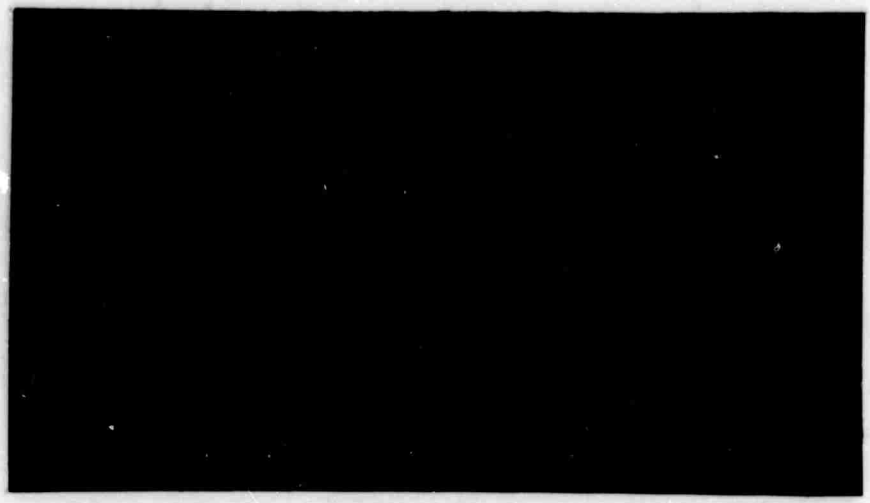
AFOSR 68-2158

AD 672706



1. This document has been approved for public release and sale; its distribution is unlimited.

THE SHUFORD-MASSENGILL CORPORATION

THE RELATIVE EFFECTIVENESS OF FIVE INSTRUCTIONAL STRATEGIES

EMIR H. SHUFORD, Jr. and H. EDWARD MASSENGILL

AN INTERIM TECHNICAL REPORT OF THE WORK PERFORMED UNDER CONTRACT NUMBER AF 49 (638)-1744, ARPA ORDER NUMBER 833, BY THE SHUFORD – MASSENGILL CORPORATION, P.O. BOX 26, LEXINGTON, MASSACHUSETTS, 02173.

FOREWORD

This is one in a continuing series of papers concerned with the theory and application of admissible confidence measurement techniques and one of a sub-series of papers concerned with the effects of guessing on the interpretation and use of objective test results in instructional settings. The research reported in this paper, prepared for the 1967 Meeting of The National Society for Programmed Instruction, was performed in support of the United States Air Force Office of Scientific Research contract number AF 49(638)-1744 sponsored by The Advanced Research Projects Agency of the Department of Defense (ARPA order number 833).

ABSTRACT

Logic and mathematics are used to gain some insight into the effective and efficient application of instruction.

A pupil's state of knowledge is represented by the degree of confidence he possesses in the subject matter. A cost, gain, and return from instruction are associated with each possible initial degree of confidence and each instructional sequence. Two group strategies, two individualized strategies, and a precisely-tailored instructional strategy are compared on the basis of expected return from instruction per individual for seven distributions of initial knowledge.

The relative effectiveness of instruction is found to depend critically upon the distribution of initial knowledge for the class of pupils.

The group and individualized strategies are rather comparable in performance with individualized instruction excelling only when the class is maximally heterogeneous.

Except for the rare instances in which the class is predominantly misinformed, precisely-tailored instruction yields large gains over any other instructional strategy. Thus, it appears that precisely-tailored instruction based on admissible probability measurement is the only strategy of the five that promises truly remarkable improvement in the effectiveness of teaching.

INTRODUCTION

There are several approaches to estimating the effectiveness of instruction. The armchair approach employs intuition and philosophy and results in the many speeches extolling the virtues of individualized instruction. The empirical approach employs data gathering and statistics to generate the many experimental reports from which, unfortunately, studies can be selected to prove the superiority or lack of superiority of almost any instructional procedure.

There is another approach: a way of using logic and mathematics to gain some insight into the effective and efficient application of instruction. By this means, conditions and relations may be specified and the logical consequences of this structure may be explored (Toda & Shuford, 1965). The results of such an analysis are valid to the extent that the conditions and relations are approximated in a real instructional setting (Massengill, 1964). This is the approach we shall use here.

STATE OF KNOWLEDGE

A pupil's state of knowledge can be represented by the degree of confidence he possesses in the subject matter. This degree of confidence can range from a state of being thoroughly misinformed through a state of being uninformed and up to a state of being completely informed.

A pupil's degree of confidence is coarsely reflected by his answer to an objective test question based on the subject matter. More recently, however, it has become possible to measure directly and unambiguously the pupil's degree of confidence by using one of the new admissible probability measurement procedures (Shuford, Albert & Massengill, 1966). These issues and techniques will be discussed in some detail by Edward Massengill on Saturday morning (Massengill & Shuford, 1967).

COST, GAIN, AND RETURN FROM INSTRUCTION

Now, suppose that for each different state of knowledge, p, we can make available an instructional sequence, S(p), precisely tailored to that particular state of knowledge. The application of such a sequence would be just sufficient to raise a pupil's level of knowledge from its initial state up to the level of complete certainty in the correct concept.

The lower the initial level of the pupil's knowledge, the more training would be required to bring this pupil up to a level of complete mastery of the material. For example, a misinformed pupil will be more difficult to instruct than an uninformed pupil. More training and greater difficulty of instruction usually means more time, both for the pupil and the teacher, invested in learning the subject matter. This time should be viewed as a cost, at least in the sense that it is time that could be spent learning other subject matter. Thus, we have the *cost* of application of precisely-tailored instruc-

tional sequences decreasing as the initial level of knowledge [illegible]. Since no instruction is required if the pupil has completely mastered the material, the *cost* of instruction must decrease to zero at this point. For simplicity and ease of computation we assume that the relation between *cost* and state of knowledge is linear as shown in Figure 1.

Now consider the *gain* resulting from application of a precisely-tailored instructional sequence. Remember that the result of instruction will be a pupil with complete mastery of the material. Given that the material is in the curriculum, such a state of knowledge would be more desirable than that of the student who is uninformed which is in turn a more desirable state than being completely misinformed. The uninformed pupil will know that he does not know while the misinformed student has complete confidence in a wrong notion. Thus, the greatest *gain* will result from bringing a maximally misinformed student up to a level of complete mastery and the *gain* from instruction will decline down to a level of zero for the pupil who already has complete mastery of the material. Again for the sake of simplicity and computational ease we assume that the relation between *gain* and state of knowledge is linear as is shown in Figure 1.

The cost must, of course, be subtracted from the gain from instruction to get the net *return* from instruction as shown in Figure 1. The *cost*, *gain* and *return* shown here are those values used for the computations given in Shuford & Massengill (1966) and reported below. The gain from instruction is twice the cost of instruction and so yields a positive net return. This two-to-one ratio is arbitrary. However, for those instances in which the gain is greater than the cost the results described below will probably not change too much. But in any application of considerable importance, it would be worthwhile to try to determine approximate values for cost and gain and to carry through the computations for a full analysis appropriate to that situation.

The cost, gain and return functions considered thus far have assumed that the "right" instructional sequence was applied to each pupil. What happens if the instructional sequence does not match the pupil's level of knowledge? Three such instances are shown in Figure 1. Consider what happens if a precisely-tailored instructional sequence for a pupil who is completely misinformed, S(0), were used with pupils of varying levels of knowledge. As a first approximation, the cost would remain the same but the return would decline for increasing levels of knowledge as shown in Figure 1. So, for an uninformed pupil the return would be zero and then would continue to decline into the negative or cost region as state of knowledge increases. The dashed line represents the net return that could have been obtained by correctly matching the instructional sequence to the pupil's level of knowledge.

The net return for the case in which the instructional sequence appropriate for a completely uninformed pupil, S(1/2), is used for all pupils is also shown in Figure 1. The return is maximal when the pupil is in fact completely uninformed but the return function then declines to negative values for both well informed and misinformed pupils.

And finally, the return function is also shown in Figure 1 for the case in which the instructional sequence appropriate for a pupil moderately well informed, S(3/4), is used for all pupils. As before, the maximal return occurs when the pupil is in fact moderately well informed and the return falls off and becomes negative to each side of this value. Notice that for lower levels of knowledge the return function declines down to a minimal level corresponding to the cost of applying the instructional sequence and then stays there. This comes from our assuming that no gain will result from the use of such an instructional sequence with a misinformed pupil since it will not serve to disabuse him of his wrong notions. These are just three of the very large number of net return functions, one for each possible precisely-tailored instructional sequence, and they can be represented in general mathematical notation as shown in Shuford & Massengill (1966).

The functions illustrated here and used in the computations reported below are not the only ones possible. They do however, seem to be a first approximation to those sometimes encountered in practice. This means that our results can not be expected to hold for all instructional settings but they most likely hold for some, e.g., those for which these types of cost, gain and return functions are a fair representation of the values implicit in the instructional setting.

DISTRIBUTION OF KNOWLEDGE

Consider now the problem of teaching a class of pupils. Some of the pupils will be uninformed, others will be well informed, while others might be misinformed. Their level of knowledge might range over the complete range. If we measured their initial levels of knowledge, we could tabulate a frequency distribution for the class. Such a frequency distribution can and will be approximated by a continuous distribution as described in Shuford & Massengill (1966). Such a distribution also admits to another interpretation. It can represent our uncertainty about the knowledge level of a particular pupil that we are teaching. Three such distributions are shown in Figure 2.

INSTRUCTIONAL STRATEGY

Knowing the distributions of levels of knowledge among the pupils in the class, we are still faced with the problem of what instructional strategy to use. Five instructional strategies seem to be of particular interest. Two of these are types of group instruction, two are types of individualized instruction, while the fifth is the type of highly individualized instruction heretofore rarely encountered in practice.

I. Group instruction at one of three levels. The teacher must choose between (a) the instructional sequence tailored for a completely misinformed pupil, (b) the instructional sequence for a completely uninformed pupil, and (c) the instructional sequence for a completely informed pupil. One of these sequences will be adopted and used for the whole class. The teacher will adopt

that sequence which yields the highest net return given the distribution of knowledge levels in the class (see below). This is group instruction in the sense that every member of the class is exposed to exactly the same instructional material.

II. Group instruction at an average level. The teacher chooses that instructional sequence which matches the mean level of ability of the class and applies it to all pupils in the class. [This level of instructional effectiveness may be approached by the evolutionary development of instructional material such as good textbooks, programmed instruction and computer-assisted instruction.]

III. Individualized instruction based on choice testing. As in Strategy I. the teacher may choose between the three instructional sequences but in this case, the choice may be made for each and every pupil, i.e., not all the pupils in the class have to endure the same instructional sequence. The choice among the instructional sequences is based upon a performance or choice test which gives a crude indication of the pupil's state of knowledge. More explicitly, if the pupil provides the correct answer then the teacher knows that he is *not misinformed* while if the pupil provides the wrong answer, then the teacher knows that he is *not informed*. On the basis of this information, the teacher can to a certain extent tailor the instruction to the pupil. [This instructional strategy seems to correspond to what most people mean when they talk about individualized instruction and is approximated by branching programmed instructional materials and "individualized" computer-assisted instructional programs.]

IV. Individualized instruction based on admissible choice testing. Here again the teacher chooses from among the three instructional strategies. The difference being that the pupils have been given an admissible choice test to assess their initial levels of knowledge. An admissible choice test (Massengill & Shuford, 1967) will give a rough indication of whether the pupil is *misinformed, uninformed* or *well informed* and thus, by eliminating the effects of guessing, is somewhat more useful than a choice test.

V. Precisely-tailored instruction based upon admissible probability measurement. Here the teacher can choose from among a large number of precisely-tailored instructional sequences. Furthermore, by using admissible probability measurement to assess a pupil's initial level of knowledge, the teacher can match the instructional sequence to the pupil's level of knowledge for each and every pupil in the class. There will be a perfect match between the pupils initial level of knowledge and the instructional sequence to which he is exposed. [Since the admissible probability measurement procedures are so new, this instructional strategy has probably never been attempted on a formal basis. It is, however, conceivable that this situation is approximated by the self-study of a very wise student who knows his own level of

knowledge and how to learn efficiently.]

EXPECTED RETURN FROM INSTRUCTION

The effectiveness of instruction is, of course, determined by computing the average return from instruction given the initial distribution of knowledge for the class. That instructional sequence which yields the best return is the optimal course of action which should of course be chosen. This quantity of expected return per individual from the optimal course of action has been computed for each of the five instructional strategies and for each of seven distributions of knowledge as discussed below.

Figure 2 shows the results for three symmetric distributions of knowledge. The first, uniform, distribution represents a class quite heterogeneous with respect to its initial knowledge of the subject matter to be taught. This distribution can also be interpreted as appropriate to the case in which the teacher has no information whatsoever about the knowledge level of an individual pupil. In any case, such a distribution is hardly likely ever to arise in practice. Its interest is largely theoretical in that it yields results analogous to those that are obtained when one does not take into account the initial knowledge level of the pupil. Adjacent to this distribution in Figure 2 is displayed the relative effectiveness of each of the five instructional strategies. Notice that the two methods of group instruction (I, II) yield zero returns while individualized instruction guided by choice testing (III) yields 50% and individualized instruction based on admissible choice testing (IV) yields about 62% of the return obtainable from precisely-tailored instruction (V). For this distribution, individualized instruction is clearly superior to group instruction. So, if such a freakish distribution of knowledge were ever encountered in practice, the obvious choice would be an individualized instructional strategy.

The second distribution in Figure 2 represents a class which is relatively ignorant of the subject matter to be taught. A few of the pupils are misinformed and a few of the pupils are somewhat informed, but most of them have very little knowledge of the subject matter. Such a distribution may be fairly typical of many classroom situations, but look at what happens to the relative effectiveness of the instructional strategies. The two group strategies (I, II) now yield positive net returns in the vicinity of 40% of the maximum possible while the two individualized strategies are yielding 50% (III) and about 56% (IV) of the maximum. Precisely-tailored instruction is maintaining a superiority over any of the other methods. The difference in the performance of group and individualized methods is disappearing. When one considers that individualized instruction may be more costly to put into effect than either of the group strategies and realizes that these costs should be subtracted from the total relative effectiveness, then it may very well be that in many instances individualized instruction is actually inferior to group instruction. It would just depend upon the relative costs of application.

The final distribution found in Figure 2 is for a class which is predominately

ignorant of the subject matter to be taught. Essentially none of the students are well informed and none of the students are misinformed. Such a distribution is fairly typical of the initial study of a foreign language and of other "new" topics. Also, such a distribution could result from an initial selection procedure which divides a group of pupils into several homogeneous classes. With this distribution, the relative effectiveness of both group and individualized instruction has risen to about 75% of the net return obtainable from precisely-tailored instruction. Which of the first four strategies should be used clearly depends upon the costs involved.

The distribution shown at the top of Figure 3 represents a class which is relatively misinformed. Such a distribution may be fairly typical of situations encountered in remedial training programs and retraining of workers in new techniques. Also, this kind of distribution can be encountered as a result of an initial selection process. Notice that the expected return per individual is greater than before. This is so because many of the pupils are misinformed and there is more to be gained from instruction. This is reflected in the effectiveness measures for the five instructional strategies.

The distribution shown at the bottom of Figure 3 represents a class which is predominately misinformed. Though this is a relatively rare situation, it may be characteristic of the rehabilitation of brain-washed individuals, e.g., teaching Bayesian statistics to classical statisticians. Here the return from instruction is even greater and the differences between the five strategies have almost disappeared. This equalization has occurred because all five strategies essentially recommend the same thing: treat all pupils as though they were completely misinformed.

The distribution shown at the top of Figure 4 represents a class which is relatively informed with respect to the subject matter to be taught. Such a distribution is fairly typical of supplementary training and also could be the result of an initial selection process. Here there arise considerable differences within the group and individualized strategies. Group instruction at an average level (II) is superior to group instruction at one of three levels (I) while individualized instruction based on admissible choice testing (IV) is quite superior to individualized instruction based on choice testing (III), precisely-tailored instruction (V) maintains a large superiority over all the other methods though the total return from instruction is less in this situation due to the fact that many of the pupil's are moderately well informed.

The distribution shown at the bottom of Figure 4 represents a class which is predominately well informed as to the subject matter to be taught. This situation is not too frequently encountered in regular educational programs. However, it is quite typical of refresher training for the maintainance of proficiency of critical skills. Here, the use of one of the group methods is actually detrimental while precisely-tailored instruction is the only method that yields any significant benefits whatsoever. Though the maximum possible benefit is smaller on our uniform utility scale this type of training can actually be quite critical and important.

SUMMARY AND CONCLUSIONS

We have used logic and mathematics to explore some of the factors effecting the value of group and individualized instruction. Some simple, but not too unrealistic, assumptions have been made about the nature of loss, gain and return from instruction. The results are applicable to those situations fairly represented by this value system.

The relative effectiveness of instruction depends upon the distribution of knowledge for the class of pupils. Though it is true that precisely-tailored instruction is the only strategy whose performance is never bettered by another strategy, other comparative statements must be qualified by referring to this distribution of knowledge.

The group strategies are rather comparable in performance. Teaching for the average pupil is somewhat better, however, when the class is relatively informed but is poorer than teaching at one of three levels when the class is relatively ignorant and when the class is predominantly well informed.

The individualized strategies are rather comparable in performance. Individualized instruction based on using admissible choice testing to detect the uninformed pupil is superior to using conventional choice testing when the class is very heterogeneous and when the class is relatively informed.

Individualized instruction is markedly superior to group instruction *only* when the class is maximally heterogeneous. This superiority of individualized instruction tends to disappear under conditions usually encountered in practice.

Except for that rare instance in which the class is predominantly misinformed, precisely-tailored instruction yields large gains over any other instructional strategy. Thus, it appears that precisely-tailored instruction based on admissible probability measurement is the only strategy of the five that promises truly remarkable improvement in the effectiveness of teaching.

REFERENCES

Massengill, H. E. (1964) Purposive systems: theory and application. ESD-TDR-64-531, Electronic Systems Division, L. G. Hanscom Field, Bedford, Massachusetts.

Massengill, H. E. & Shuford, E. H., Jr. (1967) *What pupils and teachers should know about guessing.* Lexington, Massachusetts: The Shuford-Massengill Corporation.

Shuford, E. H., Jr., Albert, A. & Massengill, H. E. (1966) Admissible probability measurement procedures. *Psychometrika, 31,* 125-145.

Shuford, E. H., Jr. & Massengill, H. E. (1966) *The worth of individualizing instruction.* Lexington, Massachusetts: The Shuford-Massengill Corporation.

Toda, M. & Shuford, E. H., Jr. (1965) Logic of systems: Introduction to the formal theory of structure. In *General systems theory yearbook*; 1965.

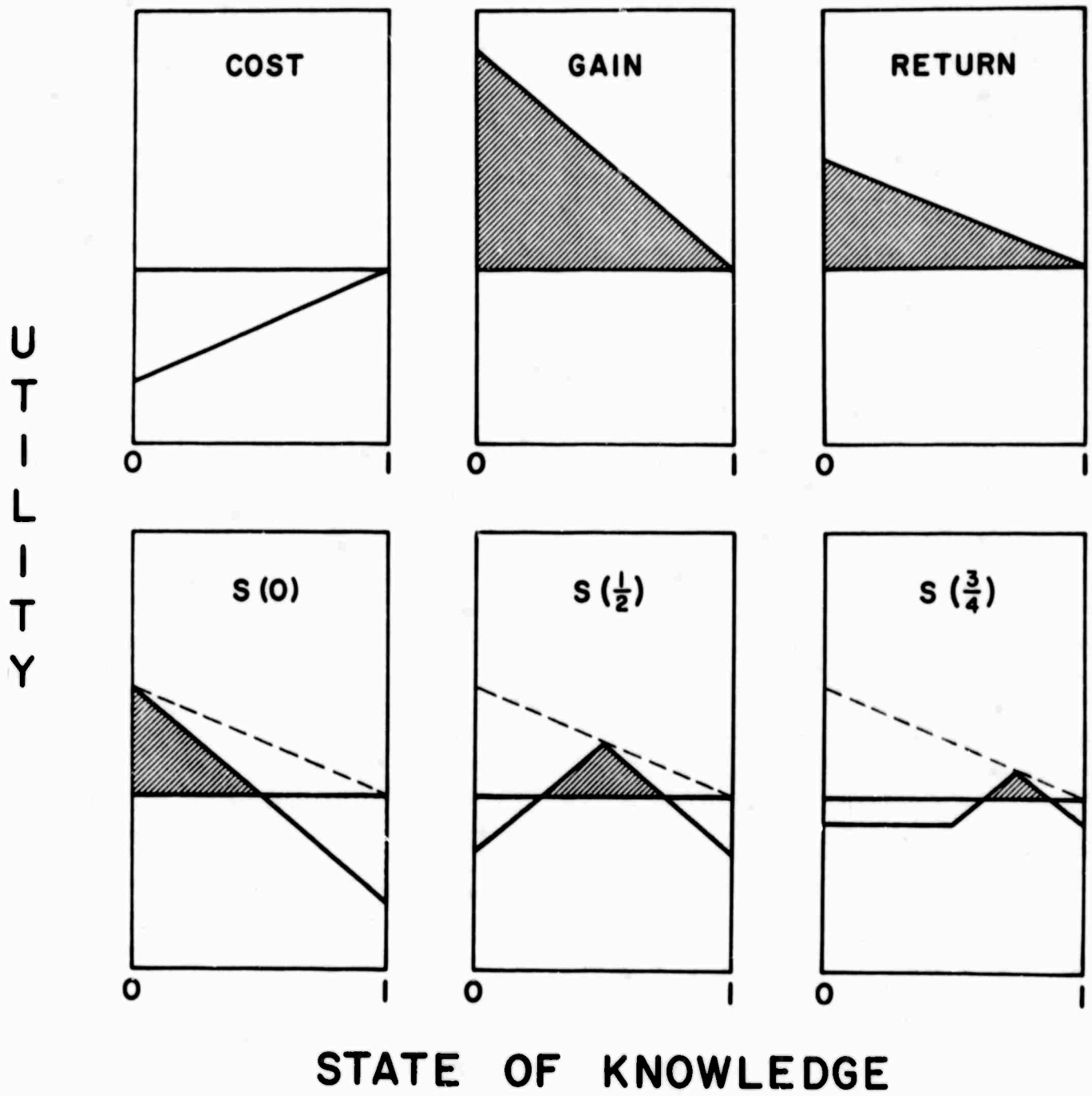


Figure 1

COST, GAIN, AND RETURN FUNCTIONS ON A COMMON UTILITY SCALE

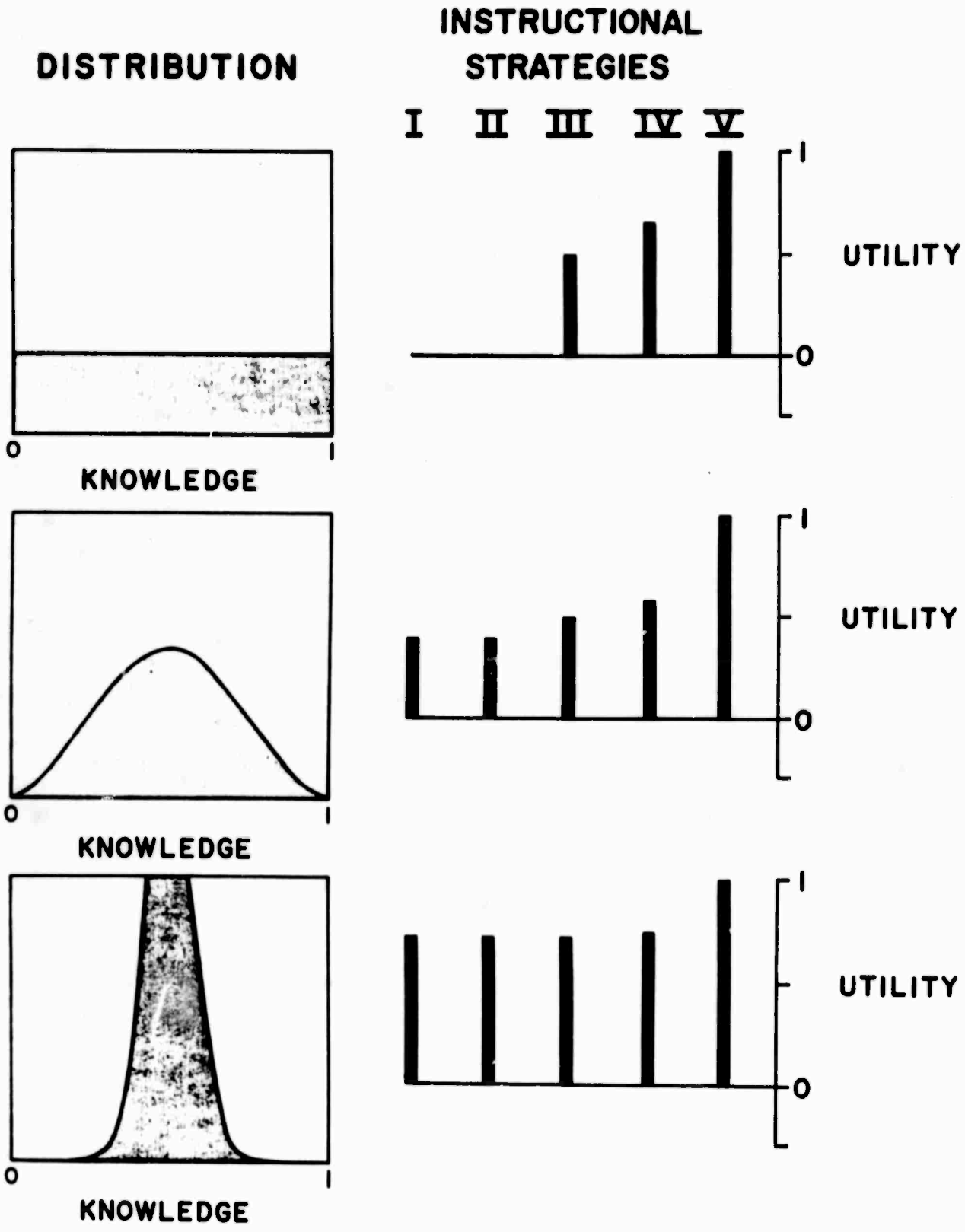


Figure 2

THREE SYMMETRIC DISTRIBUTIONS OF KNOWLEDGE AND THE CORRESPONDING EXPECTED RETURN PER INDIVIDUAL

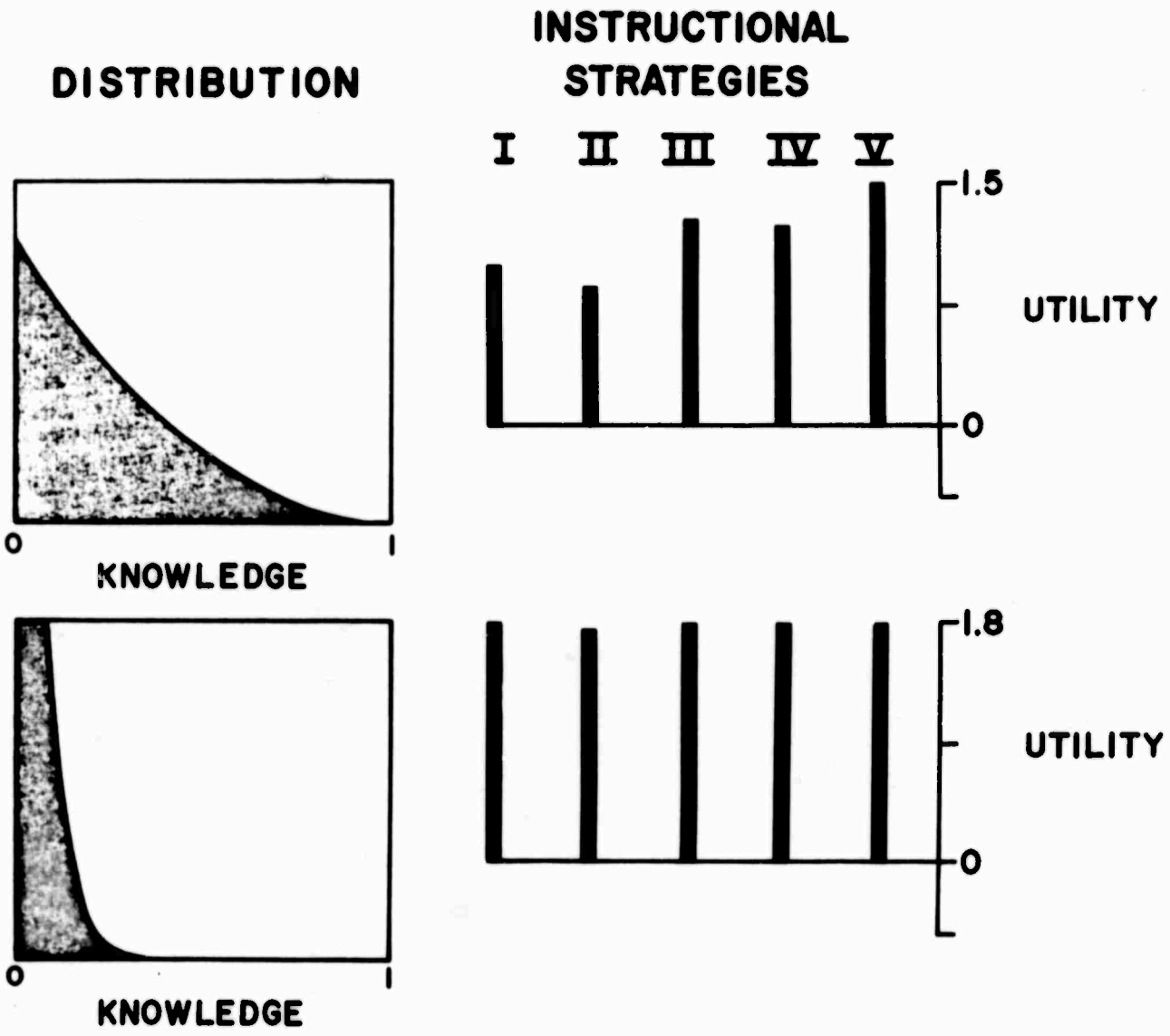


Figure 3

TWO DISTRIBUTIONS FOR MISINFORMED CLASSES

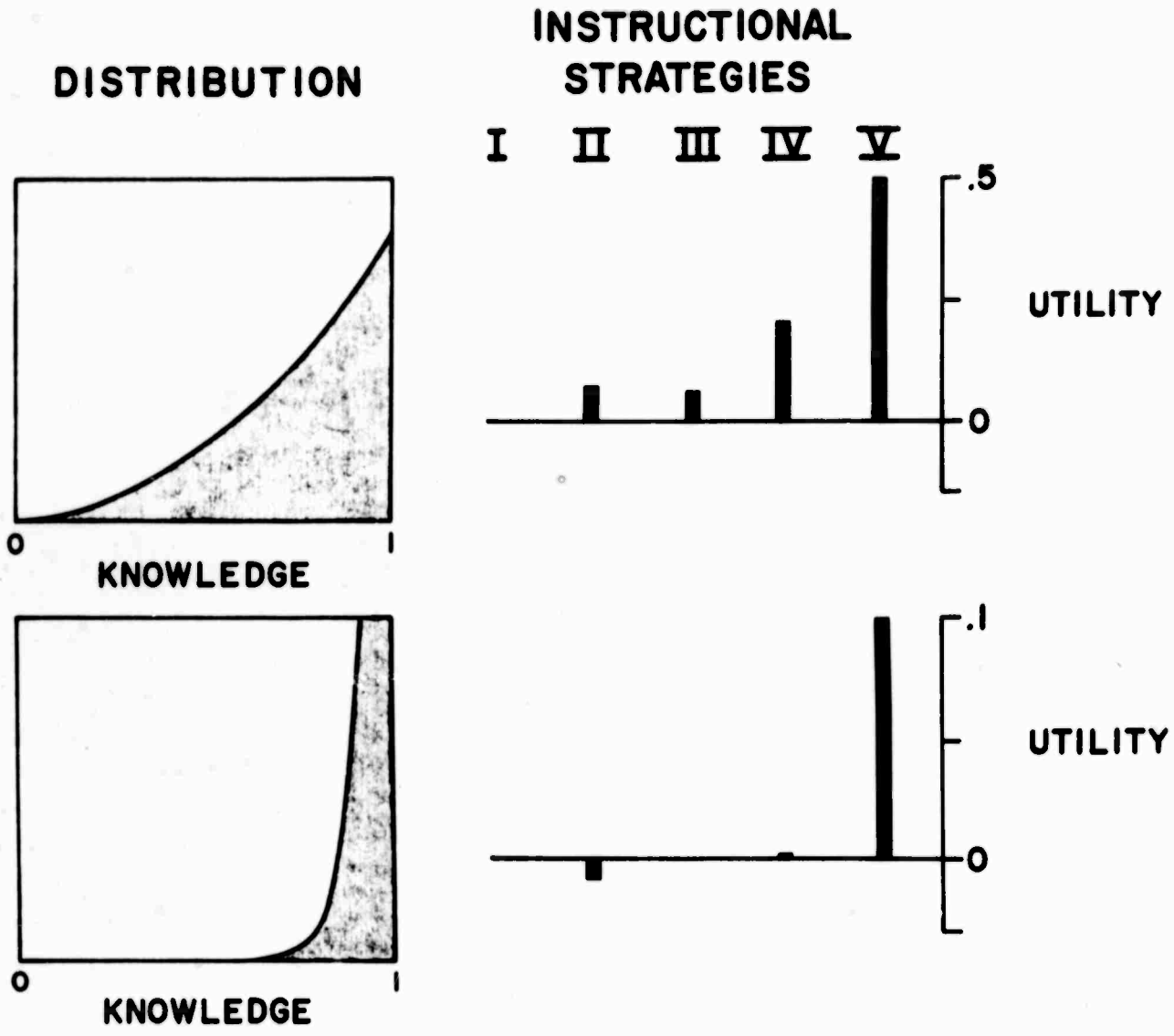


Figure 4

TWO DISTRIBUTIONS FOR WELL INFORMED CLASSES

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified

ORIGINATING ACTIVITY (Corporate author)

The Shuford-Massengill Corporation
Box 26
Lexington, Massachusetts 02173

2a. REPORT SECURITY CLASSIFICATION

UNCLASSIFIED

2b. GROUP

REPORT TITLE

THE RELATIVE EFFECTIVENESS OF FIVE INSTRUCTIONAL STRATEGIES

DESCRIPTIVE NOTES (Type of report and inclusive dates)

Scientific Interim

AUTHOR(S) (First name, middle initial, last name)

Emir H. Shuford, Jr. & H. Edward Massengill

REPORT DATE	7a. TOTAL NO OF PAGES	7b. NO OF REFS
June 1967	12	5

8a. CONTRACT OR GRANT NO	9a. ORIGINATOR'S REPORT NUMBER(S)
AF 49(638)-1744	SMC R-8
b. PROJECT NO 920F-9719 c. 6154501R d. 681313	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) AFOSR 68-2158

10. AVAILABILITY/LIMITATION NOTICES

1. This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES

TECH, OTHER

12. SPONSORING MILITARY ACTIVITY

Air Force Office of Scientific Research (SRLB)
1400 Wilson Boulevard
Arlington, Virginia 22209

ABSTRACT

Logic and mathematics are used to gain some insight into the effective and efficient application of instruction.

A pupil's state of knowledge is represented by the degree of confidence he possesses in the subject matter. A cost, gain, and return from instruction are associated with each possible initial degree of confidence and each instructional sequence. Two group strategies, two individualized strategies, and a precisely-tailored instructional strategy are compared on the basis of expected return from instruction per individual for seven distributions of initial knowledge.

The relative effectiveness of instruction is found to depend critically upon the distribution of initial knowledge for the class of pupils.

The group and individualized strategies are rather comparable in performance with individualized instruction excelling only when the class is maximally heterogeneous.

Except for the rare instances in which the class is predominantly misinformed, precisely-tailored instruction yields large gains over any other instructional strategy. Thus, it appears that precisely-tailored instruction based on admissible probability measurement is the only strategy of the five that promises truly remarkable improvement in the effectiveness of teaching.

DD FORM 1473

UNCLASSIFIED

Security Classification

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Instruction						
degree of confidence						
admissible probability measurement						
class						
pupil						