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a new 3-DIMENSIONAL display technique

M68-4

A NEW 3-DIMENSIONAL DISPLAY TECHNIQUE

by

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May 1968

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ABSTRACT

A new dynamic volumetric display technique is described in which a vibrating membrane mirror is used to form a three-dimensional image of a time-varying two-dimensional pattern. The technique offers certain simplifications over existing volumetric display systems of the electromechanical type. Among its advantages are reduced costs and increased reliability; two of its many areas of application are air traffic control and signal analysis, including radar signals.

The report describes the operation and characteristics of the basic device, the various forms which it can take, and its possible applications in modern display technology.

ACKNOWLEDGMENTS

Acknowledgments are made to A. Bark for his early and continuing interest and encouragement, and to the MITRE Research Committee, headed by Dr. F.R. Naka, for its financial support; to Dr. S.L. Smith for sustained encouragement and helpful scientific consultation; to A.J. Cantor who prepared the discussion of the mirror surface geometry presented as Appendix V; to B.A. Dawson, R. Murphy and R.S. Theriault for their many hours of technical support; to Mrs. Patricia J. Chatta for her editorial assistance throughout all stages of report preparation; and to L.F. Perry, Jr., who designed the layout for both text and figures.

In addition, many of the author's colleagues and friends, both within and outside of the Corporation, materially aided the progress of the project through helpful discussions and suggestions. Space limitations preclude a listing of individual names but their technical contributions are deeply appreciated.

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TABLES

EXPLANATORY NOTE:
Stereophotographs and 3-D Viewer

Several of the experimental display patterns generated during this research project are presented in the form of stereophotographic pairs (see centerpiece) for stereoscopic viewing. A plastic viewer is furnished with each original copy of the report to assist the reader in visually fusing the image pairs.

If the viewer is not available, the reader, after some practice, may be able to perceive the depth effect without it. In order to achieve this effect, he should hold the photographs steady at normal reading distance, with uniform lighting on the page. His eyes then should be relaxed as though gazing at a distant object, and the images should be allowed to drift past one another until the left-hand picture as seen by the left eye appears to coincide with the right-hand picture as seen by the right eye. With some concentration, these pictures can be made to fuse and thus appear as a single three-dimensional image.

If magnifying lenses with a four- or five-inch focal length are available, one of these may be placed before each eye as a focusing aid.

SECTION I

INTRODUCTION

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THREE-DIMENSIONAL DISPLAYS

Visual perception of depth in the physical world is made possible by a number of independent cues such as stereopsis, parallax, perspective, obscuration, shadowing, and so forth. Perhaps the most dramatic of these, but not always the most important, is stereopsis or binocular parallax. Stereopsis is the phenomenon which causes the eyes to change their convergence as one views objects at different depths in the immediate environment. The subjective experience of these changes plus the subtle awareness of double-images of those objects upon which the eyes are not converging constitute strong visual cues about the relative distances of nearby objects. Motion parallax, often called simply parallax, is another important cue to depth perception and is effective at greater ranges than stereopsis. This phenomenon causes objects to appear in different relative positions as the observer's vantage point changes.

The combination of various visual cues, along with tactile and other sensory cues, make us aware of the three-dimensionality of our environment, a feature which is often taken for granted until one tries to create a graphical reproduction of some real-world object.

There is a long history of man's attempts to achieve graphical representations of real or fictitious objects in which some illusion of depth is present. An ordinary landscape painting in which perspective relationships are preserved is a simple example, conveying a feeling of depth with limited cues. Other presentations make use of other cues; a bas-relief carving, such as a figure on a coin, offers slight but effective stereoptic and parallactic cues.

The various depth cues used throughout art and photography are so diverse, both in nature and in effectiveness, that it is difficult to decide what is a three-dimensional portrayal and what is not. Over the years, however, informal usage has come to imply that a representation is three-dimensional, or 3-D, if at the very least it provides stereoptic cues, and the more so if parallax is present.

VOLUMETRIC DISPLAYS

The science of stereoscopy, the presentation of specially contrived images which exhibit stereopsis, is over a century old. An historical review of stereoscopic techniques, leading up to a survey of modern methods, is contained in Appendix II. For this discussion, however, we are concerned only with direct-vision stereoscopy, a small branch of the science dealing with those methods which do not require viewing glasses or which do not restrict head movements substantially; this is in contrast to the old-fashioned parlor stereoscope and its many modifications and related devices. In particular, the emphasis here is on those direct-vision displays which offer parallax as well as stereopsis. These are the so-called "volumetric" displays because they either actually exist or appear to exist within a well-defined portion of physical space.

Several effective volumetric displays have been developed in the past two decades; some of these are discussed in Reference 1.* Certain of these displays share a working principle in which a periodically time-varying image is generated upon a surface. In one way or another, this image is made to scan a volume of space repetitively, in synchronism with its periodic changes and at a repetition rate that exceeds the critical fusion frequency of the eye. The result is a display in which the elements appear to be floating in space because of stereoptic and parallactic cues.

The fabrication and operation of these devices is not without difficulties. One technique often used is that of generating the two-dimensional (2-D) image upon a physical surface which must be oscillated or rotated at a rate of 30 cycles per second or more. As a result, there are problems of wear and power losses in the bearing surfaces, air drag, and mechanical loading of reciprocating parts during changes of direction. Alternatively, the image surface may remain fixed but may be viewed in a rigid plane mirror which is oscillating perpendicularly to its own plane; here, the same problems apply, for the total excursion of the mirror must equal exactly one-half of the volumetric depth which is to be achieved.

THE VIBRATING MEMBRANE MIRROR

This report considers the use of a vibrating membrane mirror in overcoming the above problems, and some others, to a large degree. The

*See Appendix VI.

flexible mirror is mounted upon a fixed, circular rim and is caused to oscillate between slightly convex and concave configurations, yielding large focal length changes for small excursions at the center of the mirror. The excursions may be provided through the use of a suitably driven loudspeaker behind the mirror. To our knowledge, this method has not previously been used in 3-D display technology. It offers the advantages of simple mechanical construction, durability, and savings in size, weight, power consumption, and bearing surface wear, with a consequent favorable influence upon reliability and cost.

Some of the controversy over the utility of 3-D displays is rooted in the question of cost versus effectiveness. It is frequently argued that a three-dimensional situation can be more economically depicted by use of a flat display which incorporates color- , brightness- , or alphanumeric-coding to represent the third dimension. It is believed that where cost is a critical factor, use of the display technique described herein will permit worthwhile savings over earlier types of volumetric displays. Such economy should tend to overcome what appears to be a major objection to the use of 3-D displays.

The uses for such a display, as for other 3-D displays, are illustrated by the following examples:

- spatial position representations of real objects, such as aircraft for air traffic control purposes, and missiles in order to display their positions or trajectories for control purposes;

- analysis of mathematical functional relationships;

- analysis of electrical, acoustical, electrocardiographic, or other signals;

- psychophysical testing procedures, especially those involving depth perception; and

- the creation of spatial models for educational, artistic, or entertainment purposes.

ORGANIZATION OF REPORT

The principle of the display system's operation is described in Section II of this report. Section III contains a discussion of those experiments which were conducted to achieve an understanding of and to determine applications for vibrating membrane mirrors. Some suggestions about possible further research and development work on this method follow in Section IV.

Appendix I of this report contains a discussion of the physiology and psychology of the depth perception process. As noted earlier, Appendix II contains an historical review of stereoscopic techniques.

The ultimate usefulness of any 3-D display will depend upon the wisdom with which it is selected for a particular application. The technological development of such displays has surpassed present knowledge of how to use them effectively in man/machine relationships. Many words have been written and opinions expressed both for and against the use of 3-D in various applications, but very few objective human factors studies have been performed to help place such opinions on firmer ground. Appendix III presents a discussion of those views which are generally agreed upon, those which are controversial, and some of the studies which have influenced both.

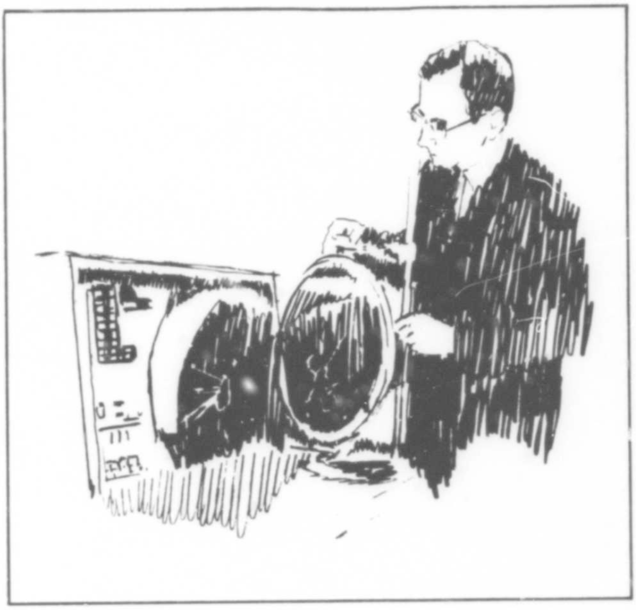
Appendix IV contains a speculative discourse on some of the areas of human endeavor where this type of 3-D display might find some use, even with the currently limited knowledge of the human factors aspects.

Appendix V is a mathematical treatment of the curvature properties of pressure-deflected membrane mirrors, for these directly influence the nature of the images formed by a vibrating membrane mirror.

Finally, references cited in this report are listed in Appendix VI.

SECTION II

PRINCIPLE OF OPERATION



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SUMMARY

The principle of the vibrating membrane mirror 3-D display is that the mirror action causes the image of an object plane to oscillate perpendicularly to itself, thus making it scan a virtual image volume which becomes a "three-dimensional raster". If the object plane consists of a time-varying repetitive 2-D image in synchronism with the mirror, and if the repetition rate is beyond the critical fusion frequency of the eye, a stationary volumetric figure will be seen in the image space.

BASIC MIRROR STRUCTURE

The heart of the display is a sheet of metallized plastic film which is stretched tightly over a circular frame to form a plane mirror, as shown in Figure 1. A film with excellent mechanical and optical properties for this purpose is aluminized Mylar.* This material is available from a number of commercial metallizers in several film thicknesses, two of which are of special interest here: the 0.00025- and the 0.00050-inch thicknesses. When suitably mounted, either film forms a mirror of rather good quality.

If such a mirror is provided with an air-tight cavity behind it, it can be made convex or concave by the application of pressure or suction in the cavity. The mirror surface configuration in these cases is essentially spherical. As the mirror undergoes changes of curvature, an observer

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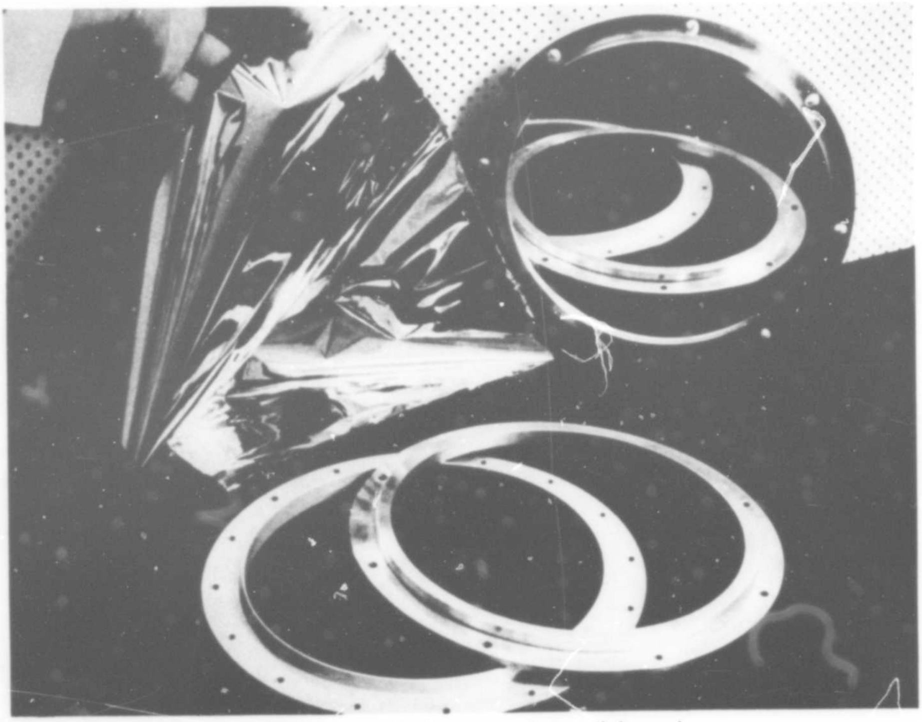
*A trademark of E.I. du Pont de Nemours and Co., Inc.



The aluminized plastic membrane (left rear) when mounted on frame in foreground forms highly effective mirror surface (right rear).

Assembly of a Membrane Mirror

Figure 1

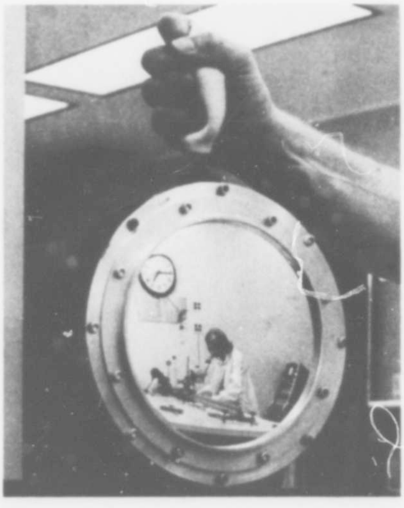
inspecting the images of room objects in it will see them change size quite spectacularly, as illustrated in Figure 2. What is less apparent but more important is that the images similarly advance and recede during the mirror excursions.

When the mirror is flat, the image seen in it has a definite location behind it, the image-to-mirror and object-to-mirror distances being equal. "Image location" simply refers to the position of the image as seen by the

Flexible mirror shows changes in magnification when deflected by air pressure. Image is of normal size when mirror is flat (right). Image becomes smaller and larger (lower left and right) as mirror is made convex and concave.

Deflection of the 7-inch diameter mirror is 1/4-inch at center. As image size is changed, image advances and recedes, making it necessary to refocus the camera.

NORMAL



CONVEX

CONCAVE

Varifocal Properties of a Membrane Mirror
Figure 2

eyes or by any other optical range-measuring device. Thus a telescope or a camera would have to be focused for the distance from the lens to the image in order to ensure that the image formed on the retina or film is sharp. It can be shown experimentally as well as theoretically that the image does have a definite location and that the location changes as the mirror changes curvature.

MIRROR AND IMAGE MOTION RELATIONSHIP

What is especially interesting is that the mirror motion is quite small compared to the depth of excursion achieved in the image space. An example can be given through a simple calculation. Because the mirror surface is essentially spherical under such a pressure deformation, the object and image distances are quite closely related through the well known "thin lens" formula:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{r}, \tag{1}$$

where

p = object distance,

q = image distance,

f = focal length of mirror, and

r = radius of curvature of mirror.

The radius of curvature of the mirror, in turn, is related to the mirror diameter and amplitude of excursion through the so-called "sagitta" formula:

$$r = \frac{(d^2 + 4h^2)}{8h} , \qquad (2)$$

where

d - mirror diameter, and

h amplitude of excursion (taken as positive in the concave direction).

The $4h^2$ term may often be neglected because it is typically very much smaller than d^2. Therefore, the expression

$$\frac{1}{p} + \frac{1}{q} = 16h/d^2 \qquad (3)$$

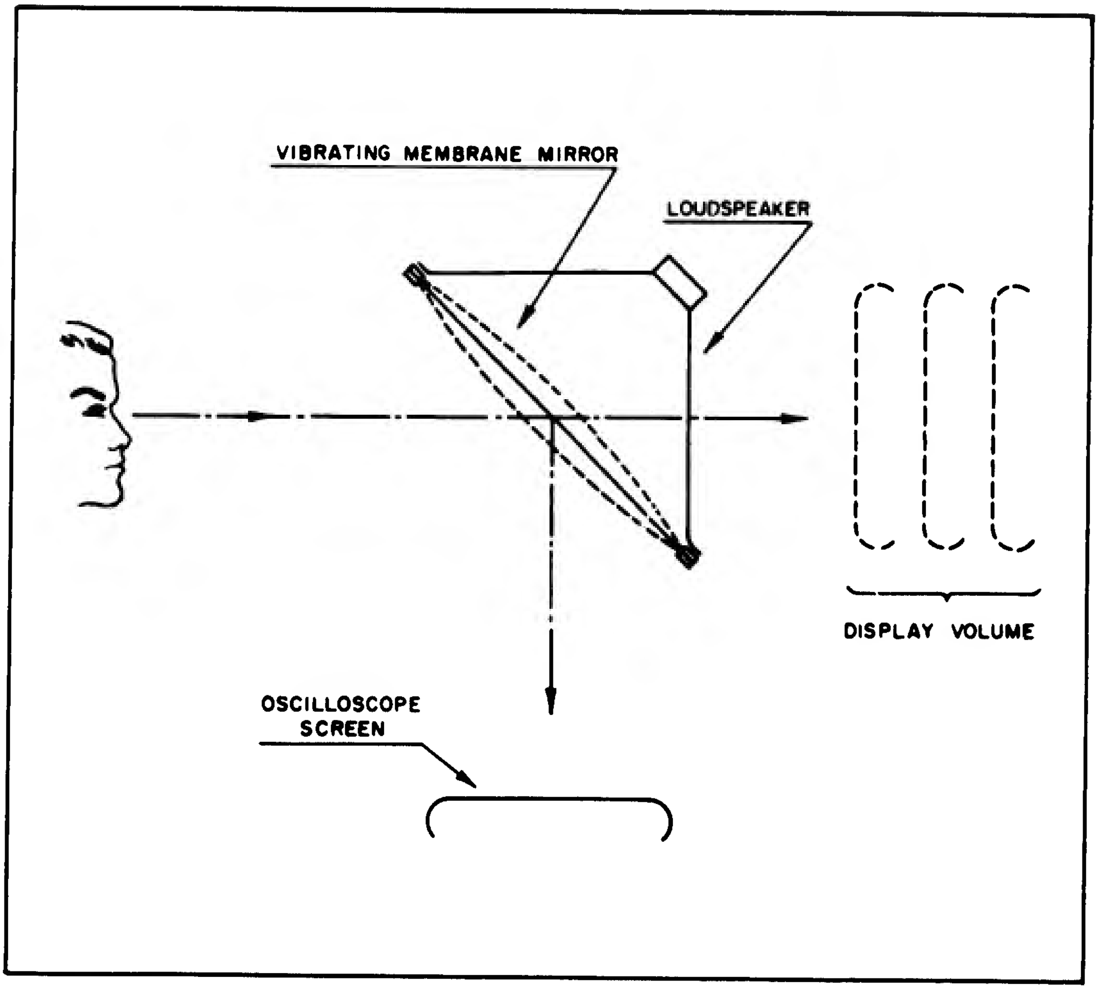
can be used to relate the object and image distances in terms of the mirror diameter and amplitude of excursion.

Assume the use of an 8-inch diameter mirror with the object 1 foot in front of it. When the mirror is planar, the image distance also will be equal to 1 foot. Let the mirror now be deflected inward, spherically, by a typical value of 1/10 inch at the center. Equation (3) will show that the new location of the image is about 17 inches behind the mirror. Conversely, if the mirror is deflected by the same amount in the opposite direction, an equal but negative value is inserted for h in Equation (3) and the image is found to be within about 9 inches of the mirror. Therefore, a 1/10-inch mirror excursion in each direction results, in this case, in a total image excursion of 8 inches.

From an engineering standpoint, the large amount of image motion gained through a small excursion of a lightweight sheet material is the single but compelling attraction of using a flexible mirror for this purpose. By contrast with the rigid mirror or projection screen that must undergo translation or rotation, the membrane mirror, by virtue of its small deflections and low inertia, serves to ease the problems of windage, driving power requirements, mechanical stresses, and bearing-surface wear.

3-D IMAGE GENERATION

Imagine that the object being imaged in the mirror is a blank surface, such as a cathode ray oscilloscope (CRO) phosphor screen. Assume that means are provided for causing the mirror to undergo its convex-concave excursions in a sinusoidal manner over some frequency range. This can be done through the use of a suitably driven loudspeaker which is placed behind it, as shown in Figure 3. The central dashed figure in the display volume represents the image of the CRO screen when the mirror is in the flat reference configuration denoted by the solid line. In this case, the screen and its image are equidistant from the mirror. The other image positions in the display volume correspond to the deflected mirror positions, shown dashed. The screen image moves closer to the mirror when the mirror is convex and recedes when the mirror is concave.



Principle of the 3-D Display Using a Vibrating Membrane Mirror
Figure 3

If the speed of mirror oscillation is increased from some value where the image excursions can be followed visually up to a frequency of about 20 cycles per second, the response speed of the human eye will no longer allow it to follow the image motion, and the instantaneous positions will become blurred or fused. At this point, the critical fusion frequency of the eye has been reached; assuming that the mirror properties remain unchanged, further changes in image motion caused by additional increases in frequency will not be discernible by the eye.

Suppose now that a steady spot of light is displayed at the center of the CRO screen. In the display volume, the spot will appear elongated in the direction of the image motion axis; that is, the rapid excursions of the spot image will make it appear as a line suspended in space, roughly parallel to the observer's viewing axis. The line will exhibit the visual properties of a luminous line in three-dimensional space, with its near and far points clearly identifiable through stereopsis and through changes in its aspect angle, due to parallax, as the observer changes his viewing position.

If the spot of light on the screen is caused to oscillate sinusoidally so as to trace out a straight line in the screen plane, which we shall call the X-Y plane, and if these oscillations are in phase with the mirror motion, then the straight line in the image space will be tilted out of the viewing axis.

At this point, it should become apparent that the mirror serves to introduce a depth-axis, or Z-axis, component to the motion of the object in the X-Y plane. The rules regarding the composition of periodic motions at right angles are applicable. Thus it is easy to demonstrate that the straight line can be made to appear elliptical or circular if an appropriate, fixed phase difference is introduced between the mirror motion and the object motion. Some simple 3-D ellipses formed in this way are shown as Stereophotographic Pair E (see centerpiece). These ellipses originated as a flat CRO pattern of four straight lines (also visible in the Stereophotograph).

Moreover, if the phase difference is made to vary continuously over time, the volumetric figure will appear to rotate in space.

Going one step further, if a low-valued integral relationship is introduced among the X-, Y-, and Z-axis frequencies, the ellipses can be made to fold back upon themselves and to form the interesting scalloped patterns known as Lissajous figures. Indeed, equipped with a CRO, a loudspeaker-driven membrane mirror and a few sine wave generators, one can spend a pleasant afternoon in the production of inspired Lissajous creations tumbling through three-dimensional space; the enjoyment is all the more complete if the CRO is a dual-trace type so that separate, counter-rotating figures can be superposed.

SOME OPTICAL PROPERTIES

It should be apparent that the vibrating membrane mirror can be used for applications other than the composition of simple periodic motions along orthogonal axes. The scanning action by which it causes an image plane to sweep out an image volume produces a "3-D blackboard". With this device, it is possible to write patterns whose complexity is limited only by the effort expended in generating the initial 2-D time-varying pattern. The section on Pattern Generators (pages 67 ff.) contains discussions of more useful and intricate display figures and the means of generating them; the discussion here is limited to some of the optical and visual properties of the basic 3-D blackboard without regard to any particular figure.

Virtual Versus Real Images

It should be pointed out that, at the mirror amplitudes ordinarily used in the experiments, the focal point of the mirror goes through an excursion that starts within a few feet of one side of the mirror, regresses to infinity, and then returns to within a few feet of the other side of the mirror. In the displays described herein, the object surface is always located within the focus of the mirror, that is, nearer to the mirror than the point of closest approach of the front focal point. In accordance with optical principles, this results in the formation of a volumetric image behind the mirror, just as in the case of a rigid plane mirror. As such, it is described optically as a "virtual" image, meaning that the light rays which cause it to be visible only appear to emanate directly from it. Actually, these rays proceed from the mirror surface itself, having reached it from the object and then having been rerouted toward the observer.

Thus, a virtual image is comprised of fictitious points from which light rays seem to be emanating. As such, it is an illusion, whose significant property is its physical inaccessibility — it cannot be touched.

By contrast, there is a class of images known as "real" images whose points are formed by the actual intersections of converging rays as they proceed from a lens or mirror. An image formed by a slide projector lens upon a projection screen is a familiar example of a real image. In this case, the slide in the projector is beyond the rear focal point of the lens. A less familiar example of a real image is sometimes used in physics demonstrations for its entertainment value. This is an "aerial" image, so called because it is formed in the air in front of a concave, or converging, mirror. The edges of the mirror are concealed by an opaque screen. A small object, often a figurine, a flower, or a coin, is located beyond the focal point of the mirror and hidden from the observer. The image is made to appear just outside an opening in the screen. Because the observer is not conscious of the mirror and because the image does not appear to be produced by any optical element, the illusion of a real object suspended in space is created. Indeed, it is so lifelike and accessible that one is tempted to grasp it, with a somewhat dismaying result. The stereophotograph labeled Pair A (see centerpiece), where a small globe appears to be cradled in its stand, depicts just such an effect. Although the stand is real, the globe is an aerial image produced by the mirror behind the aperture.

Of the many 3-D display devices developed or proposed, some offer a display format comprised of a real image. A few of these offer, in principle at least, a particular advantage: if there are no interfering mechanical parts in the display volume, the image may be "touched". More specifically, the image is accessible to the superposition of spatial coordinate grids, pointers, cursors, and other objects which would permit an operator to measure, analyze, or otherwise interact directly with the display.

In its simplest form, the vibrating membrane mirror 3-D display is not suited for direct interaction. This may be regarded as an inadequacy, although it is not an irrevocable one. Later in this report it is shown how, through the use of a simple optical relay, the virtual image can be brought out from behind the mirror and presented as a real aerial image (see Virtual-To-Real-Image Conversion, page 105). However, by use of other special optical techniques, the operator can interact with the virtual image in its behind-the-mirror location. This alternative is discussed under Superposition of Images, page 109.

Field of View

Any virtual image has the property that it appears behind some optical element whose edges act as a window frame or "limiting aperture"; an observer must position himself so that the image is not cut off by the edges of the aperture. When the image is located close to the aperture, a broad viewing angle is made available. In this case, the observer has considerable freedom of head movement or, stated differently, many observers can view the display simultaneously. If the image is made to recede from the aperture, the angle becomes smaller.

If Equation (3) is rewritten as

$$q = \frac{d^2 p}{16\,hp - d^2}\,, \tag{4}$$

it is seen that the image distance, q, varies almost directly with the object distance, p. (In practical cases, the role of p in the denominator is suppressed by the small value of h, and the d^2 term predominates.) It is also true, perhaps unfortunately, that for a fixed mirror diameter and excursion, the depth of the image volume increases approximately as the square of p.

One is therefore tempted to move the object, and hence the image, away from the mirror in order to realize the benefit of a "free" gain in image volume depth. However, this serves to place the image volume deeply behind the mirror where the viewing angle becomes restricted. Ordinarily, this would not handicap a display intended for use by a single operator; however, the gain in depth tends to be offset by the fact that the observer's stereoscopic perception decreases with increasing image distances because his interocular base line subtends correspondingly smaller angles. On the other hand, if a larger mirror is used in order to increase the viewing angle, the observer has greater freedom of head movement and can use parallactic cues as an aid to depth discrimination. In any event, an increase in the depth of the image volume can also be achieved if one drives the mirror at greater amplitudes. In the practical case, amplitude can be increased until acoustical noise becomes an objectionable factor.

It is rather difficult to give a simple formula for the most effective compromise between mirror diameter, amplitude of excursion, and object distance. Such a formula would have to take into account other factors pertaining to the display format and the viewing conditions. These factors would include the geometrical extent of the display in the X-Y plane, the density of elements in the display volume, the requirements upon depth discrimination, the number of simultaneous observers, and engineering considerations such as display system size, cost, and allowable sound levels.

The choice of display volume depth and viewing angle must therefore be left to the individual display system designer. However, when these parameters are finally established for a system, there still will be some ultimate limitation on the angle over which the display may be observed. The angle may be as small as 10 or 20 degrees or as great as 90 degrees or more. The limitation may not be important in most applications; in fact, it is a feature common to many other 3-D displays and viewing devices. It is cited here to acknowledge the fact that 3-D display systems do exist which provide complete hemispherical viewing. (This is often obtained at the sacrifice of other features such as simplicity, uniformity of display brightness, or speed of updating.) These remarks simply serve to place the membrane mirror display in its proper perspective relationship to other 3-D displays.

It may appear to the reader that the viewing angle limitation can be readily overcome if one brings the virtual image out in front of the mirror as an aerial image, a possibility mentioned earlier. It turns out, however, that the aperture restriction will nonetheless apply because an aerial image can be seen only if it is between the observer and the aperture through which it is created. As the observer changes his vantage point, the relative shifting of the image will finally cause it to be obscured by the edge of the aperture. This creates the anomaly of an object apparently being obstructed by something behind it.

The use of a larger mirror to increase the viewing angle is also not without its practical limitations which are imposed by the sizes of readily available loudspeakers. Some thought was given to the possibility of driving a rather large mirror with a speaker of modest size. Such a system might take the form of a Helmholtz resonator or "bass reflex" speaker, although no experimentation along this line was undertaken. Instead, a study was made of the practicality of an electrostatically driven membrane mirror, possibly pointing the way to diameters of several feet but with thicknesses of only a few inches; this effort is discussed in Section III under Electrostatic Drives (see page 120).

Anomalous Perspective

Another property of this display, which we call "anomalous perspective", also appears as somewhat of a limitation. It can be readily compensated for, however, and should be regarded as an inconvenience rather than as a serious flaw. The phenomenon of anomalous perspective is discussed in the following paragraphs.

It was stated earlier that, as the mirror is deformed inward and outward, the image not only grows and shrinks but recedes and advances as well. What was not stressed is that the "growing" is associated with the "receding", whereas the "shrinking" corresponds to the "advancing". This phenomenon is contrary to our everyday visual experience and has a direct impact upon the nature of the display volume.

In any image-forming optical system, it is inevitably true that as the image is made to recede from the system it becomes magnified. This is a matter of common experience; the reader may recall that when he uses a hand magnifier to view the enlarged, virtual image of a small object, the image size increases (for a while) as the lens is withdrawn from the object. Similarly, a larger image can be cast onto a projection screen as the screen is moved away from the projector.

By the same token, during a single excursion of the membrane mirror it can be observed that as the virtual image recedes from the mirror its linear magnification increases. The magnification is numerically equal to unity when the mirror is flat, at which time the object and image are equidistant from it. The magnification is less than unity when the mirror is convex and greater than unity when the mirror is concave. The value of the linear magnification is given, very simply, by the ratio of image distance to object distance, measured from the center of the mirror.

Suppose now that the vibrating membrane mirror is being used for viewing two steady, fixed spots of light on a CRO screen. It might be hoped that the images of these spots would appear as a pair of parallel lines in the display volume. What actually happens, however, is that the lateral separation of the spot images is greater at the far end of the display volume than at the near end, and the lines therefore diverge. Similarly, if one were to generate a sequence of uniformly sized alphanumeric characters or other recognizable figures upon a flat display surface for viewing in the mirror, a progressive increase in image size would be observed with increasing depth in the display volume.

As pointed out above, we denote this phenomenon as anomalous perspective to indicate its conflict with ordinary perspective relationships when real-world objects are viewed. It appears to be important only when the display volume contains material with size-reference cues which are meaningful to the observer. With simple closed geometric curves, including Lissajous figures, the distortion is less apparent.

It will be shown later (pages 45 to 47) that anomalous perspective can be eliminated completely from a given display if the sequence of images presented to the mirror during each cycle is pre-processed with respect to size; a proper demagnification schedule will compensate exactly for the change in mirror magnification. As a simple example, the spots of light on the CRO screen can be deflected so as to vary the distance between them in synchronism with the mirror motion; by this means, the diverging line images mentioned previously can easily be made parallel. Fortunately, the required demagnification schedule is a linear function of mirror position (or a sinusoidal function of time). Therefore, once the end points are chosen, it is not difficult to implement the correct schedule.

During the course of this work, no simple passive optical method has been found which will automatically eliminate anomalous perspective without the need for image pre-processing. One might hope that a second membrane mirror, vibrating synchronously but out of phase with the first, could be used in some manner as an antidote. However, in a single, short-lived experiment which was performed, it was quickly established that this method is a perfect antidote for the depth effect as well. No further work was done on automatic corrective devices and this is, perhaps, a fruitful area for future study.

CONCLUSIONS

To summarize Section II, the vibrating membrane mirror creates a 3-D raster by causing the image of a fixed plane to scan a volume of image space. Although other 3-D display systems operate in a similar fashion, attainment of this scanning action by means of small oscillations in a low-inertia optical element is attractive from an engineering viewpoint. As is true of most other 3-D display systems, the complexity of the display patterns achievable is limited only by the versatility of the 2-D pattern generator upon which the time-sequenced flat images are formed. Although viewing angle limitations, anomalous perspective, and lack of display pattern accessibility are inherent features of the basic membrane mirror system, it is shown later how these problems can be eased.

SECTION III

THE EXPERIMENTAL PROGRAM

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OBJECTIVES OF PROGRAM

The possibility of achieving a 3-D display by means of rapid, cyclical changes of curvature in a flexible mirror was speculated upon by the author some years ago. A brief exploratory study conducted at MITRE in 1963 culminated in a laboratory model which demonstrated the principle of operation. Shortly afterward, an internally sponsored research program was initiated to achieve a fuller understanding of:

- the optical and vibrational properties of membrane mirrors;
- two-dimensional pattern generation methods which would permit convenient, versatile programming of the volumetric display;
- the behavior of the basic membrane mirror display when used in combination with other optical elements in order to expand its properties; and
- the applicabilities of various forms of the display system to particular uses.

In essence, the several tasks which made up this program can be delineated as follows:

- development of frame design and membrane-mounting techniques to ensure smooth mirrors;
- study of the focal properties of membrane mirrors as they apply to object-image relationships;
- mathematical analysis of mirror surface configuration under pressure deformation;

- studies of frequency response and waveform-following characteristics of speaker-driven membrane mirrors;

- study of the cathode ray oscilloscope as a 2-D pattern generator;

- design, construction, and study of a stroboscopic optical projection system as a pattern generator, with follow-on design study;

- implementation of computer-driven displays for pattern generation;

- study of volumetric, numerical matrix generation using a bank of gas-filled, cold cathode numerical indicator tubes;

- feasibility study of electrostatically driven mirrors for "large-screen, on-the-wall" displays;

- miscellaneous studies of auxiliary optical elements which could be used to modify the properties of the basic 3-D display; and

- survey of possible applications, with special emphasis on air traffic control, psychometric testing, mathematical function analysis, and electrical signal analysis.

The results of the program are described in the remainder of Section III.

THE VIBRATING MEMBRANE MIRROR

An important initial idea in the 3-D display concept is that a metallized plastic film may be mounted on a frame in such a way as to form a mirror. If suitably mounted, the mirror will be of reasonably good optical quality, as judged by the eye, and will have the interesting property of being easily deformable. If the deformations are produced by an air pressure differential, the mirror is readily made convex or concave over a broad, continuous range of focal lengths. Moreover, because of their inherent elasticity, many types of plastic membrane can be restored quickly to their flat configurations when the pressure differential is relieved. Other features of a suitable membrane material are low inertia and the ability to withstand the rather high tensile forces provided when it is stretched tightly over a frame. These features allow the membrane to undergo small deflections repetitively at many cycles per second with the application of only a small amount of power.

In order to implement a vibrating membrane mirror system for 3-D display use, one has only to decide upon the film material, the mirror size and shape, the frame design, and the driving method. Good results have been experienced during this study through the use of aluminized Mylar stretched tightly over 6- , 8- , and 15-inch diameter circular frames, with permanent magnet loudspeakers as the drivers. Apart from a feasibility study of an electrostatic drive method for large screen use, there has been no reason in our experience to depart from this basic construction.

Details of the construction and some of the properties of the speaker-mirror assembly are given in the three following subsections.

Assembly of a Loudspeaker-Driven Mirror

Of the wide selection of available plastic film materials, Mylar appeared to be a logical choice at the outset of these experiments, and there has not yet been a need to consider other materials. Mylar, a trademark of

E.I. du Pont de Nemours & Co., Inc., is a polyester film which is tough, flexible, homogeneous, and stable. Its smooth surface provides a highly specular, or non-scattering, mirror when metallized and stretched. Moreover, it can withstand considerable stretching because of its tensile strength (25,000 psi). Age and broad variations in temperature and humidity do not adversely affect its desirable properties.

The two-piece aluminum frame shown in Figure 1 (page 12) uses the embroidery hoop principle in which the tapered cylindrical mating surfaces grip the membrane tightly and supply tensile forces as the parts are being bolted together. The taper angle is about 15 degrees and apparently is not critical. The spacing between the tapered surfaces is just sufficient for the surfaces to grasp the membrane firmly when the frame flanges meet; tightening of the bolts ensures the desired stretching. This frame design concept was used without modification throughout this program. The implication is either that the design is not critical or that the original one was the result of a fortunate guess.

During assembly of the mirror, the rear section of the frame (identifiable by its raised tapered surface) is placed face upward on a flat surface and an oversized piece of aluminized Mylar is placed over it. In most cases, the Mylar thickness is not critical and may be in the 0.00025- to 0.00050-inch range. However, if its surfaces differ in their light-scattering properties, the clearest surface is placed face upward.

The corners of the Mylar are taped to the working surface so that the membrane is fairly smooth, and the front frame section is laid over it. The frame sections are provided with indexing marks to permit alignment of the bolt holes because the lower set of holes will not be visible if the film overhangs them. The frame sections are then pressed together, the film material in the bolt holes is punched out, and the bolts are applied.

This operation leaves the membrane essentially wrinkle-free. However, small ripples resulting from the manufacturing process may be observed; these are not removed by the tension provided. It has been found that a heat treatment, either in an oven or with a "sunlamp", is effective in removing the ripples. One simply heats the membrane, slowly and uniformly, until a snap is heard. At this point, the membrane apparently undergoes an abrupt softening and shrinking, with a consequent increase in tension and a noticeable improvement in mirror quality. Heating beyond this point may cause the membrane to melt and rupture.

The assembled mirror then is affixed to a suitable loudspeaker mounted on a support and is ready for use. The mechanical connection need not be airtight; the speaker cone vibrations are readily coupled to the mirror, even through an air gap. Loudspeaker quality requirements are by no means severe because the speaker will be operated in the low-frequency sine wave region where even the most economical speakers exhibit a useful response. One exception to this applies to speakers less than 8 inches in diameter that are used in the 30-cycle per second region. Their limited low-frequency response and power-handling capability may not provide the desired amplitude of mirror excursion.

Care must be taken to avoid fingerprints on the mirror surface because their removal is difficult. The mirror must be swabbed gently in a warm detergent solution, rinsed in clear water, and dried with an air jet. Even with care, such treatment often leaves the mirror slightly hazy, and it is usually preferable to replace the entire membrane.

Optical Properties

The following discussion extends the earlier treatment of optical properties presented in Section II in which some fundamental principles were enumerated. In contrast to that treatment, this subsection discusses some detailed experimental considerations.

Mirror Surface Geometry

It was stated earlier that during static pressure displacements the mirror surface is spherical. However, whether the mirror is stationary or vibrating, its shape is a speculative matter. The predicted shape depends upon the assumptions made in the theoretical analysis and upon how well these assumptions are met both by the physical properties of the membrane and by the manner in which it is mounted and driven. In general, the theoretical shape of a vibrating circular membrane is not expressible as a simple curved surface, although at low frequencies and amplitudes the shape is often claimed to be very nearly paraboloidal. At higher frequencies, approaching and exceeding resonance, Bessel functions and harmonic functions must be used for a more accurate description of the mirror geometry. This is especially true at frequencies where multiple-mode oscillation, or "breakup", occurs.

Appendix V presents a theoretical discussion of the mirror surface geometry at rest and in motion, and a discussion of the breakup phenomenon as observed experimentally begins on page 63.

The subject of vibrations in a circular membrane is an interesting one from a physical standpoint and has been treated mathematically by a number of authors, as in References 2 and 3. This subject is important to an understanding of a wide variety of commonplace acoustical and other vibrational phenomena. Its applications range from the design of the speaker and microphone in a telephone handset to an understanding of the strains in a pressurized aircraft window under vibration conditions. From the standpoint of the vibrating membrane mirror, the theoretical treatment of the circular membrane is helpful in allowing one to understand the curvature properties of the mirror, for these in turn dictate the nature of the image which will be seen by the observer.

Two points must be brought out, however, about the relationship between theory and experiment. One is that the mathematical models are useful in describing reality under a specific set of simplifying physical assumptions; and because the laboratory conditions are often more complex than the assumed ones, the mathematical description of an actual phenomenon frequently must be taken as a general guide rather than as a statement of fact. For example, most analyses of vibrations in a circular membrane assume that the membrane is perfectly uniform in surface density and in tension, and that these properties remain constant throughout each excursion. They assume also that the membrane is perfectly elastic, that there are no damping forces, and that the amplitudes of vibration are vanishingly small. In practice, the experimental conditions most often are measurably different from the ideal ones, and the actual result therefore will depart, to some extent, from the predicted one.

The second point to be made is that the curvatures used in the vibrating membrane mirror are so shallow that, at frequencies below breakup, there is very little difference between the predicted paraboloid and a corresponding spherical surface which will fit it reasonably well. Using such a sphere as an example, the maximum deviation in corresponding surface positions between the two is given by the following equation (Reference 4):

$$T_{max} = d^4/512\,r^3 \quad , \tag{5}$$

where

T_{max} = maximum displacement between paraboloid and corresponding spherical surface,

d = diameter of either one, and

r = radius of curvature of spherical surface.

As an example, consider an 8-inch diameter membrane mirror as it might be typically used in forming a 3-D display. An excursion amplitude of 1/4 inch is far beyond the value ordinarily used in our experimental displays. However, if such a value is assumed, the corresponding radius of curvature is about 32 inches, according to the sagitta formula, Equation (2). Through use of Equation (5), we find T_{max} to be less than 0.00025 inch in this case.

In devices for precision-imaging of remote objects, e.g., astronomical telescopes, such deviations would seriously degrade an image. However, when an optical element is used with nearby objects in order to form virtual images at low magnification ratios, slight deviations in the surface configuration will have very little noticeable effect upon the image. Indeed, all optical elements become effectively quite similar at short object distances, whether the elements are spherical, paraboloidal, ellipsoidal, or otherwise, and whether they are weakly or strongly positive or negative.

Thus, for practical engineering purposes, the vibrating membrane mirror as ordinarily used in 3-D displays may be thought of as being spherical, with slight departures from sphericity when the mounting procedure does not ensure perfectly uniform tension. The mirror therefore forms essentially flat, distortion-free images of flat, nearby surfaces, and the object and image distance relationship is best described by the "thin lens" formula already given (see page 14):

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{r} , \qquad (1)$$

where

p = object distance,

q = image distance,

f = focal length of mirror, and

r = radius of curvature of mirror.

Object-Image Relationships

When the mirror is flat, it corresponds to a sphere of infinite radius. The focal length is then infinite, and the object and image distances are numerically equal. According to the sign convention in the above equation, q will then have a negative value because p is taken as positive for real objects. The significance of a negative q is that the image is virtual, erect and located behind the mirror.

When the mirror is curved, it is equivalent to a lens placed in contact with a plane mirror, the lens being convex when the mirror is concave, and conversely. The sign convention is such that a positive value is used for f when the mirror is concave, giving rise to enlarged, more remote images. As the mirror is made convex, causing the image to diminish and to approach the mirror, f and r are taken as negative.

Under these conditions, q always will be negative if f is negative or if p is less than f. Stated differently, the image will be behind the mirror

- whenever the mirror is convex, or
- whenever the mirror is concave and the object is between it and the focal point.

The linear magnification, M, of the mirror is simply the ratio of image to object distances, q/p, and its value is always negative for virtual images. In such a case, its absolute value is less than or greater than unity depending upon whether the mirror is convex or concave.

Definition of Anomalous Perspective

The increasing magnification with increasing image distance gives rise to the anomalous perspective phenomenon mentioned earlier, in which image elements in the display volume increase in size as they recede. It has been shown (see page 22) that the image distance, q, depends upon the object distance, p, the mirror diameter, d, and the excursion amplitude, h, as follows:

$$q = \frac{d^2 p}{16\,hp - d^2}, \tag{4}$$

Dividing by p, we obtain the expression for the linear magnification:

$$M = q/p = d^2/(16\,hp - d^2), \tag{6}$$

whose value is (-1) for $h = 0$, and whose absolute value increases as the mirror moves inward, with h increasing positively.

Methods for the correction of anomalous perspective are discussed in various sections of this report. For the present, it will be interesting to develop an analytical definition of this quantity so that the appropriate amount of correction for a given situation may be prescribed. What is needed first is a ratio of magnifications at the far and near boundaries of the image volume, a quantity which we shall call Q and which, for a given mirror diameter, will depend only upon the object distance and the mirror amplitude.

The magnification ratio, Q (which might also be called an anomalous perspective index), can be expressed simply as the ratio of maximum to minimum image distances when the mirror is vibrating symmetrically about the plane configuration. In this case, the extreme values of h are numerically equal but opposite in sign. The maximum image distance is therefore

$$q_1 = d^2 p / (16\, hp - d^2) \quad ,$$

whereas the minimum image distance is

$$q_2 = d^2 p / (-16\, hp - d^2) \quad ,$$

from which we have

$$Q = q_1/q_2 = (d^2 + 16\,hp)/(d^2 - 16\,hp) \quad , \tag{7}$$

where h is taken as positive. One has thus only to measure the values of d, h, and p in a given situation in order to arrive at a value for Q.

Measurement of Anomalous Perspective

In practice, it may not be easy to measure h directly while the mirror is in motion. An approximate method is used here in which the end of a millimeter scale is placed barely in contact with the center of the vibrating mirror, the mirror vibrations are halted, and the distance between the end of the scale and its mirror image is estimated. This distance is twice the amplitude of excursion, for symmetrical excursions, and the estimate can be made to within perhaps 10 percent.

An alternative method of determining Q can be used in which the radius of mirror curvature is measured directly. However, this method is quite sensitive to slight deviations from sphericity caused by possible membrane non-uniformities or by uneven tension. With the mirror in vibration, a small light source is placed just before the eye on the mirror axis, and the observer moves away from the mirror. An elongated image of the source will be seen in the mirror and, as the observer recedes, one end of the image will grow until it fills the mirror completely. At this point, the eye and the source are quite close to the center of curvature of the mirror at the moment of extreme concavity, leading directly to a value for r, the minimal positive radius of curvature. At distances greater than this, the mirror will remain filled with light as the radius passes through greater values (unless the light source is flashed stroboscopically). In practice, the sphericity requirement is not always met and the mirror illumination is quite non-uniform at the large object distances used, so that the center of curvature is not well defined.

The measured value of r is halved, yielding the minimal positive focal length, f, which is then used in the following formula (obtained through manipulation of the thin lens equation):

$$Q = \frac{f + p}{f - p} \quad . \tag{8}$$

Typical values in a laboratory situation might be

$$d = 7 \text{ inches, and}$$
$$h = 0.030 \text{ inch,}$$

leading to a value of approximately 200 inches for r, or 100 inches for f, from the sagitta formula, Equation (2). Using a typical object distance of p = 12 inches, one finds by either Equation (7) or Equation (8) that

$$Q = 1.27 \; .$$

Correction of Anomalous Perspective

The value of Q must be known if anomalous perspective is to be corrected by appropriate size-processing of the 2-D pattern sequence. If the correct processing schedule is chosen, with its end points exactly in the ratio of Q, anomalous perspective will be eliminated because all image planes will be of equal size if the sequence of original, unprocessed object planes were uniformly sized. Natural perspective will then prevail and the image planes will be seen in the perspective relationship appropriate to the actual viewing distance. Introduction of an enhanced perspective into the display volume, if necessary, can be achieved by overcorrecting for Q so that the more distant planes are actually (besides "apparently") smaller. This might be done in cases where a small display volume is used to represent some large-scale, physical world, spatial situation such as aircraft or satellite positions with respect to the earth's surface or to planetary bodies. The enhancement of perspective will add to the observer's impression that he is viewing a small model of a large volume of space.

In the expressions for Q, if the values of any two parameters are fixed, the value of the third is determined. For a given Q and f in Equation (8), for example, only one value of p will satisfy the equation. However, if only one of the values such as Q is chosen, there is a family of combinations of the other two variables, f and p, corresponding to it. This can be seen by rearranging Equation (8):

$$f = \frac{(Q + 1)}{(Q - 1)} \, p = (\text{constant})\,(p) \; .$$

If the nature of a particular display system is such that for mechanical or other reasons the size-processing schedule is not easily changed, the same schedule nonetheless may be used for other values of f and p, as related through the preceding formula, without a sacrifice of perspective relationships.

The depth of the display volume, Δq, is simply the magnitude of $\left(q_1 - q_2\right)$. In terms of d, h, and p, this is expressed as

$$\Delta q = \left| \frac{32hd^2 p^2}{(16hp)^2 - d^4} \right| . \tag{9}$$

Insertion of the values used for d, h, and p in the foregoing sample calculation yields

$$\Delta q = 2.86 \text{ inches}$$

as the depth of the display volume under these circumstances. Approximately centered in the display volume is the "q_0-plane", where $q_0 = p$. This plane is the image of the object plane formed at the instant when the mirror is flat. Since p was taken as 12 inches in the sample calculation, we find the depth limits of the display volume to be about 10.6 and 13.4 inches, respectively, the ratio of which is very close to the value of Q calculated earlier.

Alternatively, Δq may be expressed in terms of the object distance, p, and the minimal positive focal length, f:

$$\Delta q = \left| \frac{2p^2 f}{p^2 - f^2} \right| . \quad (10)$$

A final word about the appropriate choice of demagnification schedule will conclude this discussion of optical properties.

If the time sequence in which the 2-D patterns are generated is linear and if the mirror motion is sinusoidal, the image planes will become noticeably bunched toward the near and far ends of the image volume. This effect is produced by the relatively slower motion of the mirror near its extreme curvatures where it is in the process of changing direction. Th implication of this bunching is that the demagnification schedule must be similarly sinusoidal with time. The progression will be slow when the mirror is moving slowly and more rapid at higher mirror speeds.

If, on the other hand, the 2-D images are presented in a sinusoidal time sequence corresponding to the mirror motion, the spacing of image planes along the depth axis will be uniform and the demagnification schedule also should be uniform along the sequence of 2-D images. Thus, if Q = 1.5 for a particular display and if six image planes are to be generated, the linear dimensions of the 2-D patterns should follow the progression 1.5, 1.4, 1.3, 1.2, 1.1, 1.0, with the largest pattern intended for the nearest image plane.

Vibrational Properties

Besides the optical properties, another matter of interest relative to speaker-driven mirrors is the range of frequencies and wave shapes which they might faithfully reproduce in response to the speaker input signals. The extent of this range will determine the amount of flexibility available in the programming of the 2-D patterns.

The various 3-D display devices built during this program included mirror loudspeakers that were driven sinusoidally. This is the simplest waveform available and one which the mirror had no difficulty in following. This implies, however, that the sequence of 2-D patterns presented to the mirror during each cycle must also be shown in a sinusoidal time sequence if the depth scale in the display is to be linear; that is, the 2-D patterns must be shown slowly when the mirror is near the limits of its excursion and more rapidly when it is near the middle. If a sequence of 2-D images is presented linearly with time while the mirror is moving sinusoidally, the compression of image planes near the front and rear of the display volume is readily noticeable.

The requirement of sinusoidal time inputs imposes a constraint on the pattern generator, be it a CRO, a computer, or an optical projection system. Such systems are usually programmed more easily with some linear time sequence such as with a sawtoothed or triangular wave. Moreover, if the mirror should perhaps be used with a CRO in order to analyze the relationships between three arbitrary waveforms simultaneously, the restriction to sinusoidal mirror inputs is severe.

It is thus logical to ask whether the mirror would respond effectively if the speaker were driven with another waveform. We can almost guess that, with the present speaker-mirror configuration, the answer would be in the negative, and indeed this has been borne out by experiment.

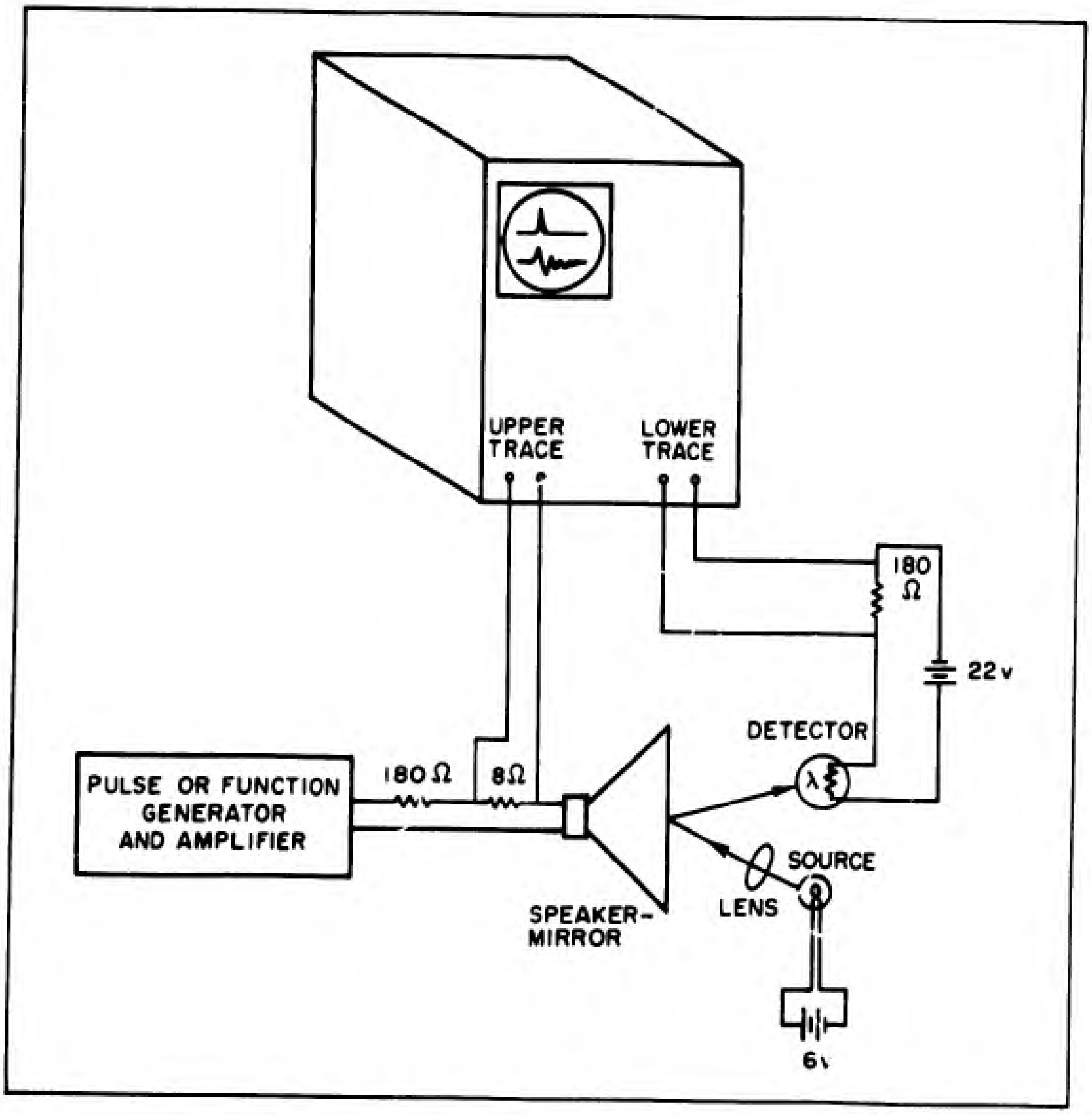
This part of the program was devoted to a study of the frequency response and waveform-following characteristics of several speaker-mirrors. It was verified that the properties of the present speaker-mirror configurations were not suitable for preserving the fine details of complicated waveforms. Triangular and square wave inputs were essentially rounded off to their sinusoidal fundamentals, and the speaker and mirror resonances often introduced undesirable ringing, especially when a resonant frequency existed at a low integral multiple of the driving frequency.

Our observation is, therefore, that the present form of speaker-mirror combination preferably should be driven sinusoidally and at frequencies of less than a few hundred cycles per second.

The following two subsections discuss the measurement program and its results.

Frequency Response

Three sizes of speaker-mirror were used in these experiments, with respective diameters of 6, 8, and 15 inches. Half-mil (0.00050 inch) Mylar was used in all cases. No attempt was made to measure the tension of each mirror, which presumably varied from one to another. The speakers were excited variously with square pulses, square waves, triangular waves, and sine waves. The mirror motions were monitored photoelectrically, with



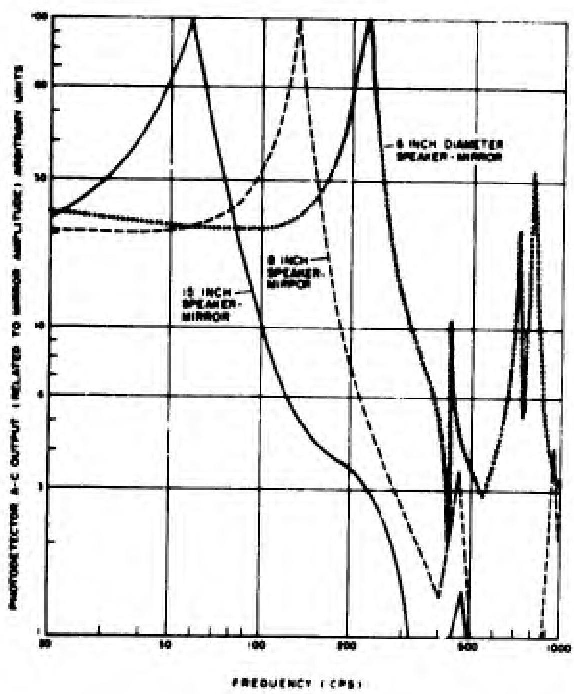
Experimental Arrangement for Measuring Frequency Response Properties of Speaker-Mirror Combination
Figure 4

a light beam from a battery-operated source impinging near the center of the mirror. The mirror motions were translated into intensity modulation at the detector by virtue of the focusing action of the mirror as it changed curvature. The experimental arrangement is depicted in Figure 4.

The photodetector output signals included the combined frequency response effects of the mirror and the loudspeaker. Feedback effects in the waveform generator circuitry were suppressed by use of a current-limiting resistor in the amplified output. A cadmium sulfide photoconductor served as the detector in most of the measurements after it was established that the various mirror responses did not contain any appreciable high-frequency components beyond the response range of this detector. We checked this by making a few mirror response measurements with an EG&G Model No. 561 Mike-Lite photodetector which uses a silicon photodiode and has a response time in the nanosecond range.

Two types of measurement were made. The frequency responses of the mirrors to sinusoidal speaker inputs were probed over the range from 20 cycles per second to 1 kilocycle, with a constant input of 1 volt across the series combination of the speaker coil and an 8-ohm non-inductive resistor. Then the waveforms developed by the mirrors in response to various speaker input waveforms were examined at selected frequencies.

The frequency response measurement results appear in Figure 5, while Figures 6 through 16 show some of the waveform response data. These results are meant to show trends only, for they are highly variable functions of the components, equipment and test conditions. They depend upon the choice and quality of the speaker and the manner in which it is mounted, as well as upon the mirror material, thickness, tension, and mounting method. In addition, the mechanical and acoustical properties of nearby equipment and of the laboratory itself will have their effects, as well as that part of the mirror being monitored photoelectrically. Even the air cushion between the



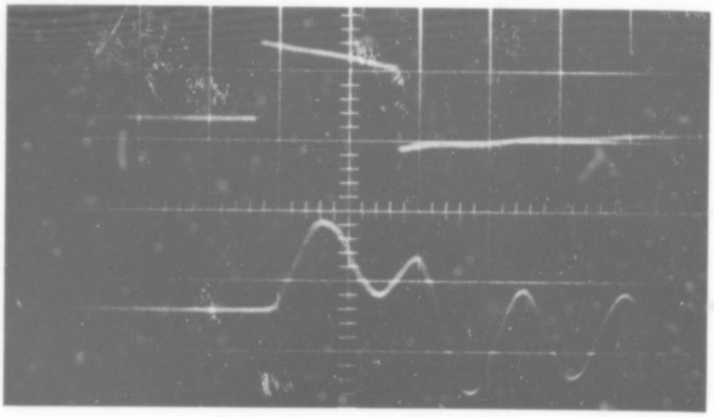
Frequency Responses of Three Speaker-Mirror Combinations with Constant Voltage Across Speaker Coil and Series Resistor
Figure 5

speaker cone and mirror will have an influence, its mass varying with the steepness of the cone and with the thickness of the mirror frame, and its acoustical properties varying as a function of the tightness with which it is contained. Finally, the shapes of the frequency response curves are highly dependent upon whether the measurements were made under conditions of constant voltage, constant current, or constant power in the speaker coil; indeed, the choice in this case was constant voltage across the coil plus load resistor in order to enhance artificially the variations with frequency.

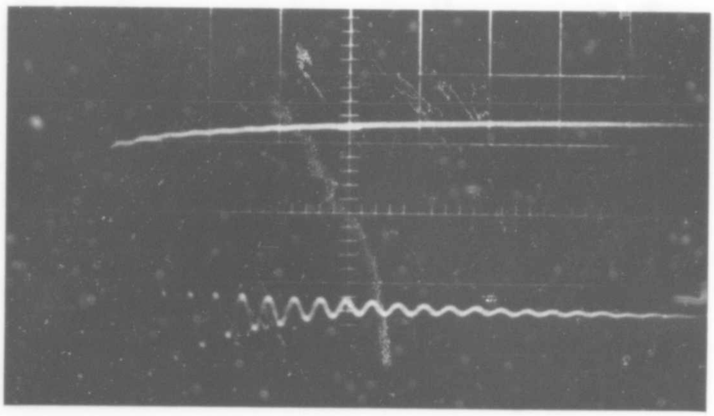
The frequency response curves of Figure 5 dramatically illustrate the fundamental resonance effects in the speaker-mirrors used. As expected, the fundamental resonance occurs at lower frequencies for larger speaker-mirrors and, compared to the other resonance peaks, it is more prominent than it is for smaller speaker-mirrors. The waveform response figures (Figures 6 through 16) show the output wave shapes. Each upper trace shown depicts the voltage variation with time across an 8-ohm resistor in series with the voice coil, while the lower trace indicates the photodetector output current versus time. The time axis is positive to the right. The apparent phase shifts between some of the pairs of periodic waves are not physically meaningful because the phasing of the upper and lower traces was arbitrary.

The ringing effects in the 6- and 8-speaker-mirrors in response to square pulses and to square waves are shown in Figures 6, 7, and 8. Ringing occurs when the mirror, driven by such waves at one frequency, responds at its natural resonant frequencies too if these happen to be simple multiples of the driving frequency. It will be noted that the ringing is at the fundamental resonant frequency except where the 6-inch speaker was driven at 250 cycles per second, as shown in Figure 8. In this case, the ringing effect shown at the higher resonant frequency of 800 cycles per second is the most prominent. The ringing effect in an 8-inch speaker-mirror, caused by subjecting the mirror to a sharp rap from the eraser end of a pencil, is shown in Figure 9.

The ringing is most evident when the driving frequency is at or near a large subharmonic of a resonant frequency. At other driving frequencies, the mirror response is a sinusoidal approximation of the wave

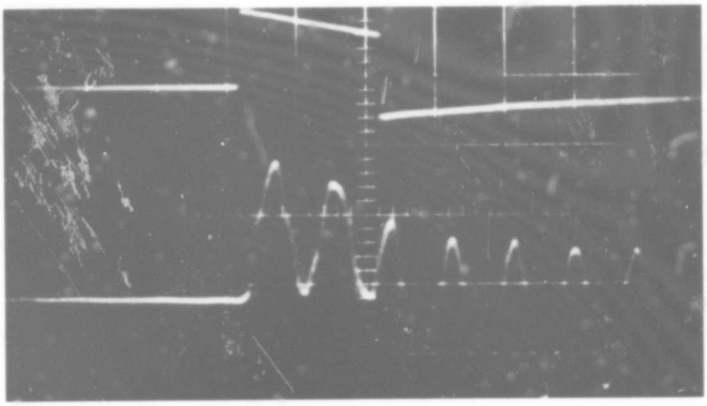


a. 5 msec per large division



b. 20 msec per large division

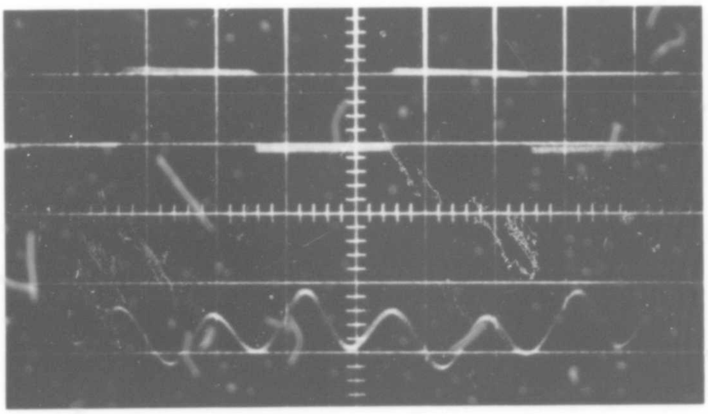
Response of an 8-inch Speaker-Mirror to a 10-millisecond Pulse
Figure 6



5 msec per large division

Ringing at 225 cps in a 6-inch Speaker-Mirror
Induced by a 10–Millisecond Square Pulse

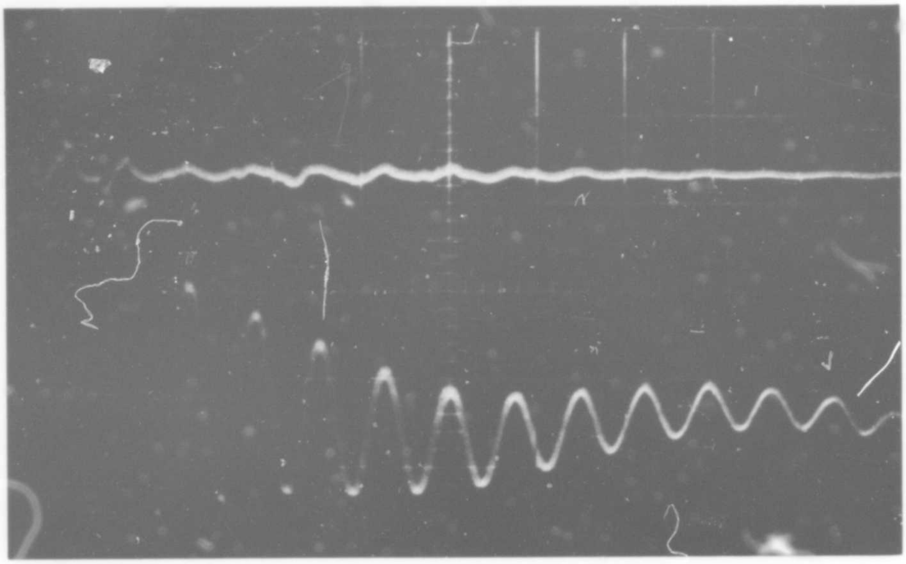
Figure 7



1 msec per large division

Ringing at 800 cps in a 6-inch Speaker-Mirror Induced by a 250-cps Square Wave

Figure 8



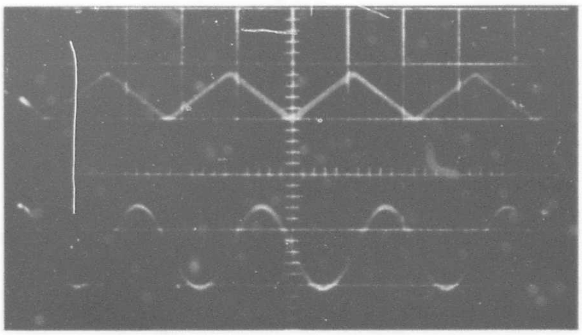
10 msec per large division

Ringing is at the fundamental resonant frequency of 130 cps. Upper trace shows terminal voltage generated in unloaded speaker voice coil.

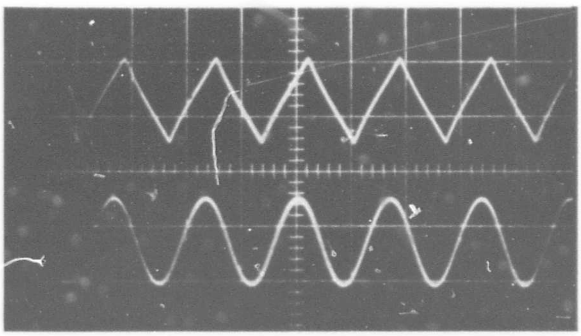
Response of 8-inch Speaker-Mirror to a Mechanical Impulse
Figure 9

shape if the wave contains many harmonics to which the mirror response is very low. For example, Figures 10 and 11 show apparently pure sinusoidal responses of two mirrors driven by triangular waves at somewhat above their fundamental resonant frequencies. However, in Figure 12a, where the driving frequency begins to approach a submultiple of the fundamental resonant frequency, the output sinusoid becomes contaminated with a small resonant component. This component is stronger in Figure 12b, where the driving frequency is closer to this submultiple.

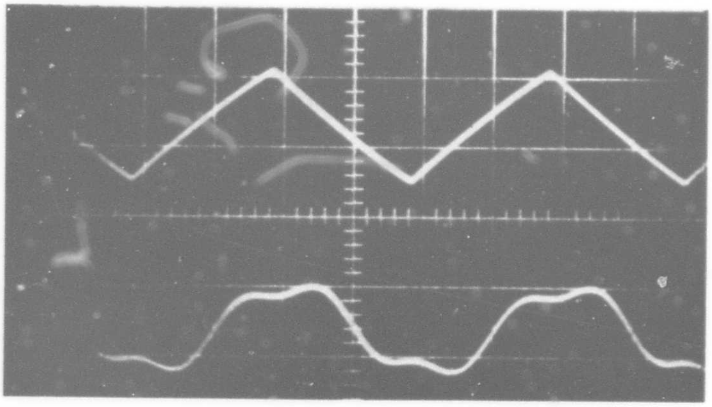
When the input signal is a low-frequency sinusoid, there is good agreement between the input and output wave shapes, as shown in Figures 13 and 14. Finally, Figures 15 and 16, shown with the same vertical scale as Figure 14, verify that the 15-inch system response drops to about one-tenth of its value as the driving frequency is shifted from fundamental resonance to 100 cycles per second. This can be seen in the frequency response curve of Figure 5 as well.



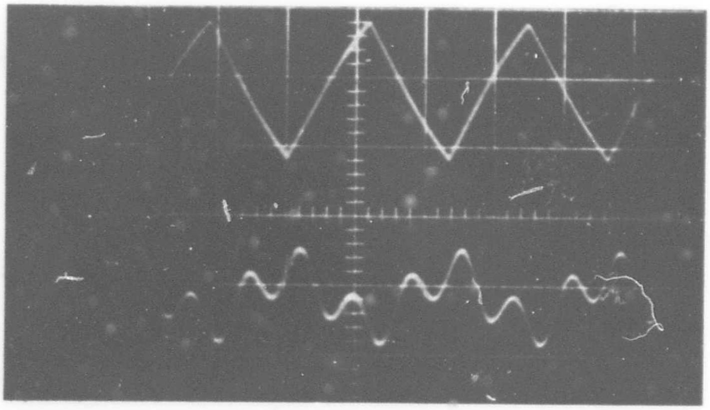
Response of 6-inch Speaker-Mirror to a 250-cps Triangular Wave
Figure 10



Response of 15-inch Speaker-Mirror to a 60-cps Triangular Wave
Figure 11



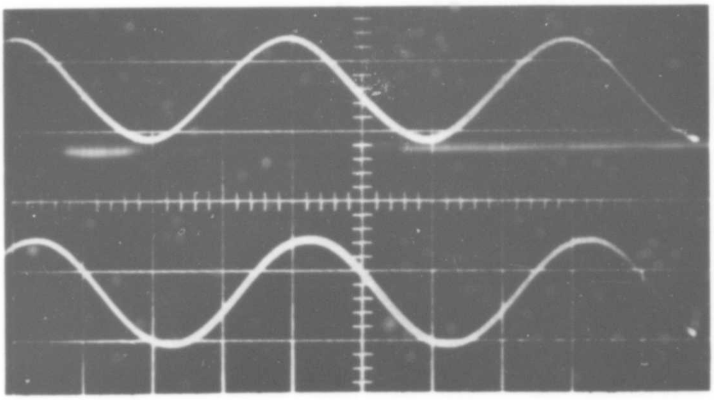
a. Response to a 50-cps triangular wave. 5 msec per large division.



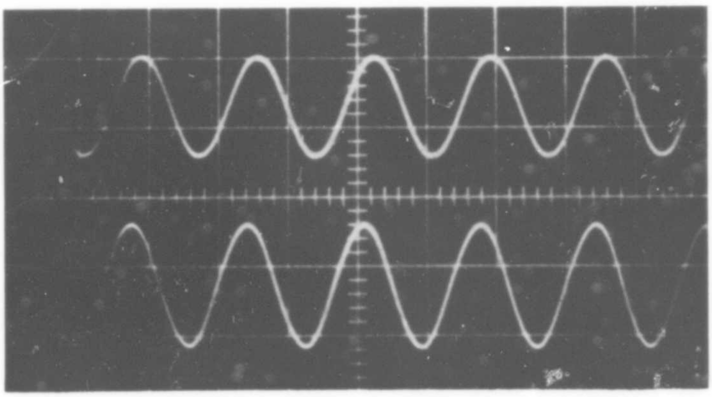
b. Response to a 44-cps triangular wave. 10 msec per large division.

Evidence of Ringing in an 8-inch Speaker-Mirror Driven at Slightly More Than One-Third the Fundamental Resonant Frequency of 130 cps

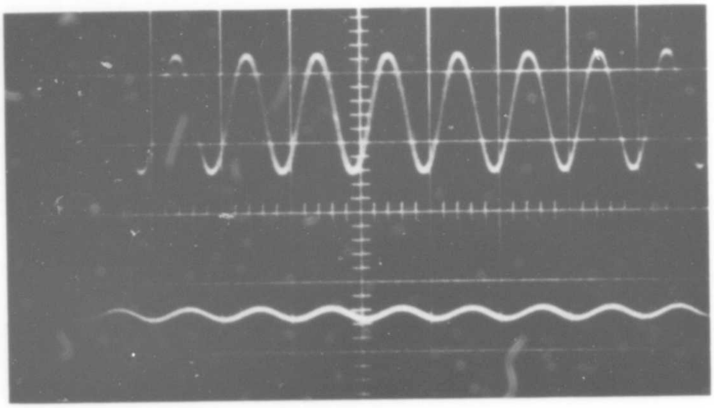
Figure 12



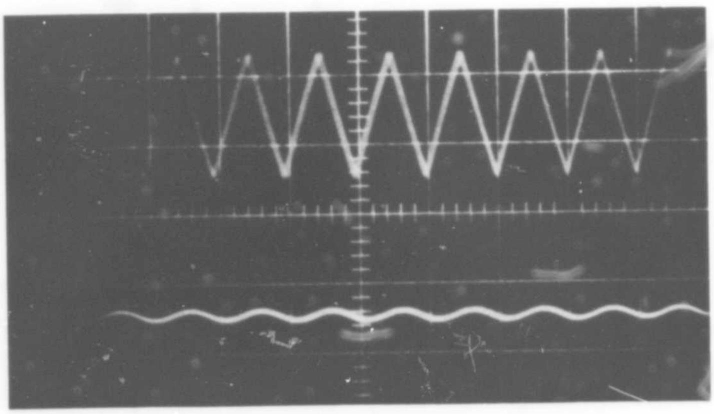
Response of 8-inch Speaker-Mirror to a 25-cps Sinusoid
Figure 13



Response of 15-inch Speaker-Mirror to a 60-cps Sinusoid
Figure 14



Response of 15-inch Speaker-Mirror to a 100-cps Sinusoid
Figure 15



Response of 15-inch Speaker-Mirror to a 100-cps Triangular Wave
Figure 16

The significant points brought out by these measurements are:

1. The high-frequency characteristics of the present speaker-mirror configurations are such that the mirror shows very limited response to other than simple waveforms such as sinusoids below a few hundred cycles per second.
2. The high resonant peaks of these systems cause undesirable ringing when the impressed signal is not a sinusoid but contains harmonics of the resonant frequencies.

This is not to say that the vibrating mirror principle cannot be used for 3-D purposes at high frequencies or with complicated waveforms. If one is willing to apply sufficient energy, the present form of speaker-mirror can be driven over a fairly broad sinusoidal frequency band. The penalty for forcing such a response is low efficiency and high acoustical levels in the speaker.

A more logical solution would be to redesign the speaker-mirror system in accordance with good acoustical design practice. In addition to the speaker parameters at the designer's disposal, other parameters would be available to him such as the membrane thickness, tension, and elasticity, as well as the design and mounting of the supporting frame. Going a step further, one might visualize an entirely new design in which the speaker armature is coupled directly to the mirror. In this case, the mirror would have to be more rigid than a membrane in order not to be deformed at the point of contact. It might be a thin sheet of metallized glass or plastic, flexible enough to undergo the required small vibrations at low driving powers.

Generally speaking, a more rigid membrane is better suited for high frequency purposes, especially if the diameter is small. As an example, frequency responses as high as 18 kilocycles per second have been achieved in 2-centimeter diameter metallic foil membranes driven electrostatically (Reference 5). This work, performed by North American Aviation, Inc., under the NASA-sponsored MIROS program, was concerned with methods of modulating light beams for communications purposes.

In any case, no attempt was made during our program to achieve speaker-mirrors with a broad band, flat response, and it is therefore likely that some benefit would result from further work.

A final question about the response of the membrane mirror deals with the maximum amplitude at which it may be driven. Mylar is a high tensile strength, resilient material and can withstand considerably more deformation than is normally used in our 3-D displays. One wonders, therefore, what conditions might impose an upper bound on practical mirror amplitudes if one sought to deepen the display volume indefinitely.

In a brief experiment, an 8-inch speaker-mirror was driven at about 7 watts at its 130-cycle per second resonant frequency; this was several times the level normally used in our displays. The volume level of the speaker was uncomfortable and could not be tolerated for more than a few moments. The amplitude of the mirror excursion was almost one-quarter inch and the sinusoidal response appeared to be good. There was no sign of permanent stretching in the membrane. Certainly, a quarter-inch deflection in an 8-inch diameter Mylar membrane would not be sufficient to rupture it. The manufacturer asserts that the product can withstand 100 percent elongation before rupturing occurs, whereas it can be shown that a quarter-inch spherical deflection of an 8-inch diameter membrane causes less than 0.3 percent elongation along the diameter.

In this experiment, the discomfort of the operator set the limit on mirror amplitude. It is likely that acoustical isolation of the system would permit an increase in amplitude until a new limit was found, but it is not known whether the limitation would be voice coil damage, mirror damage, mirror waveform distortion, or some other factor.

High-Frequency Breakup

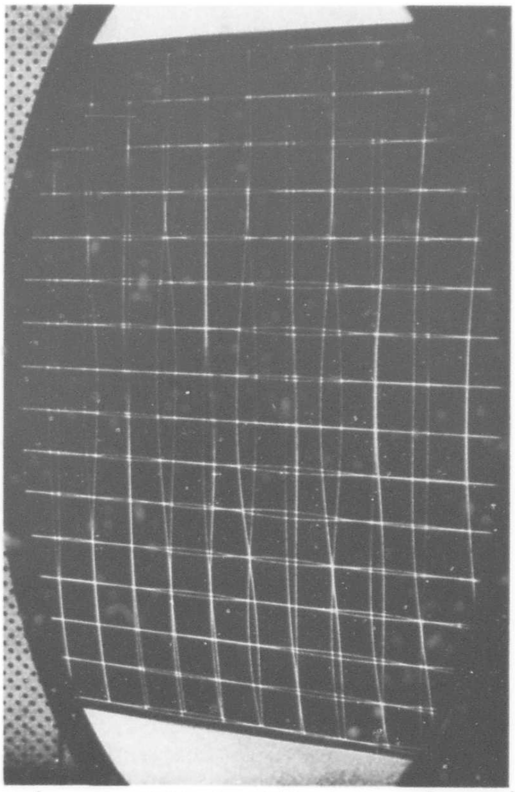
The foregoing section discusses the difficulty of coupling substantial high-frequency energy into the present form of speaker-mirror. Even if this could be done, for 3-D display purposes one would still be confined to the use of frequencies below a few hundred cycles per second because of membrane breakup, that is, the tendency of the membrane to oscillate in multiple modes at higher frequencies.

The utility of the vibrating membrane mirror for 3-D displays depends upon the mirror vibrating in a single mode. This is the simplest mode of vibration, in which the only nodal points are contained in the edge of the mirror and the maximum amplitude of motion occurs at the center. The mirror then becomes alternately concave and convex, approximating a spherical surface which is useful for purposes of image formation.

Single-mode oscillation occurs at the fundamental resonant frequency, below this frequency, and over a limited band of frequencies above it. As the frequency is increased, however, a point is reached at which other nodal regions begin to form. In a simple case, such a region may be a concentric circle part way in from the rim, and the mirror motions on opposite sides of the nodal circle will be in opposite directions. At higher frequencies, the nodal elements may form intersecting diameters or other lines across the mirror, and these may be combined with sets of circles, ellipses, or other closed curves distributed over the mirror surface.

In any event, once the single mode of oscillation ceases, the mirror is no longer interesting from an optical standpoint because the image becomes distorted. An example of such distortion is shown in Figure 17. The mirror surface irregularities appear as deviations in the virtual image of the grid of 1-inch squares. The photograph was made stroboscopically, and the timing was such that this is effectively a double-exposure which shows the grid image as it appeared at opposite extremes of the mirror excursions.

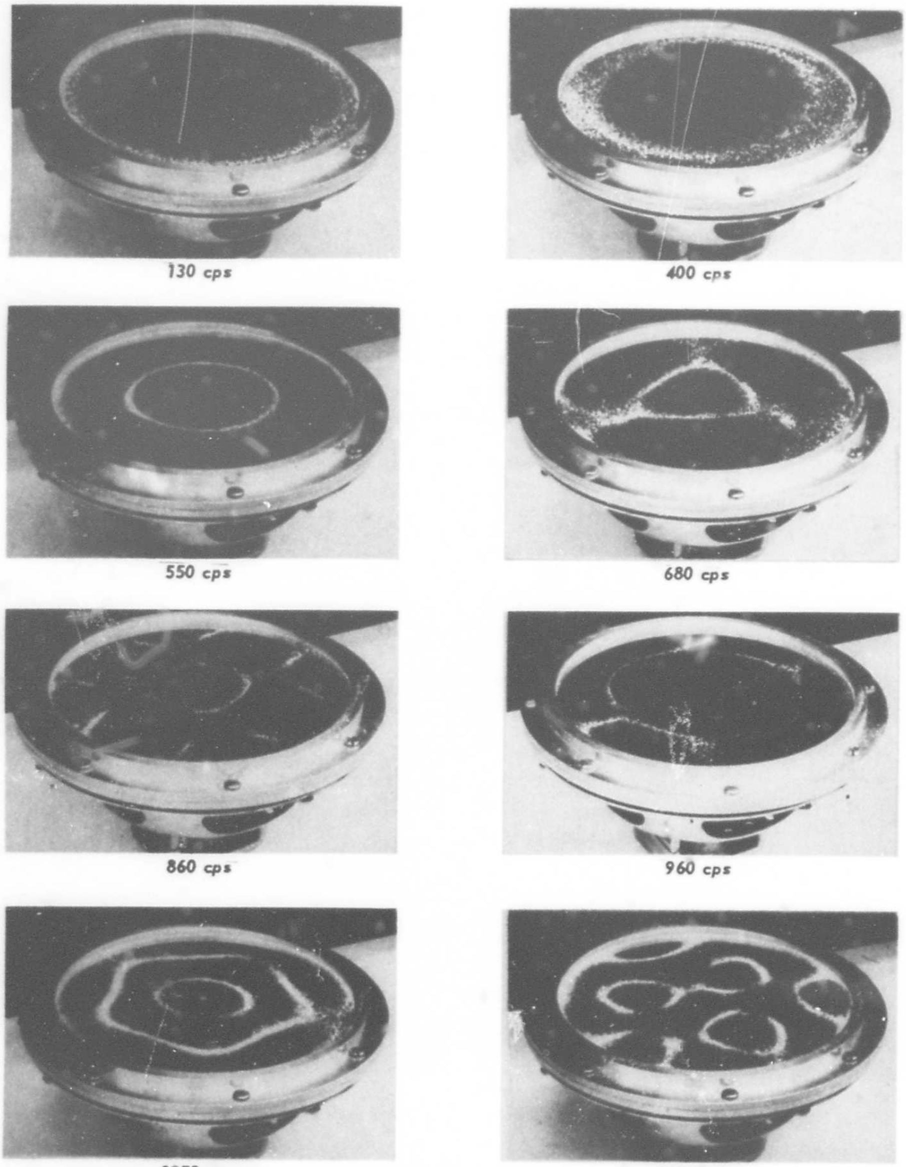
Another way of demonstrating membrane breakup is to mount the mirror horizontally and to view it directly after sprinkling a granular material such as sand or lycopodium powder over its surface. The granules tend to gather along the nodes and to make them quite visible. Direct views of several vibrational modes in an 8-inch membrane are presented in Figure 18. For high contrast, a black background was imaged in the mirror and granulated sugar was used as the nodal indicator. The mass of the sugar may have slightly influenced the shapes of the patterns.



Surface ripples are revealed by irregularities in image of rectangular grid. Photograph was made by double-flash exposure at extremes of mirror cycle.

15-inch Diameter Mirror Undergoing Breakup at 720 cps
Figure 17

What is of interest is that this speaker-mirror was capable of single-mode oscillation up to about 400 cycles per second, beyond which breakup began to occur as a nodal contour formed just inside the rim and moved inward. In similar tests, it was determined that the 6-inch combination cited previously was usable up to about 600 cycles per second, while the 15-inch system exhibited breakup beyond 200 cycles per second.



Nodal contours are revealed by sugar granules. Breakup begins at 400 cps when a nodal circumference appears just inside of mirror rim.

Several Vibrational Modes in an 8-inch Mirror

Figure 18

For these three mirrors, the breakup frequencies are comfortably high and need not be of concern in typical 3-D display applications. One would not normally wish to drive a mirror at a frequency higher than that necessary to eliminate flicker. This would dictate mirror frequencies in the 30- to 60-cycle per second range, depending upon the brightness, contrast, and other features of the display pattern, and also upon one's definition of "flicker." The penalty for exceeding the flicker fusion frequency is that the display information must be generated at higher repetition rates with no gain in the quality or rate of information transferred to the observer. Thus, the observer becomes "bandwidth limited" once the display is flicker-free.

Although the present study has not gone beyond a 15-inch speaker-mirror, it is clear that breakup can become a problem in the 30- to 60-cycle per second range if one were to use a "large screen" 3-D display requiring a mirror of perhaps several feet in diameter. Again, good acoustical design principles might be invoked and the problem eased by use of thinner, tighter membranes and special damping methods.

PATTERN GENERATORS

The term "pattern generator" refers to any device capable of presenting a two-dimensional, periodically time-varying image to the vibrating membrane mirror in order to achieve a volumetric display.

One of the simplest pattern generators is a cathode ray oscilloscope (CRO) which can repetitively trace some curve at the mirror frequency.

With simple input signals, one can generate a variety of straight lines, circles, ellipses and Lissajous or other figures which are either stationary or rotating in the display volume. A more versatile pattern generator would be a computer-driven CRO display system capable of generating line segments and alphanumeric symbols at very high rates. Additionally, one might use an optical projection screen upon which patterns generated by standard graphic techniques are made to appear in rapid sequence, offering the broader possibility of 3-D displays in color or with a continuous-tone capability.

One purpose of this program was to explore ways of implementing various pattern generators which might be useful in operational problems. Some of the more obvious ones, including those mentioned above, were studied and are discussed in the ensuing sections. Others were considered in concept only and are discussed in Section IV.

Cathode Ray Oscilloscopes

In Section II, the use of a CRO as a basic pattern generator was discussed in order to lay the groundwork for an understanding of the membrane mirror principle. Beyond using the CRO as a generator of simple geometric curves for 3-D study or demonstration purposes, there is little more that need be said about it if one is confined to the use of sinusoidal input signals. Exceptions include, perhaps, the use of such a system for routine phase, amplitude, or frequency comparisons between three such signals simultaneously or, perhaps, as a teaching device.

On the other hand, the CRO becomes interesting if one regards it, in combination with the mirror, as a possible analytical tool for general laboratory use in the simultaneous comparison of three arbitrary periodic signals. If the sinusoidal-input constraint is removed, one can imagine several laboratory situations in which such comparisons would be useful, as in the analysis of radar signals, multi-dimensional speech or electrocardiographic signals, and others. It is thus easy to visualize a CRO-mirror combination as a single entity comprising a "three-input" oscilloscope. Such a tool would afford a direct visual presentation of signal relationships in a way which could be meaningful to an operator after some experience and which should accelerate his rate of information absorption.

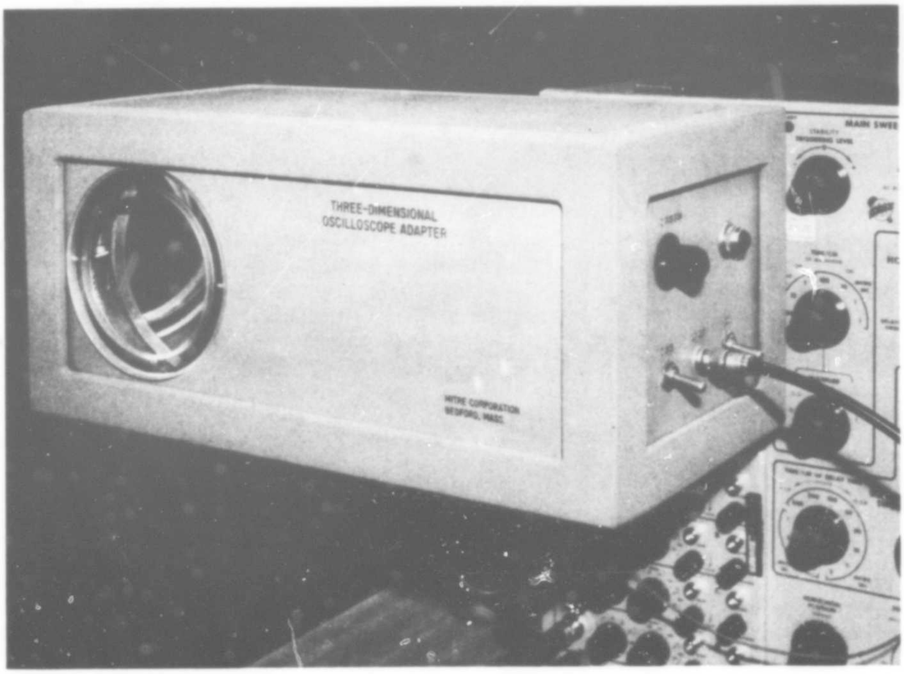
A somewhat fanciful pictorial description of one form which such an instrument might take is shown as Stereophotographic Pair B (see centerpiece). The device does not exist, of course, and the "artist's conception" was produced through the trickery of multiple-exposure photography.

To be sure, our speaker-mirror combinations in present use are restricted to low-frequency sinusoidal input signals; this is in contrast to the input channels of an ordinary CRO which can accommodate complicated waveforms. Therefore, a restriction on any 3-D CRO using these speaker-mirror combinations is that at least one of the three input signals be a low-frequency sinusoid. Presumably, this restriction could be eased if an effort were made to design a mirror-driver combination which would offer a broader frequency-response capability.

Waveform limitation notwithstanding, a task was undertaken with the purpose of assembling and evaluating an experimental model of a 3-D oscilloscope. In the interest of simplicity, the 3-D part of the system was conceived of as an external attachment which would fit many laboratory CROs and which contained the necessary optical and electronic parts. This design concept was selected over the more elaborate alternative of "re-packaging" an existing CRO with the mirror behind the present viewing port and with the cathode ray tube concealed within the CRO housing, as is implied in the illusion shown in Stereophotographic Pair B. The evaluation of this model was to have consisted of a determination, within the various MITRE electronics laboratories, of just what applications might exist in the Corporation's area of technological interest. These include radar systems, communications systems, and electronic data processing and display techniques in general.

As of this writing, the model has been implemented and is shown in Figure 19. Its evaluation will be performed under another project.

The design concept for the model involves a double-mirror system in a periscope-like configuration so that the viewing direction is parallel to the long dimension of the oscilloscope. The first mirror is a front-surface-aluminized (to eliminate ghost images) glass mirror, mounted near the CRO tube screen and at a 45-degree angle to it. Hence, it diverts light from the screen through 90 degrees toward the second mirror, which is the vibrating membrane mirror and which is parallel to it. At this point, the original 90-degree deviation is canceled and the volumetric image is observed by looking



Experimental 3-D Oscilloscope Adapter
Figure 19

into the membrane mirror in the direction of the CRO axis. A housing with appropriate viewing ports and with a low-voltage 60-cycle per second power supply for the speaker completes the model. External speaker connections are provided in the event that other driving signals are to be used.

The simple nature of the CRO-generated patterns most often used during this program is such that anomalous perspective has not been a problem. In the case of the more complicated patterns which might result when a 3-D CRO is used as an analytical tool, some means of correcting anomalous perspective may be desirable. In principle, the correction should be achievable quite easily in a CRO which has provision for external, electronic gain control along the X- and Y-axes. It is merely necessary to provide a sinusoidal voltage, of appropriate magnitude and in synchronism with the speaker signal, to the gain controls in order to contract and expand the CRO image radially during each mirror cycle. The same result can be accomplished externally with any CRO if a small amplifier is inserted into each of the CRO signal input lines and if the sinusoidal voltage is applied to each amplifier as a gain control.

In a simple experiment, a method similar to this was used to remove the angular divergence between four straight lines receding into the image volume. The lines were the vibrating mirror images of four fixed pips displayed on the screen of a four-trace CRO with no input signal. When the proper sinusoidal signal was applied to the CRO terminals, the pips underwent small excursions and appeared as short line segments along radii from the center of the screen. Their motion was just sufficient to cancel the anomalous perspective, causing their images to appear as parallel lines in the image space.

Optical Projection Systems

In addition to providing the simple CRO display of Lissajous and other patterns, much can be done to generate a broad variety of figures in order to create 3-D displays of greater utility. One such application is discussed in this subsection: a static 3-D display of discrete planes produced by stroboscopic optical projection of photographic slides containing graphical material.

With the proper synchronization, the planes appear to be stacked one behind the other when the screen is viewed in the vibrating mirror. The use of photographic slides for storing the graphical material creates several possibilities in terms of higher information rates and in the generation of figures having forms and colors which cannot be achieved with the usual CRO.

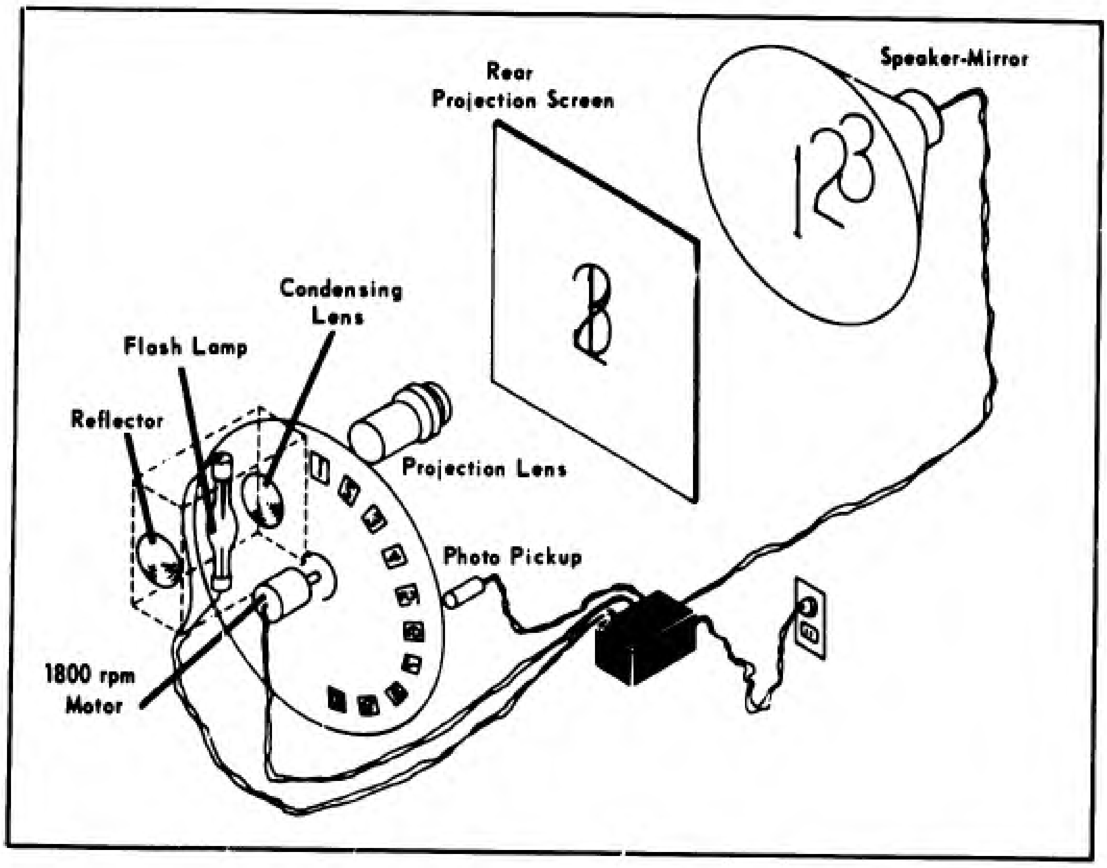
Because this display is basically a static one and requires that the slides be changed mechanically to update the display pattern, one is led to ask what advantages such a system might have over one in which a stack of real transparencies is viewed directly. One answer is that, in this system, the superposition of images is additive rather than subtractive so that darker areas in a given plane do not obstruct light areas in the planes behind it, as they would in the case of slides being viewed directly. Moreover, the luminous areas in the image planes are themselves transparent, so that lighter areas occurring in planes behind them are visible at all times. This would not be the case if such a display were achieved, for example, by embossing or scribing graphical matter in clear plastic sheets which were then stacked together and edge-lighted.

Finally, the depth of the display volume and the order in which the image planes appear along the depth axis are functions of the power applied to the loudspeaker and of the phase relationship between this signal and the slide projection sequence. Consequently, both can be varied during operation through the use of simple electronic controls.

The purpose of the task discussed in this section was to study the problems of building and using a self-contained optical projection 3-D display system which could be used in the laboratory for general-purpose static demonstrations. The approach adopted was to build and operate a simple breadboard model, which we called the Mark I display system, as a prelude to arriving at a system design concept for an improved device, designated as the Mark II. The Mark I display system uses a straightforward design concept in which the photographic slides are mounted in apertures near the rim of a rapidly spinning disc so that they move through the projection system gate at high speed. A primary goal of the Mark II design concept was a system configuration in which the slides would be stationary and could thus be replaced by others during operation. This design improvement was brought about by the introduction of an unconventional projection system approach which uses a rotating reflecting element to sweep the light beam past the slides in turn. The Mark II concept will be described in greater detail later.

The Mark I Display System: General

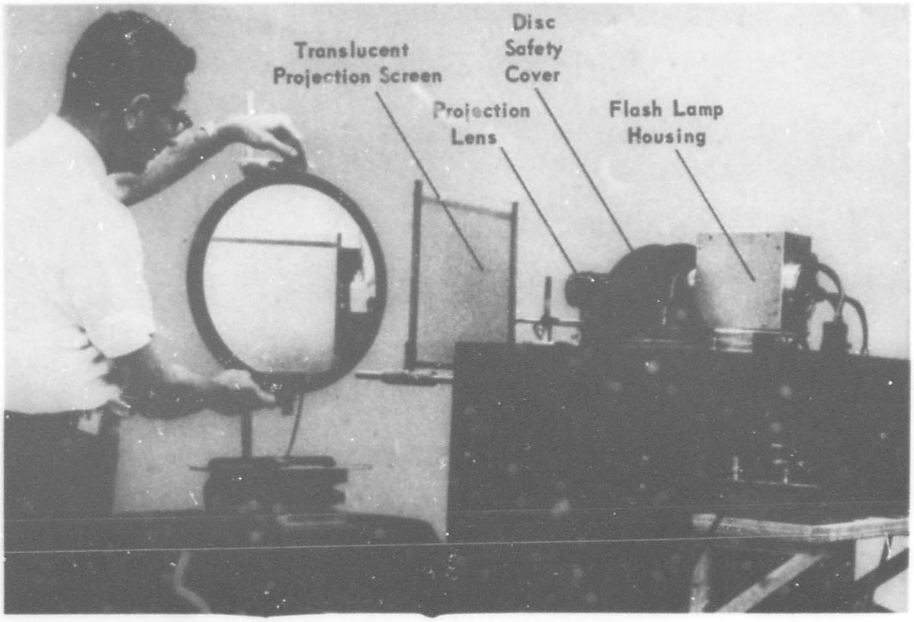
The principle of operation of the Mark I System is depicted in Figure 20, and a photographic view of the equipment is shown in Figure 21. An 18-inch diameter metallic disc is provided with a set of ten square apertures spaced evenly around half of it, just inside the rim, and the prepared 2"x 2" glass slides are mounted in the apertures. The disc is rotated at a nominal 1800 revolutions per minute, and the flash lamp is triggered each time a slide is centered on the projection system axis. The slide images thus appear



Schematic Drawing of Mark I Experimental Display System
Figure 20

in rapid sequence upon the projection screen, where they are superimposed in a common plane. When the rear of the translucent screen is viewed in the vibrating membrane mirror, which is frequency-locked to the disc, the slide images appear separated in depth.

The reason for using only half of the disc for slides is that the images are projected during a one-way excursion of the mirror; the solid half of the disc moves through the projection axis during the mirror flyback in order to blank the display. If one insisted upon using the full mirror cycle for image projection, the slides would have to be shown in reverse



Author adjusts 15-inch diameter speaker-mirror. Rear of projection screen is viewed in mirror.

Mark I Experimental Display System
Figure 21

order during the return of the mirror. Although a greater image brightness would result, the problem of exactly registering a slide image upon a duplicate of itself would be quite severe because of mechanical vibrations and because of critical requirements concerning the positioning of the slides and the timing of the flashes. As will be seen later, exact superposition of duplicate images during mirror flyback also was a problem with computer-generated displays; thus, blanking was the desirable alternative.

In actuality, the Mark I disc contains two separate sets of ten slides each to permit double use. The sets can be activated independently so that two alternative display situations can be presented by use of a toggle switch. Stereoscopic representations of two of the display situations used for evaluation and demonstration purposes are shown in Stereophotographic Pairs C and D (see centerpiece).

The alphanumeric display of Pair C illustrates a possible use in psychology laboratory tests of human depth perception. It addresses itself to the question of how densely one might be able to stack a group of display elements along a depth axis and yet ensure that they are individually recognizable and locatable to an observer aided by stereoscopic, parallactic and, possibly, color cues. The parallax cue, of course, is not shown in the stereophotograph.

A hypothetical air traffic control (ATC) situation display similar to the type used at a Federal Aviation Agency air route traffic control center is depicted in Pair D. The purpose of an ATC display in depth would be to provide the controller with altitude separation information in a more direct and rapid way than current methods permit. (This subject is treated at greater length in Appendixes III and IV.)

The aircraft are represented by data blocks containing flight information such as the flight number and velocity vector of the aircraft, and the assigned and reported altitudes in hundreds of feet. Color coding is used to designate aircraft which are climbing, descending, or in a holding maneuver. In the actual display, the data blocks appear to be hovering in space at various depths, as can be verified if the reader will view the figure stereoscopically. The altitude scale in this display is intentionally exaggerated in order to avoid the slab-like appearance characteristic of a real volume of controlled airspace. The ground plane, at the rear of the display, is made up of a geographic coastline, an air traffic terminal area, and ATC sector boundaries. The straight lines connected by circles appearing just above the ground plane represent federal airways and their intersections.

Another version of the same display is discussed under Computer-Driven Displays, starting on page 93. Its distinguishing features are:

1. it is monochromatic;
2. the screen resolution is somewhat lower;
3. more elaborate equipment is used; and
4. the display situation elements can, in principle, be presented and updated in real time.

We turn now to a few details of construction of the Mark I System.

The Mark I Display System: Details

The most important single matter in building an effective display of this type is that perfect synchronization be achieved among the disc speed, the flash lamp pulses, and the mirror motion to ensure that the volumetric image is stationary. Unwanted phase variations between the disc and the flash lamp will cause the image to shift laterally in the display volume.

Variations in the mirror frequency will produce longitudinal excursions of the image. For example, a simple but not very effective way of implementing such a display is to use the disc speed as a reference and to synchronize manually an electronic stroboscope to the framing rate of the slides. At the same time, the proper mirror frequency would be derived independently from a sine-wave generator. However, it was learned early in these experiments that the vagaries of present-day frequency sources do not permit the required degree of long-term stability, and the decision was made to derive all frequencies from a single source.

The most logical reference frequency is that of the disc itself. Because of its large mass, windage effects, and line-frequency variations to the synchronous motor which drives it, the disc frequency would be the most difficult to control precisely; one can more readily provide electronic synchronization so that the mirror and flash lamp frequencies will track the disc frequency quite well.

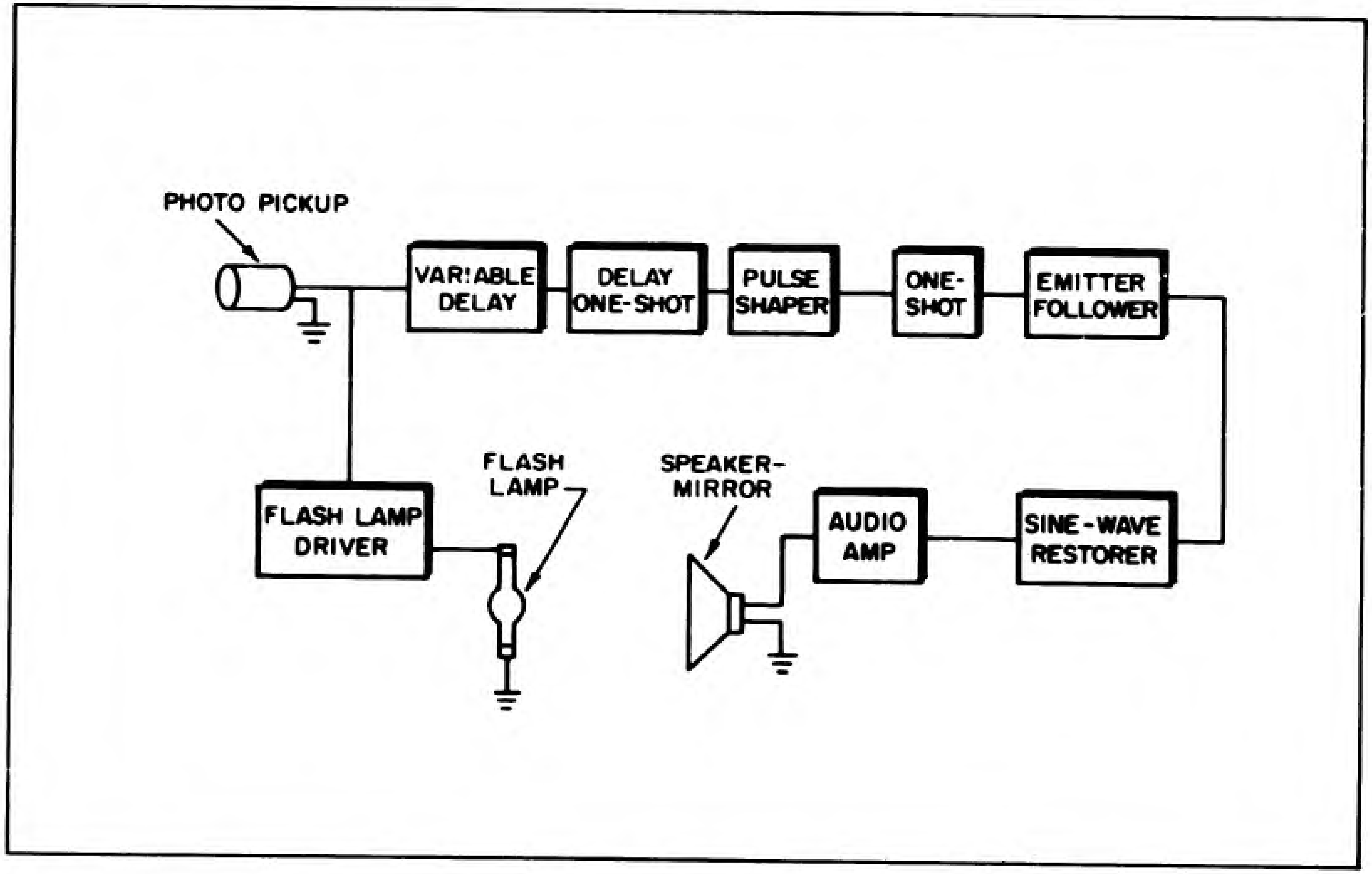
The method used was to affix reflective cue marks around the periphery of the disc, one for each slide, for the purpose of generating the flash lamp triggering pulses via a fixed photoelectric transducer or "photo pickup." At the same time, the leading pulse of each pulse train from the photo pickup serves also to trigger each cycle of a sinusoidal voltage which drives the mirror loudspeaker; essentially, the method is to derive a rectangular envelope for the pulse train and to pass this through a narrow band electronic filter centered on the disc frequency. A block diagram of the

Mark I system circuitry, indicated as the "black box" in Figure 20, is given in Figure 22.

As stated earlier, although not shown in Figure 20, the Mark I disc contains sets of slides for two independent displays. Switching from one display to the other is accomplished through activation of one of two photo pickups mounted at different radial positions, with each photo pickup following its own set of reflective cue marks.

The problem of anomalous perspective, treated earlier, was anticipated and an experimental set of slides was prepared incorporating a uniform progression of magnification from one slide to the next. A ratio of 3:2 was chosen for the length scales in the first and last slides to accommodate the situation where their corresponding images in the display volume are in a 2:3 ratio of distances from the mirror surface. Thus, when the mirror amplitude and the screen-to-mirror distance are such that the tenth image plane appears 3/2 farther behind the mirror than does the first image plane, all of the planes appear to be of equal size. It has already been shown (see page 45) that once a magnification ratio is chosen for the slides, there is a family of combinations of screen-to-mirror distances and mirror amplitudes for which the anomalous perspective will be exactly canceled.

The displays shown in Stereophotographic Pairs C and D have not been corrected for anomalous perspective and its effect is especially noticeable in Pair C.



Mark I System Circuitry (Block Diagram)
Figure 22

The slides are mounted to the disc by being inserted into the channels of aluminum slide frames which are permanently bonded to the disc. The open ends of the frames face the center of the disc. The slides are taped in place at the open ends and may be easily removed and replaced. Centrifugal force holds them snugly in place during disc rotation. A protective cover over the disc reduces the hazard to personnel should any untoward rupturing of mechanical parts occur; the centrifugal accelerations near the edge of the disc are many hundreds of earth gravity units. It is quite important also that the disc be balanced carefully in order to reduce vibrations which would cause image jitter and motor-bearing wear.

The electronic stroboscope used initially in the Mark I display system was a General Radio Company Type 1531-A Strobotac. It was operated in the external-trigger mode, using pulses from the Type 1536-A Photoelectric Pickoff which were fed through a Type 1531-P2 Flash Delay. The resulting volumetric image was satisfactory in terms of stability and clarity. In particular, the 1-microsecond flash duration was short enough to provide a crisp image, considering that the slides move through the projector gate at nearly 120 feet per second and are magnified more than five times on the screen.

It was decided, however, that an increase in image brightness would be desirable and could be obtained by use of a compact-arc xenon discharge lamp operated at a higher average power. Although the flash duration would be about twice as great, it appeared that a gain of about 25 times in screen brightness could be obtained through use of a suitable driving circuit. Some of this gain would be due to the small size of the luminous arc, which makes it more suitable for projection system applications than large area sources. Accordingly, a flash lamp driving circuit for use with a PEK Type X-150 compact arc lamp was custom-fabricated by U.S. Scientific Instruments, Inc., Watertown, Mass., and has substantially improved the screen brightness while maintaining nearly the image sharpness experienced with the Strobotac. The circuit delivers approximately a 1/4-joule pulse to the lamp for a continuous average power input of 75 watts.

When a translucent projection screen with wide-angle diffusion properties is used, the apparent brightness of the display characters on the screen axis is about 20 foot-lamberts; it is about double this if a somewhat less diffusing screen is used in order to achieve a greater brightness on axis. These values apply to the particular choice of screen material used, which is provided with a controlled amount of light absorption in order to provide images of higher contrast under room lighting conditions. The brightness values were obtained by using a Spectra Brightness Spot Meter to read a steady incandescent source which had been matched visually to the apparent character brightness in the display. The reason that the instrument was not used directly on the characters is that its saturation and response time properties apparently are not exactly matched to those of the human eye. The difference becomes evident in the case of an intermittent target with the unusual temporal and brightness properties of the Mark I display. The interpulse period of the flash lamp is some 800 times the pulse halfwidth; moreover, at the peak of the pulse, the power dissipated in the flash lamp is in excess of 100 kilowatts. Hence, for most of the time the screen is in a darkened condition which is punctuated by power bursts many hundreds of times higher than the average level. Simple tests showed that the brightness meter was overburdened under this condition and gave readings which were substantially lower than visual brightness estimates, probably because of a difference in signal decay properties between the photodetector and the eye.

It is quite important that the flash duration be held to a low value if sharp images are to result. With the slide speeds and character sizes that we have used, a flash duration of about 2 microseconds in halfwidth appears to be an acceptable upper limit. At a projection system magnification of five, the 1400-inch per second slide speed becomes 7000 inches per second on the screen. Thus, for each bright object point in a slide, a 2-microsecond flash creates a Gaussian luminous line segment nearly 15 thousandths of an inch long. At a typical viewing distance of 20 inches from eye to mirror to screen, this streak subtends an angle of 2-1/2 minutes of arc, which is somewhat above the threshold of perception for the average observer; that is, each streak is large enough to be recognized as a streak rather than as a point.

The lines and characters which make up the present displays typically are no less than 0.010 inch in stroke width on the screen. With streak lengths of 0.015 inch, such strokes can appear to be more than doubled in width in the direction of slide image motion. However, the overall stroke lengths and character dimensions are no less than 0.100 inch, in relation to which the amount of character distortion caused by the increased stroke widths is small. Indeed, upon viewing the display, the observer is hardly conscious of the streaking unless he is looking for it.

It should be pointed out that the rather large energy pulses which pass through the flash lamp give rise to considerably more radio frequency radiation than the lower-powered Strobotac. The result was the occurrence of some unexpected electrical interference with the triggering circuit, causing instability in the flashing rate and mirror frequency. These effects were finally eliminated through a careful redesign of the triggering circuitry in which better use was made of shielding and filtering techniques.

Because the mirror is driven sinusoidally and the slides are projected linearly with time, some compression of the image planes along the depth axis is noticeable near the front and rear of the image volume. If desired, this bunching can be avoided by repositioning the slide apertures in the disc so that the central ones are more closely spaced. With the proper spacings, the slides will be projected sinusoidally versus time and the image plane spacing will appear uniform. Alternatively, one can maintain uniform slide spacing by compressing the series of slide apertures about the mid-point in order to use only the "straight-line portion" of the mirror excursion, but this is done at the expense of otherwise usable depth in the image space.

Some of the more pertinent data on the Mark I system design and performance are presented in Table I.

The Mark II Display System

This system, depicted in the drawing of Figure 23, exists as a design concept only. It is intended as an articulation of our present thoughts on how to design such a system in view of the experience gained in building and operating the Mark I model. The design concept presented is very general rather than detailed, and it is meant simply as a guide to possible future work.

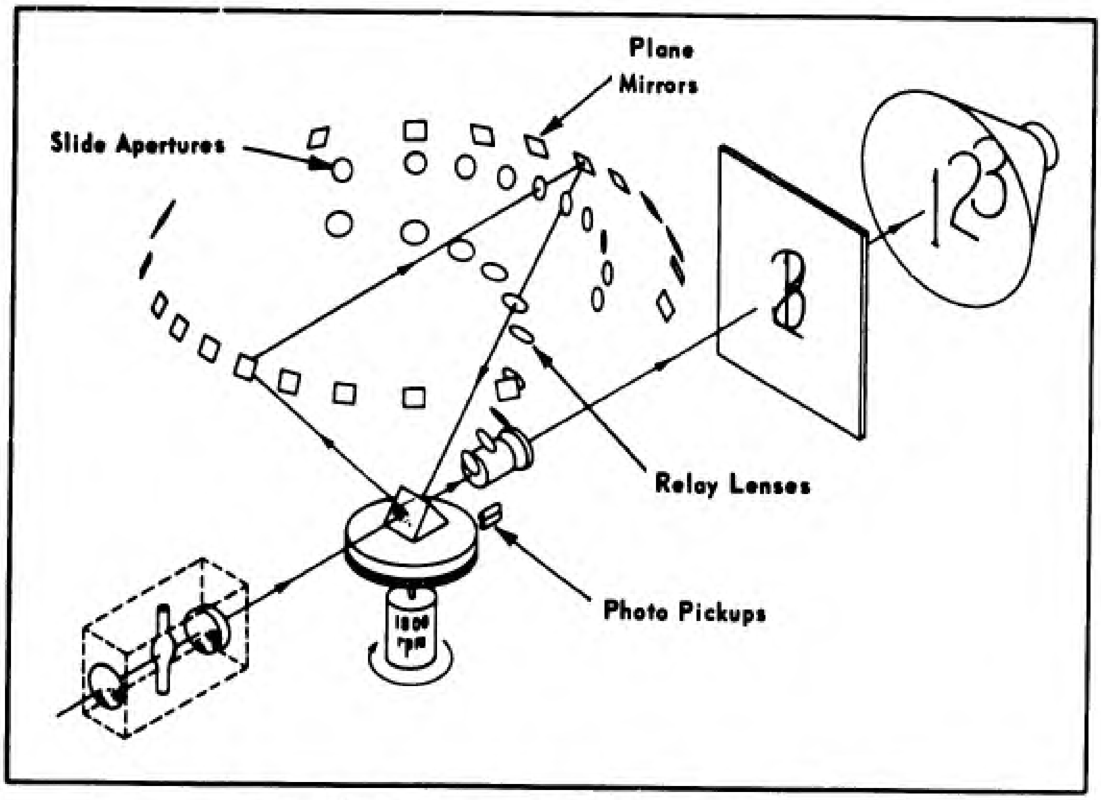
The following set of desiderata was used as the starting point in arriving at this concept.

1. The slides were to be stationary, scanned by a moving optical projection axis, to effect easier changing of slides during operation of the system.
2. Moving parts were to be small and minimal in number in order to reduce noise, vibration, and personnel hazards.
3. Automatic correction for anomalous perspective was to be built into the system so that the slides could be scaled uniformly and therefore be interchangeable.
4. The time sequence in which the slides were projected was to be sinusoidal, matched to the mirror excursions, for the image plane spacing to be uniform.

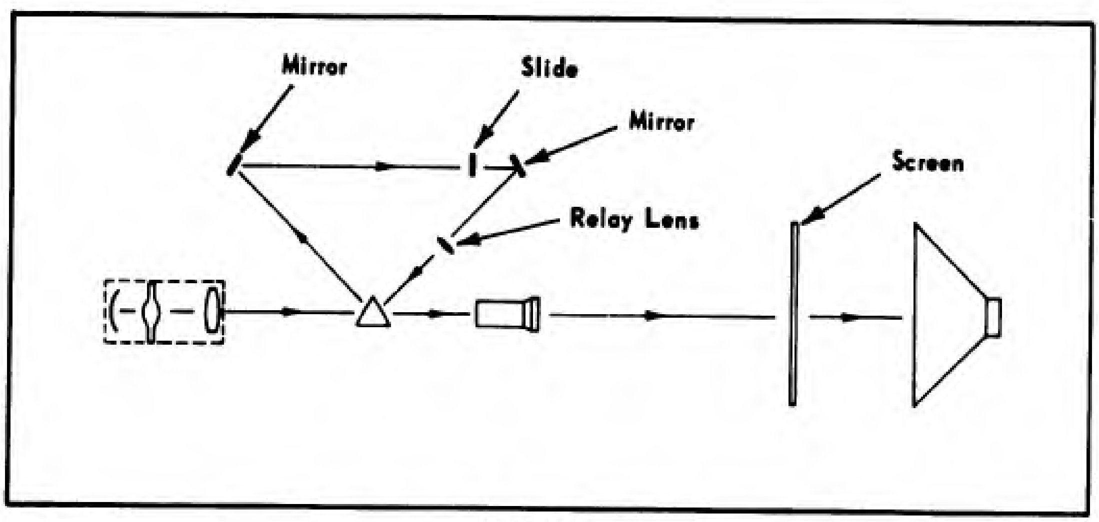
The requirement for a moving projection system axis suggested a number of ideas involving rotating axes whose end points are fixed but whose central portion is made to revolve in crankshaft fashion by means

Table I
Mark I Display System Data

COMPONENTS	SPECIFICATIONS
Speaker-mirror	
Diameter	15 in.
Speaker Power (rms)	5.7 w
Mirror Frequency	30 cps
Mirror Deflection	±4 mm at center
Focal Length Limits	Infinity to ± 15 feet
The Display	
Typical Depth	7 in. for 24-in. object distance
Height and Width	10 in.
Apparent Screen Brightness	20 ft-L
Optical System	
Source	Compact-arc lamp between curved reflector and aspheric condenser
Projection Lens	Buhl 2-in. focal length, f/2.0 slide projector lens
Projection Screen	Polacoat Plastic Lenscreen LS75PL 1/16
Slide Disc	
Diameter	18 in.
Rotation Rate	1800 rpm, nominal
Radius of Slide Centers	7.5 in.
Flash Lamp	
Designation	PEK type X-150 Xenon Arc Lamp
Power Rating	150 w
Luminous Efficiency	21 lumens/w
Arc Size	0.040 - 0.050 in.
Flash Lamp Driving Circuit	
Pulse Rate	Average rate of 300/sec in bursts of ten each at 600/sec rate, with equal dark interval during mirror flyback
Output Energy and Power	0.25 joule/pulse, 75 w average power
Pulse Width	2 μsec at half-height
Pulse Shape	Gaussian, skewed toward trailing edge



a. Simplified Schematic of Mark II Display System Concept



b. Side View

Mark II Display System Concept (Schematic Representation)

Figure 23

of rotating mirrors. The concept selected is unconventional but lends itself well to the above requirements, particularly because it permits the use of a single, small moving part. This part consists of a pair of plane mirrors mounted back to back in a prismatic configuration and affixed to a disc whose edge carries optical cue marks for triggering purposes.

The mirror pair serves as a commutator which switches the optical axis sequentially through the ten projection subsystems shown in Figure 23. Each subsystem consists of two fixed mirrors, a photographic slide, and a relay lens, as shown in Part b. of the figure. The purpose of the relay lens is to form an aerial image of the slide (i.e., to transfer the slide optically) nearer to the projection lens. This is desirable for practical reasons concerned with magnification ratios, system dimensions, and screen brightness.

The relay lens is also the key to another degree-of-freedom in the system, notably the correction of anomalous perspective. By using ten relay lenses which are graduated in focal length, and by positioning each properly in its projection subsystem, one can achieve a sequence of aerial images with the proper magnification progression to cancel the anomalous perspective associated with a given combination of mirror amplitude and screen-to-mirror distance or, indeed, by a family of such combinations.

In the Mark II system, the method for synchronizing the flash lamp and membrane mirror to the slide scanning rate differs in one respect from the Mark I method. The earlier method used a dual purpose set of cue marks in which the individual pulses triggered the flash lamp, and the loudspeaker signal was derived from the pulse train envelope. A basic feature of the method was the use of a monostable multivibrator, or "one-shot," a solid-state switch which is turned on by the leading pulse and which then turns itself off at the end of some fixed interval after the last pulse in the train. The output of this switch is the rectangular wave whose sinusoidal fundamental becomes the speaker input.

The one-shot is a rather sensitive device which passes through an unstable bias condition in the course of turning itself off, and at this time it is responsive to small spurious signals in the circuit. In the Mark I system, there was sufficient radio frequency interference from the 75-watt flash lamp circuit to cause the one-shot to behave erratically, with disruptive effects upon the mirror motion. The necessary steps taken to shield the circuit boxes and cables might be avoided in the Mark II system if the monostable multivibrator were to be replaced with a bistable multivibrator, or "flip-flop." Such a switch is not only turned on but turned off by trigger pulses, instead of by a slowly decaying voltage as in the case of the one-shot. Therefore its "off" action is more positive.

Consequently, a dual-track cue mark system is visualized for the Mark II system. The first would be the customary track section for triggering the flash lamp driver, with one cue mark per slide. The other track section would have only a pair of "on-off" cue marks for the flip-flop, separated in time by one-half of the mirror period.

The track length around the edge of the cue mark disc would actually be composed of two identical sections of track, for the disc would rotate only once for each two cycles of the speaker-mirror. The reason for this is that the moving, or crankshaft, part of the projection axis moves at twice the angular rate of the mirror shaft, completing two slide scans per revolution. Once per cycle, each mirror serves a dual role: first as the transmitter, and then as the receiver of the rotating portion of the beam.

The cue mark disc may be either a solid circular plate or a thin, hollow drum. Reflective cue marks may be affixed to the rim and illuminated by self-contained light sources in the photo pickup units if the reflective material is specular; if it is diffuse, off-axis light sources may be used for illumination. Alternatively, the rim of the disc can be replaced by a film strip containing transparent cue marks reproduced photographically from a large-scale drawing and transilluminated from within. This would allow quite accurate cue-mark positioning, which is important to slide-image registration, while also permitting a small-diameter, lightweight cue mark disc.

The sinusoidal time-sequencing of the slide images is achieved through the appropriate angular distribution of the projection subsystem axes; a non-uniform spacing is indicated in the figure. The flash lamp cue marks must be similarly spaced. Also, provision must be made for mounting the various slides at appropriate inclinations about their perpendicular axes. One of the properties of a rotating projection system of this type is that if all slides are mounted vertically, the successive screen images in the sequence will be rotated with respect to one another as the slides are scanned by the rotating mirror.

The Mark II system is similar to Mark I with regard to the light source, projection screen, and speaker-mirror. Some minor modifications are recommended, however. It may be desirable to double or treble the flash lamp pulse energy to make up for the longer projection system path and its attendant lower illumination efficiency. The loss of efficiency is due to the fact that light from a smaller solid angle about the source will ultimately reach the screen, although this problem may be countered with additional relay lenses. Further, the 30-cycle per second Mark I display is not entirely flicker-free because of the short pulse duration relative to the interpulse period. For the Mark II system, a 40-cycle per second repetition rate should be considered. This would have the additional advantage of allowing the use of standard audio frequency circuits and components for the various system amplifiers, coupling transformers, and so forth. Standard audio parts exhibit generally better performance at 40- than at 30-cycles per second.

All mirrors in the system should be front-surface-coated in order to eliminate ghost images which arise from the front surfaces of ordinary mirrors. Such coatings may be either of the low-reflectance type on ordinary glass mirrors or of the high-reflectance type in order to form a "first-surface" mirror.

Computer-Driven Displays

Having examined the use of the oscilloscope and the somewhat more complex and flexible stroboscopic projection system as 2-D pattern generators, we proceed to the still more elaborate and versatile general-purpose digital computer as a driver for the display generator.

It should be made clear that in contrast with the strobe-optical system, which uses photographic slides, computer-driven displays in current use do not ordinarily possess continuous-tone and color capabilities. This feature stems more from a lack of demand than from technical infeasibility, for most visual computer outputs can be displayed with perfect effectiveness in a monochromatic line format. Therefore the computer, if required, could be made to provide mirror-inputs in the form of continuous-tone color images. Essentially, this would amount to having the computer drive a color-television type of oscilloscope tube at the rate of about 30 depth-frames per second. A depth-frame would be comprised of a suitable number of depth-planes, each containing a three-color, intensity-modulated image. The demands upon computer storage capacity and speed would not be insurmountable but they would be severe compared to the ease with which photographic methods can respond to such demands.

The main value of the computer is the ease with which a simple computer-generated pattern is presented and updated in real-time. Signals from an external real-world situation can be introduced to change a given display more rapidly than the eye can follow. On the other hand, the most modern optical film projection display systems require at least several seconds to process an update command.

Thus, for the updating of monochromatic line drawings and alphanumeric character displays in real-time, the electronic computer has no equal. For this reason, a study was undertaken to determine the potential value of a computer for 3-D display applications.

The computer used was the IBM 7030, one of the more powerful systems available in recent years. It was programmed to drive a Data Display, Inc., DD-13 display console which was used as the pattern generator. A 12-inch square image was formed on the 19-inch diameter cathode ray tube (CRT) screen and was viewed in a 15-inch diameter vibrating membrane mirror. The DD-13 console is capable of presenting alphanumeric characters and arbitrary line vectors on the tube face at rates high enough to trace complicated planar images corresponding to several 3-D display depth-planes during the course of a mirror cycle.

A photograph of the display equipment appears as Figure 24. The image appearing in the mirror at the center of the photograph will be discussed presently. It appears in 3-D in Stereophotographic Pairs F and G (see centerpiece). Two possible computer applications were selected for this exercise: one was a visual representation of a 3-D mathematical function, and the other was a hypothetical air traffic control (ATC) situation similar to that used for the Mark I system.

Computer programmer adjusts speaker-mirror at DD-13 display console.

A Computer-Driven Display System
Figure 24

The mathematical function selected, the ambiguity function, is familiar to radar waveform designers. It was the subject of a similar study by Galvin (Reference 6), who used a CRT to generate a 2-D photograph of the mathematical surface in perspective projection.

Simulation of a Mathematical Surface

In the present study, the function was displayed as a family of vertical profiles in 25 planes equally spaced along the depth axis of the display. Each profile comprised a slightly different curve; collectively, the family provided a visual impression of a contoured ambiguity surface. Each curve was synthesized as a connected sequence of 81 vectors displayed in a small fraction of a millisecond while the mirror motion was essentially "frozen." After each curve was traced, a delay of about two-thirds of a millisecond allowed the mirror to move to a new position before the next trace was begun. In order to synchronize the mirror motion to the display sequence, the computer provided a timing signal to the mirror-driving circuitry for each complete set of traces. The circuitry is similar to that used in the Mark I display system. The approximately 30-cycle per second timing signals are used to generate a sinusoidal speaker-mirror input signal.

Initially, the sequence of curves was traced during one-half of the mirror cycle and then shown in reverse order during the return motion (mirror flyback). Each curve seen in the display volume thus was produced by two different traces per mirror cycle. However, small but objectionable mismatches were observed between the corresponding traces for each curve. One contribution to the mismatches apparently was caused by system mechanical and electronic instabilities which produced time-varying, random lateral fluctuations between consecutive traces. The other contribution resulted from a slight tilting of each trace with respect to the X-Y plane due to the finite motion of the mirror during the course of a trace. Because all curves were traced from left to right and only the ordering of the curves was reversed during mirror flyback, corresponding traces were tilted in opposite directions, producing double images to which the eye was quite sensitive. The most practical solution was to eliminate the tracing sequence during flyback. (This procedure was adopted for the [illegible] used in the Mark I [illegible]

One observation about the resultant display, significant from a human factors standpoint, should be mentioned here. This is the apparent overburdening of the 3-D perceptual processes when the spatial surface being portrayed is viewed edge-on. For viewing-positions not along the mirror axis, the ambiguity surface could be perceived quite effectively as a spatial entity because of stereoscopic and parallactic cues. However, as the observer approached the mirror axis so that the spatial curves appeared to fall behind one another, it was found that his depth perception mechanism was overtaxed with visual information, and the perceptibility of the spatial image deteriorated. The problem was aided considerably when the computer was reprogrammed to eliminate every second trace; the thirteen curves which remained could be distinguished even when viewed from a position along the mirror axis.

Our observation is, therefore, that the limit of depth perception is easily reached in a display pattern synthesized from a family of curves which are rather similar in their X-Y projections and viewed directly along the Z-axis. Nonetheless, the perceived volumetric effect was generally quite convincing and appears to offer interesting possibilities for real-time monitoring of three-parameter functional behavior as the parameter values are manipulated, especially if the surface were to be presented in the form of elevation contours and viewed from above.

Simulation of an Air Traffic Control Situation

The ATC display was somewhat more effective. Its elements consisted largely of isolated character groups (data blocks) that appeared to float in space within the image volume. The display was less "busy" and presented the observer with a perceptual situation similar to those which he encounters in ordinary visual situations, such as when observing aquatic life in a water tank.

The ATC display pattern shown in Stereophotographic Pairs F and G (see centerpiece) is intentionally similar to the photo-optically generated ATC pattern of Stereophotographic Pair D, although it is monochromatic. As in the case of the Mark I display, the altitude axis is expanded relative to the other two scales. A 6-second updating feature written into the program demonstrated rather effectively how the apparent aircraft positions might change with time when sensed by a 10-revolution-per-minute radar antenna.

Eleven data blocks, each representing an aircraft, are presented at display depths corresponding to six altitude levels above a ground plane. As the observer views the display along a horizontal axis, he witnesses the ATC situation as though looking down upon it from above. The ground plane includes a geographic coastline, an air traffic terminal area, and several ATC sector boundaries. Just above the ground plane is a pattern of federal airways with their intersections marked by circles. The individual characters in the data blocks are displayed in reverse order upon the DD-13 screen so that they will be readable in the mirror. They were synthesized from those alphanumeric characters which are symmetrical about their vertical axes because the present equipment is not designed to display characters in reverse.

The altitude separations of the data blocks are clearly discernible in the display. However, an additional program subroutine was incorporated as an aid in making altitude-separation judgments between aircraft with wide lateral separations. Through a light-pen action, the operator can interrogate the display as to the relative-altitude status of all aircraft appearing in the display at any moment. The display responds with a distribution of altitude markers along a single axis, with each marker indicating an altitude which happens to be represented by one or more aircraft at the time. The marker axis can be made to intersect any data block in the display so that the vertical separations between any aircraft and its neighbors can be examined. The markers are simply X's, without identification, forming a sort of "step-ladder in the sky" with irregular rung spacings.

The purpose of including this feature in the program was to allow a preliminary assessment of its utility, and our findings are that it does appear to be a simple and useful technique. There are, of course, many other forms of altitude scales and cursors which might be devised, and these should be studied in more detail if this work were to be continued.

This exercise in the computer programming of depth displays was undertaken to determine whether it is at all practical to use a computer for such purposes and whether the resulting displays would be effective visually. Both of these questions appear to have be . . answered in the affirmative. Provided that one does not attempt to display too much material at one time, the programming of the computer is straightforward and the 3-D perceptibility of the display is completely effective for trained and untrained observers alike.

NIXIE Tubes

A final study under the heading of pattern generators concerned the feasibility of achieving rapid intensity-modulation of electronic, gas-filled, cold-cathode indicator tubes in order to create volumetric rasters of arbitrary alphanumeric characters through operator control.

An example of such a tube is the NIXIE tube, a trademark of the Burroughs Corporation, which is widely used as a numerical readout element in a variety of electronic measuring instruments. It may be familiar to the reader as a glass-enclosed, in-line array of wire cathodes formed into numerical figures, any of which can be made luminous by the occurrence of an orange glow discharge around it when its terminal is energized.

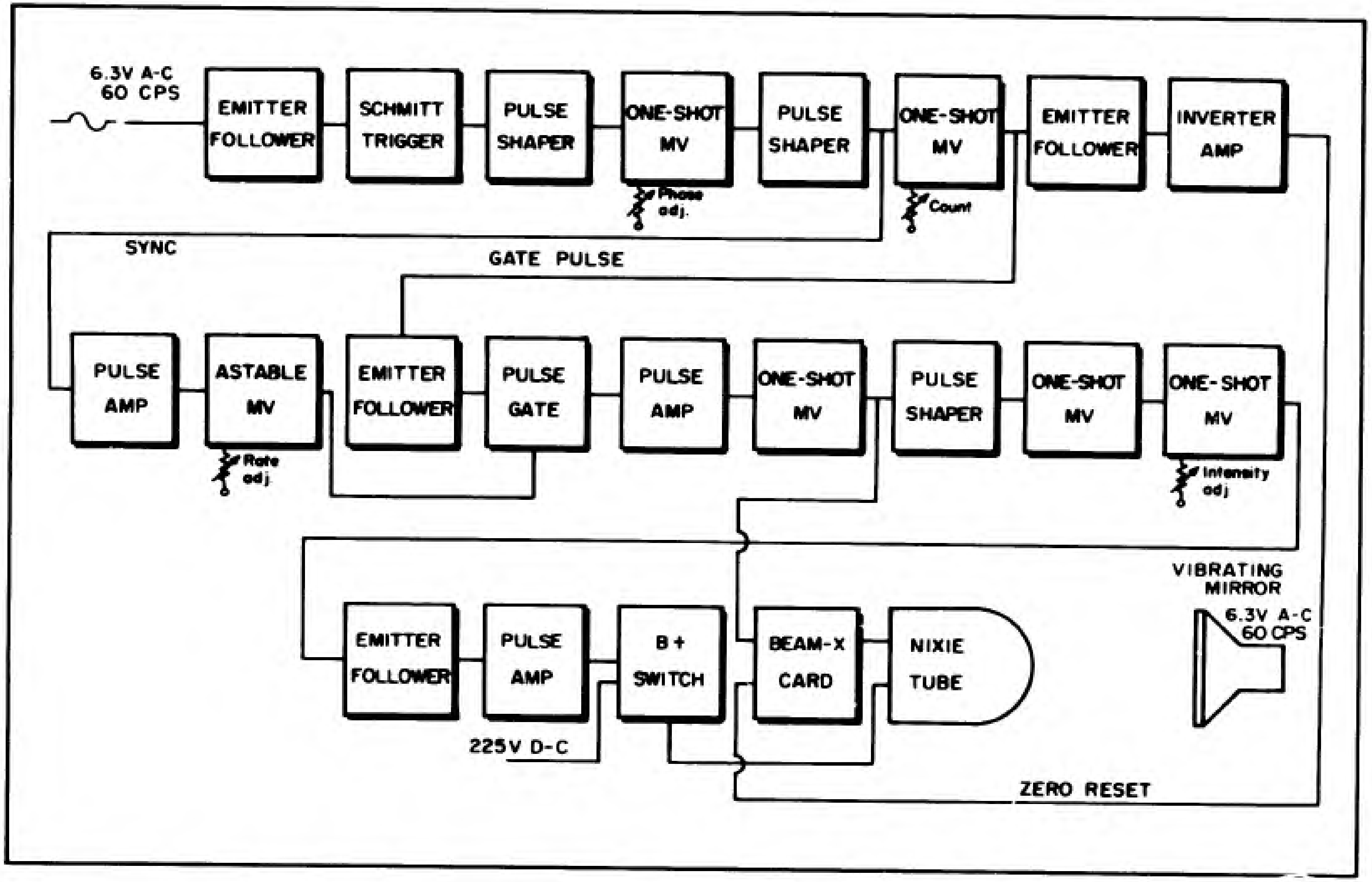
The original objective of this task was to be able to provide a flat matrix of characters which, when illuminated by rapidly modulated tungsten filament lamps, would comprise an alphanumeric pattern generator for use with the vibrating mirror. Wide availability and economy were the attractive features of tungsten light sources, but a brief study indicated that their use was highly questionable for this application because of frequency response considerations. In order to achieve crisp, ghost-free characters in the display volume, one would have to use a light source which could be nearly 100-percent modulated at several kilocycles per second, and this is not readily done with even the smallest of tungsten filaments because of their appreciable heating and cooling times. Although external modulation methods using mechanical or other modulators (Reference 7) are available, it was apparent that the use of a gaseous discharge type of light source would be a preferable alternative in terms of simplicity and effectiveness. For our purposes, the NIXIE tube was a particular convenience in that it already provided a complete set of separately wired, numerically shaped electrodes.

It therefore became the purpose of this task to implement an appropriate NIXIE tube driving circuit in order to assess its practicality and also to examine the brightness and modulation properties of the tube for this purpose. The study was confined to a single circuit and tube and did not examine the properties of a matrix of such elements.

To clarify the manner in which such a display was intended to operate, let us assume that, ideally, the driving circuit has the following capabilities. A steady train of short pulses, generated at the mirror frequency, is directed at any desired electrode. This causes one character to appear in the image volume created by the mirror. Its position along the depth axis is controllable in terms of the pulse phasing with respect to the mirror. Additionally, a second pulse train of the same frequency but with a controllable phase lag is generated at will. The second group of pulses is interspersed with the first but, through a repetitive electronic switch, could be diverted to a different NIXIE electrode, causing a second character to appear in the image space; its depth also would depend upon phase considerations. Repetition of this process, through operator control, would produce an arbitrarily-ordered depth sequence of all ten available numerals in the image space. With improved pulse switching techniques, it should be possible to repeat a character any number of times per mirror cycle and to generate any number of image positions along the depth axis, possibly even varying their individual positions and allowing them to pass through one another.

The experimental circuit built was by no means this elaborate; it was intended only to answer some fundamental questions. The nature of the circuit is indicated in Figure 25. Its capability was limited to pulsing the numerals of a No. B8091 NIXIE tube in an ordered sequence via a Burroughs BEAM-X Switch, with fixed time-delays between the pulses to the respective numerals. However, any number of the ten numerals could be activated serially per mirror cycle. The system thus was able to generate, for example, the numerals zero through nine, or any partial sequence of consecutive numbers, making them appear serially along the depth axis.

As with the other pattern generators, the numerals were activated during the one-way motion of the mirror and blanked out during its return.



Experimental Pulse Circuit for NIXIE Tube (Block Diagram)
Figure 25

The overall phasing of the circuit was variable with respect to the mirror so that the entire sequence could be shifted along the depth axis. Synchronization between the circuit and the mirror was achieved via the 60-cycle per second line frequency.

The visual results achieved with the NIXIE-generated display were for the most part satisfactory, although the tube had to be operated at low light-output levels so that the numerals would appear crisp. At higher levels, achieved by using longer and higher input pulses, the spatial images became somewhat diffuse, even to the extent of merging with one another. This appeared to be caused by the finite decay time of the glow discharge after an intense input pulse, although the decay time can presumably be shortened by use of appropriate quenching techniques.

In general, it can be said that the generation of alphanumeric or other volumetric matrices by this technique appears to be practical. A study of more sophisticated circuitry for driving the NIXIE tube is indicated for follow-on work, as well as a consideration of standard discharge lamps and of mechanical or other means for providing the sequence of characters to be illuminated by each lamp.

VARIATIONS OF THE BASIC DISPLAY

The reference system for this discussion, the "basic display," is conceived of as a loudspeaker-driven membrane mirror, generally 15 inches or less in diameter, which oscillates about a flat reference configuration.

A single pattern generator, located within the minimum positive focal length of the mirror, is used to form a virtual, erect volumetric image which is viewed directly in its behind-the-mirror location.

Through modification of the above system, especially by adding special purpose optical or other components, one can achieve variations in the form and nature of the display pattern to meet special needs. For example, one might transform the virtual image to a real image located before the mirror where it is accessible to direct spatial measurements. It is also possible, through the use of optical beam splitters, to superpose images from separate pattern generators and stroboscopically illuminated solid objects, thus creating rather complex but highly versatile displays.

A few of the many possible modifications were examined very briefly during this study and are discussed under the subheadings of this section. It was not believed necessary to implement working models of such devices because the physical principles are well understood and, apart from design decisions, major technical problems in building the devices are not anticipated.

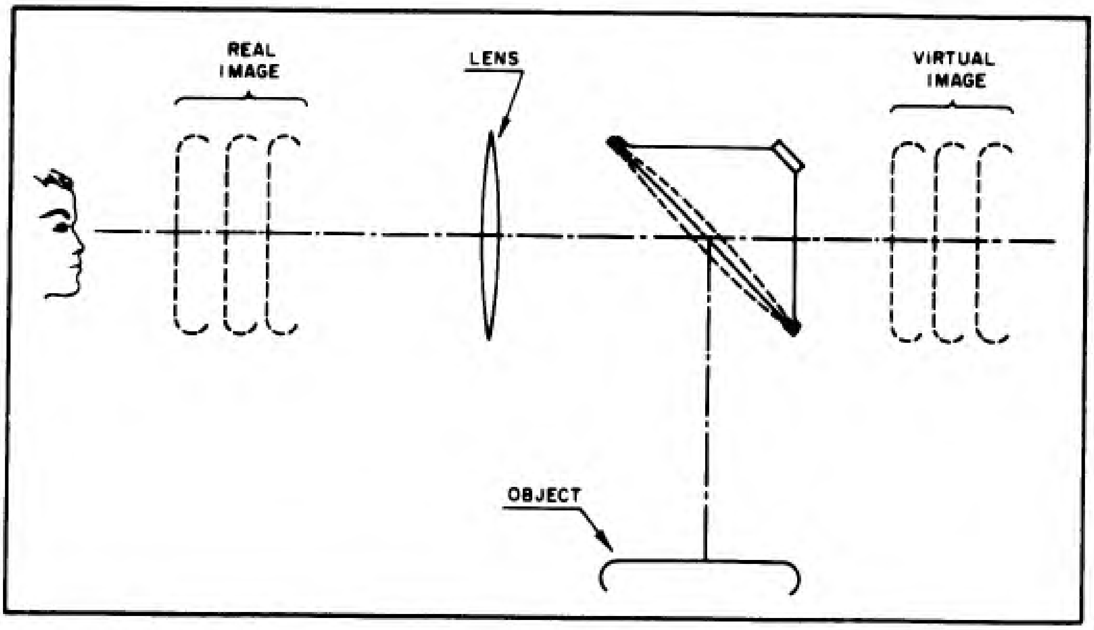
Virtual-to-Real Image Conversion

Section II includes a discussion of the fact that the virtual image exists behind the mirror where it is inaccessible but that, through a simple device, it may be transposed to a position in front of the mirror where it appears to exist in the air, as does the globe of Stereophotographic Pair A.

Two observations about optical phenomena need be cited in order to make this proposition plausible.

The first is that the formation of a real image by an optical element, such as a lens or a mirror, requires only that the object be located beyond the focal point of that element. Photographic cameras form real images in the plane of the film, provided that the object is at a greater distance from the lens than one focal length. This is approximately the lens-to-film distance when the camera is focused for infinitely distant objects. If the camera back is removed while the shutter is open, a real aerial image is visible in the vicinity of the film plane. This is an image appearing in the space which is bounded by the circumference of the lens aperture and which contains the film plane. Normally, the aperture is too small to allow simultaneous viewing by both eyes; however, with a large aperture lens one can verify stereoscopically not only that an aerial image exists but that it is a three-dimensional reproduction of the object scene. The ability of a lens to form 3-D aerial images is exhibited also by its reflecting counterpart, the concave mirror. In the case of the mirror, the object and image are on the same side of the element; in the case of the lens, they are on opposite sides.

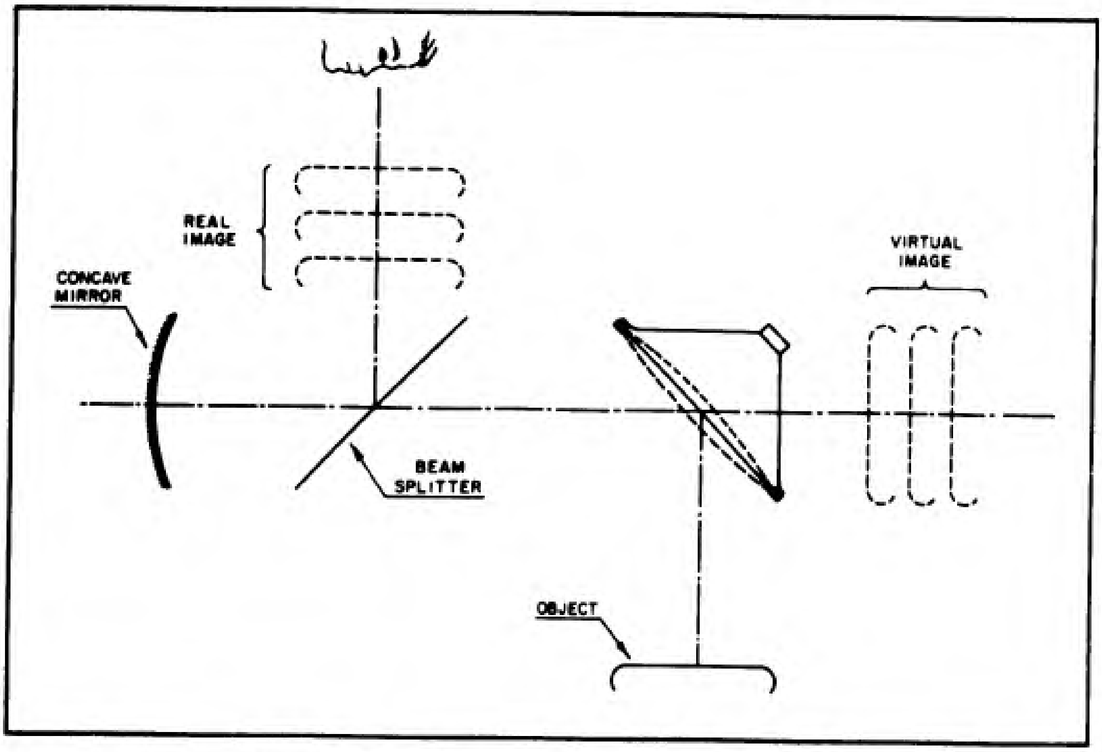
The second observation to be made is that a lens or a mirror is no more discriminating than the human eye in discerning between an actual object and a virtual image of that object formed by, let us say, a plane mirror. Stated differently, a virtual image can constitute a real object for an optical element and thus be transformed into a real image, aerial or otherwise.



Virtual-to-Real Image Conversion Using a Lens
Figure 26

(Actually, the term aerial image refers to a real image which is viewed directly by eye rather than being intercepted by a photosensitive surface or by a diffusing screen.)

Equipped with these two statements about optical phenomena, we are in a position to understand how a virtual 3-D image, as seen in a vibrating membrane mirror, can be transported to a position before the mirror in order to become accessible to mechanical cursors, metric scales, and so forth. It is merely necessary that the virtual image be re-imaged by another optical element which is placed at an appropriate object distance. An example of how this might be done is given in Figure 26 which indicates a lens as the re-imaging element, or optical relay. Alternatively, depending upon design preferences, through the use of beam splitters (see next sub-



Virtual-to-Real Image Conversion Using a Concave Mirror (Beam Splitter Moves Image into the Open)

Figure 27

section) and conventional plane mirrors, one might employ a concave mirror for the same function. When it can be used, the concave mirror is preferable because, as a matter of practicality, it happens that a mirror usually provides a wider viewing angle and better image quality than does a lens of equivalent cost. An example of how a concave mirror might be used in such a situation is indicated in Figure 27.

Apart from quality of manufacture, the choice of optical relay element is primarily a question of the focal length appropriate to a given re-imaging situation. In order to arrive at the necessary focal length, one need merely apply the thin-lens formula, Equation (1), inserting for p the optical path length from the center of the virtual image volume to the desired position of the relay and using for q the distance from the relay to the desired position of the center of the real image, with all distances being taken as positive. In addition, consideration must be given to the diameter of the usable aperture of the relay, for this will determine the viewing field angle over which the relay is useful. It must be recalled that the aerial image exists only within a region of space whose diameter is limited by that of the relay element, up to a certain point. This point is reached when the relay is large enough to form an aerial image of the membrane mirror frame, whereupon this frame becomes the limiting aperture. In any case, the aerial image will be visible to an observer only over a viewing angle which maintains the image between the viewer and the confines of the limiting aperture, be it the aperture of the relay lens or of the membrane mirror. This point is stressed to dispel any possible impression that the transformation of a virtual to a real aerial image will make the image completely visible from all viewing angles.

Superposition of Images

This subsection will consider another useful optical technique for expanding the versatility of the basic membrane mirror display. At the heart of this technique is a simple optical element known as a "beam splitter" or partial mirror. In an unsophisticated version, a beam splitter is familiar to us in the form of ordinary window glass, a sheet of which is able to transmit an image of one object while simultaneously reflecting, in the same direction, an image of a different object. Under the proper viewing conditions, the visual effect can be that of a combination of both images, and this phenomenon has been used as the basis of many superposition illusions in the motion picture and television entertainment fields. The effect may often be noted within an automobile interior at night, when external light sources on one side of the vehicle are reflected as virtual images in windows on the opposite side and are superposed upon the direct view of the night sky. Indeed, this effect is alleged to be the origin of more than one report of unidentified flying objects.

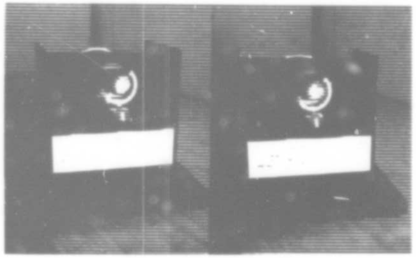
In serious laboratory applications where a transmitted and a reflected image are to be combined as a single, virtual image, ordinary glass sheeting is at a disadvantage in two respects. First, both glass surfaces are equally effective in producing reflected images and, because the surfaces are separated by a finite thickness of glass, the resulting reflected image is actually a composite of two identical images with a noticeable displacement between them. Second, depending upon the angle at which the glass is viewed, the intensity of the reflected image is usually a small fraction of that of the transmitted image, so that a brightness disparity results unless it is made up for in the illumination balance between the original objects.

As a result, laboratory beam splitters have been developed in which

1. the reflected image is produced substantially by only one reflecting surface and
2. the ratio of reflected to transmitted intensities can be controlled over a broad range as a result of proper surface preparation.

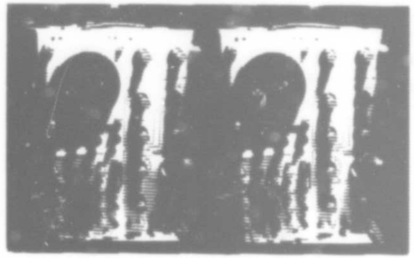
Several techniques are available for providing highly reflective, spectrally neutral coatings on glass or on other dielectric surfaces so that almost any desired balance can be achieved between the ratio of transmitted and reflected components. At the same time, low-reflectance coatings have been developed which when applied to the rear surface of a glass sheet make it optically ineffective. In addition, one can obtain, from commercial sources, microinch-thick plastic membranes whose front and rear surfaces are nearly coincident, thus providing substantially single images. These films can be supplied in a broad range of reflectance properties, providing considerable control over the transmitted-to-reflected energy ratio. One supplier of such films is National Photocolor Corporation of South Norwalk, Conn.

In addition to spectrally neutral coatings, special optical coatings can be provided in which the reflected light is predominantly of one color while the transmitted component is of the complementary color. Many of the major optical companies can supply a glass surface which, for example, appears yellow in transmission but blue by reflection. This type of surface is designated as dichroic; it finds several uses in modern technology such as in motion picture color photography where special dichroic surfaces are used to separate the image of a scene into its various color components for individual recording. In addition to this "beam splitting" application, such mirrors can be used as "beam combiners" although the name beam splitter seems to persits, even in this application. Thus, the transmitted image of an object of one color can be efficiently combined with the reflected image



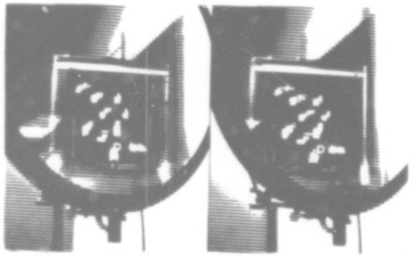
Pair A. An Aerial Image

The small globe which appears to be cradled in its stand provides an illustration of a real or aerial image. This image is formed in front of a mirror in contrast to the more familiar virtual image which is formed behind a mirror (see page 21). The actual globe is concealed within the box. Its 3-D image, which is what the reader sees in the cradle, is formed by means of a concave mirror.



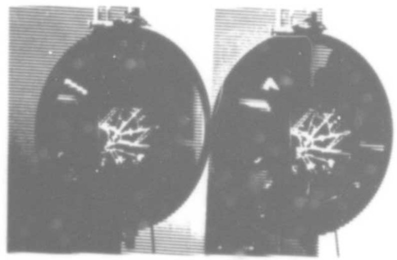
Pair B. A Conceptual 3-D CRO

This stereophotograph was prepared by multiple-exposure (or "trick") photography (see page 69). It is intended to depict an example of a multicolor 3-D CRO which can be built with today's technology.



Pair C. A 3-D Alphanumeric Display

The above display was achieved with the Mark I electromechanical demonstrator shown in Figure 21 (see page 77 for a discussion of this technique). Its possibilities for application include psychology laboratory tests of human depth perception.



Pair D. A 3-D Air Traffic Control Display

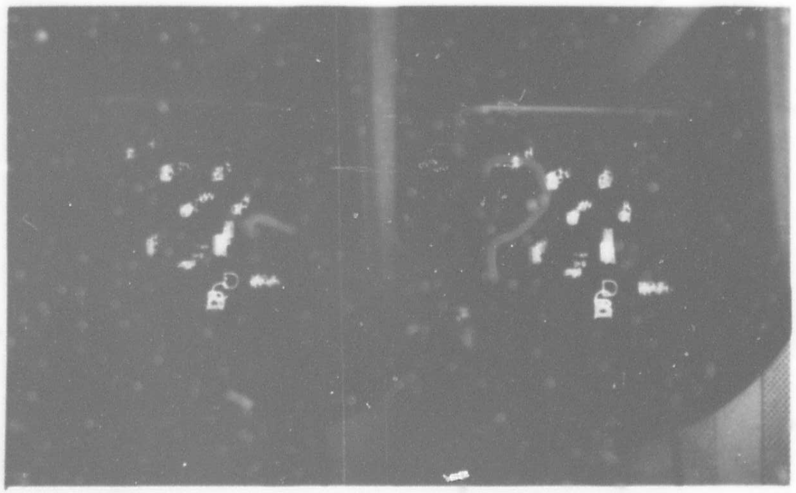
Each aircraft in this Mark I display is represented by a cluster of symbols and identifying data. Depth separations of the clusters represent altitude separations as they might be depicted in an ATC display of the future. The use of color coding, as illustrated below, would permit further separation of aircraft according to flight status such as climbing, holding or descending (see page 77).

NOTE

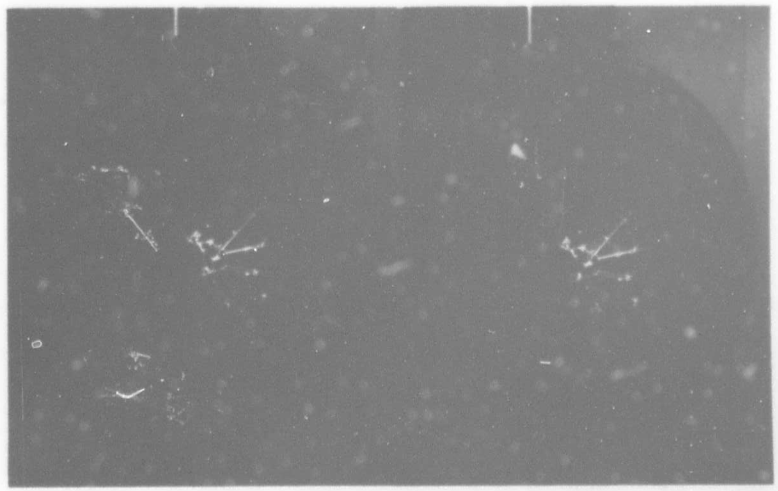
Original copies of this document include a centerpiece and a 3-D viewer. The latter is contained in an envelope bound just inside the back cover. If the viewer is missing, the reader can still achieve a 3-D effect from the centerpiece if he follows the instructions on page xii.

Pair A. An Aerial Image

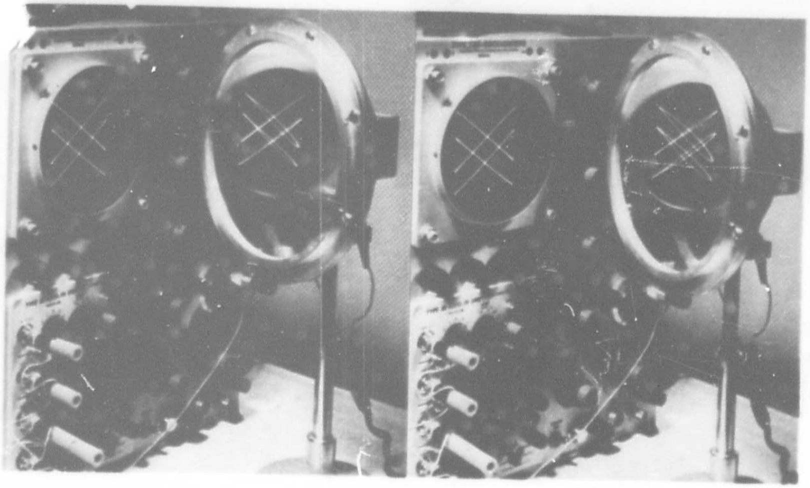
Pair B. A Conceptual 3-D CRO



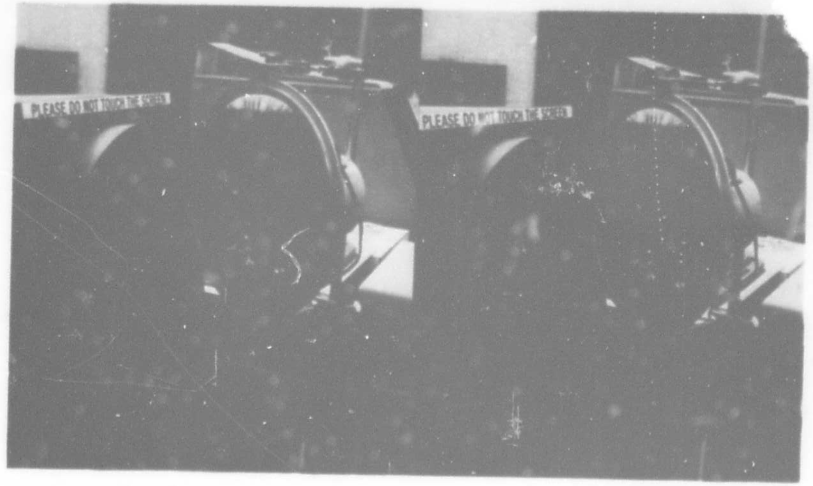
Pair C. A 3-D Alphanumeric Display



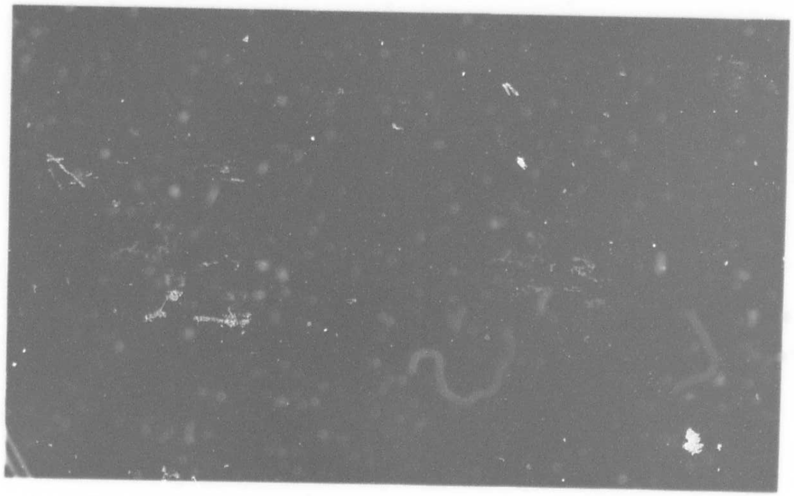
Pair D. A 3-D Air Traffic Control Display



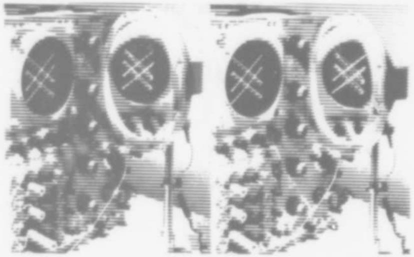
Pair E. A Vibrating Membrane Mirror Display



Pair F. A Computer-Generated Air Traffic Control Display

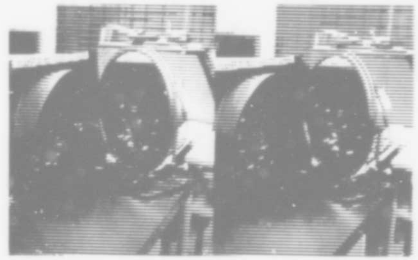


Pair G. A Computer-Generated Air Traffic Control Display: Close-up of 3-D Display



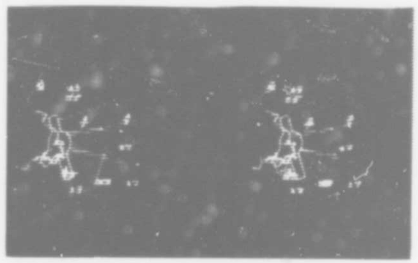
Pair E. A Vibrating Membrane Mirror Display

This stereophotographic pair shows a simple example of 2-D to 3-D conversion by use of a CRO and a vibrating membrane mirror (see page 18).



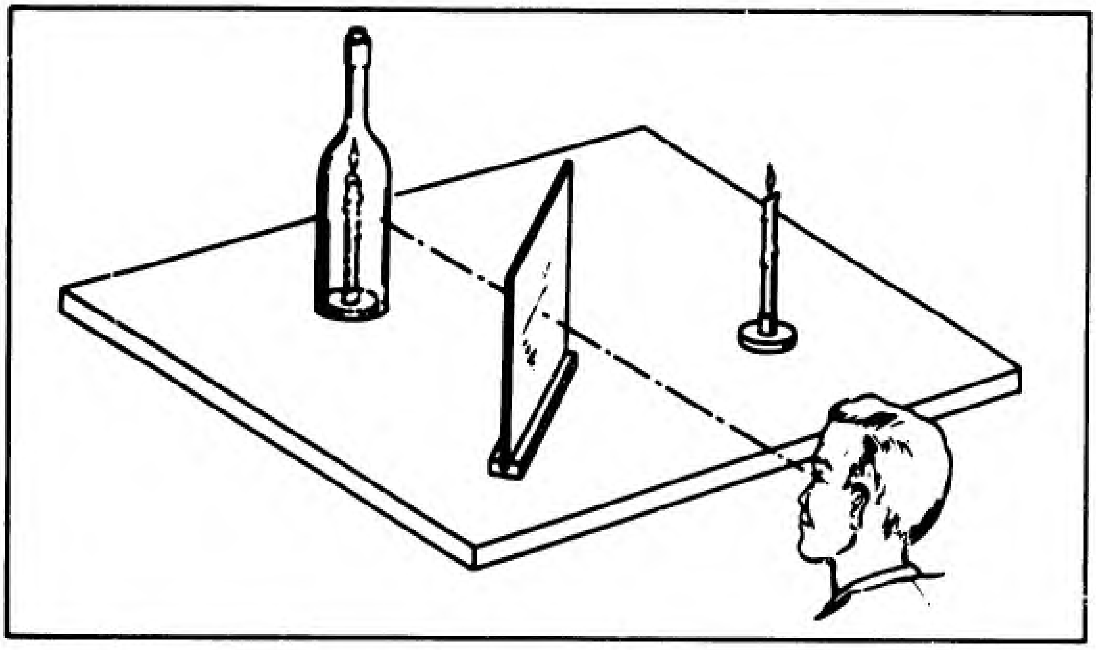
Pair F. A Computer-Generated Air Traffic Control Display

The display shown above is similar to the display pattern shown in Pair D but was generated electronically (see page 98) instead of photographically. The electronic display offers the advantage of being readily updatable in real time.



Pair G. Close-Up of a Computer-Generated 3-D Air Traffic Control Display

This stereophotographic pair presents a closer view of the display seen in Pair F. A spatial coordinate grid can be generated easily and superimposed on the display volume to facilitate comparison of aircraft positions in three dimensions (see page 98).



Optical Properties of a Beam Splitter (Illustrative Example)
Figure 28

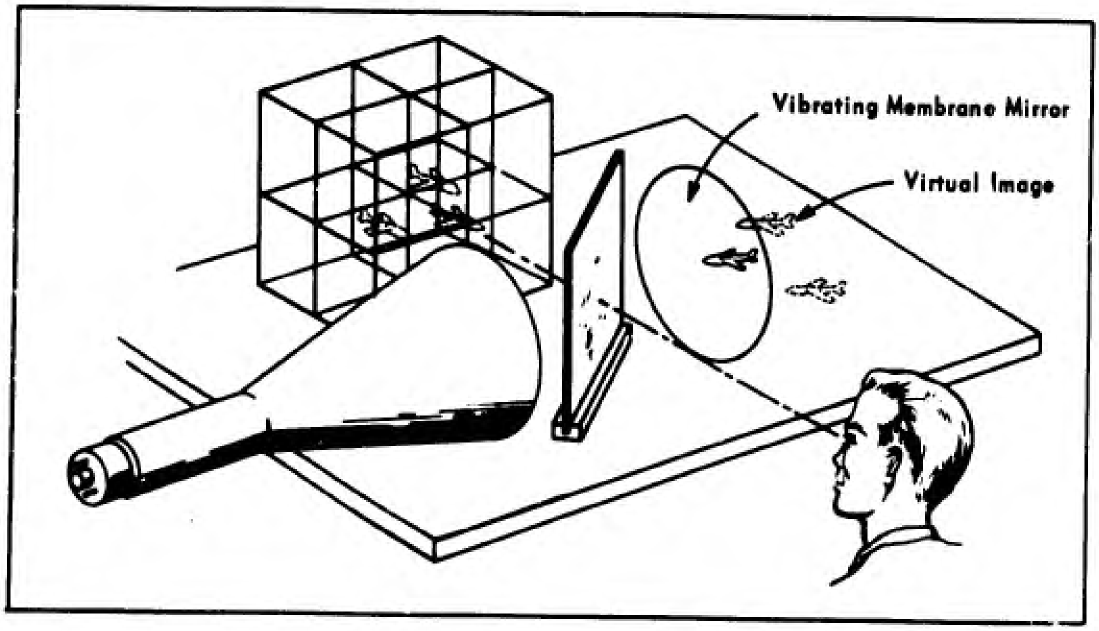
of an object of the complementary color, and it is this use of dichroic mirrors which is of special interest to the discussion which follows.

The operation of a beam splitter as an image superposition device is illustrated in Figure 28. We assume that the beam splitting element is of the dielectric multilayer type which is so prepared that at a 45-degree viewing angle, it provides a 50-percent neutral reflectance, a 50-percent neutral transmittance, and negligible loss of light by absorption. In the configuration shown, the element constitutes a partial mirror, providing a virtual image of the candle. The image appears to exist in space at a distance behind the mirror equal to the object distance. However, it appears only half as bright as does the real candle, the other half of the light having been lost in transmission through the mirror. Similarly, the direct image of the bottle as seen through the mirror will appear only half as bright as the real bottle; in this case, the balance of the light is lost by reflection. If the candle and bottle are equidistant from the beam splitter, the candle image can be placed within the bottle where it will respond to such tests of visual depth perception as stereopsis and parallax.

The optical symmetry of this situation is illustrated by the fact that if the observer swings toward the left through a 90-degree arc about the mirror, he will receive the other 50 percent of the light which was absent in the first situation. In this case he will see an image of the bottle superposed upon the actual candle. The resultant visual effect, although reversed from left to right, will be the same.

Figure 29 illustrates a simple possible use for a neutral density beam splitter in combining a virtual 3-D image with a real spatial coordinate grid. The targets are shown within the grid as they might appear to the observer through the beam splitter. The perceptual depth cues, of course, would be more effective than can be represented in a flat drawing. Alternatively, the grid could be replaced by a mechanical cursor or pointer which, by means of calibrated control knobs, might be used as a measuring or indicating device.

Because of its three-dimensional image superposition properties, the beam splitter lends itself well to combining flat images from separate pattern generators before they are introduced to the vibrating membrane



Superposition of a Virtual Image upon
a Real Grid by Use of a Beam Splitter

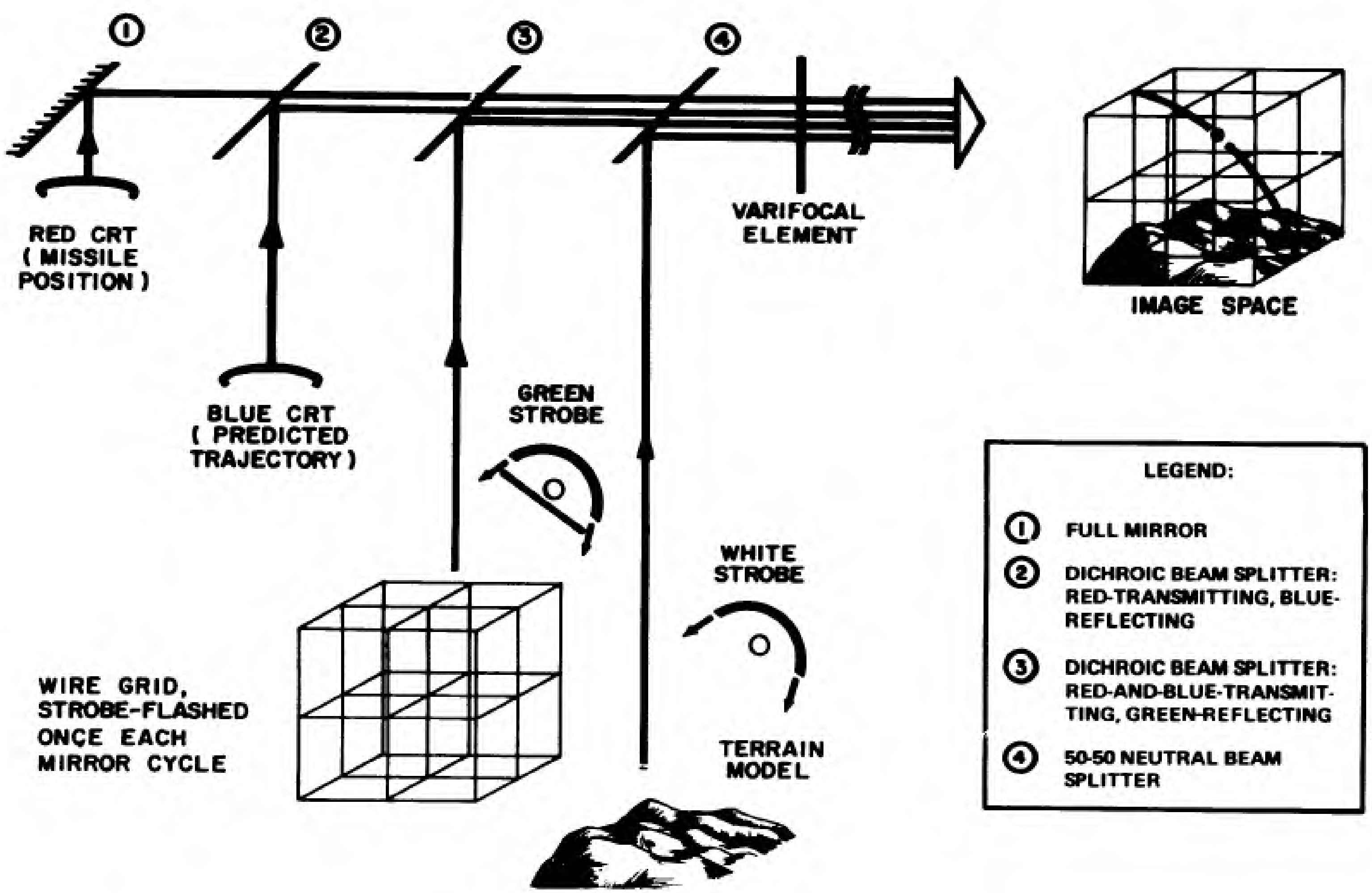
Figure 27

mirror. This technique can be useful in building display patterns with pattern generators which must operate in different scanning modes, in different colors, or from separate sources of input information. Two-dimensional patterns created by CRO displays and optical projection systems may be combined. Moreover, virtual images of solid objects may also be combined with the display; they need only be stroboscopically illuminated once per mirror cycle, usually when it is in the flat configuration, and introduced into the system via an additional beam splitter.

One beam splitter is generally necessary for each input pattern added to the display. In principle, there is no limit to the number which can be used, although in practice the addition of each new beam splitter exacts a toll in energy from the remaining light beams. Also, as more input devices are added, the increase in path lengths from the respective pattern generators to the mirror becomes a problem. All such optical paths must be equal if the separately generated patterns are to be imaged at the same depth in the mirror. If the path lengths must be increased to provide physical space for additional pattern generators, the volumetric image recedes, with a corresponding loss of viewing angle. In such a case, an optical relay may be found useful in presenting the mirror with a nearby real image of the aggregate of input patterns.

A conceptual portrayal of how a family of image generators may be combined through the use of beam splitters to create a composite display is shown in Figure 30. The example selected here is the hypothetical case of a missile trajectory display during the early-launch phase; the actual missile position is being compared with its intended trajectory for guidance and range-safety purposes. The predicted trajectory is generated by a computer-driven CRT while radar or optical tracking information provides the basis for the display of instantaneous missile positions. The various ingredients of the composite display may be separately colored, in which case they are more effectively combined through the use of dichroic beam splitters if light loss is a problem.

In addition to combining pattern generator displays as inputs to the membrane mirror, one can also combine various vibrating mirror outputs into a single depth display through the use of beam splitters. This technique



Superposition of Stereoscopic Images
Figure 30

would be of use if, for some reason, it were desired to combine depth images generated by separate mirrors which differed, for example, in size or in frequency.

Finally, a luminous marker such as an arrow or a circle can very easily be incorporated into the depth display behind the mirror and can be moved freely throughout its volume. In this case we visualize an optical projection screen as a special purpose pattern generator for the marker. The screen image is combined with the display volume via a beam splitter. The marker is imaged upon the screen by use of a stroboscopically flashed projector synchronized with the vibrating mirror. Control over the X-Y position of the marker in the display is exerted through proper aiming of the projector. The marker's position along the depth axis is controlled by varying the phase relationship of the stroboscope with respect to the mirror.

Alternatively, the marker may be projected directly upon the pattern generator surface, if this is convenient, thereby eliminating the need for a beam splitting system.

In summary, it may be said that the utility of the basic display can be greatly enhanced by the use of neutral or dichroic beam splitters. The images produced by separate pattern generators, real objects, and 3-D display patterns, in monochrome or in color, may be combined in a variety of ways to meet special needs, the diversity of which is limited only by the imagination of the display system designer.

Electrostatic Drives

The final topic to be discussed in Section III is the possibility of achieving "large-screen" displays whose diameters are not limited by the practical considerations involved in the use of large loudspeakers as drivers.

It is obvious that the metalic coating which provides the membrane mirror with its reflectance properties is also an excellent electrical conductor. This property makes the mirror suitable for use as one plate of a capacitor if a fixed mating plate is added parallel and close to it. If a potential difference is applied across the plates, an inward deformation of the membrane should result from the electrostatic forces of attraction between the capacitor plates. Conversely, if like charges are applied to the capacitor plates, an outward deformation of the membrane should occur.

The electrostatically driven membrane mirror is attractive for two reasons. First, the implementation of vibrating membrane mirrors with diameters of several feet may be possible without the need for correspondingly large and costly loudspeaker systems as drivers. Second, the capacitor-like structure of such a device offers a weight- and space-saving feature in that its thickness need not be more than an inch or two, making it suitable for direct "on-the-wall" mounting.

A simple experiment performed during the program established the basic feasibility of such a system. A more detailed engineering study would be required, however, to ready the device for practical use.

In the experiment, a 15-inch diameter mirror-capacitor structure was assembled, with a copper-clad plastic disc as the stationary member. The plate separation was about 1/8 inch, and the stationary plate was coated with a self-vulcanizing rubber compound to discourage gap breakdown.

Laboratory power supplies were wired to the assembly in order to provide a 3-kilovolt rms, 60-cycle per second signal, initially riding on a dc bias voltage. The purpose of the bias voltage was to suppress the frequency doubling phenomenon (second harmonic) which would otherwise occur because the membrane would tend to move toward the fixed plate during both halves of the ac cycle. Through theory and experiment, it was then found that unless the driving frequency coincides with the mechanical resonance of the system, one must use bias voltages several times higher than the ac amplitude to ensure that the frequency doubling will not be objectionable. Failure to do so leads to a contamination of the fundamental by a small second harmonic component, as well as by higher ones, with the result that the volumetric image is distorted.

On the other hand, if the bias voltage is eliminated, the predicted mechanical motion is almost purely sinusoidal at twice the driving frequency. The mirror deflection is in a single direction only, with respect to its reference position, and, although a shift in the mean position of the image is introduced, it should not be objectionable. It seems preferable, then, to avoid the use of a bias voltage and to drive the mirror at one-half the desired frequency of motion in order to enjoy the benefits of a rather pure waveform.

In the experiments performed, the 3-kilovolt signal was sufficient to drive the mirror slightly and to provide a modest extension in depth to a CRO-generated Lissajous pattern. The volumetric image was seen to be less distorted without the bias voltage than with it. The depth dimension was enhanced as the driving voltage was increased to about 10 kilovolts, although some image distortion was noted even without the bias. At the higher ac voltage, sporadic electrical breakdown of the air gap caused some portions of the mirror coating to vaporize, and the voltage was not increased further.

No additional work was done on this type of mirror drive, although it is believed that the gap breakdown problem can be eliminated with conventional insulating methods. One might thereby obtain usefully large deflections in a 15-inch diameter capacitor-mirror with driving voltages of perhaps 10 to 20 kilovolts.

Questions which should be further examined concern the extrapolation of these results to much larger mirrors and to higher frequencies. Also of interest are the questions of possible membrane breakup, required driving powers, and acoustical levels.

As indicated on page 62, North American Aviation, Inc., have achieved higher mirror frequencies through the use of metallic foil membranes (Reference 5). Moreover, their studies show that air pressurization about the membrane is helpful not only in extending the bandwidth but in reducing the high-voltage gap breakdown problem as well. The air-tight mirror-capacitor assembly used has a front window which is coated with a transparent conductive material. The window thus serves as an additional capacitor plate so that the system may be operated in a push-pull mode, which, it is claimed, eliminates frequency doubling and provides certain other advantages.

These results are cited as an indication of possible directions in which improvements may be found.

DISCUSSION

We have seen how the basic membrane mirror operates and have noted some of the display devices which can be created from it. The remainder of this report, including Section IV, is addressed to the questions:

1. What new knowledge has resulted from this work?
2. What is the significance of this knowledge?
3. What should be done next?

In the author's view, the following observations can be made as a result of the program.

1. The use of a loudspeaker-driven flexible mirror appears to be a practical, reliable, and economical way of creating a volumetric raster from a fixed object plane.

2. The device is readily adaptable to displays that range from simple to sophisticated – from CRO-generated figures for teaching purposes to large-scale, computer-generated displays for air traffic control or other applications.

3. By means of electrostatic or other driving methods, membrane mirrors should be adaptable for use as large-screen, on-the-wall displays several feet in diameter.

4. Modern optical and electronic technologies offer effective techniques for applying the mirror to special uses. Problems associated with anomalous perspective, with combining multiple volumetric images, and with providing coordinate grids or cursors can be readily solved. The availability of high-intensity flash lamps and of electronic drivers and synchronization methods makes high-speed, photo-optically generated patterns practical.

5. The operating limitations of the present form of speaker-mirror are better understood, particularly in regard to high-frequency response and waveform-following characteristics.

The significance of these findings appears to be that the vibrating membrane mirror is one more tool which should be added to the technology of volumetric displays.

If considerations of cost and complexity have impeded the acceptance of these devices, it is possible that the new method, because of its simplicity and economy, can help to overcome such barriers. It should be remembered however, that in many applications the predominant factor is the cost of the equipment required to generate the 2-D patterns. If a multimillion-dollar computer must be used to provide the initial 2-D display, it matters less whether an economical or a costly volumetric raster generator is used. In this case, the economy of the membrane mirror system becomes less important, although advantages may nonetheless be afforded through reliability, maintenance, and performance considerations. The latter includes the possibility of large-screen displays with a greater display depth than is practical with similar devices.

The most important question concerns what more can be done to hasten the possible exploitation of this device. Apart from the matter of technical improvements, which might expand its versatility, there is the larger question of whether 3-D display devices indeed have any place in the area of man/machine relationships. This question is pondered in Appendix III, where the point is made that human factors studies of the value of 3-D displays have not kept pace with technological developments.

From childhood, man learns to extract information from his environment through the medium of three-dimensional perception and he continues this practice in his day to day existence. It is the author's belief that a machine which can communicate visually with an operator in terms known to him will be more effective than one that demands he learn new perceptual and interpretative skills.

We must, however, await the findings of further studies in this area to be sure that such speculation is on firm grounds. In the meantime, we can only content ourselves with offering some suggestions as to the form such research should take and as to the possible technological refinements in the vibrating membrane mirror concept which might make it more useful.

These matters are discussed in Section IV.

SECTION IV

RECOMMENDATIONS

GENERAL

As has been pointed out, the greatest single deficiency in the area of 3-D display technology appears to be a background of knowledge of how to use such systems effectively in man/machine relationships. As recently as 1963 it was stated by Leibowitz (Reference 8) that the intuitive arguments supporting the value of 3-D displays had not yet been documented with reliable empirical data; this situation appears to be unchanged at the present writing.* The substance of the findings is that those few human factors studies which have been performed were confined to non-specific laboratory situations and the results are not extrapolable to operational realities.

Therefore, the very first observation to be made is that there is a great need for human factors research on the value of 3-D displays. The objectives of this research should be to make better use of a technology in which there has been an appreciable national investment and to discourage developmental efforts which may not be pointed in useful directions.

Second, although general laboratory studies of human responses to visual depth cues are undoubtedly of value for their own sake, it is stressed that the uniqueness of each operational display situation dictates that it must be examined as a separate case. Therefore, the immediate emphasis should be placed on applied research, directed at specific applications, and carried out under realistic operational conditions.

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*The subject is treated more thoroughly in Appendix III and has been discussed by other authors, such as Vlahos in Reference 1.

These arguments apply, of course, only where one is thinking of large-scale applications of and multimillion dollar investments in 3-D man/machine systems, such as for air traffic control, space vehicle navigation, and so forth. A CRO-driven 3-D display for demonstrating Lissajous figures as a teaching aid need not be subjected to a human factors evaluation and, indeed, there are many simple applications for which the user himself is adequately qualified as a human factors expert and can make swift and valid decisions. Appendix IV lists a number of possible applications which have occurred to the author, many of which are ready for immediate use.

FUTURE TECHNICAL WORK

The remaining points to be made here concern possible technological improvements in membrane mirror 3-D displays with regard to both systems and to components which might lead to new system concepts.

3-D Live-Scene Portrayal

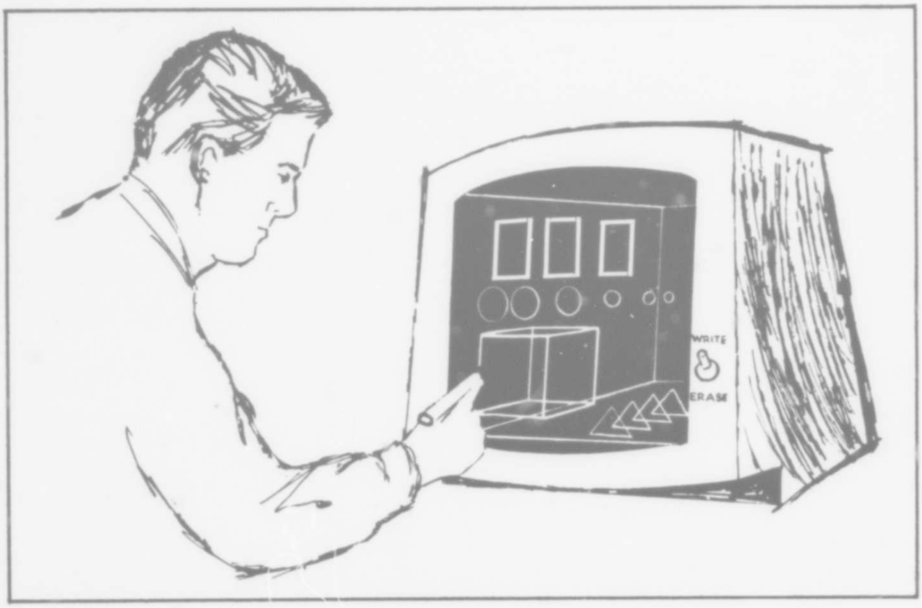
During the course of this work, the question has frequently been asked as to whether this method can be applied to the three-dimensional portrayal of live, natural scenes, giving rise to the possibility of 3-D television and motion pictures. Some thought will reveal that the method does not lend itself naturally to such presentations, although it is conceivable that the technical problems could be solved after some effort. Appendix IV ends with a discussion of these problems and also of some rather discouraging cost/effectiveness considerations which enter after the technical problems have been solved.

Accordingly, a recommendation of this report is that since cost considerations limit the attractiveness of these systems they should receive critical scrutiny before any developmental efforts are considered. Because of the tremendous data handling burden placed upon the 2-D pattern generator, such systems would find it difficult to compete with the somewhat less effective but much more economical transmission of stereoscopic image pairs, a technique which is already quite practical. The unique value of the membrane mirror system lies in its use with much lower information-density displays comprised of line patterns and alphanumerics shown at moderate data rates.

A 3-D Blackboard

A somewhat less remote possibility of an application is suggested here in the form of a direct-write, direct-erase version of an ordinary blackboard. Let us imagine a console containing a cubic volume of empty space. The writing tool resembles a pen-like flashlight which contains a tiny light source in its tip. Using this "optical stylus*," an operator can trace a volumetric figure in the empty space, whereupon the system provides a luminous spatial replica of the successive pen positions, permanent

* The term "optical stylus" is used to emphasize the difference between this device, which is a light source, and a light-pen, which really is a photosensor.



Conceptual Model of a 3-D Blackboard
Figure 31

until erased. A sketch of one possible form of the system is shown in Figure 31. Such a system is well within the realm of modern electro-optical technology because it simply extends, to three dimensions, the already existing man/machine graphical communications systems which permit one to draw figures with a light-pen directly on the face of a computer-driven CRO display screen (References 9 and 10). What is required for implementation is an optical, 3-D position-sensor which feeds a data processing and storage system to generate an appropriate 2-D pattern sequence. This sequence is converted to a virtual volumetric image via a membrane mirror, and an optical relay system transforms this into a real volumetric image coincident with the stylus positions traced out by the operator. In the erase-mode, the system converts the stylus into an aerial vacuum cleaner, so to speak, which can selectively obliterate elements of the spatial trace. For a final touch, one might add a three-color capability, controlled by manual switch action. As a practical matter, the field-of-view would be confined more narrowly about the operator than is depicted in the drawing.

Although such a system is not readily visualized as a standard operational accessory for general-purpose computer work, it is likely to find some uses as a research tool in specialized studies of man/machine relationships. Therefore, the recommendation made here is that the "3-D blackboard" concept should be explored for applicability to such studies when appropriate. It is a system which should not be difficult to implement and which should be useful as an effective interface device in man/machine graphical communications, for it would offer data input and output modes which are well tailored to human capabilities.

Other Programming Methods

Returning to more conventional means of programming the volumetric displays, it is suggested that several possible methods may be practical other than those examined during this work. For example, in the discussion of cathode ray oscilloscopes it was shown how a set of three periodic electrical signals might be combined and presented as a luminous volumetric trace, a simple example of which is the Lissajous pattern family. It should therefore be possible to create spatial curves of arbitrary shape by using hand-drawn parametric curves or families of points as the input data. Such functions could be prepared manually, one each for the X-Y, the Y-Z, and the X-Z planes and, after having been transduced to electrical signals by electro-optical image scanning or other techniques, could be programmed in serial fashion for a 2-D pattern generator by use of magnetic tape or other means.

Miscellaneous Improvements

Certainly, the 3-D oscilloscope as an analytical laboratory tool offers interesting possibilities and, as stated previously, some of these are being studied at The MITRE Corporation under an allied project. An important limitation of the present form of 3-D CRO is the limited frequency response of the speaker-mirror. Attention should be directed to other means of implementing flexible mirrors and mirror-drivers with better frequency characteristics. Also, we believe that the possibilities of mirror-driving assemblies several feet in diameter have by no means been fully explored; considerably more work can be done in examining the use of horn-type speakers, Helmholtz resonators, electrostatic drivers, and other devices. The large-screen mirror also carries with it the problems of breakup and consequent image distortion, which begin at lower frequencies than with small mirrors; this matter would have to be studied in some detail.

Obviously, there are other areas of study and application to which the membrane mirror display concept might lead, and the reader is likely to think of several in his favorite areas of interest, just as the author has.

For the moment, however, we shall close this discussion with a summary of the recommendations which have been presented so far.

SUMMARY OF RECOMMENDATIONS

1. Human factors studies of large-system 3-D display efficacies under meaningful test conditions should be performed before further considerable effort is expended on display technology development.

2. Many possible smaller-scale applications of the membrane mirror display are ready for immediate use and should be given direct consideration.

3. Three-dimensional television and motion pictures using this method are remotely possible from a technical standpoint but impose such severe bandwidth demands upon the 2-D pattern generator as to be economically infeasible.

4. The "3-D blackboard" appears to be technically feasible and should be examined as a possible research tool for man/machine graphical communications studies.

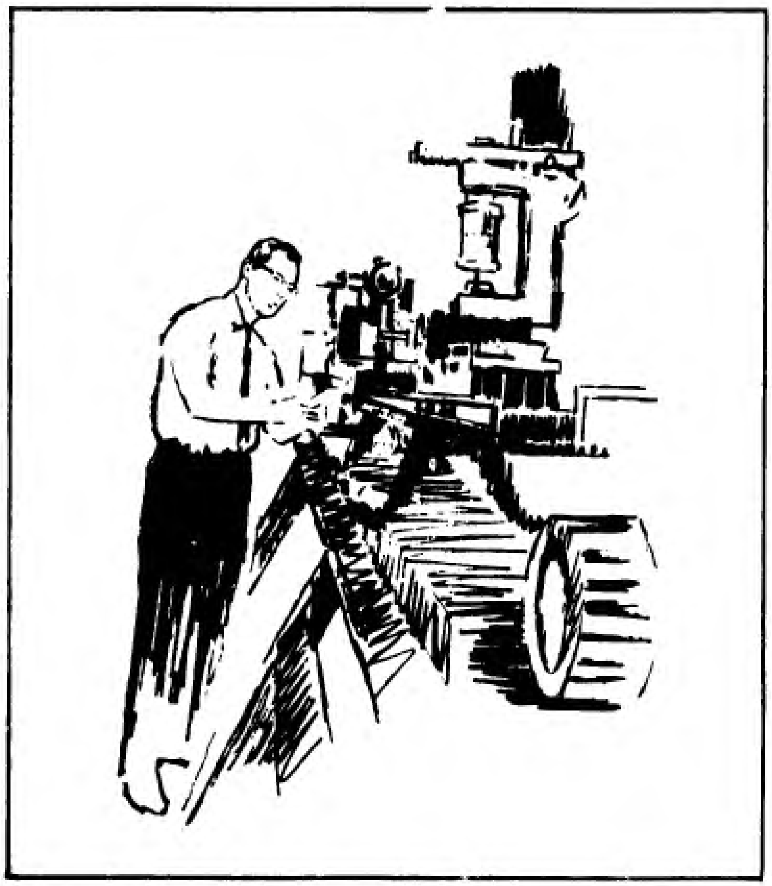
5. New methods should be sought for programming the 2-D pattern generator in order to expand its versatility, such as through the use of graphically prepared parametric curves.

6. There is need for considerable further work on improving the frequency response properties of the basic membrane mirror system, with a special view to applications involving high frequencies or complicated waveforms and the use of large-screen displays where breakup can become a problem.

7. Areas in which there is room for basic technical improvements include:

- automatic correction of anomalous perspective;
- superposition of manually controlled cursors and scales;
- pulsing and switching circuits for NIXIE tubes;
- other electronically controlled character generators; and
- larger electrostatically driven mirrors.

APPENDIXES



APPENDIX I

THE DEPTH PERCEPTION PROCESS

INTRODUCTION

In the words of a noted optical physicist, "Perception is our awareness of the things around us" (Reference 11).

The process of sensory perception, of which depth perception is a special part, is more involved than the above description of it. Through our senses we are continually bombarded with a multitude of signals which, when properly sorted and interpreted, provide information about our environment. The interpretation of these signals, whether they are visual, auditory, or other, depends upon learning and experience. From infancy we have learned to integrate the sensations from the various channels into a coherent impression of the world and of the objects in it. We have learned also to rely heavily upon the visual process as a surveillance mechanism which can reach out into the surroundings and bring us information about the locations and identities of remote objects. This information serves largely to guide our hand and foot motions so that we can navigate through the environment and manipulate objects. Another writer has pointed out that the human being is capable of but a single mechanical interaction with his environment: he can only move objects from one place to another. This simple capability, with which we have built cities and earth satellites, is vital to our everyday functioning and to our very survival. Such a capability would not be possible if we did not possess some form of perception.

SPATIAL PERCEPTION

Spatial perception, an awareness of where objects are, is a necessary ingredient in these interactions with the environment. Although spatial perception is accomplished largely by the visual process, the other senses can play an important part also, as in the case of sightless individuals. The popularity of "blindman's buff" and of stereophonic recordings attests to the abilities of the tactile and aural senses to provide spatial cues.

The visual sense is certainly the most refined and useful one in helping us to find objects in three-dimensional space. Visual spatial perception is made up of two distinct mechanisms. The question of whether an object is above or below or to the right or left of the line-of-sight is a matter of the position of its image on the retina. Its spatial location in the vertical and horizontal directions is thus determined directly by the brain from information provided by either eye. Less direct is the determination of its position along the depth axis, for the visual mechanism is not an "active" ranging system, as is a radar for example. Being passive, the eyes must rely upon several indirect cues and upon their proper interpretation by the brain.

Monocular Cues

Most of the depth cues are the so-called monocular ones which are effective with either eye. The only important additional cue afforded by both eyes working together is stereopsis, which provides a unique sensation of depth unlike any of the monocular sensations. (Another binocular cue is sometimes listed but its importance is often questioned; this is "convergence," a sensation of muscular pull as we shift our gaze from distant to nearby objects.)

The most important monocular cues for depth perception are:

(1) motion parallax, or apparent relative motion between nearby and distant objects as the observer moves;

(2) linear perspective, in which receding parallel lines seem to converge and receding objects diminish in angular size;

(3) aerial perspective, in which distant objects appear "hazy" due to light-scattering processes in the atmosphere;

(4) obscuration, overlay, or interposition, indicating which of two objects is in front of the other;

(5) the effects of lighting and shadows; and

(6) accommodation, the ability of the eye lens to vary its focal length for very nearby objects, forming defocused images of distant ones.

All of the above cues in a real scene can be reproduced through the single lens of a motion picture or television camera.

Binocular Cues

By adding stereopsis, we form a rather complete list of the important cues in visual depth perception. Stereopsis is not necessary to depth perception, and its effective range is much shorter than for all other cues except accommodation. The primary usefulness of the stereoptic ability is apparently confined to within the immediate arm's reach of the individual, and, as a precision range-comparison mechanism, stereopsis serves him well in guiding his hands toward objects. It is also useful just outside this range as an additional aid in guiding one's footsteps through his environment. Despite its short-range effectiveness, stereopsis is highly important to us in grasping and manipulating objects, an activity which perhaps consumes more of our time than any other.

Beyond distances of a dozen feet or so, the stereoptic ability to discern small depth separations between objects falls off rapidly, although large separations can be perceived at greater distances. Generally speaking, however, the maximum distance at which an object can be discerned stereoptically from an infinitely distant one is in the 250- to 1000-yard range, depending upon the observer and viewing conditions.

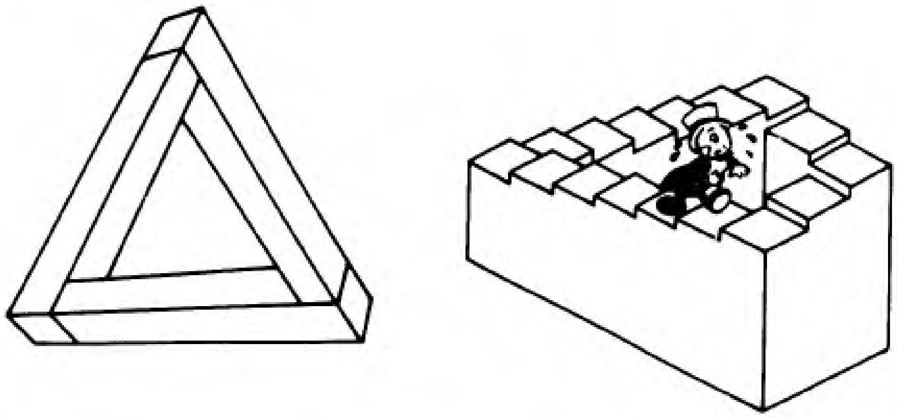
The word "stereopsis" stems from Greek roots meaning "to see solid." This phenomenon has been the subject of study from the time of Euclid, who pointed out that the two eyes see slightly different views of the world. Even today its mechanism and role in depth perception are subjects of continuing study, as evidenced by the number of scientific papers which treat them each year (see, for example, References 11 through 16).

The phenomenon of stereopsis is described in a number of texts, as in References 17 through 20, and will be dealt with only briefly here. The distance between a pair of human eyes, which is normally about 2-1/2 inches, is responsible for our slightly disparate views of objects. Simple geometry can show that the right- and left-eye views differ the most for nearby objects and become more nearly alike for distant ones. Geometrical reasoning also shows that the optic axes of the eyes can converge upon only one point in space at a time, this point being known as the fixation point. In binocular depth perception, the axes automatically converge upon, or fixate, the object which is being viewed at a given time and produce a single visual impression of it. In this process, the details of objects at all other distances can be perceived as double, the more so if they are widely separated in depth from the object being fixated.

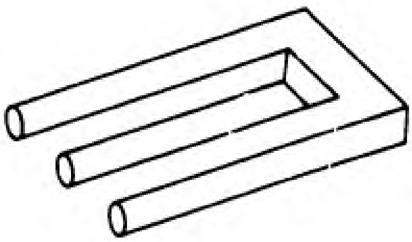
It is the observer's subtle awareness of these double images and of their relative motion as he shifts his fixation point to various distances which gives rise to the sensation of stereopsis.

DISCUSSION

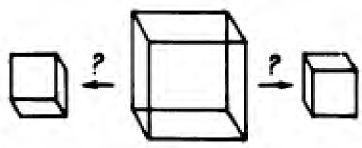
In any natural viewing situation, some or all of the depth perception processes may work together, and we rely heavily upon them to provide basic information about the locations of objects relative to ourselves. When several such processes are providing depth cues at a given time, it is especially important that all such cues be completely compatible with one another. We would be disturbed, for example, to perceive an object receding from us and growing larger in angular size instead of diminishing. Such conflicting cues are the basis of the anomalous perspective phenomenon which is discussed earlier in this report. In artificially contrived cases, purposely conflicting visual cues become the basis for many intriguing optical illusions, such as the ones illustrated in Figure 32. The observer



Perspective drawings of "impossible" structures relying upon conflicting obscuration cues. Based on L. S. Penrose and R. Penrose in Brit. J. Psychol. 49, 31 (1958).



Visual cues at left end of structure are in conflict with those at right end. If both ends of drawing are covered, central structure is perceptually meaningless.



The Necker cube which can reverse itself spontaneously in depth because of the absence of further cues. At least one other cue is needed to resolve the ambiguity, such as obscuration (as shown in small cubes), stereopsis, or parallax.

Visual Illusions Illustrating Perceptual Confusion Because of Conflicting or Missing Cues

Figure 32

immediately recognizes the non-real-world aspect of such presentations and can find entertainment value in the upsetting of the visual-response complex which he has learned from childhood. In serious 3-D display work, however, one must take pains to avoid the presentation of conflicting cues which may be inherent in the mechanism of a given display.

The requirement that diverse visual cues be compatible provides an important basis for comparing the visual efficacies of the many 3-D display methods which have been implemented. The ideal 3-D display would be one which could reproduce the real world visually in so authentic a manner as to defy detection by any visual test which might be put to it. For mass entertainment purposes perhaps the most effective, and certainly the oldest, 3-D display method is the stereoscopic image-pair in which separate left- and right-eye views of natural objects are presented to the respective eyes. In its modern motion picture form, this method is capable of conveying the important cues of perspective, stereopsis, obscuration, shadowing, and aerial perspective, all in color and with action. Except for its need for special viewing glasses, its failure to reproduce parallax, and the difficulty of updating the pictures in real-time, this method might have ended the quest for improved 3-D display methods.

On the other hand, the stereoscopic image-pair loses much of its effectiveness if it is called upon to reproduce other than natural objects. Many electro-optically generated 2-D displays are in current use for purposes of training, control, decision-making, etc. The display material most often consists of discontinuous, line-structured material rather than continuous-tone natural objects. If such displays were to be presented stereoscopically, the conventional image-pair would not be able to make use of several natural depth cues such as perspective, shadowing, and obscuration unless these were intentionally incorporated into the display material. For example, in a display of alphanumeric symbols or cartographic data, the cues associated with the apparent sizes of familiar objects would normally be absent, so that linear perspective relationships would not be available as an aid to depth perception. The observer would therefore have to place greater reliance upon other cues such as stereopsis and parallax. In such cases, the inability of the stereoscopic image-pair to reproduce parallax becomes a serious deficiency. Thus, for displaying line material in depth, the various volumetric displays are at a substantial advantage because of their ability to provide parallax.

On the subject of conflicting depth cues, it must be pointed out that in the implementation and operation of any 3-D display, be it volumetric, stereoscopic, or other, there is the possibility that incompatible cues can be introduced into the display inadvertently. This report has made frequent reference to the anomalous perspective property which is peculiarly inherent in the vibrating membrane mirror but which is readily corrected. This property is not the only offender in the realm of 3-D reproduction; various other image-space distortions leading to unnatural depth perception can occur in 3-D displays. For example, if the members of a stereoscopic image-pair are unintentionally transposed, a pseudoscopic effect occurs in which the stereoscopic cues are in direct conflict with the other distance cues. Also, if an image-pair happened to be prepared by use of a base-line (or camera lens separation) much longer or shorter than the normal 2-1/2-inch interocular distance, the observer would experience hyper- or hypo-stereoscopic effects, respectively, which would distort the perceived spatial relationships (and hence the apparent sizes) of the images.

In selecting a 3-D display system for a given application, one must, therefore, be especially mindful of any inherent system features which might tend to distort the image space. The presence of such errors and the matter of how readily they can be corrected are as important in the choice of a system as are matters of cost, size, reliability, and other such considerations.

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APPENDIX II

THREE-DIMENSIONAL DISPLAYS: PAST AND PRESENT

STEREOPHOTOGRAPHY

Early History

The science of presenting information in the form of three-dimensional visual displays was probably born when a cave man first carved a relief figure on a stone wall.

Early in man's history, it was realized that a greater sense of solidity and realism, when desired, could be conveyed by sculptural representations than by flat ones. Nonetheless, the carving remained the only true "volumetric" display for many centuries, albeit with a long update interval. The evolution of new art styles, especially during the Renaissance, adequately fulfilled many of the requirements for achieving convincing depth displays through the use of monocular depth cues in drawings and paintings, these cues including perspective, shadowing, and obscuration.

In the meantime, many thinkers such as Euclid, Ptolemy, Leonardo da Vinci, and Galen were pondering the unique phenomenon of binocular vision and the distinctive sensation of depth to which it gave rise.[21] It remained for Sir Charles Wheatstone, in about 1832, to create artificially an impression of visual depth by presenting to the eyes a pair of slightly different drawings of objects in such a way that they could be combined into a single stereoscopic image. The incidental advent of photography, a few years later, allowed Wheatstone to apply his principle to the stereoscopic, continuous-tone reproduction of natural scenes.

The Stereoscope

Wheatstone's original viewing equipment for combining the image was cumbersome but was improved upon by others, including Sir David Brewster. The subsequent work culminated in the old-fashioned parlor stereoscope which, curiously, bears Brewster's name. This simple device, capable of presenting all depth cues except motion parallax, has remained popular to the present time, appearing today in the forms of various molded plastic viewers used by children and adults alike, including photographic hobbyists, for purposes of entertainment, education, and advertising.

The principle of Brewster's stereoscope is illustrated by the plastic viewer furnished with original copies of this report. The lenses are essentially a pair of "reading glasses" which permit close viewing of the photographs without visual discomfort; in addition, each lens contains a prismatic component which allows the eyes to converge normally as though examining a single nearby object when, in reality, they are viewing two laterally separated objects.

The stereoscope is perhaps the simplest and most practical device for presenting separate images to the respective eyes. The image pairs are prepared most readily through use of a stereoscopic camera which is actually a pair of synchronized, side-by-side cameras with a lens separation about equal to the 2-1/2-inch average interocular separation of the eyes. After being processed and mounted, the "stereo pair" is inserted into the viewing device and is ready for use.

Improvements in the Stereoscope

Later developments in stereoscopy led to other methods of presenting the stereo components to the separate eyes without the use of prismatic lenses, although some sort of viewing accessory was nonetheless required. In one case (the anaglyph), the black-and-white images were specially prepared as a set of green-and-white and red-and-white images, respectively, and were superposed in a common image area. The viewing device was a pair of filter spectacles, composed of a green and a red optical filter, so designed that the eye using the green filter would see only the red image and the other eye would see the complementary image.

The anaglyph offers several advantages over the stereoscope for certain applications. The elimination of viewing lenses, with their attendant restriction to nearby placement of the stereo pair, allowed the image-pair to be presented in a large-screen format for audience use. From this, it was but a short step to anaglyphic motion pictures, and many such experimental films have been produced, several of them having been shown publicly in the late 1930s. Moreover, the use of an economical, paper-thin viewing device made it possible to use anaglyphic methods on a large-scale commercial basis in printed material. The most recent text on stereoscopy [20] contains several anaglyphic representations as illustrative matter, each copy being provided with a simple anaglyphic viewer.

An improvement on the anaglyph was the introduction of polarization coding for the separate members of the stereo pair as a substitute for color-coding. This innovation, made practical by E.H. Land's invention of sheet-polarizing materials in 1928, [22] liberated the color dimension for use in full-color still- and motion-picture stereo reproductions and freed the observer from the uncomfortable retinal-rivalry effects that result when the two eyes view separately-colored images.

Two projection systems were used in this process, one for each member of the stereo pair. Each projector was provided with a polarizing filter such that the separate polarization axes were at right angles to each other. The similarly right-angled separation of polarization axes of the polarizing filters in the viewing spectacles ensured that only the right-eye image would be visible to the viewer's right eye, and similarly for the left eye. The projection screen had to be coated with aluminum paint in order not to depolarize the images as would be done by ordinary white pigments.

The early laboratory success of this method led to its hasty adoption by the motion picture industry in the early 1950s in a series of films which must be described as experimental. Although they enjoyed a brief period of popularity, they disappeared abruptly within a few years. Their apparently premature demise was attributed, by many critics, to the public's objection to using the polarizing spectacles. Other complaints centered on viewing discomforts imposed by projector mismatches in synchronization, brightness, focus, and vertical registration. The latter problems could have been circumvented through careful projection system design or by the use of the vectographic method evolved by Land,[22] Mahler,[23] and others. In this method, the polarization-coding was contained within the image pigments themselves, and so the image-pair members could be superposed upon a single film and shown by a single projector.

DIRECT-VISION 3-D METHODS

Parallax Stereograms

All of the above methods required the use of viewing glasses, which imposed discomfort upon the observer. For more than a century after Wheatstone's work, ideas continued to be presented by which the separate images could be directed to the appropriate eyes through the use of various optical contrivances. Although such methods eliminated the spectacles, they nonetheless imposed a certain amount of head movement restriction, even though they were often described as "direct-vision" or "free-vision" methods. They led, finally, to the adoption of parallax methods of viewing, based on the use of a fine grid of opaque, vertical bars on a transparent sheet. The sheet was mounted a short distance in front of a specially synthesized composite picture of the scene. The composite picture was an interlaced array of vertical sections from the separate views of the scene, the sections being comparable in width to the bars. Under the proper viewing conditions, the left eye would see a composite image of the left-eye view of the scene through the clear strips between the bars, whereas the right eye, because of its different position, would see its corresponding image through the same strips. Head movement restrictions were still imposed, but the somewhat more natural viewing situation contributed greatly to the illusion of realism. The development of such methods is reviewed in Reference 21.

Many modifications of the basic method were proposed, including methods of projecting the images for viewing by large audiences. The various methods are best described as parallax stereogram processes, their important feature being (in contrast to later methods) that the picture information was confined to a single "stereo pair" of left- and right-eye views of the scene being presented.

A significant idea by Lippmann in 1908,[24] followed by important work by others, led to the achievement of the parallax panoramagram. In this method, a multiplicity or continuum of image-pairs was available to the eyes as the observer shifted his viewing position over some limited angle. The representation was so arranged that, as the observer moved, he witnessed changes in the aspect angles of the image details just as he would in viewing the real scene. This was apparently the first success of stereophotography in reproducing motion parallax.

Despite this large forward step, the basic method was still limited by viewing angle restrictions, lack of image brightness and resolution, the presence of ghost images in some cases, the presentation of the images as transparencies only, and the difficulty or expense of preparation. Over the ensuing decades, small improvements in the method were described by many workers, including the present author.[25] Lippmann's method and the various refinements called for the use of such devices as cylindrical or spherical lenticular elements to replace the opaque grid structure, and the use of special photographic techniques in which, during exposure, the camera was rotated in an arc about the scene. Still further developments led to opaque-print reproductions for more natural viewing; these, in turn, yielded techniques for producing motion picture and television representations in 3-D, using lenticular grids. The application of these techniques, however, has not been widespread.

In recent years, refinements in the opaque-print parallax panoramagram of the lenticular-grid type have led to economical means of mass production with little sacrifice of quality. Such 3-D pictures are beginning to enjoy a measure of extended utility for illustrative and commercial purposes in books, magazines, and advertising literature. They are unique in that they do exhibit a parallax capability over a small viewing angle. Although they may not be recognized as such, they are indeed direct descendants of Lippmann's original process.

Electro-Optical Displays

The term "electro-optical display" is used here to denote those 3-D displays in which the images are generated electronically instead of photographically. The distinguishing feature of such a display is the rapidity with which it may be updated in response to a changing real-world situation.

It is difficult to say when the electro-optical display had its origin. It began within the past few decades when various workers experimented with presenting slightly different views of CRO-generated patterns to the separate eyes. Although it has followed a course of development quite different from stereophotography, it has drawn heavily upon the knowledge which accrued during the evolution of the photographic methods.

The early work with CROs, which is reviewed in a 1948 paper by Parker and Wallis,[26] led to three different types of display which were intended to present 3-D data using a 2-D surface. The first method presents admittedly 2-D representations of solid objects by showing them in perspective projection. Schmitt[27] discusses the projective transformations necessary for one to produce convincingly solid images of simple geometric figures on a CRT face by appropriate processing of the electrical input signals. The images are, of course, flat but exhibit perspective relationships as do ordinary photographs of natural scenes. Further work on this method has continued up to the present time.[28-31]

The second method, which is stereoscopic, also is discussed by Schmitt and involves the simultaneous presentation of slightly different views of the object on separate CRTs.[27] The images are combined stereoscopically by means of prisms or polarizing filters and may also be projected on large screens for audience viewing.

Both the perspective-projection CRO and the stereoscopic CRO methods underwent considerable further development. They are mentioned again in Appendix IV under "The 3-D Oscilloscope."

The third type of 3-D CRO display is volumetric, exhibiting the features of stereopsis and motion parallax without the need for viewing devices. Again, the origin of the method is obscure, but the central idea is that one can achieve a volumetric display by presenting a cyclically changing image upon a flat surface which is undergoing rotation or oscillation at 20 or more cycles per second. An early proposal of this method (see Reference 26) calls for the use of either an oscillating flat mirror or a rotating diffusion screen to achieve the volumetric scan. Later workers effected improvements in the method and also experimented with movable phosphor screens. Such work is discussed in References 32 through 35.

The notion of scanning a volume with a moving plane was an important forward stride in the history of 3-D displays and one to which the rapid writing rate of the CRO was eminently suited. Clearly it is on this branch of the evolutionary 3-D "tree" that the vibrating membrane mirror hangs.

Holography

We pause here to consider the interesting and new 3-D display process called "holography" which is entirely different from any of its predecessors, both in technique and in properties. Although there is no evolutionary connection between holography and the processes discussed so far, the method is of enough importance to deserve a place in any discussion of 3-D technology.

In ordinary two-dimensional photography, during the moment when the camera shutter is open, the process occurring is one in which a distribution of light intensities (or colors) from an object scene is being "frozen" into a recording medium for later use. The photograph captures the 2-D aspects of the scene but leaves behind the 3-D cues of stereopsis and parallax. From the physical standpoint, what is being documented on the film is a projective transformation of the scene; what is missing in the film is a record of the spatial directions taken by the light rays from various points in the scene as they proceed to the film. The absence of the light-ray geometry is responsible for the "flatness" of the ordinary photograph.

It is known that if the geometry of these light rays could somehow be preserved, a perfect 3-D record of the object scene could be achieved. Although it has not been explicitly described as such, the work of Lippmann and several of the later investigators can be regarded as nothing more than attempts to "freeze" light rays in space. To some extent, the methods were successful but in an artificial way which introduced other problems such as light loss and resolution limitations. Nonetheless, the early work was important in highlighting the need for a more effective method of preserving light-ray geometries or, in modern optical parlance, "phase-surface relationships."

It is to the credit of Dr. Dennis Gabor that he recognized, in 1948, the possibility of freezing light rays in space, simply and effectively, through the introduction of a "reference" light-wave family.[36] The reference waves, through the well-known phenomenon of optical interference, could be made to interact with the light from the scene in such a way as to form a permanent photographic record which preserved all of the 3-D cues in a most convincing manner.

The effectiveness of Gabor's early "holograms," as he called them, was limited by the inadequacies of available light sources, but the validity of the idea was widely recognized. It remained for the advent of the laser, in 1960, with its interesting coherence properties, to bring out the promise contained in the original idea. The subsequent work of Leith and Upatnieks,[37] Stroke,[38] and many others has, in a very few years, brought the original hologram concept to a high state of development.

The physics of the holographic process has been adequately described in many publications (see, for example, Reference 38) and will not be detailed here. Some of the many popular treatments of the subject are presented in References 39 through 46.

At the present writing, holography is in a state of rapid technological growth. The most effective of today's holograms are monochromatic, although strides are being made in colored holograms. Present holograms exhibit undesired "sparkle" effects, although there is no logical obstacle to the elimination of sparkle. Also, holograms are now presented as static, after-the-fact representations, as in the case of still photographs, but there is no reason that motion-picture or real-time "TV" holograms cannot someday be achieved. Despite the present limitations, the viewing of a well made hologram is a rewarding experience which testifies to the fact that 3-D technology has not only made great strides since the early carvings of relief figures on stone walls but that progress continues to be made.

CONCLUDING REMARKS

This review has touched upon a few of the more general classes of 3-D display in only a superficial way. Within the frameworks of stereophotography, parallax panoramagrams, and electro-optical displays, there are many forms and modifications of the basic devices. These are too numerous to be considered here, and the interested reader is referred to the cited references for further information.

In addition to the 3-D display methods discussed, there are several methods which bear no historical or technical relationship to the others, as in the case of the hologram, but which do deserve mention. One example is that of a display in which a servo-driven pen, movable in three dimensions, deposits a trail of ink in a transparent, gelatinous medium. Another display uses the intersection of two steerable electron beams to ionize a gas and to create a luminous spot which can be moved rapidly throughout the gas volume. These and other methods are described in several standard display system texts, a recent example of which is indicated as Reference 47.

It is apparent that 3-D display technology offers a host of assorted methods and devices to suit the needs of the individual user within the constraints of his requirements, be they considerations of cost, size, or weight of the display system, size and viewing angle of the display format, update time, data capacity, color requirements, continuous-tone requirements, or others. There is not yet an all-purpose, universal display system, nor can any system be claimed as "superior" to another except in the context of a particular application. The choice of display method for a given application is a matter of compromise and can be determined only by the user.

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APPENDIX III

SOME PROS AND CONS OF THREE-DIMENSIONAL DISPLAYS

BACKGROUND

One of the major problems in the design of dynamic displays is to find newer and better ways of presenting visual information to the human observer. At first glance, it would appear that a dynamic 3-D display would be more desirable than a 2-D display because of its greater data handling ability. The human depth perception mechanism is well developed and is constantly being exercised. It seems reasonable, then, that a 3-D presentation would be a natural way of matching information to the perceptual channels of an observer, particularly when the subject matter is itself a real-world situation involving, for example, the relative positions and velocities of aircraft, missiles, satellites, or other spatial arrays of objects.

Despite the innate appeal of such an argument, 3-D displays have experienced fairly slow acceptance for operational use compared with the pace at which this technology has advanced in the past decade. There are several reasons for this, the most important being that very few studies have yet been performed to show just how much an operator's performance would be improved by a 3-D display. Also, it is certainly not true that all flat displays would be more effective if they were depth displays, for many of them are by no means overburdened by large data handling requirements;

however, the gray area near the crossover region between increased effectiveness and reduced simplicity needs to be examined. The complexity of 3-D displays must not, of course, be ignored; such displays are certainly more expensive, larger, and heavier than their flat counterparts. Finally, it has not been established (and there is reason to doubt) that every operator with normal depth perception will respond instantly and perfectly to the depth cues provided by any 3-D display.

It is clear that in a flat display there are other means of coding the 2-D data in order to provide a "third," nonphysical dimension (Z-axis) when necessary. Besides X- and Y-position information, one may invoke the use of coding dimensions such as color or brightness, symbol size or shape, flashing rate, alphanumerics, and line lengths or angles which often are more cost/effective than is a depth axis. However, in comparing the relative efficacies of the various coding dimensions for a particular display problem, one must take into account the following factors:

- the number of absolutely identifiable steps which the method affords;
- the ease of interpretation, with special regard to speed and accuracy;
- the distraction and fatigue which the method might impose upon the operator; and
- the amount of two-dimensional space which the coding method might consume in the display.

THE AIR TRAFFIC CONTROL CASE

The Problem

Let us consider, for example, the task of a controller at an FAA air route traffic control center as he monitors the radar-derived positions of a number of aircraft enroute between airports. The radar screen is mounted horizontally in a table top, and the controller and his assistants position themselves around it and watch it from above. Permanent markings in the display depict control sector boundaries and provide certain navigational information. The aircraft within radar range (150 to 200 miles) appear as blips on the screen, and their positions are updated with each sweep of the radar antenna several times per minute.

One of the main responsibilities of the controller is to be alert to possible conflicts in aircraft positions, directing that one or both aircraft change course if a collision appears likely. He must, of course, visualize the air traffic situation as it appears in three-dimensional space. However, the pictorial information available to him is two-dimensional, showing ground coordinates only. For altitude information, he relies upon numerical data reported verbally by the pilot. An assistant then writes this data on a plastic chip which he places on the radar screen next to its corresponding blip. The assistant then must move the chip manually as the blip moves. Alternatively, altitude information about a flight may be telemetered automatically from the aircraft's altimeter and made to appear numerically on the phosphor screen itself.

In either case, the controller relies upon two distinct coding methods and perceptual processes which he must integrate mentally to develop an appreciation of the three-dimensional air traffic situation. Two of the coordinates are presented to him in a pictorial form to which he is accustomed, allowing him to assimilate information about the relative ground positions of several aircraft almost simultaneously; the third coordinate is presented in a form that requires him to find, read, and interpret numerals and to remember their values. He does this in serial fashion, aircraft by aircraft, which implies a much lower rate for the assimilation of data.

To be sure, an experienced controller becomes adept at interpreting these mixed forms of coding, but many of them admit that keeping track of altitudes is still a serious problem. Moreover, the increasing speeds and traffic densities of modern and future aircraft will place even greater demands on the controller's ability to acquire and assimilate information in order to make valid predictions.

One is thus led to ask whether the innate perceptual ability of the human controller cannot be better utilized if altitude information were presented to him in a form (volumetric or otherwise) which is more compatible with his everyday training and experience.

The Use of Computers

Of course, there are other possible answers to the problem of the overburdened controller. For example, because of the increasing use of and confidence in electronic computers for air traffic control purposes, it is appropriate to ask why a computer cannot replace the controller altogether in a conflict-prediction role. The computer is rapid, accurate, and can process large amounts of data quite efficiently. The answer to this question has not yet been found; but, undoubtedly, the following thoughts must be taken into account:

(1) If a suitable form of coordinate data presentation were to be adopted for air traffic controllers, it is conceivable that a human controller could be as effective as the computer when such factors as the following are considered:

- the space requirements and the capital and operating costs of the human "computer" versus his electronic counterpart;
- the ready availability of "back-up" human computer systems in the event of a system malfunction or inability to reach a decision;
- the flexibility and adaptability of the human to a wide range of unpredictable situations with which the computer (considering its high degree of specialization) may not be able to cope.

(2) In the event that an electronic computer does detect and indicate an imminent aircraft conflict, it is most likely that the matter will be referred to a human controller for final evaluation before any action is taken. The computer, then, would not be a decision-maker but only an information sorter; it would be competing with the human in an area in which the human is well qualified.

Indeed, the use of computers for conflict-prediction is currently under active consideration. Undoubtedly there will be extensive tests to compare the performance of the computer and the human. The tests can be equitable only if the mode of data input to the human makes as full use of his perceptual mechanisms as it does in the case of the computer; otherwise, if the test results should favor the computer, one might always ask whether the bottleneck in the human was in the data input channel rather than in the data processing mechanism.

The Advantages of a Third Dimension

It seems worthwhile, then, to try to exploit the perceptual capabilities of the observer to their utmost before deciding that a machine might be superior. The answer to the question of how to display altitude information other than alphanumerically is not yet at hand and can result only from extensive comparison tests. In the meantime, one cannot doubt that the use of a third physical dimension would be a logical extension of the two already in use, and that a scale model of the actual situation would provide most of the necessary decision elements to the controller at one glance. This, it seems, cannot help but improve his performance as well as his and the pilots' confidence in the decisions which result.

In view of the criteria listed on page 164 for evaluating a third coding dimension, it is probably safe to make the following remarks about the use of a physical depth axis:

(1) The number of absolutely identifiable steps in the third coding dimension is not important in an aircraft altitude display. The purpose of such a display is to indicate relative positions of aircraft, and so the question of visual resolution along the coding axis is important, whether it be a color or brightness axis, a flashing rate axis, or some other. Regarding a physical depth axis, the question is this: for each distance from the observer to a display element, what depth separation is necessary between a pair of elements along the viewing axis in order for them to be seen as distinct? The same type of question might be asked if color or some other form of coding were used, and the answer, because it varies from one situation to another, would have to rely upon experiments performed under realistic display viewing conditions. In ordinary 3-D viewing, it is known that our ability to resolve depth stereoscopically along the viewing axis is somewhat less than our transverse resolution capability. Some compensation may be provided by the observer's normal head movements, bringing parallax to bear, and by the fact that in any computer-generated volumetric display it should be possible to expand the Z-axis scale independently of the other two.

(2) The ease of interpretation would be a strong argument in favor of depth-axis coding for a display which was to represent real-world spatial events.

(3) For normal observers, there is no reason for a properly designed volumetric display to impose distractions or fatigue upon the operator.

(4) By the very fact that it is three-dimensional, a volumetric display avoids the problem of cluttering a planar display in order to present third-dimensional information.

RESULTS OF HUMAN FACTORS STUDIES

As stated above, a few laboratory studies have been conducted to assess the human's ability to use stereoscopic coding. The results of these studies provide only some rudimentary scientific information about depth perception under specific test conditions; the experimenters caution that the findings cannot easily be generalized to more complex, operational display situations. For example, Guttmann and Anderson[48] have compared the efficacies of a volumetric display and an orthogonal, dual-projection 2-D display, that is, a pair of flat displays representing, for example, the plan and elevation views of a 3-D situation. Under their particular test conditions, they found the volumetric display to be slightly superior in allowing subjects to make judgments of absolute velocities of a single, moving target. In judgments of absolute and relative positions and of relative velocities of one and two targets, performance was better with the pair of 2-D displays. Most of the errors associated with the use of the volumetric mode arose from the difficulty of making depth judgments along the viewing axis. It should be noted that the subjects were allowed up to 10 seconds in which to make their judgments.

These results are not easily extrapolated to operational displays such as would be used for air traffic control. First, although a 10-second decision time is ample for composing a mental image of an object's spatial location from dual flat projections, such decision times are not available to a controller who may have to make many such decisions within a few seconds. As the decision time becomes shorter, a crossover point is certain to be reached at which the subject does not have time to look at the second of the dual displays, whereupon this method becomes useless; in an equally short decision time, the subject would be likely to make at least some sort of judgment with a volumetric display, however approximate.

Second, a dual-projection display becomes easily overburdened if more than two or three similar objects are being depicted, for it is then impossible to identify, in the two displays, those pairs of elements which represent the same object. This leads to an ambiguity or "ghosting" problem which would be intolerable in an operational air traffic control display.

Finally, the experiments above were performed without the aid of any spatial coordinate markers within the volumetric display, whereas in an operational display it would be not only simple but quite reasonable to program one, either permanently or for occasional reference.

In another study, Cohen[49] investigated some of the properties of stereoscopic presentations using CRT pairs viewed through special stereoscopic equipment. His test results are based on about 20 subjects, and his report presents some observations and recommendations about the relationships between binocular disparity and depth discrimination. At the end, he expresses the view that 3-D CRT displays, such as for radar information, would indeed be practical and worthwhile, although no comparison is made of the effectiveness of binocular disparity versus any other coding dimension. Squires[50] performed experiments similar to Cohen's, also with stereoscopic radar in mind, and he concluded that, with careful screening of the subjects, the performance can be quite superior.

All of the above tests were performed in laboratory environments which were not intended to duplicate actual operating conditions. By contrast, Bassett et al. [51] have used a volumetric display with a test subject in a more realistic viewing situation. The display was produced by projecting a CRT image onto a rotating translucent screen, and the subject was not constrained as tightly to one head position as in the other experiments. In commenting on their work, the authors say:

> While the results do not point conclusively to specific applications, the utility of volumetric 3-D in making fine position and motion discriminations has been demonstrated. Further study would be required to ascertain utility in practical situations such as air traffic control, space surveillance, etc... In general it may be said that these experiments demonstrate the potential utility of a three-dimensional volumetric display device in operational systems requiring the rapid and accurate monitoring of both stationary and moving targets. There is no question, however, that a good deal of continued research is necessary.

Considering the recent date (1965) of this report and the fact that perhaps a dozen or more companies have some time ago implemented various volumetric display ideas as working models, further research on the utility of these devices appears overdue. A few years ago, some recommendations were made on the direction which this research might take and on the questions which need to be answered.* Particularly, it was proposed that the excellent simulation capabilities of the new volumetric displays be used in a study of decision tasks such as:

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*These recommendations and questions were offered by Working Group VI on Visual Displays, a subcommittee of the Armed Forces--N.R.C. Vision Committee. Their validity does not appear to have diminished with time.

- detection of moving targets;
- localization in a volume; and
- perception of relative positions of multiple targets.

The detection of motion of a target surrounded by fixed targets, spatial grids, or visual noise might be included. It would also be important to study judgments of position in relation to reference grids and to the display volume boundaries. It was further suggested that the perception of spatial relations among targets be examined, particularly the prediction of collisions between moving targets. In recognition of the greater cost of volumetric displays, the panel also recommended a study of the efficacies of coding methods in which 3-D information could be displayed in 2-D form.

CONCLUSIONS

To return to the theme of this appendix, one can argue the pros and cons of 3-D displays only in a superficial way at present. It is undoubtedly true that volumetric displays will ultimately be used in certain applications, but, for the moment, we do not know which they will be. Empirical knowledge to help resolve this issue is overdue but is hopefully on its way. In the meantime, we shall continue to be confronted with problems of how to display three-dimensional information, and new and improved volumetric display devices will continue to be introduced; but until this gap in our knowledge is filled, we shall not know how to match the one with the other.

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APPENDIX IV
APPLICATIONS

INTRODUCTION

The basic vibrating membrane mirror can be applied to 3-D displays in many ways. A distinction must be made, however, between two types of use for the device. In one case, the mirror is combined with other components to achieve new display capabilities, and these combinations are not rightly called "applications" but rather "embodiments"; an example would be the 3-D oscilloscope without reference to a particular end use. On the other hand, a display "application" is considered here to be one in which the display serves as an implement in filling some particular need of a user.

It is appropriate that we begin a discussion of applications by reviewing the array of tools available for use, and for this reason we simply list the possible embodiments which have been discussed earlier:

- three-dimensional oscilloscope
- stroboscopic optical projection system displays
- computer-driven displays
- three-dimensional numerical matrix
- graphically-generated displays
- large-screen, on-the-wall displays
- multiple-image displays

GENERAL AREAS OF APPLICATION

The possible applications for 3-D displays fall into many general areas. In these areas, one can frequently distinguish between two general applications for the display: (1), information analysis or (2), as a small-scale pictorial model of a real-world situation. In either case, its purpose is usually to make the most efficient use of human perceptual capabilities in conveying as much information as possible in a short time. In the information analysis case, there is the additional benefit that large amounts of data can be scanned swiftly to pinpoint those areas which may require a more detailed examination. Also, the operator is quickly provided with an overall appraisal of what may be a complicated situation whose gross features might not be apparent under a prolonged, detailed examination of the input data. Furthermore, where a functional relationship between three related variables is presented as a synthetic 3-D model, as is often done through the use of perspective drawings, the observer can visualize the relationship in a language with which he is familiar, and he can often gain insights into the properties of the relationship which might not otherwise be apparent to him.

These are some of the philosophical considerations which form the basis for selecting the most useful applications for 3-D displays. Some of the general areas in which applications may be found are given below:

- command and control systems (as for air traffic control, aerospace vehicle control, and missile trajectory monitoring)

- radar and sonar signal analysis and display
- analysis of other electrical signals
- mathematical function analysis
- acoustical signal analysis
- man/machine graphical communications
- briefing and educational aids
- simulation problems
- medical diagnostics (electroencephalograms, vectorcardiograms)
- process control
- product quality control (such as transistor performance characteristics)
- mental and visual therapy
- personnel training and screening
- data display (such as in meteorology or oceanography for displaying volumetric distributions of temperature and pressure)
- terrain map-matching for navigation purposes
- computer-assisted design of structures
- art, entertainment, and commerce

On the basis of intuition or experience, we can select from these areas many specific problems which could very likely be aided by the use of 3-D displays. In some instances, the required instrumentation is elaborate, and the need should be examined very carefully beforehand. In others, a moderate investment of resources can achieve useful results immediately.

Below we have listed a number of specific ways in which the membrane mirror display, as well as other 3-D displays, might be used profitably. A discussion of each item is presented in the following paragraphs. The list is not comprehensive; the items serve as examples only.

- 3-D oscilloscope as a general purpose analytical tool
- air traffic control
- analysis of vectorcardiographic signals
- molecular models in three dimensions
- representations of radar backscatter patterns from satellites

SPECIFIC AREAS OF APPLICATION

The 3-D Oscilloscope

Background

Methods of using cathode ray tubes for presenting tridimensional data have been proposed for more than 20 years. The number of papers appearing in the technical literature on this general subject suggests that it is a lively one and has received considerable study. References 26 through 30, 32, and 52 through 54 form a partial list of such papers.

The ideas which have been proposed fall into two general classes. In one case, the method is to present either a perspective or an isometric projection of a solid figure upon a flat CRT screen. The method is not claimed to be truly three-dimensional but rather a two-dimensional representation of a solid object, whether real or fictitious. Devices belonging to the other class create stereoscopic impressions of three-dimensional objects. Two slightly different perspective views are presented separately to the eyes via a viewing device, as with the stereophotographs contained in this report.

Although many wide-scale uses have been suggested for the proposed methods, the applications have, until recently, been limited largely to laboratory uses for special purposes. A commercial device (References 52 and 53) is now available, however, which is intended for general laboratory use. It is supplied in two forms, for the presentation of either perspective projections or stereoscopic image pairs.

A few laboratory oscilloscopes have been proposed which will provide the parallax feature as well as perspective and stereopsis, as in Reference 32. Parallax is, of course, a useful but not always necessary addition to stereoscopic cues, just as the latter, in turn, are frequently but not always an improvement upon monocular cues; in many cases, a flat image conveys all of the information necessary. However, our limited experience with the 3-D oscilloscope adapter indicates that there are certain display situations where stereopsis alone is not adequate to resolve perceptual ambiguities. A thin ring suspended vertically and viewed on edge, for example, is not as easily perceived stereoscopically as when the observer is free to view it from different aspect angles by means of parallax. Also, stereoscopic perception is not effective with nontextured lines or edges parallel to the observer's interocular axis (and it is body contours and texture which form the basis of stereoscopic vision). Thus, we cannot readily perceive depth in a family of wires strung between telephone poles when our eyes are aligned with the wires.

Discussion

The inclusion of a parallax feature in a display can therefore be important at times. In the membrane mirror type of 3-D CRO it is an added feature, at no extra cost, resulting from the volumetric nature of the display. What is important is that, if the frequency bandwidth of the mirror can be increased by simple means, the system can be highly competitive with other systems because it can, in many cases, utilize raw data directly. This would eliminate the data storage and processing devices now required for the preparation of perspective projections of the objects to be portrayed.

In other cases, data handling devices can be used very profitably with the membrane mirror CRO. If we consider the cases where both raw and prepared data can be used with this system, we can arrive at an interesting list of possible laboratory applications:

- radar range, azimuth and elevation displays
- frequency or vibration spectral analysis versus time or other parameters of interest

- phase, amplitude, or frequency comparisons between three periodic electrical signals
- displays of vacuum tube (or transistor) characteristics, such as plate current vs. plate voltage vs. grid voltage
- displays of reactive and resistive components vs. frequency in various impedance devices or networks
- displays of mathematical functions of two variables
- representations of spatial field distributions such as temperature, pressure, electrostatic or magnetic field strengths, optical or RF energy as in radiation patterns, fog droplet concentrations, and cloud formations
- displays of medical diagnostic data as in electrocardiography
- time compression of long-term data for "quick-look" observations of trends

The common theme connecting the above items makes the general purpose of any 3-D CRO quite clear: its utility is in visually portraying a functional connection between three quantities whose interrelationship is in some way significant. This includes the case where the interrelationship depends upon an additional parameter such as time, temperature, voltage, and the like.

As observed earlier, some of the above items may require moderate processing of the data before it is displayed and others will require that special scanning systems be implemented as the primary data sensors, but in most cases the 3-D CRO will be the predominant instrument in the system. In other situations, the data processing requirements are more complicated and the display equipment becomes an appendage to the more overwhelming computer and sensor system required. Examples of such situations are treated separately under their own headings.

Air Traffic Control

Most civilian air traffic in the United States, both commercial and private, is controlled on a 24-hour basis by the Federal Aviation Agency's nearly 15,000 controllers who are assigned to 20-odd air route traffic control centers and to hundreds of airports.

The single purpose of the controller, as with the traffic policeman, is to assist in the orderly flow of traffic in order to forestall accidents, be they mid-air collisions or other mishaps during taxiing, take-off, enroute flight, or landing. Unlike the traffic policeman, the controller is seldom in direct visual contact with the situation for which he is responsible. He must depend upon radar sensors, telemetry systems, and voice communications in order to synthesize a mental picture of the traffic situation.

Control over this situation is especially difficult because, except for ground maneuvers, the air traffic pattern occupies one more dimension than that for automobile traffic. Moreover, the controller is not afforded the luxury of being able to stop an aircraft in flight in order to let another one pass. The following example illustrates the complexity of his task: to perform the function of conflict-prediction for collision avoidance purposes, he must have complete, moment-to-moment information on all aircraft under his control with regard to identification, individual and relative ground positions and altitudes, headings, airspeeds, and intended maneuvers. It is clear that at peak traffic periods his perceptual channels can be highly occupied.

Appendix III includes a discussion of the cognitive problems facing today's controller and of the tools presently available to him; it also considers briefly whether new display methods might improve his efficiency. This question, now speculative, will someday be resolved, but a large body of intuitive opinion holds that the possibility of 3-D air traffic situation displays is attractive for control purposes.

One possible method of processing the aircraft position data for a volumetric display of this type is discussed by Withey in his paper on a 3-D CRO using a vibrating phosphor screen (see Reference 32).

In addition to mid-air collision avoidance, 3-D displays could be helpful in ground-controlled approach procedures and, in military applications, for vectoring defensive aircraft to intercept hostile aircraft, or for terrain clearance purposes in low-altitude flights (References 55 and 56). In such cases, the display would probably be ground-based for analysis by an operator who would interpret the situation and communicate verbally with the pilot. The compactness of the membrane mirror display offers the possibility of an airborne 3-D display for direct use by the pilot. It could be operated as a slave system using ground-based signals transmitted by telemetry.

Vectorcardiography

The electrocardiograph is an important tool to the physiologist in clinical research and to the physician in medical practice who uses it for the early detection or diagnosis of heart disorders.

A fairly modern extension of the electrocardiogram is the vectorcardiogram which permits three separate heart signals to be compared simultaneously. Most frequently the heart signals are derived from electrical voltage sensors taped to the patient's skin at various places on his body, often the extremities, with a common sensor in the chest or back region. The three periodic signals are compared in amplitude and phase, and they can be used to provide information about heart activity from moment to moment or from patient to patient. The problem of interpreting the three sets of curves is a difficult one and requires a skilled diagnostician with an ability to visualize spatial relationships.

It is reasonable to expect that the process could be aided materially if the simultaneous signals were presented as a synthetic spatial figure which the diagnostician could learn to interpret rapidly after some practice. The membrane mirror display might be well suited to this task because of its portability and economy. Two of the three signals might be applied as the horizontal and vertical inputs to a CRO while the third is used to drive the mirror via the loudspeaker. The signals could be applied in real-time, although the heart rate is far below the 20 to 30 cycles per second which are required for a stationary, flicker-free image, and the volumetric image would consist of a point of light periodically traversing an irregular course. (The use of a long-persistence phosphor on the cathode ray tube screen would merely leave a luminous trail in the depth dimension rather than along the trajectory.) However, if the heart signals were tape-recorded and then played back at a higher speed, a stationary volumetric image would be seen. By means of beam splitters, other vectorcardiographic images could be superposed upon this image for comparison or reference.

The restriction on the frequency response of the present form of loudspeaker-driven mirror may or may not be a limitation, depending upon whether the heart signal applied to the mirror contained high-frequency components beyond its useful range.

Molecular Models in Three Dimensions

Levinthal, in Reference 57, and Phillips, in Reference 58, discuss the use of computers in helping to provide an understanding of the structures and functions of large, complicated biological molecules. Especially important to the function of a molecule is its three-dimensional structure, and the authors discuss various means of portraying this structure pictorially. Several interesting and effective methods are available for this purpose, from the building of physical models to the computer-generated display of a sequence of perspective projections as the body rotates. The apparent value of such portrayals to the investigator suggests that dynamic volumetric displays might have their place in this field, and that this might be another useful application for the membrane mirror display.

Radar Backscatter Patterns from Satellites

With regard to the problem of identifying artificial earth satellites from the ground, radar data can be of great help in determining body shapes if the body is rotating so that it can be observed by radar throughout a sequence of aspect angles. At any instant, the magnitude of the reflected signal depends upon the radar cross section of the body at one viewing angle, and the sequence of signal magnitudes is a function of the electromagnetic shape of the body. If the body motion is known so that the sequence of reflected signals can be correlated with various aspect angles, some deductions can often be made about the body shape.

In the simple case of a satellite tumbling about a single axis, a sequence of constantly changing aspect angles will be presented to a ground radar site as a result of the translational motion of the object along its orbital path. If the origin of a fictitious reference sphere were attached to the body, the sequence of aspect angles would, in an ideal case, trace out a spiral pattern on the sphere as its surface was intersected by the constantly changing viewing axis.

Computer simulations of such reference spheres for simple objects have been described by Cope and Somin in Reference 59. The transparent sphere is presented in perspective projection on a flat CRO screen, and the radar return signal strength versus aspect angle is depicted by means of intensity modulation of the electron beam. The visual presentation was found to be useful in helping an operator to distinguish between the separate effects of body shape and body motion when neither was fully known. The visual integrity of the pattern depends upon the validity of the assumptions made about the body motion. The pattern deteriorates noticeably when the body motion input variables are intentionally varied from their correct values.

The perspective projection of a sphere with markings on it is a simple and effective means of presenting satellite backscatter information to an observer. Of course, one can imagine unusual body configurations in which isolated "glint" effects can lead to ambiguities; it may not be clear from the display whether such a specular return belongs to the near or to the far hemisphere represented in the display. This ambiguity can be reduced to a simpler one if the computer is programmed so that the flat image of the sphere appears to rotate slowly about some axis. Such a device has been used by others as an effective means of conveying a sense of solidity to an object shown in a flat display. The remaining ambiguity, when the object is transparent, concerns the direction in which it appears to be rotating, and this, in turn, determines whether the object or its mirror-image will be perceived.

If necessary, volumetric displays may be used to remove the rotational ambiguity. Another advantage would be a more realistic, stationary portrayal of the backscatter pattern which would permit details to be scrutinized closely in three dimensions.

Other Uses for the Vibrating Membrane Mirror

Because the concept of a loudspeaker-driven membrane mirror does not appear to have been described in earlier literature and because it can be readily implemented to provide a handy laboratory tool, it is tempting to consider other possible uses for it apart from the 3-D application.

Without going into detail, we simply offer here a few diverse, likely applications which have been reflected upon by the author and his colleagues during this investigation and which may strike some responsive chord among those skilled in the art:

- as a beam scanner for line-raster display systems such as facsimile transmission systems or large-screen displays using laser beams;
- as a beam scanner for other systems such as spectrophotometers, densitometers, radiometers, and automatic star trackers;
- as a passive electro-mechanical stroboscope;
- as a chopper or modulator for radiometers or other sensors or for signaling or detection devices;
- as an audio frequency modulator of light beams, using electrical input signals (a voice-activated version of such a modulator is described in Reference 60); and
- as a passive optical-ranging or automatic-focusing device.

In some of these proposed uses, it is obvious that the entire mirror surface would not be utilized. Instead, a small portion effectively simulating a flat element would provide the scanning function, preferably a segment near the mirror boundary where the angular deflection would be the greatest. For scanning functions involving narrow light beams whose collimation was not to be disturbed, either the light beam must be extremely narrow or the mirror surface must be large in order to minimize any focusing problems which might result from mirror curvature. In any event, the vibrating membrane mirror appears useful for these purposes, from an engineering viewpoint, because of its ready implementation, low power-requirements, and bearingless action.

It might be added that in the nonvibrating mode of operation, the flexible mirror may nonetheless be interesting as a simple variable-focus device for teaching purposes, or as a zoom lens for use in amateur photography or telescopy. The membrane need merely be mounted upon a frame and attached to a chamber within which the air pressure can be varied by some convenient means, as shown in Figure 2 of this report.

Moreover, if the rear face of the chamber is transparent, the back surface of the mirror can be viewed directly. In this configuration, the device might be of some use in telescope-mirror grinding operations, where it is desired to examine the focal properties of a concave glass surface while it is still in the rough-grind stage. The mirror need merely be "blown up" against the concave face to provide an artificial specular surface which may be of sufficient quality for practical use.*

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*This suggestion was contributed by B.A. Forest of The MITRE Corporation.

AND OF THE FUTURE:
3-D TELEVISION AND MOTION PICTURES

As a final note, we offer some comments upon a technical problem which does not appear amenable to immediate solution by the membrane mirror display: notably the photographic reconstruction of live, natural subjects in depth.

At first thought, the entertainment field would appear to be an obvious and commercially important application for this display, and the author has frequently been asked about such a possibility. However, additional thought will reveal some important technical and economic objections, and the intent of the following paragraphs is to make clear why this application does not yet appear to be practical.

The Photographic Problem

The technical difficulty is that any 3-D display which composes a volumetric image by the presentation of successive 2-D images at various depths requires, obviously, that an appropriate sequence of 2-D images be available to start with. The peculiar requirement on these images is that each of them must represent a "slice" of the real scene. That is to say, each image must be composed only of the contours of the objects in a given depth plane within the scene, and of no others.

The problem is that no simple photographic method yet exists for reproducing the picture information in a live scene in only one depth plane at a time. However, it is not beyond the scope of present technology to find a means of doing so if necessary. For example, a wide-aperture lens with an intentionally short depth of field could be focused on successive object planes in order to produce a sequence of photographs, each of which presented sharp object detail in only one narrow depth layer at a time, as has been described in References 61 through 64. Picture information in other depth planes would be present, although highly defocused, and could possibly be removed by electronic image processing techniques which are now available for edge-enhancement purposes. However, such a system is far from realization, and even if it were effective, it could not be called "simple."

Laser Techniques-Promises and Problems:

Alternatively, a somewhat more remote possibility might be provided through further improvements in laser range-gating techniques whose uses have been studied for taking pictures of distant objects through backscattering media such as snowstorms or haze (References 65 through 67). In this method, an extremely brief laser pulse is directed at an object field. The wave front has sufficient lateral extent to illuminate all objects of interest in the field, except for possible shadowing effects. As the wave front progresses and intercepts objects at various depths, a sequence of return waves can be intercepted and imaged by a lens located near the laser source. By use of a high-speed shutter and film-transport system, the sequence of returning waves might be decomposed into recorded images of discrete object planes or, more correctly, slabs. The thinness of each slab will depend upon the brevity of the laser pulse and of the exposure duration. Each image is the analog of a radar return signal from a target, except that it carries 2-D information, the entire sequence constituting the raw material for a volumetric reproduction.

Image-Recording Problems . . .

The above method is speculative and imposes a severe burden on the image-recording method, at least with present technology. It must be remembered that the sequence of returning images will occupy a time interval equal to only twice the transit time of the incident wave through the object field. Thus, if the field is 30-feet deep, the image sequence will have been completed within a 60-nanosecond (60×10^{-9} sec) time interval, since light travels approximately one foot in a nanosecond. If the object field is to be sliced into, let us say, 100 depth slabs for recording purposes, then only 0.6 nanosecond is available for exposure and frame-changing purposes. If the frames are to be recorded on film, such short exposures would present problems because of reciprocity-law failure. Moreover, the mechanical problems associated with continuous filming at the rate of nearly 1.7 billion frames per second would be severe. It is also doubtful that present electro-optical image recording systems could meet such demands. If, for example, we choose to dissect each frame into a conservative 100 by 100 elements, then 1.7×10^{13} elements per second must be processed and transmitted by the video camera during the time that the wave sequence is returning, and this is a formidable information rate by modern standards.

. . . and Information Rates

In practice, the average information rate could be much lower, for it would not generally be necessary to illuminate the object field more than 30 times per second, the frame rate which is used in commercial television broadcasts. Nonetheless, corresponding to each of the flat frames of modern TV, one might have to transmit 100 or more depth frames in order to provide an image which appears continuous along the depth axis. This would apply in the motion picture case also, and it would apply whether the frames were derived from the laser-illumination method or by the technique mentioned earlier in which a lens with a short depth of field is used to scan the object field.

In any event, the required information rates impose an economic as well as a technical burden, for they directly affect the operating costs of such systems. In the motion picture case, one would have to consider the problems of projecting film at an average rate of perhaps 2400 frames per second. Aside from the technical problems involved, the 100-fold increase in film consumption and in processing costs would have to be considered carefully. In the television case, except for the possibility of some bandwidth compression because of image redundancy, the required transmission bandwidth might have to be 100 times as great as at present, requiring a greater portion of the broadcast spectrum than is now set aside for the 82 commercial TV channels in the United States.

It should be remembered that workable techniques are now available for the reproduction of stereoscopic images, whether by television or by motion pictures, which require that the picture information be transmitted at only twice the standard rate. The resulting images are convincingly three-dimensional by virtue of their stereoptical cues, even though they require the use of viewing glasses and, moreover, do not offer the parallax feature. Although both of these restrictions might conceivably be removed by use of the membrane mirror display, one must ask seriously whether the developmental and operational costs are justified.

APPENDIX V

MATHEMATICAL ANALYSIS OF MIRROR CURVATURE

A. J. Cantor*

THREE METHODS OF CALCULATING THE SHAPE OF A THIN CIRCULAR MEMBRANE UNDER PRESSURE IN THE STATIC LIMIT

Introduction

Although the membrane is normally operated under vibration, it is reasonable to assume that at moderate frequencies it follows the driving force of the loudspeaker. Hence, its continuously varying shape can be assumed, to a first approximation, to be a succession of static deformations. One need then only calculate the membrane shape for the static case in order to arrive at an approximate description of its surface configuration during vibration. The upper limit on the validity of this approximation is established by the breakup frequency.†

*The author wishes to thank A. Bark, W. Gottschalk, and G. Sauermann for their contributions to this study, which is intended to serve as a stimulus for further analysis of the optical imaging of the membrane. The discussion is introductory in nature; the full complexity of the problem is not treated.

†Breakup phenomena occur as a result of asymmetries in the driving force or in the mechanical properties of the speaker-mirror system. They are not treated in this discussion because they are not of interest in the usual 3-D display application of the membrane.

Consider an "ideal" membrane stretched over a circular frame and held to some shape by a constant pressure difference between its two sides. An "ideal" membrane is defined as one wherein

(a) the tension at any point is the same in all directions in the membrane; and wherein

(b) the tension is the same at all points in the membrane.

This is possible if the membrane is infinitely thin and perfectly homogeneous and isotropic. Needless to say, an ideal membrane is rarely encountered in practice; it is best approximated by the surface of a liquid (see page 206 ff.).

Method One

In the case of a thin membrane, the apparent pressure[68] acting normal to the surface at some point is, because of tension, $2t/R$. In the static case, this apparent pressure just balances the pressure difference:

$$p = \frac{2t}{R} = t\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad , \tag{11}$$

where

p	is	the pressure difference,
t	is	the tension in the membrane,
R	is	the average radius of the surface curvature at a given point, and
R_1, R_2	are	the maximum and minimum surface curvatures at a given point.

Quoting from Joos:[69]

> The lines of curvature are characterized by the fact that neighboring normals to the surface, at points along these lines, intersect each other, and that the directions of their tangents are those of the normal sections of greatest and least curvature.

Hence, the average radius of curvature is constant over the surface, and since we have an ideal case in which the pressure and the tension are the same everywhere, the membrane over a circular frame is a spherical section because

$$R_1 = R_2 = R_0 = \frac{2t}{p} \tag{12}$$

satisfies Equation (11) for constant p and t and is consistent with the circular boundary.

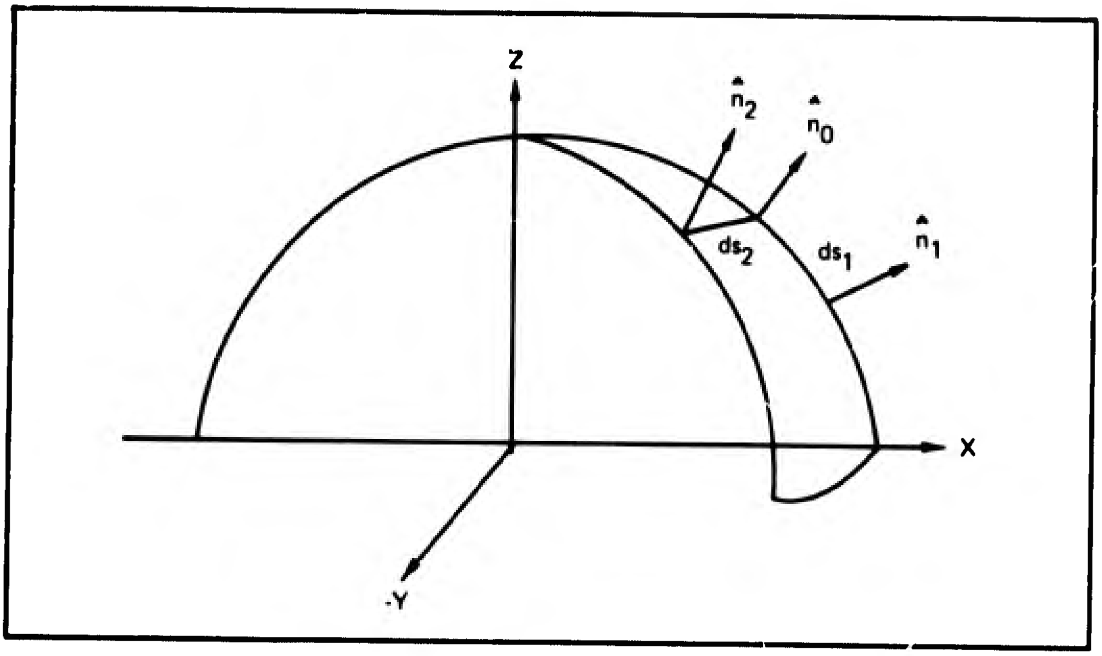
A more analytical deduction from Equation (11) is obtained by calculating $1/R_1$ and $1/R_2$ explicitly. The membrane (see Figure 33) is, by symmetry, a figure of rotation around the z-axis. Let the shape of the cross-section in the x-z plane be some function $z = f(x)$ which must be determined. In general,

$$z = f(r) \quad , \tag{13}$$

where

$$r = \sqrt{x^2 + y^2} \quad . \tag{14}$$

Three vectors normal to the surface, $\hat{n}_0$, $\hat{n}_1$, and $\hat{n}_2$, also are shown in Figure 33. Starting from the arbitrary point x, $y = 0$, $z = f(x)$, whose normal is $\hat{n}_0$, we establish the direction of $\hat{n}_1$ by a displacement ds_1 in



Definition of Coordinate System for Membrane Section

Figure 33

the x-z plane; the components of this displacement are dx_1 and dz_1 $(dy_1 = 0)$. This direction must give one of the principal curvatures, call it R_1, because $\hat{n}_0$ and $\hat{n}_1$ intersect in the x-z plane. If $d\theta_1$ is the small angle between $\hat{n}_0$ and $\hat{n}_1$, then

$$\hat{n}_0 \cdot \hat{n}_1 = \cos d\theta_1 \approx 1 - \frac{(d\theta_1)^2}{2} . \qquad (15)$$

Hence, the radius of curvature follows from $ds_1 = R_1 \, d\theta_1$:

$$\frac{1}{R_1} = \frac{d\theta_1}{ds_1} , \qquad (16)$$

and

$$d\theta_1 = \sqrt{2\left(1 - \hat{n}_0 \cdot \hat{n}_1\right)} . \qquad (17)$$

Identical formulas hold for R_2 in terms of ds_2 , $d\theta_2$ and $\hat{n}_2$, where ds_2 is drawn along a line perpendicular to the meridian used to find R_1 . Obviously, $\hat{n}_0$ and $\hat{n}_2$ will intersect somewhere on the central axis. The components of ds_2 now will be dx_2 , $dy_2 = 0$; $dz_2 = 0$.

The normal vector $\hat{n}$ can be determined quite easily. The slope of a tangent to the surface depends only on z or r. Define the angle as α; then:

$$\tan \alpha = \frac{df}{dr} = \dot{f} \quad . \tag{18}$$

Hence, the z-component of $\hat{n}$ is

$$\hat{n}_z = \cos \alpha = \frac{1}{\sec \alpha} = \frac{1}{\sqrt{1 + \tan^2 \alpha}} = (1 + \dot{f}^2)^{-1/2} \quad . \tag{19}$$

The x- and y-components can be expressed in terms of the angle ϕ measured from the x-axis in a horizontal plane:

$$n_x = \beta(r) \cos \phi \quad ; \tag{20}$$

$$n_y = \beta(r) \sin \phi \quad . \tag{21}$$

The condition

$$\hat{n}^2 = n_x^2 + n_y^2 + n_z^2 = 1$$

determines

$$\beta(r) = -\dot{f}(1 + \dot{f}^2)^{-1/2} \quad ; \tag{22}$$

the minus sign applies because $\dot{f}$ is negative on a convex surface.

A word of explanation appears warranted before proceeding with the explicit calculation. Note that if v is the parameter which is being varied to proceed from $\hat{n}_0$ to $\hat{n}_1$ or $\hat{n}_2$, then a Taylor expansion gives

$$\hat{n}(v + dv) = \hat{n}(v) + (dv)\,\frac{\partial \hat{n}(v)}{\partial v} + \frac{(dv)^2}{2}\,\frac{\partial^2}{\partial v^2}\,\hat{n}(v) \tag{23}$$

to sufficient accuracy. Hence, the cosine of the angle between $\hat{n}(v)$ and $\hat{n}(v + dv)$ depends on

$$\hat{n}(v + dv) \cdot \hat{n}(v) = \hat{n}^2(v) + (dv)\,\frac{\partial}{\partial v}\left[\frac{\hat{n}^2(v)}{2}\right] + \frac{(dv)^2}{2}\,\hat{n}(v) \cdot \frac{\partial^2}{\partial v^2}\,\hat{n}(v) \quad . \tag{24}$$

The first term is just unity, the second vanishes, and the third may be rewritten by making use of:

$$\begin{aligned} \hat{n}(v) \cdot \frac{\partial}{\partial v^2}\,\hat{n}(v) &= \frac{\partial}{\partial v}\left[\hat{n} \cdot \frac{\partial}{\partial v}\,\hat{n}(v)\right] - \left[\frac{\partial \hat{n}(v)}{\partial v}\right]^2 \\ &= -\left(\frac{\partial \hat{n}}{\partial v}\right)^2 \quad . \end{aligned} \tag{25}$$

Hence, the angle $d\theta(v)$ between $\hat{n}(v)$ and $\hat{n}(v + dv)$ is

$$d\theta(v) = dv\,\sqrt{\left[\frac{\partial \hat{n}(v)}{\partial v}\right]^2} \quad . \tag{26}$$

In order to calculate R_1, let $v = r$, ϕ = constant; then:

$$\frac{\partial n_x}{\partial r} = -\cos\phi \, \frac{\ddot{f}}{\left(1+\dot{f}^2\right)^{3/2}} \quad ; \qquad (27a)$$

$$\frac{\partial n_y}{\partial r} = -\sin\phi \, \frac{\ddot{f}}{\left(1+\dot{f}^2\right)^{3/2}} \quad ; \qquad (27b)$$

$$\frac{\partial n_z}{\partial r} = \frac{-\dot{f}\,\ddot{f}}{\left(1+f^2\right)^{3/2}} \quad ; \qquad (27c)$$

thus, since $\ddot{f}$ is negative,

$$d\theta_1 = d\theta(r) = dr \, \frac{-\ddot{f}}{\left(1+\dot{f}^2\right)} \; . \qquad (28)$$

Also

$$ds_1 = \frac{dr}{\cos\alpha} = dr \left(1+\dot{f}^2\right)^{1/2} \quad ; \qquad (29)$$

therefore:

$$\frac{1}{R_1} = \frac{-\ddot{f}}{\left(1+\dot{f}^2\right)^{3/2}} \; . \qquad (30)$$

To calculate R_2, let $v = \phi$, and let z and r be constant; then:

$$\frac{\partial n_x}{\partial \phi} = \frac{\sin\phi \, \dot{f}}{\left(1+\dot{f}^2\right)^{1/2}} \quad ; \qquad (31a)$$

$$\frac{\partial n_y}{\partial \phi} = \frac{-\cos\phi \, \dot{f}}{\left(1+\dot{f}^2\right)^{1/2}} \quad ; \text{ and} \qquad (31b)$$

$$\frac{\partial n_z}{\partial \phi} = 0 \quad . \tag{31c}$$

Hence,

$$d\theta_2 = d\theta(\phi) = d\phi \frac{-\dot{f}}{(1+\dot{f}^2)^{1/2}} \quad , \tag{32}$$

and $ds_2 = r\, d\phi$. Thus,

$$\frac{1}{R_2} = \frac{-\dot{f}}{r(1+\dot{f}^2)^{1/2}} \quad . \tag{33}$$

Therefore, the equation

$$-\frac{\ddot{f}}{(1+\dot{f}^2)^{3/2}} - \frac{\dot{f}}{r(1+\dot{f}^2)^{1/2}} = \frac{2}{R_0} \tag{34}$$

can be solved:

$$z = f(r) = \sqrt{R_0^2 - r^2} - \sqrt{R_0^2 - a^2} \quad , \tag{35}$$

or

$$x^2 + y^2 + \left(z + \sqrt{R_0^2 - a^2}\right)^2 = R_0^2 \quad , \tag{36}$$

i.e., a spherical surface. The parameter a is the radius of the circular frame for $z = 0$ when $r = a$.

When $a << R_0$, Equation (35) describes a parabola:

$$z \approx \frac{a^2 - r^2}{2R_0} \quad . \tag{37}$$

Method Two

The close connection between membranes and liquid surfaces permits another argument to be used in calculating the shape of the surface. Surface tension originates with a term in the expression for the "free energy" of a system which is proportional to the surface area of the system. Hence, one way to calculate the shape of a system is to demand that the free energy (or the surface area) be a minimum, subject to the remaining constraints of the system. In the present instance, the constraint is that the pressure be a constant. Of the "virtual" surface deformations that are possible, only those which preserve volume are allowed since they preserve the pressure within the enclosed volume. Hence, we demand that the area of the surface

$$A = \int_0^a 2\pi r \, ds_1 = \int_0^a 2\pi r \frac{ds_1}{dr} \, dr$$

$$= \int_0^a 2\pi (1 + \dot{f}^2)^{1/2} \, dr \qquad (38)$$

be a minimum, subject to the condition that the volume

$$V = \int_0^a 2\pi r z \, dr$$

$$= \int_0^a 2\pi r f \, dr \qquad (39)$$

be a constant when small changes in f(r) are considered; namely, $\epsilon(r)$; otherwise, these are arbitrary and vanish at the boundary $r = a$. The condition that $\delta A = 0$, subject to V = constant, can be simplified by applying the single Lagrangian multiplier, λ, to the single variational equation

$$\delta_f (A + \lambda V) = 0 \tag{40}$$

with no constraint on $\epsilon(r) = \delta f(r)$ other than that it vanish at $r = a$ and that it be continuous and differentiable. (The identification of the parameter λ is made later, based on knowledge of the physical situation.) Now

$$\delta_f V = \int_0^a 2\pi r \epsilon(r)\, dr \quad . \tag{41}$$

and

$$\delta_f A = \int_0^a 2\pi r \left(1 + \dot{f}^2\right)^{-1/2} \dot{f} \epsilon \, dr \quad . \tag{42}$$

Integrating by parts to free $\epsilon(r)$ from its derivative and using the boundary values to show that the boundary contributions vanish, we arrive at

$$\delta_f A = -\int_0^a 2\pi \frac{d}{dr}\left[r \left(1 + \dot{f}^2\right)^{-1/2} \dot{f}\right] \epsilon \, dr \quad . \tag{43}$$

Substituting Equations (41) and (43) into (40) and setting coefficients of $\epsilon(r)$ equal to zero gives

$$\lambda r = \frac{d}{dr}\left[r \left(1 + \dot{f}^2\right)^{-1/2} \dot{f} \right] \quad . \tag{44}$$

It can be verified that this equation is the same as Equation (34) with

$$\lambda = \frac{-2}{R_0} \quad . \tag{45}$$

Method Three

The forces produced by pressure and tension are first balanced:

$$\vec{F}_p + \vec{F}_t = 0 \quad . \tag{46}$$

Then a ring of radius r on the surface of width ds_1 is considered as in the calculation of area in Method Two. By symmetry, the horizontal forces produced separately by pressure and tension sum to zero when integrated over the ring. The vertical forces also must balance. The force of pressure is normal to the surface and consists of p times the area. Applying Equation (29), we find the vertical component to be:

$$F_p^{vert} = p\, 2\pi r\, ds_1 \cos \alpha (r) = p\, 2\pi r\, dr \quad . \tag{47}$$

By contrast, the tension is directed tangentially along the surface and must be multiplied by the length of the line along which it acts. The effective force on the element ds is caused by competitive pulls on the opposite edges of that element. Hence, the vertical component of the tension force is

$$
\begin{aligned}
F_t^{vert} &= -t\,2\pi r \sin\alpha(r) - \left[-t\,2\pi(r+dr)\sin\alpha(r+dr)\right] \\
&= 2\pi t\left[(r+dr)\left(\sin\alpha + dr\frac{d}{dr}\sin\alpha\right) - r\sin\alpha\right] \\
&= 2\pi t\,dr\left(\sin\alpha + r\frac{d}{dr}\sin\alpha\right) \quad . \qquad (48)
\end{aligned}
$$

Thus, Equation (46) becomes

$$
\begin{aligned}
-\frac{2}{R_0} = -\frac{p}{t} &= \frac{1}{r}\left(\sin\alpha + r\frac{d}{dr}\sin\alpha\right) \\
&= \frac{1}{r}\frac{d}{dr}(r\sin\alpha) \quad . \qquad (49)
\end{aligned}
$$

From $\cos\alpha = \left(1+\dot{f}^2\right)^{-1/2}$, we can derive

$$
\sin\alpha = \frac{\dot{f}}{\left(1+\dot{f}^2\right)^{1/2}} \quad . \qquad (50)
$$

It can be seen that Equation (49) is the same as Equations (34) and (44), so that once again the result is a spherical shape.

OTHER CONSIDERATIONS

On the Tensor Theory of the Membrane

The peculiarly simple properties of ideal membranes are generally assumed in most textbook treatments of vibrating membranes. The writer has yet to find an explanation of the two key facts about ideal membranes: namely, that

(a) they are characterized by a single quantity, the tension, at each point; and

(b) it is possible, in the general case, to have the tension the same at all points.

These are, in fact, properties of liquid surface tension; their applicability to elastic sheets deserves some comment.

The general state of stress in a deformable body is characterized by a symmetric tensor, the stress tensor, $\underset{\approx}{P}$, whose components give the perpendicular and tangential forces (per unit area) acting on any surface in the body. $\underset{\approx}{P}$ consists of nine components, six of which are, in general, independent. However, it is impossible for stresses to exist in a membrane if any component is perpendicular to the surface because the membrane is a flexible, two-dimensional body. Hence $\underset{\approx}{P}$ becomes, in effect, a symmetric tensor $\underset{\approx}{T}$ in two dimensions with four components, three of which are independent, giving the forces (per unit length) acting on any line in the (two-dimensional) body.

For a body in equilibrium under external forces (per unit volume) $\underset{\sim}{q}$, the equilibrium condition is [69]

$$\underset{\sim}{q} + \nabla \cdot \underset{\approx}{P} = 0 \quad . \tag{51}$$

Restricted to a thin "shell," this may be written

$$\underset{\sim}{q} + \nabla \cdot \underset{\approx}{T} = 0 \quad , \tag{52}$$

the perpendicular component of which is

$$\frac{T_1}{R_1} + \frac{T_2}{R_2} = q_n \quad . \tag{53}$$

Now, $T_1 = T_{11}$ and $T_2 = T_{22}$ are the diagonal elements of $\underset{\approx}{T}$, and the off-diagonal elements will be written

$$T_{12} = T_{21} = S \quad . \tag{54}$$

Also R_1 and R_2 are the same curvatures introduced earlier (for full details, see Reference 70). The other two equations depend on the choice of coordinate system. For a circularly symmetric shell or membrane, let θ be the angle which the normal $\hat{n}$ makes with the central axis of symmetry (which it must intersect), and let ϕ be the angle around the central axis. Let $R_1 T_1 q_1$ refer to the θ-direction (the meridian), and $R_2 T_2 q_2$ to the ϕ-direction (the parallel circle); then the other two equations are (see Reference 70, page 104):

$$\frac{1}{R_1}\frac{\partial T_1}{\partial \theta} + \frac{\cot an\ \theta}{R_2}\left(T_1 - T_2\right) + \frac{1}{R_2 \sin\theta}\frac{\partial S}{\partial \phi} + q_1 = 0 \quad ; \tag{55}$$

$$\frac{1}{R_1}\frac{\partial S}{\partial \theta} + \frac{2\cot an\ \theta}{R_2} S + \frac{1}{R_2 \sin\theta}\frac{\partial T_2}{\partial \phi} + q_2 = 0 \quad . \tag{56}$$

For the boundary conditions, it is necessary to specify T_1 and S everywhere on the circular boundary, but not T_2. [71] In this case, the "loading" $\underset{\sim}{q}$ is caused by pressure, and

$$q_1 = q_2 = 0 , \qquad q_n = p \quad . \tag{57}$$

Furthermore, if the loading at the boundary is symmetrical in ϕ, neither S, T_1, nor T_2 will depend on ϕ. This leads to simpler equations:

$$\frac{\partial T_1}{\partial \theta} + \frac{R_1 \cot an\, \theta}{R_2} \left(T_1 - T_2\right) = 0 \quad , \tag{58}$$

and

$$\frac{\partial S}{\partial \theta} + \frac{2 R_1 \cot an\, \theta}{R_2} S = 0 \quad . \tag{59}$$

Note that R_1 and R_2 depend on θ, so these equations are not immediately integrable. There is, in fact, one relation between R_1 and R_2 in the present instance: namely,

$$\frac{d}{d\theta} \left(R_2 \sin \theta\right) = R_1 \cos \theta \quad . \tag{60}$$

It first can be argued that S is zero on the boundary and therefore is zero everywhere. This can be seen from symmetry alone, or from Equation (59): if $S(\theta)$ is written as a Taylor series expansion relative to the boundary $\left(\text{at } \theta = \theta_0\right)$, every derivative $(\partial^n S/\partial \theta^n)$ evaluated at the boundary is zero because $S = 0$ at the boundary and because of Equation (59).

Next it can be argued that if the shape is spherical, then $R_1 = R_2 = R_0$, in which case Equations (53) and (58) become

$$T_1 + T_2 = R_0\, p \quad , \tag{61}$$

and

$$\frac{\partial T_1}{\partial \theta} + \cot an\, \theta \left(T_1 - T_2\right) = 0 \quad . \tag{62}$$

It follows that

$$\frac{\partial T_2}{\partial \theta} = - \frac{\partial T_1}{\partial \theta} = \text{cotan}\ \theta \left(T_1 - T_2 \right) \quad ; \tag{63}$$

hence:

$$\frac{\partial \left(T_1 - T_2 \right)}{\partial \theta} + 2\ \text{cotan}\ \theta \left(T_1 - T_2 \right) = 0 \quad . \tag{64}$$

Letting

$$M(\theta) = T_1 - T_2 \quad , \tag{65}$$

we can solve Equation (64) as shown below:

$$M(\theta) = M(\pi/2)\ \frac{1}{(\sin \theta)^2} \quad . \tag{66}$$

If $M(\theta)$ is not to become infinite at $\theta = 0$ (the center of the membrane), it must follow that $M(\pi/2) = 0$, and

$$T_1 = T_2 = t = \left(R_0\, p/2 \right) \quad . \tag{67}$$

Thus, it has been shown that a spherical shape for the membrane in a circular frame is consistent with the assumptions. Furthermore, this has been demonstrated for a membrane initially under deformation. Since, however, the theorem also is valid for the case of the flat circular membrane, it may be used for a larger class of problems than static deformations by assuming that the extra tension produced by deformation may be neglected because it is much smaller than the initial tension. Thus for small displacements of the membrane from its equilibrium position, whatever the shape may be, the ideal membrane conditions are applicable.

PROPERTIES OF MYLAR MEMBRANES

This subsection discusses certain features of the Mylar membrane when it is stretched over a circular frame. The emphasis is on order-of-magnitude calculations of the elastic and nonelastic properties of the membrane. Physical parameters of the Mylar were obtained from Reference 72.

Vibrations of the Membrane

The infinitely small displacements of a freely vibrating membrane satisfy the well-known wave equation

$$T \left(\frac{\partial^2 Z}{\partial x^2} + \frac{\partial^2 Z}{\partial y^2} \right) = \rho \frac{\partial^2 Z}{\partial t^2} , \tag{68}$$

where

T	is	the tension within the membrane when it is in its rest position on the x-y plane;
ρ	is	the areal density of the membrane (assumed to be of uniform thickness); and
$Z(x, y)$	is	the displacement of the membrane from the x-y plane at the point x, y .

The quantity

$$c = \sqrt{\frac{T}{\rho}} \tag{69}$$

is the phase velocity of waves described by Equation (68). For a circular membrane of radius a, we introduce polar coordinates r, θ and find that a typical harmonic solution of Equation (68) has the form

$$Z = k J_n \left(\frac{\omega}{c} r\right) \cos n\theta \cos \omega t \quad , \tag{70}$$

where

k = an arbitrary constant,

$\omega = 2\pi\nu$ = the frequency of the particular vibration,

n = a positive integer that gives the angular behavior of the mode of vibration, and

J_n = the n^{th} Bessel function of the first kind.

Since the membrane is fixed to the boundary at $r = a$, there is a condition

$$J_n \left(\frac{\omega}{c}\right) a = 0 \tag{71}$$

on the possible frequencies of vibration, which means that $(\omega/c)a$ must equal one of the zeros of the Bessel functions, say the i^{th} consecutive zero of J_n, designated here as $g_i^{(n)}$. For example:

$$g_1^{(0)} = 2.405 , \quad g_1^{(1)} = 3.832 , \quad g_1^{(2)} = 5.136 ;$$

$$g_2^{(0)} = 5.520 , \quad g_2^{(1)} = 7.016 , \quad g_2^{(2)} = 8.417 . \tag{72}$$

The lowest frequency of vibration occurs in the circularly symmetric $(n = 0)$ mode, $\omega a/c = 2.405$.

A membrane consisting of 1/2-mil thick aluminized Mylar stretched in a standard way over a frame with radius $a = 8.5$ cm was found to have a fundamental frequency ν_0 of 130 Hz. Using a figure of 1.40 for the specific gravity of Mylar (uncoated), we get $\rho = 0.90 \times 10^{-3}$ gm/cm^2, from which the tension

$$T = \frac{4\pi^2}{5.784} \rho \nu_0^2 a^2 \tag{73}$$

is found to be 3×10^3 dynes/cm.

Static Membrane Under Pressure

An ideal circular membrane deformed by the constant pressure difference, p, operating against its tension, T, will assume the shape of a section of a sphere with radius of curvature, R, given by

$$R = \frac{2T}{p} \quad . \tag{74}$$

The shape of the membrane is, for "not-too-large" displacements, closely approximated by a paraboloid:

$$Z = \sqrt{R^2 - r^2} - \sqrt{R^2 - a^2} \approx \frac{a^2 - r^2}{2R} \quad . \tag{75}$$

The displacement at the center, Z_0, is

$$Z_0 = \frac{a^2}{2R} = \frac{a^2 p}{4T} . \tag{76}$$

A circular Mylar membrane, prepared in a fashion identical to that discussed above and used as one face of an air-tight chamber, was observed to be capable of displacements Z_0 of the order of 1/2 centimeter under pressure increases of less than 1/100 atmosphere. Since 1 atmosphere is 1×10^6 dynes/cm^2, the implied tension, according to Equation (76), is less than 4×10^5 dynes/cm. While the pressure could not be determined more precisely, the elastic properties show that it was less than 1/100 atmosphere by a factor of from 5 to 10.

The membrane may be displaced 1/2 centimeter any number of times with no noticeable loss of elasticity.

Elastic Properties

Mylar has a tensile strength of 25,000 pounds per square inch (psi), which can be converted to 2×10^9 dynes/cm^2. The tensile strength is a measure of the maximum stress to which Mylar can be subjected before rupture occurs. For a membrane 1/2-mil thick (which is equal to 1.3×10^{-3} cm), this represents a "rupture tension" of about 2×10^6 dynes/cm, and would appear to indicate that a membrane with 1/2-centimeter displacement is close to the point of rupture.

The elastic modulus of 5-mil Mylar, 760,000 psi (mean), starts falling off rapidly below 3/2 mil and becomes 600,000 psi at 3/4 mil. A value of 500,000 psi is assumed for 1/2-mil thickness. This converts to 40 x 10^9 dynes/cm^2, which represents a tension of approximately 4 x 10^7 dynes/cm.

The approximate amount that the membrane may be stretched can be calculated from the increased area:

$$A = \int_0^a 2\pi r \left[1 + \left(\frac{dZ}{dr}\right)^2\right]^{1/2} dr$$

$$= \int_0^a 2\pi r \left[1 + \frac{1}{2}\left(\frac{dZ}{dr}\right)^2\right] dr$$

$$= \int_0^a 2\pi r \left(1 + \frac{1}{2}\frac{r^2}{R^2}\right) dr$$

$$= \pi a^2 + \frac{\pi}{4}\frac{a^4}{R^4}$$

$$= \pi \left(a + \frac{a^3}{8R^2}\right)^2 \quad . \qquad (77)$$

The fractional increase in radius is

$$\frac{\Delta a}{a} = \frac{a^2}{8R^2} - \frac{Z_0^2}{2a^2} \quad . \qquad (78)$$

which is approximately 2×10^{-3}. This number multiplied by the elastic modulus (times a constant greater than unity, depending on the value of Poisson's ratio) produces in the membrane, because of stretching, an additional tension somewhat greater than 8×10^4 dynes/cm. Thus the extra tension in the membrane produced by stretching can be bracketed between 8×10^4 and 4×10^5 dynes/cm.

CONCLUDING COMMENTS ON THE CIRCULAR MEMBRANE

The analysis indicates that:

1. the static circular membrane under pressure has a spherical shape;

2. to the extent that the membrane follows the driving pressure in the time-dependent case, it will still have a spherical shape;

3. if the inertia of the membrane is comparable to the driving force, the shape of the membrane will not be spherical;

4. in the absence of a driving force (the freely vibrating membrane), the shape is defined (in the circularly symmetric mode) by $J_0 \left[(\omega/c)\,r\right]$ for small displacements;

5. for small displacements, the shape of the sphere closely resembles that of a paraboloid: and

6. the term $J_0\left[(\omega/c)r\right]$ approximates a paraboloid only over its central region. The lower the frequency, the greater the paraboloidal region, but not even in the limit of zero frequency does the paraboloidal region extend over the whole membrane. In fact, the lowest frequency ω_0 is given by $(\omega_0/c)\ a = 2.405$, and the form of J_0 near $r = 0$ is

$$
\begin{aligned}
J_0\left[(\omega_0/c)\,r\right] &= 1 - (1/4)\left[(\omega_0/c)\,r\right]^2 \\
&= 1 - 1.446\left(r^2/a^2\right) \qquad , \qquad (79)
\end{aligned}
$$

which is a paraboloid over the central region of the sphere but not at $r = a$, the boundary.

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APPENDIX VI

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Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) The MITRE Corporation Bedford, Massachusetts	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

A New 3-Dimensional Display Technique

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Final Technical Report

5. AUTHOR(S) (First name, middle initial, last name)

Traub, Alan C. Dr.

6. REPORT DATE May 1968	7a. TOTAL NO. OF PAGES 229	7b. NO. OF REFS 72
8a. CONTRACT OR GRANT NO. Funded under The MITRE Corporation's Independent Research Program. b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) M68-4 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Funded under The MITRE Corporation's Independent Research Program.

13. ABSTRACT

A new dynamic volumetric display technique is described in which a vibrational membrane mirror is used to form a three-dimensional image of a time varying, two-dimensional pattern. This technique offers certain simplifications over existing volumetric display systems of the electromechanical type. Among its advantages are reduced costs and increased reliability; two of its many areas of application are air traffic control and signal analysis, including radar signals. The report describes the operation and characteristics of the basic device, the various forms it can take, and its possible applications in modern display technology.

DD FORM 1 NOV 65 1473

Security Classification

14

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Display systems						
Stereoscopy						
3-D displays						
Volumetric displays						
Control panels						
Optical equipment						
Viewing screens						
Membrane mirrors						
Computer displays						
Command and control systems						
Visual signals						
Teaching aids						
Air traffic control displays						
Human engineering						
Visual perception						
Radar signals						
Signal analysis						
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Vectorcardiography						
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Vibrations in membranes						
Analytical techniques						