AIR TECHNICAL INTELLIGENCE TRANSLATION

(Title Unclassified)
INTERIOR BALLISTICS

by

M. E. Serevryakov

State Printing House of the Defense Industry

Moscow, 1949, 2nd Edition

672 Pages

(Part 8 of 10 Parts, Pages 698-776)

AIR TECHNICAL INTELLIGENCE CENTER
WRIGHT-PATTERSON AIR FORCE BASE
OHIO

MAD-83064
F-TS-7327/V

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SECTION EIGHT - EMPIRICAL METHODS OF SOLUTION

Even with a certain schematization in the assumptions made, the analytical solution of problems in internal ballistics leads to rather complex correlations, which require time-consuming and complicated computations to obtain pressure and velocity curves. For this reason, many investigators have approached the solution of problems in internal ballistics either on the basis of simple algebraic correlations with coefficients determined by reference to experimental data or on the basis of very simple tables or formulas resulting from the treatment of experimental data obtained in firing tests.

Such simple formulas and tables, which leave out of account the great complexity of the phenomenon of the shot, and which coordinate their data with experiment with the aid of certain coefficients, form the basis of empirical methods of solution.

We shall briefly consider some of the most widely known formulas and tables employed in practice.

CHAPTER 1 - MONOMIAL AND DIFFERENTIAL FORMULAS

1. MONOMIAL EMPIRICAL FORMULAS

Monomial formulas usually express the dependence of the initial velocity of the projectile and the maximum gas pressure upon various loading conditions. Like other empirical formulas, they were widely employed prior to the development of exact analytical methods and of tables derived on the basis of these methods.

Such formulas include the monomial formulas of N. A. Zabudsky, which are derived in his works "On the Pressure of Smokeless-Powder Gases in Gunbarrels" [7] and "On the Pressure of Powder Gases in the Bore of the Three-Inch Gun and on the Remaining Velocity" [8]

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on the basis of treatment of a large number of firing tests. These formulas are:

$$v_0 = H_1 \frac{\omega^{\frac{3}{4}}}{d_{km}^{\frac{1}{2}} \ell_0^{\frac{1}{4}} q^{\frac{5}{16}}} \quad ; \quad p_m = K_1 \frac{\omega^2 q^{\frac{3}{4}}}{d_{km}^2 \ell_0}$$

for the 3-inch, 4.2-inch, and 6-inch guns, and:

$$v_0 = H \frac{\Delta^{\frac{1}{4}} \omega^{\frac{1}{2}}}{(2e_1)^{\frac{1}{3}} q^{\frac{1}{4}}} \quad ; \quad p_m = K \frac{\Delta^{\frac{9}{10}} \omega^{\frac{9}{10}} q^{\frac{4}{5}}}{(2e_1)^{\frac{7}{5}}}$$

for the 1902 model 3-inch gun.

Here, H_1, H, K_1, and K are numerical coefficients, which are determined from the results of firing tests under known loading conditions.

Taking into account the influence of the increase in pressure p_m upon the change in the exponents, N. A. Zabudsky proposed that, at high pressures (2200 atm and higher at that time), the exponents of q and ℓ_0 be taken as unity instead of $\frac{4}{5}$ and $\frac{9}{10}$:

$$p_{max} = K \frac{\omega^{\frac{9}{5}} q}{(2e_1)^{\frac{7}{5}} \ell_0} \quad \text{or} \quad p_{max} = K \frac{\Delta \omega^{\frac{4}{5}} q}{(2e_1)^{\frac{7}{5}}} .$$

Closely related to monomial formulas are differential formulas. By taking the logarithm of a monomial formula and differentiating it term by term, it is possible to obtain a dependence of the relative change in initial velocity and of the maximum gas pressure upon the change in loading conditions.

For example:

$$\frac{dv_0}{v_0} = a_1 \frac{d\omega}{\omega} - a_2 \frac{dq}{q} - a_3 \frac{dl_0}{l_0} + \dots .$$

2. EMPIRICAL DIFFERENTIAL FORMULAS OF IKOPZ

Wide acceptance and practical use have been accorded in this country to the formulas of the Test Commission of the Okhta Powder Works (IKOPZ), which were derived empirically on the basis of a large number of firing tests conducted during the development and adoption into service of smokeless powders between 1895 and 1910. G. P. Kisnemsky and N. A. Zabudsky took an important part in these tests.

The IKOPZ differential formulas, which are also known as correction formulas, give the dependence of the relative change in maximum pressure and initial velocity upon changes in the weight of the charge, the thickness of the powder, the volume of the chamber, the weight of the projectile, and the volatile content and temperature of the powder in the following form:

$$\frac{\Delta p_m}{p_m} = 2\frac{\Delta\omega}{\omega} - \frac{4}{3}\frac{\Delta e_1}{e_1} - \frac{4}{3}\frac{\Delta W_0}{W_0} + \frac{3}{4}\frac{\Delta q}{q} - 0.15\ (\Delta H\%) + 0.0036\ (\Delta t^{o});$$

$$\frac{\Delta v_0}{v_0} = \frac{3}{4}\frac{\Delta\omega}{\omega} - \frac{1}{3}\frac{\Delta e_1}{e_1} - \frac{1}{3}\frac{\Delta W_0}{W_0} - \frac{2}{5}\frac{\Delta q}{q} - 0.04\ (\Delta H\%) + 0.0011(\Delta t^{o}).$$

If any one of the loading conditions does not change, its change is equal to zero, and the corresponding term in the right-hand part drops out; if only one factor changes, the right-hand part contains only one term, which characterizes the influence of this factor alone.

Coefficients in excess of unity show that the relative change in pressure is greater than the change in the given factor; coefficients smaller than unity show that the pressure or the velocity vary less than the given factor.

A plus sign indicates that the pressure and the velocity change in the same direction, i.e., increase or decrease as the factor increases or decreases; a minus sign indicates that p_m and $v_д$ change in the direction opposite to the direction of the change in the given factor.

Inspection of the formulas shows that changes in all factors affect the change in pressure much more than the change in the velocity of the projectile.

The formulas presented above find widespread practical use in the selection of the charge and thickness, in applying corrections for the volume of the crusher gage, and in firing at a powder temperature other than 15°C, which is considered to be normal, and to which the results of firing must be reduced in determining the initial velocity, since the firing tables are computed at $t = 15°C$.

Example 1. In firing a 1902 model 76-mm gun with an inserted crusher gage and at a powder temperature of +12°C, the following results were obtained: $p_m = 2380$ kg/cm^2 and $v_д = 593$ m/sec. To determine p_m and $v_д$ at $t = +15°C$, without an inserted crusher gage, with normal loading, if the volume of the chamber is $W_0 = 1654$ cm^3 and the volume of the crusher gage is $W_{cr.} = 35$ cm^3.

We shall consider $W_{cr.} = \Delta W_0$; consequently, the firing was conducted with a chamber volume $W'_0 = W_0 - \Delta W_0 = 1654 - 35 = 1619$ cm^3 and at $t = +12°C$.

Reduction to the normal chamber volume requires the following correction:

w_0 .1619

We introduce the following corrections:

$$\frac{\Delta p_m}{p_m} = -\frac{4}{3}\ 0.022 + 0.0036 \cdot 3 = -0.029 + 0.011 = -0.018 = -1.8\%;$$

$$\frac{\Delta v_0}{v_0} = -\frac{1}{3}\ 0.022 + 0.0011 \cdot 3 = -0.007 + 0.003 = -0.004 = -0.4\%.$$

Since all the coefficients are approximate, the corrections are also computed with a precision of only two significant figures.

Introduction of the corrections gives:

$$\Delta p_m = -0.018 \cdot 2380 = -43;\ p_m = 2380 - 43 = 2337.$$

or, rounded off to the nearest 5 kg

$$p_m = 2335\ \text{kg/cm}^2.$$

$$\Delta v_0 = -0.004 \cdot 593 = -2.4\ \text{m/sec};\ v_0 = 593 - 2.4 = 590.6\ \text{m/sec}.$$

The formulas presented above make it possible to solve not only direct, but also inverse problems, for example: by what amounts is it necessary to change the thickness of the powder and the weight of the charge in order that the pressure be changed by so many per cent and the initial velocity by so much; or by how many per cent is it necessary to change the volatile content of the powder in order that the pressure and velocity be changed by the required amounts if the weight of the charge is changed in a certain manner.

Example 2. In firing a 1910 model 107-mm gun, a regulation charge containing 2.050 kg of new powder gave a (regulation) $p_m = 2320$ kg/cm^2 and a velocity $v_0 = 570.5$ m/sec instead of $v_0 = 579$ m/sec, which was required in accordance with the technical conditions. The question is whether the charge can be corrected by changing the volatile content; and, since both the pressure and the velocity will change as a result of this, how should the charge be changed so as to retain the same pressure?

Consequently, the problem is to determine $\frac{\Delta\omega}{\omega}$ and $\Delta H\%$ under such conditions that $\frac{\Delta p_m}{p_m} = 0$ and $\frac{\Delta v_0}{v_0} = \frac{3.5}{570.5} = 0.015 = 1.5\%$.

We formulate two equations:

$$\frac{\Delta p_m}{p_m} = 2\,\frac{\Delta\omega}{\omega} - 0.15\ (\Delta H\%),\quad \frac{\Delta v_0}{v_0} = \frac{3}{4}\,\frac{\Delta\omega}{\omega} - 0.04\ (\Delta H\%).$$

By substituting the values $\frac{\Delta p_m}{p_m} = 0$ and $\frac{\Delta v_0}{v_0} = 0.015 = 1.5\%$, converting 0.15 and 0.04 into percentages (i.e., 15 and 4), and designating $\frac{\Delta\omega}{\omega} = x$ and $\Delta H\% = y$, we obtain:

$$0 = 2x - 15y;\quad 1.5 = \frac{3^2}{4}x - 4y.$$

Upon solving this system, we find

$$x = \frac{15}{2}\,y;$$

$$1.5 = \frac{3}{4}\,\frac{15}{2}y - 4y = \left(\frac{45}{8} - 4\right)y = \frac{13}{8}\,y;$$

$$y = \frac{3}{2}\,\frac{8}{13} = \frac{12}{13} \approx 0.9\%;\quad x = \frac{12}{13}\,\frac{15}{2} = \frac{90}{13} \approx 7\%.$$

Consequently, the volatile content ΔH must be increased by 0.9%, and the charge must be increased by 7%.

Example 3. By how many per cent is it necessary to change the thickness of the powder and the weight of the charge in order that the pressure remain unchanged and the velocity may be increased by 2%?

$$\frac{\Delta p_m}{p_m} = 0 = 2\,\frac{\Delta\omega}{\omega} - \frac{4}{3}\,\frac{\Delta e_1}{e_1}\ \text{or}\ 0 = 2x - \frac{4}{3}y;$$

$$\frac{\Delta v_0}{v_0} = 2\% = \frac{3}{4}\frac{\Delta\omega}{\omega} - \frac{1}{3}\frac{\Delta e_1}{e_1}, \quad 2 = \frac{3}{4}x - \frac{1}{3}y;$$

$$x = \frac{2}{3}y; \quad 2 = \frac{3}{4}\frac{2}{3}y - \frac{1}{3}y = \frac{1}{6}y;$$

$$y = 12\%; \quad x = \frac{2}{3}\,12 = 8\%.$$

Consequently, to satisfy the imposed requirements, the thickness of the powder must be increased by 12%, and the charge must be increased by 8%.

The expression $x = (2/3)y$ or $\Delta\omega/\omega = (2/3)(\Delta e_1/e_1)$ obtained from the first equation shows that, in order that the pressure remain unchanged, the thickness of the powder and the charge must be changed in such a manner that:

$$\frac{\Delta\omega}{\omega} = \frac{2}{3}\frac{\Delta e_1}{e_1}.$$

As is seen from the examples presented above, empirical differential formulas make it possible to solve very rapidly and simply many of the problems that are continually encountered in firing-ground or powder-works practice. It is only necessary to keep in mind that these formulas were originally derived for medium-power guns ($v_0 \approx$ 400-600 m/sec), and that, in individual cases, the coefficients may deviate in either direction from the average values given in the formulas. Nevertheless, these formulas are entirely suitable for estimates and tentative computations.

As has been shown by N. A. Zabudsky, the values of some coefficients of p_m and v_0 increase with increasing pressure.

The same is noted in the French literature, where the coefficients change as the density of loading increases. For example, in the formulas

$$\frac{\Delta p_m}{p_m} = m_\omega \frac{\Delta\omega}{\omega} \text{ and } \frac{\Delta v_0}{v_0} = \ell_\omega \frac{\Delta\omega}{\omega},$$

the coefficients

$$m_\omega = \frac{1}{1 - 0.9\Delta} \text{ and } \ell_\omega \approx \log 10\Delta,$$

i.e., it is thereby taken into account that both the pressure and the velocity change more sharply with increasing Δ. At $\Delta = 0.55$, $m_\omega \approx 2$, and $\ell_\omega = 0.74 \approx \frac{3}{4}$, i.e., the values of the coefficients coincide with the values of the coefficients of the Test Commission of the Okhta Powder Works.

3. CORRECTION FORMULAS AND TABLES OF PROFESSOR V.E. SLUKHOTSKY

The influence of the density of loading and of the relative length of the gun upon the coefficients of differential formulas has been considered in greater detail by V. E. Slukhotsky [9].

The correction formulas may be represented in the following form as functions of the parameters X:

$$\frac{\Delta p_m}{p_m} = m_x \frac{\Delta X}{X} \text{ and } \frac{\Delta v_Д}{v_Д} = \ell_x \frac{\Delta X}{X},$$

where m_x and ℓ_x are numerical coefficients, as in the IKOPZ formulas.

There are presented below excerpts from the tables of V. E. Slukhotsky. The coefficients m_x are presented for the maximum pressure p_m within the limits of 2000-4500 kg/cm^2 and for values of Δ in the range of 0.50-0.80 kg/dm^3.

	m_{I_K}				m_ω				m_f			
p_m \ Δ	0.5	0.6	0.7	0.8	0.5	0.6	0.7	0.8	0.5	0.6	0.7	0.8
2000	1.49	1.40	1.32	1.24	2.04	2.17	2.29	2.38	1.80	1.78	1.72	1.64
2500	1.50	1.46	1.40	1.33	2.14	2.28	2.43	2.57	1.81	1.81	1.76	1.67
3000	1.50	1.50	1.46	1.40	2.22	2.39	2.56	2.74	1.78	1.81	1.78	1.69
3500	1.45	1.51	1.50	1.44	2.30	2.49	2.69	2.90	1.73	1.78	1.78	1.70
4000	1.36	1.48	1.50	1.46	2.38	2.59	2.82	3.05	1.65	1.73	1.76	1.71
4500	1.24	1.42	1.48	1.47	2.45	2.69	2.94	3.19	1.58	1.68	1.74	1.71
	m_q				m_{W_0}				ℓ_{W_0}			
2000	0.69	0.73	0.76	0.78	1.36	1.45	1.52	1.59	$\Lambda_\ell = 4$	6	8	10
2500	0.72	0.78	0.81	0.83	1.48	1.58	1.67	1.74	0.34	0.23	0.16	0.14
3000	0.72	0.80	0.84	0.86	1.57	1.68	1.78	1.86				
3500	0.70	0.80	0.86	0.88	1.63	1.75	1.86	1.96				
4000	0.66	0.79	0.87	0.89	1.66	1.80	1.92	2.03				
4500	0.59	0.76	0.86	0.89	1.68	1.83	1.96	2.08				

The correction coefficients m_x and ℓ_x are given for the cases of corrections of the following quantities: I_K - pressure impulse of powder gases for the period of burning of the powder; ω - weight of the charge; f - propellant force of the powder; q - weight of the projectile; and W_0 - volume of the chamber.

Since the values of the coefficients ℓ_x depend not only upon p_m and Δ, but also upon the quantity $\Lambda_\ell = \ell_\ell/\ell_0$, tables for various Λ_ℓ have been formulated for determining the values of the coefficient ℓ_x. In the tables presented below, the values of 4, 6, 8, and 10 have been taken for Λ_ℓ. For each value of Λ_ℓ, there is given its own table of values of ℓ_x as a function of p_m and Δ (p_m 2000-4500 kg/cm^2, Δ = 0.5, 0.6, 0.7, and 0.8).

$\Lambda_{д}$		4				6				8				10			
	Δ / P_n	0.5	0.6	0.7	0.8	0.5	0.6	0.7	0.8	0.5	0.6	0.7	0.8	0.5	0.6	0.7	0.8
l_{I_K}	2000	0.38	0.55	—	—	0.30	0.45	0.49	—	0.25	0.38	0.46	—	0.22	0.33	0.46	—
	2500	0.24	0.39	0.53	—	0.18	0.29	0.44	0.48	0.16	0.26	0.37	0.46	0.14	0.22	0.32	[illegible]
	3000	0.17	0.28	0.41	0.50	0.12	0.21	0.32	0.46	0.10	0.17	0.27	0.39	0.09	0.15	0.23	[illegible]
	3500	0.12	0.20	0.31	0.43	0.09	0.15	0.23	0.35	0.07	0.12	0.19	0.29	0.07	0.11	0.17	[illegible]
	4000	0.09	0.15	0.23	0.33	0.07	0.11	0.17	0.25	0.06	0.09	0.14	0.21	0.05	0.08	0.13	[illegible]
	4500	0.07	0.12	0.18	0.26	0.05	0.09	0.13	0.18	0.05	0.08	0.11	0.15	0.04	0.07	0.10	[illegible]
l_ω	2000	0.86	0.97	—	—	0.76	0.87	0.95	—	0.73	0.83	0.92	—	0.72	0.80	0.89	[illegible]
	2500	0.76	0.86	0.97	—	0.68	0.77	0.86	0.92	0.66	0.73	0.81	0.88	0.65	0.71	0.77	[illegible]
	3000	0.68	0.77	0.86	0.94	0.63	0.69	0.75	0.82	0.61	0.66	0.71	0.77	0.60	0.65	0.69	[illegible]
	3500	0.63	0.70	0.77	0.84	0.59	0.63	0.68	0.73	0.58	0.61	0.65	0.62	0.56	0.60	0.63	[illegible]
	4000	0.60	0.65	0.71	0.76	0.56	0.59	0.63	0.66	0.55	0.58	0.60	0.62	0.54	0.56	0.58	[illegible]
	4500	0.58	0.62	0.67	0.71	0.54	0.56	0.59	0.62	0.53	0.55	0.57	0.58	0.52	0.54	0.55	[illegible]
l_r	2000	0.69	0.77	—	—	0.66	0.72	0.73	—	0.63	0.69	0.72	—	0.62	0.67	0.72	[illegible]
	2500	0.63	0.69	0.75	—	0.61	0.66	0.71	0.72	0.59	0.64	0.69	0.71	0.57	0.62	0.66	[illegible]
	3000	0.59	0.64	0.69	0.72	0.57	0.61	0.66	0.71	0.56	0.60	0.64	0.68	0.54	0.57	0.61	[illegible]
	3500	0.57	0.60	0.64	0.69	0.55	0.58	0.62	0.66	0.54	0.57	0.60	0.64	0.53	0.55	0.58	[illegible]
	4000	0.55	0.58	0.61	0.64	0.54	0.56	0.59	0.62	0.53	0.55	0.57	0.60	0.52	0.54	0.56	[illegible]
	4500	0.54	0.56	0.59	0.62	0.53	0.55	0.57	0.59	0.52	0.54	0.56	0.57	0.52	0.53	0.55	[illegible]
l_q	2000	0.28	0.18	—	—	0.32	0.26	0.19	—	0.34	0.29	0.21	—	0.36	0.31	0.26	0.21
	2500	0.34	0.29	0.20	—	0.37	0.32	0.27	0.22	0.39	0.34	0.29	0.23	0.40	0.36	0.31	0.26
	3000	0.38	0.33	0.28	0.22	0.40	0.36	0.32	0.27	0.42	0.38	0.34	0.29	0.43	0.39	0.35	0.30
	3500	0.41	0.37	0.33	0.28	0.42	0.39	0.35	0.32	0.44	0.41	0.37	0.33	0.44	0.41	0.38	0.34
	4000	0.43	0.39	0.36	0.32	0.44	0.41	0.38	0.35	0.45	0.43	0.40	0.37	0.45	0.43	0.40	0.37
	4500	0.44	0.41	0.38	0.35	0.45	0.43	0.40	0.33	0.46	0.44	0.42	0.40	0.46	0.44	0.42	0.40

4. IDEA OF FORMULAS AND TABLES OF KISNEMSKY

While working in the Test Commission of the Okhta Powder Works and investigating the question of the applicability of the tables of Heidenreich to our guns and powders, G. P. Kisnemsky arrived at the necessity of substantially changing these tables and formulated his own tables on the basis of tests of our powders and guns.

Since, as a rule, no account is taken in empirical formulas of the influence of the weight of the charge and of the thickness of the powder, he proposed several formulas to eliminate this disadvantage.

For example, to establish the efficiency of the charge in the gun, Kisnemsky proposed the following formula:

$$v_0 = h\,(\omega - \omega_0)^{\frac{1}{2}},$$

where h is a proportionality factor determined by the system of the gun, and ω_0 is that part of the charge whose energy is consumed in the production of harmful work during the shot.

To determine this part of the charge, Kisnemsky gave two formulas, which took into account the influence of the thickness and propellant force of the powder and of some other data relating to the design of the gun and the loading conditions:

$$\omega_0 = 0.001\,(s l_{д} q)^{\frac{1}{2}}\,(2e_1)^{\frac{1}{3}}$$

or

$$\omega_0 = \omega - (W_0 + s l_{д})\,\frac{p_{д}}{f + \alpha p_{д}}.$$

CHAPTER 2 - EMPIRICAL FORMULAS AND TABLES

1. IDEA OF FORMULAS OF LEDUC.

The empirical formulas of Leduc (1903) were employed for rapid computation of pressure and velocity curves and were utilized for the solution of various problems in internal ballistics. The advantage of these tables consisted in their simplicity. At present, however, in view of the availability of tables formulated on the basis of more exact analytical formulas, the empirical formulas of Leduc, like the tables of Heidenreich, have lost their importance and possess some interest merely from the point of view of the method on which they are based.

On the basis of a study of extensive experimental and computational data, Leduc assumed for the velocity of the projectile a hyperbolic correlation of the following type:

$$v = \frac{a\ell}{b + \ell},$$

where a and b are constants, which depend upon the loading conditions.

It follows from this formula that, as ℓ approaches infinity, v approaches a, and the constant a expresses the limiting velocity of the projectile. As a matter of fact:

$$v_{np} = \left(\frac{a\ell}{b + \ell}\right)_{\ell = \infty} = \left(\frac{a}{\frac{b}{\ell} + 1}\right)_{\ell = \infty} = a.$$

The constant a has the dimension of velocity, and the constant b has the dimension of length.

To establish the dependence of pressure upon the path of the projectile, use is made of the usual equation of motion of the projectile in the following form:

$$ps = \varphi mv \frac{dv}{d\ell};$$

By substituting therein the expression for the velocity of the projectile and for its derivative with respect to the path, $dv/d\ell = ab/(b + \ell)^2$, we obtain the following formula for the pressure as a function of the path:

$$p = \frac{\varphi m a^2 b}{s} \frac{\ell}{(b+\ell)^3} .$$

To find the maximum pressure p_m and the path ℓ_m traversed by the projectile prior to that instant, we equate to zero the derivative of the pressure with respect to the path traversed by the projectile:

$$\frac{dp}{d\ell} = \frac{m}{s} a^2 b \frac{(b+\ell)^3 - 3\ell(b+\ell)^2}{(b+\ell)^6} = \frac{m}{s} a^2 b \frac{b - 2\ell}{(b+\ell)^4} ;$$

$$\frac{dp}{d\ell} = 0 \text{ at } b = 2\ell_m ; \quad \ell_m = \frac{b}{2} .$$

Upon substituting this value for ℓ_m into the equation, we obtain:

$$p_m = \frac{4}{27} \frac{\varphi m a^2}{s b} .$$

We obtain the velocity of the projectile in the instant of maximum pressure from the equation at $\ell_m = b/2$:

$$v_m = \frac{a \ell_m}{b + \ell_m} = \frac{a \frac{b}{2}}{b + \frac{b}{2}} = \frac{a}{3} .$$

Consequently, the constant a (or the limiting velocity) equals three times the velocity of the projectile in the instant of maximum pressure ($a = 3v_m$).

To find the correlation between the path and time, we start out on the basis of the fact that:

$$dt = \frac{d\ell}{v} = \frac{b + \ell}{a\ell} d\ell .$$

Integration within the limits from t_1 to t and from ℓ_1 to ℓ gives:

$$t = t_1 + \frac{1}{a} \left(b \ln \frac{\ell}{\ell_1} + \ell - \ell_1\right),$$

where ℓ_1 generally stands for a certain sufficiently small segment of the path and t_1 stands for the time corresponding to it.

Taking $\ell_1 = \ell_m$ and consequently $t_1 = t_m$, and moreover assuming that the projectile moves along this segment with a constant acceleration equal to the arithmetic mean between the initial acceleration (which equals zero) and the final acceleration (which equals $4/27 \cdot a^2/b$), we can write $\ell_m = b/2 = (1/2)(2/27)(a^2/b)(t_m^2)$ from which $t_1 = t_m = \sqrt{27/2}(b/a)$.

By substituting this value for t_1 into the equation for t, and taking into account that $\ell_1 = \ell_m = b/2$, we find the time necessary for the projectile to traverse the path ℓ through the bore:

$$t = \sqrt{\frac{27}{2}}\frac{b}{a} + \frac{1}{a}\left[b\,\ln\frac{2\ell}{b} + \ell - \frac{b}{2}\right] = \frac{b}{a}\left[\left(\sqrt{\frac{27}{2}} - \frac{1}{2}\right) + \frac{\ell}{b} + \ln\left(\frac{2\ell}{b}\right)\right].$$

Completion of the above operations and transformation of the result in terms of decimal logarithms finally gives us:

$$t = \frac{b}{a}\left[3.174 + \frac{\ell}{b} + 2.303\,\log\frac{2\ell}{b}\right].$$

To utilize the formulas of Leduc, it is necessary to know the constants a and b. If the values of v_ℓ and p_m are known from experiment or have been computed on the basis of exact formulas, the constants a and b are defined by the following system of equations:

$$v_\ell = \frac{a\ell_\ell}{b + \ell_\ell};$$

$$p_m = \frac{4}{27}\frac{\varphi m}{s}\frac{a^2}{b},$$

from which:

$$a = \frac{27p_m s \ell_\ell}{8\varphi m v_\ell}\left(1 \pm \sqrt{1 - \frac{16\varphi m v_\ell^2}{27 p_m s \ell_\ell}}\right).$$

In this formula, only the minus sign need be retained before the expression under the radical, since the plus sign gives for ℓ a value beyond the limits of the bore.

By designating $\eta_\ell = p_{av.}/p_m = \varphi m v_\ell^2/2p_m s \ell_\ell$ and introducing this expression into the equation for a, we obtain:

$$a = \frac{27}{16\eta_\ell}\left(1 - \sqrt{1 - \frac{32}{27}\eta_\ell}\right) v_\ell .$$

Knowing a, we find the quantity b from the following equation:

$$b = \left(\frac{a}{v_\ell} - 1\right)\ell_\ell .$$

The formulas of Leduc have the disadvantage that, in order to utilize them and to determine the constants a and b, it is necessary to know in advance p_m and v_ℓ; this reduces their value considerably. For this reason, Leduc made the attempt to predetermine the constants a and b in advance in conformity with the conditions of loading and the characteristic properties of the powder employed.

The formulas for a and b have the following form:

$$a = \alpha \left(\frac{\omega}{q}\right)^{\frac{1}{2}} \Delta^{\frac{1}{12}} ; \qquad (112)$$

$$b = \beta \left(\frac{W_0}{q}\right)^{\frac{3}{8}} \left(1 - \frac{3}{4}\Delta\right) . \qquad (113)$$

The quantities α and β characterize the powder; the quantity α characterizes the potential of the powder, depends principally upon its nature, and fluctuates within narrow limits; on the other hand, the quantity β characterizes the rate of burning of the powder, depends principally upon the thickness of the powder grain, and may fluctuate within rather wide limits (2-65).

In the case of pyroxylin powders, the value of α may be assumed to be equal to 2080 kg · m · sec, and consequently:

$$a = 2080 \left(\frac{\omega}{q}\right)^{\frac{1}{2}} \Delta^{\frac{1}{12}}.$$

Knowing a, the value of b can be determined from:

$$b = \left(\frac{a}{v_{\text{Д}}} - 1\right) \ell_{\text{Д}},$$

and, in case of necessity, the quantity β may be found from Equation (113).

The author has conducted a treatment of the results of firing tests and powder tests in a pressure bomb for the purpose of determining the dependence of the coefficient β upon the thickness of the powder or upon the pressure impulse. The following relations were established.

For powders with one perforation:

$$\beta = 2.15 \int_{\psi = 0.05}^{\psi = 0.95} pdt;$$

For powders with seven performations:

$$\beta = 4.25 \int_{0.05}^{0.85} pdt,$$

where the integrals $\int$ pdt were obtained by treatment of bomb tests.

In addition, the author has proposed the following simplified relations for a and b:

$$a = 0.16 \sqrt{2gf \frac{\omega}{q}}; \quad b = 2\ell_0 \Delta \ (*).$$

2. IDEA OF TABLES OF HEIDENREICH

The tables of Heidenreich (1900) were formulated on the basis of a treatment of a large number of velocimetric recoil curves obtained by firing various guns under a variety of loading conditions.

They consist of two separate tables.

Table 8 presents values which make it possible to determine the elements of a shot for the instant of maximum pressure and for the instant of passage of the projectile through the muzzle face (p_m and $\ell_Д$).

Table 8

$\eta = \frac{p_{av.}}{p_m}$	$\Sigma(\eta) = \frac{\ell_m}{\ell_Д}$	$\Pi(\eta) = \frac{p_Д}{p_{av.}}$	$\Phi(\eta) = \frac{v_m}{v_Д}$	$\Theta(\eta) = \frac{t_m}{t_{av.}}$	$T(\eta) = \frac{t_Д}{t_{av.}}$
0	0	-	-	0	-
0.05	0.0046	-	-	0.033	-
0.10	0.0104	0.200	0.288	0.069	0.646
0.15	0.0177	0.240	0.306	0.108	0.695
0.20	0.0262	0.274	0.322	0.150	0.744
0.25	0.0360	0.306	0.337	0.196	0.792
0.30	0.0471	0.338	0.352	0.246	0.842
0.35	0.0597	0.368	0.367	0.300	0.893
0.40	0.0740	0.400	0.383	0.358	0.946
0.45	0.0903	0.432	0.399	0.420	1.000

(*)For a more detailed description of the application of Leduc's formulas to various cases encountered in practice, cf. M. E. Serebryakov, G. V. Oppokov, and K. K. Greten, "VNUTRENNYAYA BALLISTIKA" (Internal Ballistics), 1939, pp. 333-341./6/

Table 8 (Cont'd.)

$\eta = \dfrac{p_{av.}}{p_m}$	$\Sigma(\eta) = \dfrac{\ell_m}{\ell_A}$	$\Pi(\eta) = \dfrac{p_A}{p_{av.}}$	$\Phi(\eta) = \dfrac{v_m}{v_A}$	$\Theta(\eta) = \dfrac{t_m}{t_{av.}}$	$T(\eta) = \dfrac{t_A}{t_{av.}}$
0.50	0.1090	0.465	0.416	0.487	1.056
0.55	0.132	0.501	0.435	0.560	1.116
0.60	0.160	0.541	0.457	0.642	1.180
0.65	0.192	0.585	0.482	0.734	1.249
0.70	0.231	0.635	0.511	0.835	1.322
0.75	0.283	0.697	0.546	0.958	1.406
0.80	0.360	0.779	0.592	1.115	1.507
0.825	0.422	0.838	0.636	1.225	1.575
0.85	0.605	1.000	0.747	1.485	1.715
0.825	0.855	1.181	0.908	1.735	1.815
0.80	0.980	1.254	0.987	1.835	1.845
0.79	1.000	1.266	1.000	1.850	1.850

In using Table 8, the initial quantity is $\eta = p_{av.}/p_m$, where $p_{av.}$ is defined by the following formula:

$$p_{av.} = \frac{q\left(1 + \frac{1}{2}\frac{\omega}{q}\right) v_A^2}{2\ gs\ \ell_A},$$

where $(1 + (1/2)\omega/q)$ is a coefficient which takes into account the work required to move the projectile.

Table 8 presents the following functions of η:

$$\Sigma(\eta) = \frac{\ell_m}{\ell_A};\quad \Phi(\eta) = \frac{v_m}{v_A};\quad \Theta(\eta) = \frac{t_m}{t_{av.}}\quad \Pi(\eta) = \frac{p_A}{p_{av.}};\quad T(\eta) = \frac{t_A}{t_{av.}},$$

where $t_{av.} = 2\ell_A/v_A$ is the time of motion of the projectile through the bore with the average velocity $v_Д + 0/2 = v_Д/2$.

Once the numerical value of:

$$\eta_Д = \frac{\varphi\left(1 + \frac{1}{2}\frac{\omega}{q}\right) v_Д^2}{2gs\ell_Д p_m}$$

for a given gun is known, it is found in the first column, and the values for all the remaining functions are written out as indicated in the same line.

With $\ell_Д$, $p_{av.}$, $v_Д$, and $t_{av.}$ known, these numbers are used to find the quantities ℓ_m, v_m, t_m, $p_Д$, and $t_Д$, i.e., the elements of the shot for the instant of p_m and $v_Д$.

In order to use the tables, it is necessary first to know p_m and $v_Д$, as well as q, s, $\ell_Д$, and ω/q, which also constitutes a disadvantage of these tables.

Table 9 presents data for finding intermediate values for the pressure, velocity, and time as a function of the relative path of the projectile $\Lambda = \ell/\ell_m$.

Table 9 contains numerical values of the following functions:

$$H(\lambda) = \eta; \quad \Psi(\lambda) = \frac{p}{p_m}; \quad \Omega(\lambda) = \frac{v}{v_m} \text{ and } Z(\lambda) = \frac{t}{t_m},$$

which represent curves for the pressure, velocity, and time of motion of the projectile as functions of the path ℓ/ℓ_m.

For $\lambda = 1$ ($\ell = \ell_m$, $p = p_m$), the values of the last three functions equal unity, and this line corresponds to the pressure maximum on the pressure curve. The lines above this line give values for p, v, and t on the ascending branch of the pressure curve; the lines below this line give values for these quantities on the descending branch of the curve.

In this connection, the limit of descent in Table 9 is the line for which $\lambda = \ell_\Omega/\ell_m$ or $\eta = \eta_\Omega$.

Table 9

$\lambda = \frac{\ell}{\ell_m}$	$H(\lambda) = \eta$	$\Psi(\lambda) = \frac{p}{p_m}$	$\Omega(\lambda) = \frac{v}{v_m}$	$Z(\lambda) = \frac{t}{t_m}$
0.25	0.445	0.690	0.375	0.689
0.50	0.615	0.890	0.624	0.830
0.75	0.723	0.970	0.828	0.924
1.00	0.790	1.000	1.000	1.000
1.25	0.833	0.966	1.145	1.063
1.50	0.848	0.893	1.268	1.119
1.75	0.849	0.828	1.372	1.170
2.00	0.843	0.769	1.460	1.218
2.5	0.818	0.668	1.609	1.306
3.0	0.786	0.590	1.726	1.387
3.5	0.753	0.527	1.824	1.463
4.0	0.721	0.475	1.909	1.536
4.5	0.691	0.433	1.981	1.606
5.0	0.663	0.397	2.046	1.672
6	0.614	0.340	2.158	1.801
7	0.572	0.297	2.250	1.923
8	0.536	0.263	2.328	2.042
9	0.504	0.236	2.395	2.156
10	0.476	0.214	2.453	2.267
11	0.451	0.195	2.504	2.376
12	0.429	0.179	2.551	2.483
13	0.409	0.166	2.592	2.588
14	0.391	0.154	2.630	2.692
15	0.375	0.144	2.665	2.794

Table 9 (cont'd.)

$\lambda = \frac{\ell}{\ell_m}$	$H(\lambda) = \eta$	$\Psi(\lambda) = \frac{p}{p_m}$	$\Omega(\lambda) = \frac{v}{v_m}$	$Z(\lambda) = \frac{t}{t_m}$
16	0.360	0.135	2.698	2.895
17	0.347	0.127	2.730	2.994
18	0.335	0.120	2.760	2.092
19	0.323	0.114	2.787	3.189
20	0.312	0.108	2.812	3.286
25	0.270	0.086	2.921	3.758
30	0.238	0.071	3.004	4.214
35	0.213	0.060	3.070	4.659
40	0.194	0.052	3.132	5.095
50	0.164	0.041	3.220	5.946
75	0.120	0.027	3.373	7.995
100	0.096	0.020	3.480	9.966

By copying from the tables the values of the relative quantities p/p_m, ℓ/ℓ_m, v/v_m, and t/t_m and multiplying them by p_m, ℓ_m, v_m, and t_m, respectively, we obtain the current values for p, ℓ, v, and t, with the aid of which we can plot curves for $p(\ell)$, $v(\ell)$, $p(t)$, and $v(t)$.

Example of Computation of Pressure and Velocity Curves

The following conditions are given:

Caliber $d = 76.2$ mm;

Weight of projectile $q = 6.5$ kg;

Weight of charge $\omega = 0.905$ kg;

Cross-sectional area of bore $s = 0.4693$ dm^2;

Muzzle velocity $v_A = 5880$ dm/sec;

Maximum pressure $p_m = 2320$ kg/cm^2;

Length of path of projectile $\ell_A = 18.44$ dm.

We find $p_{av.}$ ($g = 98.1$ dm/sec):

$$p_{av.} = \frac{q\left(1 + \frac{1}{2}\frac{\omega}{q}\right) v_A^2}{2\, g s \ell_A} = \frac{6.5\left(1 + \frac{1}{2}\,\frac{0.905}{6.5}\right) 5880}{298.1 \cdot 0.4693 \cdot 18.44} = 134500 \text{ kg/dm}^2 = 1345 \text{ kg/cm}^2$$

We compute $\eta = p_{av.}/p_m = 1345/2320 = 0.58$. From Table 8, interpolating for $\eta = p_{av.}/p_m = 0.58$, we find:

Table 10

η	$\Sigma(\eta) = \frac{\ell_m}{\ell_A}$	$\Phi(\eta) = \frac{v_m}{v_A}$	$\Theta(\eta) = \frac{t_m}{t_{av.}}$
0.55	0.132	0.435	0.560
0.58	0.149	0.448	0.609
0.60	0.160	0.457	0.642

Having the values of $\Sigma(\eta)$, $\Phi(\eta)$, and $\Theta(\eta)$, we compute ℓ_m, v_m, and t_m:

$$\ell_m = \ell_A \Sigma(\eta) = 18.44 \cdot 0.149 = 2.75 \text{ dm};$$

$$v_m = v_A \Phi(\eta) = 588 \cdot 0.448 = 264 \text{ m/sec};$$

$$t_{av.} = \frac{2\ell_A}{v_A} = \frac{2 \cdot 18.44}{5880} = 0.00628 \text{ sec};$$

$$t_m = t_{av.}\Theta(\eta) = 0.00628 \cdot 0.609 = 0.00382 \text{ sec}.$$

Having these values, we compute p, v, and t with the aid of the following formulas:

$$p = p_m \Psi(\lambda); \quad v = v_m \Omega(\lambda); \quad t = t_m Z(\lambda).$$

The value of λ for the muzzle face is:

$$\frac{\ell_A}{\ell_m} = \frac{18.44}{2.75} = 6.71.$$

We find from Table 9:

$$\Psi(\lambda); \Omega(\lambda); \quad Z(\lambda).$$

In conformity with this table, we compute the current values of l, p, v, and t.

Table 11

l, dm	p, kg/cm^2	v, m/sec	t, sec.
0.688	1600	99	0.00263
1.375	2060	164	0.00317
2.06	2250	218	0.00353
2.75	2320	263	0.00382
3.44	2240	301	0.00406
4.12	2070	334	0.00427
4.82	1920	361	0.00447
5.50	1785	384	0.00465
6.87	1550	423	0.00498
8.25	1370	454	0.00530
9.62	1220	480	0.00559
11.00	1100	502	0.00587
12.37	1005	521	0.00613
13.75	920	538	0.00638
16.50	790	568	0.00687
18.44	715	588	0.00722

The results of the computation are plotted in figs. 155 and 156 (the computations were performed with the aid of a slide rule).

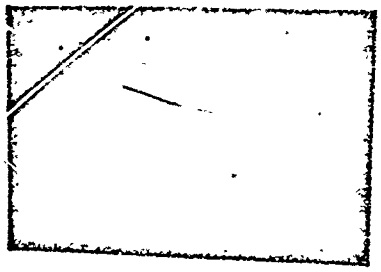

Fig. 155

p(t) and v(t) Curves.

Fig. 156

p(ℓ) and v(ℓ) Curves.

SECTION NINE - TABULAR METHODS OF SOLUTION OF PROBLEMS IN INTERNAL BALLISTICS

CHAPTER 1 - IMPORTANCE OF TABULAR METHODS OF SOLUTION IN ARTILLERY PRACTICE

In adopting the analytical method for the solution of the principal problem of pyrodynamics, i.e., for the determination of the gas-pressure curve in the bore and of the velocity of the projectile as a function of its path, it is necessary to perform a large number of computations, which require a considerable expenditure of time. Moreover, the analytical formulas do not make it possible to solve inverse problems connected with the design of the system or with determining the thickness of the powder. For this reason, in solving such problems, it has been necessary to go through a large number of variants of the direct problem and then to choose from among them by interpolation the variant suitable for the case under consideration. This introduced extraordinary complications into the solution of problems connected with the design of guns and the selection of powders, making it necessary to resort to tables and simple formulas of empirical origin which do not take into account all the circumstances surrounding the phenomenon of the shot. For example, the formulas of Leduc do not take cognizance of the weight of the charge and the thickness of the powder, just as the thickness of the powder also fails to be reflected in the tables of Heidenreich.

Neither the tables nor the formulas mentioned above make it possible to determine the position of the end of burning of the powder and to find out whether or not it burns entirely in the barrel. While this may not be essential for computing the strength of the barrel or for designing the gun carriage, it is of decisive importance in choosing the thickness of the powder to assure attainment of the necessary ballistic data.

For this reason, when, in 1910, Professor N. F. Drozdov formulated his tables for determining the maximum pressure p_m and the initial velocity $v_Д$, involving the coincidental determination of the position of the end of burning of the powder (ℓ_K/ℓ_0), this simplified considerably the solution of the direct problem of internal ballistics and permitted the rapid and simple solution of a number of inverse problems relating to the ballistic design of the barrel, such as determination of the length of path of the projectile necessary to assure attainment of the required initial (muzzle) velocity at a given density of loading and under the conditions of complete combustion of the powder in the bore $(\ell_K < \ell_Д)$, determination of the thickness of powder necessary to assure the attainment of a predetermined maximum pressure, solution of diverse variants involving changes in the weight of the charge and in the thickness of the powder at the same maximum pressure to determine the most advantageous conditions of loading, etc.

The tables formulated by Professor Drozdov played an important part in perfecting and accelerating the solution of the problem of ballistic design and received widespread recognition. For convenience in use, they were later interpolated for smaller variations in the arguments entering into them. In 1933, they were perfected and expanded by the author himself. They also served as a model for the compilation in 1933 of the "Tables of the Chair of Internal Ballistics" for powders with a constant burning area.

A further development and continuation of the tables of Professor N. F. Drozdov is represented by the "ANII Tables," published in 1933, which make it possible, under given conditions of loading, to determine not only p_m, ℓ_K, ℓ_m, and $v_Д$, but also all curves for the gas pressure, the velocity of the projectile, and the time of motion as functions of the path of the projectile. These tables made it possible still further to accelerate the solution of a number of problems connected with the field of ballistic design.

Following the introduction of the tables of Professor N. F. Drozdov, and then of the ANII Tables, into artillery practice, the empirical tables of Heidenreich lost all of their importance.

In 1942, with the ANII Tables as a model, there were compiled under the editorship of V. E. Slukhotsky and S. I. Ermolaev detailed "GAU Tables," which were published in three parts, with the subsequent addition of a special Part 4 for the ballistic computation of guns.

They are more convenient and do not contain the errors present in the ANII Tables.

Part 4 of the tables (TBC) is especially convenient for ballistic computations.

In the present chapter, we shall discuss tables which represent numerical values of the principal elements p, v, ℓ, and t, obtained on the basis of formulas for the analytical solution of the direct problem for a large number of variants of loading conditions. Such tables make it possible, almost without computations, to find all elements of a shot, such as the gas pressure and the velocity of the projectile as functions of the path of the projectile and as functions of time, there being determined among others the elements for maximum pressure, for the end of burning of the powder, and for the muzzle face.

Some such (abbreviated) tables give directly only certain individual elements of the shot, including the maximum pressure, its position, and the position of the end of burning of the powder, making it necessary to conduct additional relatively simple computations for the calculation of the muzzle velocity.

Contrary to the practice adopted by some authors, tabular methods cannot be interpreted to include those analytical methods for the solution of the direct problem comprising tables of various functions of internal ballistics, such as, for example, the D and ε functions of Professor Oppokov, the $\int_0^{\beta} z_x^{B/B_1} d\beta$ function of Sviridov, etc., which play an auxiliary part in the solution of the direct problem and in the computation of the elements of the shot.

PROCEDURE FOR FORMULATION OF TABLES(*)

In formulating tables on the basis of analytical methods, there is conducted a large number of computations leading to the solution of the direct problem of internal ballistics for various conditions of loading, which are chosen within definite limits.

To render the tables adaptable to guns of any desired caliber, the initial equations are reduced to such a form that they contain relative quantities wherever possible. For example, instead of weights of charges ω, which vary within very wide limits, use is made of densities of loading Δ, which vary but little for definite types of guns; instead of absolute paths of the projectile, there are determined the relative quantities $\Lambda = \ell/\ell_0$, where $\ell_0 = W_0/s$ is the adjusted length of the chamber. The quantity $\Lambda = \ell/\ell_0$ may be considered either as the relative path of the projectile or as the number of volumes of expansion of the gases, $\Lambda = s\ell/s\ell_0 = W/W_0$, which, in our artillery systems, varies only within circumscribed limits.

Instead of the absolute pressures p, there are sometimes determined the ratios of the pressure to the propellant force of the powder, p/f or p/p_1, where $p_1 = f\Delta/1 - \alpha\Delta$.

Moreover, the constants which characterize the conditions of

(*)Cf. Professor D. A. Ventsel, "VNUTRENNYAYA BALLISTIKA" (Internal Ballistics) [10] loading are grouped together in the form of dimensionless parameters independent of the caliber of the gun. Such parameters include, for example:

$$B = \frac{s^2 I_K^2}{f\omega\varphi m}$$

(in the method of Professor Drozdov)

or

$$H = \frac{2f\omega\varphi m}{s^2 I_K^2} = \frac{2}{B}$$

(in the method of Bianchi-Grave)

or

$$C = \frac{\theta}{H} = \frac{B\theta}{2}$$

(in the same method at $\varkappa = 1$)

To make it possible to construct the tables, it is necessary to establish the number of variables and parameters entering into the system of equations of internal ballistics. For this purpose, we shall consider the principal formulas for the elements of the shot (p, v, ℓ, and ψ).

For the first period, in the case of a powder of degressive form, we have:

$$-\varkappa\lambda = \chi - 1;$$

$$\varphi = \varphi_0 + \chi\sigma_0 x + \chi\lambda x^2;$$

$$\chi\sigma_0 = k_1 = \chi + 2\chi\lambda z_0;$$

$$z_0 = \frac{2\psi_0}{\chi(\sigma_0 + 1)} \approx \frac{\psi_0}{\chi};$$

$$\psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}} = \frac{\frac{\delta}{\Delta} - 1}{\frac{f\delta}{p_0} + \alpha\delta - 1};$$

$$v = v_{np}\sqrt{\frac{B\theta}{2}}\,x = \sqrt{\frac{\omega}{\varphi q}}\,\sqrt{fgB}\,x;$$

or

$$v\sqrt{\frac{\varphi q}{\omega}} = \sqrt{fgB}\,x;$$

$$\Lambda = \frac{\ell}{\ell_0};$$

$$\Lambda_\psi = 1 - \frac{\Delta}{\delta} - \Delta\left(\alpha - \frac{1}{\delta}\right)\psi;$$

$$p = f\Delta\frac{\psi - \frac{B\theta}{2}x^2}{\Lambda_\psi + \Lambda};$$

$$\Lambda = \Lambda_{\psi_{av.}}\left(Z_x^{-\frac{B}{B_1}} - 1\right),$$

where

$$B_1 = \frac{B\theta}{2} - \chi\lambda,$$

B_1 entering into the quantities $\gamma = B_1\psi_0/k_1^2$ and $\beta = B_1x/k_1$, in accordance with which the function $\log Z_x^{-1}$ is determined.

The chamber volume W_0 and the cross-sectional area of the chamber s enter only into the expression $\Lambda = \ell/\ell_0$ by way of the quantity $\ell_0 = W_0/s$. The weight of the charge and the weight of the projectile enter in the form of the ratio ω/q into the formula for the velocity v; the coefficient $\varphi = a + b\,\omega/q$ depends upon the same ratio.

Consideration of the quantities p, $v\sqrt{\frac{\varphi q}{\omega}}$, and $\Lambda = \ell/\ell_0$ shows them to represent functions of the argument x and of the following eight parameters:

f, α, and δ, which characterize the nature of the powder;

$\theta = c_p/c_w - 1$, which characterizes the composition of the gases and the conditions of their expansion in the bore of the gun;

χ, which characterizes the form of the powder;

p_0, which characterizes the arrangement of the belt of the projectile and of the grooves in the bore;

B and Δ, which characterize the conditions of loading.

The values of the same variables p, $v\sqrt{\varphi q/\omega}$, and Λ at the pressure maximum and at the end of burning of the powder depend upon the same eight parameters.

In the second period, p and $v\sqrt{\varphi q/\omega}$ are defined by the following expressions:

$$p = p_K\left(\frac{\Lambda_K + 1 - \alpha\Delta}{\Lambda + 1 - \alpha\Delta}\right)^{1+\theta},$$

where

$$p_K = f\Delta\,\frac{1 - \frac{B\theta}{2}(1 - z_0)^2}{\Lambda_1 + \Lambda_K} \text{ and } \Lambda_1 = 1 - \alpha\Delta;$$

and

$$\left(v\sqrt{\frac{\varphi q}{\omega}}\right)^2 = \frac{2gf}{\theta}\left\{1 - \left(\frac{\Lambda_K + 1 - \alpha\Delta}{\Lambda + 1 - \alpha\Delta}\right)^{\theta}\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]\right\},$$

where the argument is Λ; the remaining constants and the parameters B and Δ are the same as in the first period.

The large number of constants and parameters makes it necessary, in formulating the tables, to assume that some of the constants, which vary within definite limits, such as f, α, δ, $\varkappa$, p_0, etc., are constant average values, which narrows down the field of applicability of the tables. Some authors choose the alternative of introducing more complex variables and parameters, which makes it possible to reduce the number of entries in the tables, but also complicates the use of the latter.

If f, α, δ, $\varkappa$, θ, and p_0 are assumed to be constant, the quantities p, Λ, and $v\sqrt{\varphi q/\omega}$ will be functions of only the two parameters

Δ and B, and it becomes possible to formulate tables with only the two entries Δ and B.

Let us designate the quantity v $\sqrt{\varphi q/\omega}$ as $v_{tab.}$. After determining $v_{tab.}$ from the tables, the actual velocity of the projectile v is found by multiplying $v_{tab.}$ by the factor $n = \sqrt{\omega/\varphi q}$, which is known for the predetermined loading conditions:

$$v = v_{tab.} n,$$

where $\varphi = a + b\,\omega/q$.

The time of motion is expressed by the following integral:

$$t = \int_0^{\ell} \frac{d\ell}{v}.$$

If, in this integral, ℓ is expressed in terms of Λ and v in terms of $v_{tab.}$, we obtain for the time of motion the following expression:

$$t = \ell_0 \sqrt{\frac{\varphi q}{\omega}} \int_0^{\Lambda} \frac{d\Lambda}{v_{tab.}},$$

in which the integral $\int_0^{\Lambda} d\Lambda/v_{tab.}$ is likewise a function of the same Δ, B, and Λ. The tables give the following quantity:

$$t_{tab.} = \frac{10^6}{\ell_0} \sqrt{\frac{\omega}{\varphi q}}\, t.$$

It should be pointed out that $v_{tab.}$ is given in the tables in m/sec^{-1}, while ℓ_0 in the formula for the time is expressed in dm.

The transition from tabular values for conditional time, $t_{tab.}$, to actual time values is carried out in accordance with the following formula:

$$t = l_{0\ dm} \sqrt{\frac{\varphi q}{\omega}}\, 10^{-6}\, t_{tab.}$$

In formulating the tables, there are first established the limits of variation of the parameters Δ and B which may be encountered in practice, together with the intervals of variation of these parameters that are convenient for interpolation of intermediate values. For example, Δ is taken between the limits of 0.20 and 0.80 or 0.10 and 0.90, and B is taken between 1 and 3 or 0 and 4.

The intervals for Δ are best chosen to be equal to 0.04, in order that later, by interpolating half-way and then half-way again, there may be obtained variations in the tabular data for the values of Δ at intervals of 0.01. The intervals for B may be taken to be 0.4, in order that two half-way interpolations may give tabular data for the values of B at intervals of 0.1.

We thus obtain two series of values for the principal parameters:

$$\Delta = 0.20;\ 0.24;\ 0.28;\ \ldots\ 0.72;\ 0.76;\ 0.80.$$

$$B = 1.0;\ 1.4;\ 1.8;\ 2.2;\ 2.6;\ 3.0.$$

Thereupon, with one of the values of Δ (for example 0.20) as a basis, there is carried out for all the values of B written out above a complete computation of the solution of the problem of internal ballistics, involving the determination of p, $v_{tab.}$, Λ, and ψ, both for the maximum-pressure values and for the end of burning of the powder, and in some tables also for a series of intermediate points in the first and second periods until a definite value of Λ is obtained.

This is then repeated for all the chosen values of Δ.

Upon completion of the computations, the values of the quantities which must be entered into the tables (for example p_m, ℓ_m/ℓ_0, ℓ_K/ℓ_0) are plotted on a large scale on cross-sectional paper, and curves showing the variations of these quantities, for example of p_m as a function of Δ at given values of B, are constructed. Thereupon, for the same values of Δ, there are constructed curves showing the variation of p_m as a function of B; these two systems of curves make it possible to carry out interpolations for intermediate values of Δ and B, and thus to formulate a full table of variations of p_m as a function of Δ at intervals of 0.01 and as a function of B at intervals of 0.1 or 0.05. Analogous curves are also constructed for the other quantities (ℓ_m/ℓ_0, ℓ_K/ℓ_0, etc.) as well, and interpolations are carried out in a similar manner.

The data obtained after interpolation are entered into tables, which make it possible to solve both direct problems on the determination of p_m, p_K, ℓ_m, ℓ_K, and $v_\text{д}$ and inverse problems connected with the ballistic computation of guns.

CHAPTER 2 - TABLES FOR DETERMINING PRINCIPAL ELEMENTS OF SHOT (p_m, ℓ_m, ℓ_K, $v_\text{д}$)

1. TABLES OF PROFESSOR N. F. DROZDOV

The tables were compiled for strip-type powders possessing the following form characteristics: $\chi = 1.06$ and $\chi\lambda = -0.06$.

In the tables, the following characteristics were assumed to be constant.

Propellant force of powder $f = 950{,}000$ kg-dm/kg.

Covolume $\alpha = 0.98$ dm^3/kg.

Density of powder $\delta = 1.6$ kg/dm^3.

Coefficient $\varphi = 1.05$.

Forcing pressure $p_0 = 300$ kg/cm^2.

Adiabatic index $k = 1 + \theta = 1.2$ or $\theta = 0.2$.

Acceleration due to gravity $g = 98.1$ dm/sec^2.

$\alpha = 1/\delta = 1/\delta_1 = 0.355 = 1/2.82$.

The initially developed tables were brief and consisted of three tables; a basic table B and auxiliary tables A and C. They were subsequently modified and rendered more convenient and universal.

The basic data entered into the tables (cf. Tables I, II, III, and IV in the Appendix) are the density of loading Δ and the parameter of the loading conditions B:

$$B = \frac{s^2 e_1^2}{u_1^2} \frac{1}{f\omega\varphi m} = \frac{s^2 I^2_K}{f\omega\varphi m}.$$

The upper horizontal row contains quantities Δ from 0.07 to 0.80 at intervals of 0.01; the left-hand vertical column contains quantities B from 0.7 to 3.0 at intervals of 0.05.

The numbers in Table I give values for the maximum pressure p_m.

The numbers in Table II give the ratio ℓ_K/ℓ_0, where ℓ_K is the path of the projectile at the end of burning and ℓ_0 is the adjusted length of the chamber.

The numbers in Table III give the ratio ℓ_m/ℓ_0, where ℓ_m is the path of the projectile in the instant of attainment of maximum pressure.

The numbers in Table IV give values of the quantity $\log [1 - B\theta/2\ (1 - z_0)^2]$, which enters into the expression for the velocity of the projectile in the second period.

The muzzle velocity is computed with the aid of the usual formula:

$$v_A = \sqrt{\frac{2g}{\varphi} \frac{f}{\theta} \frac{\omega}{q} \left\{ 1 - \eta_1^{\theta} \left[1 - \frac{B\theta}{2} (1 - z_0)^2 \right] \right\}}, \qquad (114)$$

where

$$\eta_1 = \frac{\frac{l_K}{l_0} + 1 - \alpha\Delta}{\frac{l_A}{l_0} + 1 - \alpha\Delta}$$

represents the ratio of free volumes of the initial air space in the instant of the end of burning and in the instant of emergence of the projectile.

Under given loading conditions, it is necessary for computing v_A to find the value of l_K/l_0 from Table II and the quantity log $[1 - B\theta/2\ (1 - z_0)^2]$ from Table IV, and then to substitute these into formula (114).

If there are first substituted into formula (114) the assumed values for the constants 2fg and $\varphi\theta$, that formula will be written as

$$v_A = 29{,}790 \sqrt{1 - \eta_1^{0.2}\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]}\ \sqrt{\frac{\omega}{q}}. \qquad (115)$$

On the basis of the predetermined values for Δ and B (for example $\Delta = 0.60$ and $B = 2.0$), there is found the maximum pressure ($p_m = 2{,}255$ kg/cm^2). At the same B = 2, Table II is used to find the value $l_K/l_0 = 2.96$, and Table IV is used to find log $[1 - B\theta/2\ (1 - z_0)^2] = \bar{1}.9097$.

Substitution of these values for l_K/l_0 and log $[1 - B\theta/2\ (1 - z_0)^2]$ into formula (115) makes it possible to compute the muzzle velocity of the projectile.

Inspection of Table I indicates that, at a given density of loading Δ, p_m decreases and l_K/l_0 increases with increasing B. In a given gun, there may vary in the quantity $B = (s^2e_1^2/u_1^2)(1/f\omega\varphi m)$ the following principal items: the thickness of the powder $2e_1$, the weight of the charge ω, and the mass of the projectile m; f, φ, and s are constant, and u_1 does not experience a strong enough variation.

If the density of loading Δ remains constant in a given gun while B increases, this is due principally to an increase in the thickness of the powder $2e_1$. The table shows that, as the thickness $2e_1$ increases, the pressure p_m decreases, the end of burning moves closer to the muzzle, and incomplete combustion may result from a large increase in B:

$$\frac{\ell_K}{\ell_0} > \frac{\ell_g'}{\ell_0}.$$

Analysis of Table I shows that, in a given gun, it is possible to obtain one and the same pressure p_m at different Δ by varying B at the same time. Identical pressures p_m are arranged in the table along slanting lines from the upper left to the lower right; for example, the pressure $p_m = 2,400$ kg/cm^2 is obtained at the following combinations of Δ and B, ℓ_K/ℓ_0 varying at the same time.

Table 12

$p_m = 2400$ kg/cm^2.

Δ	0.40	0.50	0.60	0.70
B	1.00	~1.39	1.87	2.40
$\frac{\ell_K}{\ell_0}$	0.85	1.43	2.54	4.55

If Δ increases as a result of an increase in the weight of the projectile (and not as a result of a decrease in the chamber volume W_0), an increase in ω should reduce B; for this reason, to maintain the same p_m, it is necessary to increase the thickness of the powder

$2e_1$ in such a manner as to have its change not only compensate for the influence of the increase in ω, but also augment B to the values indicated in Table 12. Since the thickness of the powder increases, while p_m remains the same, the end of burning moves closer and closer to the muzzle (ℓ_K/ℓ_0 increases from 0.85 to 4.55). As the charge increases up to a certain limit, the initial velocity of the projectile v_ℓ will likewise increase; as the charge increases further, the velocity will cease increasing because of the incomplete combustion of the powder.

If Δ increases as a result of a decrease in the volume of the chamber while the weight of the charge remains unchanged (large base of projectile), the same p_m can be maintained by changing B as is indicated in the table, but the thickness of the powder will change less than in the first case, since, in this connection, it is not necessary to compensate for the increase in the weight of the charge ω.

If Δ is changed only by changing the weight of the charge ω, without any change in the thickness of the powder and in the other conditions of loading, the parameter B will simultaneously change in the reverse ratio ($B_2:B_1 = \omega_1:\omega_2$). For this reason, under otherwise identical conditions, the pressures corresponding to the change in the charge will be arranged along lines running from the lower left toward the upper right.

For example, if, at B = 2 and Δ = 0.50, p_m = 1750, then, at Δ = = 0.40, B = 2.4 and p_m = 1180; at Δ = 0.60, B = 1.67 and p_m = 2670.

Comparison of the results indicates that, as Δ changes from 0.40 to 0.50, i.e., by 25%, the pressure changes by $(1750 - 1180)/(1180) \cdot 100 =$ = 48% (almost twice as much); and as Δ changes from 0.50 to 0.60, i.e., by 20%, the pressure changes by $(2670 - 1750)/(1750) \cdot 100 = 52.5\%$ (more than 2.5 times as much).

Consequently, as the density of loading increases, the same relative increase in the charge is associated with a larger and larger increase in pressure. For this reason, in selecting a charge in practice, its weight must be increased very cautiously if the density of loading is high.

A. Application of Tables to Solution of Various Problems

With the aid of the tables, it is possible to solve very rapidly a number of problems possessing great practical importance.

a) Determination of thickness of powder to assure attainment of predetermined maximum pressure p_m. If the data for the gun W_0, s, and $\ell_д$, and for the weights of the charge ω and of the projectile q are known, and if p_m is predetermined, then, to determine the thickness of powder to assure attainment of the predetermined pressure, there is first computed $\Delta = \omega/W_0$; this Δ is used to enter the corresponding column in Table I; and the predetermined pressure is found in this column. In accordance with the value of p_m, the quantity B is found in the same row of the left-hand column, and the thickness $2e_1$ is found with the aid of the following formula:

$$2e_1 = \frac{2u_1}{s}\sqrt{Bf\omega\varphi m}\ dm.$$

Using the same value of Δ and the value of B found from Table I, Table II is used to find ℓ_K/ℓ_0 and ℓ_K, and the latter is compared with $\ell_д$ to determine whether all of the powder burns ($\ell_K < \ell_д$) or does not burn ($\ell_K > \ell_д$) in the bore. The procedure adopted for the solution may be represented by the following scheme.

Example

Table I + Table II	Table I	Table II
$\Delta \downarrow$, $\Delta \downarrow$; $B \leftarrow p_m B \rightarrow \frac{\ell_K}{\ell_0} \rightarrow \ell_K \lessgtr \ell_\partial$; $\downarrow$ $2e_1 = \frac{2u_1}{s}\sqrt{Bf\omega\varphi m}$	$\Delta = 0.60$; $B = 1.90 \leftarrow p_m = 2365$; $2e_1 = \frac{2u_1}{s}\sqrt{1.90 \cdot 95 \cdot 10^4 \cdot \omega 1.05 \cdot m}$	$\Delta = 0.60$; $B = 1.90 \leftarrow \frac{\ell_K}{\ell_0} = 2.60$

The quantity u_1 may be determined either with the aid of the following pyrostatic formula:

$$u_1 = \frac{0.175(N - 6.36)10^{-8}}{0.04(220^\circ - t_n^\circ) + 3h + h'}$$

or with the aid of the tabulation presented below, which gives an approximate dependence upon the thickness of the powder (which is connected with the varying volatile content) for pyroxylin powders.

Table 13

	Ordinary pyroxylin powders						Phlegmatized powders	
$2e_1$, mm	0.1	0.5	1.0	2.0	3.0	4.0	BT/0.3	0.7
$u_1 \cdot 10^7 \frac{dm}{sec} : \frac{kg}{dm^2}$	90	80	75	70	65	62	72	70

It is also possible to use the following approximate empirical formula, which gives the dependence of u_1 upon the total volatile content:

$$u_1 = 0.0000120 - 0.0000010(H\%).$$

b) Determination of length of path of projectile to assure attainment of required initial velocity under predetermined loading conditions. Given are the gun caliber d, the bore cross section s (including the grooves), the volume of the chamber W_0, the weight of the projectile q, the weight of the charge ω, and the required initial velocity of the projectile v_{Π}; let the magnitude of the maximum pressure p_m likewise be predetermined. It is necessary to find the length of the path of the projectile ℓ_{Π}.

To start with, $\Delta = \omega/W_0$ is computed; in the column of Table I corresponding to this Δ, we find the predetermined p_m, whereupon we use the latter to determine B. At these values for Δ and B, we find ℓ_K/ℓ_0 from Table II, $\log \left[1 + B\theta/2\,(1 - z_0)^2\right]$ from Table IV.

From formula (115) for the velocity in the second period, we find ℓ_{Π}:

$$\ell_{\Pi} = \ell_0 \left\{ \left(\frac{\ell_K}{\ell_0} + 1 - \alpha\Delta\right) \frac{\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]^{\frac{1}{\theta}}}{\left(1 - \frac{v_{\Pi}^2}{v_{\Pi p}^2}\right)^{\frac{1}{\theta}}} - (1 - \alpha\Delta) \right\}. \quad (116)$$

All quantities entering into the right-hand side of formula (116) are known.

Formula (116) makes it possible to determine that path of the projectile along the bore which will assure the attainment of the predetermined initial velocity under the given pressure p_m; the thickness of the powder will be determined in accordance with the scheme of the first problem.

Analysis of the tables of Professor N. F. Drozdov and the above exemplary problems that can be solved with their aid show their importance for the practice of artillery and their convenience and flexibility for ballistic design and for the choice of powder, whereas empirical tables express merely the general character of the variation of pressure and velocity as functions of the path and time without making it possible to draw any conclusions about the powder.

A certain disadvantage of the tables resides in the fact that they were formulated for strip-type powder possessing the definite characteristics $\chi = 1.06$ and $\chi\lambda = 0.06$, and for a constant propellant force of the powder $f = 950{,}000$ kg · dm/kg. Practice has shown, however, that, under conditions of equal charges, a powder with seven perforations gives results in firing that are practically identical with the results obtained with strip-type powder if the thicknesses of the powders are related as follows:

$$2e_1 \text{ strip-type} = \frac{10}{7}\, 2e_1 \text{ with 7 perforations or}$$

$$2e_1 \text{ with 7 perforations} = \frac{7}{10}\, 2e_1 \text{ strip-type}$$

In using the tables of Professor Drozdov for computing the action of a powder with seven perforations, its wall thickness must be multiplied by 10/7, whereupon the entire problem is solved as in the case of strip-type powder.

It is true, of course, that the end of burning of this powder and the ℓ_K/ℓ_0 obtained from the tables will not correspond to the actual values for a powder with seven perforations having decomposition products of greater thickness ($\rho = 0.532\ e_1$); but, as has been shown by firing tests from a gun equipped with lateral crusher gages, the pressure curves of standard powders - strip-type and with seven perforations - coincide almost completely.

If the full thickness $e_1 + \rho$ of the grain with seven perforations is computed in relation to an equivalent strip-type powder:

$$\left(e_1 \text{ with 7 perforations} = \frac{7}{10}\, e_1 \text{ strip-type}\right),$$

then:

$$(e_1 + \rho)_{\text{with 7 perforations}} = 1.532\, e_{1\ \text{with 7 perforations}} =$$

$$= 1.532 \frac{7}{10} e_{1\ \text{strip-type}} = 1.07\, e_{1\ \text{strip-type}} .$$

Consequently, the full thickness of the powder with seven perforations together with the thickness of the decomposition products is somewhat greater than the thickness of the equivalent strip-type powder, and for this reason its end of burning will be found to be somewhat farther than in the case of strip-type powder, and the ℓ_K/ℓ_0 determined from the tables will be somewhat smaller than the true value. As concerns the utilization of the tables at a propellant force of the powder $f \neq 950{,}000$, to determine the pressure p_m it is possible, taking from the tables p_m at a given B, to multiply it by the ratio $f_1/950{,}000$, where f_1 is the new propellant force of the powder. On the other hand, to determine the velocity $v_\text{Д}$, the value obtained by computation must be multiplied by $\sqrt{\frac{f_1}{950{,}000}}$.

These values will be approximate, since the paths ℓ_m and ℓ_K will also change simultaneously with p_m and $v_\text{Д}$; but, for purposes of taking into account the order of the corrections of the values of p_m and $v_\text{Д}$, they may be employed as a first approximation and in the presence of a force not too different from the normal value of 950,000 kg · dm/kg = = 95 t · m/kg.

At $\varphi \neq 1.05$, it is possible also to apply a correction to the value of $v_\text{Д}$ in accordance with the following formula:

$$v_\text{Д} = v_{\text{Д}\,1.05} \sqrt{\frac{1.05}{\varphi}} .$$

At a given B, the quantity p_m is clearly independent of φ.

2. TABLES OF CHAIR OF INTERNAL BALLISTICS

In 1933, on the initiative of Professor I. P. Grave, the Chair of Internal Ballistics of the Dzerzhinskii Artillery Academy compiled tables for any values of f and φ and for a powder with a constant burning area, which is closely approached by long tubular powder.

The tables were computed on the basis of the analytical formulas of Bianchi, as modified by Professor Grave.

The following constants were assumed in the tables:

$\alpha = 0.98, \quad \delta = 1.6, \quad \varkappa = 1, \quad \lambda = 0, \quad \theta = 0.2, \quad p_0/f = 0.035.$

The basic quantities used were the density of loading Δ and the parameter of the loading conditions $H = 2f\omega\varphi m/s^2 I_K^2 = 2/B$ or the reciprocal quantity $C = \theta/H$. It is not difficult to see that, at $\theta = 0.2$, $C = 0.1 B$.

In the tables, the left-hand column contains the quantity C, which varies uniformly from 0.10 to 0.40 at intervals of 0.01, and the next column contains the corresponding quantity $H = 0.2/C$, which varies nonuniformly from 2.0 to 0.5. The loading densities in the upper horizontal row vary from 0.10 to 0.90 at intervals of 0.01.

On each page of the tables, for six loading densities and for all values of C from 0.10 to 0.40, there are written the corresponding values of the ratios p_m/f, p_K/f, l_m/l_0, and l_K/l_0 and of the two auxiliary quantities D and B, which enter into the formula for the muzzle velocity of the projectile:

$$D = 1 - \varphi r_K = 1 - C(1 - \psi_0)^2, \quad B = \left(1 - \alpha\Delta + \frac{l_K}{l_0}\right)^\theta,$$

where

$$v_{\text{Д}} = \sqrt{\frac{2f\omega}{\varphi\theta m}\left[1 - \frac{BD}{\left(1 - \alpha\Delta + \frac{l_{\text{Д}}}{l_0}\right)^{\theta}}\right]} \qquad (117)$$

It is not difficult to see that this formula coincides with the previously derived formula for the velocity in the second period and with the formula presented in the initial table of Professor Drozdov, since, for a powder with a constant burning area, $1 - C(1 - \psi_0)^2 = 1 - B\theta/2(1 - z_0)^2$, and:

$$\frac{B}{\left(1 - \alpha\Delta + \frac{l_{\text{Д}}}{l_0}\right)^{\theta}} = \eta_1^{\theta}.$$

The quantity φ enters into the parameter H or C.

Having found for a given set of Δ and C the values of p_m/f and p_K/f and knowing the propellant force of the powder f, we obtain the values for p_m and p_K; and having found l_m/l_0 and l_K/l_0 and knowing $l_0 = W_0/s$, we find l_m and l_K.

We thus find the nodal points of the pressure curve: the maximum pressure p_m and its position in the bore, i.e., the path of the projectile l_m, as well as the pressure p_K at the instant of complete combustion of the powder and the corresponding path of the projectile l_K; thereupon, having found B and D from the tables by means of formula (117), we determine with the aid of a simple computation the value of the muzzle velocity $v_{\text{Д}}$.

If, in the presence of the identical constants, values for p_m and $v_{\text{Д}}$ are computed from the tables of Professor Drozdov and from the tables of the Chair, the tables of the Chair are found to give lower values for p_m and $v_д$.

For example, for $f = 950,000$, $\varphi = 1.05$, $\Delta = 0.60$, $B = 2.0$, $C = 0.2$, and $l_д/l_0 = 5$, we obtain on the basis of the tables of Professor Drozdov:

$$p_m = 2255 \text{ kg/cm}^2, \; \frac{l_m}{l_0} = 0.630; \; \frac{l_K}{l_0} = 2.96;$$

$$\sqrt{1 - \eta_1^{\theta}\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]} = 0.510;$$

whereas the tables of the Chair, in the presence of the same constants and loading conditions, give:

$$\frac{p_m}{f} = 0.2155; \; p_m = 2045; \; \frac{l_m}{l_0} = 0.6942; \; \frac{l_K}{l_0} = 3.156; \; B = 1.29; \; D = 0.8142,$$

$$\sqrt{1 - \frac{BD}{\left(1 - \alpha\Delta + \frac{l_д}{l_0}\right)^{\theta}}} = 0.500 .$$

Since $v_{np} = \sqrt{2f\omega/\varphi\theta m}$ is identical under identical conditions, it follows that the numbers 0.510 and 0.500 are proportional to the muzzle velocities computed in accordance with the tables of Professor Drozdov and of the Chair, respectively.

Using the tables of Professor Drozdov, p_m was found to be 10% (2255/2045 = 1.10) higher, and the velocity $v_д$, 2% (510/500 = 1.02) higher, than using the tables of the Chair.

The formula for x_m:

$$x_m = \frac{\varkappa\sigma_0}{\dfrac{B(1 + \theta)}{\left(1 + \dfrac{p_m}{f\delta_1}\right)} - 2\varkappa\lambda}$$

shows that, as $\varkappa$ increases, x_m, ψ_m, and consequently also p_m all increase, it being shown by the computations that, under otherwise identical conditions, the maximum pressure is almost proportional to the magnitude of the form characteristic χ, which, according to Professor Drozdov, equals 1.06, while, in the tables of the Chair, χ = 1 for a powder with a constant burning area.

For this reason, 6% of the 10% difference in pressure must have been obtained at the expense of χ; as concerns the remaining 4% difference, it is explained by the modification introduced by Professor I. P. Grave to integrate the differential equation connecting the path ℓ and x. This approximate supplementary modification leads to an increased value for the path ℓ and to a lower pressure as compared with ℓ and p_m obtained by the exact method of Professor Dr[illegible]lov.

But if a comparison is made of the results of computations of the velocity v_A under the identical maximum pressure p_m and under the identical loading conditions Δ, W_0, and ℓ_A, then the tables of the Chair give values for v_A that are somewhat higher than the v_A obtained in accordance with the tables of Professor Drozdov; in this connection, the quantity ℓ_K - the path of the projectile at the end of burning of the powder - is smaller than is obtained with the aid of the tables of Professor Drozdov.

The fundamental disadvantage of the tables of the Chair resides in the fact that, if the propellant force of the powder changes, the forcing pressure p_0 changes simultaneously and proportionally, since, in the tables, p_0/f = const. = 0.035.

In any case, it should be pointed out that even the most exact method, and especially any tables formulated on the basis of such a method at definite values for the constants, cannot yield perfect agreement with experimental data obtained by firing different guns.

This is due to the fact that the relations employed to account for the phenomena accompanying the shot, as well as all methods of solution, are to one or another degree approximate with respect to the actual phenomenon of the shot.

This circumstance demands preliminary computations for the selection of some constants, with the aid of which the computed data are obtained close to or coincident with the experimentally observed results of firing tests (p_m, v_ℓ, and ℓ_m); and each method demands the selection of its own constants, which must give the best agreement with the results of firing tests.

For example, to obtain the data for p_m and v_ℓ, the geometric law of burning demands certain constants, while the physical law of burning demands other constants.

As concerns the application of the tables of the Chair of Internal Ballistics to the investigation of the influence of various loading conditions, as well as to the solution of a series of direct and inverse problems, all the statements made above relating to the use of the tables of Professor Drozdov remains in force for the tables of the Chair as well.

For example, to determine the thickness of the powder to assure attainment of a predetermined pressure p_m provided the propellant force of the powder f is known, it is necessary first to find p_m/f, whereupon, using the table, the predetermined value of p_m/f is found at the corresponding Δ, C or H are taken accordingly in this row, and finally the thickness of the powder $2e_1$ is determined in accordance with the following formulas:

$$2e_1 = \frac{2u_1}{s}\sqrt{10Cf\omega\varphi m} = \frac{2u_1}{s}\sqrt{\frac{2f\omega\varphi m}{H}} .$$

The following scheme is employed in using the tables:

$$\begin{tikzcd}[row sep=small, column sep=small]
 & & & & \Delta \arrow[d] \\
(H) & - & C \arrow[dd] & - & \dfrac{\dot{p}_m}{f} \\
 & & & & \\
 & & 2e_1 = \dfrac{2u_1}{s}\sqrt{100 f \omega q m} & &
\end{tikzcd}$$

Once the thickness of the powder with a constant burning area (tubular powder) has been found, the wall thickness of a grain with seven perforations can be determined by multiplying $2e_1$ by the coefficient 0.75, the reverse transition involving multiplication by 4/3.

CHAPTER 3 - DETAILED TABLES FOR CONSTRUCTION OF PRESSURE AND VELOCITY CURVES

1. ANII Tables (1933).

The tables of Professor N. F. Drozdov and of the Chair of Internal Ballistics make it possible to find the maximum pressure p_m, its location l_m, the pressure p_K at the end of burning, and its location l_K. But in order to determine the initial (muzzle) velocity of the projectile, it is necessary to perform additional computations, since this velocity depends upon the length of the bore. The tables also do not give intermediate values for p, v, and l, and they do not take into account the time of motion of the projectile through the bore of the gun.

The ANII Tables (1933) represented a further step forward and considerably facilitated the conduct of the ballistic computation of guns.

They were formulated on the basis of the same constants as those used by Professor Drozdov, and the computations leading to them were based on his formulas; they were computed by the method of numerical integration. For a given density of loading Δ and a given parameter B, using a strip-type powder with the characteristics $\chi = 1.06$ and $\chi\lambda = -0.06$, they make it possible to find curves for the gas pressure, the velocity of the projectile, and the time of motion as functions of the path of the projectile through the bore, thus permitting the complete solution of the fundamental problems of internal ballistics, both direct and inverse.

The arrangement of the tables is apparent from the scheme presented on page 748. They are formulated for loading densities ranging from 0.05 to 0.95; one page is reserved for each Δ at intervals

Scheme of ANII Tables

Pressures

$\Lambda = \frac{l}{l_0}$ $\Delta = 0.61$

B \ Λ	0	0.1	0.2	· · ·	14	15	Λ_K	p_K	Λ_m	p_m
0.7	300	2824	4516		212	195	0.448	6439	0.448	6439
0.8	300	2591	4000		214	197	0.525	5724	0.525	5724
0.9	300	2379	3592		216	199	0.593	5100	0.539	5100
·	·	·	·	Second Period	·	·	·	·	·	·
·	·	·	·		·	·	·	·	·	·
·	·	·	·	First	·	·	·	·	·	·
2.0	300	1317	1782	Period	242	222	2.944	1442	0.628	2315
·	·	·	·		·	·	·	·	·	·
·	·	·	·		·	·	·	·	·	·
·	·	·	·		·	·	·	·	·	·
3.0	300	1094	1333		267	247	10.12	390	0.575	1630
				* **						

Velocities

B \ Λ	0.1	0.2	0.3	· · ·	14	15	Λ_K	v_K	Λ_m	v_m
0.7	234	427	590	Second	2044	2057	0.448	791	0.448	793
0.8	225	391	542	Period	2032	2046	0.525	813	0.525	821
0.9	218	364	504		2019	2035	0.593	834	0.593	834
·	·	·	·		·	·	·	·	·	·
·	·	·	·	Tabular Velocities	·	·	·	·	·	·
·	·	·	·		·	·	·	·	·	·
2.0	162	268	259		1868	1888	2.944	1296	0.628	596
·	·	·	·	First	·	·	·	·	·	·
·	·	·	·	Period	·	·	·	·	·	·
·	·	·	·		·	·	·	·	·	·
3.0	149	235	302		1716	1741	10.12	1584	0.575	485
				* **						

Time

B \ Λ	0.1	0.2	0.3	· · ·	14	15	Λ_K	t_K	Λ_m	t_m
0.7	59	92	110	Second	901	950	0.448	131	0.448	131
0.8	60	95	117	Period	923	973	0.525	150	0.525	150
0.9	62	98	120		944	993	0.593	175	0.593	166
·	·	·	·		·	·	·	·	·	·

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0.9	300	2379	3592	Second Period	216	199	0.593	5100	0.539	5100
•	•	•	•		•	•	•	•	•	•
2.0	300	1317	1782	First Period	242	222	2.944	1442	0.628	2315
•	•	•	•		•	•	•	•	•	•
3.0	300	1094	1333	* **	267	247	10.12	390	0.575	1630

Velocities

B \ Λ	0.1	0.2	0.3	. . .	14	15	Λ_K	v_K	Λ_m	v_m
0.7	234	427	590	Second Period	2044	2057	0.448	791	0.448	793
0.8	225	391	542		2032	2046	0.525	813	0.525	821
0.9	218	364	504		2019	2035	0.593	834	0.593	834
•	•	•	•	Tabular Velocities	•	•	•	•	•	•
2.0	162	268	259		1868	1888	2.944	1296	0.628	596
•	•	•	•	First Period	•	•	•	•	•	•
3.0	149	235	302	* **	1716	1741	10.12	1584	0.575	485

Time

B \ Λ	0.1	0.2	0.3	. . .	14	15	Λ_K	t_K	Λ_m	t_m
0.7	59	92	110	Second Period	901	950	0.448	131	0.448	131
0.8	60	95	117		923	973	0.525	150	0.525	150
0.9	62	98	120		944	993	0.593	175	0.593	166
•	•	•	•	Tabular Times	•	•	•	•	•	•
2.0	74	121	153		1125	1179	2.944	457	0.628	222
•	•	•	•	First Period	•	•	•	•	•	•
3.0	80	134	173	* **	1285	1343	10.12	1027	0.575	242

*Position of Pressure Maximum
**Position of End of Burning of Powder

$\psi_0 = 0.03173$
$z_0 = 0.03009$

of 0.01; the basic numbers are the parameter B and the relative length of path of the projectile $\Lambda = l/l_0$, where l_0 is the corrected length of the chamber (in these tables, as in the tables of Professor Drozdov, it is designated as l_1). The parameter B varies from 0.7 to 3, and at Δ greater than 0.80 it varies from 1.2 to 4; Λ varies from 0 to 15 at unequal intervals, which at first are smaller (0.1) and subsequently increase (to 1).

The velocities and times are given in the tables in arbitrary units, and in order to obtain the actual velocity of the projectile the tabular velocity values v_{tab} must be multiplied by $\sqrt{\omega/q}$. To obtain the actual time of motion of the projectile, the tabular t must be multiplied by $l_0 \sqrt{q/\omega} \cdot 10^{-6}$ if l_0 is expressed in decimeters:

$$t = t_{tab} l_0 \sqrt{\frac{q}{\omega}} \cdot 10^{-6}; \quad v = v_{tab} \sqrt{\frac{\omega}{q}}.$$

In each of the three tables, there are recorded on the right-hand side the exact values of the quantities corresponding to the maximum pressure and to the end of burning; i.e., p_K, p_m, v_K, v_m, t_K, and t_m are given for $\Lambda_m = l_m/l_0$ and for $\Lambda_K = l_K/l_0$.

The heavy broken line marks those intervals between neighboring values of Λ between which the end of burning of the powder is located.

To the left and downward from this broken line are located the values of p, v, and t corresponding to the first period of the shot; those corresponding to the second period are located to the right and upward from the broken line.

As B increases, and consequently as the thickness of the powder at a given Δ increases, the end of burning shifts closer and closer to the muzzle face.

The exact value of $l_K/l_0 = \Lambda_K$ is contained in the fourth column from the right. If $\Lambda_K < \Lambda_д = l_д/l_0$, this signifies that the burning of the powder is complete; if $\Lambda_K > \Lambda_д$, it means that the powder does not burn completely in the bore.

The thin vertical lines in the range of $\Lambda = 0.1$-0.7 show that the maximum pressure p_m is located in this region (in the particular interval marked by the thin vertical line).

At the bottom of each page, there are presented the values of ψ_0 and z_0 corresponding to the instant of initial pressure at the given Δ.

A detailed description of the use of the tables is presented in the tables themselves.

<u>Example and procedure for computation.</u> Given a 76-mm 1902 model gun:

$$W_0 = 1.654\ dm^3;\ s = 0.4693\ dm^2;\ l_д = 18.44;\ q = 6.5\ kg;$$

$$\omega = 0.900\ kg;\ 2e_1 = 1\ mm = 0.04\ dm;\ u_1 = 0.0000075\ \frac{dm}{sec};\ \frac{kg}{dm^2}$$

<u>Computation of constants.</u>

$$\Delta = \frac{\omega}{W_0} = \frac{0.900}{1.654} = 0.543;\ l_0 = \frac{W_0}{s} = \frac{1.654}{0.4693} = 3.53\ dm;$$

$$\Lambda_д = \frac{l_д}{l_0} = \frac{18.44}{3.55} = 5.20;\ I_K = \frac{e_1}{u_1} = \frac{0.005}{0.0000075} = 667\ \frac{kg \cdot sec}{dm^2}$$

$$B = \frac{s^2 I_K^2}{f\omega\varphi m} = \frac{0.4693^2 \cdot 667^2 \cdot 98.1}{95 \cdot 10^4 \cdot 0.90 \cdot 1.05 \cdot 6.5} = 1.645 \approx 1.65;\ \varphi = 1.03 + \frac{1}{3}\frac{0.900}{6.5} = 1.076;$$

$$\sqrt{\frac{1.05\omega}{1.076q}} = \sqrt{\frac{0.900 \cdot 1.05}{6.5 \cdot 1.076}} = 0.368$$

Table 14

$\Delta = 0.54$

B \ Λ		0.2	0.4	0.682 $\Lambda_m =$ $= 0.679$ 0.676	1.0	1.808 $\Lambda_K =$ $= 2.[illegible]9$ 2.[illegible]0	5.0	$\Lambda_Д = 5.20$	5.5
p kg/cm^2	$B = 1.60$	1774	2228	2368	2296	1878	672		606
	$B = 1.65$	1735	2176	$p_m =$ $= 2310$	2240	1770	675	$p_Д = 650$	609
	$B = 1.70$	1696	2124	2253	2184	1661	679		612
$v_{tab.}$	$B = 1.6$	288	475	676	849	1115	1589		1624
	$B = 1.65$	285	469	664	838	1147	1579	1593	1615
	$B = 1.70$	282	463	653	828	117[illegible]	1569		1605
$v_{tab.} \sqrt{\frac{\omega}{q} \frac{1.05}{\varphi}}$		105	172	243	307	422	577	$v_Д = 587$ m/sec	
$t_{tab.}$	$B = 1.6$	112	166	213	255	336	565		596
	$B = 1.65$	113	168	215	258	350	571	583	601
	$B = 1.70$	114	169	216	260	365	577		607
$t = t_{tab.} \sqrt{\frac{1.076\ q}{1.05\ \omega}}\ l_0 \times$ $\times 10^{-6}$ sec.		0.00108	0.00162	0.00206	0.00248	0.00335	0.00549	0.00560 sec. $= t_Д$	
$l = l_0 \Lambda dm$		0.705	1.41	2.39	3.53	6.80	17.65	$18.44 = l_Д$	

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$$l_0 \sqrt{\frac{\varphi q}{1.05\omega}} \cdot 10^{-6} = 3.53 \frac{10^{-6}}{0.368} = 0.0000096.$$

All computations are summarized in Table 14, with special consideration of the necessity of interpolating for B between 1.6 and 1.7.

The curves for p and v may be constructed either as functions of the path l or as functions of the time t.

All problems which were solved with aid of the tables of Professor N. F. Drozdov are solved in exactly the same manner with the aid of the ANII Tables. By using these tables, it is possible to determine considerably more rapidly the muzzle velocity v_A and the length of the path necessary in planning to obtain the required muzzle velocity.

To solve this last problem at a given Δ, the quantity p_m is used to find B and then the thickness of the powder $2e_1$. Knowing the predetermined value of v_A, $v_{A\,tab.} = v_A \sqrt{\omega/q \; 1.05/\varphi}$ is found, and the table of velocities is used at the value of B found to seek the column containing the value of $v_{A\,tab.}$. By ascending the column, Λ_A and then l_A are determined.

Schematically, this procedure for the solution will be represented as follows:

Pressures

$$\Delta \downarrow$$

$$2e_1 = \frac{2u_1}{s}\sqrt{Bf\omega\varphi m} \longleftarrow B \longleftarrow p_m$$

Velocities

$$\Lambda_A \rightarrow l_A = \Lambda_A \cdot l_0$$

$$B \longrightarrow v_{tab.} \uparrow$$

The ANII Tables are likewise subject to the rule for conversion from the thickness of the strip-type powder, for which they are computed, to the thickness of a grain with seven perforations:

$$2e_1 \text{ with 7 perforations} = 0.7 \cdot 2e_1 \text{ strip-type}$$

Defects of ANII Tables. In the ANII Tables, the time of passage by the projectile of the first segment from 0 to $\Lambda = 0.1$ is computed incorrectly; these values are nearly twice as small as the values computed in accordance with the more exact formulas proposed by Professor E. L. Bravin [11], who noticed this error. This error distorts the first segment of the curves for the pressure, velocity, and path as functions of time and shifts all curves toward the origin by an amount equal to the magnitude of the error.

Professor Bravin proposed a formula to permit computation to a great degree of exactness the first element of time during the passage of the path $\Lambda = 0.1$, provided that there are given curves for the pressure and velocity as functions of the path, which is exactly what is available in the ANII Tables.

Having an initial pressure $p_0 = 300$ kg/cm^2, the pressure p', and the velocity v' (the latter two corresponding to the path $\Lambda' = 0.1$), it is possible to compute the time interval t' in accordance with the following formula:

$$t' = \frac{3l'}{v'} \frac{p_0 + p'}{2p_0 + p'} .$$

By subtracting from t' the quantity t'_Λ in the ANII Tables corresponding to the same path $\Lambda' = 0.1$, there is found the constant correction $\Delta t' = t' - t'_\Lambda$, which must be added to each time value found in the given row of the ANII Tables. Professor Bravin derived tables of corrections $\Delta t'$ to be applied to the ANII Tables for various Δ and B.

Aside from this error inherent in the computations, the ANII Tables suffer from poorly performed interpolation and contain many misprints. For this reason, in using them, it is recommended either to construct curves, which will make it possible, by the departure of points, immediately to detect errors and misprints, or else to pay close attention to the consistent character of the variation of the quantity being determined with the aid of the tables (p, $v_{tab.}$, $t_{tab.}$).

In spite of these defects, the ANII Tables represent a good aid in the solution of the most diverse, both direct and inverse, problems in internal ballistics and in the ballistic design of guns.

Some additional applications of tables of the type of the ANII Tables are cited in the chapter on the ballistic design of guns.

2. GAU Tables (1942)

There have now been published the more convenient and exact GAU Tables of 1942, which are formulated on the same general principle as the ANII Tables, but with a different arrangement of the fundamental parameters and elements of the shot. Moreover, the range of variation of the parameter B has been considerably expanded in the GAU Tables (from zero to 4.0).

The GAU Tables were formulated under the direction of Professor V. E. Slukhotsky and S. I. Ermolaev [12].

They consist of four parts. The first part comprises the tables of pressures, the second the tables of nominal velocities $v_{tab.} = v\sqrt{\varphi q/\omega}$, and the third the tables of nominal times $t_{tab.} = t\, 1/l_0$

$\sqrt{\omega/\varphi q}$.. With the aid of these tables, it is possible to conduct all ballistic computations of a gun in designing an artillery system. However, for convenience in computation, the three parts mentioned above are supplemented by a fourth, which comprises special tables for ballistic computation (TBR).

The tables of pressures, velocities, and times are characterized by the density of loading, whose values are given from 0.05 to 0.95 kg/dm^3 at 0.01 kg/dm^3 intervals.

The basic numbers in the tables of pressures, velocities, and times are the quantities:

$$B = \frac{s^2 I_K^2}{f\omega\varphi m}$$

and the relative path of the projectile:

$$\Lambda = \frac{l}{l_0}.$$

The values of Λ in each table are varied in the range of 0-20 at varying intervals. In each of the three tables, there are also contained exact values for the quantities Λ_m, Λ_K, p_m, p_K, v_{Tm}, v_{TK}, t_{Tm}, and t_{TK}, which correspond to the instant of attainment of maximum powder-gas pressure in the barrel and to the instant of the end of burning of the powder. The pressures are given in kg/cm^2.

The tables of pressures have the following form (cf. scheme).

In the tables of the first part, there are presented the true values for the pressures corresponding to the predetermined values of Δ, B, and Λ. By taking in the tables Λ from 0.1 to $\Lambda_Д = l_Д/l_0$ for the given gun, we shall obtain the corresponding values for the pressure in kg/cm^2 and shall be able to plot by points the p-Λ or p-l curve, since $l = l_0 \Lambda$.

Pressures (kg/cm^2), $\Delta = 0,...$

Λ \ B	0.1	0.2	0.3	. . . .	. . .	4.0
0.1						
0.2						
.						
.						
.						
1.0						
1.5						
.						
.						
.						
.						
19						
20						
Λ_K						
p_K						
Λ_m						
p_m						

The one or two thin horizontal lines in the tables show that the maximum pressure is located in the given interval of Λ; the heavy "stairway" indicates the boundary between the first and second periods.

The tables of nominal velocities $v_{tab.}$ in the second part and of nominal times $t_{tab.}$ in the third part are arranged in exactly the same manner as the tables in the first part, except that, in the last line and in the third line from the bottom, there are presented, respectively, v_m and v_K in the second part and t_m and t_K in the third part.

The actual velocities of the projectile are defined by the following expression:

$$v = v_{tab.} \sqrt{\frac{\omega}{\varphi q}} .$$

The actual times are defined by the following formula:

$$t = t_{tab.}\, l_0 \sqrt{\frac{\varphi q}{\omega}} \cdot 10^{-6} ,$$

where l_0 is in decimeters.

The tables are formulated on the basis of the following data:

Propellant force of powder	$f = 950{,}000$ kg · dm/kg
Covolume	$\alpha = 1.00$ dm^3/kg
Specific gravity of powder	$\delta = 1.6$ kg/dm^3
Initial pressure	$p_0 = 300$ kg/cm^2

In addition, the following assumptions were made in formulating the tables: $\theta = 0.2$, $\varkappa = 1.06$, $\varkappa\lambda = -0.06$. The velocities and times of motion of the projectile through the bore were computed on the condition that $\varphi = 1$.

The results obtained by the computations should be summarized in the form of a "Table of Principal Elements of Shot from Gun" at the following data:

$d = 107$ mm; $W_0 = 4{,}600$ dm^3; $s = 0.9165$ dm^2; $l_g = 34.20$ dm; $q = 17.0$ kg; $\omega = 3.0$ kg; $p_m = 2500$ kg/cm^2; $\frac{\omega}{q} = 0.1765$; $\varphi = 1.05 + \frac{1}{3}\frac{\omega}{q} = 1.109$;

$$l_0 = \frac{w_0}{s} = 5.019 \text{ dm}; \quad \Lambda_{\Delta} = \frac{l_{\Delta}}{l_0} = 6.814; \quad \Delta = 0.65 \text{ (rounded off to 0.01)};$$

$$n_v = \sqrt{\frac{\omega}{\varphi q}} = 0.399; \quad n_t = l_0 \sqrt{\frac{\varphi q}{\omega}}\, 10^{-6} = 12.56 \cdot 10^{-6}.$$

Summary of Results Obtained

Λ	0	0.2	0.4	$\Lambda_m = 0.623$	1.0	2.0	$\Lambda_K = 3.19$	5.0	$\Lambda_{\Delta} = 6.814$
l dm	0	1.00	2.01	3.13	5.02	10.04	16.01	25.10	34.20
p kg/cm^2	300	2008	2415	2500	2361	1827	1392	850	602
$v_{tab.}$	0	294	472	623	816	1136	1568	1565	1686
$v = n_v v_{tab.}$	0	117	188	249	325	453	546	624	673
$t_{tab.}$	0	204	258	299	351	452	546	667	779
sec $\cdot 10^3$	0	2.57	3.24	3.76	4.41	5.68	6.86	8.38	9.79

CHAPTER 4 - TABLES BASED ON GENERALIZED FORMULAS WITH REDUCED NUMBER OF PARAMETERS AND WITH RELATIVE VARIABLES

1. FORMULAS AND TABLES OF PROFESSOR B.N. OKUNEV.

Toward the end of the thirties, there were published several investigations in which groupings of parameters and relative variables were introduced for the purpose of reducing the large number of parameters and characteristic constants, as well as for the purpose of avoiding absolute values for the principal elements of the shot.

Such investigations include those by the Soviet workers Professor N.F. Drozdov [16], Professor B.N. Okunev [13], M.S. Gorokhov and A.I. Sviridov [14], and Professor G.V. Oppokov [15].

As an example, we shall consider the method of Professor Okunev, in which there introduced the relative variables $p_1 = p/p_1$, where $p_1 = \frac{f\Delta}{1 - \alpha\Delta}$; $\nu = \frac{v}{v_{np}}$; $\tau = t/T$, where $T = \frac{\varphi q}{gs}\frac{v_{np}}{p_1}$; $X = \frac{\Lambda}{1 - \alpha\Delta} = l/l_1$; and the generalized parameters $R = \sqrt{\frac{2}{B\Theta}} = \sqrt{\frac{H}{\Theta}}$ and $\Lambda_\Delta = \frac{1 - \frac{\Delta}{\delta}}{1 - \alpha\Delta} = l_\Delta/l_1$.

Professor Okunev' quantity Λ_Δ is the reciprocal of the quantity ∂, which was introduced by us in pyrostatics for the computation of $\psi : \Lambda_\Delta = 1/\partial$.

$$X = \frac{\Lambda}{1 - \alpha\Delta} = \frac{l}{l_1}.$$

Let us divide by $1 - \alpha\Delta$ the numerator and denominator of the formula for pressure:

$$p = f\Delta \frac{\psi - \frac{B\Theta}{2} x^2}{\Lambda_\psi + \Lambda},$$

where

$$\Lambda_\psi = 1 - \frac{\Delta}{\delta} - \Delta\left(\alpha - \frac{1}{\delta}\right)\psi.$$

By transferring p_1 to the left, and keeping in mind that

$$X_\psi = \frac{\Lambda_\psi}{1 - \alpha\Delta} = \frac{1 - \frac{\Delta}{\delta}}{1 - \alpha\Delta} - \left(\frac{1 - \frac{\Delta}{\delta}}{1 - \alpha\Delta} - 1\right)\psi = \Lambda_\Delta - (\Lambda_\Delta - 1)\psi,$$

where

$$\Lambda_\Delta = \frac{1 - \frac{\Delta}{\delta}}{1 - \alpha\Delta}.$$

we obtain the tabular pressure:

$$p_{tab.} = \frac{\psi - \frac{B\Theta}{2} x^2}{X_\psi + X},$$

where

$$p_{tab.} = \frac{p}{p_1}.$$

After dividing both sides of the differential equation:

$$\frac{dL}{dx} + \frac{B}{B_1}\frac{x}{\xi(x)} L = -\frac{\Delta}{\delta}(\alpha\delta - 1)(k_1 - 2\varkappa\lambda x)$$

by $1 - \alpha\Delta$, we obtain the following equation:

$$\frac{d(X_\psi + X)}{dx} + \frac{B}{B_1}\frac{x}{\xi(x)}(X_\psi + X) = -(\Lambda_\Delta - 1)(k_1 - 2\varkappa\lambda x).$$

Consideration of the expressions derived above shows that the quantity p_T and the quantity X are in the first period functions of the argument x and not of eight parameters, as previously, but of only five:

$$\theta, \varkappa, z_0, \Lambda_\Delta, B.$$

In the second period:

$$\frac{p}{p_K} = \left(\frac{1 - \alpha\Delta + \Lambda_K}{1 - \alpha\Delta + \Lambda}\right)^{1+\theta} = \left(\frac{1 + X_K}{1 + X}\right)^{1+\theta}.$$

The values of $p_{tab.}$ and X at the pressure maximum and at the end of burning depend upon the same five parameters. It should, however, be noted that, instead of p_0, the parameters include z_0. At a predetermined z_0, different p_0 will be obtained at different values of $\varkappa$, Λ_Δ, and B, so that, in formulating the tables, it is not possible to base them on definite z_0, it being instead necessary to accept z_0 as one of the variables, whose variation, it is true, is encompassed within a narrow range. If, furthermore, the form of the powder, i.e., $\varkappa$, and the ratio of heat capacities $1 + \theta$ are predetermined, the values of $p_{tab.}$ and X at the pressure maximum and at the end of burning can be summarized in the form of tables with three entries: Λ_Δ, B, and z_0. In this connection, the remaining quantities which have not received definite values are the propellant force of the powder f, the covolume of the powder α, and the density of the powder δ. Unfortunately, it is impossible to vary the propellant force of the powder within wide limits, since the quantity $1 + \theta$, which is connected with it, is predetermined.

This principle was used by Professor B.N. Okunev in the formulation of his tables [13]. In the first table, the values for $p_{tab.}$, X, $\nu = v/v_{\Pi p}$, and $\tau = t/T$ are given as functions of Λ_Δ, $R = \sqrt{2/B\theta}$, and z_0 in the supporting points of the pressure curve. In the expressions for ν and τ, $v_{\Pi p}$ is the limiting velocity, and $T = \varphi q/gs\, v_{\Pi p}/p_1$. In the second table, $p_{tab.}$, X, and τ are given as functions of the parameters Λ_Δ, R, and z_0 and of the argument ν. This table makes it possible to construct curves for the pressures and velocities as functions of paths and times.

In respect to these tables, there remains in force what was said above. In varying f, it is necessary to vary $1 + \theta$, but this quantity, in the tables of Professor B.N. Okunev, has the definite value $1 + \theta = 1.20$; for this reason, the tables relate to a definite value of the propellant force of powder f corresponding to this value.

It should also be pointed out that the necessity of placing tabular pressures in the tables creates great complications in all those cases when it is necessary to solve problems under the condition of maintaining p_m constant, as is usually the case in ballistic design.

2. METHOD OF PROFESSOR N.F. DROZDOV [16]

In his work, Professor N.F. Drozdov chose as the relative variable the ratio of the current pressure to the initial pressure: $\Pi = p/p_0$. Likewise, taking the parameter of Professor Okunev, $\Lambda_\Delta = \dfrac{1 - \frac{\Delta}{\delta}}{1 - \alpha\Delta}$ Professor Drozdov replaces it by $\zeta = 1 - 1/\Lambda_\Delta = 1 - \partial$ and introduces two additional parameters:

$$R = \frac{p_0\left(\alpha - \frac{1}{\delta}\right)}{f} = \frac{p_0}{f\delta_1} \text{ and } R_1 = \frac{R}{1 + R} = \frac{1}{\frac{f\delta_1}{p_0} + 1}$$

which are functions of f, α, δ, and p_0.

For the normal tabular values of these constants:

$$R = 0.01121;\ R_1 = 0.01108.$$

Transforming his equations for a powder with a constant burning area, Professor Drozdov reduces the relative maximum pressure and the pressure at the end of burning, as well as the velocity of the projectile in the first period, to the following expressions, where:

$$\gamma = B_1\psi_0/k_1^2,\ \beta = B_1 x/k_1:$$

$$B_1 = \frac{B\theta}{2} - \chi\lambda;\ \text{at } \chi = 1\ \frac{B}{B_1} = \frac{2}{\theta};$$

$$\beta_m = \frac{1}{\frac{B}{B_1} + 2}(1 + \Pi_m R) = \frac{\theta}{2(1 + \theta)}(1 + \Pi_m R);$$

$$\Pi_m = \frac{p_m}{p_0} = \frac{\dfrac{\gamma + \beta_m - \beta_m^2 \, Z_m^{\frac{B}{B_1}}}{\gamma}}{1 - R\dfrac{s_m}{\gamma}},$$

where $s' = \int_0^\beta Z^{\frac{B}{B_1}} d\beta$ is found from the tables in the Appendix (Appendix IV) for B/B_1 from 6 to 10.

These expressions are convenient for investigating the question of the influence of variations in powder characteristics and of various parameters generally upon the value of the maximum pressure, as well as for solving the problem of the transition from one set of powder characteristics to another.

For the end of burning:

$$\Pi_K = \frac{p_K}{p_0} = \frac{\frac{\gamma + \beta_K - \beta_K^2}{\gamma} Z_K^{\frac{B}{B_1}}}{1 - R \frac{s_K'}{\gamma}} = \frac{(\gamma + \beta_K - \beta_K^2) Z_K^{\frac{B}{B_1}}}{\gamma - R s_K'} .$$

The velocity of the projectile in the first period:

$$v = \beta \sqrt{\frac{B}{B_1} \frac{R_1}{R} \frac{p_0 s l_0 \left(1 - \frac{\Delta}{\delta}\right)}{\gamma \varphi m}} ,$$

where

$$R_1/R = 1/1 + R = 1/1 + p_0/f\delta_1 .$$

In the second period:

$$\Pi_A = \frac{p_A}{p_0} = \Pi_K \left(\frac{\Lambda_K + 1 - \alpha\Delta}{\Lambda_A + 1 - \alpha\Delta}\right)^{1+\theta} \Pi = \Pi_K \eta_1^{1+\theta} ;$$

$$v_{\text{д}}^2 = v_{\text{пр}}^2 \left[1 - \eta_1^{\theta} K \right],$$

where

$$K = \left[1 - \frac{B\theta}{2} (1 - z_0)^2 \right].$$

On the basis of the relations obtained, Professor N.F. Drozdov solved a number of problems relating to the influence of the parameters entering into the equation for the maximum pressure upon the magnitude of this pressure, utilizing for this purpose a large number of new tables formulated by him and appended to his above-mentioned work.

Thus, he determined the influence of the following factors upon the variation of the maximum pressure p_m: variation of the initial pressure p_0, variation of the propellant force of the powder f, the quantity θ, the powder density δ, the covolume α, and the transition from a powder with one set of form characteristics to a powder with another set of characteristics.

For certain partial loading conditions, his computations led to the following results.

a) Influence of Initial Pressure p_0 upon Value of p_m.

p_0	Δp_0	p_m	Δp_m
200		2364	
	100		158
300		2522	
	150		240
450		2762	

From this, it is possible to derive the relation:

$$\Delta p_m \approx 1.6\Delta p \ .$$

i.e., the difference between maximum pressures is greater than the difference between initial pressures.

The difference increases with diminishing p_0.

p_0	Δp_0	p_m	Δp_m	k_p
450		2762		
	150		240	1.60
300		2522		
	100		158	1.58
200		2364		
	92		180	1.96
108		2184		
	54		146	2.70
54		2038		

b) Influence of Variation of α.

α	$\Delta\alpha$	p_m	Δp_m
0.98		2767	
	0.08		-104
0.90		2663	

As the covolume α diminishes by 0.01, the pressure p_m decreases by 13 kg/cm^2.

c) Influence of Variation of δ.

δ	$\Delta\delta$	p_m	Δp_m	$\frac{1}{\delta}$	$\Delta\left(\frac{1}{\delta}\right)$
1.60		2522		0.625	
	-0.04		+24		+0.015
1.56		2546		0.640	

As the specific volume of the powder increases by 0.01, the pressure p_m increases by 16 kg/cm^2.

Consequently, similar variations in α and $1/\delta$ result in nearly the same change in the quantity p_m.

d) Influence of Variation of χ.

χ	1.00	1.02	1.045	1.06
p_m	2377	2425	2481	2500

p_m increases almost proportionally to the quantity :

$$\frac{\Delta p_m}{p_m} \approx \frac{\Delta \chi}{\chi}.$$

e) Influence of Θ.

Θ	p_m	Δp_m	$k=1+\Theta$
0.25	3366		1.25
		121	
0.20	3487		1.20
		114	
0.16	3601		1.16

$$\frac{\Delta p_m}{p_m} \approx -1.1\frac{\Delta k}{k}.$$

f) Influence of f.

B = const.

$f, \frac{T\cdot m}{kg}$	Δf	p_m	Δp_m
104.5		2724	
	-9.5		-202
95		2522	
	-15.8		-328
79.2		2194	

$$\frac{l_K^2}{f} = \text{const.}$$

$$\frac{\Delta p_m}{p_m} \approx 0.8\,\frac{\Delta f}{f}$$

l_K = const.

$f \frac{T \cdot m}{kg}$	Δf	p_m	Δp_m
104.5		2997	
	-9.5		-475
95		2522	
	-8.7		-374
86.3		2148	

$$\frac{\Delta p_m}{p_m} \approx 1.75 \frac{\Delta f}{f}$$

Subsequently, these new tables of Professor N.F. Drozdov were utilized in certain investigations connected with the consideration of variations in the initial pressure.

3. FORMULAS AND TABLES OF M.S. GOROKHOV AND L.I. SVIRIDOV [14]

By utilizing the parameters and variables γ and β of the method of N.F. Drozdov, and by introducing a new parameter D_1, it becomes possible to transform the formula for the path of the projectile to the following form:

$$\theta = \frac{l}{l_\Delta} + 1 = N(\gamma,\beta)\left[1 - D_1 L(\gamma,\beta,n)\right] + \rho,$$

where

$$l_\Delta = l_0 \quad 1 - \frac{\Delta}{\delta} \quad ; \quad D_1 = D\,\frac{n(1+\theta)}{2+n} \quad ; \quad D = \frac{\alpha - \frac{1}{\delta}}{\frac{1}{\Delta} - \frac{1}{\delta}}\,\frac{k_1^2}{B_1} \quad ;$$

$$k_1^2 = x^2 - 4x\lambda\psi_0; \quad B_1 = x\lambda + B\,\frac{\theta}{2}; \quad n = \frac{B}{B_1} \quad ;$$

$$N(\gamma,\ \beta,\ n) = Z^{-n}(\gamma,\beta);$$

$$L(\gamma,\ \beta,\ h) = \gamma + \int_0^\beta \frac{d\beta}{N(\gamma,B,n)} - \frac{\gamma+\beta-\beta^2}{N(\gamma,\beta,n)} \quad ; \quad \rho = \frac{\theta}{2}\,Dn\beta^2.$$

For the function $\log Z^{-1} (\gamma, \beta)$, detailed four-place tables relating it to γ and to β have been set up. For the function $L (\gamma, \beta, n)$, tables relating it to β, γ, and n have been set up; in this connection, the parameters and variables vary within the following ranges:

$$0.00 \leqslant \gamma \leqslant 0.13; \; 0.00 \leqslant \beta \leqslant 0.70; \; 3 \leqslant n \leqslant 14.$$

The velocity of the projectile in the period of burning is determined with the aid of the usual formula.

The formula for the pressure can be transformed to the following form:

$$\Pi = \frac{\alpha - \frac{1}{\delta}}{f} p = \frac{D\xi(\gamma, B)}{\theta - \rho - D\xi(\gamma, \beta)},$$

where

$$\xi (\gamma, \beta) = \gamma + \beta - \beta^2.$$

To determine the maximum pressure, it is necessary to determine the β_m corresponding to it with the aid of the following formula:

$$\beta_m - \frac{1}{2 + n} = D_1 \left\{ \left(\gamma + \int_0^{\beta_m} \frac{\alpha\beta}{N(\gamma, \beta, n)} \right) \left(\beta_m - \frac{1}{2 + n} \right) + \frac{\gamma + \beta_m - \beta_m^2}{(2 + n) N(\gamma, \beta, n)} \right\}.$$

Detailed three-place tables have been set up for $\beta_m - 1/2 + n$. With the aid of the tables mentioned above, the fundamental problem of internal ballistics is solved for any values of the constants α, δ, f, θ, p_0, and χ present as definite values in the tables of N.F. Drozdov and of the GAU Artillery Committee.

The above-mentioned formulas and tables were first published in the work by M. Gorokhov on "Internal Ballistics" in 1943 [17].

Analogous formulas and tables (at the predetermined value $\theta = 0.2$) were obtained by Gorokhov and Sviridov in 1939 [14, 17].

Generalized formulas, accompanied by the use of tables, make it possible to solve problems in cases involving deviations from the usually accepted values for the constants (α, δ, f, p_0, θ, and $\varkappa$), as well as in the case of a combined charge consisting of any desired number of grades of powders and in the case of its being necessary to take into account the afterburning of decomposition residues from the burning of powders of the progressive form.

In this connection, in the course of the first phase, when all the powders burn simultaneously, the formulas remain the same as those presented above; but in the course of the second and subsequent phases, the formulas become somewhat more complex.

CHAPTER 5 - FUNDAMENTALS OF THEORY OF SIMILITUDE

1. THEORETICAL PRINCIPLES.

Ballistically similar guns are those in which the gas-pressure curves ($p-l$) and the projectile-velocity curves ($v-l$) are geometrically similar, i.e., can be made to coincide merely by changing the scale alone.

Algebraically, the condition of similitude may be expressed by the equations $F_1(p, l) = F_2(\alpha_1 p, \alpha_2 l)$ for pressure curves and $\Phi_1(v, l) = \Phi_2(\beta_1 v, \beta_2 l)$ for velocity curves, where α and β are coefficients of the change in scale leading to coincidence of the $p-l$ and the $v-l$ curves.

The theory of similitude in internal ballistics was developed in the Soviet Union by Professor I. P. Grave [18]; the field was further developed by Professor B.N. Okunev [19], who contributed generalized relations.

Conclusions based on the theory of similitude can find application in the transition from guns of one caliber to those of another; they can also be applied to the ballistic design of new systems by making it possible to utilize data for already existing guns on the assumption that the conditions of the shot remain unchanged regardless of the size of the caliber of the gun, an assumption scarcely justified by actual facts.

Investigation shows that the p-l and v-l curves at different densities of loading can be similar only if $\alpha = 1/\delta$, which is not actually the case. For this reason, the case of different Δ is not considered, and only the case of Δ = const. is utilized.

The fundamental equations in the theory of similitude are the following equations of internal ballistics reduced in terms of relative variables for the formulation of ballistic tables:

$$\frac{p}{f} = \Delta \frac{\psi - \frac{B\theta}{2} x}{\Lambda_{\psi} + \Lambda};$$

$$\Lambda = \Lambda_{\psi_{av.}} (Z_x^{\frac{B}{B_1}} - 1);$$

$$v\sqrt{\frac{\varphi q}{\omega}} = v_{tab.} = \sqrt{fgB}\, x;$$

$$\psi = \psi_0 + \chi\sigma_0 x + \chi\lambda x^2;$$

$$\psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}};$$

$$z_0 = \frac{2\psi_0}{\chi(\sigma_0 + 1)} \approx \frac{\psi_0}{\chi};$$

$$\Lambda_\psi = 1 - \frac{\Delta}{\delta} - \Delta\left(\alpha - \frac{1}{\delta}\right)\psi;$$

$$B_1 = \frac{B\Theta}{2} - \chi\lambda;$$

$$B = \frac{s^2 I_K^2 g}{f\omega\varphi q}.$$

In order that the p-l and v-l curves in two guns be similar at the same Δ, it is necessary that the values of p, v, and Λ for one and the same value of x be the same. For this condition to be fulfilled, the following conditions must in turn be satisfied.

1) The nature of the powder (f, α, δ, Θ) must be the same.

2) The form of the powder (χ, λ) must be the same.

3) p_0/f or ψ_0 must be the same.

4) The parameter of the loading conditions B must be the same.

Only under these conditions will B_1, ψ_0, ψ, Λ_ψ, $\Lambda_{\psi av.}$, $v_{tab.}$, v $\left(\sqrt{\varphi q/\omega}\text{ is a scale factor}\right)$, $\gamma = B_1\psi_0/k_1^2$, $\beta = B_1 x/k_1$, $\log Z_x^{-1}$, Λ, and p also be the same, and consequently the curves for the pressure, p-Λ, and for the velocity, v-Λ, will coincide in all points, i.e., p-l and v-l will be similar for the entire first period.

For the end of burning ($\psi = 1$), we shall have the same values for v_K, p_K, Λ_K, and Λ_1.

For the second period:

$$p = p_K\left(\frac{\Lambda_K + 1 - \alpha\Delta}{\Lambda + 1 - \alpha\Delta}\right)^{1+\theta};$$

$$\left(v\sqrt{\frac{\varphi q}{\omega}}\right)^2 = v_{tab.}^2 = \frac{2gf}{\theta}\left\{1 - \left(\frac{\Lambda_K + 1 - \alpha\Delta}{\Lambda + 1 - \alpha\Delta}\right)^{\theta}\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]\right\}.$$

Here, the independent variable is $\Lambda > \Lambda_K$, and, since all remaining parameters in the two guns are the same, the p-Λ and $v_{tab.}$-Λ curves will also coincide, i.e., will be similar, in the second period as well.

In this connection, it is possible to draw the conclusion that, as a matter of fact, all our ballistic tables, beginning with the tables of Professor Drozdov and ending with the most complete GAU tables, are an expression and practical application of the theory of similitude. Indeed, at a given loading density Δ and at a given value of B, the quantity p_m and the p-Λ curve, as well as the $v_{tab.}$-Λ curve, depend neither upon the caliber of the gun nor upon its absolute dimensions, but only upon the ratio of Λ_m to Λ, while $v_{tab.\Lambda}$ depends upon Λ_A. The factors $\sqrt{\omega/\varphi q}$ take into account the influence of the ratio ω/q and are scale factors used to reduce different v-Λ curves to one and the same common $v_{tab.}$-Λ curve.

The parameter of the loading conditions $B = s^2 I_K^2 g/f\omega\varphi q$ is dimensionless. For the analysis of the influence of various conditions of loading upon its magnitude, and, through it, upon the pressure and velocity curves, it is more convenient to write it differently:

$$B = \frac{n_s^2 g}{f} \frac{\left(\frac{I_K}{d}\right)^2}{\frac{\omega}{q}\varphi\, c_q^2}.$$

From an equality of the parameter B for two guns of different caliber, it follows that, in making the transition from one caliber to the other, with the weight of the projectile and ω/q remaining unchanged, the ratio $I_K/d:c_q$ or $I_K:q/d^2$ must also remain constant.

Consequently, it is possible to draw the following conclusion.

For similar guns of different calibers firing projectiles of the same weight, and with p_m = const., the pressure impulse I_K must be inversely proportional to the square of the caliber of the guns.

2. SOME THEOREMS OF THEORY OF SIMILITUDE.

Definitions. 1. Geometrically similar barrels are barrels in which the linear dimensions of the parts of the bore are proportional to the calibers, the cross sections of the bore are proportional to the squares of the calibers, and the chamber and bore volumes are proportional to the cubes of the calibers.

2. Similarly charged guns are guns in which the weights of the projectiles and of the charges are proportional to the cubes of the calibers.

Theorem 1. In geometrically similar and similarly charged guns, similar p-ℓ and v-ℓ curves can be obtained only if the powder thicknesses or the impulses I_K are proportional to the calibers of the bores.

From the condition of equality of the parameters B' and B", we have:

$$\frac{\left(\frac{I'_K}{d'}\right)^2}{\frac{\omega'}{q'}\varphi' c'^2_q} = \frac{\left(\frac{I''_K}{d''}\right)^2}{\frac{\omega''}{q''}\varphi'' c''^2_q}$$

or

$$\left(\frac{I''_K}{d''}\right)^2 : \left(\frac{I'_K}{d'}\right)^2 = \frac{\omega''}{\omega'}\,\frac{q'}{q''}\,\frac{\varphi''}{\varphi'}\,\frac{c''^2_q}{c'^2_q},$$

but the condition of similarly charged guns gives us:

$$c_q'' = c_q'; \frac{\omega''}{\omega'} = \frac{d''^3}{d'^3}; \quad \frac{q'}{q''} = \frac{d'^3}{d''^3} \quad \frac{\omega''}{\omega'}\frac{q'}{q''} = 1.$$

Since

$$\omega'' = \omega'\left(\frac{d''}{d'}\right)^3; \quad q'' = q'\left(\frac{d''}{d'}\right)^3,$$

it follows that

$$\frac{\omega''}{q''} = \frac{\omega'}{q'} \text{ and } \frac{\varphi''}{\varphi'} = 1.$$

Consequently

$$\frac{I_K''}{d''} = \frac{I_K'}{d'},$$

and the theorem is proved.

Theorem 2. In similar and similarly charged guns, equal relative paths Λ will be associated with equal pressure and projectile velocities.

The equality of pressures follows from the ballistic similarity of curves at the same Δ and equal B, and since, for similarly charged guns:

$$\frac{\omega'}{\varphi' q'} = \frac{\omega''}{\varphi'' q''} \text{ and } v'_{tab.} = v''_{tab.},$$

it is also true that

$$v' = v''.$$

Theorem 3. In using one and the same gun for firing projectiles of different weights while maintaining the charge and the maximum pressure constant, the velocities of the projectiles are inversely proportional to the square root of the ratio of the products of the projectile weights multiplied by the corresponding coefficient φ. As a matter of fact, under the predetermined conditions, at given Δ and B and $\Lambda_{\text{д}}$, $v'_{tab.} = v''_{tab.}$ or:

$$v'_{Д} : \sqrt{\frac{\omega}{\varphi' q'}} = v''_{Д} : \sqrt{\frac{\omega}{\varphi'' q''}} ,$$

from which

$$\frac{v''_{Д}}{v'_{Д}} = \sqrt{\frac{\varphi'}{\varphi''} \frac{q'}{q''}} .$$

In order to maintain p_m = const., we obtain from the condition B' = B":

$$\frac{I'^2_K}{\varphi' q'} = \frac{I''^2_K}{\varphi'' q''}.$$

or

$$\frac{I''_K}{I'_K} = \sqrt{\frac{\varphi'' q''}{\varphi' q'}} ; \quad I''_K v''_{Д} = I'_K v'_{Д} ,$$

i.e., the impulses I_K must be directly proportional to the square root of the product of the projectile weights multiplied by the corresponding coefficients φ.

In limiting consideration to these most important among the theorems of similitude and refraining from a discussion of the other numerous theorems, it is possible merely to point out that, as a rule, when a transition is made from a gun of one caliber to one of another, a change in the thickness of the powder is accompanied by a certain change in its nature, as well as in the initial pressure and in the heat loss.

This makes it necessary to consider the theorems of the theory of similitude in internal ballistics as being capable of giving merely approximate conclusions and relations.