FTD-HT-23-653-68

AD 683953

# FOREIGN TECHNOLOGY DIVISION

CALCULATION OF HEAT TRANSFER ON FRONTAL SURFACE OF BLUFF BODY IN HYPERSONIC FLOW

by

Yu. N. Yermak and V. Ya. Neyland

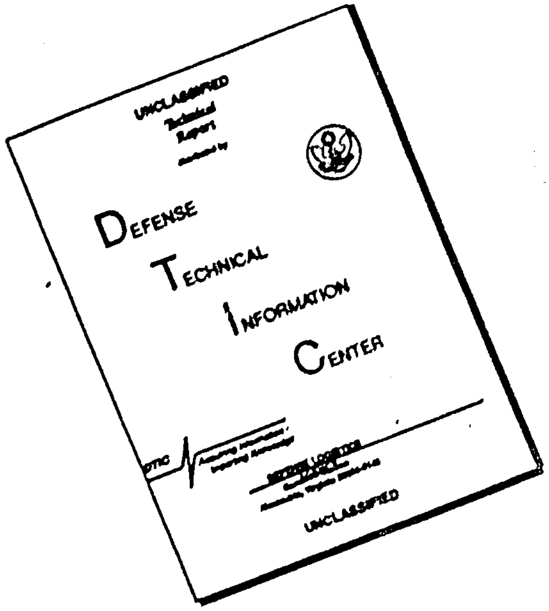






8

# DISCLAIMER NOTICE




THIS DOCUMENT IS BEST QUALITY AVAILABLE. THE COPY FURNISHED TO DTIC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.

FTD-HT-23-653-68

# EDITED TRANSLATION

CALCULATION OF HEAT TRANSFER ON FRONTAL SURFACE OF BLU BODY IN HYPERSONIC FLOW

By: Yu. N. Yermak and V. Ya. Neyland

English pages: 5

Source: AN SSSR. Izvestiya. Mekhanika Zhidkosti i Gaza (Academy of Sciences of the USSR. News. Fluid and Gas Mechanics), No. 6, 1967, pp. 153-156.

Translated under: F33657-68-D-0865 P0002

UR/0421-67-000-006



PREPARED BY:

TRANSLATION DIVISION
FOREIGN TECHNOLOGY DIVISION
WP-AFB, OHIO.

 Date 14 Nov 1968

## DATA HANDLING PAGE

| 01-ACCESSION NO. | 98-DOCUMENT LOC | 39-TOPIC TAGS |
|---|---|---|
| TP8001979 | | hypersonic aerodynamics, aerodynamic heat transfer, Navier Stokes equation, viscous flow, asymptotic solution, similarity theory |

**09-TITLE**
CALCULATION OF HEAT TRANSFER ON FRONTAL SURFACE OF BLUFF BODY IN HYPERSONIC FLOW

**47-SUBJECT AREA**
01

| 42-AUTHOR/CO-AUTHORS | 10-DATE OF INFO |
|---|---|
| YERMAK, YU. N.; 16-NEYLAND, V. YA. | -----67 |

| 43-SOURCE | 68-DOCUMENT NO. | 69-PROJECT NO. |
|---|---|---|
| AN SSSR. IZVESTIYA: MEKHANIKA ZHIDKOSTI I GAZA. (RUSSIAN) | FTD-HT 23-653-68 | 72301-74 |

| 63-SECURITY AND DOWNGRADING INFORMATION | 64-CONTROL MARKINGS | 97-HEADER CLASN |
|---|---|---|
| UNCL, 0 | NONE | UNCL |

| 76-REEL/FRAME NO. | 77-SUPERSEDES | 78-CHANGES | 40-GEOGRAPHICAL AREA | NO. OF PAGES |
|---|---|---|---|---|
| 1887 0695 | | | UR | 5 |

| CONTRACT NO. | X REF ACC. NO. | PUBLISHING DATE | TYPE PRODUCT | REVISION FREQ |
|---|---|---|---|---|
| F33657-68 D-0865/2 | 65-AP8004016 | 94-00 | Translation | NONE |

| STEP NO. | ACCESSION NO. |
|---|---|
| 02-UR/0421/67/000/006/0153/0156 | |


ABSTRACT

Viscous hypersonic flow at high Reynolds numbers R ($R = \rho_0 u_0 \mu_0 / r$) over a blunt-nosed body is considered on the basis of Navier-Stokes equations. An asymptotic solution of the Navier-Stokes equations is obtained for the case where substantial viscous effects are present in the entire inner layer of $\varepsilon^{3/2}r$ thickness ($\varepsilon$ is the shock density ratio). This means that the thickness of the boundary and the inner inviscid layers are of the sameoorder of magnitude. By substituting the nondimensional coordinates and functions of the viscous flow region in the Navier-Stokes equations and taking the similarity parameter $\Delta = 1/R_1\varepsilon^{5/2}$ constant into account, a system of equations is derived. A distinctive feature of the boundary value problem considered here is that it contains a new similarity parameter $\Delta$. It is demonstrated that the asymptotic solution where $\Delta$ tends to zero coincides with that of Fay and Riddel. The numerical results of computations on an M-20 computer for various at three different temperature parameters $g_w$($g_w$ = 1 corresponds to adiabatic wall), presented in tabular form, show that at $\Delta$ = 1 the heat flux is 17-23% higher than in the case of an ordinary boundary layer. The value $\Delta \sim 1$ corresponds, for example, to flight conditions at M-20, Re = $10^{+4}$, and $\varepsilon$ = 0.1. Orig. art. has: 13 formulas.


AFSC FORM AUG 66 4 (TEST FORM, FTD OVERPRINT) AFSC (AAFB)

## CALCULATION OF HEAT TRANSFER ON FRONTAL SURFACE OF BLUFF BODY IN HYPERSONIC FLOW

**Yu.N. Yermak, V.Ya., Neyland**

*(Moscow)*

The present paper considers hypersonic flow of a viscous gas past the nose of a bluff body at high Reynolds numbers $R$ ($R = \rho_0 u_0 \mu_0 / r$, where the subscript 0 is attached to the values of quantities in the oncoming flow). A rather large number of works, among which we may cite [1-3], has been devoted to this problem. The purpose of the investigations usually consists in determining heat flow at the critical point of the body. These studies have been made within the framework of boundary-layer theory in the first [1, 2] or second [3] approximation. When we remember that at hypersonic flow velocity, the ratio of the gas densities in front of and behind the compression shock $\varepsilon = \rho_0/\rho_1$ is small, the solution of the problem can be approached by the so-called Newtonian theory, within whose framework we seek an asymptotic solution to the Navier-Stokes equations (see, for example, [3]) as $M_0 \to \infty$, $\varepsilon \to 0$ and $R \to \infty$. In first approximation in this theory, an inviscid shock layer with a thickness of the order of $\varepsilon r$ is formed between the body and the shock wave. The flow in it is described around the critical point by the Euler equations, in which the term taking account of the longitudinal pressure gradient vanishes. To obtain these equations, it is necessary to introduce dimensionless coordinates with consideration of shock-layer thickness and the longitudinal dimension of the minimum influence region near the compression shock:

$$x = r\varepsilon^{1/2}\bar{x}, \qquad y = r\varepsilon\bar{y} \tag{1}$$

where $r$ is the radius of curvature of the body's contour and $x$, $y$ are orthogonal curvilinear coordinates bound to the surface of the body in the direction of the normal to it. The expansions for the flow function in the neighborhood of the critical point can be presented in the form

$$\begin{gathered} u = u_0\varepsilon^{1/2}[\bar{u}(\bar{y})\bar{x} + \ldots], \qquad v = u_0\varepsilon[\bar{v}(\bar{y}) + \ldots] \\ \rho = \rho_0\varepsilon^{-1}[\bar{\rho}(\bar{y}) + \ldots], \quad p = \rho_0 u_0^2[\bar{p}(\bar{y}) + \ldots], \quad h = {}^1/{}_2 u_0^2[\bar{h}(\bar{y}) + \ldots] \end{gathered} \tag{2}$$

Then we obtain in first approximation from the Navier-Stokes equations

$$2u + v' = 0, \qquad u^2 + vu' = 0 \tag{3}$$

Here we have dropped the prime on the dimensionless quantities. Considering the boundary conditions at the compression shock, we can obtain

$$u = \sqrt{-v}, \qquad y = y_2 + \sqrt{-v} - 1, \qquad p = \varrho = 1 \tag{4}$$

since on the wave $u = -v = 1$ for $y = y_2$. Near the body's surface $v \to 0$, $u \to 0$.

The inertial terms in the momentum equation in a layer with a thickness of the order of $r\varepsilon^{3/2}$ acquire the same order of magnitude as the dropped term with the longitudinal pressure gradient. Hence to obtain the uniform exact solution in this region as well, it is necessary to introduce new dimensionless coordinates and flow functions:

$$x = r\varepsilon^{1/2}\bar{x}, \qquad y = r\varepsilon^{3/2}Y, \qquad u = u_0\varepsilon[U(Y)\bar{x} + \ldots], \qquad v = u_0\varepsilon^2[V(Y) + \ldots] \tag{5}$$

Such a layer was examined in [4] in solving the problem of hypersonic flow of an inviscid gas past a flat plate set transverse to the oncoming flow. After substitution of (5) in the Navier-Stokes equations, we obtain, in first approximation,

$$2U + V' = 0, \qquad U^2 + VU' = 2 \tag{6}$$

Integrating (6), we obtain

$$U = (Y + \sqrt{2}C) / C, \qquad V = C(2 - U^2) \tag{7}$$

Use of the familiar principle of splicing the external and internal asymptotic expansions enables us to determine the constants: $C = 1$ and $y_2 = 1 + 0(\varepsilon^{1/2})$. Thus we obtain the velocity gradient at the body's surface as $u_0\sqrt{2\varepsilon}/r$. This value is also obtained in the conventional Newtonian theory. It is used as the boundary condition for the boundary layer, whose equations are obtained by asymptotic expansion of the Navier-Stokes equations in a region with a thickness of order $R^{-1/2}$ immediately adjacent to the body's surface.

Thus, in the conventional scheme for finding the asymptotic solution of the Navier-Stokes equations, the limit transition $\varepsilon \to 0$, $M_0 \to \infty$, $R \to \infty$ is accomplished in such a way that the entire shock layer will decompose into two inviscid regions and a boundary layer. This limit transition describes flow conditions well for Mach number $M_0$ and $\varepsilon$ moderate and $R$ large.

The present work examines another type of limit transition, $M_0 \to \infty$, $\varepsilon \to 0$, $R \to \infty$, in which viscous effects remain essential throughout an inner layer with thickness $\varepsilon^{3/2}r$. This means that the thickness of the boundary layer and the internal inviscid layer are of the same order of magnitude.

Let us introduce the parameter $R_1 = R\mu_1/\mu_0$. In the limit transition under examination, the parameter $\Lambda = 1/R_1\varepsilon^{3/2}$ remains finite as $M_0 \to \infty$, $\varepsilon \to 0$ and $R_1 \to \infty$. It characterizes the ratio of the thicknesses of the boundary and inner inviscid layers. As estimates show, the thickness of the compression shock remains negligibly small, i.e., passage across the compression shock is described, as before, by the Hugoniot solution. The asymptotic expansions and the solutions retain their previous form (1)-(3) in a layer whose thickness is of the order of $\varepsilon r$.

The dimensionless coordinates and functions in the viscous flow region have the form (5). Substituting them into the Navier-Stokes equations and considering the condition $R_1\varepsilon^{1/2} = \mathrm{const}$, we obtain, in first approximation, the following equations, which have been expanded in $x$:

$$2\rho U + (\rho V)' = 0, \quad \rho V g' = \Delta[(\mu/\sigma)g']', \quad \rho(U^2 + VU') = 2\Delta(\mu U')' \qquad (8)$$

The boundary conditions for System (8) are known only at the wall, $U = V = 0$, and the missing ones are obtained by splicing with Solution (4). Introducing the stream function and Dorodnitsyn's variables, we transform the system to

$$\Delta(Nf'')' + 2ff'' + 2/\rho - (f')^2 = 0, \quad \Delta[(N/\sigma)g']' + 2fg' = 0 \left(N = \frac{\rho\mu}{\rho_0\mu_0}, \quad \eta = \int_0^Y \rho\, dY\right) \qquad (9)$$

Here differentiation is with respect to the Dorodnitsyn variable. The boundary conditions at the wall have the usual form

$$f(0) = f'(0) = 0; \quad g(0) = g_w \qquad (10)$$

The missing boundary condition for the momentum equation, which is obtained by splicing the solutions of System (9) with (4), takes the form

$$(\partial^2 f/\partial\eta^2)_{\eta\to\infty} = (\partial u/\partial y)_{y\to 0} = 1, \quad \text{or} \quad f''(\infty) = 1 \qquad (11)$$

The external boundary condition for the energy equation has remained as before:

$$g(\infty) = 1 \qquad (12)$$

The boundary-value problem (9)-(12) differs from that considered earlier in [1, 2] and others in that the new similarity parameter $\Delta$ has appeared; its physical sense is indicated above. The external boundary condition for the momentum equation has also changed. It is evident from physical considerations that the asymptotic solution (9)-(12) must be identical to the solution obtained in [1, 2] and other papers as $\Delta \to 0$. It is not difficult to show that this is indeed the case, since the asymptotic solution (9)-(12) takes the following form as $\Delta \to 0$:

$$f'' \to \varphi''/\sqrt{\Delta}, \qquad g' \to h'/\sqrt{\Delta} \qquad (13)$$

where $\varphi$ and $h$ are solutions to the boundary-value problem

$$(N\varphi'')' + 2\varphi\varphi'' + 2/\rho - (\varphi')^2 = 0, \qquad [(N/\sigma)h']' + 2\varphi h' = 0$$
$$\varphi(0) = \varphi'(0) = 0, \quad h(0) = h_w = g_w, \quad \varphi'(\infty) = \sqrt{2}, \quad h(\infty) = 1$$

Equations (9) were solved on the M-20 electronic computer [EC] (ЭВМ) with boundary conditions (10), (11) and (12). We present the results of calculations for various Δ and three values of the temperature factor $g_w$ (the value $g_w$ = 1 corresponds to an adiabatic wall).

$g_w = 0.1$, $\varphi_w'' = 0.88$, $h_w' = 0.41$

| | | | | | | |
|---|---|---|---|---|---|---|
| $\Delta =$ | $10^{-2}$ 0.37 | $10^{-1}$ 0.11 | $10^{-1}$ 0.22 | 0.11 | 0.43 | 0.51 |
| $f_w'' =$ | 14.82 | 8.76 | 6.31 | 3.11 | 1.80 | 1.70 |
| $\sim\varphi_w''/\sqrt{\Delta} =$ | 14.39 | 8.39 | 5.40 | 2.67 | 1.33 | 1.23 |
| $g_w' =$ | 6.72 | 3.92 | 2.79 | 1.31 | 0.69 | 0.64 |
| $\sim h_w'/\sqrt{\Delta} =$ | 6.69 | 3.90 | 2.765 | 1.24 | 0.62 | 0.57 |
| $\Delta =$ | 0.63 | 0.79 | 0.90 | 1.00 | 3.86 | 11.33 |
| $f_w'' =$ | 1.58 | 1.46 | 1.41 | 1.36 | 0.96 | 0.80 |
| $\varphi_w''/\sqrt{\Delta} =$ | 1.07 | 0.98 | 0.92 | 0.88 | 0.447 | 0.262 |
| $g_w' =$ | 0.58 | 0.53 | 0.50 | 0.48 | 0.27 | 0.18 |
| $h_w'/\sqrt{\Delta} =$ | 0.517 | 0.45 | 0.43 | 0.41 | 0.209 | 0.13 |

$g_w = 10^{-4}$, $\varphi_w'' = 0.24$, $h_w' = 0.13$

| | | | | | | |
|---|---|---|---|---|---|---|
| $\Delta =$ | $10^{-4}$ 0.16 | $10^{-3}$ 0.24 | $10^{-2}$ 0.17 | $10^{-2}$ 0.19 | $10^{-1}$ 0.51 | 0.10 |
| $f_w'' =$ | 63.56 | 5.25 | 2.09 | 1.96 | 1.26 | 0.92 |
| $\sim\varphi_w''/\sqrt{\Delta} =$ | 61.53 | 4.85 | 1.80 | 1.72 | 1.05 | 0.76 |
| $g_w' =$ | 34.51 | 2.80 | 1.09 | 1.02 | 0.63 | 0.45 |
| $\sim h_w'/\sqrt{\Delta} =$ | 33.33 | 2.61 | 1.01 | 0.93 | 0.57 | 0.41 |
| $\Delta =$ | 0.19 | 0.48 | 0.88 | 1.00 | 2.39 | 4.48 |
| $f_w'' =$ | 0.72 | 0.51 | 0.42 | 0.40 | 0.32 | 0.28 |
| $\sim\varphi_w''/\sqrt{\Delta} =$ | 0.55 | 0.347 | 0.256 | 0.24 | 0.15 | 0.11 |
| $g_w' =$ | 0.34 | 0.22 | 0.17 | 0.16 | 0.11 | $10^{-1}$ 0.89 |
| $\sim h_w'/\sqrt{\Delta} =$ | 0.298 | 0.187 | 0.138 | 0.13 | $0.84 \cdot 10^{-1}$ | $0.61 \cdot 10^{-1}$ |

$g_w = 1$, $h_w' = 0$, $\varphi_w'' = 2.206$

| | | | | | | |
|---|---|---|---|---|---|---|
| $\Delta =$ | $0.81 \cdot 10^{-3}$ | $0.44 \cdot 10^{-2}$ | $0.20 \cdot 10^{-1}$ | $0.46 \cdot 10^{-1}$ | 0.13 | 0.25 |
| $f_w'' =$ | 77.94 | 33.79 | 16.24 | 10.91 | 6.82 | 5.10 |
| $\sim\varphi_w''/\sqrt{\Delta} =$ | 77.40 | 33.2 | 15.65 | 10.25 | 6.13 | 4.40 |
| $\Delta =$ | 0.59 | 0.7 | 0.814 | 1.00 | 1.9[illegible]8 | 10 |
| $f_w'' =$ | 3.57 | 3.346 | 3.158 | 2.92 | 2.30 | 1.499 |
| $\sim\varphi_w''/\sqrt{\Delta} =$ | 2.87 | 2.63 | 2.44 | 2.206 | 1.56 | 0.69 |

It was assumed in all calculations that σ = 0.74 and that the variation of viscosity $\mu \sim g^{\omega}$, where ω = 0.76.

It can be seen that for Δ = 1, the heat flow is 17-23% larger than the corresponding value for the ordinary boundary layer. The quantity Δ ∿ 1, for example, corresponds to flight regimes with $M > 20$, ε = 0.1 and Reynolds numbers $R$ of the order of $10^{+4}$.

Received 23 December 1966

## REFERENCES

1. Sibulkin, M., Heat Transfer Near the Forward Stagnation Point of a Body of Revolution. J. Aeronaut. Sci., 1952, Vol. 19, No. 8.

2. Fay, I.A., Riddel, F.R., Theory of Stagnation Point Heat Transfer in Dissociated Air. J. Aeronaut Sci., 1958, Vol. 25, No. 2.

3. Van-Dayk, M., Teoriya szhimayemogo pogranichnogo sloya vo vtorom priblizhenii s primeneniyem k obtekaniyu zatuplennykh tel giperzvukovym potokom [Theory of the Compressible Boundary Layer in Second Approximation with Application to Hypersonic Flow Past Bluff Bodies]. Collection "Issledovaniye giperzvukovykh techeniy" [Research on Hypersonic Flows]. Izd. "Mir," 1964.

4. Koul, Zh., Braynerd, Zh., "Obtekaniye tonkikh kryl'yev giperzvukovymi potokami pri bol'shikh uglakh ataki" [Hypersonic Flows Past Thin Wings at Large Angles of Attack]. Collection "Issledovaniye giperzvukovykh techeniy." Izd. "Mir," 1964.