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LIFE CYCLE COST MODELING

JOHN L. HAMILTON

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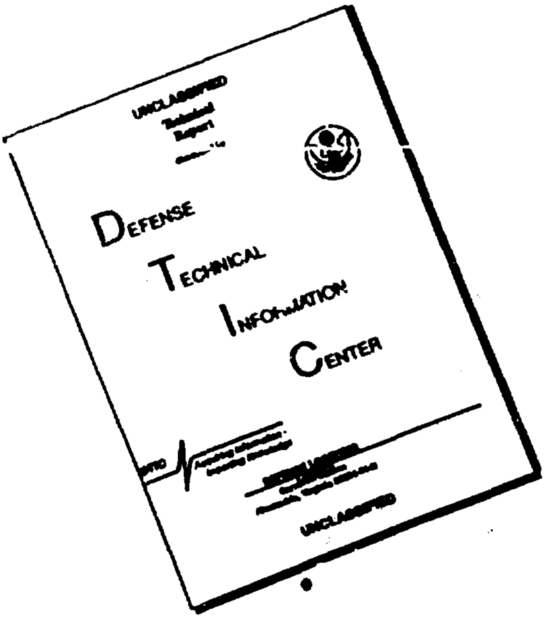
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Life Cycle Cost Modeling

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US Army Materiel Command

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ABSTRACT

This report discusses the mathematical aspects of life cycle cost modeling with emphasis on treatment of parameters, time-phasing of models, and sensitivity analysis. Learning curves, percentage factors, and simple additive cost categories are discussed. General and specific time-phased equations with constant and changing learning curves are presented in detail. The use of partial differential equations for sensitivity analysis is developed.

LIST OF FIGURES

TABLE OF CONTENTS

I. INTRODUCTION

Since the inception of the Army Cost Analysis Program, a great number of materiel/weapon system cost methodologies and models have been developed. These models have covered the range from simple, hand-performed accounting structures through complex, mathematical, computer models covering the life cycle cost stream of materiel and activities. Very few if any of these models are compatible, much less comparable. Cost categories are radically different, and such factors as time-phasing and sensitivity analyses appear at the whim of the analyst. The result is that communication among the various cost analysis efforts and activities is difficult. Coordination among these different groups requires an excessive amount of time spent in the definition of terms and specifications of requirements.

The purpose of this technical report is to attempt to provide a general basis for cost analysis model building. Major subjects covered will be:

(1) The basic model with the treatment of different forms of mathematical variables

(2) A discussion of time-phased versus non-phased costs

(3) Sensitivity analysis

II. THE BASIC COST MODEL

In the development of a standardized cost model or set of equations, it is first necessary to determine the cost categories required. Each category fits into one of several major groupings depending on its logical use in the cost model. These general cost category groupings and their treatment will be discussed in this section. The specific cost categories vary between studies and materiel system types.

Cost Independent of Quantity.

A major cost group is that group of fixed costs which is incurred independent of the quantity of a system procured. Examples are Research and Development (RDTE) and acceptance testing. These costs can be considered as a total figure when allocated over total procurement, as an addition to unit cost. They should be treated as total or program costs since the allocation of a large fixed cost over a small procurement will bury the true investment cost. These costs, or portions of them, are often sunk and must be treated as such. A typical equation for this type of cost category is:

$$C = \frac{RDTE}{N} \qquad (1)$$

This equation indicates that RDTE cost per unit, C, equals the total research and development cost, RDTE, divided by the number of units in the system, N.

Costs Dependent on Quantity

A second major cost group is that which is variably dependent upon the quantity of the systems produced or procured. The most obvious example is cost-quantity relationship which is describable by learning curves, i.e., negative exponential curves.* Approximations of the two relevant hardware cost equations are:

$$\text{Hardware Unit Cost} = \frac{A}{1-B}\left[(P+N)^{-B} - (P)^{-B}\right] \qquad (2)$$

$$\text{Hardware Total Cost} = \frac{A}{1-B}\left[(P+N)^{1-B} - (P)^{1-B}\right] \qquad (3)$$

WHERE: A = Cost of first unit of production

B = Learning curve exponent

N = Number of units

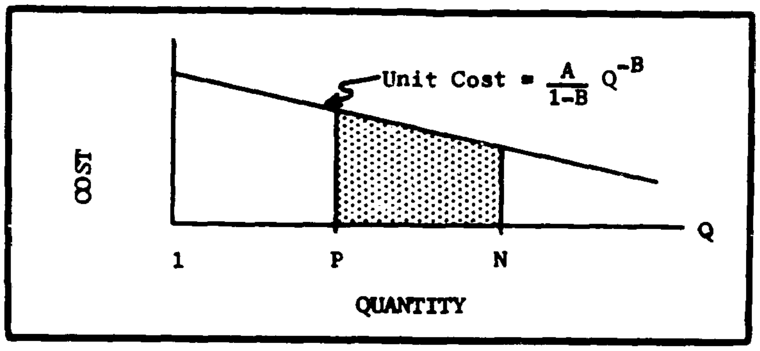
P = Beginning quantity on learning curve

*For an excellent discussion of learning curves see <u>Alpha and Omega and the Experience Curve</u> (Ref. 3)

Figure 1 shows the location on the beginning quantity which is the past production relative to current procurement.

Figure 1

The Learning Curve



The shaded area is Hardware Total Cost determined by equation (3).

The difficult problem of determining quantity exists with this portion of the cost model. The actual quantity required to satisfy an asset objective may be greater or less than the objective depending on whether or not the useful life of the system is equal to the time span of the study. If they are not equal, some form of allocation of hardware cost must be made.

At this point the determination must be made as to whether a steady state or variable inventory will be assumed.

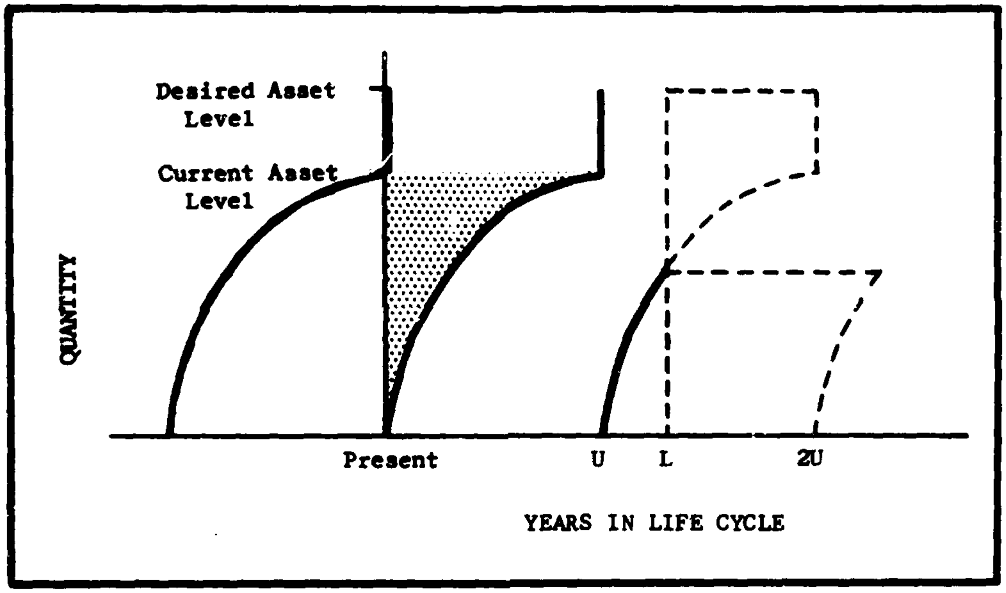
The variable inventory system is treated as a simple accounting inventory problem. The "value" of current assets plus the cost of any replacements and additions during the study time less the "value" of assets remaining at the study time span. Questions as to age and value of current assets, actual useful life of the individual items, replacement rates for attrition, and variable quantity requirements over time have dictated, up to this time, that the variable inventory method be a manual computation method. A fairly sophisticated computer program with the ability to handle inputs for all of these would be required. Since this report is directed toward a simplified set of equations more readily adoptable to the computer, this manual approach will not be considered further.

The steady state inventory system assumes that a fixed inventory level will be maintained throughout the study time span. The questions of inventory age and replacement are of no real concern since the hardware equation is now value used during the study time span equals life cycle costs. Beginning and ending inventories are equal so need not be considered.

Figure 2 illustrates a steady state showing inherited assets (shaded area) and remaining value (area in dotted lines). The study period is that segment between Present and L.

Figure 2

Hardware Allocation in Constant Inventory System with Useful Life Less than Life Cycle



The only adjustment needed is to raise or lower the current asset level to the desired asset level. The steady state model will assume the immediate procurement shown by the heavy line if the desired level is greater than the current level. Immediate procurement is a weak assumption only if that quantity is great relative to the capabilities of the producers. In Figure 2, the study time span (Present to 1) is greater than the useful life (U) of the system under study. The heavy lines represent procurement to replace attrition which occurs at some rate as described by the curves. The inherited assets are replaced by time U at the very latest. The newly procured assets begin to fail at U and are replaced again. The shapes of these attrition replacement curves are unimportant since a steady state exists. The result is that allocated hardware quantity (X) assuming a desired asset level (N) and given a useful life (U) and stduy time span (2) is given by equation 4.

$$X = (N)\left(\frac{L}{U}\right) \tag{4}$$

If U is greater than L, N will be greater than X and vice versa.

Cost Factors Expressed as Percentages

Closely related to the learning curve costs are those costs which can best be represented as a percentage of hardware cost. Repair parts costs are often represented this way.

$$\text{REPAIR PARTS COST} = (R)(\text{Hardware Unit Cost}) \qquad (5)$$

Where: R = Repair parts factor
Hardware Unit Cost computed from equation (2)

Initial provisioning of repair parts can also be expressed as a percentage of hardware cost.

Factors for availability such as discussed on page 9 and illustrated in equation 7, page 10 are also expressable as percentages.

One percentage figure can be used without the usual criticisms found with "blanked" data. The reason is that the constant percentage figure is applied to factors, such as hardware costs, which will be varied for varying systems. That is, a ten percent provisioning factor applied to a cargo truck hardware cost will give a lower provisioning cost than the same ten percent factor applied to the higher cost of a truck wrecker.

Additive Costs

There are other costs which are simply added into the model. An example is Government Furnished Equipment which is generally not sensitive to changes in quantity or any other factor which is relevant to the cost model.

Operating Costs

Operating costs generally have the same types of cost groups as discussed above but are combined differently. The operating cost factors will be needed as a cost per year. Therefore, to obtain a life cycle operating cost, the cost factors must be multiplied by the study life cycle length:

$$\text{Life Cycle Operating Costs} = L\left(C_1 + C_2 + \cdots + C_N\right) \qquad (6)$$

Where: $C_1, C_2, \cdots, C_N$ are the operating cost categories in their simplest mathematical forms

Some of the operating cost factors are also sensitive to availability or usage factors. A vehicle which is available only 87 percent of the time uses only 87 percent of the POL used by a vehicle which is available 100 percent of the time. A factor should be applied to the equation to make corrections such as this.

$$\text{Life Cycle Operating Costs} = L(C_1 + C_2 + F \times C_3) \qquad (7$$

Where: F = Availability percentage

C_1 & C_2 = Operating cost categories insensitive to availability

C_3 = Operating cost category sensitive to availability

The complete study life cycle cost equation will consist of RDTE, investment, and operation and maintenance costs. The equation will be summed after each of the terms has been solved. There are two basic methods of summing -- time-phased and non-phased. They will be discussed in the next section.

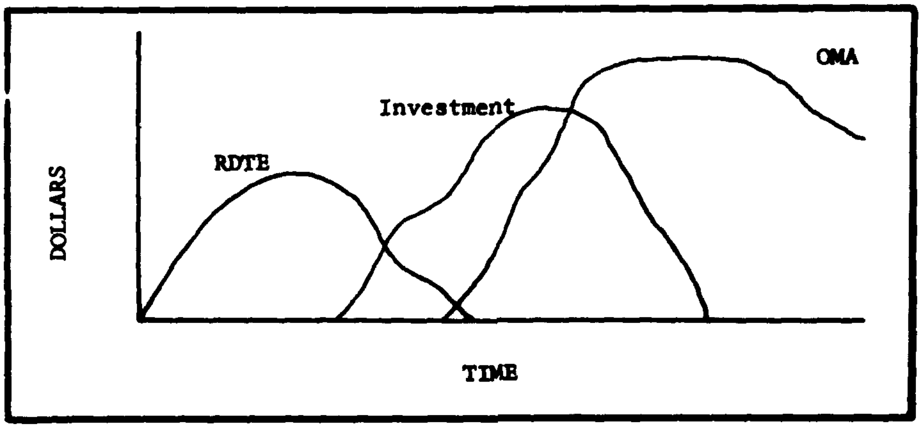
III. TIME-PHASE AND NON-TIME-PHASE IMPLICATIONS OF THE MODEL

After the decision has been made to use a mathematical cost model, a further decision must be made as to whether the model will be time-phased or non-time-phased.

A time-phased model is one which presents costs by year. The advantage of this type is that the impact of varying development and procurement rates can be studied. Figure 3 illustrates a chart which could be constructed from the information provided. It is very difficult to construct this type of chart for a system in its early developmental stages since investment and operating costs will be developed on a

Figure 3

Time-Phased Life Cycle Cost Chart
(hypothetical data)



"paper" system. Similarly, this type of model does not provide much useful information if the system has already been procured. A time-phased model is more difficult to construct and therefore more expensive. The additional information provided is of no value if it is not to be used. The typical equation would be:

$$\text{Life Cycle Cost} = \sum_{i=1}^{L} (\text{RDTE}_i + \text{Investment}_i + \text{OMA}_i) \tag{8}$$

Where: i = Year of estimate

The Learning Curve would be time-phased according to the following:

$$\text{Hardware Total Cost} = \sum_{i=1}^{L} \frac{A}{1-B} \left[(P_{i-1} + N_i)^{1-B} - (P_{i-1})^{1-B} \right] \tag{9}$$

Where: L, A, B, P and N have been defined previously

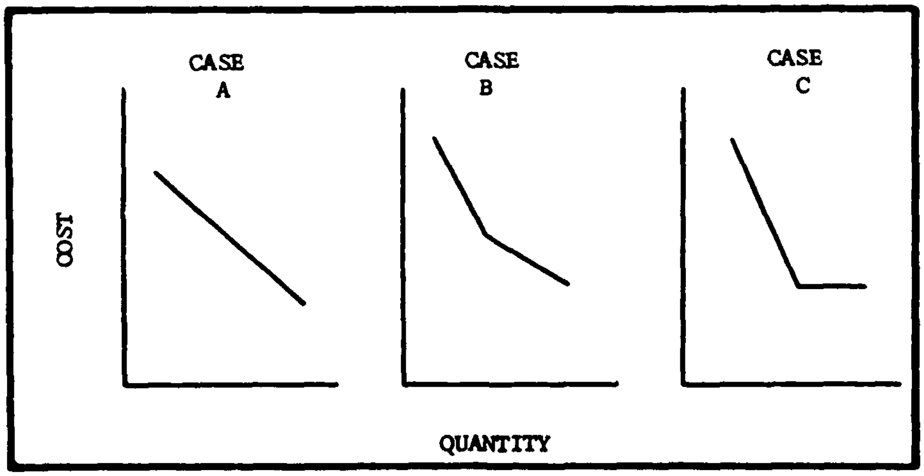
The non-time-phased cost model will provide a cost figure which is the same in total as the time-phased model but which is not broken down by year. This type is easiest to develop, provide input data for, analyze for sensitivity, and manipulate. A typical equation of this type is:

$$\text{Life Cycle Cost} = \text{RDTE} + \text{Investment} + \text{OMA} \qquad \mathbf{(10)}$$

Whether the model is time-phased or not, there may be learning curve considerations in the investment cost equation which make the two types of models similar. The learning curve may not have the same slope over the entire production of the system being studied. Figure 4 shows three cases of the learning curve.

Figure 4

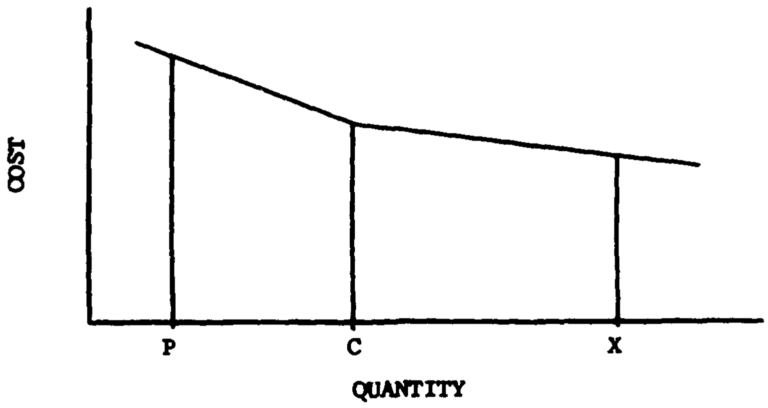
Samples of Learning Curves



Case (a) depicts the usual learning curve with constant slope over the P to N range. Case (b) shows a change of slope between P and N and Case (c) shows that the slope goes to zero in the relevant range. Case (c) is common. The usual statement is, "The slope is 90% over the first 60,000 units of production and then goes to zero." The meaning is that no more learning is likely after a great number of that particular system has been produced. The result in either case (b) or (c) is that an equation must be used which allows for the changes in slopes. Figure 5 illustrates the learning curve and the equation.

Figure 5

Hardware Equation with Varying Slopes



$$\text{Hardware Unit Cost} = \frac{A}{1-B_1}(P + C)^{-B_1} - (P)^{-B_1}$$

$$+ \frac{A}{1-B_2}(C + X)^{-B_2} - (C)^{-B_2} \qquad (11)$$

Point C is the point of change of slope. The equation is summed over the points of change just as the time phased equation was summed over the years. For each change P takes the value of the C for the last change so a general equation is:

$$\text{Hardware Unit Cost} = \sum_{i=1}^{n} \frac{A}{1-B} (C_{i-1} + C_i)^{-B_i} - (C_{i-1})^{-B_i} \qquad \textbf{(12)}$$

Where: n = Number of slope changes

C_i = Quantity at slope i

IV. SENSITIVITY ANALYSIS

One of the most important sections of a cost study is the sensitivity analysis performed on the model. Sensitivity analysis is the study of the effect on the total of a change in magnitude of a part or a particular factor. There are two main reasons why this type of analysis is performed. The first is that it provides an indication of the accuracy of the study since it shows the effect on the total of an assumed error in any part. The second is that it shows which parts of the model have the greatest impact on the total and which therefore require the greatest emphasis during the data gathering phases of the study. These two considerations are of course related but the calculations to satisfy each are made at different times during the study. Sensitivity is not the same as analysis of the propagation of errors in which the errors of the parts are propagated to find the possible error or dispersion in the composite distribution.

An easy method of studying the effects of changes is simply to solve the equation several times, altering each variable one at a time, and noting the effect of each change on the total.

A more effective method is to use partial differential equations to obtain equations representing the sensitivity of total cost to each variable. A simplified total cost equation might be:

$$\text{Total Cost} = \text{RDTE} + \frac{A}{1-B}(N)^{1-B} + N(L)\left[F \times C_1 + C_2 + \frac{C_3}{N}\right] \qquad (13)$$

Where: RDTE = Total RDTE costs

A = First unit cost

B = Learning Curve exponent

N = Procurement Quantity

L = Life cycle length

F = Availability percentage

C_1 = Operating cost dependent on availability

C_2 = Simple operating cost factor

C_3 = Fixed operating cost

The partial derivative of total cost with respect to each of the variables would produce a new set of equations. Each of them will provide a measure of the sensitivity of total cost of one particular variable. For example, the sensitivity of total cost to operating cost C_1 in equation (13) would be represented by equation (14).

$$\frac{\partial \text{ Total Cost}}{\partial C_1} = N(L)(F) \qquad (14)$$

Where: ∂ is the mathematical sign for the partial derivative

A change of one dollar in C_1 would change Total Cost an amount equal to N(L)(F).

A dollar amount as provided by equation (14) is not always the most significant measure of sensitivity. A change of $4 million in Total Cost resulting from a change of one dollar in an operating cost category may seem large but it is not significant in a $500 million program. The obvious solution is to use ratios of sensitivity to total cost. Equation (14) could be rewritten as a ratio this way:

$$\begin{array}{c}\text{\% change in Total Cost for a} \\ \text{one unit change in } C_1\end{array} = \frac{N(L)(F)}{\text{Total Cost}} \qquad (15)$$

Equation (15) would quickly indicate whether a change in a variable would cause a significant percentage change in Total Cost.

Of course a change in C_1 of one dollar is not really of interest. C_1 would probably be accurate only with a range of plus or minus some D dollars. The result of equation (14) or (15) would be multiplied by D to obtain the effect of expected or foreseeable variations in C_1. The fact that equations have been used for sensitivity allows the analyst to vary D at will. He will be able to respond to questions concerning changes in variables quickly and accurately.

V. SUMMARY

This report has described treatments of various types of parameters which will be encountered in model building for life cycle costs of Army materiel/weapon systems. The explanations have been general to avoid the suggestion that they are regulatory. Use of the methodology presented in this report will aid in the standardization of studies and add to their comparability.

The section on time-phased and non-time-phased models describes the advantages and disadvantages of each type with suggestions as to when each type should be used.

Two types of sensitivity analysis were described. Changing values of variables to discover resulting changes in total costs is the less flexible method. The use of partial differential equations to derive sensitivity equations is the preferred method.

The techniques described will provide a common basis for model building and simplify future problems resulting from the present lack of comparability between models.

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13. ABSTRACT

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