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THE PRESENT VALUE OF THE PAST

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THE PRESENT VALUE OF THE PAST

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One of the seminal ideas in economics is that future events have a present value (either as costs or benefits), and that this value is calculable (through a private or social rate of interest). On a theoretical plane, it plays a central role in capital and savings theory, and in the theory of investment choice. On an operational plane, it is of central importance in cost-benefit analyses, planning-programming-and-budgeting systems, and systems analysis.

Having acknowledged this pivotal role, I will argue that present valuation is nevertheless not of much help in some problems, and can lead to erroneous results in others. To understand and to resolve certain problems, it is as important to have a method for evaluating the past as the future. Actions taken in the present can differentially affect past valuations (either by the actor or by those whom the action affects), and these valuations can influence both actual and preferred choices.

The main purposes of this paper are: (I) to elaborate the heuristics of evaluating the past; (II) to formulate an economic model in which valuation of the past is central, and is subject to

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I am indebted to Kenneth Arrow and Lloyd Shapley for several initial discussions of the general problem, as well as to them and to Nathan Leites and John McCall for comments on a previous draft.

influence by present choices; and (III) to illustrate the model by applying it to several problems which it can handle, but which the standard approach cannot.

### I. The Heuristics of Past Valuation

Those of us, economists and others, who regard ourselves as practical, problem-solvers, normally view the past as concluded, done, inert. It isn't. In part, what is at issue is the unreliability of memory -- individual as well as institutional -- and the influence that present actions often exert on what is remembered. In general, the past is more supple, malleable, and reinterpretable than we are comfortable to admit. While we know that the future is uncertain, we feel quite uncomfortable to consider that the past may also be. The discomfort arises in part because such a protean view of the past seems to border on solipsism. I will argue that the discomfort can be surmounted; the border is wide enough to transit.

Three premises underlie the model presented below. The first premise is that prior events, somehow aggregated, enter as arguments in the utility function of various individual and institutional actors. In other words, people are not only forward-looking maximizers, but backward-looking maximizers (or minimizers, insofar as regret is concerned). Stated differently, the present values that they maximize often include the past as well as the future.<sup>1</sup>

<sup>1</sup>A similar idea is developed in a different context by Richard Zeckhauser and Stephen Fels, "Discounting for Proximity with Perfect and Total Altruism," Harvard Institute of Economic Research, November 1968.

Second, the process of aggregating prior events can be viewed as mediated through a backward-looking discount rate, which I shall call a decay rate. Of the three factors that are commonly acknowledged to influence the discount applied to future events, two also apply to prior events: namely, uncertainty (i.e., for the decay rate, the analogue is imperfect memory), and pure time preference.<sup>1</sup> (The third factor, the opportunity cost of capital, is absent from the valuation of the past.) Together these two factors determine the decay rate. In principle, different decay rates may apply to different events and to different individuals, just as interest rates differ for different transactions and transactors. The model described below ignores this complexity, working instead with a single decay rate (in effect corresponding to the prime interest rate), which is subject to change in specified ways.<sup>2</sup>

The connection between these two premises is obviously close. Thus, the stream of prior events that affects utility is subject to attrition in recall; this rate of attrition is reflected in the decay rate. (Of course, events from the past may also wax rather than wane in recollection: the decay rate may be negative.)<sup>3</sup>

<sup>1</sup>A beneficent event yesterday is preferred to the same event a fortnight ago, as the event today would be preferred to that a fortnight hence.

<sup>2</sup>Empirical referents for decay rates are elusive. Some coarse indicators: attenuation over time in footnote references to journal articles from prior years; variations in the statute of limitations according to the gravity of offense (presumably, memory is considered to be more reliable the graver the offense); the proportion of the relevant population alive now that was alive "then" (e.g., if birth and death rates are high, decay may tend to be rapid).

<sup>3</sup>Cf. below, p. 11.

Third, the decay rate may be affected by present action. For example, it is a common experience that a current event may increase or decrease the vividness with which a prior event is recalled (corresponding, respectively, to a decrease or an increase in the decay rate on that event). Revisiting a place or a person, or confronting a new situation that contains familiar characteristics, can have this effect. Events which happened a considerable time ago may then seem to have happened recently, while others which happened recently may seem to have happened a considerable time ago. One contributory explanation is the aging process itself -- which is indeed a kind of "present action," though a relentless one. In an almost truistic sense, this explanation holds good: the chance of error in dating events is increased because age provides more time over which to err.

The third premise leads to a corollary which is crucial for the simple structural model we will next turn to: actions taken in the present, which contribute to increases in utility with respect to arguments that have a present or future subscript, may diminish utility with respect to arguments that have a past subscript, and vice versa.

## II. A Model for Present Valuation of the Past (PVP)

The PVP model can be described in terms of five equations.

Consider first a utility function

$$(1) \quad U_t = U(Y_t, \tilde{V}_t),$$

where  $Y_t$  is that part of present welfare corresponding, say, to

income in period  $t$ ,<sup>1</sup> and  $\tilde{V}_t$  is the present value of some set of prior events (i.e., costs or benefits).<sup>2</sup>

The next equation defines the present value of these prior events.

$$(2) \quad \tilde{V}_t = \sum_{\tau=t-1}^{t-m} (1+r^*)^{\tau-t} V_\tau,$$

where  $r^*$  is the decay rate, and the  $V_\tau$ 's are the values of prior events over a relevant mnemonic span,  $t-1$  to  $t-m$ .<sup>3</sup> In other words,

<sup>1</sup> $V_t$  can alternatively be viewed as the present value of future income discounted at an appropriate rate over the relevant time period.

<sup>2</sup>Prior accomplishments would be an example of benefits, while perceived prior inequities or discrimination would be an example of costs. Whether the events are costs or benefits depends, of course, on the zero point, which in turn depends on the subject's point of view; a chicken in the pot in year  $\tau$  is a benefit if zero is no chickens, a cost if zero is two chickens.

As suggested by some of the examples discussed later,  $\tilde{V}_t$  may often turn out to be closely associated with status, reputation and self-esteem. The utility function is thus similar to what one finds in games that combine status and welfare, cf., Martin Shubik, "Games of Status," RAND D(L)-17685, August 29, 1968. It also is similar to that used by William M. Gorman, ("Convex Indifference Curves and Diminishing Marginal Utility," Journal of Political Economy, 65, February 1957), and Zeckhauser-Fels (op. cit.). In the Gorman-Zeckhauser-Fels terminology,  $V_t$  corresponds to present consumption ("felicity"), while  $\tilde{V}_t$  corresponds (almost) to the altruistic terms relating to other generations' utilities.

<sup>3</sup>Alternatively, equation (2) may be written:  $\tilde{V}_t = \sum_{\tau=1}^m (1+r)^{-\tau} V_{t-\tau}$ .

$m$  may be set equal to infinity or, more realistically, truncated at some point, depending on the context. From several computer simulations of the model, using linear functions and plausible values for  $V_\tau$ ,  $r^*$ , and the other parameters, the  $\tilde{V}$ -results were not sensitive to extending the period beyond  $m \approx 20$ .

A weighted-average version of (2) may be preferable in cases where the zero-point is not sharply defined, or is intrinsically undefinable (as, for example, in the case of  $V_\tau$ 's that represent past injustices. See below pp. 12 ff.). In this case, we may use instead of (2)

$$(2.1) \quad \tilde{V}_t = \sum_{\tau=t-1}^m c_{t-\tau}(r^*) V_\tau,$$

where  $\sum_{\tau=t-1}^m c_{t-\tau}(r^*) = 1$ ,

and, for any individual coefficient,  $c_{t-\tau}(r^*) = \frac{(1+r)^{\tau-t}}{\sum_{\tau=t-1}^{t-m} (1+r)^{\tau-t}}$ .

(2) shrinks these prior events into units of equivalent present value and sums them, while (1) shows the tradeoffs between these units and units of current income.

Next, we assume

$$(3) \quad r^* = g(P_{ti}) ,$$

where  $P_{ti}$  denotes the  $i^{\text{th}}$  member of the set of feasible policies for changing income in period  $t$ . A choice from the policy set also determines  $\dot{Y}_t$ . Generally, policies that lower  $r^*$  will be those that improve information retrieval (improve memory), or reduce the preference for present over past. We can thus write

$$(4) \quad \dot{Y}_t = h(P_{ti}) .$$

A variant of equation (3) is

$$(3.1) \quad r^* = f(\dot{Y}_t) = g(h(P_{ti})) ,$$

which covers the special case where the effect of a policy choice on the decay rate depends only on the income change, and not on the manner in which it is produced.

In some cases,  $f' > 0$ , while in others  $f' < 0$ , as will be discussed later.

To complete the model, we have

$$(5) \quad Y_t = Y_{t-1} + \dot{Y}_t .$$

With these equations, including (3.1) as a special case, we have five unknowns:  $U_t, r^*, \tilde{V}_t, Y_t, \dot{Y}_t$ ; and  $(m+2)$  exogenous variables:  $m(V_r), Y_{t-1}, P_{ti}$ .

Two differences in the possible shapes of the utility function (1) and the decay functions (3) and (3.1), as well as in the interaction between them, are worth distinguishing.  $\tilde{V}_t$  may represent a

stream of prior costs (hence  $\frac{\partial U}{\partial \tilde{V}_t} < 0$ ), or benefits ( $\frac{\partial U}{\partial \tilde{V}_t} > 0$ ). And  $f'$  may be negative or positive, as noted above. If the  $V_\tau$ 's are costs, and  $f'$  is negative, raising  $Y$  (i.e.,  $\dot{Y} > 0$ ), lowers  $r^*$  and increases  $\left| \tilde{V}_t \right|$ . The effect on  $U_t$  depends on the marginal rate of substitution between  $Y_t$  and  $\tilde{V}_t$ , and on  $f'$ . Thus, the larger is  $\left| \frac{\partial U / \partial \tilde{V}_t}{\partial U / \partial Y_t} \right|$  and  $\left| f' \right|$ , the greater the fall in  $U_t$  from raising  $Y$ .

If the  $V_\tau$ 's are benefits, and  $f'$  positive, raising  $Y$  increases  $r^*$  and lowers  $\tilde{V}_t$ . The effect on  $U_t$  again depends on the rate of substitution between  $Y_t$  and  $\tilde{V}_t$ , and on  $f'$ . Again, the larger is  $\left| \frac{\partial U / \partial \tilde{V}_t}{\partial U / \partial Y_t} \right|$  and  $f'$ , the greater the fall in  $U_t$  from raising  $Y$ .\*

The two other combinations of  $V_\tau$  and  $f'$  are reinforcing, rather than offsetting. Thus, if the  $V_\tau$ 's are costs and  $f'$  positive, or if the  $V_\tau$ 's are benefits and  $f'$  negative, raising  $Y$  increases both arguments in the utility function. The past and present blissfully harmonize; whereas discordance between past and present are represented by the previous examples.

The examples we now turn to illustrate the discordance.

### III. Some Examples of PVP: Sunk Costs and Social Inequities

#### 1. Sunk Costs

According to a familiar economic theorem, sunk costs should not influence decision, only marginal costs. If the present value of

\*The effects on  $U_t$  can be seen more easily by looking at the total derivative of  $U_t$  with respect to  $Y_t$ , assuming (3.1). Thus,

$$\frac{dU_t}{dY_t} = \frac{\partial U}{\partial Y_t} + \frac{\partial U}{\partial \tilde{V}_t} \cdot \frac{\partial \tilde{V}_t}{\partial r^*} \cdot f'(Y_t)$$



future benefits associated with a particular option A exceed that associated with B, choose A; don't worry about any prior costs associated with each of the options, so the normative argument runs. To the ordinary man's puzzled plaint, "Yes, I hear what you say, but it's just not right: prior costs really do matter," the usual response is either an evasion ("people are just irrational, or anyhow non-rational"), or a tautology (marginal costs can be redefined so as to include any pain that may be associated with an otherwise preferred choice because of its connection with prior events). Neither response is very satisfying. The former is not, because it accepts without explaining the gap between the normative and the descriptive; the latter because, while bridging this gap, it lacks a mechanism to clarify what is going on (it averts the illogical by substituting the tautological). The PVP model provides such a mechanism.

To formulate the sunk-costs problem in terms of PVP, we let the argument  $\tilde{V}_t$  represent a benefit realized in the present but based on a set of prior costs.<sup>1</sup> Prior costs may have a present value when they affect (a) the decision maker directly, e.g., his self-esteem; and (b) the reactions of others towards him, e.g., his reputation and credibility.<sup>2</sup> The magnitude of prior outlays that have been incurred for a particular purpose or policy is often the best

<sup>1</sup>In a sense,  $\tilde{V}_t$  corresponds to the book value of tangible assets as a proxy for their market value (replacement costs). The concern of decision makers with protecting or increasing  $\tilde{V}_t$  is thus analogous to concern with protecting asset value.

<sup>2</sup>In some situations, the prime importance of such values is suggested by the fact that the utility function may be lexicographic in this dimension antecedent to all other arguments. Cf. Shubik's treatment, op. cit., pp. 2-3.

proxy for this present value, with the decay rate,  $r^*$ , determining how this value behaves. Hence, the choice of an incomes policy, or other welfare instrument,  $P_{t1}$ , will (should) be influenced by its effect on  $r^*$ .

Thus, suppose  $P_{t1}$  will raise  $Y_t$  more than does  $P_{t2}$ , but will at the same time also increase  $r^*$  (and hence lower  $\tilde{V}_t$ ) more than does  $P_{t2}$ . If

$$\dot{U}_1 = \dot{U}(\tilde{V}_t, P_{t1}) < \dot{U}(\tilde{V}_t, P_{t2}) = \dot{U}_2,$$

the choice of policy  $P_{t2}$  is preferred to  $P_{t1}$ . The result applies both normatively (for a decision unit with a specified utility function), and descriptively (for observers concerned with predicting behavior).

The sunk-costs example applies to a large class of problems of which the following are illustrations: predicting the R&D policy that a particular military service will pursue (e.g., the advanced manned bomber system) from information about its prior R&D outlays on similar systems; evaluating the worth of a particular country (e.g., Vietnam) to the United States and predicting the course of U.S. policy, from information about prior outlays and policies<sup>1</sup>;

<sup>1</sup>Note the following statement by Henry Kissinger: "However fashionable it is to ridicule the terms 'credibility' or 'prestige', they are not empty phrases; other nations can gear their actions to ours only if they can count on our steadiness. The collapse of the American effort in Vietnam would not mollify many critics... (and) those whose safety or national goals depend on American commitments could only be dismayed. In many parts of the world... stability depends on confidence in American promises." Henry A. Kissinger, "The Vietnam Negotiations," Foreign Affairs, January 1969, p. 219. Also, see the discussion of how sunk-costs affect the value of other countries to the United States, Charles Wolf, Jr., United States Policy and the Third World, Boston, Chapter 1, page 19.

explaining salary structures (especially, but not exclusively, in academic and research institutions) in terms of individuals' prior, as well as current, productivity; and understanding a broad range of individual and small-group behavioral phenomena some of which will be mentioned briefly below. Although in each of these cases, there is a strong element of serial correlation that often permits prediction, an underlying mechanism to explain what is happening is suggested by FVP, while obviously lacking in the coefficients of time-lagged regression models. The mechanism that these examples share is one in which present action is influenced by a desire to protect or preserve a present benefit whose magnitude is often indicated by the scale of prior (i.e., sunk) costs.

A classic example is provided by Agamemnon's strategem for persuading the Greeks to persevere in the Trojan wars by pointing out that withdrawal would cause dishonor to those whose lives had already been lost.<sup>1</sup>

A hypothetical example, as readily applicable to the behavior of the firm as to that of optimizing individuals inside or outside it, can be put in the form of a question. If two alternative actions (e.g., investment choices, R&D projects, academic appointments, support for political candidates) have an equal expected yield but the decision maker has previously expended resources on one, which will (should) he choose? The descriptive answer is easy; the normative answer involves more elusive considerations of prestige,

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<sup>1</sup>See Richmond Lattimore's translation of Homer's Iliad, Chicago, 1951, Book 9, pp. 198-199.

credibility and the desire for personal vindication, which can be readily assimilated to PVP. When the yields aren't equal, the proper analytic precept is not to choose the higher, but rather to show exactly how much higher it is, so that this margin ( $\dot{Y}_t$ ) can be compared with the possible loss of other (prior) values ( $\tilde{V}_t$ ) in arriving at a U-maximizing choice.

PVP in the benefit sense (i.e.,  $\partial U / \partial \tilde{V}_t > 0$ ) also applies to a familiar phenomenon sometimes associated with aging. People often find it tolerable, or even pleasant, to acknowledge a currently inadequate performance on their own part (athletics comes readily to mind), which ostensibly affords contrast to a claimed superior performance in prior years (e.g., "But you should have seen me ten years ago!"). The transaction that seems to be at work is an exchange between a lowered present and an enhanced prior performance. In terms of PVP, the mechanism can be viewed as a smaller (or even negative)  $\dot{Y}_t$  which generates a lower decay rate (or even a negative  $r^*$ ), thereby sustaining (or inflating) the present sense of the past accomplishment.

To take another common experience, it often seems that the older a person is the shorter does an impending period appear to be. As best I can tell, my fifteen-year-old son views next Christmas as much farther in the future than it seems to me, and I am prepared to believe that the next Christmas seemed farther in the future when I was fifteen than it does to me now. People seem to normalize for time in relation to their past, which they have accumulated in different amounts. One possible implication for the PVP model is this: an increase in  $\dot{Y}_t$  may seem smaller in relation to changes

in  $r^*$  and  $\tilde{V}_t$  the older a person is. Thus, the proposition that "you can't trust anyone over 30" (or under 30, for that matter!) has operational significance with respect to the probably different marginal rates of substitution between  $r^*$  and  $Y_t$  for people of different age groups. People who have lived longer simply have more sunk costs tied up in their  $\tilde{V}_t$ , and changes in  $r^*$  are therefore relatively more important to them than changes in  $Y_t$ .

## 2. The Negative Present Utility of Past Injustice

Over a century ago, Tocqueville posed a problem which has puzzled social scientists ever since. Tocqueville's paradox was based on his studies of the French Revolution which led him to observe:

...That in none of the decades immediately following the Revolution did our national prosperity make such rapid forward strides as in the two preceding it ... It is a singular fact that the steadily increasing prosperity, far from tranquilizing the population, everywhere promoted a spirit of unrest. Moreover, those parts of France in which the improvement in the standard of living was most pronounced were the chief centers of the Revolutionary movement ... It was precisely in those parts of France where there had been most improvement that popular discontent ran highest.<sup>1</sup>

Tocqueville's paradox can be formulated in more general terms: why is it that improvements in welfare and in social justice often seem to intensify resentment and unrest? The paradox has been elaborated by Brinton, Olson, and Mitchell, among others, and most eloquently by Eric Hoffer and Robert Waelder; it has been referred

<sup>1</sup>Alexis de Tocqueville, The Old Regime and the French Revolution, New York, Doubleday and Company, Inc., 1955, pp. 174-176.

to by Pettigrew in discussing negro attitudes in the United States.<sup>1</sup> Attempts to explain the paradox have been advanced by these authors as well as others, but the explanations, though often illuminating, have generally not been satisfying. Although they rarely ascribe it to "irrationality," (as in the case of sunk costs), they usually are quite unclear -- as in that case -- about a precise mechanism to account for the paradox. With minor modifications, the PVP model provides such a mechanism.

In this case,  $\tilde{V}_t$  is the discounted aggregate of prior costs or injuries. In the context of racial problems in the United States, or in other upperclass-underclass relations, the "costs" that are relevant may relate to inequalities accumulated over some plausible mnemonic span, say  $n$  years in the past. In the U.S. context, the sense of prior injustices, represented by  $\tilde{V}_t$ , is intimately linked with the black's sense of identity and self-respect. It is likely to be a heavily weighted argument in his utility function.<sup>2</sup>

As a possibly reasonable proxy for  $V_t$ , the measure that might be used is  $(1-q_t)$ , where  $q_t$  is the ratio of black to white income in

<sup>1</sup>Crane Brinton, The Anatomy of Revolution, New York, Vintage Books (rev. 1962); Eric Hoffer, The True Believer, New York, Harper & Bros., 1952; Edward Mitchell, "Inequality & Insurgency: A Statistical Study of South Vietnam," World Politics, Vol. 20, April 1968; Mancur Olson, Jr., "Rapid Growth as a Destabilizing Force," Journal of Economic History, Vol. 23, No. 4, December 1963; and Robert Waelder, Revolution and Progress, New York, New York, International Universities Press, 1967. Pettigrew refers to Tocqueville in discussing relative deprivation as it effects negro attitudes. T. F. Pettigrew, A Profile of the American Negro, Princeton, New Jersey, Van Nostrand Press, 1964.

<sup>2</sup>This point accords with a perception at least as old as Aristotle that inequality is a principal cause of rebellion. The term "inequity" might be more appropriate, in the sense of unfair treatment compared to that received by others equally qualified. Cf., Ariscotie, Politics, New York, Modern Library, 1943, p. 211.

the United States in period  $\tau$ , normalized for differences in the size of the two populations.

Again, as in the case of sunk costs, the choice of a policy for influencing present income,  $P_{ti}$ , affects  $\tilde{V}_t$  by changing the decay rate.<sup>1</sup> If, for example, current income rises, the sense of past injustice may become more vivid and painful. Either of several psychological reactions may account for this heightened sensibility: (a) a perception that the income-raising policy itself is an acknowledgment of how egregious past injustice must have been; (b) a keener recognition that the past injustice need not have been suffered passively; and (c) a wish to avert or to ease the self-degrading fear of having "sold out" in accepting the improvement.

As a result, the negative present utility of prior inequities may be magnified. The reduced decay rate thus acts as a surrogate for the black's bitterness at not having received his due in the past (or at his ancestors' not having done so). A lower  $r^*$  not only changes  $\tilde{V}_t$ , but does so by changing its time profile, as well. If the past was characterized by continued improvement (i.e., declining inequity), a lower  $r^*$  increases the relative weight of the earlier period when inequity was greater.

Thus, it is not necessarily the largest  $\dot{Y}_t$  that will make the biggest positive contribution to  $U$ . A lower  $\dot{Y}_t$  may be a preferable

<sup>1</sup> Policy may affect  $\tilde{V}_t$  by influencing the zero point as well as  $r^*$ , and may do so intentionally or inadvertently, and with or without an effect on  $\dot{Y}_t$ . A striking example of its use in Maoist China in propaganda designed to influence  $r^*$  and the zero point, but not  $\dot{Y}_t$ , is contained in William Hinton, Fanshen: A Documentary of Revolution in a Chinese Village, New York, Monthly Review Press, 1967.

choice because it doesn't lower  $r^*$  (and hence  $\tilde{V}_t$ ) as much.<sup>1</sup> Moreover, there may be no current incomes policy which in fact raises  $U_t$ . The choice may be between allowing  $Y_t$  to stagnate (hence a stagnant  $U_t$ ), or raising  $Y_t$  but actually lowering  $U_t$  (hence, resentment and violence) if  $U_t$  is to be raised at some future time when

$$\partial U / \partial Y_t > \left| \frac{\partial U}{\partial \tilde{V}_t} \cdot \frac{\partial \tilde{V}_t}{\partial r^*} \cdot f'(Y_t) \right|.$$

The dilemma this creates for anti-poverty programs at home (or abroad) is clear. Advancing the laudable objective of reducing or eliminating poverty in black ghetto areas may for a time intensify racial hostilities rather than allay them. A phase of increased violence may be part of the price that has to be paid for reducing poverty, rather than an alternative to it.<sup>2</sup>

<sup>1</sup>In this case,  $\tilde{V}_t$  has a negative sign in the U-function; lowering  $r^*$  raises the absolute value of  $\tilde{V}_t$ .

<sup>2</sup>The mechanism at work in this example may acquire a reverse twist in other examples, in which present income is lowered in order to reduce the present value of prior costs. Consider the drunkard who drinks to forget the past: his optimal policy is to continue until the marginal current loss from drinking, e.g., the hangover, equals the marginal benefit from forgetting the past!