ESD-TR-68-276

WP-2214

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THE DOPPLER MAP OF AN EDGE

M. R. Weiss

NOVEMBER 1968

Prepared for

DEPUTY FOR SURVEILLANCE AND CONTROL SYSTEMS
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

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FOREWORD

This technical report has been prepared by The MITRE Corporation, Department D-85, under Contract AF 19(628)-5165, Project 4966. The contract is sponsored by the Electronic Systems Division, Air Force Systems Command, L. G. Hanscom Field, Bedford, Massachusetts.

REVIEW AND APPROVAL

This technical report has been reviewed and is approved.

HENRY J. MAZUR, Colonel, USAF
Director, Space Defense & Command Systems Pgm Ofc
Deputy for Surveillance & Control Systems

ABSTRACT

Previous work on the use of narrowband interferometry for doppler processing has modeled the target as a collection of scattering centers fixed with respect to a center of mass. The present paper is intended to suggest a model for a curved edge taking into consideration the "slippery" nature of the scattering center.

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SECTION I

INTRODUCTION

Previous work on the effects of polarization-dependent scatterers on doppler processed data has investigated the results for the case of a point dipole. The main result of this analysis was that, aside from the conventional doppler shift (associated with radial velocity), an additional shift of $\pm \frac{1}{\pi} \dot{\varphi}$ is introduced into the doppler maps of the diagonal polarization terms. Under certain restrictive conditions it is possible to use an analysis similar to that for the dipole in determining the doppler map result for an edge.

SECTION II

DISCUSSION

For an edge, using the results of Keller's geometrical theory of diffraction[1], we have for linear polarization

$$E_H = \frac{\rho^{1/2} e^{i\pi/e} \sin\pi/q \cos^{1/2}\theta}{qk^{1/2}} \left[(\cos\pi/q - 1)^{-1} - \left(\cos\pi/q - \cos\frac{2\theta + \pi}{q}\right)^{-1} \right]$$

$$E_V = \frac{\rho^{1/2} e^{i\pi/4} \sin\pi/q \cos^{1/2}\theta}{qk^{1/2}} \left[(\cos\pi/q - 1)^{-1} + \left(\cos\pi/q - \cos\frac{2\theta + \pi}{q}\right)^{-1} \right] \tag{1}$$

where E_H represents the received horizontal electric field for a horizontally polarized incident wave (E vector along the edge), and E_V represents the vertical-vertical result (E vector normal to the edge). The necessary angles are defined in Figure 1, which shows a cross-sectional view of the edge, where in Equation (1)

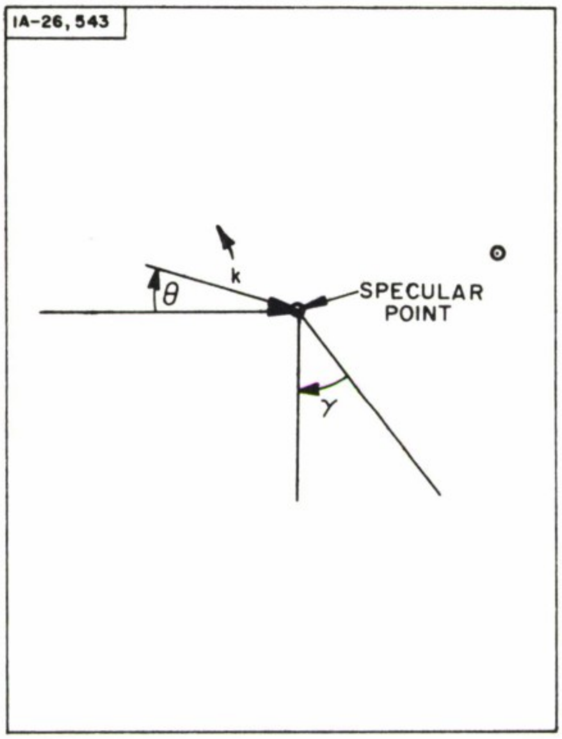

Figure 1. Edge Cross-sectional View

ρ = radius of curvature of the specular point

$$q = \frac{2\pi - \gamma}{\pi} \quad , \qquad \gamma = \text{wedge angle}$$

If, as in previous analyses,[2,3] we consider the scatterer to be rotating with constant angular velocity $\vec{\omega}$ about a center of gravity O while O is being translated with velocity $\vec{v}(t)$, scattering from an edge C on the body may be depicted as in Figure 2.

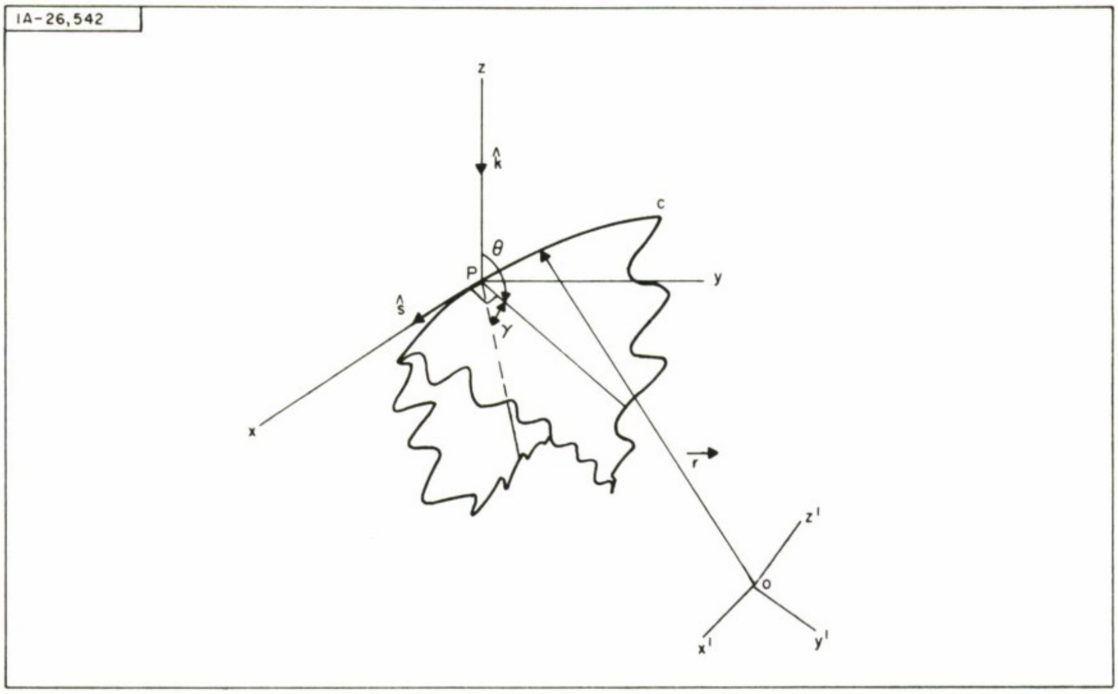

Figure 2. Portion of Edge on Scattering Body

$\vec{k}(t) = \frac{2\pi}{\lambda}\,\hat{k}(t)$ is a vector directed along the line of sight

$\vec{r}(t)$ is the radius vector from O to the edge

$\vec{r}_p(t)$ hence is the radius vector from O to the specular point of scattering

$\hat{s}(t)$ is the instantaneous tangent vector at the specular point

θ and γ are defined consistent with Figure 1 (both being measured in the $y - z$ plane)

If we further define $\hat{e}_1$ and $\hat{e}_2$ to represent directions for a horizontally and vertically polarized incident field then we may introduce a polarization angle φ which is given simply by $\varphi = \sin^{-1} \hat{s} \cdot \hat{e}_1$.

Based on Equation (1) and the quantities defined in Figure 2, the monostatic circular polarization scattering elements are given by*

$$C_{11} = \frac{1}{2} (E_H - E_V)\, e^{2i\varphi(t)}\, e^{-2i\vec{k}(t) \cdot \vec{r}_p(t)}$$

$$C_{22} = \frac{1}{2} (E_H - E_V)\, e^{-2i\varphi(t)}\, e^{-2i\vec{k}(t) \cdot \vec{r}_p(t)} \tag{2}$$

$$C_{12} = C_{21} = \frac{1}{2} (E_H + E_V)\, e^{-2i\vec{k}(t) \cdot \vec{r}_p(t)}$$

As in the case of the dipole analysis, we are interested in performing Taylor series expansions for the various time-dependent functions in the exponentials of Equations (2). It is at this juncture that we must consider the differences between a "point scatterer" model and the case of scattering from the specular point of an edge. Whereas in the case of point scatterers we were able to consider motion in terms of points rigidly connected to O, for the edge we must, to some extent, account for the "slippery" nature of the specular point.

Because of slipping of the specular point with motion along the edge it is no longer possible to characterize all three components of velocity in terms

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*Note that except for a common phase term of $\pi/4$ (which has been suppressed) there is no phase dependence with polarization due to E_H and E_V. Also, the Faraday rotation effect which would appear in C_{12} and C_{21} has been neglected.

of the angular velocity vector about the center of mass. In directions orthogonal to the edge, however, the specular point does behave in the conventional manner (i.e., as a rigid point).

For a first-order approximation we assume that the edge has constant radius of curvature (i.e., circular edge) in the neighborhood of the specular point.* If one can assume a circular edge at least during the doppler processing time interval (which it must be if the time interval is small enough), the radius vector to the specular point may then be expanded as

$$\vec{r}_p(t) \simeq \vec{r}_p(t_o) + \frac{d}{dt}\vec{r}_p(t)\Big|_{t=t_o}(t - t_o) \tag{3}$$

where

$$\frac{d}{dt}\vec{r}_p(t) = \hat{k}(\vec{r}_p \times \vec{\omega}) \cdot \hat{k} + \hat{n}(\vec{r}_p \times \vec{\omega}) \cdot \hat{n}$$

with $\hat{n} = \hat{k} \times \hat{s}$, and $\vec{\omega}$ the angular velocity vector about the center of mass. Note that in Equation (3) the specular point has no apparent velocity in the tangential direction given by $\hat{s}$. In addition, $\hat{k}(t)$ can be expanded as

$$\hat{k}(t) \simeq \hat{k}(t_o) + \frac{d}{dt}\hat{k}(t)\Big|_{t=t_o}(t - t_o) \tag{4}$$

where

$$\frac{d}{dt}\hat{k} = \vec{\omega}_A \times \hat{k} \ .$$

*The existence of a specular point implies curved edges only (i.e., straight edges are excluded from this analysis).

The doppler maps are produced by Fourier time transforms of the radar returns given by Equation (2). Of interest are the location of peaks in this doppler map which are given by the coefficient of the $(t - t_o)$ term in the exponential of (2) and the phase at the peak, given by the constant term in the exponential.

Consequently, using the results given in Equations (3) and (4), expanding $\varphi(t)$ in Equation (2), and defining $\vec{\Omega} \equiv \vec{\omega} - \vec{\omega}_A$:

$$\Phi_{11} = -2\left[\vec{k}(t_o) \cdot \vec{r}_p(t_o) - \varphi(t_o)\right]$$

$$\Phi_{22} = -2\left[\vec{k}(t_o) \cdot \vec{r}_p(t_o) + \varphi(t_o)\right] \tag{5a}$$

$$\Phi_{12} = \Phi_{21} = -2\vec{k}(t_o) \cdot \vec{r}_p(t_o)$$

$$\Delta f_{11} = -\frac{1}{\pi}\left[\vec{k}(t_o) \cdot \vec{r}_p(t_o) \times \vec{\Omega} - \dot{\phi}\right]$$

$$\Delta f_{22} = -\frac{1}{\pi}\left[\vec{k}(t_o) \cdot \vec{r}_p(t_o) \times \vec{\Omega} + \dot{\phi}\right] \tag{5b}$$

$$\Delta f_{12} = \Delta f_{21} = -\frac{1}{\pi}\vec{k}(t_o) \cdot \vec{r}_p(t_o) \times \vec{\Omega}$$

Two observations are noteworthy. First, the only component of specular point velocity appearing in the doppler shifts given in (5b) is along the viewing direction, $\hat{k}$. The slipping of the edge specular point plays no immediate role in the resultant doppler shift. Secondly, the results given in Equations (5) are identical to results arrived at with a point dipole model, the only differences occurring in the amplitude of the Fourier transform itself. As a result, the rate of change of polarization angle, ϕ, and the polarization angle $\phi(t_o)$ can be determined directly from Equations (5a). In fact, if one neglects the difference in specular points as viewed from the bistatic sites, an analysis identical to that given in References [2] and [3] applies.

The extension of these results to the bistatic receivers is accomplished by the technique described in Reference [2] or, alternately, by using the bistatic monostatic equivalence theorem. For an edge, however, each receiver is presented with return from a different specular point. This difficulty can be accounted for with the following analysis.

The assumption that the edge C is circular in the neighborhood of the specular point of course implies that C is a plane curve in this region. Then, in accordance with Figure 3, we can define u as the angle between

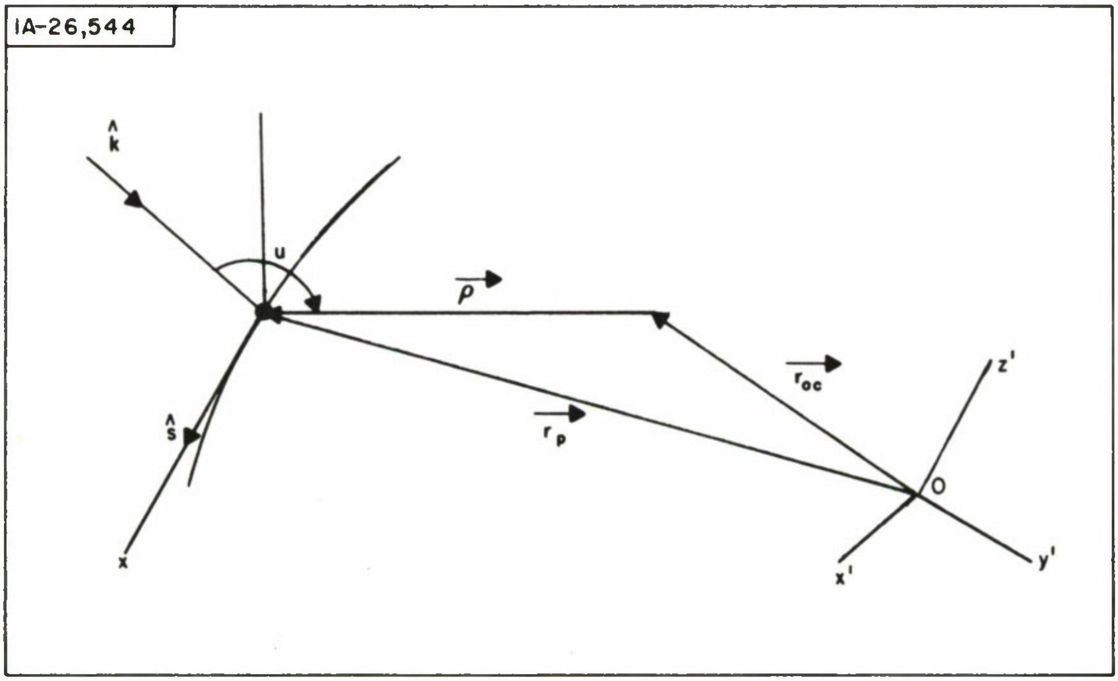

Figure 3. Definition of Edge Plane

$\hat{k}$ and $\vec{\rho}$, where*

$\hat{k}$ = vector directed along line of sight

$\hat{\rho}$ = vector lying in the plane of the edge, directed from center of curvature (c. c.) to the specular point

$|\vec{\rho}| = \rho$ = radius of curvature of edge

$\vec{r}_{oc}$ = vector from center of rotation, O, to center of curvature

From Figure 3,

$$\vec{r}_p = \vec{r}_{oc} + \vec{\rho} = \vec{r}_{oc} + \hat{k}\rho \cos u + \hat{n}\rho \sin u \tag{6}$$

Substituting this expression in the doppler shift given by Equation (5) (with the φ term removed),

$$\Delta f = -\frac{1}{\pi}\vec{k} \cdot \vec{r}_{oc} \times \vec{\Omega} \frac{1}{\pi}\vec{k} \cdot \left[(\hat{k}\rho \cos u + \hat{n}\rho \sin u) \times \vec{\Omega}\right] \tag{7}$$

and, since $(\hat{k} \times \hat{n}) = -\hat{s}$, we get

$$\Delta f = -\frac{2}{\lambda}\left[\hat{k} \cdot \vec{r}_{oc} \times \vec{\Omega} - \hat{s} \cdot \vec{\Omega}\rho \sin u\right] \tag{8}$$

Equation (8) is in a convenient form because the radius vector $\vec{r}_{oc}$ does not change for the bistatic sites (as $\vec{r}_p$ itself does). Instead, the change in definition of specular points for the bistatic sites is manifested by a change in the vectors $\hat{k}$, $\hat{s}$ and (by virtue of $\vec{\omega}_A$) $\vec{\Omega}$ and the angle u. Because of

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*Note that $\theta \leq u \leq \theta + \gamma$ if the surfaces forming the edge are convex.

the assumption of a circular edge, however, the quantities $\hat{k} \cdot \hat{s}$ and u change in a known way.

We may now, by inspection, write the doppler shift and phase term for a bistatic receiving site as, *

$$\Phi_1 = \frac{-4\pi}{\lambda}\left[\hat{k}_1 \cdot \vec{r}_{oc} + \rho \cos u_1\right] \cos \theta_1$$

$$\Delta f_1 = \frac{-2}{\lambda}\left[\hat{k}_1 \cdot \vec{r}_{oc} \times \vec{\Omega}_1 - \hat{s}_1 \cdot \vec{\Omega}_1 \rho \sin u_1\right] \cos \theta_1 \tag{9}$$

where for a given bistatic receiver,

$\hat{k}_1$ = a vector directed along the bisector of the transmitter-receiver

$\vec{\Omega}_1 = \vec{\omega} - \vec{\omega}_1$; $\frac{d\hat{k}_1}{dt} = \vec{\omega}_1 \times \hat{k}_1$

$\hat{s}_1$ = a vector normal to $\hat{k}_1$ and coplaner with $\hat{s}$ (i.e., tangent to the edge at the specular point corresponding to $\hat{k}_1$)

u_1 = angle between $\hat{k}_1$ and $\vec{\rho}$ (reference Figure 3)

θ_1 = 1/2 the bistatic angle between transmitter-receiver

* The $\varphi(t_o)$ and $\dot{\varphi}(t_o)$ terms which can be determined from the scattering matrix data are assumed absorbed in the definitions of Φ_1 and Δf_1. Note also that unsubscripted quantities ($\hat{k}$, u, $\vec{\Omega}$, etc.) refer to the monostatic site while the 1 subscript refers to the remote site.

SECTION III

SUMMARY

As a result, for the case of two remote receivers and one monostatic transmitter-receiver, we have at our disposal three sets of equations of the form given in (9) and one additional equation for $\dot{\varphi}$ from the scattering matrix data. These seven equations are, however, functions of eight unknown quantities given by:

(a) Three components of the body angular velocity, $\vec{\omega}$.

(b) Three components of the vector from the center of mass to the center of curvature of the edge, $\vec{r}_{oc}$.

(c) The radius of curvature of the edge, ρ.

(d) The plane of the edge, expressed through the angle u (recall that u_1 for a remote receiver can be expressed as a function of u).

Since those equations represent an incomplete solution for the unknowns, additional information is required. It seems most reasonable to depend on point-scattering centers on the body for the determination of angular velocity $\vec{\omega}$. If one then utilized the known $\vec{\omega}$ in Equations (9), there would remain six equations for five unknowns. While no closed-form solution is obvious even in this case, an iteration procedure does appear plausible.

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UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D		
(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)		
1. ORIGINATING ACTIVITY *(Corporate author)* The MITRE Corporation Bedford, Massachusetts	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP N/A	
3. REPORT TITLE THE DOPPLER MAP OF AN EDGE		
4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)* N/A		
5. AUTHOR(S) *(First name, middle initial, last name)* M. R. Weiss		
6. REPORT DATE November 1968	7a. TOTAL NO. OF PAGES 17	7b. NO. OF REFS 3
8a. CONTRACT OR GRANT NO. AF 19(628)-5165 b. PROJECT NO. Project 4966 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) ESD-TR-68-276 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)* WP-2214	
10. DISTRIBUTION STATEMENT This document has been approved for public release and sale; its distribution is unlimited.		
11. SUPPLEMENTARY NOTES N/A	12. SPONSORING MILITARY ACTIVITY Deputy for Surveillance and Control Systems, Electronic Systems Division, AF Systems Command, L. G. Hanscom Field, Bedford, Mass.	

13. ABSTRACT

Previous work on the use of narrowband interferometry for doppler processing has modeled the target as a collection of scattering centers fixed with respect to a center of mass. The present paper is intended to suggest a model for a curved edge taking into consideration the "slippery" nature of the scattering center.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
ELECTROMAGNETIC SCATTERING						
POLARIZATION SCATTERING MATRIX						
DOPPLER MAPPING						