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SYNCHRONIZATION STUDY FOR THE IRIG 102-64 DATA FORMAT

J. T. Barrows

MARCH 1969

Prepared for

AEROSPACE INSTRUMENTATION OFFICE

ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

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Project 705B
Prepared by

THE MITRE CORPORATION
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FOREWORD

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REVIEW AND APPROVAL

This technical report has been reviewed and is approved.

GEORGE T. GALT, Colonel, USAF
Director, Aerospace Instrumentation
Program Office

ABSTRACT

An analysis is made of synchronization properties of patterns suitable for incorporation in the 240 bit standard IRIG 102-64 word format. Sequences of length m = 7, 13 and 17 bits are used as prefix of an (240-m) bit randomly generated word. An analysis which includes a digital simulation is made to determine the synchronization capabilities of each of those sequences. In the first part of the study, for a randomly generated 240 bit word, the average number of false sync sequences generated and the probability of obtaining at least one false sync were computed for each prefix and confirmed by simulation tests. The sync sequences were then appended to the (240-m) bit word and the sync process was defined. The process was simulated for each prefix to determine the number of words lost before acquiring sync and the number of false syncs.

Results indicate that of the three prefixes considered, the 17-bit sequence afforded the best trade-off between the number of false syncs and the time required for sync acquisition.

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SECTION I

INTRODUCTION

The proposed IRIG standard word format consists of a 240 bit word of which 7 bits are to be alloted for word synchronization and 9 bits for error detection.

This study investigates some of the problems associated with this format and compares the performance of a 7-bit sync prefix with that of a 13-bit and a 17-bit prefix with respect to:

a) determining the probability and time to acquire word sync.

b) determining the probability of false sync and the probability of falling out of sync.

For an m-bit sync prefix it was proposed that the remaining (240-m) bits of the data words be assumed random to simulate a worst case condition. This is not necessarily the worst case when attempting to acquire sync since, in practice, there are sub-blocks within the words (such as the 17-bits alloted to time in seconds) which will vary slowly with time. These sub-blocks will periodically contain digits which look like the sync prefix and if the detector locks on to such a subsequence, many words could be lost before the mistake is found. By using the complement of the prefix in alternate words this problem can be minimized and the assumption of random data for this analysis will be a realistic one. Once sync is acquired the slowly varying subsequences can be used as an additional check on sync maintenance.

Section II discusses the three sync prefixes under consideration and their properties. In subsection IIA the probabilities and expected values of the number of occurrences of a randomly generated sync prefix in an n-bit word are calculated. In subsection IIB the probability of obtaining at least one m-bit sync prefix in an n-bit word (whose bits are randomly generated) is calculated. These calculations are compared with experimental results and listed in Table 1. In Section III the method used for acquiring sync is described. The time-to-sync for each of the three prefixes is determined by computer simulation for various channel error rates and sync criteria. Also the number of false sync occurrences was determined for each case. The results are listed in Tables 2 through 7.

SECTION II

DESCRIPTION AND PROPERTIES OF SYNC PREFIXED

The first phase of the analysis of the process is effected by considering a randomly generated bit stream segmented into 240 bit words. The bit stream passes through a sync window (correlation detector) in which the m-bit prefix is compared with the m-bit segment of the data stream present in the window at each bit time and a correlation number is calculated. In this study m is always an odd number and each of the 2^m possible m-bit segments is associated with one of m+1 possible correlation numbers (correlation coefficients can be obtained by dividing the correlation numbers by m).

Let $\sigma(m,i) = S_i$ be the m-bit sequence present in the center of the window at bit time i and let $\sigma_o(m)$ be the sync prefix. The function of the window can be described as

$$C(\sigma(m,i)) = \rho_k = m - 2k$$

for some $k = 0, 1, 2, \ldots, m$. Specifically $C(\sigma_o(m)) = \rho_o = m$, at bit time 1 within a word when the system is in sync.

The properties of prefixes for m = 7, 13 and 17 were investigated. The 7-bit and 13-bit prefixes are Barker sequences [1, 2] and are respectively

$$\sigma_o(7) = 111\text{-}1\text{-}11\text{-}1$$

$$\sigma_o(13) = 11111\text{-}1\text{-}111\text{-}11\text{-}11$$

where the -1 has been used instead of zero as a binary digit. One of the properties of these sequences is that, for no error occurrences in the prefix, there is zero probability of obtaining a sync signal when it is anywhere in the window except the center.

The 17-bit prefix used is a quadratic residue sequence [3] generated by the Legendre symbol $(x/p) = \pm 1$, for $x = 0, 1, 2, \ldots, 16$, according as x is or is not a quadratic residue of the prime $p = 17$, and is given by

$$\sigma_o(17) = 111\text{-}11\text{-}1\text{-}1\text{-}111\text{-}1\text{-}1\text{-}11\text{-}111$$

This sequence is slightly less optimal than a Barker sequence (which doesn't exist for $m = 17$); that is, after shifting this prefix 15 or 16 bits (in either direction) from the center of the window there is a small but nonzero probability of obtaining a correlation number of $\rho_o = 17$. (This assumes as throughout this analysis that the bits on either side of the prefix are randomly generated.)

The first series of tests consists of examining a randomly generated 240-bit word to determine the number of occurences of all possible correlation numbers within the word for each of the three prefixes. Each trial was performed on at least $M = 1000$ randomly generated words. From this we can find the relative frequency of occurence of randomly generated m-bit patterns in a word which have a correlation number within some predetermined error tolerance of a given prefix. This will establish a rationale for choosing a synchronization scheme which will work in the presence of channel errors and for comparing the merits of the three prefixes. Also, theoretical calculations of various probabilities can be compared with these results which will dictate the number of trials needed to obtain valid statistics in later tests, and will indicate the possibilities of constructing a mathematical model of the sync process.

A. Occurrence of Correlation Numbers in a Randomly Generated n - bit Word.

Given an arbitrary m - bit binary sequence S to be correlated with prefix σ_o, the correlation numbers $C(S) = \rho_k = m - 2k$ for $k = 0, 1, 2, \ldots, m$, are possible. There is one way out of 2^m possibilities of obtaining $C(S) = \rho_o$, $\binom{m}{1} = m$ ways of obtaining $C(S) = \rho_1$, etc. Thus the probability of occurrence of any ρ_k when the m bits are chosen at random is given by

$$P\{C(S) = \rho_k\} = \binom{m}{k}/2^m$$

where $\binom{m}{k} = \binom{m}{m-k}$ is the binomial coefficient. Both the theoretical and experimental values of this function are listed in Table 1 for $k = 0$, 1 and 2 for each prefix ($m = 7$, 13 and 17) and are designated $P\{\rho = m\}$, $P\{\rho = m - 2\}$ and $P\{\rho = m - 4\}$ respectively.

Also, the expected or average number of sync patterns in a 240 bit random word is tabulated as E ($\rho = m$ per word).

B. <u>The probability of at least one randomly generated m-bit sync pattern in an n - bit word</u>.

Starting at bit 1, the probability that the sequence S_1 within the window is σ_o is

$$P\{S_1 = \sigma_o\} = 2^{-m}$$

Therefore

$$P\{S_1 \neq \sigma_o\} = 1 - 2^{-m}.$$

At step (bit) 2, the probability that $S_2 = \sigma_o$, given that $S_1 \neq \sigma_o$ is given by

$$P\{S_2 = \sigma_o | S_1 \neq \sigma_o\} = \tfrac{1}{2}\, P\,\{\text{the last m-1 bits of } S_1 = \text{first m-1 bits of } \sigma_o\}$$

$$= \tfrac{1}{2} \times 2^{-(m-1)} = 2^{-m}$$

Therefore

$$P\{S_2 \neq \sigma_o / S_1 \neq \sigma_o\} = 1 - 2^{-m}$$

and

$$P\{S_2 \neq \sigma_o,\ S_1 \neq \sigma_o\} = \left[1 - 2^{-m}\right]^2.$$

Continuing in this manner for n bits

$$P\{S_i \neq \sigma_o \text{ for } i = 1, 2, \ldots, n\} = \left[1 - 2^{-m}\right]^n$$

and

$$P\{S_i = \sigma_o \text{ at least once in n steps}\} = 1 - \left[1 - 2^{-m}\right]^n.$$

(Note that in order to get n correlation numbers we need a word length of n + m bits.)

Now let

A_o be the event such that an n-bit random word, $\bar{w}$, contains at least one occurrence of a correlation number $\rho = \rho_o = m$;

A_1 be the event such that $\bar{w}$ contains at least one occurrence of $\rho = \rho_1 = m - 2$;

A_2 be the event such that $\bar{w}$ contains at least one occurrence of $\rho = \rho_2 = m - 4$.

In addition to calculating $P(A_o) = 1 - [1-2^{-m}]^n$ for n = 240, the probabilities $P(A_o)$, $P(A_o \cup A_1)$ and $P(A_o \cup A_1 \cup A_2)$ have been obtained from the computer tests for the three prefixes and are also listed in Table 1.

A number of conclusions can be drawn from Table 1.

1. The probability of a sync pattern occurring in a randomly generated 240-bit word decreases rapidly as the length of the prefix increases. The probability of obtaining the 7-bit prefix is alarmingly high with the odds better than 5:1 of obtaining at least one occurrence in a word.

2. The probabilities $P(A_o)$, $P(A_o \cup A_1)$ and $P(A_o \cup A_1 \cup A_2)$ indicate that we could allow a threshold of 2 errors to sync on the 17-bit sequence, 1 error in the 13-bit sequence and no errors in the 7-bit sequence.

3. Comparison of the theoretical and experimental values of relative frequencies for each of the 3 prefixes (the first three lines of Table 1) shows that 1000 trials yields adequate statistics for the 7-bit sequence as governed by the least probable event. 2000 and 3000 trials for the 13-bit and 17-bit sequence respectively, give reasonably accurate results. To obtain an increase in accuracy would require many more trials and therefore a proportionally larger amount of computer time.

	7 Bit Sync Pattern		13 Bit Sync Pattern		17 Bit Sync Pattern	
	Theor.	Exper. (1000 Trials)	Theor.	Exper. (2000 Trials)	Theor.	Exper. (3000 Trials)
P (ρ = m)	.0078	.0077	1.2×10^{-4}	1.17×10^{-4}	7.63×10^{-6}	8.3×10^{-6}
P (ρ = m-2)	.0547	.0548	1.59×10^{-3}	1.55×10^{-3}	1.297×10^{-4}	1.19×10^{-4}
P (ρ = m-4)	.164	.165	9.52×10^{-3}	9.7×10^{-3}	1.038×10^{-3}	1.1×10^{-3}
E (ρ = m per word)	1.87	1.85	0.029	0.028	1.83×10^{-3}	1.92×10^{-3}
P (A_o)	0.847	0.85	0.027	0.028	.0018	0.002
P ($A_o \cup A_1$)	---	1.00	---	0.326	---	0.0307
P ($A_o \cup A_1 \cup A_2$)	---	---	---	0.952	---	0.257

TABLE 1

Probabilities Associated with a Randomly Generated Sync Pattern in a 240 Bit Word

SECTION III

SYNC ACQUISITION

The general procedure for establishing word sync for an n-bit word containing an m-bit sync prefix is initiated by opening the sync window at a randomly selected bit position within a word. This accounts in part for the fact that there is a time uncertainty as to when the transmission began.

1. The window calculates the correlation at each bit time and declares a preparatory sync condition whenever it detects a $\sigma(m,b)=S_b$ starting at position b such that

$$C\ [\sigma(m,b)] \geq \rho_s$$

for some predetermined value of s. The subscript s is the error threshold and for an adequate prefix we can allow a limited number of errors in the prefix and still accept it (s = o implies only error free acceptance, s = 1 implies that we will accept no more than one error, etc.).

2. The window is then closed for n bits (one word length) and the correlation number of the subsequence in the window at bit time (b + n) is determined.

3. Step 2 is repeated up to N times. If

$$C\ [\sigma\ (m,\ b + n\tau)] \geq \rho_s$$

for $\tau = 1, 2, \ldots, N$, the system is declared to be in word sync. However, if for some $\tau < N$ we have

$$C\ [\sigma\ (m,\ b + n\tau)] < \rho_s$$

the sync criterion has not been satisfied and the window starts at bit $b + n\tau + 1$ to calculate the correlation numbers bit-by-bit as in step 1 and the process repeats until the criterion has been satisfied.

For an m-bit prefix the sync criterion is fixed by the pair of numbers (s, N). s is called the "error threshold number" and N is the "consecutive sync count". If b is some multiple of n = 240 then a legitimate sync condition can be found in step 3, otherwise the system will be in false synchronization.

This sync scheme was simulated on the 7030 computer for each prefix. In each case tested the time-to-sync was found in terms of the number of words lost or wasted before sync was acquired. The first word was included in this count even though only part of it passed through the sync window.

The parameters involved in this simulation are:

$\sigma_o(m)$	sync prefix; m = 7, 13, and 17
N	consecutive sync count; N = 2 and 3
p	channel bit error probability; p = 0, 10^{-4}, 10^{-3}, 10^{-2}
s	error threshold;

$s = o$ or $\rho_s = 7$ for $\sigma_o(7)$

$s = o$ or $\rho_s = 13$
$s \leq 1$ or $\rho_s \geq 11$ } $\sigma_o(13)$

$s = o$ or $\rho_s = 17$
$s \leq 1$ or $\rho_s \geq 15$ } $\sigma_o(17)$
$s \leq 2$ or $\rho_s \geq 13$

For each of the parameter combinations (42 cases in all), 1000 sync acquisition trials were performed and the number of times Y words were wasted before acquiring sync was determined. Also the number of false sync acquisitions was noted in each case.

7 BIT PREFIX

	OCCURRENCES							
	N = 2				N = 3			
WORDS WASTED	$p=10^{-2}$	$p=10^{-3}$	$p=10^{-4}$	$p=0$	$p=10^{-2}$	$p=10^{-3}$	$p=10^{-4}$	$p=0$
1	2	5	3	4	0	0	0	0
2	393	451	457	465	2	2	5	3
3	279	287	285	284	364	446	451	464
4	135	148	140	144	245	302	310	287
5	76	62	67	55	145	134	145	161
6	35	21	26	34	69	78	60	58
7	32	11	9	7	47	20	24	19
8	16	2	3	2	38	11	2	5
9	7	2			36	5	3	3
10	7	0			16	2		
11	3	0			13			
12	3	1			7			
13	0				9			
14	0				2			
15	0				2			
16	1				1			
17	1				0			
18					0			
19					1			
20					1			
21	TEN FALSE SYNCS	TEN FALSE SYNCS	TEN FALSE SYNCS	FIVE FALSE SYNCS	0			
22					1			
23					0			
24					1			

TABLE 2

13 BIT PREFIX

WORDS WASTED	OCCURRENCES N = 2 p=10^{-2}	N = 2 p=10^{-3}	N = 2 p=10^{-4}	N = 2 p=0	N = 3 p=10^{-2}	N = 3 p=10^{-3}	N = 3 p=10^{-4}	N = 3 p=0
1	5	2	3	3	0	0	0	0
2	767	959	975	986	4	2	3	4
3	104	28	22	11	568	941	982	987
4	80	10			83	31	13	9
5	20	1			92	14	1	
6	15				84	12	0	
7	6				36		1	
8	2				17			
9	1				14			
10					6			
11					1			
12					2			
13					1			
14					0			
15					0			
16					1			
17					0			
18					1			

TABLE 3

13-BIT PREFIX-THRESHOLD: $\rho \geq 11$

	OCCURRENCES					
	N = 2			N = 3		
WORDS WASTED	10^{-2}	10^{-3}	10^{-4}	10^{-2}	10^{-3}	10^{-4}
1	6	6	5	0	0	0
2	811	823	833	6	6	5
3	148	151	140	791	808	824
4	28	16	19	164	167	153
5	6	4	2	21	17	17
6	1			13	2	1
7				5		
			1 False Sync			

TABLE 4

17 BIT PREFIX

	OCCURRENCES							
	N = 2				N = 3			
WORDS WASTED	$p=10^{-2}$	$p=10^{-3}$	$p=10^{-4}$	p=0	$p=10^{-2}$	$p=10^{-3}$	$p=10^{-4}$	p=0
1	5	3	3	5	0	0	0	0
2	678	972	995	995	5	3	3	6
3	123	12	1		567	943	994	993
4	124	13	1		90	15	2	1
5	31				107	19	1	
6	23				97	19		
7	8				43	1		
8	4				29			
9	2				27			
10	0				10			
11	1				12			
12	1				5			
13					2			
14					0			
15					3			
16					2			
17					0			
18					0			
19					1			

TABLE 5

17-BIT PREFIX THRESHOLD: $\rho \geq 15$

	OCCURRENCES					
	N = 2			N = 3		
WORDS WASTED	$p=10^{-2}$	$p=10^{-3}$	$p=10^{-4}$	$p=10^{-2}$	$p=10^{-3}$	$p=10^{-4}$
1	6	6	5	0	0	0
2	954	974	979	6	6	5
3	29	19	16	948	977	980
4	10	1	0	20	16	15
5	1	0		14	0	0
6	0			12	1	
7				0	0	

TABLE 6

17-BIT PREFIX THRESHOLD: $\rho \geq 13$

	OCCURRENCES					
	N = 2			N = 3		
WORDS WASTED	$p=10^{-2}$	$p=10^{-3}$	$p=10^{-4}$	$p=10^{-2}$	$p=10^{-3}$	$p=10^{-4}$
1	6	6	5	0	0	0
2	848	846	870	6	6	5
3	133	125	113	881	873	885
4	12	20	11	102	106	101
5	1	1	1	10	13	8
6		1		0	2	1
7		1 False Sync		1		

TABLE 7

To facilitate the reading and interpretation of the results given in Tables 2, 3, 4, 5, 6 and 7 we take a specific case. For the 7-bit prefix let N = 3 and $p = 10^{-2}$, given in Table 2. Since we require the detection of three consecutive errorless prefixes in this case, the minimum number of words wasted is 2, which occurred twice out of 1000 trials. This can happen only if the sync window was opened at bit position 1 of the first word and then proceeded to find 2 more errorless prefixes in succession. Also for this case, 10 data words were wasted 16 (out of 1000) times before word sync was acquired. This means that either the window detected random sync patterns and skipped ahead 240 bits before opening again and/or when it calculated the correlation number at a valid prefix position, errors were present in the prefix. In each of these 16 occurrences, not until the beginning of the ninth word did the window detect valid prefixes (without errors) in three successive words.

For a given sync count criterion, N, we would like to have most, if not all, sync tests wasting no more than N + 1 words, which is the minimal value assuming the window first opens between bit 2 and bit 240 of the first word. At high error rates all prefixes fall far short of this goal, with σ_o (13) and σ_o (17) having roughly the same distributions and σ_o (7) having a much more diffuse distribution than the other prefixes at all error rates. Also, a disturbing number of false syncs appeared in every case for σ_o (7) with N = 2. Increasing the sync count to N = 3 has the effect of eliminating the false syncs for σ_o (7) but spreads the number of words wasted over a larger range in every case which one might intuitively expect since errors in the prefix become more likely.

By introducing an error threshold for σ_o (13) and σ_o (17) it is seen in Tables 4, 6 and 7 that the distributions are tightened up for all error rates with the improvement inversely proportional to the error rate.

It should be noted in Table 7 that a false sync is obtained. Although not enough data was accumulated to reach any definite conclusions, it seems reasonable to expect (from the results listed in Table 1) that as the error threshold is lowered there is a higher probability of false syncs. Thus in general, for a given error rate p, there is definitely an optimum selection of m, N and ρ_s. For example, for $p = 10^{-3}$ by choosing σ_o (17), N = 2 and $\rho_s \geq 15$, 98% of the data streams tested acquired true word sync in no more than 2 words.

The results show that the 17-bit sync prefix is superior to the others tested even though its' correlation properties are not as good as the 7 and 13-bit Barker sequences. This is in part due to the obvious fact that the longer the prefix is relative to the total word length, the smaller the probability of obtaining a randomly generated sync sequence within a word. The 17-bit prefix also has the advantage that a limited number of (transmission) errors can be allowed in the prefix without unduly increasing the probability of acquiring a false sync indication. This will reduce the amount of data lost before sync is acquired.

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13. ABSTRACT

An analysis is made of synchronization properties of patterns suitable for incorporation in the 240 bit standard IRIG 102-64 word format. Sequences of length m = 7, 13 and 17 bits are used as prefix of an (240-m) bit randomly generated word. An analysis which includes a digital simulation is made to determine the synchronization capabilities of each of those sequences. In the first part of the study, for a randomly generated 240 bit word, the average number of false sync sequences generated and the probability of obtaining at least one false sync were computed for each prefix and confirmed by simulation tests. The sync sequences were then appended to the (240-m) bit word and the sync process was defined. The process was simulated for each prefix to determine the number of words lost before acquiring sync and the number of false syncs.

Results indicate that of the three prefixes considered, the 17-bit sequence afforded the best trade-off between the number of false syncs and the time required for sync acquisition.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
DIGITAL DATA TRANSMISSION SIMULATION						