] with a length to diameter ($\frac{L}{D}$) ratio of 1000 was calculated and is shown in Fig. 1. The dipole length is chosen in this example to position the center frequency at $\beta L = 1.8$. Figure 2 is an expanded plot of the dipole input susceptance. A tangent line through the point $\beta L = 1.8$ is drawn and the slope obtained is 5.45 milli mho/radian. The equal negative slope required of a matching network and the resultant constant input susceptance are also shown. The matching network is chosen to place the zero susceptance point at $\beta L = 1.8$.

The simple case of the linear slope contribution from both the match- ing network and dipole must be modified by the actual condition where the

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1.   Henry Jasik, <u>Antenna Engineering Handbook</u>, McGraw-Hill (1961) pp. 3-2.

susceptance slope is linear over only a limited region. The desired slope parameter $\frac{d(B)}{d(\beta \ell)}$ = -5.45 milli mho/radian has already been determined and can be used to calculate the desired response of a transmission line network. The normalized input admittance of a shorted transmission line section is given by

$$\frac{Y}{Y_o} = G + jB = \coth(\alpha\ell + j\beta\ell) \tag{1}$$

which can be expanded to

$$\frac{Y}{Y_o} = \frac{\sinh\alpha\ell \; \cosh\alpha\ell - j\sin\beta\ell \; \cos\beta\ell}{\sinh^2\alpha\ell \; \cos^2\beta\ell + \cosh^2\alpha\ell \; \sin^2\beta\ell} \tag{2}$$

where $\quad \alpha$ = attenuation coefficient, $\frac{\text{nepers}}{\text{wavelength}}$

$\qquad \beta$ = phase constant, $\frac{2\pi}{\lambda}$

$\qquad \ell$ = transmission line length

Separating the real and imaginary components and simplifying the denominator gives

$$\frac{G}{Y_o} = \frac{\sinh\alpha\ell \; \cosh\alpha\ell}{\sin^2\alpha\ell + \sin^2\beta\ell} \tag{3}$$

$$\frac{B}{Y_o} = \frac{-\sin\beta\ell \; \cos\beta\ell}{\sinh^2\alpha\ell + \sin^2\beta\ell} \tag{4}$$

Similar expressions can be derived for the impedances. The expressions Eq. 3, 4 for $\alpha = 7.4 \frac{dB}{\lambda}$ are plotted in Figs. 3 and 4. The susceptance curves when $\alpha = 0$ exhibit only positive slopes; it is a negative slope that is of interest to us. Negative slopes occur near $\beta\ell = \pi, 2\pi, ---$ when α is not equal to zero.

2

Approximate relations can be derived for the susceptance slopes. For angles near $\pi \pm \Delta\theta$, where $\Delta\theta = \Delta(\beta\ell)$

$$\sin\beta\ell \approx \pm\sin\Delta\theta \approx \pm\Delta\theta$$

$$\cos\beta\ell \approx \cos\Delta\theta \approx -1 \tag{5}$$

and $\dfrac{B}{Y_o}$ can be written:

$$\frac{B}{Y_o} \approx \frac{(\pm)\sin\Delta\theta\cos\Delta\theta}{\sinh^2\alpha\ell + \sin^2\Delta\theta} \approx \frac{\pm\Delta\theta}{(\alpha\ell)^2 + (\Delta\theta)^2} \approx \frac{-\Delta\theta}{(\alpha\ell)^2} \tag{6}$$

The approximate slope can now be expressed as:

$$\frac{d(B)}{d(\beta\ell)} \approx -\frac{Y_o}{(\alpha\ell)^2} \tag{7}$$

From the last expression it is apparent that the two parameters Y_o and α can be used to influence the susceptance slope. In addition, the slope is linear for constant $\alpha\ell$ as can be seen in Fig. 3. From the previous discussion the dipole susceptance slope is:

$$\frac{dB}{d(\beta\ell)} = 5.45 \ \frac{\text{milli mhos}}{\text{radian}}$$

The required shorted stub susceptance slope is equal to the negative of the dipole susceptance slope:

$$\frac{d(B/Y_o)}{d(\beta\ell)} = -\frac{1}{(\alpha\ell)^2} = 0\ 5.45 \ \frac{\text{milli mhos}}{\text{radian}}$$

We choose for this case a value of characteristic admittance which simplifies the calculation.

$$Y_o = 1 \text{ milli mho}$$

3

and from the above equation we get

$$\alpha l = 0.428$$

since at $\beta l = \pi$, $l = \dfrac{\lambda}{2}$, $\alpha = 0.856 \dfrac{\text{neper}}{\lambda} = 7.44 \text{ dB}/\lambda$

This is the required stub attenuation factor. This choice of stub parameters results in a susceptance curve which at $\beta l = \pi$ will exhibit a negative slope equal to the dipole positive slope. See Fig. 3. The zero loss case is also shown for reference.

It is appreciated that the realization of a characteristic admittance of $Y_o = 1$ milli mho may not be very practical; therefore, an example with a more easily achieved characteristic admittance is also presented later on in this section.

The analysis can be repeated for the open stub case and the same expression results as for the shorted stub (Eq. 7). As before, the required stub susceptance slope is

$$\frac{d(B/Y_o)}{d(\beta l)} = -5.45 = \frac{1}{(\alpha l)^2}$$

and

$$\alpha l = 0.428.$$

The open transmission line admittance is plotted in Figs. 5 and 6. It should be noticed that for an open line the first resonance that produces a negative susceptance slope is at $\beta l = \dfrac{\pi}{2}$ (see Fig. 5).

Since $l = 0.250 \lambda$

$$\alpha = \frac{0.428}{0.250} = 1.71 \frac{\text{nepers}}{\lambda} = 14.9 \text{ dB}/\lambda$$

which gives the required stub attenuation factor.

A third case will be described where Y_o will be changed to 10 milli mhos, and it is desired that the slope value be as before.

4

$$\frac{d(B/Y_o)}{d\beta \ell} = - \frac{1}{(\alpha \ell)^2} = 5.45$$

$$(\alpha \ell)^2 = \frac{Y_o}{5.45} = 1.835$$

$$\alpha \ell = 1.355.$$

For the open line $\alpha = 1.355 \times 4 = 5.420$ nepers$/\lambda = 47.08$ dB$/\lambda$.

For the shorted line $\alpha = 1.344 \times 2 = 2.710$ nepers$/\lambda = 23.54$ dB$/\lambda$.

Both the open-circuit transmission line of one-quarter wavelength and the short-circuit transmission line of one-half wavelength would have equal total loss of 11.77 dB.

The short-circuit transmission line input susceptance was calculated, and the results near resonance are plotted in Fig. 7. The deviation from the predicted slope occurs because the approximate formula (Eq. 7) was derived for small values of $\alpha \ell$. Although the slope could be calculated more exactly using the original equations (Eq. 1 - 4), it was felt that the approximation technique was sufficiently accurate for these examples. The asymmetry of the susceptance curve is due to the very large attenuation constant used in this example.

III.   COMBINED INPUT ADMITTANCE PROPERTIES

The results of the previous section will now be used to show the effects of combining the dipole with various transmission line matching networks. The equivalent circuit is shown in Fig. 8.

The dipole input admittance was combined with the shorted stub input admittance shown in Figs. 3 and 4. The zero susceptance point $(\ell = \frac{\lambda}{2})$ was positioned at the dipole electrical length of $\beta L = 1.8$. The resultant input admittance (Y_{in}) is plotted in Fig. 9 to a normalized value of 10 milli mhos. Figure 10 is an input admittance plot of the dipole and open stub combination with the zero susceptance point $(\ell = \frac{\lambda}{4})$ of the open stub also

5

positioned at the dipole electrical length of $\beta \ell$ = 1.8. The open stub input
admittance plot is seen in Figs. 5 and 6. Both input admittance plots show
a definite tendency towards the desired constant susceptance vs. electrical
length characteristic. Note that the open stub case with its shorter length
($\frac{\lambda}{4}$) is less frequency sensitive than the shorted stub case. The susceptance
curves for the lossy stubs have an S-shape characteristic (see Figs. 3, 5),
and the objective is to place the desired band of frequencies along the linear
portion of the S curve. A slight deviation from linearity would not signifi-
cantly degrade the results so that the peak-to-peak portion of the S curve
establishes the range of frequencies which can be used. The peak-to-peak
separation of the short-circuit lossy stub (see Fig. 3) expressed as a ratio
of electrical lengths is 3.5/2.75 = 1.27. The peak-to-peak electrical length
ratio of the open-circuit lossy stub (see Fig. 5) is 1.85/1.2 = 1.54. It is
seen that the shorter length lossy stub peak-to-peak bandwidth of 54 percent
is twice the 27 percent peak-to-peak bandwidth of the longer stub. The longer
stub is therefore more frequency sensitive, and this is why it is usually de-
sirable to operate at the first resonance of the lossy stub.

The final case calculated is the shorted stub with Y_o = 10 milli mhos
(see Fig. 11) and combined with the same dipole used in the previous exam-
ples. Again the tendency towards a constant susceptance is noted. This plot
is much closer to the center of the chart than in the previous two plots of in-
put admittance and is due mainly to the increased loss in the transmission line
matching network. An additional improvement of the input admittance shown
could now be accomplished by adding a lumped element, for instance, with the
appropriate inductive susceptance component. Prior to the manipulation of
the susceptance slopes, the addition of a lumped element would only have im-
proved the response over a narrow band of frequencies.

The percent efficiency (η) of the equivalent circuit in Fig. 8 is

$$\eta = \frac{G}{G + G'} \times 100 \text{ percent}$$

G = input conductance of the dipole

G' = input conductance of the lossy stub

6

This is equal to the ratio of power radiated to the total power at the input. As would be expected, the efficiency improves for longer dipole lengths where the dipole conductance rapidly increases. The center of the frequency band investigated $(\beta L = 1.8)$ resulted in an efficiency of approximately 15% for the open and shorted stub with dipole combinations shown in Figs. 9 and 10. The efficiencies calculated at the upper end of the band $(\beta L = 2.4)$ are approximately 80%. For the case plotted in Fig. 11 which uses the stub with $\alpha = 23.6$ dB/λ, the approximate efficiency at midband is 4% and 31% at the upper end.

IV.   DISCUSSION

The previous material has been confined to the problem of a short dipole with a lossy stub. This approach to matching problems may be applied to other microwave networks when the input susceptance behavior is similar to that of the short dipole. Also, in circuits where lossy stubs must be used the concept of controlling the slope with the attenuation coefficient and the characteristic admittance may prove useful.

V.   CONCLUSION

A technique has been described and examples calculated showing how the susceptance characteristics of a dipole and a lossy stub can be combined to yield results useful in impedance matching problems. The stub attenuation and characteristic admittance can be chosen to produce a wide range of susceptance slopes.
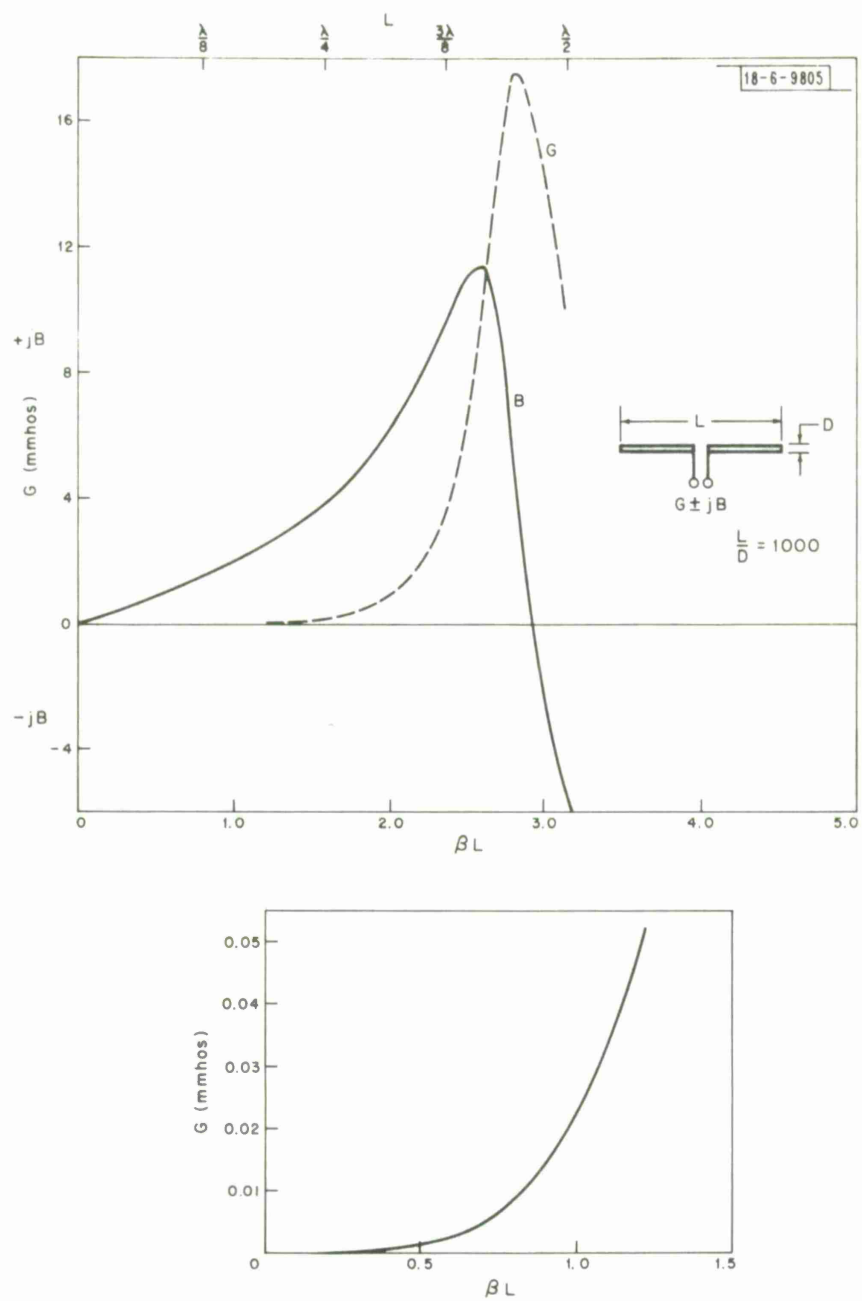
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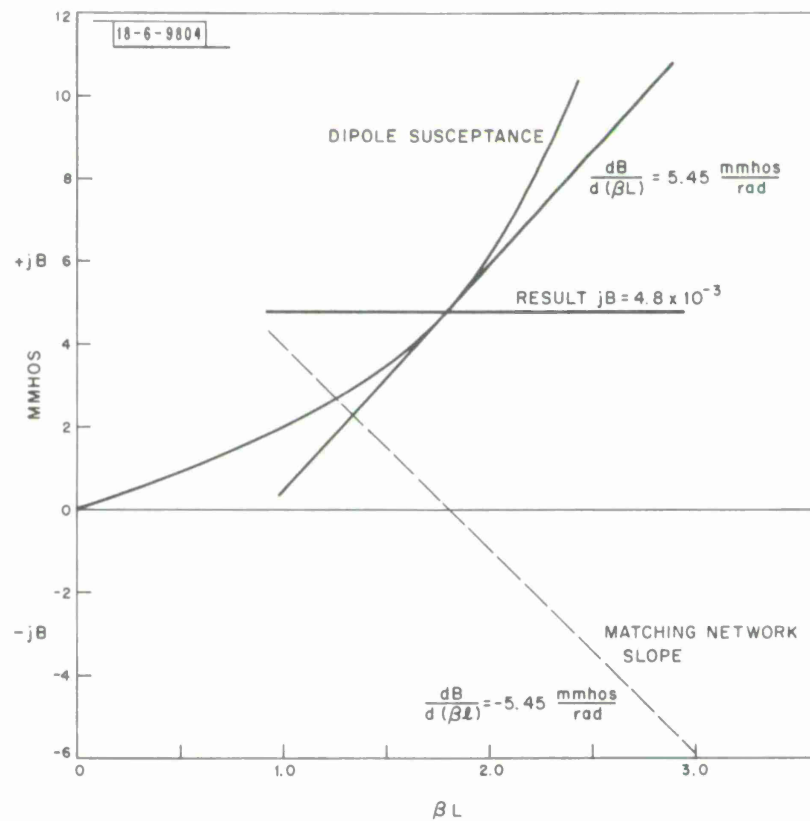Fig. 1. Input admittance of a center-fed dipole.

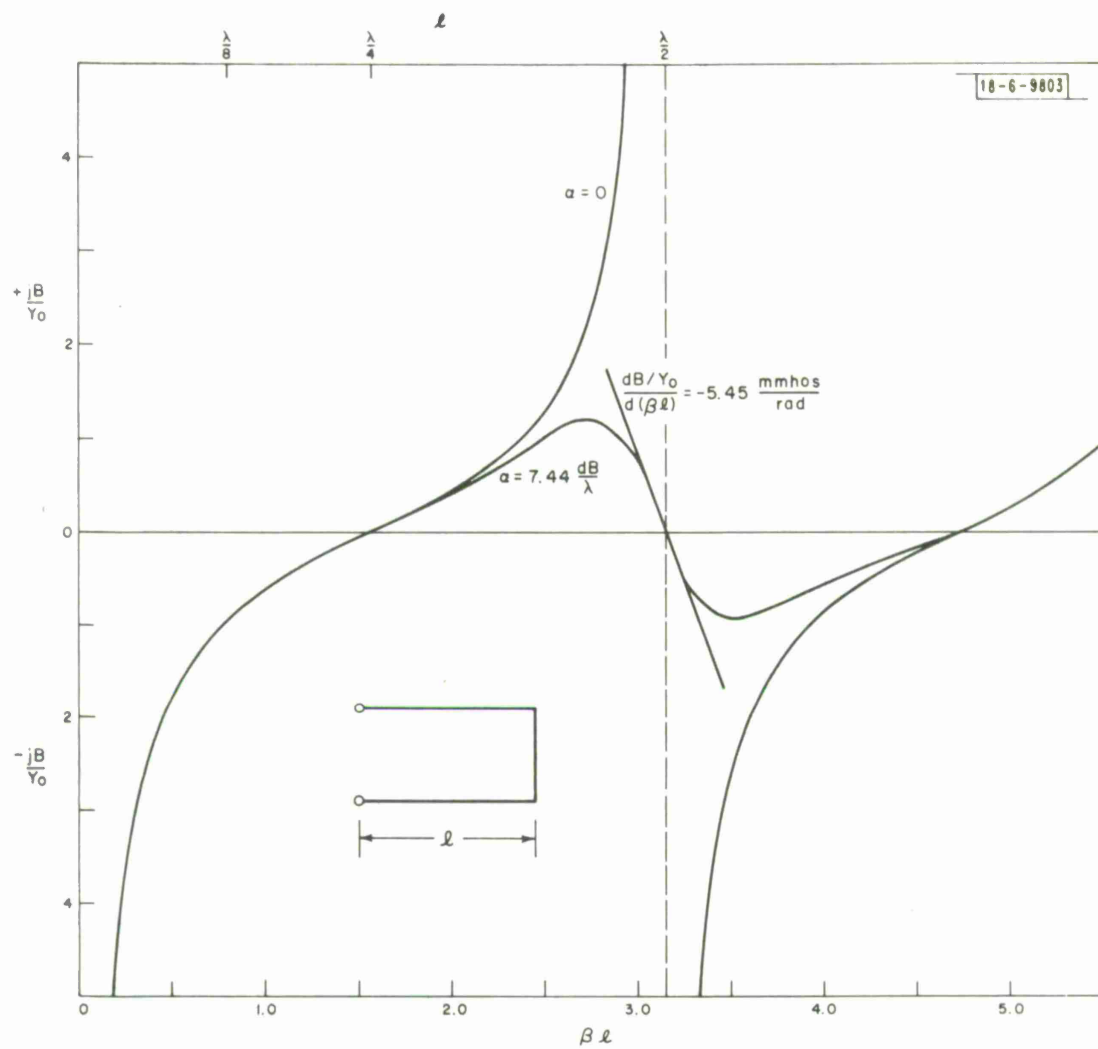Fig. 2. Input susceptance of a center-fed dipole.
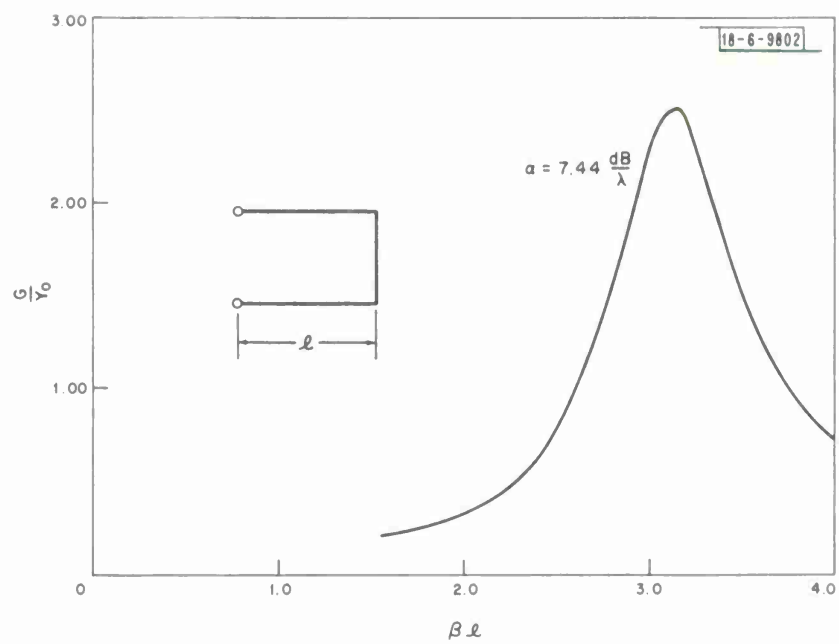
Fig. 3. Input susceptance of a shorted line.
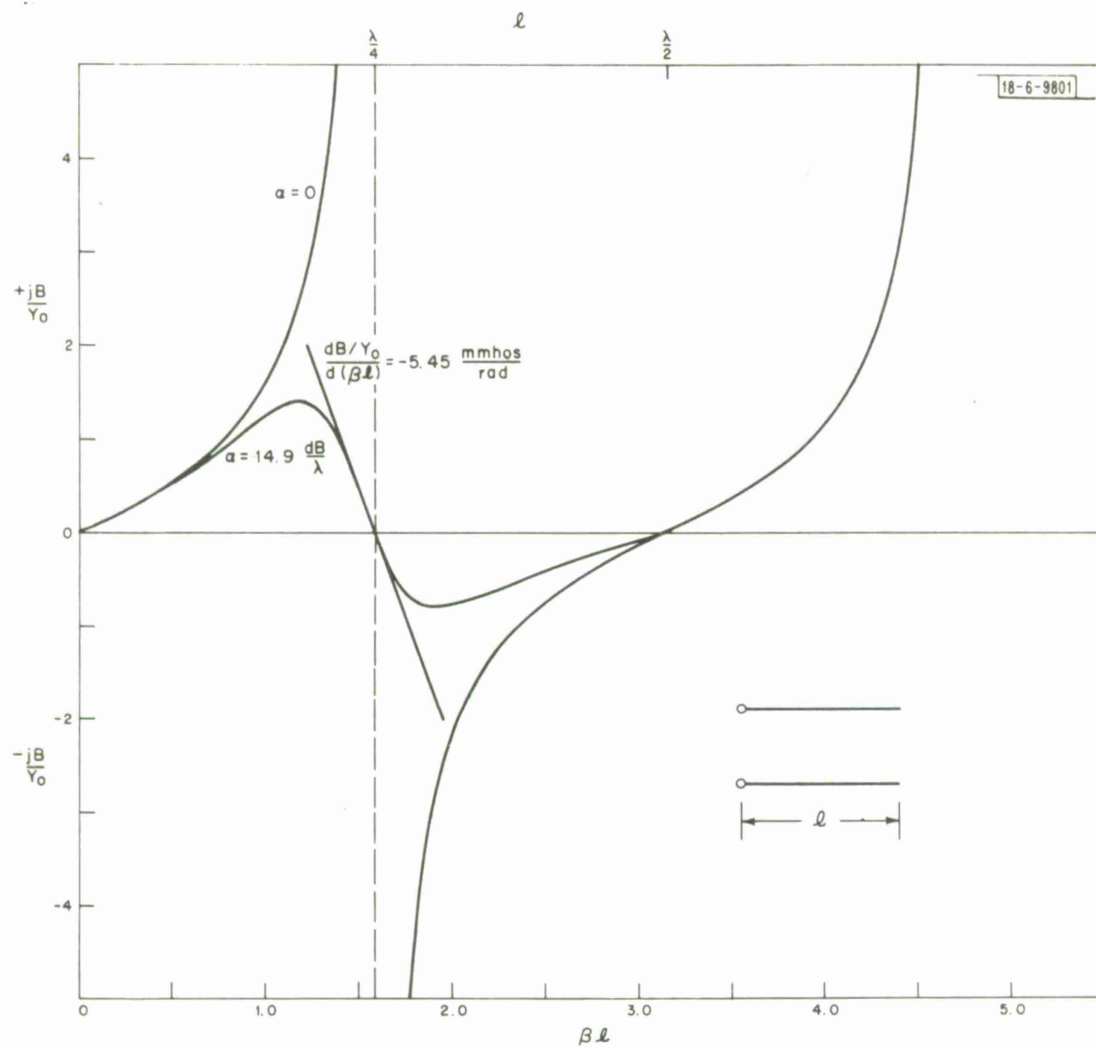
Fig. 4.   Input conductance of a shorted line.

Fig. 5. Input susceptance of an open line.

12

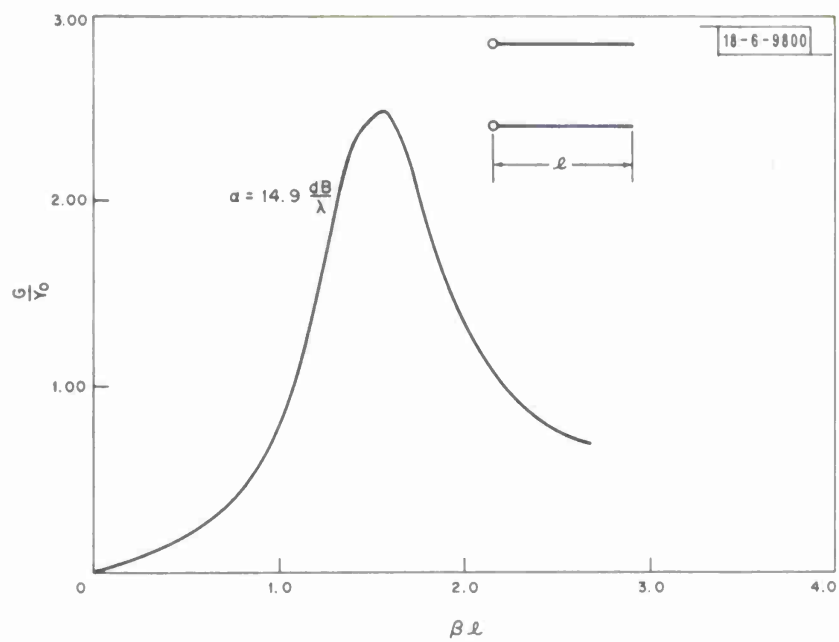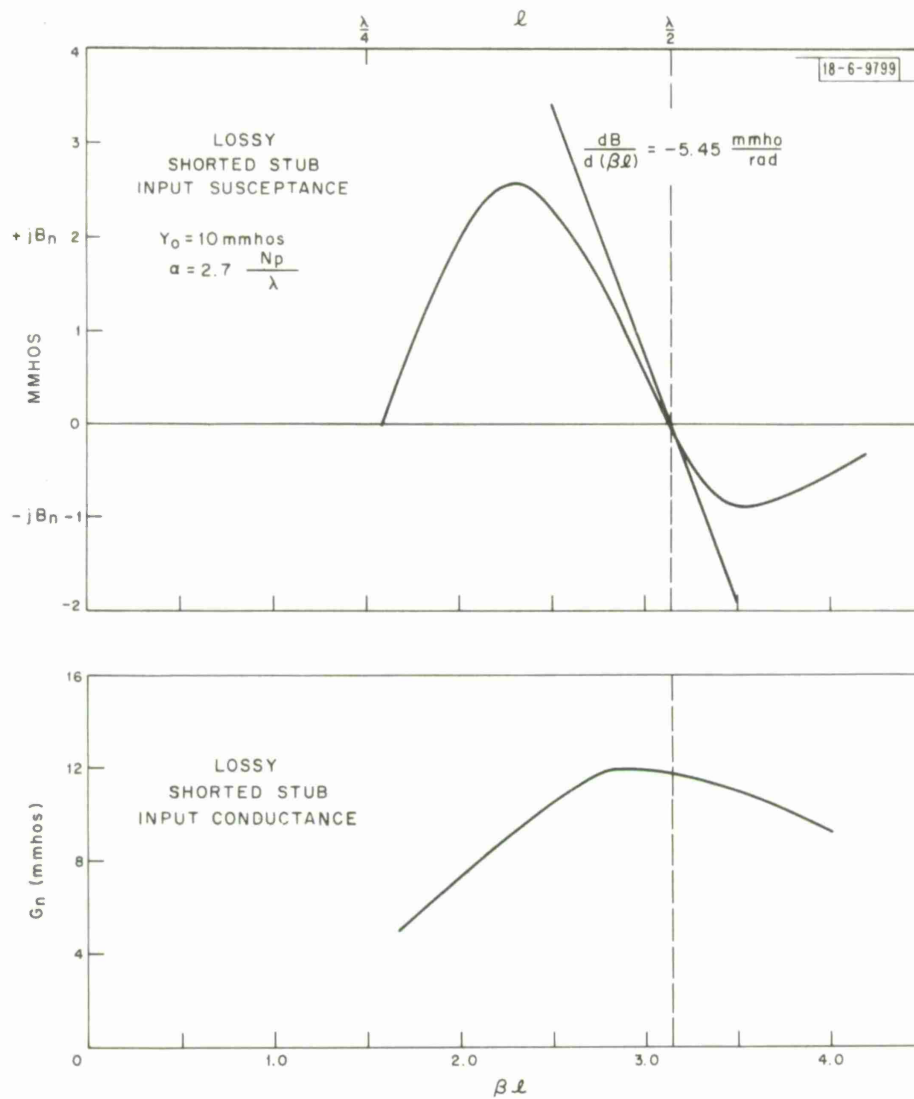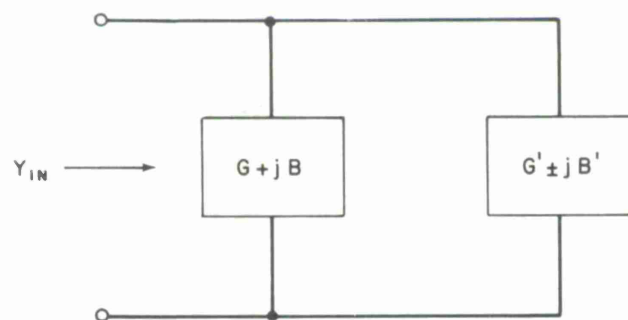Fig. 6.   Input conductance in an open line.

Fig. 7.   Input admittance of a lossy shorted line.

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$Y_{IN} \longrightarrow$ G + j B   G' ± j B'

$Y = G + jB$ IS THE DIPOLE ADMITTANCE
$Y' = G' \pm jB'$ IS THE STUB ADMITTANCE
$Y_{IN} = Y + Y'$
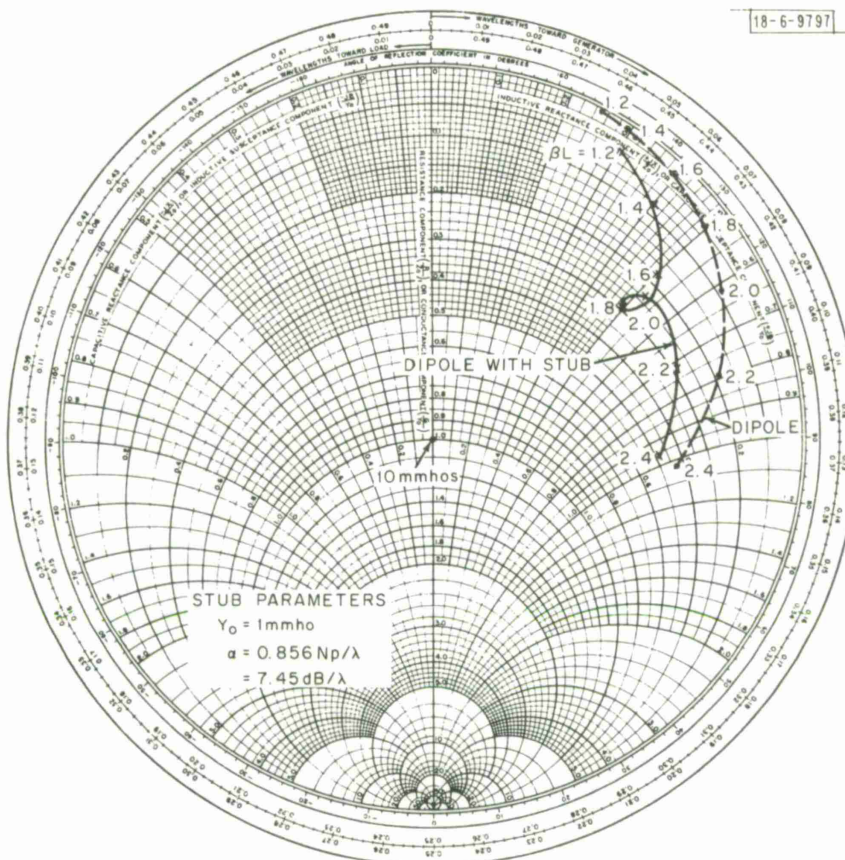
Fig. 8. Equivalent circuit of a stub and dipole combined.

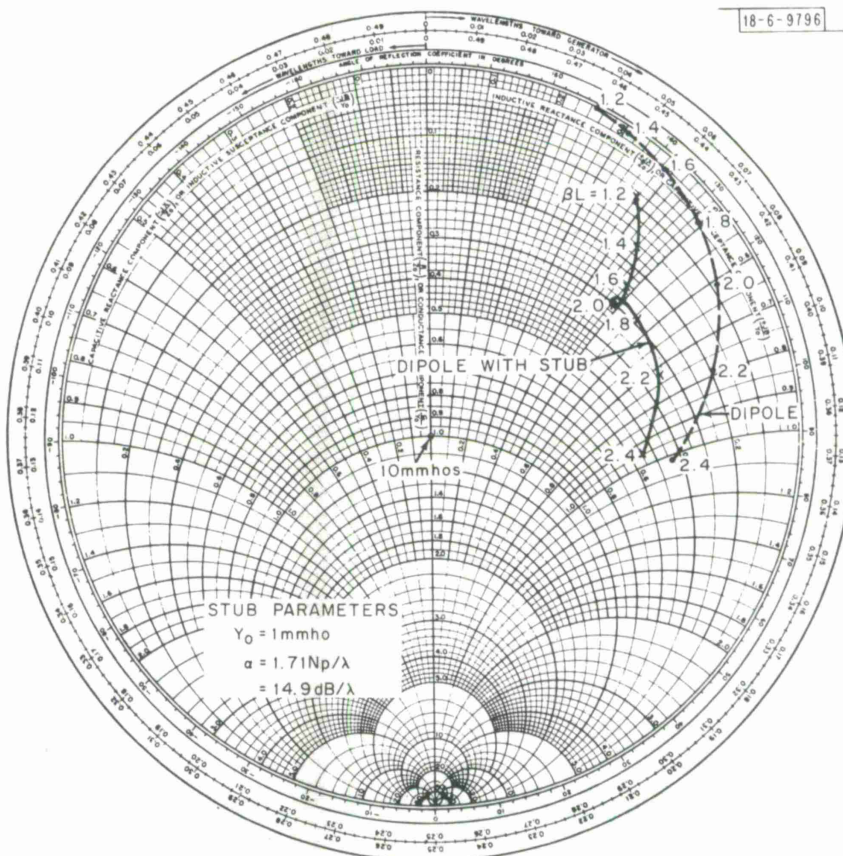Fig. 9. Input admittance of the dipole and Y_o = 1 milli mho shorted stub.

Fig. 10. Input admittance of the dipole and
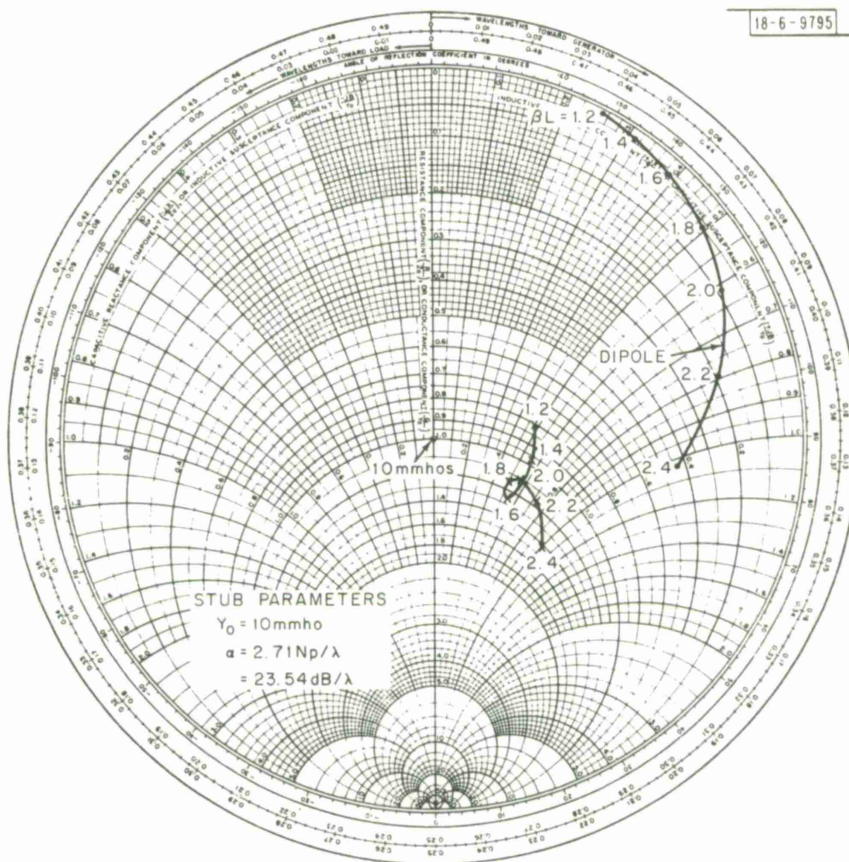$Y_o = 1$ milli mho open stub.

Fig. 11.   Input admittance of the dipole and
$Y_O = 10$ milli mho shorted stub.

18

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13. ABSTRACT

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14. KEY WORDS

Navy communications	impedance	broadband
dipoles	network theory	matching problems