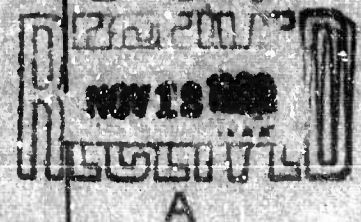TECHNICAL REPORT
69-30-CM

RELIABILITY OF SOME CRITICAL PERFORMANCE CHARACTERISTICS OF COTTON DUCK FABRIC

by

Clarence J. Pope
and Stanley J. Werkowski

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TECHNICAL REPORT
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RELIABILITY OF SOME CRITICAL PERFORMANCE
CHARACTERISTICS OF COTTON DUCK FABRIC

by

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and

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ABSTRACT

Reliability analyses are made on physical characteristics of a cotton duck material (with a fire, weather, water, mildew - resistant treatment) used in the construction of military tentage and equipage items. The breaking strength in both the lengthwise (warp) direction and crosswise (filling) direction and also the water resistance property were selected as critical characteristics for analyses. The reliability analyses relate to a commodity situation where these techniques may be useful in assessing how well characteristics of a commercial material meet established requirements. The Chi Square (x^2) test for goodness of fit was used to test normality of the distribution of laboratory data for each of the characteristics evaluated. Different methods are used to obtain one-sided lower reliability limits. Point estimates of reliability are also computed. Results of the different methods are compared, and the relative advantages of these methods are discussed.

RELIABILITY OF SOME CRITICAL PERFORMANCE
CHARACTERISTICS OF COTTON DUCK FABRIC

1. Introduction

Reliability is generally associated with a "time to failure"
criterion and would thus be defined as the probability that an item
will perform successfully for a specified time under specific
conditions of usage. Time is a critical parameter in these
applications. The use of a "time to failure" criterion to
establish reliability for textile materials may be very limited.
These materials, however, present commodity situations where the
"stress-strength" analysis technique offers more meaningful
information, e.g., the probability that a critical performance
characteristic will fall within specification limits.[1] The
determination of reliability is dependent directly upon some
specific level of performance for textile materials. The
performance level selected is customarily an engineering estimate
and may be modified to be consistent with production capabilities
and/or field serviceability needs.

This report is concerned with the application of reliability
techniques [2] [3] in predicting performance reliability of a
basket weave cotton duck with a fire, weather, water, mildew-
resistant treatment. The parameters selected for evaluation are
the breaking strength in the warp and filling directions, and also
the water resistance property. These strength characteristics are
directly related to durability, while water resistance is an
important criteria for functionality.

2. Methodology for Computing Estimates of Reliability

Data were acquired from tests of duck fabric produced by
five different manufacturers. A histogram was constructed for each
strength factor using a total of 425 individual test results in
each instance (Figs 1 and 2). The class interval for each
distribution is 10 breaking strength units expressed in pounds.

Both sets of strength data appeared to typify the characteristic
bell-shape of a normal distribution. This assumption was also
supported by the Chi Square (x^2) goodness of fit tests as shown in
Appendices A and B. Accordingly, the methods [2] [3] used in this
study to derive reliability point estimates and confidence limits
are those applicable to data from a normal population.

CLASS INTERVAL (Pounds)	FREQUENCY
109.5-119.5	1
119.5-129.5	3
129.5-139.5	20
139.5-149.5	67
149.5-159.5	95
159.5-169.5	111
169.5-179.5	81
179.5-189.5	37
189.5-199.5	7
199.5-209.5	2
209.5-219.5	1
	425
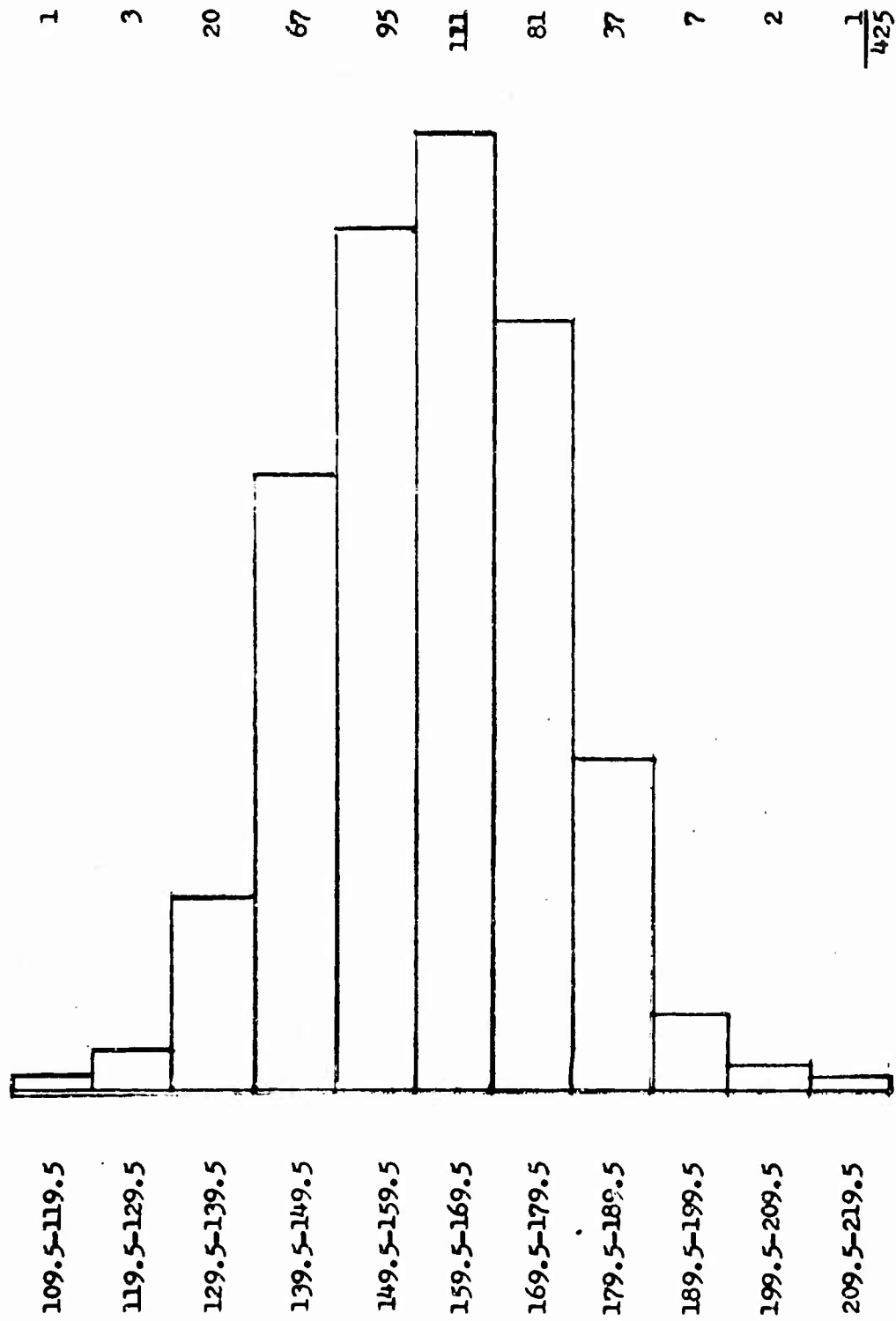
Figure 1. Filling Breaking Strength Histogram

2

FREQUENCY

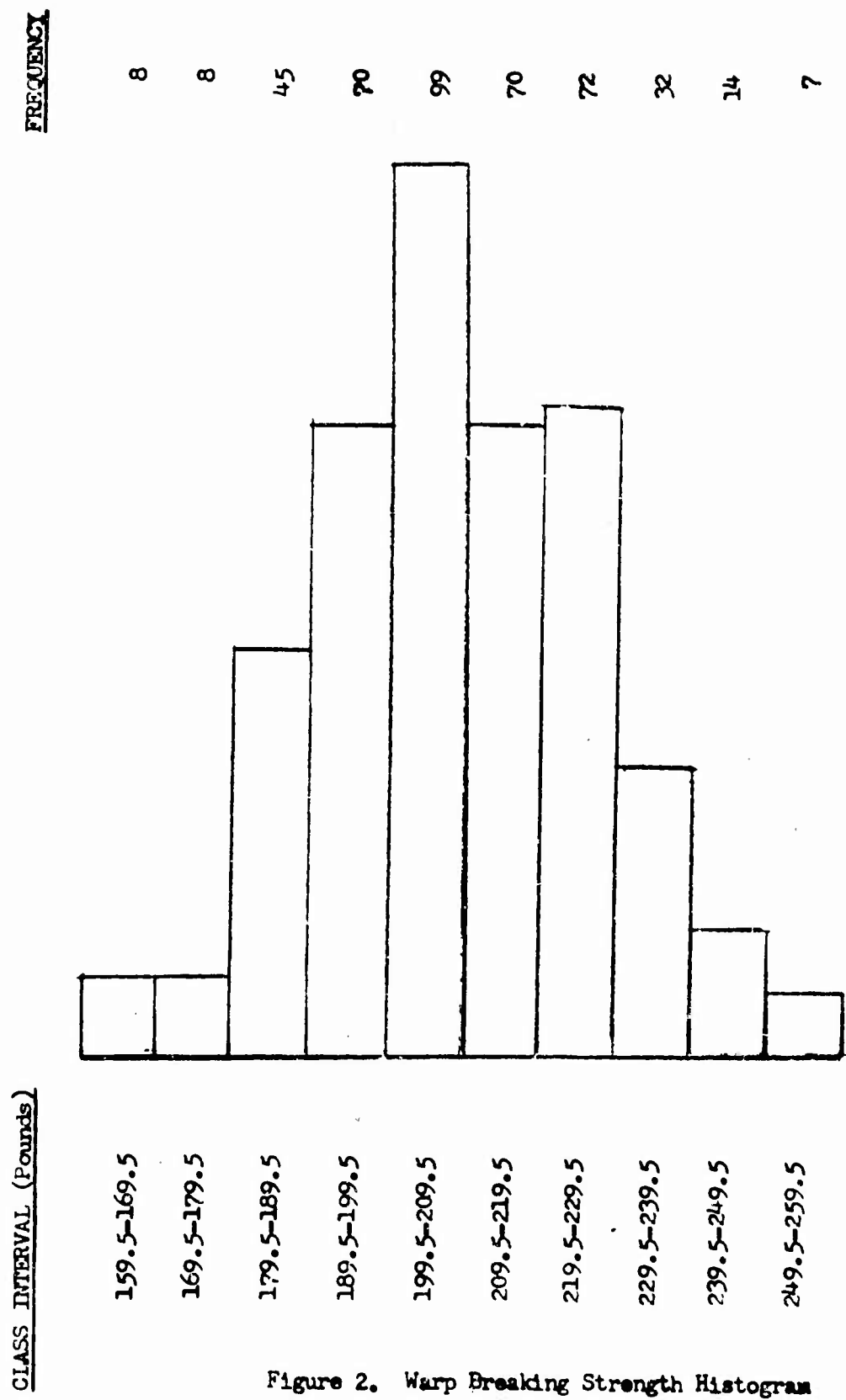| CLASS INTERVAL (Pounds)	FREQUENCY
159.5-169.5	8
169.5-179.5	8
179.5-189.5	45
189.5-199.5	90
199.5-209.5	99
209.5-219.5	70
219.5-229.5	72
229.5-239.5	32
239.5-249.5	14
249.5-259.5	7

Figure 2. Warp Breaking Strength Histogram

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Figure 3 is a histogram showing data for the water resistance property of the duck material. It is apparent that the profile of the histogram does not suggest normality. In this case, the use of the methods discussed here for reliability would be misleading. However, the 200 cc being the maximum amount of water penetration permitted by the specification, reliability point estimate can be obtained by the ratio of total number of values not exceeding 200 cc to the total number of values, as follows:

$$\text{Reliability} = \frac{238}{255} = .93$$

Reliability, in this case, is the probability that the duck material will permit not more than 200 cc of water penetration when subjected to the specification test. Compliance with this performance characteristic will occur 93 percent of the time. However, no confidence level can be associated with this reliability estimate.

The breaking strength requirements for duck in MIL-C-41808A are 150 pounds for the warp direction and 140 pounds for the filling direction. Since these requirements are cited on a minimum basis, it is appropriate to consider reliability in terms of a one-sided lower limit. Breaking strength values falling above the minimum specified would indicate successful performance of the material for this characteristic.

The procedure for deriving a point estimate of reliability from data representing a normal distribution requires the computation of the standard normal deviate (k) as follows:

$$k = \frac{\bar{x} - L}{S}$$

where: $\bar{x}$ = mean breaking strength   (1)

L = specification lower limit or minimum

S = standard deviation

The standard normal deviate is used to obtain the percent area in a normal distribution which exceeds the specification minimum (Fig. 4). This area is the probability that a strength value will meet the performance requirement. This probability is also the point estimate of reliability, and may be obtained from a cumulative normal distribution table[4] which is included in most statistical texts.

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FREQUENCY

(CLASS INTERVAL (cc of water))

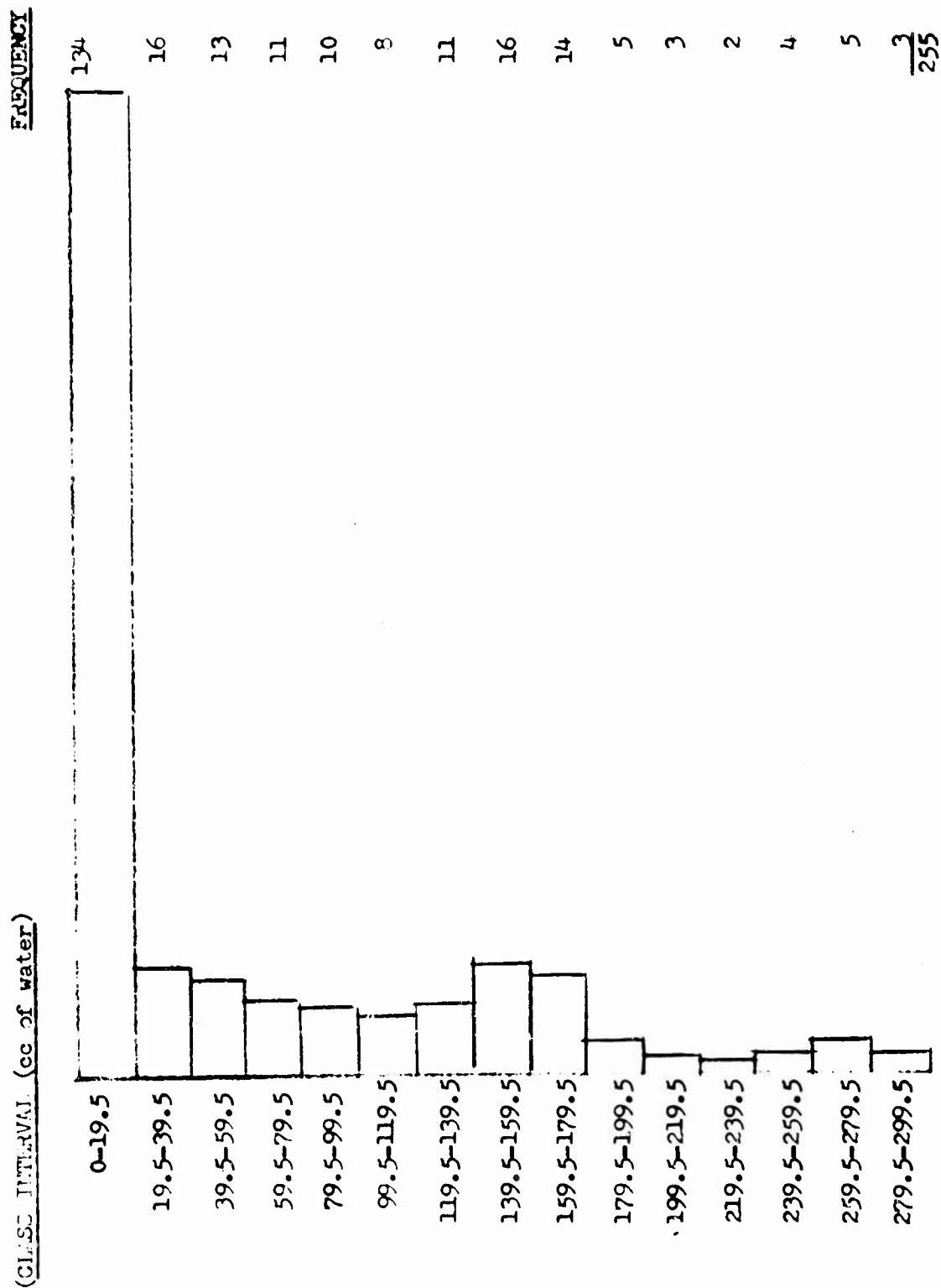| Class Interval	Frequency
0-19.5	134
19.5-39.5	16
39.5-59.5	13
59.5-79.5	11
79.5-99.5	10
99.5-119.5	9
119.5-139.5	11
139.5-159.5	16
159.5-179.5	14
179.5-199.5	5
199.5-219.5	3
219.5-239.5	2
239.5-259.5	4
259.5-279.5	5
279.5-299.5	3
	255

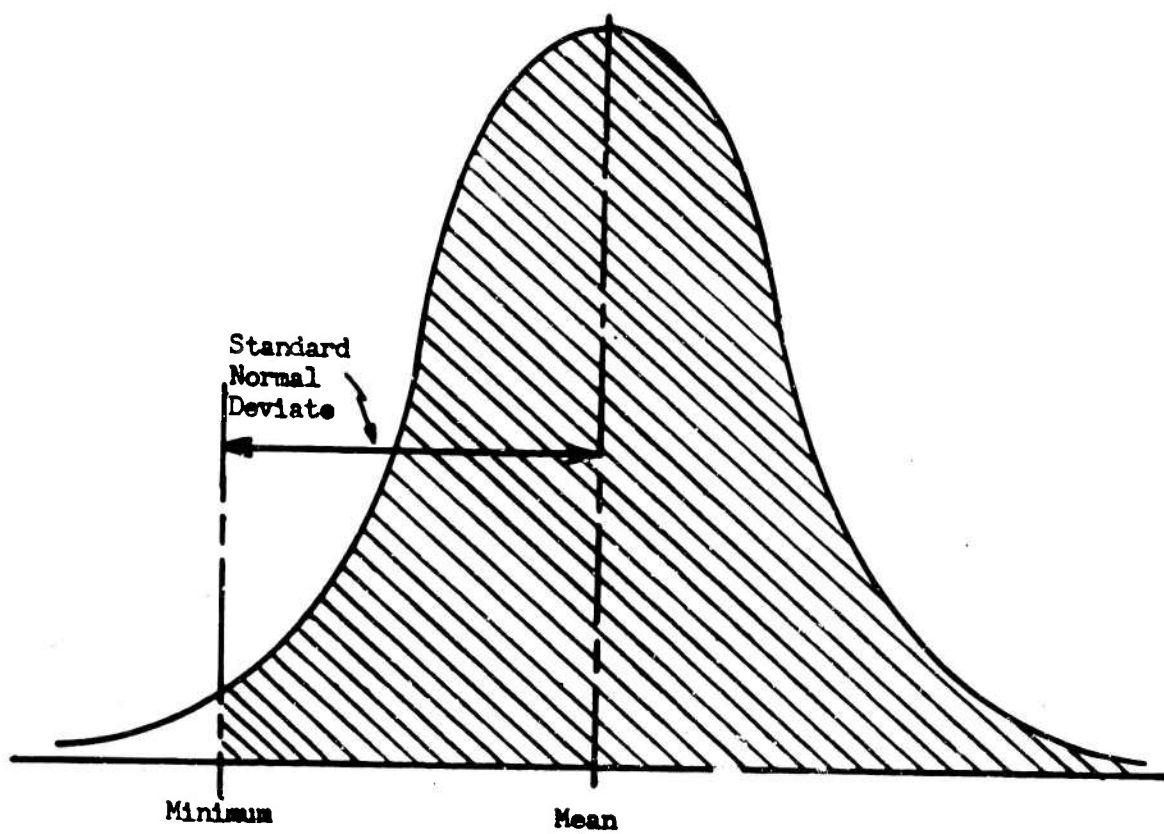Figure 3. Water Resistance Histogram

Figure 4. Area Under Normal Curve

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By means of a lower reliability limit based on a confidence
level, a more precise statement can be made about reliability.
One-sided lower reliability limits were computed for the strength
factors as shown in Appendices A and B. The first method of
establishing the lower limit with a 95 percent confidence was
found by computing K_{RL} from the following formulae:

Method 1

$$K_{RL} = k - A \tag{2}$$

$$A = K_{(1 - \gamma)} \left[\frac{1}{n} + \frac{k^2}{2n-2} \right]^{\frac{1}{2}} \tag{3}$$

A further definition of the quantities in equation (3) is:

$K_{(1 - \gamma)}$ = represents the standard normal deviate
for a particular confidence level. Since
the 95 percent one-sided confidence level
was used in this study, $K_{(1 - \gamma)} = K_{(1 - .95)}$
$= K_{(.05)} = 1.645$. This value was obtained
from the cumulative normal distribution table.

n = number of test specimens (values)

k = standard normal deviate computed by equation (1)

The quantity K_{RL} is the standard normal deviate for the
lower one-sided reliability limit. By entering this value in the
cumulative normal distribution table,[4] the lower reliability limit
at the 95 percent confidence level is obtained.

The lower one-sided limit for reliability can also be
established by a second, more involved method which is based on
the non-central t-statistic[3]. The mathematical relationships
are as follows:

Method 2

$$k^2 (a) - 2k (K\alpha) + b = 0 \tag{4}$$

$$a = 1 - K_{\gamma}^{2}/2(n - 1) \tag{5}$$

$$b = K_{\alpha}^{2} - K_{\gamma}^{2}/n \tag{6}$$

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where: K_α = standard normal deviate of the lower one-sided
reliability limit.

  k = standard normal deviate of the reliability
point estimate. This value is derived by
equation (1)

  K_γ = standard normal deviate for the 95 percent
confidence level which is equal to 1.645.

  n = number of test specimens (values)

By solving equations (5) and (6) and substituting in equation (4),
the standard normal deviate (K_α) is obtained and can be
converted to the lower reliability limit by entering the K_α
value in the cumulative normal distribution table (4).

3. Results

The reliability limit calculations using the two methods are
shown in Appendices A and B and the results are summarized in
Table I. The reliability of the filling breaking strengths is
91.5 percent at the 95 percent confidence level. There is a
95 percent assurance that at least 91.5 percent of the duck
fabrics will have breaking strengths above the minimum
specification requirement (140 pounds). Actually, 401 breaking
strength values of the 425 total were above the minimum.

The reliability of the warp breaking strength is 99.9
percent. Thus, 95 percent of the time, at least 99.9 percent of
the duck fabrics can be expected to have warp breaking strengths
above the specification requirement (150 pounds). For this
particular data, all duck fabrics tested had strength values
above the specification minimum.

4. Summary

Reliability estimates obtained by both methods were the same.
Method 1, however, is the method of choice since it is simpler,
involving fewer computations.

This study has shown that the warp and filling breaking
strength and water resistance requirements for cotton duck
fabrics are consistent with manufacturing capabilities. Reliability
analyses provide a quantitative means to assess, with assurance,

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how well materials are meeting requirements. Similar analysis of other fabric characteristics such as tearing strength, wind resistance, and shrinkage may be feasible and should be investigated.

The data for this study were pooled from several sources. Similar analyses may be made for individual sources (suppliers), and would serve to assess the capability of each source to meet established requirements.

TABLE I

RELIABILITY OF BREAKING STRENGTH OF A COTTON DUCK*

	Warp (%)	Filling (%)
Method 1	99.9	91.5
Method 2	99.9	91.5

*
_____

Based on one-sided 95 percent limit

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5. References:

1. Kennedy, S. J. and L. I. Weiner, "The Application of the Concept of Reliability to Textile Products," Textile Series Report No. 153 - U. S. Army Natick Laboratories, September 1967.

2. Drnas, T. M., "Methods of Estimating Reliability," American Society for Quality Control Journal, September 1966.

3. Lieberman, G. J., "Tables for One-Sided Statistical Tolerance Limits," American Society for Quality Control Journal, April 1958.

4. Dixon, W. J. and F. J. Massey, "Introduction to Statistical Analysis," McGraw-Hill Book Company, Inc., New York, 1951.

APPENDIX A

FILLING BREAKING STRENGTH

1. Calculation of Mean and Standard Deviation

Interval	Coded Score(X)	f	fx	fx^2
109.5-119.5	-5	1	-5 ⎤	25
119.5-129.5	-4	3	-12	48
129.5-139.5	-3	20	-60 ⎟ -306	180
139.5-149.5	-2	67	-134	268
149.5-159.5	-1	95	-95 ⎦	95
159.5-164.5-169.5	0	111	0	0
169.5-179.5	1	81	81 ⎤	81
179.5-189.5	2	37	74 ⎟ 189	148
189.5-199.5	3	7	21	63
199.5-209.5	4	2	8	32
209.5-219.5	5	1	5 ⎦	25
		425	-117	965

$$\tilde{x} = \frac{-117}{425} = -.275$$

$$\bar{x} = 164.5 - [(.275)(10)] = 161.75$$

$$s^2 = \frac{965 - \frac{(-117)^2}{425}}{424} = 2.20$$

$$S = (\sqrt{2.20})(10) = 14.8$$

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2. Calculation of Chi Square Test for Goodness of Fit to a Normal
   Distribution [1]

Interval	$\dfrac{I - \bar{X}}{S}$	Relative Freq.	Theoretical Freq. F[2]	Actual Freq. f[3]	$\dfrac{(f - F)^2}{F}$
119.5 or less	-2.85	.0022	0.9 ⎤ 6.2	1 ⎤ 4	.780
119.5-129.5	-2.18	.0147	5.3 ⎦	3 ⎦	
129.5-139.5	-1.50	.0668	22.2	20	.218
139.5-149.5	-0.83	.2000	56.6	67	1.910
149.5-159.5	-0.15	.4400	102.0	95	0.480
159.5-169.5	0.52	.7100	114.8	111	0.127
169.5-179.5	1.20	.8849	74.3	81	.604
179.5-189.5	1.88	.9640	33.6	37	.345
189.5-199.5	2.56	.9940	12.8 ⎤	7 ⎤	1.835
199.5-209.5	3.23	.9992	2.1 ⎥ 15.3	2 ⎥ -10	
above 209.5			.4 ⎦	1 ⎦	
			$\overline{425}$		$\overline{6.299}$

1] The letter I is the upper value of each class interval

2] The theoretical frequency is obtained by multiplying the relative
   frequency for each interval by 425 and subtracting from this
   result the theoretical frequency for the preceding interval.
   For example, for the 119.5-129.5 interval, multiply
   $(.0147)(425) = 6.2$. Subtract 0.9 from 6.2 to obtain 5.3, as
   shown above.

3] The theoretical frequency for any interval should not be less
   than 5.0. χ^2 (.95), (5) = 11.07

Since $11.07 > 6.299$, there is no reason to reject the assumption
that the data represent a normal distribution at 95 percent
significance level with 5 degrees of freedom.

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3. Calculation of Point Estimate of Reliability

$$k = \frac{\bar{x} - L}{S} = \frac{161.75 - 140}{14.8} = 1.47$$

$$\hat{R} = 92.9\% \text{ (From Tables)}$$

4. Reliability at 95 Percent Confidence Level (One-Sided Lower Limit)

   Method 1

$$K_{RL} = k - A$$

$$A = K(1 - \gamma)\left[\frac{1}{n} + \frac{k^2}{2n - 2}\right]^{\frac{1}{2}}$$

At 95 Percent Confidence Level

$$K(1 - \gamma) = K(1 - .95) = K_{.05} = 1.645 \text{ (Tables)}$$

$$n = 425$$

$$k = 1.47$$

$$A = 1.645\left[\frac{1}{425} + \frac{(1.47)^2}{848}\right]^{\frac{1}{2}} = 0.115$$

$$K_{RL} = 1.47 - 0.115 = 1.355$$

$$RL = 91.5\% \text{ (From Tables)}$$

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Method 2

$$k^2a - 2kK_\alpha + b = 0$$

$$k = 1.47$$

$$a = 1 - \frac{(1.645)^2}{2(424)} = .99687$$

$$b = K^2_\alpha - .006$$

$$(1.47)^2 (.99687) - 2(1.47) K_\alpha + K^2_\alpha - .006 = 0$$

$$2.154 - 2.94 K_\alpha + K^2_\alpha - .006 = 0$$

$$K^2_\alpha - 2.94 K_\alpha + 2.148 = 0$$

$$K_\alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$K_\alpha = \frac{2.94 \pm \sqrt{(2.94)^2 - 4(1)(2.148)}}{2}$$

$$K_\alpha = 1.355$$

$$\hat{R} = 91.5\% \text{ (Tables)}$$

APPENDIX B

WARP BREAKING STRENGTH

1. Calculation of Mean and Standard Deviation

	Coded Score (x)	f	fx	fx^2
159.5-169.5	-4	8	-32	128
169.5-179.5	-3	8	-24	72
			-216	
179.5-189.5	-2	45	-90	180
189.5-199.5	-1	70	-70	70
199.5- 204.5 -209.5 0		99	0	0
209.5-219.5	1	70	70	70
219.5-229.5	2	72	144	288
229.5-239.5	3	32	96 +401	288
239.5-249.5	4	14	56	244
249.5-259.5	5	7	35	175
		425	185	1495

$$\hat{x} = \frac{185}{425} = .435$$

$$\bar{x} = 204.5 + (.435)(10) = 208.85$$

$$s^2 = \frac{1495 - \frac{(185)^2}{425}}{424} = 3.33$$

$$S = (\sqrt{3.33})(10) = 18.2$$

2. Calculation of Chi Square Test for Goodness of Fit to Normal
   Distribution

Interval	$\dfrac{I - \bar{x}}{S}$	Relative Freq.	Theoretical Freq. F^2	Actual Freq. f^3	$\dfrac{(f-F)^2}{F}$
169.5 or less	-2.16	.0160	6.8	8	0.212
169.5-179.5	-1.61	.0548	16.4	8	4.300
179.5-189.5	-1.06	.1469	39.1	45	0.891
189.5-199.5	-0.51	.3085	68.7	70	0.024
199.5-209.5	0.03	.5120	86.5	99	1.807
209.5-219.5	0.58	.7188	87.9	70	3.648
219.5-229.5	1.13	.8690	63.8	72	1.053
229.5-239.5	1.68	.9534	35.9	32	0.446
239.5-249.5	2.23	.9870	14.2	14	0.003
above 249.5	2.78	.9972	5.7	7	0.295
			425		12.679

$$\chi^2_{.95(7)} = 14.07$$

$$14.07 > 12.68$$

Since $14.07 > 12.68$, there is no reason to reject the assumption
that the data represent a normal distribution at 95 percent
significance level with 7 degrees of freedom.

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3. Calculation of Point Estimate of Reliability

$$k = \frac{\overline{X} - L}{S} = \frac{208.85 - 150}{18.2} = 3.230$$

$\hat{R}$ (Tables) = 99.9%

Method 1

$$K_{RL} = k - A$$

$$A = K_{(1 -)} \left[\frac{1}{n} + \frac{k^2}{2n - 2} \right]^{\frac{1}{2}}$$

At 95 Percent Confidence Level

$$K_{(1 - \gamma)} = K_{(1 - .95)} = K_{.05} = 1.645 \text{ (From Tables)}$$

$$n = 425$$

$$k = 3.23$$

$$A = 1.645 \left[\frac{1}{425} + \frac{(3.23)^2}{848} \right]^{\frac{1}{2}} = 0.199$$

$$K_{RL} = 3.230 - 0.199 = 3.031$$

$$RL = 99.9\% \text{ (From Tables)}$$

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Method 2

$$k^2 a - 2kK_\alpha + b = 0$$

$$k = 3.23$$

$$a = 1 - \frac{(1.645)^2}{2(424)} = .99687$$

$$b = K_\alpha^2 - .006$$

$$(3.23)^2 (.99687) - 2(3.23) K_\alpha + K_\alpha^2 - .006 = 0$$

$$10.400 - 6.46 K_\alpha + K_\alpha^2 - .006 = 0$$

$$K_\alpha^2 - 6.46 K_\alpha + 10.394 = 0$$

$$K_\alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$K_\alpha = \frac{6.46 \pm \sqrt{(6.46)^2 - 4(1)(10.394)}}{2}$$

$$K_\alpha = 3.032$$

$$\hat{R} = 99.9\% \quad \text{(Tables)}$$

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DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)	2a. REPORT SECURITY CLASSIFICATION
U. S. Army Natick Laboratories Natick, Massachusetts 01760	UNCLASSIFIED
	2b. GROUP

3. REPORT TITLE

RELIABILITY OF SOME CRITICAL PERFORMANCE CHARACTERISTICS OF COTTON DUCK FABRIC

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Evaluation Report, June 1967 - May 1968

5. AUTHOR(S) (First name, middle initial, last name)

Clarence J. Pope and Stanley J. Werkowski

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
September 1968	18	4

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO. 1T024401A329	69-30-CM
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	TS-158

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13. ABSTRACT

Reliability analyses are made on physical characteristics of a cotton duck material used in the construction of military tentage and equipage items. The breaking strength in both the lengthwise (warp) direction and crosswise (filling) direction and also the water resistance property were selected as critical characteristics for analyses. The reliability analyses relate to a commodity situation where these techniques may be useful in assessing how well characteristics of a duck material meet established requirements. The Chi Square (x^2) test for goodness of fit was used to test normality of the distribution of laboratory data for each of the characteristics evaluated. Different methods are used to obtain one-sided lower reliability limits. Point estimates of reliability are also computed. Results of the different methods are compared, and the relative advantages of these methods are discussed.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Testing	8					
Physical Properties	8,9		9			
Cotton	9		9			
Duck	9		9			
Tents	9		9			
Armed Forces Equipment	4		4			
Warp Ends	9		9			
Filling Yarns	9		9			
Breaks	8					
Reliability	8		9			
Strength	8		9			
Analysis	8		9			
Parameters	8		8			