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ABSOLUTE CALIBRATION OF ELECTROACOUSTIC TRANSDUCERS BY THE RECIPROCITY METHOD IN A QUASI-SPHERICAL FIELD

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I. FUNDAMENTAL RELATIONS

Introduction

The calibration of electroacoustic transducers with respect to a field involves the pressure in a free plane wave. In actual practice, however, the calibration is always produced in a quasi-spherical field of real transducers. The results give, within a certain degree of accuracy, the desired value of the sensitivity of the transducer for both reception and transmission. Of course, a sufficiently detailed examination of the conditions of the calibration should be made before the practical measurements are carried out.

The theory of the reciprocity method of absolute calibration of electroacoustic transducers in a spherical wave has been examined in a series of articles /1-4/. Nonetheless, the theory has not been complete until now. The fundamental deficiency of the theory as it appears in the literature is the failure to take into account all of the conditions of measurement, particularly the mutual effect of the finite dimensions of the participating transducers on each other, and the difference between the actual conditions of calibration in a spherical wave and the ideal conditions for a free plane-wave field. Thus, some of the questions concerning minimum calibration distance and the quantitative determination of sensitivity in a spherical-wave field have not been investigated.

In the present article, the complete and accurate theory of the reciprocity method of calibrating electroacoustic transducers in a spherical-wave field is presented, and also analytical and experimental investigations of the fundamental conditions for practical application of the method. The development of the theory is based on the electroacoustical reciprocity theorem, a direct proof of which is given first. As a result of the analysis, "calibration distance" for real transducers is introduced, the range of minimum distance for calibration with a specified degree of accuracy is fixed, and analytical means of improving the accuracy are presented.

Electroacoustical Reciprocity Theorem

The derivation of the equations for reciprocity calibration is based on the application of the electroacoustical reciprocity theorem, which

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involves relations between voltage and current at the terminals of one linear reversible transducer in an infinite or a bounded sound-conducting medium and the distribution of pressure or particle velocity on the surface of another (usually auxiliary) transducer in the same medium.

The form of the reciprocity theorem that is often used in practical applications can be directly obtained from the well-known theorems for acoustic fields /3/ and electroacoustical transducers /4/.

We will consider the general form of the acoustical system of a transducer as a single vibrating surface. We will designate by I', V', $\dot{\xi}_n'$, p' and I'', V'', $\dot{\xi}_n''$, p'' arbitrary values of current and voltage on the electrical terminals and, corresponding to them, the distribution on the surface S of normal particle velocity and pressure for two conditions as indicated by a single or a double prime.

The theorem of reciprocity for such transducers is

$$\int_S (p'\dot{\xi}_n'' - p''\dot{\xi}_n')ds - (V'I'' - V''I') = 0. \qquad (1)$$

We note that the absolute value of the quantity $(V'I'' - V''I')$ in this expression is independent of the type of electromechanical coupling.

The differential form of the reciprocity theorem for the elastic medium that surrounds the transducer, in the absence of mass forces, is given by the expression

$$\operatorname{div}\,(p'\dot{\xi}'' - p''\dot{\xi}') = 0, \qquad (2)$$

where $\dot{\xi}'$, p' and $\dot{\xi}''$, p'' are the distribution of particle velocities and pressure in the sound-conducting medium when it is assumed that only longitudinal waves are present.

We will assume the medium to be infinite, and we will put into it a system of two transducers S_1 and S_2.

We integrate relation (2) by applying Green's theorem for volumes to the exterior of the transducers involved. For this, we use expression (1) and the condition of continuity on the surface of transducer S_1, which, by the accepted convention of signs, gives

$$P = p \quad \text{and} \quad \dot{\xi}_n = -\dot{\xi}.$$

The result gives the desired electroacoustic reciprocity theorem for the acoustical side in the integral form

$$\int_{S_2} (p'\dot{\xi}'' - p''\dot{\xi}')ds - (V'I'' - V''I') = 0. \qquad (3)$$

Assuming that /1/

$$Q'' = \int_{S_2} \dot{\xi}''ds, \qquad P'' = (1/S_2)\int_{S_2} p'ds$$

and, correspondingly,

$$Q' = (1/P'') \int_{S_2} p''\dot{\xi}' ds, \qquad P' = (1/Q'') \int_{S_2} p'\dot{\xi}'' ds,$$

we will obtain the final form of the reciprocity theorem

$$(P'Q'' - P''Q') - (V'I'' - V''I') = 0, \tag{4}$$

where P and Q correspond to the values of pressure and volume velocity on the surface of transducer S_2.

To examine the meaning of the derived reciprocity relation, we consider the special case of open-circuit conditions:

$$P'Q'' - E''I' = 0, \tag{5}$$

where $Q' = 0$ and $I'' = 0$.

For calibrations in a quasi-spherical field we can consider a small pulsating sphere to be the theoretical source of the field. Under these conditions, Q represents the radial volume velocity of the pulsating sphere and P, the pressure on its blocked surface, is equal to the free-field pressure.

The reciprocity theorem in the forms (3), (4), and (5) can also be obtained in a similar manner for the condition when the transducers under consideration and the sound-conducting medium surrounding them are bounded by some arbitrarily vibrating surface. The integration of expression (3) for this condition is carried out over the entire bounding surface, or, if part of the surface is rigid, then over its nonrigid part. This interpretation of the theorem is the basis for the derivation of calculated expressions for pressure calibration in a finite tube /5/ for the particular case of a small chamber, and also for the calibration with respect to the field in a semi-infinite tube /6/. It is convenient to consider here that the auxiliary transducer is a theoretical source of the planar field in the tube, as for instance, performing planar vibrations in the cross section of the tube.

The reciprocity theorem that has been considered does not, in practice, limit the form of the surface vibrations of the transducer. In an infinite medium, the theorem permits also the presence in the medium of solid and co-vibrating obstacles and of surfaces separating the medium into sections having different acoustical impedances.

Calibration in a Quasi-Spherical Field

Let reversible electroacoustic transducer 1 be placed at point A, and at point B, a distance d on the axis of transducer 1, put as an auxiliary transducer a small pulsating sphere.

The current I_{rev} through the transducer, operating as a projector, will cause at point B a free-field pressure P_B; the radial volume velocity Q_B of the pulsating sphere, in its turn, will cause at point A a free spherical wave P_A and a corresponding open-circuit voltage E_{rev} on the terminals of transducer 1 operating as a receiver. On the basis of this, the pressure on the surface of transducer 1 can be calculated for total pressure on its blocked surface by means of the following application of Thevenin's theorem:

$$\frac{P_A(1 + \alpha_1')}{1 + (\zeta_1/Z_1)}, \tag{6}$$

where

$$\alpha_1' = (X_1 + jY_1)\left[1 + j(\lambda/2\pi d)\right]$$

$[(X_1 + jY_1) = \alpha_1$ is a dimensionless coefficient in the expression for the complete acoustical impedance of the transducer], $\zeta_1 = (\rho c/S_1)(X_1 + jY_1)$ is the reaction of the free medium on the oscillating transducer (S_1 is the surface area of transducer 1), Z_1 is the total acoustical impedance of the mechanical system of the transducer.

When such a pressure exists in the field of a free plane wave, the pressure acting on the surface of the transducer would be equal to

$$\frac{P_A(1 + \alpha_1)}{1 + (\zeta_1/Z_1)}. \tag{7}$$

Let the sensitivity of the transducer operating as a receiver in a sound field be represented by M, which is equal to the ratio of the open-circuit voltage developed at the transducer terminals to the pressure in a free field of plane progressive waves. Then the open-circuit voltage developed by the transducer as a result of the pressure acting on it is

$$E_{rev} = M_{rev}\, P_A(1 + \alpha_1')/(1 + \alpha_1) . \tag{8}$$

We now put at point B transducer 2 which is to be calibrated as a receiver. We assume that at this point there exists a quasi-spherical field from projector 1, and that

$$d \gg S_1/\lambda \tag{9}$$

where d is the distance measured from the acoustic center of the transducer that is the source of the quasi-spherical wave. The pressure acting on the surface of transducer 2 equals

$$\frac{P_B(1 + \alpha_2')}{1 + (\zeta_2'/Z_2)} \tag{10}$$

where all of the symbols have the same meaning as in expression (6) except that ζ_2' is the reaction of the medium on transducer 2 when it is in the field of transducer 1.

The pressure acting on transducer 2 in a plane progressive wave field will be

$$\frac{P_B(1 + \alpha_2)}{1 + (\zeta_2/Z_2)} . \tag{11}$$

Transducer 2 then develops an open-circuit voltage

$$E_r = M_r P_B \frac{(1 + \alpha_2')(Z_2 + \zeta_2')}{(1 + \alpha_2)(Z_2 + \zeta_2)} . \tag{12}$$

We will now determine a meaning for ζ_2, for which purpose we also assume that

$$d \gg S_2/\lambda . \tag{13}$$

Then the pressure being created by transducer 2 at point A when the volume velocity of its surface is Q_2 can be written in the form

$$j(\omega\rho Q_2/4\pi d).$$

The volume velocity of the surface of transducer 1 that is caused by this pressure will be equal to

$$j \frac{\omega\rho Q_2(1 + \alpha_2')}{4\pi d(Z_1 + \zeta_1)} .$$

Under these conditions we assume that the reaction of the medium does not change in the field of transducer 2 operating with fixed velocity and having therefore infinite impedance. Such an approximation, generally speaking, is permissible only if conditions (9) and (13) are satisfied.

This particle velocity causes on the surface of blocked transducer 2 the pressure

$$-Q_2\left(\frac{\omega\rho}{4\pi d}\right)^2 \frac{(1 + \alpha_1')(1 + \alpha_2')}{Z_1 + \zeta_1} .$$

The total pressure will be equal to

$$Q_2\left[\frac{\rho c}{S_2}\alpha_2 - \left(\frac{\omega\rho}{4\pi d}\right)^2 \frac{(1 + \alpha_1')(1 + \alpha_2')}{Z_1 + \zeta_1}\right] .$$

Since the desired value for ζ_2' equals $P_{2(total)}/Q_2$, then

$$\zeta_2' = \zeta_2 - \left(\frac{\omega\rho}{4\pi d}\right)^2 \frac{(1 + \alpha_1')(1 + \alpha_2')}{Z_1 + \zeta_1} . \tag{14}$$

We will now make use of the reciprocity theorem (5) and will put into it the values for voltage and pressure defined by the expressions (8) and (12). Finally, we will perform the experiment being discussed, which is considered the basic one in the well-known "scheme of three measurements," and which gives the product of the sensitivities of the transducers involved in the calibration in the form

$$M_r M_{rev} = C \frac{4\pi d}{\omega\rho} \frac{E_r}{I_{rev}}, \tag{15}$$

where $4\pi d/\omega\rho = 2d\lambda/\rho c = H$ is the reciprocity parameter for spherical waves.

Relation (15) differs from the well-known relation by the field coefficient C, which takes account of the mutual influence of the transducers on each other and the difference between the effective pressure on the surface of the transducer in the spherical-wave field and the pressure in a plane-wave field.

If the conditions of calibration are chosen so that C = 1, then the calibration with respect to a quasi-spherical wave field can be carried out. For this, it is sufficient, with the help of some third auxiliary projector, to represent the receiving sensitivity of one of the basic two transducers in terms of the sensitivity of the other one; that is, to determine the ratio

$$\frac{M_{rev}}{M_r} = \frac{E'_{rev}}{E'_r} m, \tag{16}$$

where m is a coefficient representing, in a general case, the conditions of measurement with respect to the current through the independent projector and the distance separating the transducers.

Combining (15) and (16), it is possible to obtain, for example, the sensitivity of the receiver being calibrated

$$M_r = \left[\frac{E_r}{I_{rev}} \frac{E'_r}{E'_{rev}} \frac{H}{m}\right]^{\frac{1}{2}}, \tag{17}$$

but it is also possible to determine both the receiving and transmitting sensitivities of all the remaining transducers involved in the calibration /1,3,6/.

Conditions of Calibration

We will consider the conditions under which the field coefficient C becomes approximately equal to one. We will write the expression for it

$$C = \frac{(1+\alpha_1)(1+\alpha_2)}{(1+\alpha'_1)(1+\alpha'_2)}\left[1 - \frac{(1+\alpha'_1)(1+\alpha'_2)}{(Z_1+\zeta_1)(Z_2+\zeta_2)}\left(\frac{\rho c}{2d\lambda}\right)^2\right]. \tag{18}$$

We will assume that the distance between the transducers does not change and that this distance is large compared to the dimension 2a of the largest one of them; that is

$$d \gg 2a. \tag{19}$$

Then

$$\frac{1+\alpha_1}{1+\alpha_1'} \approx 1 \quad \text{and} \quad \frac{1+\alpha_2}{1+\alpha_2'} \approx 1 \tag{20}$$

since, if $d \gg \lambda/2\pi$, then $\lambda/2\pi d$ is a small quantity and $\alpha \approx \alpha'$; if now $d < \lambda/2\pi$, then α and α' are small by comparison with unity by virtue of the assumption of constant phase ($d \gg S/\lambda$) and constant amplitude ($d \gg 2a$) on all of the surface of the transducer. Having taken this into consideration, we will obtain, from expression (18), the condition under which the calibration will be independent of the input acoustical impedance of the transducers participating in the measurement; the form of this condition is

$$(Z_1 + \zeta_1)(Z_2 + \zeta_2) \gg \left(\frac{\rho c}{2d\lambda}\right)^2 (1 + \alpha_1')(1 + \alpha_2'). \tag{21}$$

We will consider the frequency limits and the most unfavorable case, that is, when both transducers are operating at their resonant frequencies. Then

$$Z_1 + \zeta_1 = (\rho c/S_1)X_1 \quad \text{and} \quad Z_2 + \zeta_2 = (\rho c/S_2)X_2.$$

For low frequencies, $2\pi a/\lambda \ll 1$, and condition (21) takes the form

$$d \gg \frac{1}{2\lambda}\left[\frac{S_1 S_2}{X_1 X_2}\right]^{\frac{1}{2}}. \tag{22}$$

For high frequencies, $2\pi a/\lambda \gg 1$, and condition (21) takes the form

$$d \gg (2/\lambda)(S_1 S_2)^{\frac{1}{2}}. \tag{23}$$

Here it is quite evident that the condition for the independence of the calibration from the acoustical impedance of the transducers involved in the measurement can be even more severe at high and especially at low (that is, for small values of X) frequencies than the condition for the presence of a quasi-spherical wave and the resemblance of this wave to plane wave conditions (9), (13), and (19). The physical meaning of this for resonant transducers is that the effective surface of absorption and scattering of such transducers can significantly exceed the geometrical surface.

<u>Extent of Quasi-Spherical Field</u>

From the preceding analysis, it is quite evident that in practical application of the method it is necessary first of all to determine the extent of the quasi-spherical field by taking into account the finite dimensions and the input acoustical impedance of the transducers.

It is well known from established theory that the region of Fraunhofer diffraction is limited to a certain value of the wavelength ratio, which is usually expressed merely by the inequality $\varepsilon = d\lambda/S \gg 1$. It is not difficult to see that this parameter defines the boundary of the region that has "a given degree of quasi-sphericity" in relative coordinates d/2a and 2a/λ in the following form

$$\frac{d}{2a} = \frac{\varepsilon}{4} \frac{2a}{\lambda}.$$

A quantitative analytical determination of the region of the quasi-spherical field for real transducers is difficult; it is, therefore, expedient to determine the boundary experimentally. For this purpose, we have carried out a series of measurements on the acoustic fields of projector-receiver systems as a function of the distance between them. The investigation involved the most common types of transducers with conical and piston-type radiating surfaces. The two most common cases investigated were: transducers with equal surface area, and a pair in which the surface of one was large compared to the surface of the other. To determine the relation between the transducers and the field, the measurements were made at a series of frequencies (corresponding to values of λ_0 in Figs. 1-3), which correspond to the resonant frequency for one or both of the transducers. The measurements were made in a free field in an undeadened chamber (additional details about this will be given later). In each experiment, the projector remained stationary while the receiver was moved. The field was determined from the voltage developed by the receiver for a given current into the projector. The position of the projector during the measurements was vaired to eliminate the effect of inhomogeneity of the field, especially in an undeadened chamber.

The distance d was read from the center of the base of the cone for transducers with a conical radiating surface, from the center of the vibrating surface for transducers with plane diaphragms. Further measurements of the acoustical field of a series of systems confirmed the fact that the acoustical center of radiation for these transducers can be taken as these points.

The average dependence $p = \phi(d)$ determined from these experiments are shown as follows:

Fig. 1. Conical radiators, small receiver with plane diaphragm.

Fig. 2. Conical transducers of equal size.

Fig. 3. Transducers with plane rectangular and circular diaphragms of equal size.

From an examination of the results it is possible to conclude that the boundary in a quasi-spherical field is rather sharply defined. The uncertainty about this boundary for smaller values of d/2a depends on the ratio of transducer dimension to wavelength 2a/λ and becomes more sharp as this ratio decreases. For a given ratio, particularly for small values of the ratio, the displacement of the boundary substantially depends on the magnitude of the input acoustical impedance of the transducers, which also permits distinguishing the cause of the distortion of the field.

On the basis of the data thus obtained, we can establish the boundary of the region of quasi-spherical field that is of interest to us. This boundary has been plotted in Fig. 4; it represents the locus of the points of discontinuity of the curves $p = \phi(d)$.* Curve A is plotted for a system of transducers with large input impedance, curve B for a system of equal-sized transducers with low input impedance, curve C for a system of a large and a small transducer with the large one having low input impedance.

The results of the experiment agree with the theoretical calculation that has been made. For small values of $2a/\lambda$, the form of the system of transducers and the magnitudes of their input acoustical impedance substantially affect the boundary of the quasi-spherical field and then significantly increase the minimal permissible calibration distance, for which, as for medium and high values of $2a/\lambda$, the actual boundary coincides with the theoretical one and is not affected by the reciprocal influence of the transducers even if they are of the resonant type.

It is possible to conclude from these results that the boundary of the quasi-spherical field at the most used frequencies and for the most commonly used types of transducers is restricted to comparatively short distances--smaller than two or three times the dimensions of the largest of the transducers.

Coefficients of Conformity and Accuracy of Measurements

We will consider the receiving region of a quasi-spherical field from the point of view of the resemblance of a quasi-spherical field to a plane-wave field. As a criterion of this resemblance, we can use the quantities amplitude K_a and phase K_ϕ as coefficients of conformity, which determine the difference between the integral pressure in a spherical and in a plane wave caused respectively by the variation of amplitudes and phases on the surface of a plane transducer.

Values of these coefficients for the most common systems of radiator and receiver have been calculated and reproduced in a table. In this table, $C(\pi/\varepsilon)$ and $S(\pi/\varepsilon)$ are tabulations of Fresnel integrals, $\overline{C}(\pi/\varepsilon) = \int_S C(\pi/\varepsilon)ds$, $\overline{S}(\pi/\varepsilon) = \int_S S(\pi/\varepsilon)$. Calculation of $C(\pi/\varepsilon)$ and $S(\pi/\varepsilon)$ yields integrals for the surface of one transducer, calculation of $\overline{C}(\pi/\varepsilon)$ and $\overline{S}(\pi/\varepsilon)$ yield integrals for the surface of the other transducer.

Numerical values for K_a and K_ϕ for the cases covered by the table are shown in Figs. 5 and 6, where it is assumed that $a = b$.

Attempts to calculate coefficients of conformity for systems that include projectors and receivers having conical surfaces have not been successful. We will show later that coefficients for practical systems are not necessary.

It is not difficult to see that the magnitudes of the coefficients of conformity place additional limitations on the region of quasi-spherical field within the boundaries of which calibration with respect to a field can be carried out with a known degree of accuracy. In particular, iso-line K_a intersects the minimal distance of calibration at the ordinate $d/2a = \phi(K_a)$, and the value K_ϕ gives iso-lines of conformity by means of

*Value of $2a/\lambda = 31.3$ taken from reference /7/.

the known $\mathcal{E} = \psi(K_{\phi})$. The desired accuracy of the result being established by such means, it is possible to further define the region of minimal distances of calibration.

Such a new region for a system consisting of a finite round projector and a small receiver, for example with a known general coefficient of field conformity $K = K_a K_{\phi} = (0.98)(0.95) = 0.93$, is shown in Fig. 4 by dotted lines.

The selection of values for K_a and K_{ϕ} may substantially limit the minimal distance of calibration. However, the most useful application of the computed coefficient of conformity lies not in determining the limit of the minimal distance of calibration but in the possibility of increasing in many cases the accuracy of the method for calibrations made in broader regions of a quasi-spherical field. For this, it is obviously sufficient to increase the measured sensitivity of the transducer by the factor $1/K_a K_{\phi}$ corresponding to the coordinates of measurements $2a/\lambda$ and $d/2a$.

It will later be shown that the results obtained above substantially expand the experimental possibilities of the reciprocity method. In particular, they are the basis of a generally available method of absolute calibration with respect to a field in an undeadened chamber that has been discussed by the author.

Conclusion

The theoretical and experimental investigations that have been carried out permit us to conclude that the quasi-spherical field needed for the practical application of the reciprocity method for absolute calibration of electroacoustic transducers with respect to a field exists only within a comparatively small distance between transducers. This distance is the minimal distance of calibration. The accuracy that is obtained from measurements by this method depends on the calibration distance, and can be, if necessary, increased either by increasing the calibration distance or by introducing corrections for the phase and amplitude differences between a spherical field and a plane field.

The conclusions reached here will be confirmed in the next section of the article by the results of direct calibrations of transducers.

II. RESULTS OF EXPERIMENTS

Introduction

A significantly smaller space in the literature is allotted to the experimental proof of the method of reciprocity with respect to a field than is allotted to its theory. References /3,8,11/ are based on simplified theory that excessively idealizes the conditions of measurement. In particular, the calibration distance is assumed to be large by comparison with the dimensions of the transducers. No attention is paid to the difference between the sensitivity measured in a real quasi-spherical field and sensitivity in an ideal plane-wave field; the very possibility of calibration under continuous-wave conditions is invariably made contingent upon the availability of ideal free-field conditions.

Neither do we find in the literature any comparison of the results obtained from reciprocity calibration with respect to a field with results obtained by other absolute methods of measurement, nor any description of the worker's method of measurement.

We have considered above the principal features of practical calibration with respect to a field under c-w conditions. In particular, we have given a geometrical sense to the concept of calibration distance, and have shown that measurements can be made when the spacing between the transducers is small, and have also noted means for increasing the accuracy of results obtained from calibration measurements in a quasi-spherical field.

In this section of the article, these results will be subjected to further re-examination in connection with direct calibrations. It will be shown that calibration at close distances removes some of the difficulty of achieving true free-field conditions and that sufficiently accurate measurements can be made in readily available anechoic and reverberant chambers, and further that some of the difficulties associated with providing auxiliary transducers sufficiently sensitive in a wide frequency range have been reduced. An important section is allotted to a description of the method of measurements, by means of which we have succeeded in achieving a combination of simplicity and high accuracy of measurement.

In conclusion, a comparison is made of the results obtained under various conditions (free field, partially anechoic, and reverberant chambers) and by various methods of absolute measurements (calibration with respect to pressure by the reciprocity method in a tube and a small chamber, the electrostatic method, calibration with respect to a field by the Rayleigh disk method). The results are in good agreement.

Method of Measurements

The calibrations were made under c-w conditions. As is well know, the low-frequency limit that is characteristic of the pulse method /9/ does not exist for c-w measurements. Another point on which the c-w calibration is superior is, as will be shown below, the possibility of using a simple electrical scheme for the measurements.

Fig. 7 shows the well-known scheme of three measurements which will yield a calibration under c-w conditions. In this diagram: R (П) is a receiver linear throughout the working range of pressure; P (И) is a projector linear in the working range of current; T (О) is the reversible transducer used alternately as a receiver and a transmitter, and linear in the working range of pressures and currents. According to the scheme, transducers are placed alternately in pairs at a certain distance from each other. In each experiment, both the current flowing through the transmitting transducer and the open-circuit voltage developed by the receiving transducer are measured.

The physical arrangement of the transducers has a substantial effect on the accuracy of the measurements. Analysis of all the derived equations shows that for the condition of a nonideal field, the smallest measurement error occurs when the distance between the transducers remains fixed and when the transducers occupy during the experiments one and only one point in the field.

The magnitude of the calibration distance is also important. The minimal distance is restricted to the limits established above for the region of a quasi-spherical field. This limit is shown in Fig. 4 and in Figs. 11 and 12 (dash-dotted line). As explained above, it is clear that the practical minimal distance is usually established in agreement with the limit for the highest frequency to be encountered in the calibration. The maximal distance of calibration is limited by the magnitude of the signal-to-interference ratio that exists for the measurements, which, as is well known, ought to be greater than two, and by the inhomogeneity of the field that is to be tolerated, which, as is shown by experiment, does not exceed 10-15%. The results to be presented below were obtained under the conditions discussed here. The measurement of the calibration distance was made between the acoustic centers of the transducers in accordance with the theory. As was shown above, the distance can be measured from the center of the base of the cone for transducers with conical diaphragms and from the center of the radiating surface for transducers with plane diaphragms. The direct measurement of this distance was made with a very simple measurement scheme such as that shown in Fig. 8.

In Fig. 9 is shown the electrical set-up used for the measurements. Part of it, enclosed in dotted lines, is a simple switch-box. The experiments showed that the use of this set-up simplified the process of making the measurements and increased the accuracy of the results. The remainder of the system consists of standard laboratory apparatus: sound generator G (Г), a sensitive vacuum-tube voltmeter (measuring amplifier) VTVM (Л.В.), resistance box R, and balancing transformer T_p. The chief advantage of the scheme is that all of the electrical quantities are measured in turn by one and the same apparatus. Thus, the current through the projector is measured as the voltage drop across the calibrated resistor R which is small with respect to the internal resistance of the projector. The magnitude of this resistance was chosen so that all of the voltage measurements, or at least pairs of them, were made on the same scale of the voltmeter. This simple procedure reduces to the minimum the most important of the measurement errors connected with scale errors and inaccuracies of the absolute calibration of the voltmeter.

It should be noted that the results of the measurement procedure described were invariably independent of the type and the absolute accuracy of the voltmeters used, which shows the great superiority of the method.

The linearity and reversibility of transducers T (O) and P (И) must be verified before the calibration measurements are made. For this, the amplitude characteristics of the transducers operating as transmitters were recorded; the receiver, as usual, was assumed to be linear. The degree of nonlinearity was determined visually from an oscillogram of the voltage developed by the receiver. The allowed magnitude of nonlinearity was 5-8%. In order to reduce the possible effect of nonlinearity of the transmitter, the current through it was maintained as nearly constant as possible throughout the calibration process. As is evident from formulas to be presented below, the magnitude of this current is partially excluded from the calculation. To simplify the measurement procedure and to increase the accuracy of the results, the auxiliary transducers most often used in our experiments were of the same type and reversible. In this case, an examination of their linearity was replaced by the more usual examination of their reversibility. For this, each transducer was used alternately as a receiver and a projector with a constant distance between them. As is well known /3/, reversible transducers must then satisfy the relation

$$E_P/I_T = E_T/I_P$$

or, if $I_P = I_T$, then $E_P = E_T$.

In the case when the input impedance of the vacuum-tube voltmeter being used in the measurements is insufficient for the direct measurement of the electromotive force of the receiver being calibrated, the open-circuit sensitivity of the transducer is determined by applying to the obtained result a correction which takes into account the absolute magnitude of the ratio of input impedance of the vacuum-tube voltmeter R_{IN} to the internal impedance of the transducer R_i. In Fig. 10 is shown the scheme for determining this correction, where R_0 is a small calibrated resistance across which is developed a certain voltage V_0. If V is the voltage being measured at the terminals of the transducer, then it is not difficult to see that when $R_i \gg R_0$ and $R_{IN} \gg R_0$ the correction factor is equal to V/V_0.

In conclusion, we will state the formulas* derived for determing the sensitivity of the transducers from measurements in a quasi-spherical field:

$$M_R = (4.[illegible]) \frac{1}{K} \left[\frac{E_2 E_1}{E_3} \frac{I_3}{I_2 I_1} \frac{d}{\rho f} \right]^{\frac{1}{2}},$$

$$M_T = [illegible](10^{-4}) \frac{1}{K} \left[\frac{E_3 E_1}{E_2} \frac{I_2}{I_3 I_1} \frac{d}{\rho f} \right]^{\frac{1}{2}},$$

$$S_T = (2.26)(10^3) \left[\frac{E_3 E_1}{E_2} \frac{I_2}{I_3 I_1} \frac{\rho f}{d} \right]^{\frac{1}{2}},$$

$$S_P = (2.26)(10^3) \left[\frac{E_2 E_3}{E_1} \frac{I_1}{I_2 I_3} \frac{\rho f}{d} \right]^{\frac{1}{2}},$$

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*The last two formulas have a physical meaning if the transducers R (П) and P (И) are reversible, which they usually are.

$$M_p = (4.48)(10^{-4})\,\frac{1}{K}\left[\frac{E_2E_3}{E_1}\,\frac{I_1}{I_2I_3}\,\frac{d}{\rho f}\right]^{\frac{1}{2}},$$

$$S_R = (2.26)(10^3)\left[\frac{E_2E_1}{E_3}\,\frac{I_3}{I_1I_2}\,\frac{\rho f}{d}\right]^{\frac{1}{2}}.$$

Here M is the sensitivity of the receiving transducer defined as the ratio of the open-circuit voltage developed at its terminals to the pressure in a free field of plane, progressive waves; S is the sensitivity of the transmitting transducer defined as the ratio of the pressure produced by the projector at a given point in a free field to the current flowing through its electrical circuit; d is the distance between the acoustical centers of the transducers; ρ is the density of the medium in which the calibration measurements are made; f is the frequency of measurement; and K is the phase and amplitude coefficient of conformity of the field. The value of K for the case of a system of two equal-sized plane-surfaced transducers is shown in Fig. 11, and the value for a system with a large and a small plane surface is shown in Fig. 12. The numerical coefficient is introduced for the sake of consistency of units between the practical system for electrical quantities and the CGS system for pressure and distance. The formulas quoted apply to pressure-sensitive transducers.

For determining the sensitivity of velocity-sensitive transducers, the usual correction factor is used:

$$\left[1 + (\lambda/2\pi d)^2\right]^{-\frac{1}{2}}$$

where λ is the wavelength.

Calibration in a Quasi-spherical Field

Figure 13 shows results of a calibration of a typical piezoelectric receiver with a round plane diaphragm; Fig. 14 shows results of a calibration of a typical magnetostrictive transducer with a rectangular radiating surface. In both cases, the measurements were made in a free field with auxiliary transducers of the same type and size. The calibration distance was varied during the measurements. The largest distance used was deliberately selected to satisfy the ideal conditions for measurement: equality of phases and amplitudes and absence of mutual influence of the transducers [conditions (9), (13), and (21)]. The sensitivity measured under these conditions was taken as unity (0 db) for each frequency on the graph. The sensitivity for any given distance was determined by two means: by standard formulas without taking the coefficient of conformity into account (dotted curve) and by these formulas with the coefficient of conformity included in the calibration (solid curve). As can be seen from the graphs, the value of the sensitivity corrected according to our formulas agrees with the plane-wave value for calibrations in all regions of the quasi-spherical field. This is especially obvious on Fig. 14 where the calibration results were obtained with great accuracy.

We will return now to the results shown in Fig. 15 which are typical for an air calibration of an electrodynamic receiver with a small plane diaphragm measured with the help of two equal-sized conical transducers. These measurements were made in poorly deadened spaces having an average coefficient of absorption on the order of 0.7. The validity of this method of measurement will be examined in detail later. As was mentioned above, attempts to calculate a coefficient of conformity for the case of conical surfaces was not successful. In connection with this, the sensitivity value quoted was calculated without taking into account the coefficient of conformity. Nonetheless, the results of the calibration are independent of the distance of measurement almost up to the boundary of the quasi-spherical field. The direct calibration in this manner of a small receiver in combination with conical auxiliary transducers in a quasi-spherical field is shown to be equivalent to the sensitivity measured in a plane-wave field.

Taking into consideration the small value of the coefficient of field conformity for the system of transducers just discussed (Fig. 11), we note that the result obtained does not reveal any important disagreement between theory and experiment. The small error might be explained by the effect of the decrease in the effective ratio of cone size to wavelength as the frequency of measurement increases, and also by the better "fit" of the spherical wave front to the conical surface. As a result of this, coefficients of conformity K for conical surfaces, if they could have been calculated by us, would probably have been closer to unity than for the case of the plane surfaces shown graphically in Figs. 11 and 12. Furthermore, if it is necessary, this coefficient can be determined experimentally for various systems from the results of calibrations at various distances.

Conditions of Measurement in Reverberant Chambers

It follows from the theory of the reciprocity method discussed above that the feasibility of calibration with respect to field is determined by two factors: by the quasi-spherical character of the average pressure field and the magnitude of fluctuations relative to the average field. We will investigate from this point of view the worst condition of calibration, namely, the field in a reverberant chamber. The average field in a reverberant chamber, as measured by various systems of transducers, has been shown in Figs. 1 - 3. These results are obtained for chambers with an average coefficient of absorption α_{av} on the order of 0.1 to 0.3 in a wide range of frequencies for which the linear dimension R of the cubical chamber ranged from $R/\lambda_H \approx 1.0$ to $R/\lambda_B \approx 300$. The field was measured as nearly as possible in the middle of the chamber.

On the basis of these curves, it has been established above that under these conditions, the average field is, within a certain range of distances, quasi-spherical. Naturally, as a result of this, one can make under the frequently occurring conditions of a partially reverberant room even free-field measurements near one or more reflecting or scattering surfaces.

The magnitude of the fluctuations of the field is determined from the ratio of the pressure in the reflected field to the pressure in the direct field. This ratio can be made as small as desired by decreasing the distance between the projector and the receiver as compared to the distance to the walls of the chamber. A very simple procedure for measuring directly the magnitude of the fluctuations of the field can be proposed. It consists of varying the position in the chamber of the projector-receiver system but maintaining a fixed distance between the transducers and a fixed current through the projector. The variation of the voltage developed by the receiver from its average value in a constant direct field produces the desired magnitude of the fluctuations. Fig. 16 shows the average results obtained in this manner for maximal fluctuations β as a function of the ratio of distance between transducers to the distance to the walls of the chamber. This curve is for an average coefficient of absorption of the chamber walls $\alpha_{av} = 0.1 - 0.3$ and for a range of wavelength ratios of the chamber R/λ from 1 to 300. This dependence can be represented approximately by

$$\beta = \frac{p_T}{p_R} \approx \frac{(1 - \alpha_{av})^{\frac{1}{2}}}{1 + (R/d)},$$

in the limited range of values of d/R from 0.05 to 0.2. We observe that the nature of the field as determined from these investigations and the form of the approximation for its fluctuations correspond, as one might expect, more to the existence of fairly ordered reflections than to a diffuse field.

In the description of the method of measurements, it has already been shown that if, in each of the experiments in the "scheme of three measurements," the transducers are put in one and only one place in the field, then the error in the measurements caused by the distortion of the field and in particular by the fluctuations of the field, is equal to the square root of the magnitude of the distortion of the field.* Then, the error of the measurement being established, it is possible to select from Fig. 16 the relative proportion d/R for the calibration chamber. The possibility of measurements at small distances d established above makes the choice of a chamber for calibration very easy; practical calibrations can be produced in standard laboratory spaces and in standard tanks that have not been treated with sound-absorbing materials.

It is possible to make the error as small as 0.5 db. For this, the average dimension of the chamber must be 15 - 20 times the calibration distance.

Comparison of Results

We will examine the accuracy of the method of calibration in a quasi-spherical field when it is used under free-field conditions and also in chambers whose surfaces are not sound absorbent or only partially absorbent. We will compare the results obtained by this method and under these conditions with results obtained by other usual absolute methods. This will be all the more interesting because no such series of comparisons that includes calibration by the reciprocity method with respect to field is given in the literature.

Fig. 17 shows the results of sensitivity measurements on an electrodynamic microphone made by three independent modifications of the

*This error can be made a systematic error and thus be excluded. To do this, it is sufficient to fix the points of measurements.

reciprocity method: calibration with respect to pressure in a tube (solid-line curve) and in a small chamber (dotted curve) and calibrations with respect to field in partially anechoic chambers having an average coefficient of absorption $\alpha_{av} = 0.7$ (measurements indicated by small crosses). In all three cases, the measurements were made by one of the methods mentioned above.

We show calibration results for the range of frequencies in which sensitivity with respect to field ought to agree for a given receiver with its sensitivity with respect to pressure. The error determined for the calibration measurements amounts to: 0.2 db for calibration in a tube, 0.4 db for calibration in a small chamber, and 0.8 - 0.9 db for calibration in a free field. Taking these errors into account, it is possible to consider the results as completely coinciding.

The solid-line curve in Fig. 18 shows the average results of calibration of a Rochelle salts receiver with respect to field obtained in open water; the small crosses indicate the results of measurements made in a tank that was not acoustically treated, and the dashed curve shows results of measurements of sensitivity with respect to pressure by the electrostatic method in oil and in air. As is evident from the curves, in the region where the sensitivities with respect to field and with respect to pressure coincide, the divergence of the average results obtained by various methods does not exceed 0.5 db, and the divergence of the results obtained under various conditions does not exceed 1.0 db. Thus, these results can also be considered as coinciding. We will note, therefore, that the divergence among the results decreases with an increase in the sensitivity of the transducer. For resonant transducers, the divergence practically does not exist. We will point out, also, that the measurement error for calibration by the electrostatic method amounted to less than 0.5 db, and that the error for calibration with respect to field was 0.8 - 1.3 db, but for high-impedance transducers the error was higher.

The solid-line curve in Fig. 19 shows the results of calibration of an electrodynamic microphone with respect to field obtained in a chamber that was not acoustically treated (in a laboratory room) having $\alpha_{av} \approx 0.15$ for $d/R = 0.14$. The small circles represent the results of a calibration of this microphone in an anechoic chamber of the Institute of Metrology with respect to a Rayleigh disk, which is more precise than this Institute's accepted secondary standard method in which is used a loudspeaker that has been thoroughly studied by means of a Rayleigh disk. It can be noted that the results (after taking into account the errors of measurement) are in full agreement throughout the middle and high-frequency range. At the low frequencies, some of the results diverge by as much as 1.6 db, which is more than the measurement error. On the basis of the data shown on Fig. 17, it can be said that this divergence in the results occurs because the field in the chamber of the Institute of Metrology is not good enough at these frequencies.

We have demonstrated above the accuracy of the method by giving examples of the calibration of pressure-sensitive receivers. Without citing any additional results, we will point out that results obtained in the calibration of velocity-sensitive receivers and projectors were also in good agreement.

In this connection, it is interesting to note that Ernsthousen /10/ compared various methods of calibration (except the reciprocity method), and concluded that the practical divergence of results of calibration with respect to field by various methods is 1.5 db. Thus, our results show entirely satisfactory agreement.

Conclusion

The absolute calibration of electroacoustic transducers with respect to field in air and particularly in water have, until now, been considered a more or less delicate physical experiment. In connection with persistent requirements of practice, it has been desirable to make this important branch of acoustical measurements more generally available. The engineering method of calibration by reciprocity in a quasi-spherical field that has been discussed basically solves the problem. This method guarantees good accuracy and broad frequency range of measurements in both air and water. The application of it, in general, does not require any special electroacoustic transducers nor measuring arrangements. It is possible to carry out c-w calibrations under generally available conditions, in particular in chambers having only partially absorbent surfaces or untreated surfaces. The scheme and procedure of the measurements is simple and elementary.

The author and others, after using the method that has been discussed, have now accumulated a comparatively large amount of material that confirms the fundamental principles and results presented in this article. The possibilities for accuracy by this method are, seemingly, good enough that by this method calibrations have successfully been made of microphones in the range of frequencies up to 15,000 cps and of other special transducers.

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Submitted 8 January 1952

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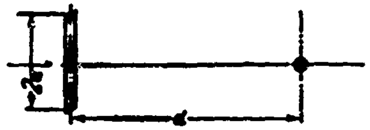
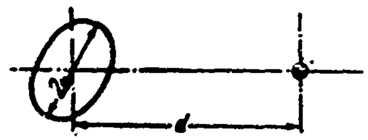
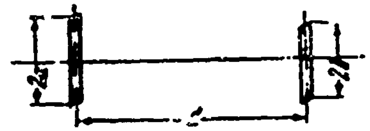
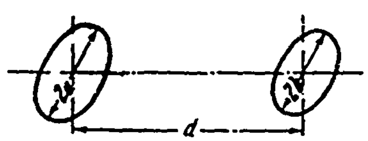
	Sketch of System	K_a	K_φ
1		$\dfrac{1}{1+\frac{1}{6}\left(\frac{a}{d}\right)^2}$	$\sqrt{\frac{\varepsilon}{2}}\sqrt{C^2\left(\frac{\pi}{\varepsilon}\right)+S^2\left(\frac{\pi}{\varepsilon}\right)}$
2		$\dfrac{1}{1+\frac{1}{4}\left(\frac{a}{d}\right)^2}$	$\sqrt{\frac{\varepsilon}{\pi}}\sqrt{\left(\cos\frac{\pi}{\varepsilon}-1\right)^2+\sin^2\frac{\pi}{\varepsilon}}$
3		$\dfrac{1}{1+\frac{1}{6}\left(\frac{a}{d}\right)^2+\frac{1}{6}\left(\frac{b}{d}\right)^2}$	$\sqrt{\frac{\varepsilon}{2}}\sqrt{C^2\left(\frac{\pi}{\varepsilon}\right)+S^2\left(\frac{\pi}{\varepsilon}\right)}$
4		$\dfrac{1}{1+\frac{1}{4}\left(\frac{a}{d}\right)^2+\frac{1}{4}\left(\frac{b}{d}\right)^2}$	Determined by numerical integration

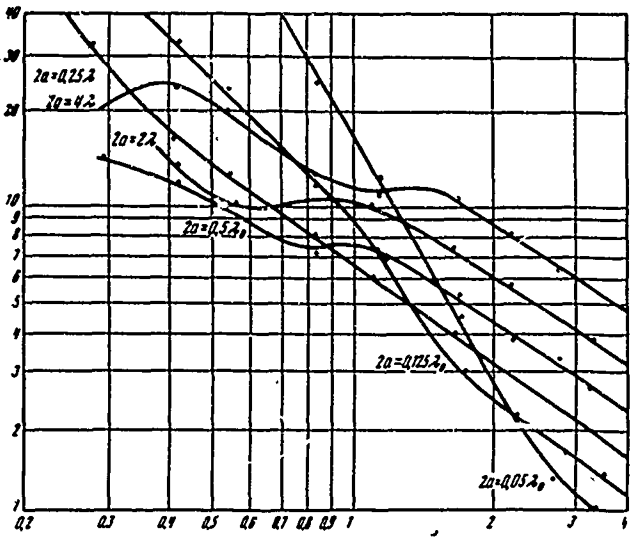


Fig. 1. Field of a system consisting of a conical projector and a small receiver with a plane diaphragm.
Abscissa: Relative distance d/2a
Ordinate: Relative pressure

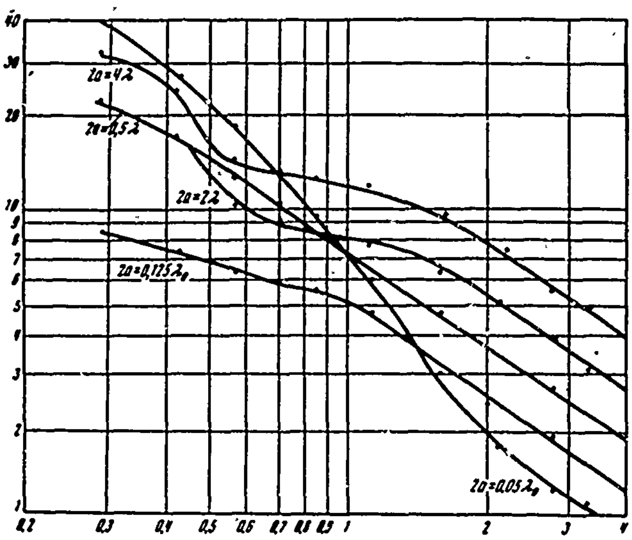


Fig. 2. Field of a system of two conical transducers of equal size.
Abscissa: Relative distance d/2a
Ordinate: Relative pressure

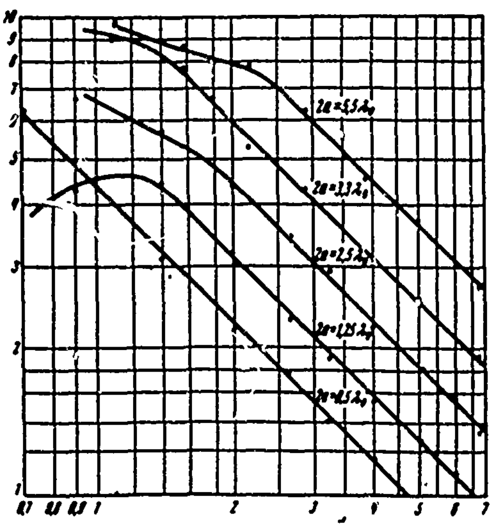


Fig. 3. Field of a system of two equal-sized rectangular and round transducers with plane diaphragms.
Abscissa: Relative distance d/2a
Ordinate: Relative pressure

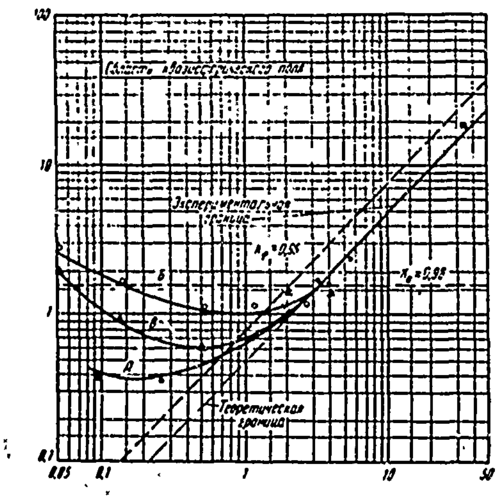


Fig. 4. Limits of region of quasi-spherical field for a system of two finite transducers.
At top of graph: Region of quasi-spherical field. At middle of graph: Experimental limits. At bottom of graph: Theoretical limits.
Abscissa: Wave ratio of transducers 2a/λ. Ordinate: Relative distance d/2a.

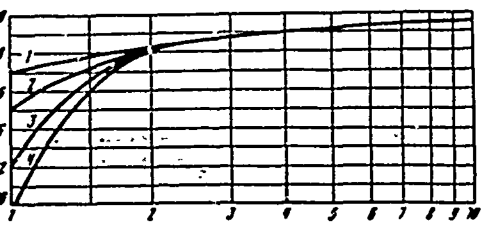


Fig. 5. Amplitude coefficient of conformity of a quasi-spherical field to a plane field.
Abscissa: Relative distance d/2a.
Ordinate: Coefficient K_a.

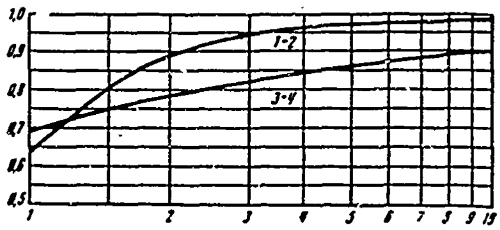


Fig. 6. Phase coefficient of conformity of quasi-spherical to plane field. Abscissa: $d\lambda/a^2$.
Ordinate: Coefficient K_ϕ.

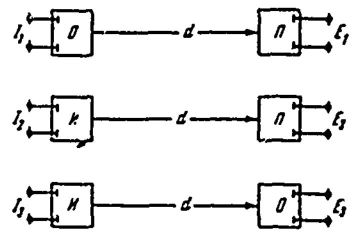


Fig. 7. Calibration scheme.
О = reversible transducer = T
И = projector = P
П = receiver = R

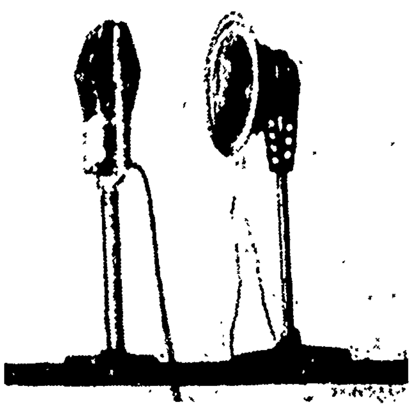


Fig. 8. Calibration of ribbon microphone in undeadened room.

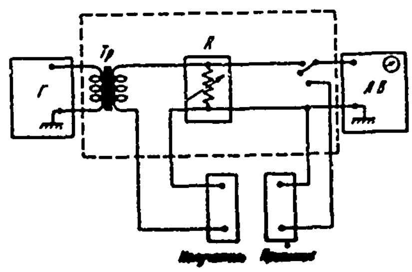


Fig. 9. **Electrical scheme of measurements.**
Г = generator (oscillator) = G
ЛВ = vacuum-tube voltmeter
излучатель = projector = P
Приемник = receiver = R

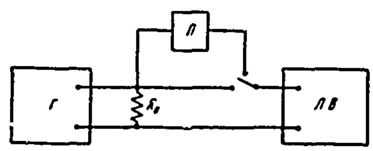


Fig. 10. Scheme for the determination of correction taking into account finite magnitude of R_{IN}/R_i.
Г = generator (oscillator) = G
П = receiver = R
ЛВ = vacuum-tube voltmeter = VTVM

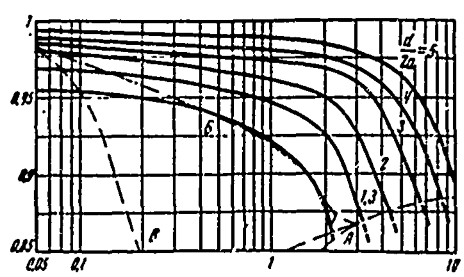


Fig. 11. Amplitude and phase coefficient of conformity of quasi-spherical to plane field for a system of transducers with one large and one small plane diaphragm.
Abscissa: wave ratio $2a/\lambda$.
Ordinate: Coefficient K.

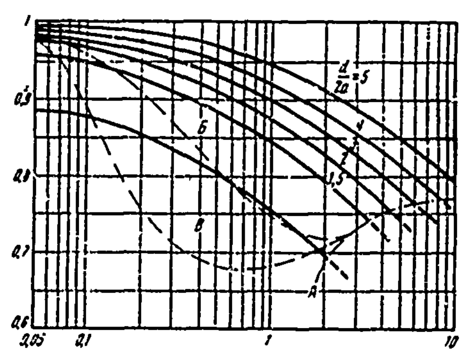


Fig. 12. Amplitude and phase coefficient of conformity of quasi-spherical to plane field for a system of transducers with plane diaphragms of equal size.
Abscissa: Wave ratio $2a/\lambda$.
Ordinate: Coefficient K.

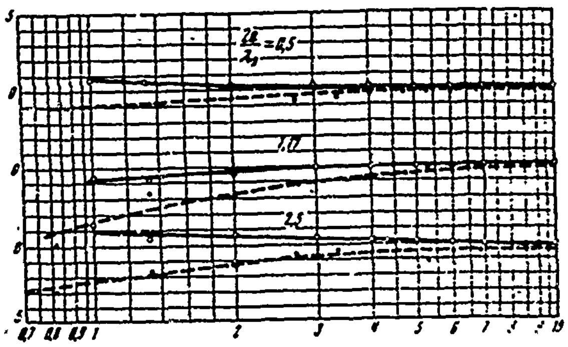


Fig. 13. Dependence of sensitivity on calibration distance for a system of transducers with equal-sized plane round diaphragms.
Abscissa: Relative distance d/2a.
Ordinate: Sensitivity, db.

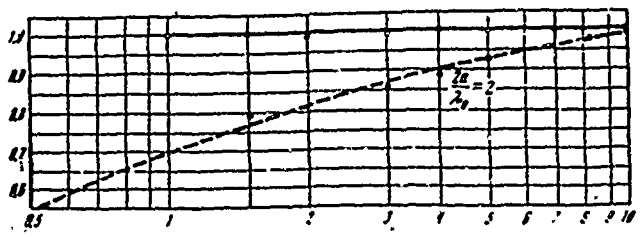


Fig. 14. Dependence of sensitivity on calibration distance for a system of transducers with equal-sized plane rectangular diaphragms.
Abscissa: Relative distance d/2a.
Ordinate: Sensitivity, db.

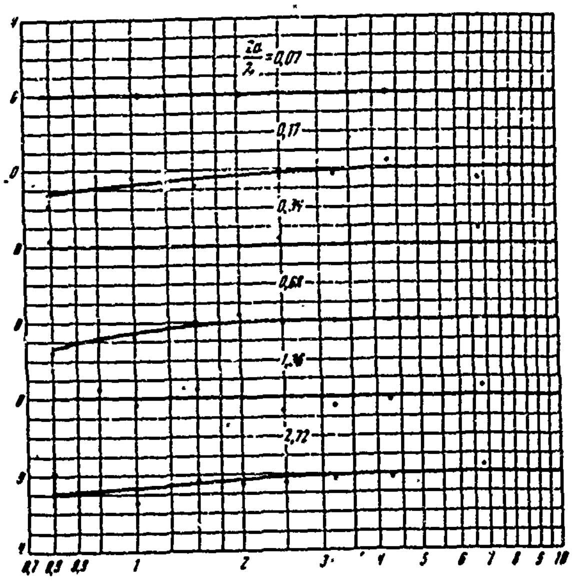


Fig. 15. Dependence of calibration of receiver with a small plane diaphragm on calibration distance. (Auxiliary transducers with large conical diaphragms.)
Abscissa: Relative distance d/2a.
Ordinate: Sensitivity, db.

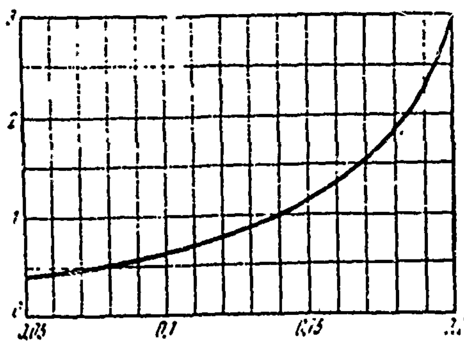


Fig. 16. Fluctuations of field in a reverberant chamber as a function of the ratio of distance between transducers to distance to the walls of the chamber.
Abscissa: Relative distance d/R.
Ordinate: Fluctuations, db.

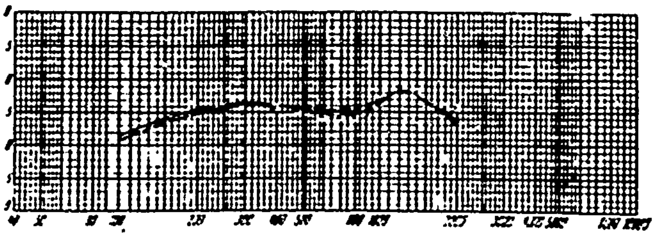


Fig. 17. Sensitivity of an electro-dynamic microphone measured by the reciprocity method.
Abscissa: Frequency, cps.
Ordinate: Sensitivity, db.

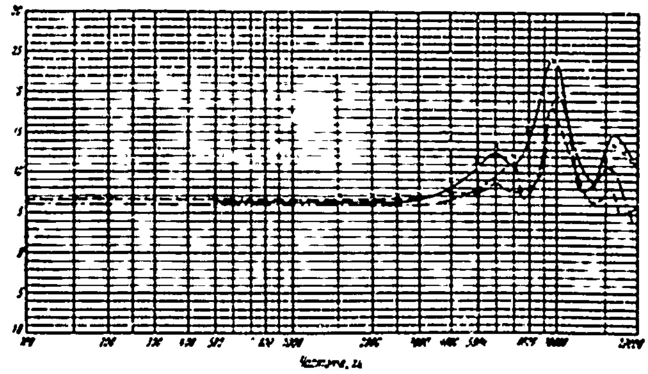


Fig. 18. Sensitivity of piezoelectric receiver.
Abscissa: Frequency, cps.
Ordinate: Sensitivity, db.

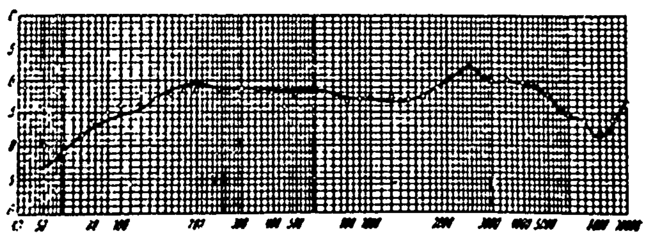


Fig. 19. Sensitivity of an electro-dynamic microphone.
Abscissa: Frequency, cps.
Ordinate: Sensitivity db.