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THE ELASTIC THEORY OF EXPLOSION INDUCED WHIPPING
COMPUTER PROGRAM SPECIFICATION

ABSTRACT

A FORTRAN II computer program is described which evaluates the equations given in N.C.R.E./R.532 for the explosion induced whipping motions of ships. The equations themselves have been repeated in full for completeness.

A shortened version of the program is also described. This version determines just the still water vibration frequencies and mode shapes of a ship, as these are often of interest in circumstances other than explosion loading.

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THE ELASTIC THEORY OF EXPLOSION INDUCED WHIPPING
COMPUTER PROGRAM SPECIFICATION

by

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This report describes a computer program, NCRE No. 61A, which evaluates the 'normal mode' equations for the explosion induced whipping motion of ships. The first part of the program determines the frequencies and shapes of the vertical heaving, pitching and whipping modes and a separate, shortened version of the program (N.C.R.E. No. 61B) has been prepared to carry out just this part of the full calculation. The programs are written entirely in FORTRAN II for running on an Atlas computer. A version is also to be written to run with the EGTRAN 3 compiler on a KDF-9 computer. As the program is rather long, no detailed listing is given here, although a flow diagram has been included (Figure 1).

2. The derivation of the equations used in the program is given in an earlier report [1]* and the equations cover only the simplest possible case; viz., the completely elastic response of a ship not too close to a pulsating but non-migrating explosion bubble. The effects of migration, i.e. the modified magnitudes of the later pulses, the simple change in the explosion geometry at later pulses and the more complicated dipole pressure term induced by the migration are all neglected. Also neglected is the interaction of the spherically diverging flow field around the bubble with the finite size of the ship structure. This effect will be most pronounced for charges very close to the ship. Multiple images, of the explosion bubble due to the proximity of the water-surface and sea-bottom can however be included (up to a maximum of four).

3. Although these complicating effects are not included in the program, considerable provision has been made to allow for their later inclusion and the program as it stands should be regarded more as a research tool to investigate such effects than as an established means for predicting whipping motions. In fact, although the program has been fairly thoroughly checked to ensure its internal consistency, the validity of the equations has yet to be confirmed by comparison of the predicted results with experimental data. The purpose of this report is to present a clear and detailed picture of the basic program before the proposed, complicating, modifications are carried out.

4. Unfortunately, the non-linear nature of the plastic response of a ship hinging during whipping motions precludes the possibility of using a normal

/mode

*[ ] = References on page 15.

mode approach in such cases. The more direct approach outlined in [1] would have to be used.

THE EQUATIONS

5. To make the report as self contained as possible, the equations are given here in full, although with little explanation. The ship is considered to be representable by 'n' lumped masses and rotary inertias connected by (n - 1) weightless uniform beams of equal length (whose properties vary however from beam to beam). The elastic and buoyant nature of the ship is then represented by the stiffness matrix

$$S = \begin{bmatrix} (A + K) & B \\ B^T & C + K_r \end{bmatrix} \qquad (2n \times 2n)$$

where the (n x n) submatrices A, B and C have the elements defined by

$$A = (a_{ij}) \quad ; \quad a_{ii} = 6(\alpha_i + \alpha_{i-1}) \qquad (i = 1, \ldots, n)$$

$$a_{i,\, i+1} = a_{i+1,\, i} = - 6\alpha_i \qquad (i = 1, \ldots, n - 1)$$

$$a_{ij} = 0 \qquad \text{otherwise}$$

$$B = (b_{ij}) \quad ; \quad b_{ii} = 3(\alpha_i - \alpha_{i-1})\ell \qquad (i = 1, \ldots, n)$$

$$b_{i,\, i+1} = - b_{i+1,\, i} = 3\alpha_i\ell \quad (i = 1, \ldots, n - 1)$$

$$b_{ij} = 0 \qquad \text{otherwise}$$

$$C = (c_{ij}) \quad ; \quad c_{ii} = \alpha_i\ell^2(2 + \epsilon_i) + \alpha_{i-1}\, \ell^2 \ (2 + \epsilon_{i-1}) \qquad (i = 1, \ldots, n)$$

$$c_{i,\, i+1} = c_{i+1,\, i} = \alpha_i\ \ell^2\ (1 - \epsilon_i) \qquad (i = 1, \ldots, n - 1)$$

$$c_{ij} = 0 \qquad \text{otherwise}$$

In these matrices, ℓ is the length of each beam connecting two masses and α_i and ϵ_i are related to the basic beam properties by

$$\epsilon_o = \epsilon_n = 0 \quad ; \quad \epsilon_i = \frac{12(1 + \nu)I_i}{A_i\ \ell^2} \quad ; \quad (i = 1, \ldots, n - 1)$$

$$\alpha_o = \alpha_n = 0 \quad ; \quad \alpha_i = \frac{2EI_i}{\ell^3(1 + 2\epsilon_i)} \quad ; \quad (i = 1, \ldots, n - 1)$$

E and ν are Young's Modulus and Poisson's Ratio for the material of the beam section. I_i and A_i are the moment of inertia of the cross section and the cross-sectional area which is effective in shear (e.g. for a solid rectangular cross-section a x b, A would be 2/3 ab). The stiffness matrix S therefore represents fully the effect of shear deflections.

6. The matrices K and K_r are diagonal matrices representing the effect of buoyancy. It is assumed that when the ship is floating in its equilibrium position, the sides, at the waterline, are nearly vertical. The diagonal element k_i is then simply

$$k_i = \rho A_{wi} \qquad (i = 1, \ldots, n)$$

where A_{wi} is the waterplane area associated with the ith mass. k_{ri} is given by

$$k_{ri} = \frac{1}{12} k_i \, \ell^2 = \frac{1}{12} \rho A_{wi} \ell^2 \qquad (i = 1, \ldots, n)$$

The symmetric matrix S is transformed into the symmetric matrix E by the transform

$$E = M_1^{-\frac{1}{2}} \, S \, M_1^{-\frac{1}{2}}$$

where $M_1^{-\frac{1}{2}}$ is a (2n x 2n) diagonal matrix with elements (μ_i) given by

$$\left.\begin{aligned} \mu_i &= (m_i + m_{wi})^{-\frac{1}{2}} \\ \mu_{n+i} &= (R_i + R_{wi})^{-\frac{1}{2}} \end{aligned}\right) \quad (i = 1, \ldots, n)$$

m_i is the mass of the ith lumped mass, m_{wi} its added water mass and R_i its rotary inertia. R_{wi} is its added water rotary inertia and is given by

$$R_{wi} = \frac{1}{12} m_{wi} \, \ell^2$$

7. The eigen-values w_i^2 (they will all be positive) of E are the squares of the natural angular velocities of the heaving, pitching and whipping modes of the ship. The frequency of the ith mode is then

$$f_i = \frac{\omega_i}{2\pi}$$

The normal mode shapes of the ship are related to the eigen vectors $\underline{e}_i$ of E by the equation

$$\begin{bmatrix} \underline{y}_i \\ \underline{v}_i \end{bmatrix} = M_1^{-\frac{1}{2}} \, \underline{e}_i$$

/Here

Here y_{ij} is the displacement of the jth mass in the ith mode and v_{ij} is the bending rotation at the jth mass due to the ith mode. The computer program will determine and print out the nodal frequencies and shapes, the shapes being normalised so that

$$\begin{bmatrix} \underset{\sim}{y}_i \\ \underset{\sim}{v}_i \end{bmatrix}^T M_1 \begin{bmatrix} \underset{\sim}{y}_i \\ \underset{\sim}{v}_i \end{bmatrix} = 1$$

$$\left(\text{i.e.} \quad \sum_{j=1}^{n} \{(m_i + m_{wi}) y_{ij}^2 + (R_i + R_{wi})\, v_{ij}^2\} = 1 \quad \right)$$

For submarines, k_i and k_{ri} are zero for all i and the first two natural frequencies will both be zero. This should however cause no difficulties since the eigen value routine uscd in the program finds accurately orthogonal eigenvectors even for repeated eigen values. Physically this case corresponds correctly to zero pitching and heaving frequencies.

8. The normal mode equations of motion are

$$\ddot{\alpha}_i + w_i^2\, \alpha_i = \lambda_i\, \ddot{V}\,(t) \quad ; \quad i = 1, \ldots, n \qquad (1)$$

In this equation α_i is the mode coefficient, V(t) is the volume of the explosion bubble and λ_i the mode forcing function coefficient defined by

$$\lambda_i = \frac{1}{4\pi} \begin{bmatrix} \underset{\sim}{y}_i \\ \underset{\sim}{v}_i \end{bmatrix}^T M_2 \begin{bmatrix} \underset{\sim}{g} \\ \underset{\sim}{h} \end{bmatrix} = \frac{1}{4\pi} \left[\sum_{j=1}^{n} (m_{wi} + \overline{m}_{wi}) y_{ij} g_j + (R_{wi} + \overline{R}_{wi}) v_{ij} h_j \right] \quad ;$$

M_2 being a diagonal matrix with elements $(\overline{\mu}_i)$ given by

$$\left.\begin{aligned} \overline{\mu}_i &= m_{wi} + \overline{m}_{wi} \\ \overline{\mu}_{i+n} &= R_{wi} + \overline{R}_{wi} \end{aligned}\right) \quad (i = 1, \ldots, n)$$

where $\overline{m}_{wi} = \rho \overline{A}_i \ell$; $\overline{R}_{wi} = \frac{1}{12}\, \overline{m}_{wi}\, \ell^2$; $\overline{A}_i$ = cross-sectional area of the water displaced by the ship at the ith mass. g_j and h_j are given by

$$g_j = g_{cj} + g_{sj} + g_{Bj} + g_{sBj}$$

$$h_j = h_{cj} + h_{sj} + h_{Bj} + h_{sBj}$$

/g_{cj}

$$g_{cj} = \frac{D - d}{R_{cj}^{3}} \; ; \; g_{sj} = \frac{D + d}{R_{sj}^{3}} \; ; \; g_{Bj} = \frac{2h - (D + d)}{R_{Bj}^{3}} \; ; \; g_{sBj} = \frac{2h - (D - d)}{R_{sBj}^{3}}$$

$$h_{cj} = \frac{-3(D - d)(X_j - X_w)}{R_{cj}^{5}} \; ; \; h_{sj} = \frac{-3(D + d)(X_j - X_w)}{R_{sj}^{5}} \; ;$$

$$h_{Bj} = \frac{-3\left[2h - (D + d)\right](X_j - X_w)}{R_{Bj}^{5}} \; ; \; h_{sBj} = \frac{-3\left[2h - (D - d)\right](X_j - X_w)}{R_{sBj}^{5}}$$

$$R_{cj}^{2} = (D - d)^2 + H^2 + (X_j - X_w)^2$$

$$R_{sj}^{2} = (D + d)^2 + H^2 + (X_j - X_w)^2$$

$$R_{Bj}^{2} = |2h - (D + d)|^2 + H^2 + (X_j - X_w)^2$$

$$R_{sB_j}^{2} = |2h - (D - d)|^2 + H^2 + (X_j - X_w)^2$$

where the geometries are indicated in Figures 2(a) and 2(b). For a given ship, the program will produce a series of sets of λ_i corresponding to a given series of attack geometries.

9. For non-migrating bubbles, the volume-time record of bubbles can be represented fairly accurately by a single universal, non-dimensional, volume-time record $v(\tau)$ and the second derivative of the volume can be similarly represented. The program, at present, uses linear interpolation in a table of values to find $\ddot{V}(\tau)$. Then $\ddot{V}(t) = K \ddot{v}(\tau)$

where $\tau = t/T_o$

For a charge of W lbs. of T.N.T. (or the T.N.T.equivalent for other explosives) at a depth D ft., K and T_o are given by

$$K = 1220 \, W^{1/3} (D + 33)^{2/3} \quad \text{cu.ft./sec.}^2$$

$$T_o = 2.94 \, W^{1/3}/(D + 33)^{5/6} \quad \text{secs.}$$

The present table for $\ddot{v}(\tau)$ is given in Table 1 and is shown graphically in Figure 3. This table, punched on tape, is attached to the end of the program tape and may easily be replaced by a more realistic version if one is available. For values of τ greater than the maximum given in the table, the program puts $\ddot{V}(\tau) = 0$.

/10.

10. As commented on in [1], the equations of motion (1) may not apply in the early compressible phase of an explosion and $\dot{V}(\tau)$ is not properly defined in this phase of the motion. To by-pass such difficulties, the compressible early shock motion is assumed to set up an instantaneous velocity distribution along the ship, and the function $\dot{V}(\tau)$ is assumed to be zero until an incompressible flow analysis shows that it would otherwise be changing from positive to negative. The first value of τ given in the table corresponds to this instant. The assumed initial velocity distribution along the ship is given by

$$\begin{bmatrix} \dot{\underline{y}}_c \\ \dot{\underline{v}}_c \end{bmatrix} = \frac{\dot{V}_c}{4\pi} M_1^{-1} M_2 \begin{bmatrix} \underline{g} \\ \underline{h} \end{bmatrix}$$

where $\dot{V}_c$ is the early bubble maximum rate of volume expansion, assumed to be achieved instaneously. From an incompressible bubble analysis $\dot{V}_c$ is found to be given by

$$\dot{V}_c = 5950 \frac{W^{2/3}}{(D + 33)^{1/6}} \quad \text{cu.ft./sec.}$$

The program will accept as initial conditions for the integration of equations (1) either an arbitrary initial velocity distribution specified on the data tape, or the one specified above. In the latter case, the distribution will also be printed in the output. With either form of initial velocity distribution, the initial conditions for the equations (1) are

$$\left.\begin{aligned} \alpha_i &= 0 \\ \beta_i = \dot{\alpha}_i &= \begin{bmatrix} \underline{y}_i \\ \underline{v}_i \end{bmatrix}^T M_1 \begin{bmatrix} \dot{\underline{y}}_c \\ \dot{\underline{v}}_c \end{bmatrix} \end{aligned}\right) \quad \text{when } t = 0$$

For the standard initial velocity distribution these are equivalent to

$$\left.\begin{aligned} \alpha_i &= 0 \\ \beta_i = \dot{\alpha}_i &= \lambda_i \dot{V}_c \end{aligned}\right) \; t = 0$$

11. Equations (1) are integrated in the program by a step-by-step Runge-Kutta integration procedure which requires the selection of a suitable time step length to maintain accuracy (or even stability). For stability it is necessary that the incremental time step should not exceed about 1/6 of the shortest period $(2\pi/w_i)$ inherent in equations (1). In addition, the forcing function $\lambda\dot{V}(t)$ is sometimes very rapidly changing (e.g. near each bubble pulse) and sometimes slowly changing (between the pulses).

/The

The time step must be short enough to represent the forcing function accurately even when it is changing most rapidly. To achieve some economy in the number of time increments used, the time increment varies during the integration. It is normally set to δt = (1/Nth of the shortest period present in equations (1)). N is given on the data tape and for accuracy it should normally be at least 8. If, during the integration, the program finds that the dimensionless increment between the current tabular values of τ in the $V(\tau)$ table is less than $\delta t/T_o$, then it reduces the increment to this smaller value. The increment is increased again as soon as possible. Thus, between the pulses, the integration step is tied to the stability of the differential equations, and near the pulses it is related to the increments necessary to define the forcing function. If the standard tabular forcing function is changed, its role in determining integration step lengths near pulses should be borne in mind.

12. Since many more incremental integration steps will be made than are necessary to see easily the form of the results, the time increments at which the results are printed is independent of the integration step length. A print increment δt_p is given on the data tape and the program prints out t, α_i and $\dot{\alpha}_i$ at the nearest integration steps to the print intervals. The output is therefore not quite at equal time intervals.

13. A knowledge of the mode shapes (χ_i, v_i) and the mode coefficients α_i and their derivatives $\dot{\alpha}_i$ determine the whipping motion completely. Generally however it is more convenient to know the total motion (displacement, velocity, bending moment, shearing force and deck stress) at specified points along the ship. This motion at a point distance x from the bows of the ship may be determined from the equations:-

$$\begin{aligned}
&\text{Displacement} \quad ; \quad y(x,t) = \sum_{i=1}^{N_2} \alpha_i(t)\, y_i\,(x) \\
&\text{Velocity} \quad , \quad \dot{y}(x,t) = \sum_{i=1}^{N_2} \dot{\alpha}_i(t) y_i\,(x) \\
&\text{Bending Moment}; \quad M(x,t) = \sum_{i=1}^{N_2} \alpha_i(t)\, M_i(x) \\
&\text{Shearing Force}; \quad S(x,t) = \sum_{i=1}^{N_2} \alpha_i(t)\, S_i(x) \\
&\text{Deck Stress} \quad ; \quad \sigma(x,t) = \frac{\bar{y}M(x,t)}{I} \;,
\end{aligned} \qquad (2)$$

/where

where N_2 is the number of modes being considered, I is the cross-sectional moment of inertia for the beam section at the point x and $\bar{y}$ is the distance of the deck from the neutral axis of that particular beam section. The quantities $y_i(x)$, $M_i(x)$ and $S_i(x)$ are called the 'positional mode coefficients' in the program and are the displacement, bending moment and shearing force at the point x when the lumped mass displacements and rotations are defined by the ith mode shape $(\underline{y}_i, \underline{v}_i)$.

14. Since each beam section is of length 'ℓ', x can be expressed as

$$x = (j - \tfrac{1}{2})\ell + \bar{x} \qquad \text{where } 0 \leq \bar{x} \leq \ell$$

and x will lie in the jth beam with shear area A_j and moment of inertia I_j. $Y_i(x)$, $M_i(x)$ and $S_i(x)$ are then given by the equations

$$\begin{bmatrix} M_L \\ -S_L \end{bmatrix} = \begin{bmatrix} 3\alpha\ell & -\alpha\ell^2(1 - \varepsilon) \\ -6\alpha & 3\alpha\ell \end{bmatrix} \begin{bmatrix} y_{i\ j+1} \\ v_{i\ j+1} \end{bmatrix} + \begin{bmatrix} -3\alpha\ell & -\alpha\ell^2(2 + \varepsilon) \\ 6\alpha & 3\alpha\ell \end{bmatrix} \begin{bmatrix} y_{ij} \\ v_{ij} \end{bmatrix}$$

$$\begin{bmatrix} y_i(x) \\ M_i(x) \\ -S_i(x) \end{bmatrix} = \begin{bmatrix} 1 & \bar{x} & \dfrac{\bar{x}^2}{2EI_j} & \dfrac{\bar{x}^3(1 - \varepsilon)}{6EI_j} \\ 0 & 0 & 1 & \bar{x} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{ij} \\ v_{ij} \\ M_L \\ -S_L \end{bmatrix} \qquad (3)$$

where y_{ij} and v_{ij} have the same meaning as earlier (displacement and rotation at jth mass in ith mode) and, again as earlier

$$\varepsilon = \frac{12(1 + \nu)I_j}{A_j\ell^2}$$

$$\alpha = \frac{2EI_j}{\ell^3(1 + 2\varepsilon_j)}$$

The sign convention for these quantities is shown in Figure 4.
If required, the program will form and print out both the positional mode coefficients and for each print time step, the summations defined by (2). These quantities can be compared directly with experiment.

ROTARY INERTIAS

15. In all the above equations it has been assumed that rotary inertia plays an important part in determining the mode shapes. Where it is thought this

/is

is not the case the order of the above equations (generally 2n) may be halved.

The (2n x 2n) stiffness matrix S is replaced by the (n x n) matrix

$$S^{*} = \left[(A + K) - BC^{-1}B^{T}\right]$$

The (2n x 2n) matrix E is replaced by the (n x n) matrix

$$E^{*} = M_1^{*-\frac{1}{2}}\, S^{*}\, M_1^{*-\frac{1}{2}}$$

where $M_1^{*-\frac{1}{2}}$ is the (n x n) diagonal matrix with elements μ_i^{*} given by

$$\mu_i^{*} = (m_i + m_{wi})^{-\frac{1}{2}} \qquad (i = 1, \ldots, n)$$

E^{*} has only n eigen values and vectors.

From each eigenvector $\underset{\sim}{e}_i^{*}$, the displacements of the lumped masses in the ith mode shape are determined by

$$\underset{\sim}{y}_i = M_1^{*-\frac{1}{2}}\ \underset{\sim}{e}_i^{*}$$

This equation however now only gives the displacements. The rotations must be determined from the auxiliary equation

$$\underset{\sim}{v}_i = - C^{-1}B^{T}\,\underset{\sim}{y}_i$$

The normalisation of the modes in this case is such that

$$\underset{\sim}{y}_i^{T}\, M_1^{*}\, \underset{\sim}{y}_i = 1$$

$$\text{i.e.} \sum_{j=1}^{n} (m_i + m_{wi})y_{ij}^{2} = 1$$

The mode forcing functions are then given by

$$\lambda_i = \frac{1}{4\pi}\, \underset{\sim}{y}_i^{T}\, M_2^{*}\, \underset{\sim}{g} = \frac{1}{4\pi}\left[\sum_{j=1}^{n}(m_{wi} + \overline{m}_{wi})\, y_{ij}\, g_j\right]$$

where M_2^{*} is the obvious (n x n) contraction of M_2. Similarly the initial velocity distribution equation is a simple contraction of the rotary inertia case.

16. The program incorporates all these modifications to the basic equation when rotary inertias are to be suppressed.

/SUBROUTINES

SUBROUTINES

17. The program contains the following subroutines:-

(a) PRINT 1;

This prints out mode frequencies and shapes.

(b) PRINT 2;

This routine prints the forcing function coefficients when these are required.

(c) RK (N);

This carried out a Runge Kutta integration on a set of 'N' first order differential equations. It requires a subroutine AUX (I, TA) to set up values of the right hand side of the equations.

(d) AUX (I, TA) ;

This routine sets up the right hand sides of the equations solved by the routine RK. The right hand sides have the form $\lambda_i \ddot{V}(t)$ and the routine needs another routine DECDER (TA) to give the current value of $\bar{V}(t)$ at time t = TA. When the explosion bubble is migrating, the λ_i values will also be time dependent. This case is indicated to the program by setting D on the data tape to minus the actual charge depth. AUX will then call in an additional routine FORMD(TA) which will determine the current depth D(t) of the bubble and then set corresponding λ_i values. In the non-migrating case the same λ_i values are retained throughout the integration. The parameter I determines whether the current entry to AUX is the 1st, 2nd, 3rd or 4th for the integration step. On the 2nd and 4th entries AUX adds ½ DELTAT (DELTAT is the current integration step length) to TA.

(e) LAMDA;

Given the attack geometry d, h, W, x_w, H and the charge depth D, which is possibly time dependent (set by FORMD prior to entry to LAMDA), this routine calculates $\underset{\sim}{g}$ and $\underset{\sim}{h}$, from these and the mode shapes, determines the values of λ_i, the mode forcing function coefficients. It also tests n, and, if this is negative, it calls in PRINT 2 to print out the values of λ_i.

(f) MATDVI;

This routine carries out the matrix division required in the evaluation of S^*, the stiffness matrix in the non-rotary inertia case

(g) NCEIGN;

This routine is a master routine for calling in a suite of subroutines which carry out an HGW type determination of the eigen

/values

values and vectors of a symmetric matrix. The routines are described in detail in [2]. Only the first N_1 eigen values (in ascending order of frequency) are found and the routines will give orthogonal vectors even for repeated roots (e.g. zero frequency pitch and heave for a submerged submarine). The suite of subroutines consists of:-

NCHTD - Householder Tridiagonalisation

NCTDBS - Tridiagonal Matrix root bisection routine

STURM - Sturm sequence evaluation

NCSEP - Separation routine for equal/nearly equal roots

NCTRII - Inverse Iteration procedure

NCORTH - Orthogonalisation of vectors at a repeated root

NCBKTR - Back transformation of vectors

(h) FORMD (TA);

This routine is called in by AUX, when the migrating bubble case is specified on the data-tape by setting D to - (charge depth). It should determine the depth of the explosion bubble centre for a given value TA of the time since the charge detonated. It should then call in the subroutine LAMDA to set values of λ_i.

At present the subroutine is merely a dummy, leaving the value of D unchanged. The program can therefore only deal with the non-migrating case.

(i) SECDER (TA);

This routine determines the value of $\ddot{V}(t)$, the bubble volume acceleration for the current value t = TA of the time since detonation. It also determines the largest time interval consistent with a reasonably accurate definition of $\ddot{V}(t)$ where this is changing rapidly (near the bubble pulses). The normal integration interval for subroutine RK is DEL. DELW is the next integration step which will be used. If the time interval defining $\ddot{V}$ is less than DEL then DELW is put equal to the $\ddot{V}$ interval. If however the interval is greater than DEL then DELW is set equal to DEL. The present SECDER routine computes a 'scale' time T_o to reduce TA to a dimensionless time τ. It interpolates linearly in a table of values to find a dimensionless acceleration $\ddot{v}(\tau)$ and converts this to the required quantity $\ddot{V}$(TA) by another scale factor K. T_o and K are described in paragraph 8. The current time interval needed to describe $\ddot{V}$ is T_o times the current increment in the table.

THE DATA TAPE

18. This can vary considerably, depending on the output required of the calculation. Table 2 shows a schematic basic data tape, together with the input formats and units. The description in Table 2 indicates where additions or subtractions to the basic data tape should be made. Several of the parameters (e.g. n, N_2, n_1 etc.) serve both as basic data and as control parameters. If set to zero they return control to an earlier point in the program so that sections of the calculation may be repeated several times with different sets of data. The program can only be properly terminated by using these control parameters to return control to the start of the data tape and then setting n = 0. These control parameters are described more fully in the following notes.

NOTES ON THE DATA TAPE

19. (a) The data tape is in two logically distinct parts:- the first, from n to $\bar{y}$ gives details of the ship, its girder strength etc., and the points along it of particular interest. The second part, from n_1 to t_F, gives details of the attack geometry and the integration procedure. The program is arranged so that the second part, from n_1, may be repeated as often as required so that, at one run, several different attack geometries may be used against the same ship geometry, the ship normal modes being evaluated only on the first calculation. To finish calculations for the one ship, a dummy value zero should replace the next n_1; this returns control to the start of the data tape, where a completely new ship can be considered if required by repeating the entire data tape with the new ship values. Alternatively, a second dummy value, zero, for the new n will stop the program.

(b) If only the mode shapes and frequencies are of interest and these are to be evaluated for several different ships or several times for the same ship with minor changes in the ship details, this is more simply accomplished by setting N_2 = 0 in each set of ship data and omitting the rest of the tape for each case. When N_2 is read, if it is zero, control is returned immediately to the start of the tape. A separate shortened version N.C.R.E. No. 61B of the program has been made which evaluates only the normal modes and frequencies. The data tape for the short version is as for the full program down to N_1 but then repeats if more than one calculation is required.

/(c)

(c) Positional Mode Coefficients

These are used during the integration to compute the total displacement, velocity, bending moment, shear force and deck stress due to all the included modes acting together. They may however be of interest by themselves and will be printed out if m is specified as a negative value.

(d) The histories of some of the quantities displacement, velocity, bending moment etc. may not be required, and to reduce the output only the relevant ones need be printed. The parameters determining the print are L_1, L_2 and L_3

$L_1 = 1$ displacement and velocity are printed

$L_1 = 0$ " " " " not printed

$L_2 = 1$ bending moment and shear force are printed

$L_2 = 0$ " " " " " " not printed

$L_3 = 1$ deck stress is printed

$L_3 = 0$ " " " not printed

(e) Number of Charge Images

As noted in paragraph 7, the free water surface and the sea bottom may influence the ship whipping if the ship or the charge is close to either. Their effect is approximated by images; the number of images being determined by $|n_1|$.

$|n_1| = 1$; charge only

$|n_1| = 2$; charge + surface image

$|n_1| = 3$; " + " + bottom image

$|n_1| = 4$; " + " + " " + image of bottom image in the surface

If n_1 is set to zero, it immediately returns the program control to the start of the data tape (n). Although these images account for the distortion of the fluid flow due to the bottom and free surface, they do not include the effect on the bubble period. Consequently, for charges very close to the bottom it would be better to put $|n_1| = 1$ or 2, ignoring the bottom image, and to use twice the given charge weight for W.

(f) Mode Forcing Function Coefficients

Apart from minor changes in the bubble period due to differing charge depths, the extent to which each mode is excited is, by the linearity of the equations (1), proportional to the mode

/forcing

forcing function coefficients λ_i. A great deal may therefore be learnt about the relative severities of different attack geometries simply by comparing the mode coefficients for the geometries. Such studies minimise the number of complete integrations required. The λ_i's will be printed if n_1 is given a negative value. To evaluate several sets of λ_i for different attack geometries, the basic geometry set d, h, W, xw, H, D may be repeated as often as desired if the values of W are all made negative. $|W|$ will then be used as the charge weight. If any positive value of W is encountered the program will continue normally, or, if a zero value is specified in a dummy geometry set, control will be returned immediately to the data-tape at the position N_2 (see Note (b)).

(g) If D in the attack geometry set is given as negative, the program will assume that bubble migration is taking place during the integration and the subroutine FORMD will be expected to give the bubble depth at any specified time. This will allow for the change in the position of the bubble between pulses but will not allow for the effect of the migration on the magnitude of the pulses. At present FORMD is a dummy routine (see subroutine (h)).

(h) Initial Velocity Distribution

This should normally be determined by the equations of paragraph 9 and for this case N_3 should be 0 or 1. If $N_3 = 0$, the initial values of $\underset{\sim}{\alpha}$ and β only will be computed, as initial conditions for the Runge-Kutta integration. If $N_3 = 1$, the actual values of $\underset{\sim}{y}_c$ and $\underset{\sim}{v}_c$ will also be computed and printed. A non-standard velocity distribution may be specified on the data tape if required by setting N_3 equal to 2. If rotary intertias have been included, both $\underset{\sim}{y}_c$ and $\underset{\sim}{v}_c$ will have to be given on the data tape, but if rotary inertia has been neglected throughout, then only $\underset{\sim}{y}_c$ should be specified.

OUTPUT

20. This depends very much on the data tape used since most of the output is optional. The output has been extensively titled and, apart from the units used is self explanatory. The maximum output consists of

(a) Mode shapes and frequencies. The displacements are in feet and the frequencies in cycles/sec.

(b) Positional mode coefficients. These values refer to the ship under unit deformation successively in each mode. The units are displacement - ft., moment - lbs.ft., shear force - lbs., deck stress (SIGMA) - lbs./sq.in. This last quantity is the longitudinal stress at the specified distance 'x' along the ship and height $\bar{y}$ above the neutral axis.

(c) Attack geometry data. These are the quantities d, h, W, x_W, H and D as given on the data tape except that negative values for W and D (used to control the course of the program) will have been suppressed. The units are as for the data tape.

(d) Forcing function coefficients. These values $\lambda_1, \lambda_2, \ldots, \lambda_{N_2}$ are in $(\text{feet})^{-3}$.

(e) Initial velocity distribution. $\underset{\sim}{\dot{y}}$ is in ft./sec. and $\dot{v}$ in sec.$^{-1}$

(f) Time histories. For each print-time increment, the time (in secs.) is followed by the values of the mode coefficients $\underset{\sim}{\alpha}$ and $\underset{\sim}{\beta}$ $(\underset{\sim}{\dot{\alpha}})$. Then for each specified position x, the displacement, velocity etc. due to all the specified modes together is given.

MAXIMUM SIZE OF CALCULATION

21. The program can cope with a ship divided into up to 40 lumped masses and 40 lumped rotary inertias (i.e. 80 degrees of freedom).

22. Up to 20 positions may be specified for positional mode coefficients and displacement etc. histories.

REFERENCES

1. A. N. Hicks, N.C.R.E. Report No. R.532. "The Elastic Theory of the Explosion Induced Whipping Motion of Ships and Submarines". April, 1968.

2. G. Room: "Two matrix program specifications: eigenvalue determination and supermatrix solution of equations" (to be published).

/TABLE 1

TABLE 1

DIMENSIONLESS VOLUME ACCELERATION $\ddot{v}(\tau)$

0.16850	+ 0.0000		2.18649	- 2.0505
0.19281	- 0.5827		2.28430	- 1.6194
0.28521	- 1.9905		2.34820	- 1.1452
0.34729	- 2.5879		2.39739	- 0.6150
0.42805	- 3.1353		2.43765	- 0.0106
0.55009	- 3.6433		2.47166	+ 0.6945
0.71611	- 3.9090		2.50104	+ 1.5392
0.76462	- 3.9090		2.52688	+ 2.5837
0.93064	- 3.6433		2.55021	+ 3.9240
1.05268	- 3.1353		2.57237	+ 5.7217
1.13344	- 2.5879		2.58638	+ 7.1310
1.19552	- 1.9905		2.59759	+ 8.2644
1.28792	- 0.5827		2.60543	+ 8.9034
1.31223	+ 0.0000		2.61352	+ 9.2785
1.37936	+ 2.5207			
1.41973	+ 6.0896		2.62008	+10.4291
1.44802	+12.9140		2.62490	+ 9.8540
1.45846	+19.2760		2.63518	+ 7.7911
1.46971	+37.2590		2.65371	+ 4.6281
1.47366	+52.6290		2.66761	+ 3.1751
1.47769	+79.1790		2.68312	+ 2.0977
1.47859	+85.4500		2.72113	+ 0.5882
1.47930	+89.5690		2.77276	- 0.4515
1.47962	+91.0060		2.80650	- 0.8710
1.48006	+92.4780		2.84889	- 1.2455
1.48036	+93.0770		2.90710	- 1.5855
1.48449	+ 9.2785			
1.49258	+ 8.9034			
1.50042	+ 8.2644			
1.51163	+ 7.1310			
1.52564	+ 5.7217			
1.54780	+ 3.9240			
1.57113	+ 2.5837			
1.59697	+ 1.5392			
1.62635	+ 0.6945			
1.66036	- 0.0106			
1.70062	- 0.6150			
1.74981	- 1.1452			
1.81367	- 1.6194			
1.91152	- 2.0505			
2.03656	- 2.2490			
2.06145	- 2.2490			

/TABLE 2

TABLE 2

SCHEMATIC DATA TAPE

Quantity	Format	Units	Program Variables	Description
DATA K15	-	-	-	Atlas data control symbols
n	(I3)	-	INLM	$\|n\|$ = number of lumped masses $n > 0$; rotary inertias included $n < 0$; " " excluded $n = 0$; program terminates
E ν L	(F5.0, F5.2, F6.2)	10^6 p.s.i., -, ft.	EM, PR, AL	Young's modulus, Poissons Ratio and length of each beam section
A_1 I_1) A_2 I_2) . .) . .) . .) A_{n-1} I_{n-1})	(F5.0, F8.0)	sq.ins., $ft.^2-in.^2$	AA(I), AI(I)	Shear areas and area moments of inertia for beam sections
m_1 m_{v1} $\bar{m}_{v1}$ k_1) m_2 m_{v2} $\bar{m}_{v2}$ k_2) $\vdots$ $\vdots$ $\vdots$ $\vdots$) m_n m_{vn} $\bar{m}_{vn}$ k_n)	(4 F7.1)	tons, tons, tons, tons/ft.	AM(I), AMW(I), AMWB(I), AK(I)	Lumped masses, added water masses of the 1st and 2nd kinds and the lumped buoyancies
R_1) R_2) $\vdots$) R_n)	(F8.0)	$tons-ft.^2$	AR(I)	Lumped rotary inertias. Should only be included when $n > 0$
N_1	(I3)	-	N	$\|N_1\|$ = number of modes to be determined $N_1 > 0$; mode frequencies and shapes to be printed $N_1 < 0$; mode frequencies and shapes not to be printed
N_2	(I3)	-	N2MOD	Number of modes to be used in remainder of calculation. If N_2 is set equal to zero, control is returned to start of calculation. Data tape format should then restart at n. (Note (b))
m	(I3)	-	MMX	Number of ship positions at which positional coefficients or displacement histories etc. are required $m > 0$; positional mode coefficients not printed (Note (c)) $m < 0$; positional mode coefficients printed
L_1 L_2 L_3	(3I3)	-	L_1, L_2, L_3	Parameters to determine print form for displacement etc. histories $L_1 = 1$ displacement and velocity printed $L_1 = 0$ " " " not printed $L_2 = 1$ moment and shear force printed $L_2 = 0$ " " " " not printed $L_3 = 1$ deck stress printed $L_3 = 0$ " " not printed (see Note (d)

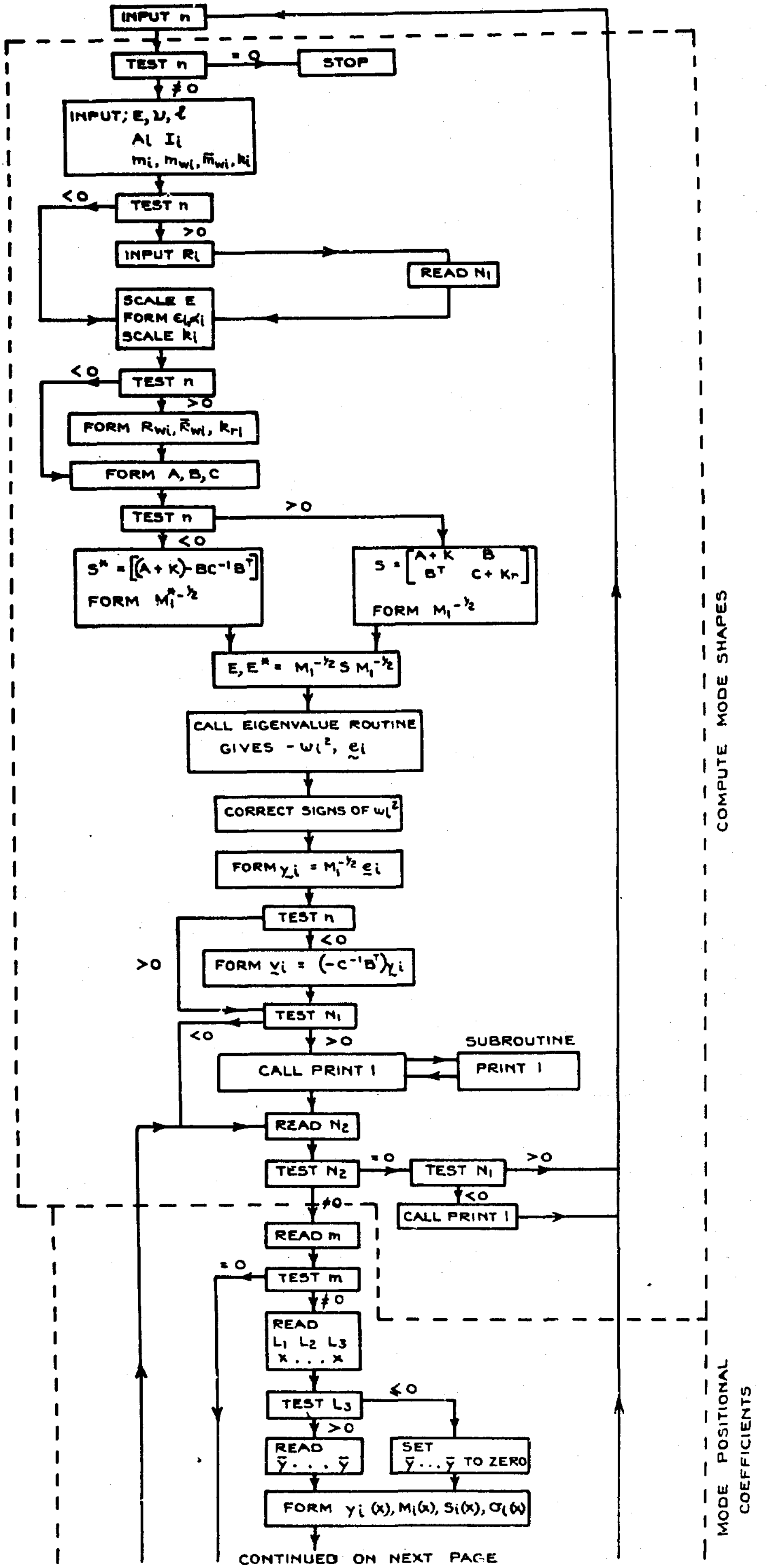
TABLE 2 (contd.)

Quantity	Format	Units	Program Variables	Description
$x_1\ x_2 \dots x_m$	(6F5.0)	ft.	XP(I)	Distances of positions at which displacement etc. required from the bows
$\bar{y}_1\ \bar{y}_2 \dots \bar{y}_m$	(6F6.2	ft.	YBP(I)	Distances of deck etc., at above positions, from the neutral axis. These should be omitted from data tape if $L_3 = 0$.
n_1	(I3)	-	NI	$\lvert n_1 \rvert$ = number of images to be used forming g and h (see Note (e)) $n_1 < 0$ forcing function coefficients to be printed $n_1 > 0$ forcing function coefficients not to be printed $n_1 = 0$ control returns to data tape at n (see Note (a))
d a w x_w H D	(6F5.0)	ft., ft., lbs., ft.,ft.,ft.	SD, SH, WWW, XW, H, D	Attack geometry (see Figures 2a and 2b) If W is set equal to zero, control is returned to the data tape at N_2. If W is put equal to - (charge weight) then after evaluating the mode forcing function coefficients control is returned to this part of the data tape so that another attack geometry may be read in (see Note (f)) If D set to - (charge depth) program will expect the explosion bubble to migrate and the subroutine FORD must give D as a function of time. At present this sub-routines is a dummy (see Note (g))
N_3	(I3)	-	N3	Parameter giving type of initial velocity to be used:- $N_3 = 0$, standard velocity distribution (see paragraph 9), $\dot{y}_c$ and $\dot{\psi}_c$ not to be printed $N_3 = 1$, standard velocity distribution, $\dot{y}_c$ and $\dot{\psi}_c$ to be printed $N_3 = 2$, non standard velocity distribution to be read from data tape
$\dot{y}_1\ \dot{\psi}_1$ $\dot{y}_2\ \dot{\psi}_2$ $\vdots\ \ \vdots$ $\dot{y}_n\ \dot{\psi}_n$	(F6.1, F7.4)	ft./sec., secs.$^{-1}$	YCD(I)	Initial Velocity Distribution. Only to be included when $N_3 = 2$. When $N_3 = 2$ and $n < 0$ (rotary inertia excluded) $\dot{\psi}_1 \dots \dot{\psi}_n$ should be omitted (Note (n))
N_4	(F4.0)	-	DEL	$N_4 < 0$; return control immediately to data tape at n_1 $N_4 = 0$; print α_c and β_c (initial mode coefficients then return control to n_1. $N_4 > 0$; N_4 = number of Runge-Kutta integration steps per cycle of highest frequency mode. (see paragraph 10)

TABLE 2 (contd.)

Quantity	Format	Units	Program Variables	Description
δt_p t_F	(2F6.3)	secs., secs.	TIMPR TF	Time increment between successive history printouts and time at which integration should be terminated.
				On termination of integration control returns to n_1.
0	(I3)			Dummy value fcr n_1; returns control to start of tape at n.
0	(13)			Dummy value for n; terminates calculation
***Z	-	-	-	Atlas control symbols

FLOW DIAGRAM.


INPUT n
TEST n
= 0
STOP
≠ 0
INPUT; E, ν, ℓ
A_i I_i
m_i, m_{wi}, $\bar{m}_{wi}$, k_i
< 0
TEST n
> 0
INPUT R_i
READ N_1
SCALE E
FORM ϵ_i, κ_i
SCALE k_i
< 0
TEST n
> 0
FORM R_{wi}, $\bar{R}_{wi}$, k_{ri}
FORM A, B, C
TEST n
> 0
< 0
$S^* = [(A+K) - BC^{-1}B^T]$
FORM $M_i^{*-1/2}$
$S = \begin{bmatrix} A+K & B \\ B^T & C+K_r \end{bmatrix}$
FORM $M_i^{-1/2}$
$E, E^* = M_i^{-1/2} S M_i^{-1/2}$
CALL EIGENVALUE ROUTINE
GIVES $-\omega_i^2$, $\underset{\sim}{e}_i$
CORRECT SIGNS OF ω_i^2
FORM $\underset{\sim}{\chi}_i = M_i^{-1/2} \underset{\sim}{e}_i$
TEST n
< 0
> 0
FORM $\underset{\sim}{y}_i = (-C^{-1}B^T)\underset{\sim}{\chi}_i$
TEST N_1
< 0
> 0
CALL PRINT 1
SUBROUTINE
PRINT 1
RZAD N_2
TEST N_2
= 0
TEST N_1
> 0
≠ 0
< 0
CALL PRINT 1
READ m
= 0
TEST m
≠ 0
READ
L_1 L_2 L_3
x . . . x
TEST L_3
< 0
> 0
READ
$\bar{y}$ . . . $\bar{y}$
SET
$\bar{y}$ . . . $\bar{y}$ TO ZERO
FORM $y_i(x)$, $M_i(x)$, $S_i(x)$, $\sigma_i(x)$
COMPUTE MODE SHAPES
MODE POSITIONAL COEFFICIENTS

CONTINUED ON NEXT PAGE

FIGURE 1.

FLOW DIAGRAM.

CONTINUED FROM PREVIOUS PAGE

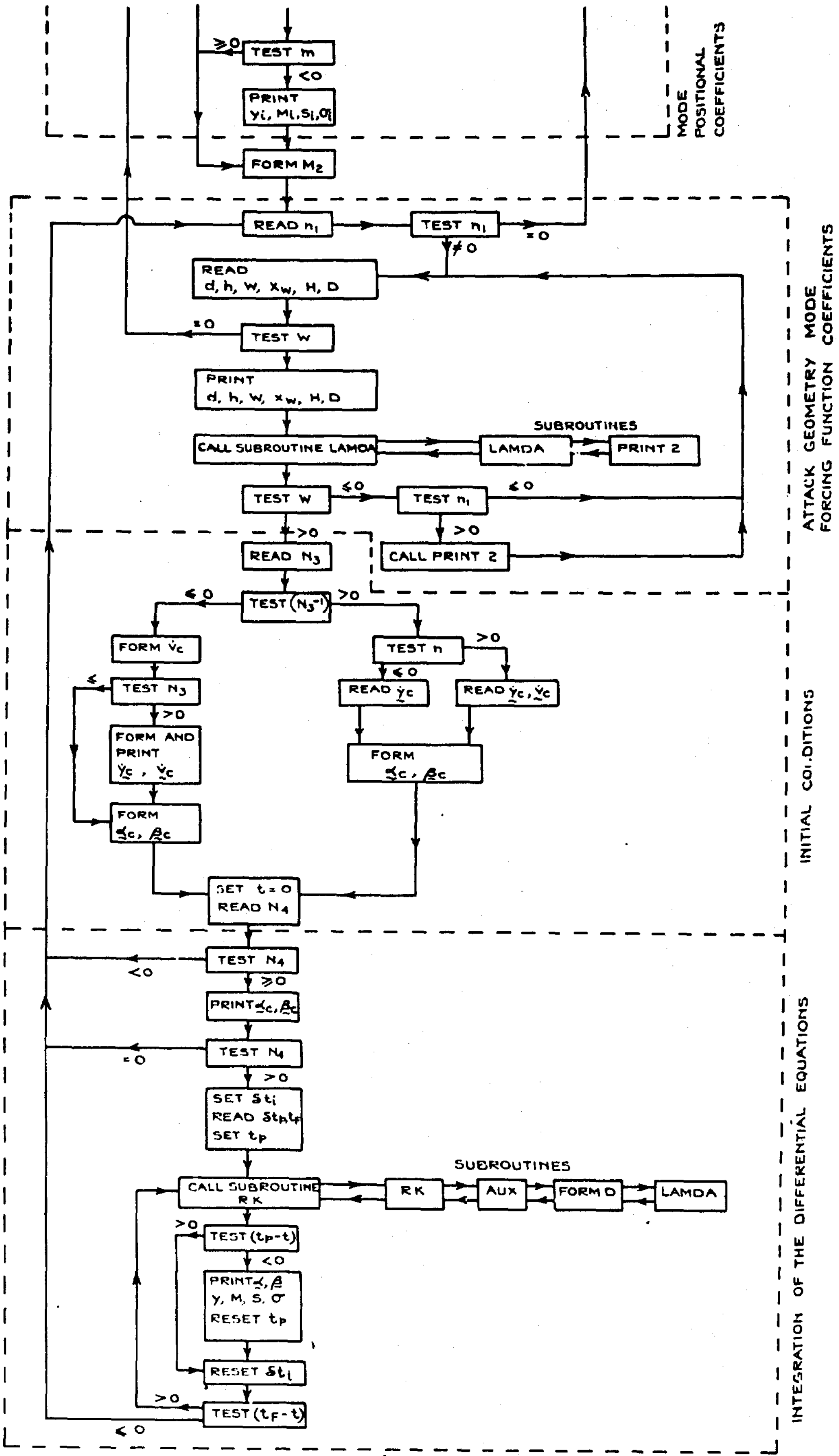

TEST m
≥0
<0
PRINT y_i, M_i, S_i, σ_i
FORM M_2
MODE POSITIONAL COEFFICIENTS
READ n_1
TEST n_1
=0
≠0
READ d, h, W, x_w, H, D
=0
TEST W
PRINT d, h, W, x_w, H, D
SUBROUTINES
CALL SUBROUTINE LAMDA
LAMDA
PRINT 2
TEST W
≤0
TEST n_1
≤0
>0
>0
CALL PRINT 2
ATTACK GEOMETRY MODE FORCING FUNCTION COEFFICIENTS
READ N_3
TEST (N_3-1)
≤0
>0
FORM $\dot{v}_c$
TEST N_3
≤
>0
FORM AND PRINT y_c, v_c
FORM α_c, β_c
TEST n
>0
≤0
READ y_c
READ y_c, v_c
FORM α_c, β_c
SET t = 0 READ N_4
INITIAL CONDITIONS
TEST N_4
<0
≥0
PRINT α_c, β_c
TEST N_4
=0
>0
SET δt_i READ δt_p, t_F SET t_p
SUBROUTINES
CALL SUBROUTINE RK
RK
AUX
FORM D
LAMDA
>0
TEST $(t_p - t)$
<0
PRINT α, β y, M, S, σ RESET t_p
RESET δt_i
>0
TEST $(t_F - t)$
<0
INTEGRATION OF THE DIFFERENTIAL EQUATIONS

FIGURE 1 (CONTD.)

ATTACK GEOMETRY.

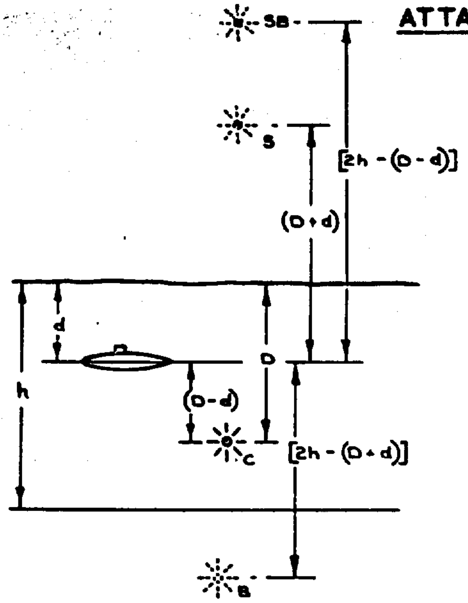


FIGURE 2 a.

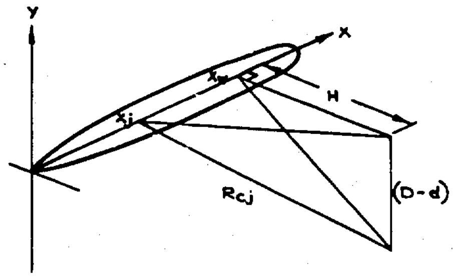


FIGURE 2 b.

DIMENSIONLESS VOLUME ACCELERATION.

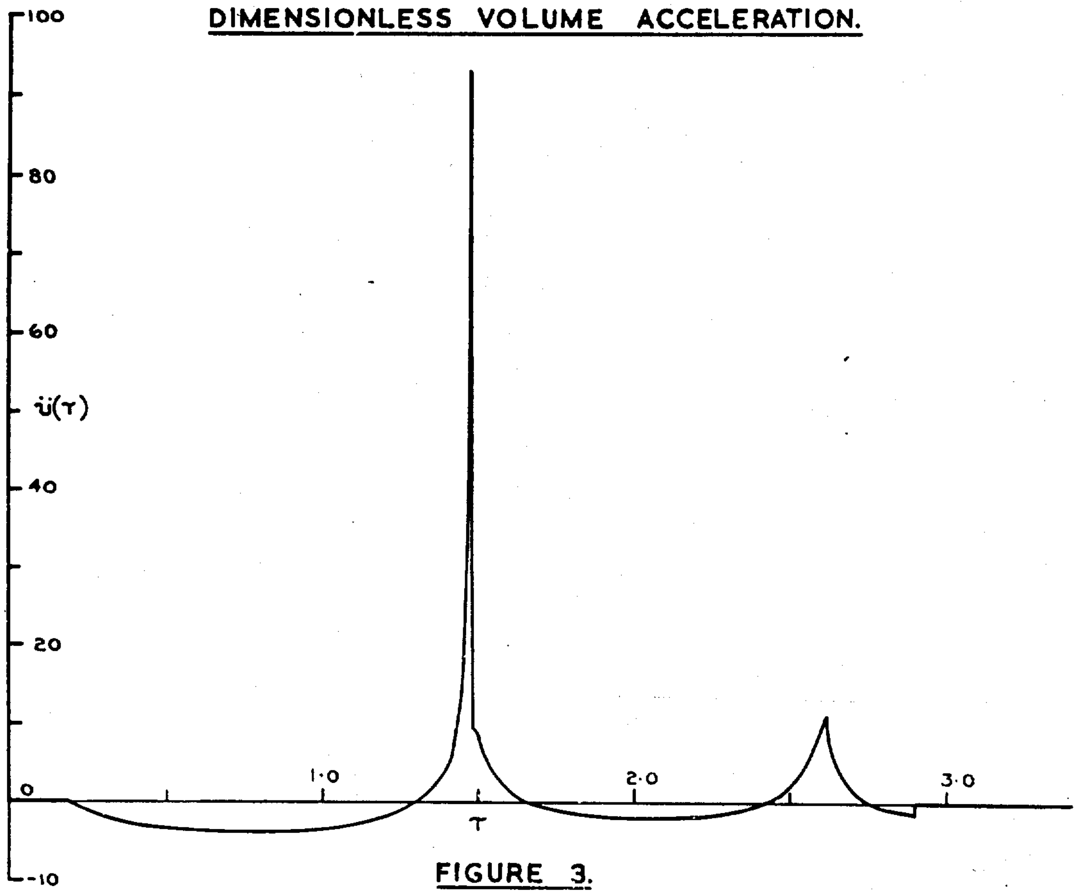


FIGURE 3.

SIGN CONVENTIONS.

$v = \frac{\partial y}{\partial x}$) BENDING

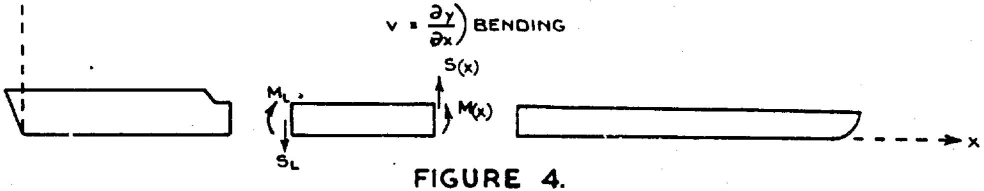


FIGURE 4.