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INTERIOR BALLISTICS

by

M. E. Serebryakov

State Printing House of the Defense Industry

Moscow, 1949, 2nd Edition

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PART TWO - THE THEORY AND PRACTICE OF SOLVING PROBLEMS IN INTERNAL BALLISTICS

(THEORETICAL AND APPLIED PYRODYNAMICS)

INTRODUCTION

On the basis of the widespread study of the phenomena and processes occurring during a discharge, internal ballistics must establish the laws relating the conditions of loading to the quantities depending upon them - called ballistic elements of discharge - and must furnish the method of solving a large number of problems encountered in practice.

The establishment of such laws, providing the means for regulating a discharge, constitutes the general problem of internal ballistics.

The conditions of loading include the following: the dimensions of the powder chamber and those of the bore of the barrel, the weight of the latter, the arrangement of the rifling in the bore, the weight and arrangement of the projectile, the pressure necessary to overcome the inertia of the projectile, the weight of the charge, the make of powder, the physico-chemical and ballistic characteristics of the powder, the characteristics of the expansion of gases.

The ballistic elements of a discharge include the path of the projectile l, its velocity v, the pressure of the powder gases p, their temperature T, all values varying with time, and also the quantity of gas $\omega\psi$ formed at a given time.

In solving the above general problem of internal ballistics, one may distinguish two fundamental and most important problems of pyrodynamics, and a series of special problems.

The first fundamental problem consists in determining by calculation the change in gas pressure and the velocity of the projectile in the barrel as a function of the path of the projectile and of time, for given loading conditions. Together with the curves p, l-v, l-p, t-v, t two most important loading characteristics of the gun are determined: the maximum gas pressure p_m in the bore, and the muzzle velocity v_A of the projectile, i.e., the velocity of the projectile at the instant it leaves the barrel of the gun. This problem may be called the direct problem of pyrodynamics.

For given conditions of loading it has a single solution - a single pressure curve with maximum p_m, a single velocity curve for the projectile, and a muzzle velocity v_A.

By varying the conditions of loading, it is possible to analyze the effect of these conditions on the variation of the gas pressure and projectile velocity curves, i.e., it is possible to solve a series of special problems related to the solution of the direct problem.

The second fundamental problem of pyrodynamics is the problem of the ballistic design of the gun; it consists in determining the design data of the barrel and conditions of loading necessary to impart some definite initial (muzzle) velocity to a projectile of a given caliber and weight. This velocity is determined from the tactical and technical requirements imposed upon the gun to be constructed.

In solving such a problem, the maximum gas pressure is usually given.

The design data and conditions of loading insuring that a projectile of a given caliber and weight will attain the desired velocity, are obtained from the solution of the above problem. Once the conditions of loading are given, gas pressure and projectile velocity curves are drawn as a function of path and time, i.e., the direct problem of internal ballistics is solved for the selected type of gun and charge.

The obtained curve p, l is used by the engineers to calculate the strength of the gun barrel and projectile shell, while the curve p, t is used to design the carriage, the time fuzes and the igniters. At the same time, the necessary thickness and shape of the powder which must be prepared at the factory, are given.

Thus the further planning of the entire system of artillery and of the necessary ammunition depends to a considerable extent upon the feasibility and rationality of the selected form of the ballistic solution.

This is why the problem of the ballistic design of guns is the principal applied problem of interior ballistics.

The problem of ballistic design is broader than the first problem; it includes the latter as a final step and is in reality an inverse problem of interior ballistics. It admits of numerous solutions, numerous combinations of gun design data and loading conditions under which a projectile of a given caliber and weight will attain the required muzzle velocity.

Because of the indeterminate character of the solution, there arises the need of developing a definite method for obtaining the necessary answer in the shortest possible time, and for selecting from among this multiplicity of solutions the most efficient and desirable solution, satisfying the tactical and technical requirements imposed upon the gun to be designed.

In this connection, special problems arise with regard to finding the most desirable solution, and for obtaining a gun of maximum power and minimum length or volume, the most suitable projectile, and the most desirable loading conditions.

The solution of these special problems permits in turn to pose the problem of the development of a general theory and method of ballistic design which would take into account the most desirable solutions and tactical and technical requirements.

Besides the indicated fundamental problems of internal ballistics, there is also a series of special and secondary problems introduced below.

For a given bore and a given projectile weight, calculate the weight ω of the projectile insuring a given muzzle velocity v_{μ}, and the thickness $2e_1$ of powder giving the required maximum pressure p_m.

Because of the complexity of the phenomenon of discharge, not all of its details can be taken into account, even approximately; some of these details must be neglected and can not be introduced into the mathematical equations expressing the relations between the separate processes occurring during a discharge.

For this reason, the equations of internal ballistics give only approximate values of p, v, l, ψ, and t. But since in practice these equations must give results agreeing with experimental data, it is necessary, in order to insure this agreement, that the problem be solved by selecting certain constant characteristics. When these are substituted into the equations, they give values of p_m and v_A

for the gases and the projectile, respectively, which values correspond to the results of firing tests.

The very manner in which the problem is posed indicates that the processes taking place during a discharge are not yet all known and analyzed. For this reason one of the main problems of internal ballistics, that must be eventually solved, is the exact determination of constants, those of the gun powder in particular, as derived from its physical and chemical properties. The determination of the powder constants involves a more exact method of pressure determination by experimental means, because all the ballistic characteristics (f, α, u_1) are determined from the latter.

In addition to the problems enumerated above, one should note the problem of determining the variation in the maximum gas pressure and in the initial projectile velocity under specific changes in loading conditions, as well as a series of other problems.

The fundamental elements of a discharge - l, v, p, T, ψ and t - are interrelated by a series of equations expressing the fundamental processes taking place during a discharge, i.e.: the burning of the gun powder and the formation of gases, the transformation of the thermal energy of the gases into the kinetic energy of the system projectile - charge - barrel, and the movements of parts of this system.

The methods of solution of theoretical pyrodynamics must make it possible to compute and establish the dependence of gas pressure and of the velocity of the projectile on the path and the time it takes the projectile to move through the gun barrel, i.e., to solve the fundamental direct problem of internal ballistics.

The methods of solving problems in pyrodynamics may be divided into analytical, numerical, empirical and tabular methods.

The empirical methods were of definite advantage, so long as the theoretical concepts of internal ballistics had not been sufficiently developed.

They were based on some relatively simple empirical equations expressing in a simplified form the experimentally obtained interrelations of the elements of a discharge. Tables were used along with these equations, which tables offered the means for computing very rapidly the elements of the curves depicting gas pressure and projectile velocity.

The empirical methods were derived from the analysis of experimental data obtained in firing weapons under different conditions, with the characteristics and constants entering into these expressions determined from the conditions of the experiment.

The disadvantage of these methods (formulas and tables) consists in the fact that they fail to take into account certain very important factors and conditions of loading, and that such methods may be applied only under the conditions and within the limits established for the given case.

The number of empirical equations and tables is very large; prior to the development of analytical solutions, they were of primary value because of their simplicity. But the appearance of exact theoretical solutions, taking into account with sufficient completeness the influence of most of the conditions of loading and singularities of the processes occurring during a discharge, made it possible to solve all the fundamental problems of pyrodynamics by means of exact analytical relations. As a result, many empirical equations and tables have lost their significance and are now used only in certain auxiliary cases.

The analytical methods are based on a series of assumptions characterizing the conditions of powder burning and the motions of the gases, projectile and gun; these assumptions are based chiefly on experimental or theoretical data expressing the physical side of the process of discharge.

For this reason, analytical methods of solution give a more profound understanding of the real nature of the phenomenon than empirical methods, and approach more closely the essence of the processes taking place during a discharge.

In the analytical method the problem is reduced to the solution and integration of differential equations of different types. This solution can be obtained with greater accuracy (in which case the resulting equations become more complex), or approximately (which results in simpler relations).

Solutions may be given for the more complex cases obtained in practice, and also for simplified, admittedly schematic cases, in which case the analysis of the relationships is simplified.

Solutions may be based on the geometric and physical laws of powder burning.

Tables of auxiliary values or functions, necessary to calculate certain intermediate values, are prepared in order to expedite and simplify the computations involved in the solution of problems.

Numerical methods of integrating a system of differential equations are used along with the analytical methods. The integration is usually performed by the method of finite differences or by expansion in Taylor's series. These methods are resorted to in especially difficult cases, when the value of one or several parameters varies throughout the process of discharge and their variation does not permit to solve the problem analytically in finite form. This happens, for example, when the cross section of a barrel bore varies (tapered bore), or when the parameter θ varies throughout the discharge accompanied by a varying gas temperature or by a change of the coefficient φ depicting to secondary work done in the process, etc.

On the basis of analytical or numerical solutions, it is possible to set up numerical tables of the fundamental elements (p, v, l, t) for different loading conditions and some general constants. These tables enable one to plot very rapidly the necessary curves p, l and v, l or p, t and v, t with a minimum number of calculations. In so doing, the process of solving the direct problem is greatly simplified and expedited. These tables are usually set up for certain average values of the constants (characteristics and shape of the powder), although in practice one may encounter a series of regressions from these average values. In that case it is necessary to introduce appropriate corrections into the results obtained.

Thus the ballistic tables for the solution of direct problems of pyrodynamics are in reality analytical equations reduced to numerical values in a series of concrete loading conditions.

However, the ballistic tables enable one to solve a series of problems which cannot be solved directly by means of analytical equations.

The basic difference between tabular values and analytical equations is the following: because of their complexity, the analytical equations do not give a direct relationship between pressure or velocity and path length, for example; these variables are usually related through some auxiliary variable.

In tables, on the contrary, the basic elements of discharge are interrelated directly: the pressure, the projectile velocity, and the time of its travel through the barrel are given in function of the path traversed by the projectile; this simplifies considerably the analysis and permits the development of a special method for solving problems which cannot be solved by analytical means.

The development of the theory of ballistic design became possible only with the introduction of tables for the solution of internal ballistic problems.

For this reason the first tables prepared in our country by Prof. N.F. Drozdov on the basis of his exact solution given in 1910, are of great importance. These very tables simplified and expedited the calculations involved in the ballistic design of weapons, and gave the engineers a reliable means of solving rapidly inverse problems in pyrodynamics.

They also served as an example for a series of more detailed tables compiled subsequently.

SECTION SIX - ANALYTICAL METHODS OF SOLUTION OF THE DIRECT PROBLEM OF INTERNAL BALLISTICS.

BASIC ASSUMPTIONS

When we examined the phenomenon of a discharge, we had pointed out its extreme complexity and the fact that some of the factors influencing the results were still insufficiently known. For this reason, when solving theoretically the fundamental equation and deriving the relations between the physico-chemical and mechanical phenomena in a discharge, it is necessary to take recourse to certain simplifications and schemes.

The basic assumptions are as follows:

1) The burning of powder obeys the geometrical law of combustion.

2) The powder burns under an average pressure p.

3) The composition of the products of combustion does not change during burning, nor during the adiabatic expansion of the gases (f and α are constant) after the powder is burned.

4) The rate of burning is proportional to the pressure:

$$u = u_1 p.$$

5) The auxiliary work done is proportional to the principal work of the forward motion of the projectile, and is represented by the coefficient φ.

6) The projectile starts moving when the pressure developed in the chamber by the partial burning of the charge equals p_0, i.e., when the pressure is sufficient to force the driving band completely into the rifling of the bore; the gradual forcing of the band and the increasing resistance encountered by it are not taken into account.

7) The work done in forcing the driving band is not accounted for separately, nor the increasing velocity of the projectile during the gradual forcing of the band.

8) The expansion of the barrel during the discharge, the gases escaping through the clearance between the driving band and the walls of the gun, and the air resistance are disregarded.

9) The cooling of the gases through heat transfer to the walls of the barrel is not accounted for directly, and may be taken into account indirectly (for example, by decreasing the force $f = RT_1$ or increasing $\theta = 1/(A + BT_{av.})$.

10) The motion of the projectile is considered only until it passes the muzzle face.

11) The quantity $\theta = (c_p/c_w) - 1$ is taken as its average value, constant throughout the discharge.

The assumptions enumerated above make our representation of a discharge more schematic, and deviate the phenomenon to a greater or lesser degree from reality. For this reason the relations obtained in the solution will express the physico-mechanical nature of the discharge only with a certain degree of approximation. Thus the values of the fundamental elements (maximum gas pressure p_m, initial or muzzle velocity v_o of the projectile) obtained from these equations may not coincide with the values obtained experimentally. Nevertheless, in order to solve practical problems, it is necessary to obtain analytical data which would agree with experiment. For this reason (keeping in mind the complexity of the discharge phenomenon, the incomplete knowledge of its elements, and the disagreement between our basic assumptions and reality) it is necessary to introduce coefficients of "agreement with experiment" into the constants obtained.

Such a method is widely used in various scientific laws (hydrodynamics, aerodynamics, etc.) dealing with complex phenomena, whose details cannot be fully analyzed.

Eventually, as our knowledge is further developed, we will find it possible to render some of the assumptions with greater accuracy and take into account some of the conditions not yet understood. As new experimental data is accumulated and new methods are applied, the deductions arrived at may be modified and even replaced by others of a more complete and exact nature.

In solving the fundamental equation of pyrodynamics, one should strive to obtain the maximum possible mathematical accuracy. However, in that case some of the expressions become excessively cumbersome, so that even exact formulas will fail to represent the true phenomena of a discharge; and for this reason certain simplifications may be used with advantage in the process of solution.

A comparison of these simplified solutions with the exact ones may show the extent of mathematical error involved with the use of the same constants and conditions.

With an appropriate selection of constants, the somewhat simplified solutions may also yield results approaching experimental data as closely as those obtained by the use of more exact equations.

CHAPTER 1 - SOLUTION OF THE FUNDAMENTAL PROBLEM WHEN THE PRESSURE TO OVERCOME THE PROJECTILE INERTIA IS KNOWN, AND WHEN BURNING PROCEEDS ACCORDING TO THE GEOMETRIC LAW.

As we have shown above, the fundamental equation of pyrodynamics includes a large number of constants characterizing the projectile, charge and powder which determine the conditions of loading, and the four variables, ψ, v, l and p, which are called the elements of a shot.

In order to establish the relation between the elements of a shot, new equations are added to the fundamental equation, which are the equations of powder burning and projectile motion; this leads to the appearance of a new variable, the time t, and to the appearance of quantity z when burning proceeds according to the geometrical law.

We obtain as a result the following system of equations:

The fundamental equation of pyrodynamics:

$$ps(l_\psi + l) = f\omega\psi - \frac{\theta}{2}\varphi mv^2. \tag{1}$$

The rate of powder burning:

$$u = \frac{de}{dt} = u_1 p. \tag{2}$$

The law of generation (inflow) of gases:

$$\psi = \varkappa z(1 + \lambda z) = \varkappa z + \varkappa\lambda z^2. \tag{3}$$

The law of motion of the projectile:

$$ps = \varphi m \frac{dv}{dt} \tag{4}$$

or

$$ps = \varphi mv \frac{dv}{dt}. \tag{5}$$

The totality of these equations affords the solution of the fundamental mathematical problem: of determining the curves p, l

and v, l and also p, t and v, t and of finding in particular the maximum gas pressure p_m and the muzzle velocity v_μ of the projectile.

We first solve the problem for regressive powder shapes, using the two-term formulas ($\varkappa > 1$, $\lambda < 0$, $\mu = 0$) and the assumptions enumerated above.

We shall solve the problem for all the periods of a shot in succession.

1. PRELIMINARY PERIOD

When establishing the relations for this period, we shall assume the simplest form of the phenomenon: the instantaneous forcing of the projectile band into the rifling.

Fundamental assumption. If the force necessary to overcome the resistance encountered by the driving band of the projectile in completely penetrating the rifling is Π_0, and the cross section of the bore is s, the quantity $p_0 = \Pi_0/s$ will be called "the pressure to overcome the inertia of the projectile" or the forcing pressure. We shall assume that the projectile is set in motion at the instant the gas pressure attains the value p_0.

Up to that moment the burning of the powder takes place in a constant volume. For this reason the preliminary period may be called pyrostatic and one may apply the already known equations of pyrostatics.

In this period, besides the forcing pressure p_0, we will be interested in the portion of the charge ψ_0 burned at the instant the projectile is set in motion, in the relative thickness of the powder $z_0 = e_0/e_1$, and in the relative surface area of the powder $S_0/S_1 = \sigma_0$.

These quantities, characterizing the end of the preliminary period, are simultaneously the initial values of the first period.

Let us introduce the fundamental equations for the preliminary period.

The igniter is burned first and the pressure developed in the chamber is p_B, which may be computed by the following formula:

$$p_B = \frac{f_B \omega_B}{W_0 - \frac{\omega}{\delta} - \alpha_B \omega_B}, \tag{6}$$

where W_0 is the volume of the chamber; ω/δ is the volume of the charge proper; and f_B, α_B, ω_B are, respectively, the force, co-volume, and weight of the igniter. Under the usual conditions of ignition, $\alpha_B \omega_B$ may be neglected.

The charge proper will ignite when the pressure reaches p_B; at this instant the pressure is determined by the general equation of pyrostatics, which takes into account the effect of the igniter. At the instant the driving band is forced in the rifling, a certain portion of the charge ψ_0 will have burned, and:

$$p_0 = p_B + \frac{f \omega \psi_0}{W_0 - \frac{\omega}{\delta} - \frac{\omega}{\delta_1} \psi_0}, \tag{7}$$

where

$$\frac{1}{\delta_1} = \alpha - \frac{1}{\delta}.$$

Inasmuch as the forcing pressure p_0 is known(*), we can determine

(*) p_0 varies between 250 and 400 kg/cm^2 for shells and between 300-500 kg/cm^2 for bullets when the entire side surface is forced into the rifling of the bore.

what part ψ_0 of the charge will have been burned at the instant the projectile is set in motion. Solving equation (7) for ψ_0, we obtain:

$$\psi_0 = \frac{(p_0 - p_B)\left(\frac{1}{\Delta} - \frac{1}{\delta}\right)}{f + (p_0 - p_B)\left(\alpha - \frac{1}{\delta}\right)} = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0 - p_B} + \frac{1}{\delta_1}}. \tag{8}$$

If we may neglect the pressure of the igniter, because p_0 is known only approximately, while p_B is small; we will obtain a simpler expression for computation:

$$\psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}}. \tag{9}$$

The quantity ψ_0 mainly depends on Δ and varies, in general, between 0.02 and 0.10.

If the amount ψ_0 of the burned portion of the powder is known, and the law of powder burning is in the form:

$$\psi = \varkappa z(1 + \lambda z) = \varkappa z + \varkappa \lambda z^2,$$

we can determine the relative thickness $z_0 = e_0/e_1$ of the powder burned at the start of motion, and the relative surface area σ_0.

We find σ_0 from the following formula:

$$\sigma_0 = \sqrt{1 + 4\frac{\lambda}{\varkappa}\psi_0},$$

and z_0, from the equation $\sigma_0 = 1 + 2\lambda z_0$:

$$z_0 = \frac{\sigma_0 - 1}{2\lambda} = \frac{(\sigma_0 - 1)(\sigma_0 + 1)}{2\lambda(\sigma_0 + 1)} = \frac{\sigma_0^2 - 1}{2\lambda(\sigma_0 + 1)} = \frac{2\psi_0}{(\sigma_0 + 1)\varkappa}.$$

Since usually $\sigma_0 \approx 1$, the following approximation is correct:

$$z_0 \approx \frac{\psi_0}{\varkappa}.$$

Besides these characteristics, we will also require the value l_{ψ_0} - the length of the free space in the chamber at the start of motion. This reduced value is determined from one of the following expressions:

$$l_{\psi_0} = \frac{W_{\psi_0}}{s} = \frac{1}{s}\left(W_0 - \frac{\omega}{\delta} - \frac{\omega\psi_0}{\delta_1}\right) = l_0\left(1 - \frac{\Delta}{\delta} - \frac{\Delta}{\delta_1}\psi_0\right) =$$

$$= l_0\Delta\left(\frac{1}{\Delta} - \frac{1}{\delta} - \frac{\psi_0}{\delta_1}\right),$$

where

$$l_0\Delta = \frac{W_0}{s}\,\frac{\omega}{W_0} = \frac{\omega}{s}.$$

2. FIRST PERIOD

In deriving the fundamental relationships for the first period, Prof. N.F. Drozdov was the first to propose the introduction of a new independent variable, $x = z - z_0$ (the relative thickness of the powder burned after the projectile is set in motion).

At the instant the projectile is set in motion $z = z_0$ and $x = 0$; at the ending of burning $z_K = 1$ and $x_K = 1 - z_0$.

Thus the limits of variation of the new argument are known in advance. Let us express all four fundamental elements, ψ, v, l, and p, as a function of this argument.

1. Relation $\psi = f_1(x)$. Substituting $z = z_0 + x$ in the formula $\psi = \varkappa z + \varkappa\lambda z^2$, we obtain:

$$\psi = \varkappa z_0 + \varkappa\lambda z_0^2 + \varkappa(1 + 2\lambda z_0)x + \varkappa\lambda x^2,$$

but

$$\varkappa z_0 + \varkappa\lambda z_0^2 = \psi_0; \; 1 + 2\lambda z_0 = \sigma_0.$$

Introducing, according to Drozdov, the additional designation $\varkappa\sigma_0 = k_1$, we obtain the desired relation:

$$\psi = \psi_0 + k_1 x + \varkappa\lambda x^2. \tag{10}$$

2. Relation $v = f_2(x)$. The velocity v enters into the equation of motion:

$$sp = \varphi m \frac{dv}{dt}.$$

In order to eliminate p and t, we add the law governing the rate of burning:

$$u = \frac{de}{dt} = u_1 p.$$

Multiplying these equations term by term and simplifying, we obtain:

$$dv = \frac{s}{\varphi m}\frac{de}{e_1} = \frac{se_1}{\varphi m u_1}dz = \frac{sI_K}{\varphi m}dz.$$

Integrating from 0 to v and from z_0 to z:

$$v = \frac{sI_K}{\varphi m}(z - z_0) = \frac{sI_K}{\varphi m}x. \tag{11}$$

Prior to the end of burning

$$v_K = \frac{sI_K}{\varphi m}(1 - z_0) = \frac{g}{\varphi}\frac{I_K}{q:s}(1 - z_0). \tag{12}$$

Consequently, the velocity of the projectile at the end of burning can be computed in advance, if the impulse of the powder pressure $I_K = e_1/u_1$ and the cross-sectional loading of the projectile are known.

Inasmuch as the quantities φ and $1 - z_0$ vary relatively little, the velocity of the projectile at the end of burning of the powder depends in the main on the ratio of the impulse I_K to the cross-sectional load q/s on the projectile, and during burning, the velocity of the projectile varies in proportion to x.

Equation (12) permits to compute v_K, but it does not tell us the point on the path the projectile at which the powder is burned, whether the speed v_K is properly chosen for the given gun, or whether the powder is fully burned before the projectile leaves the gun. For this reason, this equation alone is insufficient, and it is necessary to find also the equation for the path traversed by the projectile at the end of burning.

Equation (12) is plotted in fig. 133; it shows the curve v, l and gives the value of v_K, but it does not show the position of the projectile at the end of burning.

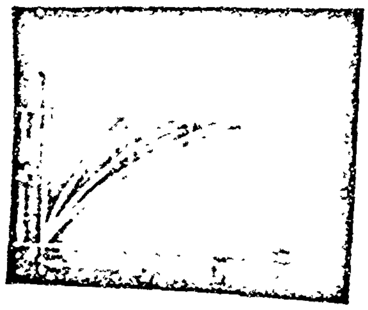

Fig. 133 - Path of the Projectile at the End of Powder Burning.

3. Relation $l = f_3(x)$. In order to determine the path of the projectile, two equations must be used: the fundamental equation of pyrodynamics and the equation of motion of the projectile in the form of elementary work:

$$ps(l_\psi + l) = f\omega\psi - \frac{\theta}{2}\varphi m v^2 = f\omega\left(\psi - \frac{v^2}{v^2_{\Pi p}}\right);$$

$$psdl = \varphi mvdv,$$

where $v^2_{\Pi p} = 2f\omega/\varphi\theta m$.(*)

We eliminate p by dividing the second equation by the first:

(*) The Russian subscripts Пp denote: path traversed inside a barrel. Editor.

$$\frac{dl}{l_\psi + l} = \frac{\varphi m}{f\omega} \frac{vdv}{\psi - \frac{v^2}{v^2_{\text{пp}}}}.$$

Since v and ψ are functions of x according to equations (10) and (11), the right-hand side of this differential equation may be represented as a function of x.

Designating this function by dF(x) and substituting for v and ψ their expressions in x we obtain:

$$dF(x) = \frac{\varphi m}{fm} \frac{\frac{s^2 I_K^2}{\varphi^2 m^2} x dx}{\psi_0 + k_1 x + \varkappa\lambda x^2 - \frac{s^2 I_K^2 \varphi m \theta}{\varphi^2 m^2 2 f\omega} x^2} =$$

$$= \frac{s^2 I_K^2}{f\omega\varphi m} \frac{x dx}{\psi_0 + k_1 x - \left(\frac{s^2 I_K^2}{f\omega\varphi m} \frac{\theta}{2} - \varkappa\lambda\right) x^2}.$$

The same group $s^2 I_K^2 / f\omega\varphi m$ of constants and characteristics appears in the numerator and the denominator. As suggested by Prof. N.F. Drozdov, it is represented by B and is called "the parameter of the loading conditions" (Prof. N.F. Drozdov's parameter):

$$B = \frac{s^2 l_K^2}{f\omega\varphi m} = \frac{s^2 c_1^2}{u_1^2 f\omega\varphi m}.$$

The influence of this parameter will be established later. Let us designate (also according to Drozdov):

$$\frac{B\theta}{2} - \varkappa\lambda = B_1.$$

Then

$$\frac{dl}{l_\psi + l} = \frac{Bxdx}{\psi_0 + k_1 x - B_1 x^2}. \qquad (13)$$

The expression obtained is the fundamental differential equation for the path of the projectile as a function of x. It is solved differently by various authors.

If we place outside the parenthesis $-B_1$ in the denominator of the right side of the equation (as suggested by Drozdov) in order to obtain a polynomial in descending powers of x, with the coefficient of x^2 equal to 1, we obtain:

$$\frac{dl}{l_\psi + l} = -\frac{B}{B_1}\frac{xdx}{x^2 - \frac{k_1}{B_1}x - \frac{\psi_0}{B_1}} = -\frac{B}{B_1}\frac{xdx}{\xi_1(x)}, \qquad (13')$$

where:

$$\xi_1(x) = x^2 - \frac{k_1}{B_1}x - \frac{\psi_0}{B_1}.$$

Prof. N.F. Drozdov was the first to solve this equation exactly, in 1903, by reducing it to the form of a linear equation of the first order:

$$\frac{dl}{dx} + \frac{B}{B_1}\frac{x}{\xi_1(x)}l = -\frac{B}{B_1}\frac{x}{\xi_1(x)}l_\psi$$

or

$$\frac{dl}{dx} + P_x l = Q_x,$$

where P_x and Q_x are functions of x.

The full solution of this equation is presented later.

A simpler, but approximate solution is obtained if we assume $l_\psi = l_{\psi\,av.} = \text{const.}$

It will be presented later with the designation of the parameters, with some of the auxiliary functions derived according to Prof. Drozdov.

During burning of the powder at the start of the projectile's motion, l_ψ varies within the limits of l_{ψ_0} and l_1:

$$l_{\psi_0} > l_\psi \geqslant l_1,$$

where

$$l_\psi = l_0\left[1 - \frac{\Delta}{\delta} - \Delta\left(\alpha - \frac{1}{\delta}\right)\psi\right].$$

The rate of change of l_ψ increases with Δ.

Assuming that $l_\psi = l_{\psi_{av.}}$ and integrating equation (13') we have:

$$\int_0^l \frac{dl}{l_{\psi_{av.}} + l} = -\frac{B}{B_1}\int_0^x \frac{xdx}{\xi_1(x)}.$$

The integral of the right-hand side is obtained by decomposing the integrand into the simplest fractions; it is a logarithmic function of x which we shall temporarily designate by $\ln Z_x$. The left-hand side is integrated also:

$$\ln\left(1 + \frac{l}{l_{\psi_{av.}}}\right) = -\frac{B}{B_1}\ln Z_x,$$

whence:

$$l = l_{\psi_{av.}}\left(Z_x^{-\frac{B}{B_1}} - 1\right). \tag{14}$$

Thus the expression for the path l as a function of x is more complex than the expressions for ψ and v.

Substituting into it $x_K = 1 - z_0$, we can find the path l_K at the end of powder burning. Comparing it with the full path $l_д$ traversed by the projectile within the bore, it is possible to determine whether the thickness of the powder and the velocity v_K of the projectile are correctly chosen for the given gun.

In computing $l_{\psi_{av.}}$ we may use in the equation $l_{\psi_{av.}} = l_0 \left[1 - \frac{\Delta}{\delta} - \Delta\left(\alpha - \frac{1}{\delta}\right)\psi_{av.}\right]$ the expression $\psi_{av.} = \frac{\psi_0 + \psi}{2}$.

The investigations of Prof. G.V. Oppokov in his book "O TOCHNOSTI NEKOTORYKH ANALITICHESKIKH SPOSOBOV RESHENIYA OSNOVNOI ZADACHI VNUTRENNEI BALLISTIKI DLIA PERVOGO PERIODA" (Concerning the Accuracy of Certain Analytical Methods of Solving the Fundamental Problem of Internal Ballistics for the First Period), 1932 [2] have shown the following. When the loading density is $\Delta = 0.5$-0.7, formula (14) is very accurate for evaluating p_m and $v_д$, if $l_{\psi_{av.}}$ is not taken to have the same value for all the values x from 0 to $1 - z_0$, and if a different value of $l_{\psi_{av.}}$ is taken for every value of x, assuming either of the following values for $l_{\psi_{av.}}$ in the formula: $\psi_{av.} = (\psi - \psi_0)/2$ (Oppokov) or $\psi_{av.} = (\psi_0 + \psi)/2$ (Serebryakov).

Inasmuch ~~as x~~ is directly proportional to v [equation (11)], equation (14) gives in fact the direct relation between the path l and the projectile velocity v.

The expression for Z_x presented below shows that this relation is expressed by a rather complex function.

4. Relation $p = f_4(x)$. The pressure p is found from the fundamental equation of pyrodynamics:

$$p = \frac{f\omega}{s} \frac{\psi - \frac{v^2}{v_{\Pi p}^2}}{l_\psi + l}.$$

If the quantities ψ, v, and l are replaced in the right-hand side by their expressions in function of x, then

$$p = \frac{f\omega}{s} \frac{\psi_0 + k_1 x - B_1 x^2}{l_\psi + l_{\psi_{av.}} (Z_x^{-\frac{B}{B_1}} - 1)} = \frac{f\omega}{s} \frac{\psi - \frac{B\theta}{2} x^2}{l_\psi + l}, \tag{15}$$

where l_ψ can be represented as a function of x as well.

Inasmuch as ψ, v, and l are already determined, it is no longer necessary in computing p to use equation (15) which is expressed in terms of x, and the numerical values of ψ, v, l_ψ, and l can be substituted in the preceding equation.

By attributing to x different values within the limits of 0 to $1 - z_0$, equations (10), (11), (14), and (15) permit one to find the values of all the elements ψ, v, l, and p of a shot entering into the fundamental equation of pyrodynamics, and to plot a curve showing the variation of p, v, ψ as a function of l, i.e., the curves p, l and v, l.

Consequently, the proposed problem concerning the solution of the fundamental equation of pyrodynamics has been resolved, and the relation between the elements has been found.

Substituting the value of $x_K = 1 - z_0$ in the above equations, we find all the elements v_K, p_K, and l_K corresponding to the instant the burning of the powder ends ($\psi = 1$). These values will be the initial values in the second period.

Note. The expression for the projectile velocity may be replaced by the following:

$$v^2 \frac{s^2 I_K^2}{\varphi^2 m^2} x^2 = \frac{s^2 I_K^2 \theta}{2 f \omega \varphi m} \frac{2 f \omega}{\varphi m \theta} x^2 = \frac{B\theta}{2} v_{\Pi p}^2 x^2,$$

whence,

$$v = v_{\Pi p} \sqrt{\frac{B\theta}{2} x}.$$

This expression brings out the effect of the limiting velocity of the projectile and that of the parameter of loading conditions, B. Since in most guns B varies within narrow limits, it follows that the velocity mainly depends upon the potential f/θ of the powder and upon the relative weight ω/q of the charge.

<u>Determination of the function</u> $Z_x = e^{\int_0^x x dx/\xi_1(x)}$. In order to evaluate the integral

$$\int_0^x \frac{x dx}{\xi_1(x)} = \int_0^x \frac{x dx}{x^2 - \frac{k_1}{B_1} x - \frac{\psi_0}{B_1}}$$

we decompose the integrand into the simplest fractions, finding the roots of equation $\xi_1(x) = 0$ and introducing the designation:

$$b = \sqrt{1 + 4\frac{B_1\psi_0}{k_1^2}} = \sqrt{1 + 4\gamma} > 1;$$

$$x = \frac{k_1}{2B_1} \pm \sqrt{\frac{k_1^2}{4B_1^2} + \frac{\psi_0}{B_1}} = \frac{k_1}{2B_1}\left(1 \pm \sqrt{1 + 4\frac{B_1\psi_0}{k_1^2}}\right) =$$

$$= \frac{k_1}{2B_1}(1 \pm b); \qquad (16)$$

$$x_1 = \frac{k_1}{2B_1}(1 + b), \quad x_2 = \frac{k_1}{2B_1}(1 - b) < 0;$$

$$\xi_1(x) = (x - x_1)(x - x_2).$$

Let us write an equation to determine the numerators of the simplest fractions:

$$\frac{x}{\xi_1(x)} = \frac{A_1}{x - x_1} + \frac{A_2}{x - x_2};$$

Equating on both sides the coefficients of identical powers of x, we find:

$$A_1(x - x_2) + A_2(x - x_1) = x;$$

$$A_1 + A_2 = 1; \quad -A_1x_2 - A_2x_1 = 0,$$

whence,

$$A_1 = -\frac{x_1}{x_2 - x_1}, \quad A_2 = \frac{x_2}{x_2 - x_1},$$

but:

$$x_2 - x_1 = -\frac{k_1}{B_1} b,$$

and, consequently:

$$A_1 = \frac{b + 1}{2b}, \quad A_2 = \frac{b - 1}{2b},$$

$$\int_0^x \frac{xdx}{\xi_1(x)} = \frac{b + 1}{2b} \int_0^x \frac{dx}{x - x_1} + \frac{b - 1}{2b} \int_0^x \frac{dx}{x - x_2} =$$

$$= \ln\left(\frac{x - x_1}{-x_1}\right)^{\frac{b+1}{2b}} \left(\frac{x - x_2}{-x_2}\right)^{\frac{b-1}{2b}} = \ln\left(1 - \frac{x}{x_1}\right)^{\frac{b-1}{2b}} \left(1 - \frac{x}{x_2}\right)^{\frac{b-1}{2b}} = \ln Z,$$

where:

$$Z = \left(1 - \frac{x}{x_1}\right)^{\frac{b+1}{2b}} \left(1 - \frac{x}{x_2}\right)^{\frac{b+1}{2b}}. \tag{17}$$

x_1 and x_2 are expressed by equations (16).

Substituting here these values of x_1 and x_2, we get:

$$Z_x = \left(1 - \frac{2}{b + 1} \frac{B_1}{k_1} x\right)^{\frac{b+1}{2b}} \left(1 + \frac{2}{b - 1} \frac{B_1}{k_1} x\right)^{\frac{b-1}{2b}}.$$

Inasmuch as the quantity $b = \sqrt{1 + 4B_1\psi_0/k_1^2} = \sqrt{1 + 4\gamma}$ is itself a function of the parameter $\gamma = B_1\psi_0/k_1^2$, the function Z_x actually depends only upon two quantities: the constant $\gamma = B_1\psi_0/k_1^2$ and the variable $\beta = B_1x/k_1$.

From these data it is possible to set up a table. Since the equation of the path contains the expression Z_x^{-B/B_1}, the tables are set up for $\log Z_x^{-1}$ to make their use more convenient.

The quantities entered (introduced) are γ and β.

It is not difficult to show by another method that $\int_0^x \frac{xdx}{x^2 - \frac{k_1}{B_1}x - \frac{\psi_0}{B_1}} = \ln Z_x$ is a function of $\gamma = B_1\psi_0/k_1^2$ and $\beta = B_1x/k_1$, if the numerator and denominator of the integrand are multiplied by B_1^2/k_1^2. Then,

$$\ln Z_x = \int_0^x \frac{\frac{B_1}{k_1} x d \frac{B_1}{k_1} x}{\left(\frac{B_1}{k_1} x\right)^2 - \frac{B_1}{k_1} x - \frac{B_1\psi_0}{k_1^2}} = \int_0^\beta \frac{\beta d\beta}{\beta^2 - \beta - \gamma}.$$

This expression actually shows that $\ln Z_x$ is a function of γ and β.

The table of the logarithms of the function ($\log Z_x^{-1}$) is presented below (Table 1).

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Procedure for Using the Table.

For every problem we will have one value for the entry parameter $\gamma = B_1\Psi_0/k_1^2$ and a series of values $\beta = B_1x/k_1$, where x varies between 0 and $1 - z_0$.

When determining $\log Z_x^{-1}$, write down the values from the columns containing the nearest smaller and larger tabular values of γ, so that the value of γ obtained from the solution would fall between them. The coefficient of interpolation will be the same along all the horizontal rows. For this reason, it is more convenient to interpolate first along the horizontal rows between which are contained the values of β selected in the problem, and then to interpolate along the columns (vertically) using the corresponding interpolation coefficients β.

In order to reduce the number of vertical interpolations (except such cases when $\log Z_x^{-1}$ is used for computing the values β_m and β_K), it is more convenient to assign tabular values of β for the intermediate values of x and to perform only the horizontal interpolation for γ, and then determine x by means of equation $x = k_1\beta/B_1$.

3. DETERMINATION OF THE MAXIMUM PRESSURE GENERATED BY THE POWDER GASES

The maximum gas pressure p_m in the barrel is the most important ballistic characteristic of a gun. Its value depends on the chosen conditions of loading, and the obtainment of the desired value of p_m serves as a criterion or control for the proper selection of the weight of the charge, the thickness of the powder, and other loading conditions.

For this reason, it is sometimes important to be able to compute the pressure p_m for the given loading conditions, without constructing the entire p, l pressure curve. In order to achieve this, it is necessary to derive first a formula for determining the value of x_m for which the gas pressure is maximum.

In this case the derivative dp/dl or dp/dt must be equated to zero. The expression for the derivative was derived earlier by differentiating the expression for p from the fundamental equation of pyrodynamics.

$$\frac{dp}{dl} = \frac{p}{l_\psi + l}\left\{\frac{f\omega}{s}\,\frac{\varkappa}{I_K}\,\frac{\sigma}{v}\left[1 + \left(\alpha - \frac{1}{\delta}\right)\frac{p}{f}\right] - (1 + \theta)\right\}.$$

Equating the expression in braces to zero, and substituting for v and σ their expressions in terms of x:

$$v = \frac{sI_K}{\varphi m}\,x, \quad \sigma = 1 + 2\lambda z = \sigma_0 + 2\lambda x,$$

we obtain the possibility of determining x_m for which the pressure is maximum:

$$\frac{f\omega}{s}\,\frac{\varkappa}{I_K}\,\frac{(\sigma_0 + 2\lambda x_m)\varphi m}{sI_K x_m}\left[1 + \left(\alpha - \frac{1}{\delta}\right)\frac{p_m}{f}\right] - (1 + \theta) = 0$$

or

$$\frac{\varkappa\sigma_0 + 2\varkappa\lambda x_m}{Bx_m}\left[1 + \left(\alpha - \frac{1}{\delta}\right)\frac{p_m}{f}\right] = 1 + \theta.$$

$$x_m = \frac{k_1}{\dfrac{B(1+\theta)}{1 + \left(\alpha - \dfrac{1}{\delta}\right)\dfrac{p_m}{f}} - 2\varkappa\lambda}. \tag{18}$$

If the powder has a constant burning area $\lambda = 0$, $k_1 = \varkappa\sigma_0 = 1$ and

$$x_m = \frac{1 + \left(\alpha - \dfrac{1}{\delta}\right)\dfrac{p_m}{f}}{B(1+\theta)}. \tag{19}$$

It is seen from these equations that in order to determine x_m it is necessary to know p_m, but inasmuch as we do not know it, we must find the real value of x_m by the method of successive approximations. First we assume a reference value $p_m^{(0)}$, substitute it in equation (18) or (19), and compute the value of x_m', following which we substitute the latter successively into all the fundamental equations

$$v = \frac{sI_K}{\varphi m}x;\quad \psi = \psi_0 + k_1x + \varkappa\lambda x^2;\quad l = l_{\psi\,av.}\left(Z_x^{-\frac{B}{B_1}} - 1\right);$$

$$p = \frac{f\omega}{s} \frac{\psi - \frac{v^2}{v_{np}^2}}{l_\psi + l},$$

and find the values of v'_m, l'_m, ψ'_m, p'_m. If p'_m coincides with $p_m^{(0)}$, it is indeed the true maximum pressure. However, if $p'_m \neq p_m^{(0)}$, then p'_m must again be substituted in (18) or (19) and a new x''_m obtained; then the whole process is repeated and a new p''_m is obtained. If x''_m is chosen correctly, p''_m should not differ from p'_m by more than 10-20 kg/cm^2 (the accuracy of a slide rule).

It must be remembered that equations (18) and (19) are used for calculating x_m and can not be employed for calculating p_m.

When carrying out the approximations, the following should be kept in mind: the relation p, x is represented by a curve shown in fig. 134, which varies slowly in the neighborhood of the maximum.

The true value of x_m is not known, and we find by means of equations (18) and (19) only a certain approximate value, which, even upon substituting the value $p_0 = 300$ kg/cm^2 for p_m in (19) will give a value of x'_m differing from the real value by not more than 10%. The value of p'_m will then be sufficiently close to the true value of p_m, and at the next approximation x'_m will practically coincide with x_m.

Whatever the quantity of $p_m^{(0)}$ assigned in the first approximation, whether smaller or larger than the real value of p_m, the values of p'_m and p''_m will be smaller in both cases than the real p_m. In the subsequent approximations the pressure values must increase, tending toward the real p_m, i.e., $p'_m < p''_m < p'''_m \rightarrow p_{m\ real}$, regardless of the value of $x_m^{(0)}$. Lecturer Belenky proved analytically that in successive approximations the quantity x'_m is monotonic increasing, tending toward x_m as the limit, while x''_m is monotonic decreasing and tends toward the same limit x_m.

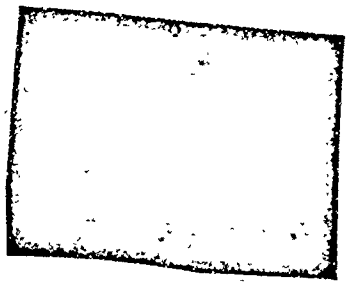

Fig. 134 - Determination of x_m and p_m (graph p, x).

This law must be used for controlling the accuracy of the calculations.

Since the pressure varies slowly in the neighborhood of the maximum, the p'_m obtained following the substitution of these values of x'_m in the working equations will be very close to the real p_m, and the second approximation will be adequate to obtain a value of p''_m sufficiently close to the real value.

Having found x_m, we substitute it into (11) for v, (10) for ψ, (14) for l, and (15) for p_m, and obtain the elements of the projectile's motion, i.e., v_m, ψ_m, l_m, and p_m at the instant of greatest pressure.

Expression (18) gives the analytical expression for x_m at which the gas pressure becomes maximum. Can this equation always be used to determine the maximum pressure?

In most cases when the chosen loading conditions are normal, this formula will give the right answer. But there are cases when it may yield a value x_m devoid of physical meaning. This occurs when:

$$x_m > x_K;$$

$x_K = 1 - z_0$ corresponds to the instant when the burning of the powder terminates, the instant when the inflow of gases ends. For this reason this formula will give realistic results while x_m is smaller than or at most is equal to $x_K (x_m \leqslant x_K)$.

When $x_m < x_K$, we have a normal case: the maximum pressure is reached before the end of burning. When $x_m = x_K$, the maximum pressure is reached at the end of burning. Finally, when $x_m > x_K$, we have the case of the so-called "unreal" maximum, i.e., a purely analytical case. In reality, when $x_m > x_K$, the powder, burning according to a definite law, stops burning on the upward branch of the pressure curve, the flow of gases stops, following which the pressure begins to drop, in spite of the fact that the analytic maximum had not yet been reached. In fact, the maximum pressure in this case will be the pressure p_K at the end of burning.

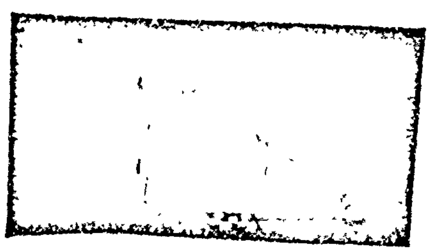

Fig. 135 - Pressure Curve with Normal Maximum.

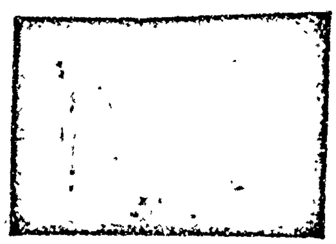

Fig. 136 - The Maximum Pressure Coincides with the End of Burning.

A large value of x_m may be obtained when the parameter of the loading conditions, $B = s^2 c_1^2 / u_1^2 f\omega\varphi m$ is small; this happens when the powder is thin.

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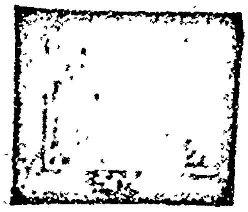

Fig. 137 - Unreal Maximum, $x_m > x_K$.

Such cases occur in practice when the firing is performed with thin powders for special purposes.

The appearance of the pressure curve in these three cases is shown in figs. 135, 136, 137.

At the end of the first period, we have:

$$\psi_K = 1; \quad l_{\psi_K} = l_1 = l_0(1 - \alpha\Delta);$$

$$x_K = 1 - z_0; \quad l_K = l_{av.K}\left(Z_{xK}^{-\frac{B}{B_1}} - 1\right);$$

$$v_K = \frac{sI_K}{\varphi m} x_K; \quad p_K = \frac{f\omega\left(1 - \frac{B\theta}{2} x_K^2\right)}{s(l_1 + l_K)}.$$

The same values will characterize the start of the second period - the period of adiabatic expansion of the gases.

4. SECOND PERIOD

The second period, starting at the end of burning of the charge and ending when the base of the projectile passes the muzzle face of the gun, constitutes a process of adiabatic expansion of the gases.

This period is considerably simpler than the first, because the whole process is reduced to the expansion of gases without the addition of energy and without heat losses.

In the second period $\psi = 1$, the number of variables is reduced, the independent variable is usually taken to be the path l of the projectile, and equations expressing the pressure p and the velocity v as a function of l are derived.

The beginning of the second period is characterized by the following data obtained at the end of the first period:

$$\psi = 1;\ v = v_K;\quad l = l_K;\ p = p_K;\quad l_\psi = l_1;\ T = T_K.$$

The fundamental equation of the second period is:

$$ps(l_1 + l) = f\omega - \frac{\theta}{2}\varphi m v^2 = f\omega\left(1 - \frac{v^2}{v_{\Pi p}^2}\right), \tag{20}$$

where

$$\frac{2f\omega}{\varphi\theta m} = v_{\Pi p}^2.$$

Since the gas temperature is lower in the second period than in the first, θ should be made larger in the second period, but most authors take an average value of θ common to both periods.

A. Derivation of the Expression for Pressure in the Second Period

$$[p = f_1(l)].$$

The equation for pressure is derived from the adiabatic equation:

$$pW^{1+\theta} = p_K W_K^{1+\theta}, \tag{21}$$

where p_K and p are the gas pressures at the beginning of the second period and at a given moment, respectively;

W_K and W are the free volumes of the initial air space at the same instants.

From equation (21), we have:

$$p = p_K \left(\frac{W_K}{W}\right)^{1+\theta}.$$

Expanding the quantities W_K and W, we obtain:

$$W_K = W_0 - \alpha\omega + sl_K = s(l_1 + l_K);$$

$$W = W_0 - \alpha\omega + sl = s(l_1 + l).$$

Substituting these values in the equation of p, we find:

$$p = p_K \left(\frac{l_1 + l_K}{l_1 + l}\right)^{1+\theta}. \tag{22}$$

At the muzzle face, we will have:

$$p_Д = p_K \left(\frac{l_1 + l_K}{l_1 + l_Д}\right)^{1+\theta}.$$

B. Derivation of the Expression for Velocity in the Second Period, $v = f_2(l)$

Let us write the fundamental equation of pyrodynamics for any moment and for the beginning of the second period:

$$ps(l_1 + l) = f\omega \left(1 - \frac{v^2}{v_{Пр}^2}\right);$$

$$p_K s(l_1 + l_K) = f\omega \left(1 - \frac{v_K^2}{v_{Пр}^2}\right).$$

Dividing one equation by the other, term by term, and replacing the ratio p/p_K from (22), we obtain:

$$\left(\frac{l_1 + l_K}{l_1 + l}\right)^{\theta} = \frac{1 - \frac{v^2}{v_{Пр}^2}}{1 - \frac{v_K^2}{v_{Пр}^2}},$$

whence

$$v = v_{Пр}\sqrt{1 - \left(\frac{l_1 + l_K}{l_1 + l}\right)^{\theta} \left(1 - \frac{v_K^2}{v_{Пр}^2}\right)}. \qquad (23)$$

If we replace v_K by its expression $v_K = \frac{sI_K}{\varphi m}(1 - z_0)$,

$$\frac{v_K^2}{v_{\text{пр}}^2} = \frac{s^2 I_K^2 (1 - z_0)^2}{\varphi^2 m^2} \cdot \frac{\varphi \theta m}{2 f \omega} = B \frac{\theta}{2}(1 - z_0)^2,$$

and then

$$v = v_{\text{пр}} \sqrt{1 - \left(\frac{l_1 + l_K}{l_1 + l}\right)^{\theta} \left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]}. \tag{24}$$

When $l = l_д$, we obtain an expression for the muzzle velocity $v_д$:

$$v_д = \sqrt{\frac{2g}{\varphi} \frac{f}{\theta} \frac{\omega}{q} \left\{1 - \left(\frac{l_1 + l_K}{l_1 + l_д}\right)^{\theta} \left[1 - \frac{B\theta}{2}(1 - z_0)^2\right] \right\}}. \tag{25}$$

This equation is of great importance for investigating the most desirable solutions when designing guns.

Equations (22) and (23) or (24) give the expressions for the gas pressure in the bore of the gun and for the projectile velocity in the second period as a function of the projectile path l.

Thus, on the basis of the assumptions made, the equations derived above express the relation between the conditions of loading and the ballistic elements of a gun discharge in both the first and the second periods. They enable one, for given loading conditions, to compute the projectile velocity and the gas pressure at different points of the projectile's motion in the bore of the gun, and to determine the maximum pressure, the muzzle pressure, and the initial (muzzle) velocity of the projectile.

Curves of p and v as a function of l will usually have the form shown in fig. 138.

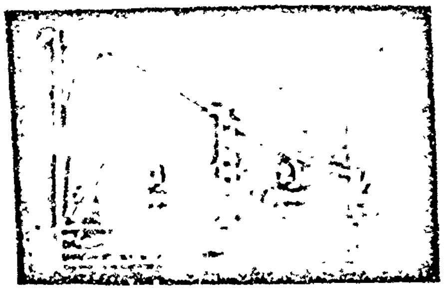

Fig. 138 - Normal p, l and v, l Curves.

1) Period I; 2) period II.

C. Equations for Calculating the Temperature of Powder Gases.

Having solved the fundamental equation of pyrodynamics and established the relation between the basic elements (p, v, l, and ψ) and the new independent variable x, and, consequently, also the relationship between these elements, an equation can be written for determining the temperature of the powder gases at any given instant, and, in particular, at the instant the projectile leaves the bore of the gun barrel.

The temperature of the gases flowing in the path of the projectile determines whether the discharge will be accompanied by a flash, or will be flashless, because according to the present concepts the flash accompanying a shot is a process involving the burning of inflammable hydrogen and carbon oxide gases making up about 50% of the entire gas mixture.

If the temperature of these gases is very high, the gases will burst into flames when mixed with the oxygen in air, and produce a flash accompanying the shot.

In order to obtain the desired equation, let us make use of the energy balance equation in which Ec_w is replaced by R/θ:

$$\frac{RT_1\omega\psi}{\theta} - \frac{RT\omega\psi}{\theta} = \frac{\varphi m v^2}{2}.$$

Since $RT_1 = f$,

$$\frac{f\omega\psi}{\theta}\left(1 - \frac{T}{T_1}\right) = \frac{\varphi m v^2}{2},$$

or

$$\frac{T}{T_1} = 1 - \frac{\varphi m \theta}{2f\omega}\,\frac{v^2}{\psi} = 1 - \frac{1}{\psi}\,\frac{v^2}{v^2_{\text{пр}}}. \tag{26}$$

Knowing v and ψ from the first period, let us find T/T_1 and then T.

Inasmuch as

$$v = \frac{sI_K}{\varphi m}\,x\ldots$$

$$\psi = \psi_0 + k_1 x + \varkappa\lambda x^2,$$

then, bearing in mind that:

$$B = \frac{s^2 I_K^2}{f\omega\varphi m},$$

we get

$$\frac{T}{T_1} = 1 - \frac{B\theta}{2}\,\frac{x^2}{\psi} = 1 - \frac{B\theta}{2}\,\frac{x^2}{(\psi_0 + k_1 x + \varkappa\lambda x^2)}.$$

This equation shows that the variation in the temperature of the gases depends upon the conditions of loading (parameter B) and upon the shape of the grain (coefficients $\varkappa$ and λ).

At the end of burning ($\psi = 1$) we will have:

$$\frac{T_K}{T_1} = 1 - \frac{B\theta}{2}(1 - z_0)^2 = 1 - \varphi r_K. \tag{27}$$

In the second period $\psi = 1$ and we obtain from equation (26):

$$\frac{T}{T_1} = 1 - \frac{\varphi m\theta}{2f\omega}\,v^2 = 1 - \frac{v^2}{v_{\Pi p}^2}, \tag{28}$$

where

$$v_{\Pi p}^2 = \frac{2f\omega}{\varphi\theta m}.$$

At the instant the projectile leaves the barrel

$$\frac{T_Д}{T_1} = 1 - \frac{v_Д^2}{v_{Пр}^2} = 1 - \varphi r_Д .$$

This value $T_Д/T_1$ varies in artillery pieces between 0.65 and 0.75.

Comparing the value v from equation (28) with the values T/T_1 and T_K/T_1 from equations (27) and (28), we obtain other expressions for T/T_1:

$$\frac{T}{T_1} = \left(\frac{l_1 + l_K}{l_1 + l}\right)^{\theta} \left[1 - \frac{B\theta}{2}(1 - z_0)^2\right] = \frac{T_K}{T_1} \left(\frac{l_1 + l_K}{l_1 + l}\right)^{\theta};$$

$$\frac{T_Д}{T_1} = \left[1 - \frac{B\theta}{2}(1 - z_0)^2\right] \left(\frac{l_1 + l_K}{l_1 + l_Д}\right)^{\theta} = \frac{T_K}{T_1} \left(\frac{l_1 + l_K}{l_1 + l_Д}\right)^{\theta};$$

$$T_Д = T_1\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right] \left(\frac{l_1 + l_K}{l_1 + l_Д}\right)^{\theta}.$$

This equation proves that the temperature of the gases at the instant the base of the projectile passes the muzzle face depends on:

1) the temperature T_1 of the burning powder;

2) the temperature of the gases at the end of burning:

$$T_K = T_1\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right],$$

this temperature decreases as B increases;

3) the ratio of free volumes $(l_1 + l_K)/(l_1 + l_\text{д})$, which depends upon the path traversed by the projectile at the end of burning and decreases as l_K increases.

D. Equations for Calculating the Time of Motion of the Projectile.

The time t does not appear directly in the solution of the fundamental problem of pyrodynamics; one may compute and draw the curves of the gas pressure p and the projectile velocity v as a function of the projectile path l, and by this means solve the fundamental problem of internal ballistics, yielding the design data of the gun (volume of powder chamber, length of projectile path).

But in order to fully clarify the phenomena taking place during a shot, it is also necessary to know the variation of the basic elements (p, v, l) as a function of the time t, particularly, because some of the existing devices permit determining the path l, the velocity v, and the gas pressure p as a function of the time t (velocimeter, piezoelectric manometer). Moreover, it is the pressures curves as a function of time which must be known when solving problems relating to the theory of gun mounts fuzes and firing devices.

The time of motion of the projectile in the barrel can be obtained most simply if the curve of the velocity v as a function of the path l is available, and by using the following equation of mechanics:

$$v = \frac{dl}{dt},$$

whence:

$$dt = \frac{dl}{v}.$$

If, having the curve v, l, we plot the curve $\frac{1}{v}$, l, then by taking the integral:

$$\int_0^l \frac{1}{v} dl,$$

We could determine the time of motion of the projectile along the given path l. But inasmuch as at the lower limit, when $l = 0$, $v = 0$, and the integrand $1/v$ becomes infinite ($1/v = 1/0 = \infty$), it is impossible to perform the integration. Therefore, the time t is divided into two parts, t' and t":

$$t = t' + t'', \tag{29}$$

where the first time interval t' - from the start of motion up to a point representing a small length of the path l' - is calculated approximately, and the second interval t", from l' to l along the path - is calculated by means of quadratic formulas:

$$t'' = \int_{l'}^{l} \frac{1}{v} dl.$$

The first time interval t' is found from the equation:

$$t' = \frac{l'}{v'_{av.}},$$

where, in the first approximation:

$$v'_{av.} = \frac{0 + v'}{2} = \frac{v'}{2}$$

and v' is the velocity of the projectile at time t' and the path distance l'; consequently:

$$t' = \frac{2l'}{v'};$$

the smaller the distance l', the greater will be the accuracy of determining t.

Substituting t' and t'' into (105), we obtain the equation giving the time of motion of the projectile in the bore in the form:

$$t = \frac{2l'}{v'} + \int_0^l \frac{1}{v} dl. \tag{30}$$

Inasmuch as the first interval of time for traversing the path l', as determined by (30), is very approximate, Prof. E.L. Bravin proposed a more exact expression for computing the average velocity of the projectile along the segment ot'. He assumed the acceleration, rather than the velocity, to be linear along this segment (fig. 139):

$$\frac{dv}{dt} = \frac{s}{\varphi m} p = \frac{s\alpha}{\varphi m}(p_0 + kt),$$

where $k = (p' - p_0)/t'$ is the angular coefficient of the straight line

p_0, p'; α is a factor, smaller than unity, determined from the condition that the areas bounded by the curve p, t and the straight line p_1p_2 replacing it along the segment $0t'$ are equal. When determining $v'_{av.}$ in terms of v', the coefficient α is reduced.

$$dv = \frac{s\alpha}{\varphi m}\left(p_0 + \frac{p' - p_0}{t'}\, t\right) dt.$$

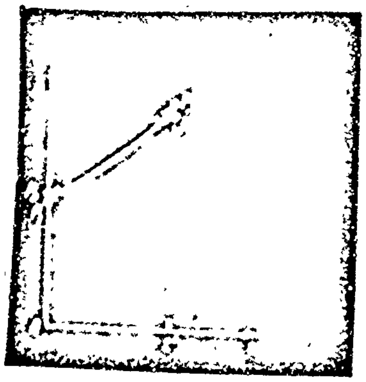

Fig. 139 - Curve p, t Along the Initial Path Segment (According to Bravin).

After integration, we obtain:

$$v = \frac{s}{\varphi m}\alpha\left(p_0 t + \frac{p' - p_0}{t'}\,\frac{t^2}{2}\right).$$

Assuming that $t = t'$, we find v', the velocity of the projectile at the time t':

$$v' = \frac{s}{\varphi m}\alpha\left(p_0 t' + \frac{p' - p_0}{2}t'\right) = \frac{s}{\varphi m}\alpha\left(\frac{p_0 + p'}{2}\right)t'.$$

The average value of the projectile velocity along this segment is found from the following equation:

$$v'_{av.} = \frac{1}{t'}\int_0^{t'} v dt = \frac{1}{t'}\frac{s\alpha}{\varphi m}\int_0^{t'}\left(p_0 t + \frac{p' - p_0}{2t'}t^2\right)dt =$$

$$= \frac{s\alpha}{\varphi m}\frac{1}{t'}\left(\frac{p_0 t'^2}{2} + \frac{p' - p_0}{6}t'^2\right) = \frac{s\alpha}{\varphi m}\cdot\frac{2p_0 + p'}{6}t'$$

or, replacing $\frac{s\alpha}{\varphi m}t'$ from the preceding equation by v', we obtain:

$$v'_{av.} = \frac{2p_0 + p'}{p_0 + p'}\frac{v'}{3}.$$

The final expression for t' will be in the form:

$$t' = \frac{l'}{v_{av.}} = \frac{3l'}{v'}\frac{p_0 + p'}{2p_0 + p'}. \tag{31}$$

This is the equation proposed by Prof. E.L. Bravin[3].

Comparing it with the previous expression for t', we note that the time t obtained by the first expression is shorter, and the difference between them increases as the length of the segment l' and the pressure p' increase. At the limit, when p' is reduced to p_0, the two expressions for t' become equal:

$$t' = \frac{3l'}{v'}\frac{2p_0}{3p_0} = \frac{2l'}{v'}.$$

If the relation p, t along the first segment is expressed by a second-degree equation, the resulting equation will be more exact:

$$t_2' = \frac{4l'}{v'}\,\frac{2p_0 + p'}{5p_0 + p'}.$$

Prof. Bravin also introduced equations for computing the time in segments measuring 0 to t_m, t_m to t_K, and t_K to $t_Д$:

$$t_m = \frac{3l_m}{v_m}\,\frac{p_0 + p_m}{2p_0 + p_m}; \qquad \text{(a)}$$

$$t_k - t_m = \frac{3(l_K - l_m)(p_m + p_K)}{v_m(p_m + 2p_K) + v_K(2p_m + p_K)}; \qquad \text{(b)}$$

$$t_Д - t_k = \frac{3(l_Д - l_K)(p_K + p_Д)}{v_K(p_K + 2p_Д) + v_Д(2p_K + p_Д)}. \qquad \text{(c)}$$

In order to compute t in the first period, the graph $\frac{1}{p}$, x can be used also.

Indeed, $x = z - z_0$;

$$\frac{dx}{dt} = \frac{dz}{dt} = \frac{1}{e_1}\,\frac{de}{dt} = \frac{u_1 p}{e_1} = \frac{p}{I_K};$$

whence

$$dt = I_K \frac{dx}{p};$$

$$t = I_K \int_0^x \frac{dx}{p}, \quad (32)$$

The function under the integral sign does not become infinite when $p_0 > 0$.

When calculating the total time of the shot, it is necessary to consider not only the time of motion of the projectile, but also the burning time of the powder in the preliminary period before the start of this motion, which is computed by the following formula:

$$t_0 = 2.303\, \tau_0 \log \frac{p_0}{p_B},$$

where $\frac{1}{\tau_0} = \frac{f\Delta}{1 - \Delta/\delta} \frac{\varkappa}{I_K}$, $I_K = e_1/u_1$, and p_B is the pressure of the igniter gases.

The time lapse between the instant the firing pin strikes the percussion cap and the end of burning of the igniter is usually not taken into account.

5. SAMPLE CALCULATION OF THE GAS PRESSURE CURVE AND OF THE PROJECTILE VELOCITY BY THE $l_{\Psi_{av.}}$ METHOD

The following data are given:

Barrel: 76 mm gun, 1936 model

Chamber capacity, W_0 in dm^3..........................1.515

Cross-sectional area of the bore, including the rifling grooves, s, in dm^2..........................0.4692

Path traversed by the projectile in the bore, $l_д$, in dm..33.91

Projectile:

Weight of projectile, q, in kg.......................6.2

Forcing pressure, p_0, in kg/cm^2....................300

Charge:

Weight of charge, ω, in kg..........................1.08

Powder constants:

Powder energy (force) f, in kg·dm/kg............950,000

Co-volume, α, in dm^3/kg..............................0.98

Density of the powder, δ, in kg/dm^3................1.6

Burning rate, u_1, of the powder when p = 1, in dm/sec:kg/dm^2......................................0.0000074

Dimensions of strip (thickness $2e_1$, in mm).........1.357

$$\varkappa = 1.06$$

$$\varkappa\lambda = -0.06$$

Polytropic index k......................................1.2

$\theta = k - 1$............................0.2

FUNDAMENTAL EQUATIONS

A. Preliminary Period

$$\psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}}; \quad \sigma_0 = \sqrt{1 + 4\frac{\lambda}{\varkappa}\psi_0}; \quad z_0 = \frac{2\psi_0}{\varkappa(\sigma_0 + 1)} \approx \frac{\psi_0}{\varkappa}.$$

B. First Period

$$v = \frac{sI_K}{\varphi m}x; \quad \psi = \psi_0 + k_1 x + \varkappa\lambda x^2;$$

$$l = l_{\psi_{av.}} \left(Z_x^{-\frac{B}{B_1}} - 1 \right); \quad p = \frac{f\omega}{s} \frac{\psi - \frac{B\theta}{2} x^2}{l_\psi + l};$$

$$k_1 = \varkappa G_0; \quad B = \frac{s^2 I^2}{f\omega\varphi m};$$

$$B = \frac{B\theta}{2} - \varkappa\lambda; \quad l_\psi = l_\Delta - a\psi;$$

$$l_{\psi_{av.}} = l_\Delta - a\psi_{av.}; \quad a = \frac{l_0 \Delta}{\delta_1} = \frac{\omega}{s\delta_1} = \frac{\omega}{s}\left(\alpha - \frac{1}{\delta} \right);$$

$$l_\Delta = l_0 \Delta \left(\frac{1}{\Delta} - \frac{1}{\delta} \right) = \frac{\omega}{s}\left(\frac{1}{\Delta} - \frac{1}{\delta} \right).$$

Z_x is determined from the double-entry table:

$$\gamma = \frac{B_1 \psi_0}{k_1^2} \text{ and } \beta = \frac{B_1}{k_1} x;$$

$$x_m = \frac{k_1}{\dfrac{B(1 + \theta)}{1 + \dfrac{p_m}{f\delta_1}} - 2\varkappa\lambda}.$$

C. Second Period

$$p = p_k \left(\frac{l_1 + l_K}{l_1 + l} \right)^{1+\theta};$$

$$l_1 = l_0 \Delta \left(\frac{1}{\Delta} - \alpha \right) = l_0 (1 - \alpha\Delta);$$

$$v = v_{\Pi p} \sqrt{1 - \left(\frac{l_1 + l_K}{l_1 + l} \right)^{\theta} \left(1 - \frac{v_K^2}{v_{\Pi p}^2} \right)}$$

or

$$v = v_{\Pi p} \sqrt{1 - \left(\frac{l_1 + l_K}{l_1 + l} \right)^{\theta} \left[1 - \frac{B\theta}{2} (1 - z_0)^2 \right]};$$

$$v_{\Pi p} = \sqrt{\frac{2g}{\varphi} \frac{f}{\theta} \frac{\omega}{q}}.$$

The calculation of the constants is effected first from the given data:

$$\Delta = 0.7128;$$

$$\psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}} = \frac{\frac{1}{0.7128} - \frac{1}{1.6}}{\frac{950000}{30000} + 0.98 - 0.625} = 0.02429;$$

$$\sigma_0 = \sqrt{1 + 4\frac{\lambda}{\kappa}\psi_0} = \sqrt{1 + \frac{4(-0.0566)0.02429}{1.06}} = 0.9972;$$

$$z_0 = \frac{2\psi_0}{\kappa(\sigma_0 + 1)} = 0.02294;$$

$$x_K = 1 - z_0 = 1 - 0.02294 = 0.97706;$$

$$k_1 = \kappa\sigma_0 = 1.06\cdot 0.9972 = 1.057;$$

$$I_K = \frac{e_1}{u_1} = \frac{0.00678}{0.0000074} = 916.2;$$

$$\varphi = 1.03 + \frac{1}{3}\frac{\omega}{q} = 1.03 + \frac{1}{3}\cdot\frac{1.08}{6.2} = 1.088;$$

$$\frac{sI_K}{\varphi m} = \frac{0.4692\cdot 916.2\cdot 98.1}{1.088\cdot 6.2} = 6253;$$

$$l_0 = \frac{W_0}{s} = \frac{1.515}{0.4692} = 3.228;$$

$$B = \frac{s^2 I_K^2}{f\omega\varphi m} = \frac{0.4692^2\cdot 916.2^2\cdot 98.1}{950000\cdot 1.08\cdot 1.088\cdot 6.2} = 2.617;$$

$$B_1 = \frac{B\theta}{2} - \kappa\lambda = \frac{2.617\cdot 0.2}{2} + 0.06 = 0.3217; \quad \frac{B}{B_1} = 8.134;$$

$$a = \frac{l_0\Delta}{\delta_1} = 3.228 \cdot 0.7128 \cdot 0.355 = 0.8168;$$

$$l_\Delta = l_0\Delta\left(\frac{1}{\Delta} - \frac{1}{\delta}\right) = 3.222 \cdot 0.7128\left(\frac{1}{0.7128} - \frac{1}{1.6}\right) = 1.790;$$

$$\gamma = \frac{B_1\psi_0}{k_1^2} = \frac{0.3217 \cdot 0.02429}{1.057^2} = 0.006995;$$

$$x_m = \frac{k_1}{\dfrac{B(1+\theta)}{1 + \dfrac{p_m}{f\delta_1}} - 2\kappa\lambda} = \frac{1.057}{\dfrac{2.617 \cdot 1.2}{1 + \dfrac{232000}{950000} \cdot 0.355} + 0.12} = 0.3512;$$

$$x_m \text{ 2nd approx.} = \frac{1.057}{\dfrac{2.617 \cdot 1.2}{1 + \dfrac{236000}{950000} \cdot 0.355} + 0.12} = 0.3517.$$

Fig. 140 - Curves p, l and v, l According to Prof. Drozdov and by the $l_{\Psi av.}$ Method.

1) v, in m/sec; 2) p, in kg/cm^2; 3) curves $p(l)$ and $v(l)$; 4) Drozdov's method; 5) $l_{\Psi av.}$ method; 6) l, in dm.

Table of the Ballistic Elements (ψ, v, l, and p) for the First Period

Initial Formulas	No.	Operations		Max. Pressure	2nd Approx.		
$\frac{B_1}{k_1} = 0.3043$	1	x	0.1972	0.3512	0.3517	0.5270	0
	2	$\beta = \frac{B_1}{k_1} x$	0.060	0.1069	0.1070	0.160	0
$v = \frac{sI_K}{\varphi m} x = 6253x$	3	v, in dm/sec	1233	2196	2199	3295	4
	4	$k_1 x$	0.2084	0.3712	0.3717	0.5570	0
$k_1 = 1.057$	5	$(+) \varkappa\lambda x^2$	-0.0023	-0.0074	-0.0074	-0.0167	-0
$\varkappa\lambda = -0.06$	6	ψ_0	0.0243	0.0243	0.0243	0.0243	0
$\psi = \psi_0 + k_1 x +$	7	ψ	0.2304	0.3881	0.3886	0.5646	0
$+ \varkappa\lambda x^2$	8	$\psi + \psi_0$	0.2547	0.4124	0.4129	0.5889	0
$\psi_0 = 0.0243$	9	$\psi_{av.}$	0.1273	0.2062	0.2064	0.2944	0
$\psi_{av.} = \frac{\psi + \psi_0}{2}$	10	$(-) \; l_\Delta$	1.790	1.790	1.790	1.790	1.
$l_{\psi_{av.}} = l_\Delta - a\psi_{av.} =$ $= 1.79 - 0,186\psi_{av.}$	11	$a\psi_{av.}$	0.140	0.168	0.168	0.240	0.
	12	$l_{\psi_{av.}} = l_c$	1.650	1.622	1.622	1.550	1.
	13	$(-) \; l_\Delta$	1.790	1.790	1.790	1.790	1.
	14	$a\psi$	0.188	0.317	0.317	0.464	0.6
$- a\psi$	15	l_ψ	1.602	1.473	1.473	1.329	1.3

Table of the Ballistic Elements (ψ, v, l_ψ and p) for the First Period

No.	Operations		Max. Pressure	2nd Approx.			End of Burning
1	x	0.1972	0.3512	0.3517	0.5270	0.723	0.9???
2	$\beta = \frac{B_1}{k_1} x$	0.080	0.1069	0.1970	0.160	0.226	0.2??
3	v, in dm/sec	1233	2196	2199	3295	4521	6110
4	(+) $k_1 x$	0.2084	0.3712	0.3717	0.5570	0.7642	1.033
5	$\varkappa\lambda x^2$	-0.0023	-0.0074	-0.0074	-0.0167	-0.0314	0.0573
6	ψ_0	0.0243	0.0243	0.0243	0.0243	0.0243	0.0243
7	ψ	0.2304	0.3881	0.3886	0.5646	0.7571	1.000
8	$\psi + \psi_0$	0.2547	0.4124	0.4129	0.5889	0.7814	1.0243
9	$\psi_{av.}$	0.1273	0.2062	0.2064	0.2944	0.3907	0.5??
10	(-) l_Δ	1.790	1.790	1.790	1.790	1.790	1.7?0
11	$a\psi_{av.}$	0.140	0.168	0.168	0.240	0.319	0.448
12	$l_{\psi_{av.}} = l_c$	1.650	1.622	1.622	1.550	1.471	1.377
13	(-) l_Δ	1.790	1.790	1.790	1.790	1.790	1.7?0
14	$a\psi$	0.188	0.317	0.317	0.461	0.618	0.317
15	l_ψ	1.602	1.473	1.473	1.329	1.172	[illegible]

$\psi_{av.} = \frac{\ \ \ 0}{2}$

$l_{\psi av.} = l_{\Delta} - a\psi_{av.} =$

$= 1.79 - 0.186\psi_{av.}$

$l_{\psi} = l_{\Delta} - a\psi$

colog Z from table
$\gamma = 0.006995$

$\frac{B}{B_1} = 8.134$

$\frac{f\omega}{s} = 2,187,000$

$p = \frac{f\omega}{s} \frac{\psi - \frac{B\theta}{2}x^2}{l_{\psi} + l}$

10	(-) l_{Δ}	1.790	1.790	1.790	1.790	1.
11	$a\psi_{av.}$	0.140	0.168	0.168	0.240	0.
12	$l_{\psi av.} = l_c$	1.650	1.622	1.622	1.550	1.
13	(-) l_{Δ}	1.790	1.790	1.790	1.790	[illegible]
14	$a\psi$	0.188	0.317	0.317	0.461	0.6
15	l_{ψ}	1.602	1.473	1.473	[illegible]	[illegible]
16	colog Z	0.0198	0.0402	0.04024	0.06521	0.C
17	$\frac{B}{B_1}$ colog Z	0.1610	0.3270	0.3273	0.5304	0.7
18	$Z_x^{\frac{B}{B_1}}$	1.449	2.123	2.124	3.391	6.0
19	$l = l_{\psi cp} \cdot$	0.7408	1.822	[illegible] 923	3.706	7.4
20	$\cdot (Z_x^{\frac{B}{B_1}} - 1)$	1.602	1.473	1.473	1.329	1.
21	$l_{\psi} + l$	2.343	3.295	3.296	5.035	8.
22	ψ	0.2304	0.3881	0.3886	0.5646	0.7
23	(-) $\frac{B\theta}{2} x^2$	0.0102	0.0323	0.0324	0.0727	0.1
24	$\psi - \frac{B\theta}{2} x^2$	0.2202	0.3558	0.3562	0.4919	0.6
25	p, in kg/cm^2	2054	2360	2362	2136	158

$l\Delta$	1.790	1.790	1.790	1.790	1.790	1.790
$a\psi_{av.}$	0.140	0.168	0.168	0.240	0.319	0.418
$l_{\psi_{av.}} = l_c$	1.650	1.622	1.622	1.550	1.471	1.372
(-) $l\Delta$	1.790	1.790	1.790	1.790	1.790	1.790
$a\psi$	0.188	0.317	0.317	0.461	0.618	0.317
l_ψ	1.602	1.473	1.473	1.329	1.172	[illegible]
colog Z	0.0198	0.0402	0.04024	0.06521	0.09506	0.1397
$\frac{B}{B_1}$ colog Z	0.1610	0.3270	0.3273	0.5304	0.7805	1.1369
$Z_x^{-\frac{B}{B_1}}$	1.449	2.123	2.124	3.391	6.033	13.69
$l = l_{\psi_{cp}} \cdot (Z_x^{-\frac{B}{B_1}} - 1)$	0.7408	1.822	1.823	3.706	7.403	17.411
	1.602	1.473	1.473	1.329	1.172	0.973
$l_\psi + l$	2.343	3.295	3.296	5.035	8.575	18.384
(-) ψ	0.2304	0.3881	0.3886	0.5646	0.7571	1.000
$\frac{B\theta}{2} x^2$	0.0102	0.0323	0.0324	0.0727	0.1368	0.2498
$\psi - \frac{B\theta}{2} x^2$	0.2202	0.3558	0.3562	0.4919	0.6203	0.7502
σ, in kg/cm^2	2054	2360	2362	2136	1580	892

COMPUTATIONS FOR THE SECOND PERIOD

Calculation of the constants of the second period:

$$v_{np}^2 = \frac{2\cdot 950000\cdot 1.08\cdot 98.1}{1.088\cdot 0.2\cdot 6.2} = 149,300,000; \qquad \frac{v_K^2}{v_{np}^2} = 0.250;$$

$$C = 1 - \frac{v_K^2}{v_{np}^2} = 1-0.250 = 0.750.$$

From the first period, $l_1 + l_K = 18.384$.

$$p_K = 892 \text{ kg/cm}^2.$$

Table of the Elements of the Second Period

Initial Formulas	No.	Operations			Muzzle Face
$p = p_K\left(\frac{l_1 + l_K}{l_1 + l}\right)^{1+\theta} =$	1	(+) l	22.60	27.44	33.91
$= 892\left(\frac{18.384}{0.973 + l}\right)^{1.2}$	2	l_1	0.973	0.973	0.973
	3	$l + l_1$	23.573	28.413	34.883
	4	$\frac{l_1 + l_K}{l_1 + l} = \eta$	0.7799	0.6471	0.5270
	5	$\log \eta$	$\bar{1}.8920$	$\bar{1}.8110$	$\bar{1}.7218$
			-0.1080	-0.1890	-0.2782
			-0.1296	-0.2268	-0.3338
$v = v_{np}\sqrt{1 - \eta^{0.2}\left(1 - \frac{v_K^2}{v_{np}^2}\right)} =$	6	$1.2 \log \eta$	$\bar{1}.8704$	$\bar{1}.7732$	$\bar{1}.6662$

Table 1 - Table of Logarithms of the Function log $Z^{-1}(\beta, \gamma)$

γ	0	0.0005	0.001	0.002	0.004	0.006	0.008	0.010	0.020	0.040	0.060	0.080	0.100
0	0	0	0	0	0	0	0	0	0	0	0	0	0
.020	0.0088	0.0080	0.0075	0.0067	0.0057	0.0049	0.0044	0.0039	0.0023	0.0017	0.0011	0.0009	0.0008
.040	0.0177	0.0168	0.0161	0.0150	0.0134	0.0123	0.0014	0.0106	0.0079	0.0054	0.0041	0.0033	0.0028
.060	0.0269	0.0258	0.0250	0.0238	0.0219	0.0204	0.0192	0.0181	0.0144	0.0103	0.0081	0.0067	0.0057
.080	0.0362	0.0351	0.0342	0.0329	0.0307	0.0290	0.0275	0.0262	0.0215	0.0161	0.0130	0.0109	0.0094
.100	0.0458	0.0446	0.0436	0.0422	0.0398	0.0379	0.0362	0.0347	0.0292	0.0225	0.0185	0.0157	0.0137
.120	0.0555	0.0543	0.0533	0.0517	0.0491	0.0471	0.0452	0.0435	0.0373	0.0294	0.0246	0.0211	0.0185
.140	0.0655	0.0642	0.0632	0.0615	0.0588	0.0565	0.0545	0.0527	0.0458	0.0368	0.0311	0.0269	0.0238
.160	0.0757	0.0744	0.0734	0.0716	0.0687	0.0663	0.0641	0.0622	0.0546	0.0446	0.0380	0.0332	0.0295
.180	0.0862	0.0848	0.0838	0.0819	0.0789	0.0763	0.0740	0.0720	0.0637	0.0528	0.0453	0.0399	0.0356
.200	0.0969	0.0955	0.0944	0.0925	0.0893	0.0867	0.0842	0.0820	0.0732	0.0613	0.0530	0.0469	0.0421
.220	0.1079	0.1065	0.1053	0.1034	0.1001	0.0972	0.0947	0.0924	0.0830	0.0702	0.0611	0.0543	0.0490
.240	0.1192	0.1177	0.1166	0.1145	0.1111	0.1081	0.1055	0.1031	0.0932	0.0794	0.0695	0.0621	0.0562
.260	0.1308	0.1293	0.1281	0.1260	0.1224	0.1194	0.1166	0.1141	0.1037	0.0889	0.0783	0.0702	0.0638
.280	0.1427	0.1411	0.1399	0.1378	0.1341	0.1309	0.1280	0.1254	0.1145	0.0988	0.0874	0.0787	0.0717
.300	0.1549	0.1533	0.1521	0.1499	0.1461	0.1428	0.1398	0.1371	0.1256	0.1090	0.0969	0.0875	0.0799
.320	0.1675	0.1659	0.1646	0.1624	0.1585	0.1551	0.1520	0.1491	0.1371	0.1196	0.1067	0.0967	0.0885
.340	0.1805	0.1783	0.1775	0.1752	0.1712	0.1677	0.1645	0.1615	0.1490	0.1306	0.1169	0.1062	0.0974
.360	0.1938	0.1922	0.1908	0.1884	0.1843	0.1807	0.1774	0.1743	0.1613	0.1419	0.1275	0.1161	0.1067
.380	0.2076	0.2059	0.2046	0.2021	0.1979	0.1941	0.1907	0.1875	0.1740	0.1536	0.1385	0.1263	0.1163
.400	0.2219	0.2201	0.2188	0.2163	0.2119	0.2080	0.2045	0.2012	0.1871	0.1659	0.1499	0.1370	0.1264
.420	0.2366	0.2348	0.2335	0.2310	0.2264	0.2224	0.2187	0.2154	0.2007	0.1786	0.1617	0.1481	0.1369
.440	0.2518	0.2500	0.2486	0.2461	0.2414	0.2373	0.2335	0.2301	0.2148	0.1917	0.1739	0.1596	0.1478
.460	0.2676	0.2658	0.2643	0.2617	0.2569	0.2527	0.2488	0.2453	0.2294	0.2052	0.1866	0.1715	0.1589
.480	0.2840	0.2822	0.2806	0.2779	0.2730	0.2687	0.2647	0.2610	0.2446	0.2193	0.1998	0.1839	0.1705
.500	0.3010	0.2992	0.2976	0.2948	0.2898	0.2853	0.2812	0.2773	0.2604	0.2340	0.2136	0.1968	0.1827
.520	0.3187	0.3169	0.3153	0.3124	0.3073	0.3026	0.2984	0.2943	0.2768	0.2493	0.2279	0.2102	0.1954
.540	0.3372	0.3353	0.3337	0.3307	0.3255	0.3207	0.3163	0.3121	0.2939	0.2653	0.2428	0.2242	0.2086
.560	0.3565	0.3545	0.3529	0.3498	0.3445	0.3396	0.3350	0.3307	0.3117	0.2819	0.2583	0.2388	0.2223
.580	0.3767	0.3747	0.3730	0.3699	0.3614	0.3593	0.3545	0.3501	0.3304	0.2992	0.2745	0.2540	0.2366
.600	0.3979	0.3959	0.3941	0.3909	0.3852	0.3799	0.3750	0.3704	0.3500	0.3174	0.2915	0.2699	0.2516

Table 1 - Table of Logarithms of the Function $\log Z^{-1}(\beta, \mu)$

01	0.002	0.004	0.006	0.008	0.010	0.020	0.040	0.060	0.080	0.100	0.150	0.200
	0	0	0	0	0	0	0	0	0	0	0	0
075	0.0067	0.0057	0.0049	0.0044	0.0039	0.0023	0.0017	0.0011	0.0009	0.0008	0.0006	0.0004
161	0.0150	0.0134	0.0123	0.0014	0.0106	0.0079	0.0054	0.0041	0.0033	0.0028	0.0020	0.0015
250	0.0238	0.0219	0.0204	0.0192	0.0181	0.0144	0.0103	0.0081	0.0067	0.0057	0.0042	0.0033
342	0.0329	0.0307	0.0290	0.0275	0.0262	0.0215	0.0161	0.0130	0.0109	0.0094	0.0070	[illegible]
436	0.0422	0.0398	0.0379	0.0362	0.0347	0.0292	0.0225	0.0185	0.0157	0.0137	0.0104	0.0084
33	0.0517	0.0491	0.0471	0.0452	0.0435	0.0373	0.0294	0.0246	0.0211	0.0185	0.0142	0.0116
32	0.0615	0.0588	0.0565	0.0545	0.0527	0.0458	0.0368	0.0311	0.0269	0.0238	0.0185	0.0152
34	0.0716	0.0687	0.0663	0.0641	0.0622	0.0546	0.0446	0.0380	0.0332	0.0295	0.0232	0.0191
38	0.0819	0.0789	0.0763	0.0740	0.0720	0.0637	0.0528	0.0453	0.0399	0.0356	0.0283	0.0234
44	0.0925	0.0893	0.0867	0.0042	0.0820	0.0732	0.0613	0.0530	0.0469	0.0421	0.0337	0.0281
53	0.1034	0.1001	0.0972	0.0947	0.0924	0.0830	0.0702	0.0611	0.0543	0.0490	0.0395	0.0381
66	0.1145	0.1111	0.1081	0.1055	0.1031	0.0932	0.0794	0.0695	0.0621	0.0562	0.0456	0.0384
81	0.1260	0.1224	0.1194	0.1166	0.1141	0.1037	0.0889	0.0783	0.0702	0.0638	0.0520	0.0440
79	0.1378	0.1341	0.1309	0.1280	0.1254	0.1145	0.0988	0.0874	0.0787	0.0717	0.0588	0.0499
21	0.1499	0.1461	0.1428	0.1398	0.1371	0.1256	0.1090	0.0969	0.0875	0.0799	0.0659	0.0561
6	0.1624	0.1585	0.1551	0.1520	0.1491	0.1371	0.1196	0.1067	0.0967	0.0885	0.0733	0.0626
75	0.1752	0.1712	0.1677	0.1645	0.1615	0.1490	0.1306	0.1169	0.1062	0.0974	0.0810	[illegible]
8	0.1884	0.1843	0.1807	0.1774	0.1743	0.1613	0.1419	0.1275	0.1161	0.1067	0.0890	[illegible]
6	0.2021	0.1979	0.1941	0.1907	0.1875	0.1740	0.1536	0.1385	0.1263	0.1163	0.0974	0.0840
8	0.2163	0.2119	0.2080	0.2045	0.2012	0.1871	0.1659	0.1499	0.1370	0.1264	0.1062	0.0917
5	0.2310	0.2264	0.2224	0.2187	0.2154	0.2007	0.1786	0.1617	0.1481	0.1369	0.1153	0.0998
6	0.2461	0.2414	0.2373	0.2335	0.2301	0.2148	0.1917	0.1739	0.1596	0.1478	0.1247	0.1082
3	0.2617	0.2569	0.2527	0.2488	0.2453	0.2294	0.2052	0.1866	0.1715	0.1589	0.1345	0.1169
6	0.2779	0.2730	0.2687	0.2647	0.2610	0.2446	0.2193	0.1998	0.1839	0.1705	0.1447	0.1260
5	0.2948	0.2898	0.2853	0.2812	0.2773	0.2604	0.2340	0.2136	0.1968	0.1827	0.1554	0.1354
3	0.3124	0.3073	0.3026	0.2984	0.2943	0.2768	0.2493	0.2279	0.2102	0.1954	0.1665	0.1452
7	0.3307	0.3255	0.3207	0.3163	0.3121	0.2939	0.2653	0.2428	0.2242	0.2086	0.1780	0.1555
[illegible]	0.3498	0.3445	0.3396	0.3350	0.3307	0.3117	0.2819	0.2583	0.2388	0.2223	0.1900	0.1662
[illegible]	0.3699	0.3614	0.3593	0.3545	0.3501	0.3304	0.2992	0.2745	0.2540	0.2366	0.2025	0.1773
[illegible]	0.3909	0.3852	0.3799	0.3750	0.3704	0.3500	0.3174	0.2915	0.2699	0.2516	0.2156	0.1889

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Table (Cont'd.)

Initial Formulas	No.	Operations			Muzzle Face
$= 12{,}220\sqrt{1-0.750\eta^{0.2}}$	7	$\eta^{1.2}$	0.7420	0.5932	0.4636
	8	$p = p_K\eta^{1.2}$	662	529	414
	9	$0.2 \log \eta$	-0.0216 $\bar{1}$.9784	-0.0378 $\bar{1}$.9622	-0.0556 $\bar{1}$.9444
	10	$\eta^{0.2}$	0.9515	0.9166	0.8798
	11	$0.750\eta^{0.2}$	0.7136	0.6874	0.6598
	12	$1-0.750\eta^{0.2}$	0.2864	0.3126	0.3402
	13	v in dm/sec	6530	6822	7117

The results of these calculations are shown in fig. 140 on p. 510 in the form of $p(l)$ and $v(l)$ curves.

CHAPTER 2 - PROF. N.F. DROZDOV'S EXACT METHOD [1]

(Written by Prof. G.V. Oppokov)

The assumption

$$l_\psi = l_{\psi_{av}},$$

made in the preceding chapter gives an approximate solution. Yet the differential equation of the projectile path in the first period

$$\frac{dl}{dx} = \frac{Bx(l_\psi + l)}{\psi - \frac{B\theta}{2}x^2} \tag{33}$$

can be integrated exactly.

Prof. N.F. Drozdov's great contribution to the field of internal ballistics lies in the very fact that he was the first to solve this equation exactly, without any additional assumptions or simplifications, as had been done before him by all the other authors without exception.

Namely, if we introduce for convenience the following designation:

$$M = \frac{Bx}{\psi - \frac{B\theta}{2}x^2}, \tag{34}$$

equation (33) takes on the form:

$$\frac{dl}{dx} - Ml = Ml_\psi. \tag{35}$$

When this differential equation of the first order is integrated, the following relationship obtains:

$$l = e^{\int_0^x M dx} \int_0^x l_\psi \cdot e^{-\int_0^x M dx} M dx. \qquad (36)$$

The integral-exponent of e in the right-hand side of equation (36) is (see pp. 478-481) equal, as before, to:

$$\int_0^x M dx = \frac{B}{B_1} \int_0^\beta \frac{\beta d\beta}{\gamma + \beta - \beta^2} = \ln Z^{-\frac{B}{B_1}}. \qquad (37)$$

The main integral is:

$$Y = \int_0^x l_\psi e^{-\int_0^x M dx} M dx = \int_0^x l_\psi d(-e^{\int_0^x M dx}).$$

Noting this peculiarity, the author integrates by parts:

$$Y = \left| -l_\psi e^{-\int_0^x M dx} \right|_0^x + \int_0^x e^{-\int_0^x M dx} dl_\psi,$$

or, [see equation (37)]:

$$Y = -l_{\psi} Z^{\frac{B}{B_1}} + l_{\psi_0} + \int_{l_{\psi_0}}^{l_{\psi}} Z^{\frac{B}{B_1}} \, dl_{\psi}.$$

But

$$\psi = \psi_0 + k_1 x + \varkappa \lambda x^2; \qquad l_{\psi} = l_{\Delta} - a\psi,$$

where $a = \frac{\omega}{s}\left(\alpha - \frac{1}{\delta}\right)$.

$$dl_{\psi} = -ak_1 dx - 2a\varkappa\lambda x dx,$$

whence

$$Y = -l_{\psi} Z^{\frac{B}{B_1}} + l_{\psi} - ak_1 \int_0^x Z^{\frac{B}{B_1}} dx - 2a\varkappa\lambda \int_0^x Z^{\frac{B}{B_1}} x dx. \tag{38}$$

It is now possible to proceed in two ways: eliminate from the equation either the first or the second integral in the right-hand side. The author selected the second course.

Namely, it follows from (34) and (37) that:

$$\int_0^x \frac{Bxdx}{\psi_0 + k_1 x - B_1 x^2} = -\frac{B}{B_1} \ln Z$$

or

$$\int_0^x \frac{x\,dx}{x^2 - \frac{k_1}{B_1}x - \frac{\Psi_0}{B_1}} = \ln Z,$$

whence

$$\frac{x\,dx}{x^2 - \frac{k_1}{B_1}x - \frac{\Psi_0}{B_1}} = \frac{dZ}{Z},$$

and, consequently

$$x\,dx = \left(x^2 - \frac{k_1}{B_1}x - \frac{\Psi_0}{B_1}\right)\frac{dZ}{Z}.$$

The desired integral is equal to:

$$\int_0^x Z^{\frac{B}{B_1}} x\,dx = \int_1^Z Z^{\frac{B}{B_1}-1}\left(x^2 - \frac{k_1}{B_1}x - \frac{\Psi_0}{B_1}\right)dZ =$$

$$= \frac{B_1}{B}\int_1^Z \left(x^2 - \frac{k_1}{B_1}x - \frac{\Psi_0}{B_1}\right)dZ.$$

Integrating by parts:

$$\int_0^x Z^{\frac{B}{B_1}} x\,dx = \frac{B_1}{B}\left(x^2 - \frac{k_1}{B}x - \frac{\psi_0}{B_1}\right) Z^{\frac{B}{B_1}} + \frac{B_1}{B}\,\frac{\psi_0}{B_1} -$$

$$- \frac{2B_1}{B}\int_0^x Z^{\frac{B}{B_1}} x\,dx + \frac{k_1}{B}\int_0^x Z^{\frac{B}{B_1}} dx,$$

because when $x = 0$, we will have:

$$Z^{\frac{B}{B_1}} = e^0 = 1.$$

We will find the desired integral from equation (38):

$$\int_0^x Z^{\frac{B}{B_1}} x\,dx = \frac{1}{B + 2B_1}\left(B_1 x^2 - k_1 x - \psi_0\right) Z^{\frac{B}{B_1}} +$$

$$+ \frac{\psi_0}{B + 2B_1} + \frac{k_1}{B + 2B_1}\int_0^x Z^{\frac{B}{B_1}} dx.$$

Now the obtained value of the integral must be substituted into (38), noting that:

$$x^2 - \frac{k_1}{B_1}x - \frac{\psi_0}{B_1} = \frac{1}{B_1}\left(\psi - \frac{B\theta}{2}x^2\right),$$

and using Prof. N.F. Drozdov's designations:

$$a_1 = -\frac{2x\lambda}{B + 2B_1}; \quad b_1 = \frac{1}{l_0}(l_{\psi_0} + aa_1\psi_0); \quad c_1 = \frac{ak_1}{l_0}(1 - a_1). \qquad (39)$$

Then finally we have:

$$Y = -l_\psi Z^{\frac{B}{B_1}} - aa_1\left(\psi - \frac{B\theta}{2}x^2\right) Z^{\frac{B}{B_1}} + l_0(b_1 - c_1\int_0^x Z^{\frac{B}{B_1}} dx). \qquad (37)$$

Let us introduce the value of the integral from (37) and the value of the integral found immediately above:

$$Y = \int_0^x l_\psi e^{-\int_0^x Mdx} Mdx$$

into (36):

$$l = Z^{-\frac{B}{B_1}}\left[l_\psi Z^{\frac{B}{B_1}} - aa_1\left(\psi - \frac{B\theta}{2}x^2\right) Z^{\frac{B}{B_1}} + \right.$$

$$\left. + l_0\left(b_1 - c_1\int_0^x Z^{\frac{B}{B_1}} dx\right)\right].$$

Upon expanding the expression in brackets and replacing

$$a = \frac{\omega}{s\delta_1}; \quad l_\psi = l_\Delta - \frac{\omega\psi}{s\delta_1},$$

after the transfer of l_ψ to the left-hand side, we obtain:

$$l + l_\Delta - \frac{\omega\psi}{s\delta_1} = -\frac{\omega}{s\delta_1}a_1\left(\psi - \frac{B\theta}{2}x^2\right) +$$

$$+ l_0(b_1 - c_1\int_0^x Z^{\frac{B}{B_1}} dx)Z^{-\frac{B}{B_1}}.$$

Dividing both sides by l_0 and noting that:

$$\frac{\omega}{s\delta_1}: l_0 = \frac{\omega}{W_0\delta_1} = \frac{\Delta}{\delta_1},$$

we obtain Drozdov's well-known equation:

$$\frac{l}{l_0} + \frac{l_\Delta}{l_0} - \frac{\psi\Delta}{\delta_1} = -\frac{a_1\Delta}{\delta_1}\left(\psi - \frac{B\theta}{2}x^2\right) +$$

$$+ (b_1 - c_1\int_0^x Z^{\frac{B}{B_1}} dx)Z^{-\frac{B}{B_1}}. \qquad (40)$$

In this equation we have:

$$\int_0^x Z^{\frac{B}{B_1}}\,dx = \frac{k_1}{B_1}\int_0^{\beta} Z^{\frac{B}{B_1}}\,d\beta.$$

The table for the last integral with three entries, β, γ, and B/B_1, was prepared by Prof. D.A. Venttsel and is reproduced in the appendix.

Table 2 - Computational Formulas Used in Drozdov's Exact Method

$\varphi = K + \frac{1}{3}\frac{\omega}{q}$; $\frac{\varphi m}{s}$; $I_K = \frac{e_1}{u_1}$; $v_{K,0} = I_K$; $\frac{\varphi m}{s}$;
$\frac{\omega}{s}$; $f\frac{\omega}{s}$; $l_\Delta = \frac{\omega}{s}\cdot\frac{1}{\delta_2}$; $a = \frac{\omega}{s}\cdot\frac{1}{\delta_1}$;
$l_{\psi_0} = l_\Delta - a\psi_0$; $\frac{\Delta}{\delta_1}$; $f\Delta$; $l_0 = \frac{W_0}{s}$; $\frac{l_\Delta}{l_0}$.
$B = I_K^2$; $\left(f\frac{\omega}{s}\cdot\frac{\varphi m}{s}\right)$; $B_1 = \frac{B\theta}{2} - x\lambda$;
$\frac{B}{B_1}$; $C = \frac{B_1}{k_1}$; $\gamma = \frac{C\psi_0}{k_1}$.

$$a_1 = -\frac{2\varkappa\lambda}{B + 2B_1};\quad b_1 = \frac{1}{l_0}(l_{\psi_0} + a_\alpha a_1\psi_0);$$

$$\frac{c_1 k_1}{B_1} = \frac{ak_1(1 - a_1)}{l_0 C};\quad a_1\frac{\Delta}{\delta_1}.$$

$$\beta = Cx; \quad v = v_{K,0}x; \quad \psi = \psi_0 + k_1 x + \varkappa\lambda x^2;$$

$$\frac{l}{l_0} + \frac{l_\Delta}{l_0} - \psi\frac{\Delta}{\delta_1} = -a_1\frac{\Delta}{\delta_1}\left(\psi - \frac{B\theta}{2}x^2\right) +$$

$$+\left(b_1 - \frac{c_1 K_1}{B_1}\int_0^\beta Z^{\frac{B}{B_1}}\,d\beta\right) Z^{-\frac{B}{B_1}};$$

$$p = f\Delta\left(\psi - \frac{B\theta}{2}x^2\right) : \left(\frac{l}{l_0} + \frac{l_\Delta}{l_0} - \psi\frac{\Delta}{\delta_1}\right).$$

The equation given by Drozdov for the maximum pressure is:

$$x_m = \frac{k_1}{B + 2B_1} + h,$$

where

$$h = \frac{c_1}{B + 2B_1}\,\frac{\psi_m - \frac{B\theta}{2}x_m^2}{(b_1 - c_1\int_0^x Z_x^{\frac{B}{B_1}}\,dx)Z_x^{-\frac{B}{B_1}}} \quad \text{and} \quad x_m = \frac{k_1}{B + 2B_1}.$$

h is a function of B and Δ for which a special table has been compiled and is presented below. This equation enables one to avoid approximations.

Table 3 - Approximate Values of the Function n

B \ Δ	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70
1.0	0.025	0.031	0.037	0.044	0.052	0.062	0.075	0.090	0.108	0.123	0.151
1.1	020	025	031	037	014	052	062	074	089	106	125
1.2	017	021	026	031	037	044	052	062	073	088	104
1.3	014	018	022	027	032	038	045	053	063	075	088
1.4	012	015	019	023	027	032	038	046	054	064	075
1.5	010	0.013	016	020	024	028	034	010	047	055	065
1.6	009	012	014	017	021	025	030	035	041	048	056
1.7	008	010	012	015	018	022	026	031	036	042	049
1.8	007	009	011	013	016	019	023	027	032	037	043
1.9	006	008	010	012	014	017	020	024	029	033	039
2.0	006	007	009	011	013	016	019	022	026	030	035
2.2	005	006	008	010	011	013	016	018	021	024	028
2.4	004	006	007	008	010	011	013	015	018	021	024
2.6	004	005	006	007	008	009	011	013	015	018	020
2.8	003	004	005	006	007	008	010	011	013	015	017
3.0	0025	0035	0045	0055	0065	0075	009	010	011	0125	014

Table 3 - Approximate Values of the Function m

0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80
0.037	0.044	0.052	0.062	0.075	0.090	0.108	0.123	0.151	0.177	0.208
031	037	014	052	062	074	089	106	125	147	173
026	031	037	044	052	062	073	088	104	122	144
022	027	032	038	045	053	063	075	088	103	122
019	023	027	032	038	046	054	064	075	088	104
016	020	024	028	034	010	047	055	065	076	0
014	017	021	025	030	035	041	048	056	066	077
012	015	018	022	026	031	036	042	049	057	066
011	013	016	019	023	027	032	037	043	050	058
010	012	014	017	020	024	029	033	039	045	051
009	011	013	016	019	022	026	030	035	040	046
008	010	011	013	016	018	021	024	028	032	037
007	008	010	011	013	015	018	021	024	027	030
006	007	008	009	011	013	015	018	020	022	025
005	006	007	008	010	011	013	015	017	019	021
0045	0055	0065	0075	009	010	011	0125	014	0155	0175

The equations of the preceding table should be applied to the first period; the equations for the preliminary and second periods remain unchanged (Table 3).

6. EXAMPLE OF CALCULATING THE GAS PRESSURE CURVE AND THE PROJECTILE VELOCITY BY PROF. N.F. DROZDOV'S METHOD

The following data are given:

<u>Barrel</u>: 76 mm gun, model 1936

Chamber capacity, W_0, in dm^3 ....1.515

Cross-sectional area of the bore, including rifling, s, in dm^2 ....0.4692

Path traversed by projectile inside the bore, l_A, in dm 33.91

<u>Projectile</u>

Weight of projectile, q, in kg 6.2

Forcing pressure, p_0, in kg/cm^2 300

<u>Charge</u>

Weight of charge, ω, in kg 1.08

<u>Powder constants</u>

Powder energy (force), f, in kg·dm/kg...............950000

Covolume, α, in dm^3/kg................................ 0.98

Powder density, δ, in kg/dm^3 .... 1.6

Burning rate of powder, u_1, when p = 1, in

dm/sec: kg/dm^2.......................................0.0000074

Dimensions of strip (thickness $2e_1$ mm)..............1.357

$$\varkappa = 1.06$$

$$-\varkappa\lambda = 0.06$$

Polytropic index k......................................1.2

$\theta = k - 1$.......................................0.2

BASIC COMPUTATIONAL FORMULAS

A. Preliminary Period

$$\psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}}; \quad \sigma_0 = \sqrt{1 + 4\frac{\lambda}{\varkappa}\psi_0}; \quad z_0 = \frac{2\psi_0}{\varkappa(\sigma_0 + 1)}.$$

B. First Period

$$v = \frac{sI_K}{\varphi m}x; \quad \psi = \psi_0 + k_1 x + \varkappa\lambda x^2;$$

$$\Lambda_\psi + \Lambda = a_1 \frac{\Delta}{\delta_1}\left(\psi - \frac{B\theta}{2}x^2\right) + \left(b_1 - \frac{c_1 k_1}{B_1}\int_0^x Z^{\frac{B}{B_1}} dx\right)Z^{-\frac{B}{B_1}}.$$

$$p = f\Delta \frac{\psi - \frac{B\theta}{2}x^2}{\Lambda_\psi + \Lambda}, \qquad (*)$$

where

$$B = \frac{s^2 I_K^2}{f\omega\varphi m}; \quad B_1 = \frac{B\theta}{2} - \varkappa\lambda;$$

$$k_1 = \varkappa\sigma_0; \quad a_1 = \frac{2\varkappa\lambda}{2 + 2B};$$

$$b_1 = \frac{l_\Delta}{l_0} - \frac{\Delta}{\delta_1}\psi_0(1 + a_1); \quad c_1 = \frac{k_1}{\delta_1}\Delta(1 - a_1);$$

$$\frac{1}{\delta_1} = \alpha - \frac{1}{\delta}; \quad \frac{l_\Delta}{l_0} = 1 - \frac{\Delta}{\delta};$$

$$\Lambda_\psi = \frac{l_\psi}{l_0} = 1 - \frac{\Delta}{\delta} - \frac{\Delta}{\delta_1}\psi;$$

$$l_0 = \frac{W_0}{s}; \quad \Lambda = \frac{l}{l_0}.$$

Z and $\int_0^\beta Z^{B/B_1}\,d\beta$ are determined from tables with the two entries:

$$\gamma = \frac{B_1\psi_0}{k_1^2}; \quad \beta = \frac{B_1}{k_1}x;$$

$$x_m = \frac{k_1}{B + 2B_1} + h,$$

where

$$h = \frac{c_1}{B + 2B_1} \; \frac{\psi_m - \frac{B\theta}{2}x_m^2}{\left(b_1 - c_1\int_0^x Z^{\frac{B}{B_1}}dx\right)Z^{-\frac{B}{B_1}}}.$$

C. Second Period

$$p = p_K \left(\frac{\Lambda_1 + \Lambda_K}{\Lambda_1 + \Lambda} \right)^{1+\theta};$$

$$v = v_{\Pi p} \sqrt{1 - \left(\frac{\Lambda_1 + \Lambda_K}{\Lambda_1 + \Lambda} \right)^{\theta} \left(1 - \frac{v_K^2}{v_{\Pi p}^2} \right)},$$

where

$$v_{\Pi p} = \sqrt{\frac{2g}{\varphi} \frac{f}{\theta} \frac{\omega}{q}}.$$

or

$$v = v_{\Pi p} \sqrt{1 - \left(\frac{\Lambda_1 + \Lambda_K}{\Lambda_1 + \Lambda} \right)^{\theta} \left[1 - \frac{B\theta}{2} (1 - z_0)^2 \right]};$$

$$\Lambda_1 = \frac{l_\Delta}{l_0} = \frac{\Delta}{\delta_1} = 1 - \alpha\Delta.$$

The computation of the constants is effected first from the known data:

$$\Delta = \frac{\omega}{W_0} = \frac{1.08}{1.515} = 0.7128;$$

$$\psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}} = \frac{\frac{1}{0.7128} - \frac{1}{1.6}}{\frac{950000}{30000} + 0.98 - 0.625} = 0.02429;$$

$$\sigma_0 = \sqrt{1 + 4\frac{\lambda}{\varkappa}\psi_0} = \sqrt{1 + \frac{4(-0.0566)0.02429}{1.06}} = 0.9972$$

$$z_0 = \frac{2\psi_0}{\varkappa(\sigma_0 + 1)} = \frac{2 \cdot 0.02429}{1.06 \cdot 1.9972} = 0.02294;$$

$$x_K = 1 - z_0 = 1 - 0.02294 = 0.97706;$$

$$k_1 = \varkappa\sigma_0 = 1.06 \cdot 0.9972 = 1.057;$$

$$I_K = \frac{e_1}{u_1} = \frac{0.00678}{0.0000074} = 916.2;$$

$$\varphi = 1.03 + \frac{1}{3}\frac{\omega}{q} = 1.03 + \frac{1}{3}\frac{1.08}{6.2} = 1.088;$$

$$\frac{sI_K}{\varphi m} = \frac{0.4692 \cdot 916.2 \cdot 98.1}{1.088 \cdot 6.2} = 6253;$$

$$l_0 = \frac{W_0}{s} = \frac{1.515}{0.4692} = 3.228 \text{ dm};$$

$$\frac{l_\Delta}{l_0} = 1 - \frac{\Delta}{\delta} = 1 - \frac{0.7128}{1.6} = 0.5545;$$

$$\Lambda_A = \frac{l_A}{l_0} = \frac{33.91}{3.228} = 10.51;$$

$$B = \frac{s^2 I_K^2}{f\omega\varphi m} = \frac{0.4692^2 \cdot 916.2^2 \cdot 98.1}{950000 \cdot 1.08 \cdot 1.088 \cdot 6.2} = 2.617;$$

$$B_1 = \frac{B\theta}{2} - \varkappa\lambda = \frac{2.617 \cdot 0.2}{2} + 0.06 = 0.3217;$$

$$B + 2B_1 = 2.617 + 2 \cdot 0.3217 = 3.2604;$$

$$a_1 = \frac{2\varkappa\lambda}{B + 2B_1} = \frac{2 \cdot (-0.06)}{2.617 + 2 \cdot 0.3217} = -0.0368,$$

$$b_1 = \frac{l_\Delta}{l_0} - \frac{\Delta_{\psi 0}}{\delta_1}(1 + a_1) = 0.5545 - 0.7128 \cdot 0.02429 \cdot 0.355 \cdot 0.9632 = 0.5486;$$

$$c_1 = \frac{k_1}{\delta_1}\Delta(1 + a_1) = 1.057 \cdot 0.355 \cdot 0.7128 \cdot 0.9632 = 0.2576;$$

$$\frac{c_1 k_1}{B_1} = \frac{0.2576 \cdot 1.057}{0.3217} = 0.8464;$$

$$\gamma = \frac{B_1 \psi_0}{k_1^2} = \frac{0.3217 \cdot 0.02429}{1.057^2} = 0.006995;$$

$$\frac{B}{B_1} = \frac{2 \cdot 617}{0.3217} = 8.134;$$

$$f \cdot \Delta = 950{,}000 \cdot 0.7128 = 677{,}200.$$

Computing x_m

$$x_m = \frac{k_1}{B + 2B_1} + h, \qquad (*)$$

where

$$h = \frac{c_1}{B + 2B_1} \frac{\psi_m - \frac{B\theta}{2} x_m^2}{\left(b_1 - c_1 \int_0^x Z^{\frac{B}{B_1}} dx\right) Z^{-\frac{B}{B_1}}}.$$

$-\frac{B\theta}{2} x_m^2$	0.3332
$\frac{1}{2B_1} \cdot (\psi_m - \frac{B\theta}{2} x_m^2)$	0.02633
$\frac{B_1}{k_1} x_m$	0.09865
colog Z	0.03646
$\frac{B}{B_1}$ colog Z	0.2966
$Z^{-\frac{B}{B_1}}$	1.98
$\int_0^\beta Z^{\frac{B}{B_1}} d\beta$	0.07438
b_1	0.5486
$\frac{c_1 k_1}{B_1} \int_0^\beta Z^{\frac{B}{B_1}} d\beta$	0.0629
$-c_1 \frac{k_1}{B_1} \int_0^\beta Z^{\frac{B}{B_1}} d\beta$	0.4857
$\left(b_1 - c_1 \frac{k_1}{B_1} \int_0^\beta Z^{\frac{B}{B_1}} d\beta\right) Z^{-\frac{B}{B_1}}$	0.9617

$$\int_0^\beta Z^{\frac{B}{B_1}} d\beta \text{ when } \frac{B}{B_1} = 8.0$$

β \ γ	0.006	0.006995	0.008
0.08	0.064	0.0645	0.065
0.09865		0.0[illegible]7	
0.100	0.075	0.0755	0.076

$$\int_0^\beta Z^{\frac{B}{B_1}} d\beta \text{ when } \frac{B}{B_1} = 9.0$$

β \ γ	0.006	0.006995	0.008
0.08	0.062	0.0625	0.063
0.09865		0.0723	
0.100	0.072	0.0730	0.074

able for Computing x_m by Means of Equation (*)

erations	
$x_m = \frac{k_1}{B + 2B_1}$	0.3242
$\frac{c_1}{+ 2B_1}$	0.07901
$k_1 x_m$	0.3427
$\varkappa\lambda x_m^2$	-0.0063
ψ_0	0.0243
$= k_1 x_m + \varkappa\lambda x_m^2 + \psi_0$	0.3607
$\frac{B\theta}{2} x_m^2$	0.0275
$\frac{B\theta}{2} x_m^2$	0.3332
$\frac{c_1}{+ 2B_1} \cdot (\psi_m - \frac{B\theta}{2} x_m^2)$	0.02633
$\frac{B_1}{k_1} x_m$	0.09865
colog Z	0.03646
$\frac{B}{B_1}$ colog Z	0.2966
$- \frac{B}{B_1}$	1.98

$\xi_\beta = 0.9325$

Interpolation

$\xi_\gamma = 0.4975$

β \ γ	0.006	0.006995	0.008
0.08	0.0290	0.02826	0.0275
0.09865		0.03646	
0.100	0.0379	0.03706	0.0362

$\log Z^{-1} = 0.03646$

$$\int_0^\beta Z^{\frac{B}{B_1}} d\beta \text{ when } \frac{B}{B_1} = 8.0$$

β \ γ	0.006	0.006995	0.008
0.08	0.064	0.0645	0.065
0.09865		0.0[illegible]7	
0.100	0.075	0.0755	0.076

perations	
$\dfrac{c_1}{B + 2B_1} \cdot \dfrac{\psi_m - \dfrac{B\theta}{2}x_m^2}{(b_1 - c_1 \dfrac{k_1}{B_1} \int_0^\beta Z^{\frac{B}{B_1}} d\beta) Z^{\frac{B}{B_1}}}$	0.0274
$\dfrac{k_1}{B + 2B_1}$	0.3242
x_m	0.3516

Continued

$$\int_0^\beta Z^{\frac{B}{B_1}} d\beta \text{ when } \frac{B}{B_1} = 8.134$$

$\frac{B}{B_1}$	8.0	8.134	9.0
$\int_0^\beta Z^{\frac{B}{B_1}} d\beta$	0.0747	0.07438	0.0723

Knowing the value of x_m from equation (*) (p. 530), one may find the value of the maximum pressure p_m.

Table for Computing the Ballistic Elements for the First Period
(Calculated on a 50-cm Slide-Rule)

Initial Formulas	No.	Operations	For Maximum Pressure				End of Burning
$\frac{B_1}{k_1} = 0.3043$	1	x	0.1972	0.3516	0.527	0.723	0.9771
$v = \frac{SI}{\varphi m} x = 6253x$	2	$\beta = \frac{B_1}{k_1} x$	0.060	0.1070	0.160	0.220	0.29?
	3	v in dm/sec	1233	2198	3295	4521	6110
$k_1 = 1.057;$	4	$k_1 x$	0.2084	0.3716	0.5570	0.7642	1.033
$\varkappa\lambda = -0.06$	5	(+) $\varkappa\lambda x^2$	-0.0023	-0.0074	-0.0167	-0.0314	-0.0573
$\psi = \psi_0 + k_1 x + \varkappa\lambda x^2$	6	ψ_0	0.0243	0.0243	0.0243	0.0243	0.0243
	7	ψ	0.2304	0.3885	0.5646	0.7571	1.00
$\frac{B\theta}{2} = 0.2617$	8	(-) $\frac{B\theta}{2} x^2$	0.0102	0.0323	0.0727	0.1368	0.2498
	9	$\psi - \frac{B\theta}{2} x^2$	0.2202	0.3562	0.4919	0.6205	0.7502
$a_1 \frac{\Delta}{\delta_1} = -0.00931$	10	$a_1 \frac{\Delta}{\delta_1}\left(\psi - \frac{B\theta}{2} x^2\right)$	-0.0020	-0.0033	-0.0046	-0.0058	-0.0070

Table for Computing the Ballistic Elements for the First Period
(Calculated on a 50-cm Slide-Rule)

Initial Formulas	No.	Operations	For Maximum Pressure				End of Burning
colog Z from Table 1, page 182	11	colog Z	0.0198	0.04024	0.06521	0.09596	0.1397
$\frac{B}{B_1} = 8.134$	12	$\frac{B}{B_1}$ colog Z	0.1610	0.3273	0.5304	0.7805	1.1363
$Z^{\frac{B}{B_1}} d\beta$ from	13	$Z^{-\frac{B}{B_1}}$	1.449	2.124	3.391	6.033	13.69
able on page 62(*)	14	$\int_0^\beta Z^{\frac{B}{B_1}} d\beta$	0.05137	0.07829	0.09833	0.1120	0.1209
= 0.006995	15	b_1	0.5486	0.5486	0.5486	0.5486	0.5486
	16	$(-)\ \frac{c_1 k_1}{B_1} \int_0^\beta Z^{\frac{B}{B_1}} d\beta$	0.0435	0.0663	0.0832	0.0948	0.1022
	17	$b_1 - \frac{c_1 k_1}{B_1} \int_0^\beta Z^{\frac{B}{B_1}} d\beta$	0.5051	0.4823	0.4654	0.4538	0.4463
	18	$\left(b_1 - \frac{c_1 k_1}{B_1} \int_0^\beta Z^{\frac{B}{B_1}} d\beta\right) Z^{-\frac{B}{B_1}}$	0.7319	1.0244	1.5782	2.7378	6.1098
$\frac{}{B_1} = 0.2530$	19	$\Lambda_\psi + \Lambda = (10) + (18)$	0.7299	1.0211	1.5736	2.7320	6.1028

$\gamma = 0.006995$	15	$(-)\left\{ b_1 \right.$	0.5486	0.5486	0.5486	0.5486	0 5486
	16	$(-)\left\{ \frac{c_1 k_1}{B_1} \int_0^\beta Z^{\frac{B}{B_1}} d\beta \right.$	0.0435	0.0663	0.0832	0.0948	0.1023
	17	$b_1 - \frac{c_1 k_1}{B_1} \int_0^\beta Z^{\frac{B}{B_1}} d\beta$	0.5051	0.4823	0.4654	0.4538	0.4463
	18	$\left(b_1 - \frac{c_1 k_1}{B_1} \int_0^\beta Z^{\frac{B}{B_1}} d\beta \right) Z^{-\frac{B}{B_1}}$	0.7319	1.0244	1.5782	2.7378	6.1098
$\frac{\Delta}{\delta_1} = 0.2530$	19	$\Lambda_\psi + \Lambda = (10) + (18)$	0.7299	1.0211	1.5736	2.7320	6.1028
$\psi = \frac{l_\Delta}{l_0} -$	20	$(-)\left\{ \frac{l_\Delta}{l_0} - 1 - \frac{\Delta}{\delta} \right.$	0.5545	0.5545	0.5545	0.5545	0.5545
$- \frac{\Delta}{\delta_1} \psi$	21	$(-)\left\{ \frac{\Delta}{\delta_1} \psi \right.$	0.0583	0.0983	0.1428	0.1935	0.2530
	22	Λ_ψ	0.4962	0.4562	0.4117	0.3630	0.3015
	23	$\Lambda = (\Lambda_\psi + \Lambda) - \Lambda_\psi$	0.2337	0.5649	1.1619	2.3690	5.8013
$= 3.228$	24	$l = \Lambda l_0$	0.7544	1.828	3.751	7.647	18.73
$= f\Delta \frac{\psi - \frac{B\theta}{2} x^2}{\Lambda_\psi + \Lambda}$	25	p, kg/cm^2	2043	2362	2116	1538	832.6

*) This denotes the page number in the original manuscript. This portion has not been translated as yet.

COMPUTING THE SECOND PERIOD

Computation of the constants of the second period:

$$\sqrt{\frac{2gf\omega}{\varphi\theta q}} = \sqrt{\frac{2\cdot 98.1\cdot 950000\cdot 1.08}{1.088\cdot 0.2\cdot 6.2}} = 12{,}220 \text{ dm/sec};$$

$$\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right] = 0.7502;$$

From the first period $\Lambda_1 + \Lambda_K = 6.1028$, $p_K = 832.6$ kg/cm^2.

Table for Computing the Elements of the Second Period

Initial Formulas	No.	Operations			Muzzle Face
$p = p_K\left(\dfrac{\Lambda_1 + \Lambda_K}{\Lambda_1 + \Lambda}\right)^{1+\theta} =$	1	$(+)\ \Lambda$	7	8.5	10.51
$= 832.6\left(\dfrac{6.1028}{0.3015 + \Lambda}\right)^{1.2}$	2	Λ_1	0.3015	0.3015	0.3015
	3	$\Lambda_1 + \Lambda$	7.3015	8.8015	10.810
	4	$\eta = \dfrac{\Lambda_1 + \Lambda_K}{\Lambda_1 + \Lambda}$	0.8358	0.6933	0.5645
	5	$\log \eta$	$\bar{1}.9221$	$\bar{1}.8409$	$\bar{1}.7517$
			-0.0779	-0.1591	0.2483
	6	$1.2 \log \eta$	-0.09348	-0.1909	-0.2979
			$\bar{1}.9065$	$\bar{1}.8091$	$\bar{1}.702$
	7	$\eta^{1.2}$	0.8063	0.6443	0.5036
	8	$p = p_K\eta^{1.2}$	671	536	419
$v = v_{\Pi p} \times$	9	$0.2 \log \eta$	-0.01558	-0.03182	0.04966
			$\bar{1}.9844$	$\bar{1}.9682$	$\bar{1}.9503$
$\times\sqrt{1 - \eta^{0.2}\left[1 - \dfrac{B\theta}{2}(1 - z_0)^2\right]} =$	10	$\eta^{0.2}$	0.9647	0.9294	0.8919
$= 12,220\sqrt{1 - 0.7502\eta^{0.2}}$	11	$0.7502\eta^{0.2}$	0.7237	0.6972	0.6691
	12	$1-0.7502\eta^{0.2}$	0.2763	0.3028	0.3309
	13	v, dm/sec	6424	6723	7030

Partial Formulas	No.	Operations			Muzzle Face
$p = p_K\left(\frac{\Lambda_1 + \Lambda_K}{\Lambda_1 + \Lambda}\right)^{1+\theta} =$	1	(+) Λ	7	8.5	10.51
$= 832.6\left(\frac{6.1028}{0.3015 + \Lambda}\right)^{1.2}$	2	Λ_1	0.3015	0.3015	0.3015
	3	$\Lambda_1 + \Lambda$	7.3015	8.8015	10.810
	4	$\eta = \frac{\Lambda_1 + \Lambda_K}{\Lambda_1 + \Lambda}$	0.8358	0.6933	0.5645
	5	$\log \eta$	$\bar{1}.9221$ -0.0779	$\bar{1}.8409$ -0.1591	$\bar{1}.7517$ 0.2483
	6	$1.2 \log \eta$	-0.09348 $\bar{1}.9065$	-0.1909 $\bar{1}.8091$	-0.2979 $\bar{1}.702$
	7	$\eta^{1.2}$	0.8063	0.6443	0.5036
	8	$p = p_K\eta^{1.2}$	671	536	419
$v = v_{\Pi p} \times$	9	$0.2 \log \eta$	-0.01558 $\bar{1}.9844$	-0.03182 $\bar{1}.9682$	0.04966 $\bar{1}.9503$
$\times\sqrt{1 - \eta^{0.2}\left[1 - \frac{B\theta}{2}(1 - z_0)^2\right]} =$	10	$\eta^{0.2}$	0.9647	0.9294	0.8919
$= 12{,}220\sqrt{1 - 0.7502\eta^{0.2}}$	11	$0.7502\eta^{0.2}$	0.7237	0.6972	0.6691
	12	$1-0.7502\eta^{0.2}$	0.2763	0.3028	0.3309
	13	v, dm/sec	6424	6723	7030
	14	l, dm	22.60	27.44	33.91

The results of the calculations are presented graphically in fig. 140.

ADDITIONAL NOTES FOR THE SOLUTION OF THE PROBLEM OF INTERNAL BALLISTICS BY PROF. DROZDOV'S METHOD

In the equation for the path derived by Prof. Drozdov:

$$\Lambda_\psi + \Lambda = a_1 \frac{\Delta}{\delta_1}\left(\psi - \frac{B\theta}{2}x^2\right) + (b_1 - c_1 \int_0^x Z^{\frac{B}{B_1}} dx) Z^{-\frac{B}{B_1}}$$

the function Z_x and the quantity $\int_0^x Z^{B/B_1} dx = \frac{k_1}{B_1}\int_0^\beta Z^{B/B_1} d\beta$ are found in the tables from the entries:

$$\gamma = \frac{B_1\psi_0}{k_1^2} \text{ and } \beta = \frac{B_1}{k_1}x.$$

For the sake of convenience all the calculations of $\log Z^{-1}$ and $\int_0^\beta Z^{B/B_1} d\beta$ are performed on another form for all values of x, i.e., for all combinations of β and γ.

$\gamma = 0.006995$; $\beta = 0.060$; $\beta_m = 0.1070$; $\beta = 0.160$; $\beta = 0.220$;

$$\beta_K = 0.2973.$$

The values of $\log Z^{-1}$ and of $\int_0^\beta Z^{B/B_1} d\beta$ are written for every combination of γ and β, as shown in the form. Then the interpolation factors ζ_γ and ζ_β are determined from the following equations:

$$\zeta_\gamma = \frac{\gamma - \gamma_n}{\gamma_{n+1} - \gamma_n} \text{ for horizontal interpolation;}$$

$$\zeta_\beta = \frac{\beta - \beta_n}{\beta_{n+1} - \beta_n} \text{ for vertical interpolation.}$$

Example. Find log Z^{-1}, if it is known that $\gamma = 0.006995$; $\beta = 0.2973$. These values of γ and β are not found in the table of the logarithms of function Z^{-1}, and we take the nearest values, i.e.:

$$\gamma = 0.006 \text{ and } \gamma = 0.008;$$

$$\beta = 0.28 \text{ and } \beta = 0.300,$$

and we find the values of log Z^{-1} for these combinations. We then calculate ζ_γ and ζ_β

$\zeta_\gamma = 0.4975$

$\zeta_\beta = 0.865$

β \ γ	0.006	0.006995	0.008
0.28	0.1309	0.1295	0.1280
0.2973		0.1397	
0.300	0.1428	0.1413	0.1398

$$\zeta_\gamma = \frac{0.006995-0.006}{0.008-0.006} = 0.4975$$

$$\zeta_\beta = \frac{0.2973-0.28}{0.300-0.28} = 0.865$$

We find:

$$\log Z^{-1}_{(1)} = 0.1309 - 0.4975(0.1309 - 0.1280) = 0.1295;$$

$$\log Z^{-1}_{(2)} = 0.1428 - 0.4975(0.1428 - 0.1398) = 0.1413.$$

Finally we obtain:

$$\log Z^{-1} = 0.1295 + 0.865\ (0.1413 - 0.1295) = 0.1397.$$

WORK FORM FOR DETERMINING log Z^{-1} FROM THE TABLES.

→	$\xi_\gamma = 0.4975$		
β \ γ	0.006	0.006995	0.008
0.060	0.0204	0.0198	0.0192

$$\xi_\gamma = \frac{0.006995-0.006}{0.008-0.006} = 0.4975$$

$$\log Z^{-1} = 0.0204-0.4975 \cdot (0.0204-0.0192) = 0.0198$$

$\xi_\beta = 0.350$ ↓

→ $\xi_\gamma = 0.4975$

β \ γ	0.006	0.006995	0.008
0.100	0.0379	0.03706	0.0362
0.1070	-	0.04024	-
0.120	0.0471	0.04616	0.0452

$\xi_\gamma = 0.4975$

$$\xi_\beta = \frac{0.1070-0.100}{0.120-0.100} = 0.350$$

$$\log Z^{-1}_{(1)} = 0.0379-0.4975 \cdot (0.0379-0.0362) = 0.03706$$

$$\log Z^{-1}_{(2)} = 0.0471-0.4975 \cdot (0.0471-0.0452) = 0.04616$$

$$\log Z^{-1} = 0.03706 + 0.350 \cdot (0.04616-0.03706) = 0.040$$

→ $\xi_\gamma = 0.4975$

β \ γ	0.006	0.006995	0.008
0.160	0.0663	0.06521	0.0641

$\log Z^{-1} = 0.06521$

→ $\xi_\gamma = 0.4975$

β \ γ	0.006	0.006995	0.008
0.220	0.0972	0.09596	0.0947

$\log Z^{-1} = 0.09596$

→ ξ [illegible]

$\xi_\beta = 0.350$

β \ γ	0.006	0.006995	0.008
0.100	0.0379	0.03706	0.0362
0.1070	-	0.04024	-
0.120	0.0471	0.04616	0.0452

$\xi_\beta = \frac{0.1070-0.100}{0.120-0.100} = 0.350$

$\log Z^{-1}_{(1)} = 0.0379-0.4975 \cdot (0.0379-0.0362) = 0.03706$

$\log Z^{-1}_{(2)} = 0.0471-0.4975 \cdot (0.0471-0.0452) = 0.04616$

$\log Z^{-1} = 0.03706 + 0.350 \cdot (0.04616-0.03706) = 0.040$

$\rightarrow \xi_\gamma = 0.4975$

β \ γ	0.006	0.006995	0.008
0.160	0.0663	0.06521	0.0641

$\log Z^{-1} = 0.06521$

$\rightarrow \xi_\gamma = 0.4975$

β \ γ	0.006	0.006995	0.008
0.220	0.0972	0.09596	0.0947

$\log Z^{-1} = 0.09596$

$\rightarrow \xi_\gamma = 0.4975$

$_\beta = 0.865$

β \ γ	0.006	0.006995	0.008
0.28	0.1309	0.1295	0.1280
0.2973	-	0.1397	-
0.300	0.1428	0.1413	0.1398

$\log Z^{-1} = 0.1397$

DETERMINATION OF $\int_0^{\beta} Z^{B/B_1} \cdot d\beta$ FROM THF TABLES

(APPENDIX 3)

Prof. N.F. Drozdov recommends to calculate the quantity $\int_0^{x} Z^{B/B_1} dx$ by the trapezoid rule. In order to simplify the calculations, $\int_0^{x} Z^{B/B_1} dx$ may be found from the tables of the function:

$$\int_0^{\beta} Z^{\frac{B}{B_1}} d\beta,$$

where

$$\beta = \frac{B_1}{k_1} x,$$

whence

$$x = \frac{\beta k_1}{B_1}.$$

Consequently

$$\int_0^{x} Z^{\frac{B}{B_1}} dx = \frac{k_1}{B_1} \int_0^{\beta} Z^{\frac{B}{B_1}} d\beta.$$

The tables of the function $\int_0^{\beta} Z^{B/B_1} d\beta$ are computed for B/B_1 =

= 5, 6, 7, 8, 9 and 10.

In our problem the ratio $B/B_1 = 8.134$, i.e., it is intermediate between $B/B_1 = 8.0$ and $B/B_1 = 9.0$. This requires an additional interpolation. Therefore the determination of $\int_0^\beta Z^{B/B_1} d\beta$ when $B/B_1 =$ $= 8.134$ is reduced to the calculation of $\int_0^\beta Z^{B/B_1} d\beta$ from the tables, first when $B/B_1 = 8.0$, then when $B/B_1 = 9.0$. Interpolating these values of the integrals along B/B_1, we finally obtain $\int_0^\beta Z^{B/B_1} d\beta$ for $B/B_1 = 8.134$ and the given values of β.

It should be remembered when performing these calculations that the interpolation of the intermediate integrals must be performed by the same procedure as that of the function $\log Z^{-1}$.

WORK FORM FOR DETERMINING $\int_0^{\beta} Z^{B/B_1} d\beta$ FROM THE TABLES

$\int_0^{\beta} Z^{\frac{B}{B_1}} d\beta$ when $\frac{B}{B_1} = 8.0$

$\rightarrow \zeta_{\gamma} = 0.4975$

β \ γ	0.006	0.006995	0.008
0.060	0.051	0.0515	0.052

$\rightarrow \zeta_{\gamma} = 0.4975$

β \ γ	0.006	0.006995	0.008
0.100	0.075	0.0755	0.076
0.1070		0.0786	
0.120	0.084	0.0845	0.085

$\downarrow$
$\zeta_{\beta} = 0.350$

$\rightarrow \zeta_{\gamma} = 0.4975$

β \ γ	0.006	0.006995	0.008

$\int_0^{\beta} Z^{\frac{B}{B_1}} d\beta$ when $\frac{B}{B_1} = 9.0$

$\rightarrow \zeta_{\gamma} = 0.4975$

β \ γ	0.006	0.006995	0.008
0.060	0.050	0.0505	0.051

$\rightarrow \zeta_{\gamma} = 0.4975$

β \ γ	0.006	0.006995	0.008
0.100	0.072	0.073	0.074
0.1070		0.07597	
0.120	0.080	0.0815	0.083

$\rightarrow \zeta_{\gamma} = 0.4975$

β \ γ	0.006	0.006995	0.008

$\int_0^{\beta} Z^{\frac{B}{B_1}} d\beta$ when $\frac{B}{B_1} = 8.134$

$\rightarrow \zeta_{\frac{B}{B_1}} = 0.134$

β \ $\frac{B}{B_1}$	8.0	8.134	9.0
0.060	0.0515	0.05137	0.0505

$\rightarrow \zeta_{\frac{B}{B_1}} = 0.134$

β \ $\frac{B}{B_1}$	8.0	8.134	9.0
0.1070	0.0786	0.07829	0.07597

$\rightarrow \zeta_{\frac{B}{1}} = 0.134$

β \ $\frac{B}{B_1}$	8.0	8.134	9.0

WORK FORM FOR DETERMINING $\int_0^{\beta} z^{B/B_1} d\beta$ FROM THE TABLES

$\int_0^{\beta} z^{\frac{B}{B_1}} d\beta$ when $\frac{B}{B_1} = 9.0$

$\int_0^{\beta} z^{\frac{B}{B_1}} d\beta$ when $\frac{B}{B_1} = 8.134$

8.0

006995	0.008
0515	0.052

$\longrightarrow \zeta_{\gamma} = 0.4975$

$\beta \backslash \gamma$	0.006	0.006995	0.008
0.060	0.050	0.0505	0.05[illegible]

$\longrightarrow \zeta_{\frac{B}{B_1}} = 0.134$

$\beta \backslash \frac{B}{B_1}$	8.0	8.134	9.0
0.060	0.0515	0.05137	0.0505

$\int_0^{\beta} z^{8.134} d\beta =$

$= 0.5137$ when $\beta =$

$= 0.060$

006995	0.008
0755 0786 0845	0.076 0.085

$\downarrow$

$\zeta_{\beta} = 0.350$

$\longrightarrow \zeta_{\gamma} = 0.4975$

$\beta \backslash \gamma$	0.006	0.006995	0.008
0.100 0.1070 0.120	0.072 0.080	0.073 0.07597 0.0815	0.074 0.083

$\longrightarrow \zeta_{\frac{B}{B_1}} = 0.134$

$\beta \backslash \frac{B}{B_1}$	8.0	8.134	9.0
0.1070	0.0786	0.07829	0.07597

$\int_0^{\beta} z^{8.134} d\beta =$

$= 0.07829$ when $\beta =$

$= 0.1070$

$\longrightarrow \zeta_{\gamma} = 0.4975$

$\longrightarrow \zeta_{\frac{B}{B_1}} = 0.134$

β

0.120	0.084	0.0845	0.085

0.120	0.080	0.0815	0.083

$\longrightarrow \zeta_\gamma = 0.4975$

β \ γ	0.006	0.006995	0.008
0.160	0.098	0.099	0.100

$\longrightarrow \zeta_\gamma = 0.4975$

β \ γ	0.006	0.006995	0.008
0.160	0.0930	0.0940	0.095

$\longrightarrow \zeta_{\frac{B}{B_1}} = 0.134$

β \ $\frac{B}{B_1}$	8.0	8.134	9.0
0.160	0.099	0.09833	0.09

$\longrightarrow \zeta_\gamma = 0.4975$

β \ γ	0.006	0.006995	0.0 8
0.220	0.112	0.1130	0.114

$\longrightarrow \zeta_\gamma = 0.4975$

β \ γ	0.006	0.006995	0.008
0.220	0.104	0.1055	0.107

$\longrightarrow \zeta_{\frac{B}{B_1}} = 0.134$

β \ $\frac{B}{B_1}$	8.0	8.134	9.0
0.220	0.113	0.1120	0.10

$\longrightarrow \zeta_\gamma = 0.4975$

$\zeta_\beta \downarrow = 0.865$

β \ γ	0.006	0.006995	0.008
0.28	0.119	0.1205	0.122
0.2973		0.1222	
0.300	0.121	0.1225	0.124

$\longrightarrow \zeta_\gamma = 0.4975$

$\zeta_\beta \downarrow = 0.865$

β \ γ	0.006	0.006995	0.008
0.28	0.110	0.1115	0.113
0.2973		0.1124	
0.300	0.111	0.1125	0.114

$\longrightarrow \zeta_{\frac{B}{B_1}} = 0.134$

β \ $\frac{B}{B_1}$	8.0	8.134	9.0
0.2973	0.1222	0.1209	0.12

.008
.100

$\longrightarrow \zeta_{\gamma} = 0.4975$

β \ γ	0.006	0.006995	0.008
0.160	0.0930	0.0940	0.095

$\longrightarrow \zeta_{\frac{B}{B_1}} = 0.134$

β \ $\frac{B}{B_1}$	8.0	8.134	9.0
0.160	0.099	0.09833	0.0940

$$\int_0^{\beta} z^{8.134} d\beta = 0.09833 \text{ when } \beta = 0.160$$

.008
.114

$\longrightarrow \zeta_{\gamma} = 0.4975$

β \ γ	0.006	0.006995	0.008
0.220	0.104	0.1055	0.107

$\longrightarrow \zeta_{\frac{B}{B_1}} = 0.134$

β \ $\frac{B}{B_1}$	8.0	8.134	9.0
0.220	0.113	0.1120	0.1055

$$\int_0^{\beta} z^{8.134} d\beta = 0.1120 \text{ when } \beta = 0.220$$

0.008
0.122
0.124

$\downarrow$

$\zeta_{\beta} = 0.865$

$\longrightarrow \zeta_{\gamma} = 0.4975$

β \ γ	0.006	0.006995	0.008
0.28	0.110	0.1115	0.113
0.2973		0.1124	
0.300	0.111	0.1125	0.114

$\longrightarrow \zeta_{\frac{B}{B_1}} = 0.134$

β \ $\frac{B}{B_1}$	8.0	8.134	9.0
0.2973	0.1222	0.1209	0.1424

$$\int_0^{\beta} z^{8.134} d\beta = 0.1209 \text{ when } \beta = 0.2973$$

CHAPTER 3 - SOLUTION OF THE PROBLEMS OF INTERIOR BALLISTICS FOR THE SIMPLEST CASES

1. SOLUTION OF THE PROBLEM FOR THE CASE OF INSTANTANEOUS BURNING OF THE POWDER

The colloidal powders now used burn gradually, in parallel layers, and when the web thickness is properly selected, permit the regulation of the flow of gases during burning, so that the maximum pressure in the bore p_m would not exceed a given value (usually of the order of 2500-3500 kg/cm^2).

The case of the instantaneous burning of the charge is anamalous and generally does not occur in practice. It can be achieved in practice only under special conditions, such as, for example, when burning a charge of dry pyroxylin in powder form, or of fine porous powder loaded very densely.

In that case, if the loading density were normal (Δ = 0.50-0.75), the pressure prior to the projectile's displacement would reach a maximum value of the order of several tens of thousands of atmospheres (20,000 to 40,000 kg/cm^2). The present ultimate strength of gun barrels is such that the walls of the barrel would burst when subjected to such pressures.

Nevertheless, the case of the instantaneous burning of a charge is very interesting; its examination has an important meaning when compared with gradual burning of powder because in so doing the importance of slow burning and of the shape and dimensions of the powder grains become evident. Moreover, the pressure curve p, l in the case of instantaneous burning becomes a sort of a "guide" for the curves depicting slow burning. These p, l curves arrange themselves with a certain regularity with respect to the instantaneous burning curves.

The analytical solution of the problem is very simple in the case of instantaneous burning, because one of the four variables entering into the fundamental equation of pyrodynamics is transformed into a constant ($\psi = 1$).

Let the gun and the loading conditions be characterized as follows:

The chamber capacity is W_0, the cross sectional area of the bore, including the rifling is s, the path of the projectile is $l_д$, the weight of the charge is ω, and the weight of the projectile is q. The energy of the powder is f, and α is the covolume; the adiabatic index is $k = 1 + \theta$, and the secondary work done is taken into account by the coefficient $\varphi = a + b\frac{\omega}{q}$.

When $\psi = 1$, the fundamental equation of pyrodynamics is:

$$ps(l_1 + l) = f\omega - \frac{\theta}{2}\varphi mv^2; \tag{41}$$

the equation of the projectile motion is:

$$psdl = \varphi mvdv, \tag{42}$$

where $l_1 = (W_0 - \alpha\omega)/s$ is the reduced length of the chamber at the end of burning.

When the powder in the chamber is burned instantaneously the maximum pressure is determined by means of the well known formula:

$$p_1 = \frac{f\Delta}{1 - \alpha\Delta} = \frac{f\omega}{W_0 - \alpha\omega} = \frac{f\omega}{s l_1} \tag{43}$$

The projectile will be set in motion when the following initial conditions obtain:

$$l = 0; \quad v = 0; \quad p = p_1.$$

Eliminating pressure p from equations (41) and (42), we obtain v as a function of l:

$$\frac{dl}{l_1 + l} = \frac{\varphi m v dv}{f\omega - \frac{\theta}{2}\varphi m v^2} = -\frac{1}{\theta}\,\frac{d\,\frac{\theta\varphi m v^2}{2}}{f\omega - \frac{\theta}{2}\varphi m v^2}.$$

We shall integrate this differential equation with the variables separab[illegible]:

$$\frac{l_1 + l}{l_1} = \left(\frac{f\omega - \frac{\theta\varphi m v^2}{2}}{f\omega}\right)^{-\frac{1}{\theta}} = \left(1 - \frac{v^2}{v_{\Pi p}^2}\right)^{-\frac{1}{\theta}}, \tag{44}$$

whence

$$v^2 = v_{\Pi p}^2\left[1 - \left(\frac{l_1}{l_1 + l}\right)^{\theta}\right],$$

where $v_{\Pi p}^2 = 2gf\omega/\varphi\theta q$ is the limiting velocity of the projectile:

$$v = v_{\Pi p}\sqrt{1-\left(\frac{l_1}{l_1+l}\right)^{\theta}}. \qquad (45)$$

This formula expresses the velocity v of the projectile as a function of its path l ; the velocity increases when l increases.

In order to determine the dependence of p on l from (44) we determine:

$$f\omega - \frac{\theta}{2}\varphi m v^2 = f\omega\left(\frac{l_1}{l_1+l}\right)^{\theta}$$

and include it into (41):

$$ps(l_1+l) = f\omega\left(\frac{l_1}{l_1+l}\right)^{\theta}.$$

Whence

$$p = \frac{f\omega}{sl_1}\left(\frac{l_1}{l_1+l}\right)^{1+\theta} = p_1\left(\frac{l_1}{l_1+l}\right)^{1+\theta}. \qquad (46)$$

But

$$\frac{l_1}{l_1+l} = \frac{W_1}{W_1+sl} = \frac{W_0-\alpha\omega}{W_0-\alpha\omega+sl}$$

is the ratio between the free volumes in the initial air space measured at the start of motion and at the given instant. Consequently, formula (46) is the equation of the adiabatic curve starting with the motion of the projectile under the pressure $p_1 = f\Delta/(1 - \alpha\Delta)$.

The change in temperature of the gases doing the work is expressed by the following relationship for the adiabatic process:

$$\frac{T}{T_1} = \left(\frac{l_1}{l_1 + l}\right)^{\theta}.$$

Consequently,

$$v = v_{\Pi p} \sqrt{1 - \frac{T}{T_1}}. \tag{47}$$

If we divide the numerator and denominator in parentheses in formulas (45) and (47) by l_1, and designate l/l_1 by y, we will get:

$$v = v_{\Pi p} \sqrt{1 - \frac{1}{(1 + y)^{\theta}}}; \tag{48}$$

$$p = p_1 \frac{1}{(1 + y)^{1+\theta}}. \tag{49}$$

The quantity y is the ratio of the relative projectile path to the reduced length of the free volume in the chamber at the end of burning, and is called the "number of free volumes of gas expansion."

Equations (48) and (49) show that under the given loading conditions (q, ω, f, α, W_0, s) the pressure p and the velocity v depend only upon the number y of free volumes of expansion. The greater y, the higher is the projectile velocity and the smaller the pressure; the greater the reduced length l_1 of the free space in the chamber, the greater will be the gas pressure for a given projectile path. Consequently, the drop in pressure as a function of the projectile path will be slower in a large chamber than in a small one.

It can be proved that the velocity of the projectile computed by means of formula (48) for the case of instantaneous burning will be always greater than the true velocity for the case of slow burning, under the same charging conditions.

Indeed, the maximum work done by a powder charge ω of energy f in setting a projectile of mass m in motion, is determined by the expression $f\omega/\theta$. This maximum work will be the same for both modes of burning (instantaneous and gradual) and is expressed by the areas under the curves sp as a function of l, when l varies between 0 and infinity. Consequently, in both cases the areas will be equal to:

$$s \int_0^\infty p\,dl = \frac{f\omega}{\theta}.$$

In the case of instantaneous burning the curve p, l starts from the maximum pressure p_1, then varies according to the adiabatic law, decreasing continuously (fig. 141, curve I). When burning is gradual, the curve II of the pressure p, l rises gradually from p_0, losing a portion of area A; and inasmuch as the total area under the curve p, l in the second case, limited by $l = \infty$, must be the same as the first, curve II must necessarily cross curve I during burning of the powder when the pressure drops, and then continues to rise. The excess area B between the curves, when $l = \infty$ is the limit, must be equal to A. But inasmuch as the actual bore has a finite projectile path $l_{д}$, the portion of the area B on this finite length is always smaller than A, and consequently, for a given path length, the work done by the gases and the velocity of the projectile will be always smaller in the case of gradual burning than in the case of instantaneous burning.

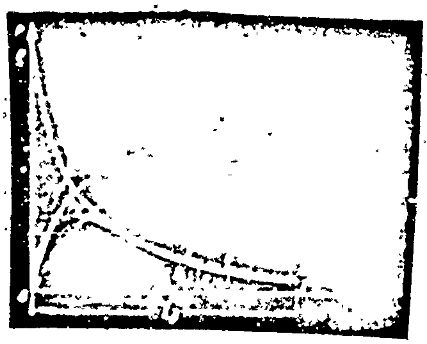

Fig. 141 - Curves p, l and v, l Depicting the Instantaneous Burning of Powder.

The actual initial (muzzle) velocity of a projectile of a medium-caliber gun, measured experimentally, represents 80-90% of the velocity computed by formula (45).

Inasmuch as the work represented by the area under the curve p, l is larger in the case of instantaneous burning than in gradual burning, especially at the start of the motion, the corresponding velocity curve rises more steeply at first. Thereafter, because of the addition of the area B, the velocity increase becomes greater in the case of gradual burning, and curve II gradually begins to approach curve I (see fig. 141), tending at $l = \infty$ to the common limit $v_{np} = \sqrt{2f\omega/\varphi\theta m}$.

2. SOLUTION OF THE PROBLEM FOR A POWDER WITH A CONSTANT BURNING AREA, WHEN THE FORCING PRESSURE IS ABSENT.

The solution of the problem of internal ballistics for degressive powders in the presence of forcing pressure results in equations which do not give an immediate relation between v, p, and l, and, therefore, exclude the possibility of an analytical examination of the basic relations. In order to obtain this possibility, it is necessary to introduce certain simplifications into the initial data, namely:

1) Consider a powder having a constant burning area

$$\varkappa = 1; \quad \lambda = 0; \quad \psi = z.$$

2) Consider the forcing pressure to be negligible; assume that the projectile is set in motion when the pressure equals the pressure of the igniter gases, and that the burning of the charge begins when the projectile is set in motion:

$$p_0 = p_B; \quad \psi_0 = 0.$$

3) Assume that $\alpha = 1/\delta$.

When these assumptions are made, the solution of the fundamental system of equations is greatly simplified.

The first assumption corresponds to the burning of long tubular powder; the second corresponds to projectiles with pre-cut bands; the third assumption simplifies the solution and permits determining the qualitative effect of the loading conditions.

Under the assumptions made, the preliminary period does not exist. The motion of the projectile begins under the following conditions:

$$p_0 = p_B; \quad \psi_0 = 0; \quad z_0 = 0; \quad l = 0; \quad v = 0.$$

Inasmuch as $\alpha = 1/\delta$,

$$l_\Delta = l_\psi = l_1 = l_0(1 - \alpha\Delta).$$

The law governing burning of powder, $\psi = f(z)$ will be expressed by the formula:

$$\psi = z = x, \tag{50}$$

and ψ may be taken as the independent variable. Then the equation of the projectile velocity will take on the form:

$$v = \frac{sI_K}{\varphi m}\psi. \tag{51}$$

The fundamental equation of pyrodynamics is:

$$ps(l_1 + l) = f\omega\psi - \frac{\theta}{2}\varphi m v^2 = f\omega\left(\psi - \frac{B\theta}{2}\psi^2\right).$$

The equation of the elementary work done is:

$$psdl = \varphi mvdv = \frac{s^2 I_K^2}{\varphi m} \psi d\psi;$$

the differential equation of the projectile path will be:

$$\frac{dl}{l_1 + l} = \frac{Bd\psi}{1 - \frac{B\theta}{2}\psi} = -\frac{2}{\theta}\,\frac{-\frac{B\theta}{2}d\psi}{1 - \frac{B\theta}{2}\psi}.$$

Integrating, we get:

$$1 + \frac{l}{l_1} = \left(1 - \frac{B\theta}{2}\psi\right)^{-\frac{2}{\theta}}, \tag{52}$$

whence,

$$l = l_1\left[\frac{1}{\left(1 - \frac{B\theta}{2}\psi\right)^{\frac{2}{\theta}}} - 1\right]. \tag{53}$$

Designating, as in the case of instantaneous burning,

$$\frac{l}{l_1} = y,$$

we obtain from equation (52):

$$\psi = \frac{2}{B\theta}\left[1 - \frac{1}{(1 + y)^{\frac{\theta}{2}}}\right], \tag{54}$$

and inasmuch as

$$v = \frac{sI_K}{\varphi m}\psi,$$

$$v = \frac{2f\omega}{\theta s I_K}\left[1 - \frac{1}{(1 + y)^{\frac{\theta}{2}}}\right]. \tag{55}$$

These equations give the direct dependence of v and ψ on the path of the projectile l, or $y = l/l_1$.

Let us write the pressure equation, taking into account the igniter pressure:

$$p_B = \frac{f_B\omega_B}{W_0 - \frac{\omega}{\delta}} \approx \frac{f_B\omega_B}{W_0 - \alpha\omega} = \frac{f_B\Delta_B}{s l_1};$$

$$p = \frac{f_B\omega_B + f\omega\psi - \frac{\theta}{2}\varphi m v^2}{s(l_1 + l)}. \tag{56}$$

Let us designate the relative energy of the igniter gases by:

$$\frac{f_B\omega_B}{f\omega} = \chi_B.$$

Carrying $f\omega$ outside the parentheses in (56) and replacing v according to (52), we obtain:

$$p = \frac{f\omega\left[\chi_B + \psi\left(1 - \frac{B\theta}{2}\psi\right)\right]}{sl_1(1 + y)} = p_1 \frac{\chi_B}{1 + y} + p_1 \frac{\psi}{(1 + y)^{1 + \frac{\theta}{2}}}, \tag{57}$$

where $p_1 = f\omega/sl_1 = f\Delta/(1 - \alpha\Delta)$ is the maximum pressure developed by the burning of the entire charge within the space of the chamber when the density of the loading is Δ;

$$1 - \frac{B\theta}{2}\psi = \frac{1}{(1 + y)^{\frac{\theta}{2}}}$$ on the basis of equation (52).

Substituting ψ by its expression in (54), we obtain the pressure p as a function of the path of the projectile:

$$p = p_1 \frac{\chi_B}{1 + y} + p_1 \frac{2}{B\theta}\left[1 - \frac{1}{(1 + y)^{\frac{\theta}{2}}}\right] \frac{1}{(1 + y)^{1+ \frac{\theta}{2}}}. \tag{58}$$

The quantity

$$p_1 \frac{\chi_B}{1 + y} = \frac{f_B\omega_B}{sl_1(1 + y)} = \frac{p_{B,0}}{1 + y} = p_B$$

represents the pressure developed by the igniter gases in the variable space of the bore. At the start of motion $y = 0$, $p_B = p_{B,0}$; as the projectile moves forward and y increases, p_B decreases $(1 + y)$ times, and may be neglected when compared with the pressure developed by the gases of the powder charge.

At the beginning of motion, $y = 0$, the second term is equal to zero (equation 58); $p = p_{B,0}$. As the projectile moves and y increases, the factor in the brackets increases, while the factor $(1 + y)^{-(1+\theta/2)}$ decreases.

The maximum pressure p_m will occur at some value y_m.

Let us designate:

$$F(y) = \left[1 - \frac{1}{(1 + y)^{\frac{\theta}{2}}} \right] \frac{1}{(1 + y)^{1+ \frac{\theta}{2}}} = (1 + y)^{-\left(1+\frac{\theta}{2}\right)} - (1 + y)^{-(1+\theta)}.$$

Differentiating F(y) with respect to y, and equating the derivative to zero, we find:

$$1 + y_m = \left(\frac{1 + \theta}{1 + \frac{\theta}{2}} \right)^{\frac{2}{\theta}} = F_1(\theta) = \text{const.} \qquad (59)$$

[when $\theta = 0.2$; $F_1(\theta) = 2.387$]

whence,

$$l_m = l_1 [F_1(\theta) - 1] = l_0(1 - \alpha\Delta)[F_1(\theta) - 1]. \qquad (60)$$

Substituting (59) in (58), (54) and (55), we obtain the expressions for all the elements of motion at the instant $p = p_{max}$:

$$p_m = \frac{p_1}{B}\ \frac{2}{\theta}\left[\frac{\theta}{2(1+\theta)}\right]\left(\frac{1+\frac{\theta}{2}}{1+\theta}\right)^{\frac{2+\theta}{\theta}} =$$

$$= \frac{p_1}{B}\ \frac{1}{1+\theta}\left(\frac{1+\frac{\theta}{2}}{1+\theta}\right)^{\frac{2+\theta}{\theta}} = \frac{p_1}{B}F_2(\theta). \quad (61)$$

When $\theta = 0.20$, $F_2(\theta) = 0.3200$.

$$\psi_m = \frac{1}{B(1+\theta)}; \quad (62)$$

$$v_m = \frac{f\omega}{sI_K}\ \frac{1}{1+\theta}. \quad (63)$$

Equations (60), (61), (62) and (63) give the direct relationship between the elements of motion and several characteristics and parameters at the instant of maximum pressure. Thus the path l_m is directly proportional to the reduced length of the chamber l_0 and to $1 - \alpha\Delta$. When Δ increases, l_m decreases. The pressure p_m is directly proportional to the pressure p_1 of instantaneous burning, determined by Nobel's equation; p_m is inversely proportional to Prof. Drozdov's parameter B. ψ_m is also inversely proportional to the parameter B.

When $\theta = 0.2$:

$$p_m = 0.320\,\frac{p_1}{B}, \quad \frac{l_m}{l_0} = \Lambda_m = 1.387(1 - \alpha\Delta).$$

The equations are very simple and accessible to analysis.

At the end of burning, $\psi = 1$.

$$v_K = \frac{sI_K}{\varphi m}; \tag{64}$$

$$1 + y_K = \frac{1}{\left(1 - \frac{B\theta}{2}\right)^{\frac{2}{\theta}}}; \tag{65}$$

$$p_K = \frac{p_1}{(1 + y_K)^{1+\frac{\theta}{2}}} = p_1\left(1 - \frac{B\theta}{2}\right)^{\frac{2+\theta}{\theta}}. \tag{66}$$

In the second period we find the following relationships:

$$p = p_K\left(\frac{l_1 + l_K}{l_1 + l}\right)^{1+\theta} = p_K\left(\frac{1 + y_K}{1 + y}\right)^{1+\theta}.$$

Substituting for p_K and $1 + y_K$ their expressions from (65) and (66), we find:

$$p = \frac{p_1}{\left(1 - \frac{B\theta}{2}\right)}\frac{1}{(1 + y)^{1+\theta}}. \tag{67}$$

This is the equation of an adiabatic curve with initial ordinate $p_1/(1 - \frac{B\theta}{2})$; the latter increases with B, i.e., with the thickness of the powder.

From the general equation for the projectile velocity, we have:

$$v = v_{\Pi p}\sqrt{1 - \left(\frac{1 + y_K}{1 + y}\right)^{\theta}\left(1 - \frac{B\theta}{2}\right)} =$$

$$= v_{\Pi p}\sqrt{1 - \left(\frac{1}{1 - \frac{B\theta}{2}}\right)\frac{1}{(1 + y)^{\theta}}}. \tag{68}$$

The Temperature of Powder Gases.

In the case of instantaneous burning we had:

$$\frac{T}{T_1} = \left(\frac{l_1}{l_1 + l}\right)^{\theta} = \frac{1}{(1 + y)^{\theta}}. \tag{69}$$

When burning is gradual:

$$\frac{T}{T_1} = 1 - \frac{1}{\psi}\frac{v^2}{v_{\Pi p}^2} = 1 - \frac{v}{\psi}\frac{v}{v_{\Pi p}^2}.$$

Substituting the values of ψ, v from (51) and (55) and $v_{\Pi p}^2 = 2f\omega/\varphi\theta m$, we get:

$$\frac{T}{T_1} = 1 - \frac{sI_K}{\varphi m}\frac{\varphi\theta m}{2f\omega}\frac{2f\omega}{\theta sI_K}\left[1 - \frac{1}{(1 + y)^{\frac{\theta}{2}}}\right]$$

or

$$\frac{T}{T_1} = \frac{1}{(1 + y)^{\frac{\theta}{2}}}. \qquad (70)$$

A comparison of (69) with (70) will show that in the expression for relative gas temperature for gradual burning of a powder with a constant area, the value of the exponent is one-half of that of instantaneous burning. Consequently, the temperature drop in the case of gradual burning proceeds almost at half the rate of instantaneous burning, because the work done in traversing a given path is considerably smaller.

The expression for the temperature T/T_1 may be written as a function of ψ only:

$$\frac{T}{T_1} = 1 - \frac{v_K^2 \psi^2}{\psi\, v_{\Pi p}^2} = 1 - \frac{v_K^2}{v_{\Pi p}^2}\psi = 1 - \frac{B\theta}{2}\psi, \qquad (71)$$

i.e., the temperature of the gases inside the barrel during burning of powder with constant area is a linear decreasing function of ψ.

At the end of burning ($\psi = 1$)

$$\frac{T_K}{T_1} = 1 - \frac{B\theta}{2}.$$

The thicker the powder, the larger is B, and the lower is the temperature T_K.

In the second period, from expression (68):

$$\frac{T}{T_1} = 1 - \frac{v^2}{v_{\Pi p}^2} = \frac{1}{1 - \frac{B\theta}{2}} \frac{1}{(1 + y)^{\theta}} \qquad (72)$$

or

$$\frac{T}{T_1} = \left(1 - \frac{B\theta}{2}\right)\left(\frac{1 + y_K}{1 + y}\right)^{\theta} = \frac{T_K}{T_1}\frac{T}{T_K}. \qquad (73)$$

These relatively simple equations enable one to perform an analysis of the variation of the elements of a shot (p, v, ψ, T) as a function of y - the relative path of the projectile - and to establish a series of relations and properties of the variation curves of these elements.

CHAPTER 4 - ANALYSIS OF THE BASIC RELATIONS FOR THE SIMPLEST CASE

$$\left(\kappa = 1,\ \psi_0 = 0,\ \alpha = \frac{1}{\delta}\right)$$

1. ANALYSIS OF THE FUNDAMENTAL CURVES p, v, T, ψ.

An analysis of the equations obtained, and of the curves represented by them, shows that they represent certain simple combinations of two types of curves (fig. 142):

a) Two polytropic curves with exponents $k = 1 + \theta$ and $k' = 1 + \theta/2$, starting from the point (1, 0), first dropping steeply and then more gradually;

b) Two curves analogous to the polytropic curves but with exponents smaller than unity ($k - 1 = \theta$ and $k' - 1 = \theta/2$) also starting from point (1, 0) and descending much more slowly with the convex side directed downward.

Fig. 142 - Basic Types of Curves for the Simplest Case.

Indeed, in the case of instantaneous burning:

$$\frac{T}{T_1} = \frac{1}{(1 + y)^{\theta}}; \tag{69}$$

in the case of gradual burning:

$$\frac{T}{T_1} = \frac{1}{(1 + y)^{\frac{\theta}{2}}} \tag{70}$$

$$\psi = \frac{2}{B\theta}\left[1 - \frac{1}{(1 + y)^{\frac{\theta}{2}}}\right] = \frac{2}{B\theta}\left[1 - \frac{T}{T_1}\right]; \tag{54'}$$

$$\frac{v}{v_{\text{пр}}} = \sqrt{\frac{B\theta}{2}}\,\psi = \sqrt{\frac{2}{B\theta}}\left[1 - \frac{1}{(1 + y)^{\frac{\theta}{2}}}\right] =$$

$$= \sqrt{\frac{2}{B\theta}}\left[1 - \frac{T}{T_1}\right]; \tag{55'}$$

$$\frac{p}{p_1} = \frac{\psi}{(1 + y)^{1+\frac{\theta}{2}}}; \tag{57'}$$

$$\frac{p_K}{p_1} = \frac{1}{(1 + y_K)^{1+\frac{\theta}{2}}}. \tag{66'}$$

In the second period:

$$\frac{p}{p_1} = \frac{1}{\left(1 - \frac{B\theta}{2}\right)} \frac{1}{(1 + y)^{1+\theta}}; \tag{67'}$$

$$\frac{T}{T_1} = \frac{1}{1 - \frac{B\theta}{2}} \frac{1}{(1 + y)^{\theta}}; \tag{72}$$

$$\frac{v}{v_{np}} = \sqrt{1 - \frac{T}{T_1}}. \tag{68'}$$

Each of these equations contains one of the polytropics indicated above in the form of a variable component.

The curves $\frac{T}{T_1}$, y for instantaneous and gradual burning are expressed directly in the first period by curves with exponents θ and $\theta/2$, according to equations (69) and (70).

The ordinates of the curves ψ (54') and v (55') are obtained from the ordinates of the curves $\Delta T/T_1 = 1 - T/T_1$ (fig. 142), measured from the horizontal 1-1, multiplied by the coefficients $2/B\theta$ and $2f\omega/I_K\theta s$, respectively, and laid off upwards along the abscissa from the origin of coordinates to y_K which corresponds to the end of burning (fig. 143). The curves ψ, y and $\frac{v}{v_{np}}$, y are inverted with respect to the curves $\frac{\Delta T}{T_1}$, y.

The ordinates of the relative pressure curve (57') are obtained by multiplying the ordinates of the ψ curve (54') by those of the auxiliary polytropic curve $y = 1/(1 + y)^{1+\theta/2}$. This same polytropic curve is the geometrical locus of the pressure p_K/p_1 at the end of burning, expressed as a function of y (66).

Fig. 143 - Curves ψ and v/v_{np} in the First Period.

It is seen from equation (66) that when B (powder thickness) increases, p_K decreases, while y_K increases [according to (65)].

The curves of the second period elements start to the right of the abscissa y_K; the curve T/T_1 represents the curve $T/T_1 = 1/(1 + y)^{\theta}$ of instantaneous burning, multiplied by the quantity $1/(1 - B\theta/2) > 1$, which means that the gradual burning curve is higher than the instantaneous burning curve, the difference in height increasing with the parameter B, i.e., the difference being greater for thicker powders.

From equation (60) it is seen that the maximum pressure is independent of the parameter B, of the thickness of the powder, and of the weight of the projectile. For a given Δ, the length l_m is proportional to the length l_0 of the chamber; when Δ increases the maximum is shifted toward the start of motion.

The real maximum pressure [equation (61)] is proportional to the maximum pressure in the case of instantaneous burning, $p_1 = f\Delta/(1 - \alpha\Delta)$, and is inversely proportional to the parameter B, or to the powder density squared. When the energy of the powder, entering in the expressions for the pressure p_1 and the denominator B, is changed, the maximum pressure varies proportionally to the energy of the powder squared, and to the weight of the projectile, because

$$B = \frac{s^2 I_K^2 g}{f\omega\varphi q},$$

and

$$p_m = \frac{p_1}{B} F_2(\theta) = F_2(\theta) \frac{f\Delta}{1 - \alpha\Delta} \frac{f\omega\varphi m}{s^2 I_K^2}.$$

When B increases, ψ_m and v_m decrease also [equations (62) and (63)].

The adiabatic pressure curve in the case of instantaneous burning, $p/p_1 = 1/(1 + y)^{1+\theta}$, acts as "guide" for the adiabatic curves of the second period when the powder burns gradually. The ordinates of these curves are obtained by multiplying the ordinates of the first curve by $1/(1 - B\theta/2) > 1$ [equation (67)]. The thicker the powder, the larger is B, the larger is this value, and the higher will the adiabatic curve of the second period lie above the adiabatic curve of instantaneous burning. The ratio of the ordinates of these adiabatics for the same value of y (or l) is constant and equal to $1/(1 - B\theta/2)$ = const.

In the case of instantaneous burning and in the second period, the projectile velocity is proportional to the square root of the temperature drop, rather than to the first power of this factor, as is the case for the first period.

For a given charging density and during the burning of the powder, the projectile velocity in a given section does not depend upon the weight of the projectile. Indeed, from equation (55)

$$v = \frac{2f\omega}{I_K \theta s}\left[1 - \frac{1}{(1 + y)^{\frac{\theta}{2}}}\right]$$

it is seen that for a given value of $y = l/l_0(1 - \alpha\Delta)$ and for one and the same powder (f, α, I_K) the projectile velocity is independent of the weight q of the projectile. The same may be said of the temperature of the gases [according to formula (70)].

If, all other conditions being equal, we vary only q which enters into parameter B, the pressure p and ψ from equations (54) and (56) and the maximum pressure p_m and ψ_m increases in proportion with q, while the location of the maximum and of the value v_m does not change.

Consequently, the velocity curves coincide point by point when superimposed on each other(*), and only when the projectile

(*) The result obtained (when $\alpha = 1/\delta$) can be confirmed by comparing it with the GAU tables (ordnance tables), compiled for $p_0 = 300$, $\alpha = 1$, and $\varkappa = 1.06$.

is heavier will the velocity v_K [from equation (64)] be attained earlier.

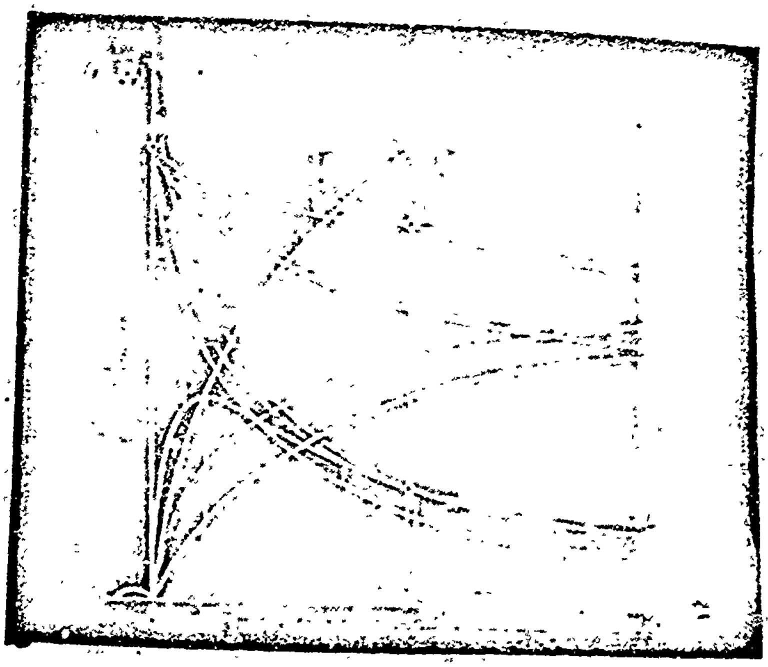

Fig. 144 - Ballistic Curves for the Simplest Case.

The fact that the velocity of the projectile does not vary with its weight may be used to verify the complete burning of the powder in the gun. If a gun using the same charge and type of powder is used to fire two projectiles of different weights and the velocity remains unchanged, it is proof that the combustion of the powder was incomplete in both cases.

Figure 144 illustrates the basic curves of the elements of a shot (ψ, v, T, p) as a function of l or y for the cases of instantaneous and gradual burning of the powder.

All the curves on the graph are marked with the number of the equation they represent.

There are two basic points on the ordinate axis, one is at -1, and the other at $-1/(1 - B\theta/2)$.

The two pressure polytropic curves with exponents k and k', and the two "polytropics" of temperature, with exponents θ for instantaneous burning (69), and $\theta/2$ for gradual burning (70) issue from the first point; the ordinate at $y = -1$ is a common asymptote for all of these curves.

The curves p/p_1 (67') and T/T_1 (72) for the second-period, issue from the second point, whose ordinate is $1/(1 - B\theta/2)$. These curves are real only to the right of the ordinates p_K/p_1 and $T_K/T_1 = 1 - B\theta/2$ with abscissa y_K.

Both of these curves lie above the corresponding curves for instantaneous burning, the ratio between the two sets being constant and equal to $1/(1 - B\theta/2) > 1$. The horizontal line whose ordinate equals unity is the origin for the curves p/p_1(1) and T/T_1 (69) and (70), the terminal point for ψ and v/v_K (54'), and is an asymptote for v/v_{np} for both instantaneous and gradual burning of powder.

The curves ψ, y and $\frac{v}{v_{np}}$, y in the first period are similar to the curve $\Delta T/T$ which is measured from the horizontal along the ordinate equal to unity.

The curve p/p_1 for gradual burning is obtained by multiplying the ordinates of curve ψ(54') and of the adiabatic curve 2 with the exponent $k' = 1 + \theta/2$.

2. THE CONDITIONS FOR MAINTAINING THE MAXIMUM PRESSURE CONSTANT.

This question is very important in the ballistic design of guns, because the condition generally imposed is that p_m must not exceed a certain given value. For this reason the designer must know how to vary the loading conditions in order to keep the maximum pressure constant.

The relations derived above permit one to establish the analytical conditions under which the maximum pressure p_m will remain constant when the weight of the charge or its density are varied in a given gun.

Indeed,

$$p_m = \frac{p_1}{B} F_2(\theta) = \frac{f\Delta}{1 - \alpha\Delta} \frac{F_2(\theta) f\omega\varphi q}{s^2 I_K^2 g}. \qquad (61')$$

For a given type of powder (f, α, u_1) and a given projectile weight q, p_m can be changed by varying either Δ or $I_K = e_1/u_1$. If Δ is increased simultaneously with $2e_1$ or I_K, p_m can be kept constant. The condition of the constancy of the pressure is obtained in the form:

$$p_m = \frac{F_2(\theta)}{B} \frac{f\Delta}{1 - \alpha\Delta} = \text{const.}$$

Let us group together the constants:

$$B\left(\frac{1}{\Delta} - \alpha\right) = \frac{f}{p_m} F_2(\theta) = \text{const.}$$

Designating:

$$\frac{f}{p_m} F_2(\theta) = a_m \approx \text{const,}$$

we obtain the condition for maintaining p_m constant:

$$B\left(\frac{1}{\Delta} - \alpha\right) = a_m = \text{const.} \tag{74}$$

In the case when $\alpha \neq 1/\delta$, when $\psi_0 = 0$ and $\varkappa = 1$, the condition p_m = const. has an analogous form:

$$B\left(\frac{1}{\Delta} - \frac{1}{\delta}\right) = \text{const.}$$

Knowing a_m and given Δ, one may find the quantity B insuring the obtainment of the given p_m, and, knowing B, one may find the corresponding value of $2e_1$ or I_K.

Condition (74) shows that in order to keep p_m constant when Δ is increased, it is necessary to increase the thickness $2e_1$ of the powder in order to offset the decrease of B obtained from increasing the weight ω of the charge together with the increase of Δ.

From the condition (74) of the constancy of the maximum pressure for a given gun, projectile and powder of definite physico-chemical properties (f, α, δ, u_1), a direct relation may be established between the weight ω of the charge, the thickness $2e_1$ of the powder or its pressure impulse I_K, and the reduced length of the free space in the chamber at the end of burning:

$$l_1 = l_0(1 - \alpha\Delta) = \frac{W_0}{s}(1 - \alpha\Delta) = \frac{W_1}{s}.$$

Indeed, substituting the value of B in (74) and replacing Δ in the denominator by ω/W, we obtain:

$$\frac{s^2 I_K^2 W_0(1 - \alpha\Delta)}{f\omega\varphi m\omega} = \frac{s^2 I_K^2 W_1}{f\omega^2\varphi m} = a_m.$$

Transposing all the constants to the right side, and designating them by K_m, we obtain:

$$\frac{I_K^2 W_1}{\omega^2} = \frac{a_m f\varphi m}{s^2} = \frac{f^2 F_2(\theta)\varphi m}{p_m s^2} = K_m. \tag{75}$$

Computing first K_m from the loading conditions by the following equation:

$$K_m = \frac{F_2(\theta) f^2 \varphi m}{p_m s^2}$$

and calculati g the value ω of the charge necessary to insure a given initial (muzzle) velocity v_A, we can determine the full pressure impulse $I_K = e_1/u_1$

$$I_K = \frac{\sqrt{K_m \omega}}{\sqrt{W_0 - \alpha\omega}}. \tag{76}$$

This equation shows that in order that the pressure in a given gun remain constant when the weight of the charge is increased, the full pressure impulse I_K or the thickness $2e_1$ of the powder must be increased at a somewhat higher rate than the weight of the charge.

Let us apply equation (76) to calculate the powder thickness for a 76 mm gun, 1902 model.

The conditions of loading are:

$W_0 = 1.654$; $s = 0.4693$; $\omega = 0.930$; $q = 6.5$; $f = 9 \cdot 10^5$, $\alpha = 1$;

$u_1 = 7.5 \cdot 10^{-6}$; $\varphi = 1.08$; $\theta = 0.2$; $p_m = 2320 \cdot 10^2$.

$$\sqrt{K_m} = \sqrt{\frac{F_2(\theta) f^2 \varphi m}{s^2 p_m}} = \sqrt{\frac{0.32 \cdot 9^2 \cdot 10^{10} \cdot 1.08 \cdot 0.0663}{0.4693^2 \cdot 2.32 \cdot 10^5}} =$$

$$= \sqrt{36.4 \cdot 10^4} = 603;$$

$$I_K = \frac{e_1}{u_1} = \frac{\sqrt{K_m}\,\omega}{\sqrt{W_0 - \alpha\omega}} = \frac{603 \cdot 0.930}{\sqrt{1.654 - 0.930}} = \frac{603 \cdot 0.930}{0.85} = 660 \text{ kg} \cdot \text{sec/dm}^2;$$

$$2e_1 = 2u_1 I_K = 2 \cdot 7.5 \cdot 10^{-6} \cdot 660 = 0.0099 \text{ dm} = 0.99 \text{ mm} \approx 1 \text{ mm}(*).$$

We have obtained the thickness of strip or tubular powder of grade SP which was used in this gun, and developed a velocity v_A =

(*) This thickness of tubular powder corresponds to the grade 7/7, while the thickness of 1.28 mm corresponds to grade 9/7.

Indeed, substituting the value of B in (74) and replacing Δ in the denominator by ω/W, we obtain:

$$\frac{s^2 I_K^2 W_0(1-\alpha\Delta)}{f\omega\varphi m\omega} - \frac{s^2 I_K^2 W_1}{f\omega^2\varphi m} = a_m.$$

Transposing all the constants to the right side, and designating them by K_m, we obtain:

$$\frac{I_K^2 W_1}{\omega^2} = \frac{a_m f\varphi m}{s^2} = \frac{f^2 F_2(\theta)\varphi m}{P_m s^2} = K_m. \tag{75}$$

Computing first K_m from the loading conditions by the following equation:

$$K_m = \frac{F_2(\theta) f^2 \varphi m}{p_m s^2}.$$

and calculating the value ω of the charge necessary to insure a given initial (muzzle) velocity v_A, we can determine the full pressure impulse $I_K = e_1/u_1$

$$I_K = \frac{\sqrt{K_m \omega}}{\sqrt{W_0 - \alpha\omega}}. \tag{76}$$

This equation shows that in order that the pressure in a given gun remain constant when the weight of the charge is increased, the full pressure impulse I_K or the thickness $2e_1$ of the powder must be increased at a somewhat higher rate than the weight of the charge.

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$$\sqrt{K_m} = \sqrt{\frac{F_2(\theta) f^2 \varphi m}{s^2 p_m}} = \sqrt{\frac{0.32 \cdot 9^2 \cdot 10^{10} \cdot 1.08 \cdot 0.0663}{0.4693^2 \cdot 2.32 \cdot 10^5}} =$$

$$= \sqrt{36.4 \cdot 10^4} = 603;$$

$$I_K = \frac{e_1}{u_1} = \frac{\sqrt{K_m}\,\omega}{\sqrt{W_0 - \alpha\omega}} = \frac{603 \cdot 0.930}{\sqrt{1.654 - 0.930}} = \frac{603 \cdot 0.930}{0.85} = 660 \text{ kg:sec/dm}^2;$$

$$2e_1 = 2u_1 I_K = 2 \cdot 7.5 \cdot 10^{-6} \cdot 660 = 0.0099 \text{ dm} = 0.99 \text{ mm} \approx 1 \text{ mm}(*).$$

We have obtained the thickness of strip or tubular powder of grade SP which was used in this gun, and developed a velocity v_A =

(*) This thickness of tubular powder corresponds to the grade 7/7, while the thickness of 1.28 mm corresponds to grade 9/7.

= 588 m/sec when the charge ω was 0.930 kg.

The same gun may be fired with a charge ω = 1.08 kg and a velocity of v_0 = 620 m/sec can be obtained at the same p_m.

Find from equation (75) the thickness of the powder in this case.

Inasmuch as $\sqrt{K_m}$ remains the same as in the first case,

$$I_{K2} = \frac{603 \cdot 1.08}{\sqrt{1.654-1.08}} = \frac{603 \cdot 1.08}{0.757} = 860,$$

the thickness of tubular powder for the same u_1 will be

$$2e_1 = 2 \cdot 7.5 \cdot 10^{-6} \cdot 860 = 0.0129 \text{ dm} = 1.29 \text{ mm},$$

and this is the thickness of our previous powder C_{42}.

The ratio

$$\frac{I_{K2}}{I_{K1}} = \frac{860}{660} = 1.3 \approx \frac{9}{7}.$$

Calculations show that the equations derived from approximate relations yield results which are close to experimental data, and may be used to calculate the variation in the powder thickness concomitant with variations in the charge, if the maximum pressure is kept the same.

3. THE POSITION OF MAXIMUM PRESSURE p_m IN THE BORE OF THE GUN, OR THE PATH l_m TRAVERSED BY THE PROJECTILE AT THE INSTANT OF MAXIMUM PRESSURE.

$$l_m = l_1 \left[F_1(\theta) - 1 \right] = l_0(1 - \alpha\Delta)\left[F_1(\theta) - 1 \right]. \tag{77}$$

For a given loading density in a gun employing a given type of powder, the maximum pressure p_m is developed at the same distance from the starting point of the projectile (l_m = const.), regardless of the powder thickness and the projectile weight. That is, the path traversed by the projectile up to the instant of maximum pressure does not depend on the powder thickness $2e_1$ nor on the weight q of the projectile.

In a given gun (W_0, s, l_0) using a given type of powder (f, α, u_1, θ) the position (l_m) of maximum pressure depends only on the density of the charge. The larger Δ, the smaller will be $(1 - \alpha\Delta)$ and the nearer to the starting point of motion will be p_m(*).

Equation (77) shows that under the condition of constancy of the maximum pressure p_m, when Δ is increased with the simultaneous increase of B and of the powder thickness $2e_1$, the maximum p_m shifts toward the origin of the projectile motion. Because the parameter B increases thereby, then, on the basis of formula (62):

(*) This conclusion is also confirmed by the GAU tables for the case of $p_0 = 300$ kg/cm^2, $\kappa = 1.06$, and $\alpha \neq 1/\delta$.

$$\psi_m = \frac{1}{B(1 + \theta)}$$

the portion of the charge burned at the instant of maximum pressure decreases.

4. END OF BURNING AND PATH l_K TRAVERSED BY PROJECTILE IN THE BORE OF THE GUN.

The location of the projectile at the end of burning is determined by equation (65), while the corresponding pressure p_K is found from equation (66).

From (65) we get:

$$l_K = l_0(1 - \alpha\Delta)\left[\frac{1}{\left(1 - \frac{B\theta}{2}\right)^{\frac{2}{\theta}}} - 1\right].$$

The equation shows that for a given loading density Δ, the quantity l_K increases with the increase of parameter B, i.e., mainly with the increase in powder thickness. When the weight of the projectile is diminished while Δ remains the same, the path traversed by the projectile at the end of burning is shifted toward the muzzle face.

Equation (66) shows that for a given loading density the values of p_K at the end of burning, when B is varied (i.e., when the powder thickness and the projectile weight are varied) lie on the curve p, y whose equation is:

$$p_K = p_1 \frac{1}{(1 + y_K)^{1+\frac{\theta}{2}}} . \qquad (66)$$

This curve is of the same type as the adiabatic curve for instantaneous burning whose exponent is however $1 + \theta/2$. This curve is known as the pressure curve of completely burned powder.

The above statements are clarified by the graph of fig. 145.

The curves а, б, в and г depict the pressure variation during the burning of powders of different thicknesses when Δ is the same.

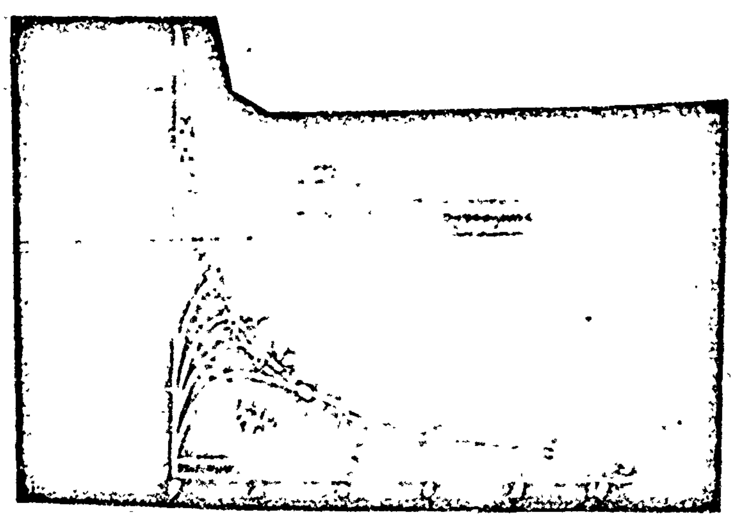

Fig. 145 - Pressure Curves for Different Powder Thicknesses.

а - thick powder; б - normal powder; в - thin powder; г - very thin powder.

2... $\bar{\Delta}$ - curve p_K, y_K; 1... o - curve p, y for instantaneous burning.

Curve 1 - $\frac{p}{p_1} = \frac{1}{(1 + y)^{1,2}}$; Curve 2 - $\frac{p_K}{p_1} = \frac{1}{(1 + y_K)^{1,1}}$.

They are disposed in such a way that their maxima are on the same ordinate at a distance y_m from the origin. The end of burning occurs at a distance which is governed by the powder thickness, the distance being the greater the thicker the powder ($y_{Kа} > y_{Kб} > y_{Kв}$); the pressure values at the end of burning increase as the powder thickness decreases ($p_{Kа} < p_{Kб} < p_{Kв}$). The points corresponding to the end of burning lie on curve 2-2 calculated from equation (66) [when $\theta = 0.2$, $1 + \theta/2 = 1.1$].

Curve 1-1 corresponds to the adiabatic variation of the pressure at instantaneous powder burning ($1 + \theta = 1.2$).

The disposition of curves 1-1 and 2-2 shows that the pressure curves for gradual burning (а, б, в, г) intersect the curve 1-1 depicting instantaneous burning. The second period curves for the cases а, б, в, and г, which are not represented on the diagram, are all disposed below the curve 2-2 and above the curve 1-1.

At the same time, since for the given powder $p_1/(1 - B\theta/2) = \text{const.}$, the nature of the pressure change p in the second period depends upon the variation of the variable factor $1/(1 + y)^{1+\theta}$, the latter varying as in the case of instantaneous burning, i.e., along the adiabatic curve with initial pressure

$$p_1' = p_1 \frac{1}{\left(1 - \frac{B\theta}{2}\right)} > p_1.$$

When the powder thickness is decreased, B and p_1' decrease also, and inasmuch as the adiabatic curves with the same exponent $1 + \theta$ do not cross, the adiabatics in the second period are disposed the lower, the thinner the powder, i.e., inversely to the disposition of the pressure curves in the first period.

If we compare the expressions for pressure in the second period and at instantaneous burning, keeping the value of y the same, we will get the following:

In the case of gradual burning in the second period

$$p'' = p_1 \frac{1}{\left(1 - \frac{B\theta}{2}\right)} \frac{1}{(1 + y)^{1+\theta}};$$

in the case of instantaneous burning

$$p' = p_1 \frac{1}{(1 + y)^{1+\theta}}$$

or

$$\frac{p''}{p'} = \frac{1}{1 - \frac{B\theta}{2}} = \text{const},$$

i.e., when the loading density is the same, the ratio of the pressure in the second period to the pressure at instantaneous burning remains constant for any path length of the projectile (greater than l_K). This ratio decreases when B decreases and the projectile weight increases.

The graphs and equations presented above for the simplest case ($\varkappa = 1$, $\psi_0 = 0$, $\alpha = 1/\delta$) permit one to estimate directly the appearance and the form of the basic relations between the ballistic elements of a shot. They depict the location and magnitude of maximum pressure, its dependence on the loading conditions (Δ, B) the position of the projectile at the end of burning and the gas pressure developed thereby, the condition of maintaining the maximum pressure constant when the weight of the charge and the powder thickness are varied, and the independence of the curve y, l in the first period of the weight of the projectile.

Such simple relations are not obtained for the more complex cases ($\varkappa \neq 1$, $\psi_0 \neq 0$, $\alpha \neq 1/\delta$). In such a case it becomes necessary to analyze the effect of the individual elements by computing a series of variations or by using the data found in ballistic tables.

CHAPTER 5 - A SURVEY OF CERTAIN OTHER METHODS OF SOLUTION

(Written by Prof. G.V. Oppokov)

1. A VARIATION OF PROF. G.V. OPPOKOV'S SOLUTION

In order to integrate equation (13), Chapter 1, (p. 473):

$$\frac{dl}{l_\psi - l} = \frac{Bxdx}{\psi_0 \pm k_1 x - B_1 x^2} \tag{78}$$

it is convenient to apply the usual method of classical mathematical analysis - the method of substitution, namely, of temporarily introducing into the process a new variable, ζ, so that:

$$l = \zeta + a \frac{B\theta}{2} x^2 - l_\Delta, \tag{79}$$

where a is the difference between the lengths of the free volumes of the chamber at the start and end of burning:

$$a = l_\Delta - l_1 = \frac{\omega}{s}\left(\alpha - \frac{1}{\delta}\right). \tag{80}$$

When this substitution of variables is effected in the new equation, it will become presently apparent that "the last term" does not contain in the denominator the difference:

$$\psi_0 + k_1 x - B_1 x^2.$$

This obviates the need for mathematical transformations in the course of integration for the purpose of replacing the obtained integral by other, more simple ones.

Indeed, it follows from equation (79) that:

$$\frac{dl}{dx} = \frac{d\zeta}{dx} + B\theta ax,$$

because l_Δ is a constant in every concrete case. Moreover:

$$l_\psi + l = l_\Delta - a\psi + l = l_\Delta - a(\psi_0 + k_1 x + \varkappa\lambda x^2) + l.$$

Substituting in the above the value of l from (79):

$$l_\psi + l = \zeta + a\frac{B\theta}{2}x^2 - a(\psi_0 + k_1 x + \varkappa\lambda x^2)$$

or

$$l_\psi + l = \zeta - a(\psi_0 + k_1 x - B_1 x^2).$$

We shall substitute into the differential equation (78) the obtained values of dl/dx and $(l_\psi + l)$:

$$\frac{d\zeta}{dx} + B\theta ax = \frac{Bx}{\psi_0 + k_1 x - B_1 x^2}\left[\zeta - a(\psi_0 + k_1 x - B_1 x^2)\right].$$

We now remove the brackets and effect the necessary simplifications:

$$\frac{d\zeta}{dx} + B\theta ax = \frac{Bx}{\psi_0 + k_1 x - B_1 x^2}\zeta - Bax.$$

Grouping the terms:

$$\frac{d\zeta}{dx} - \frac{Bx}{\psi_0 + k_1 x - B_1 x^2}\zeta = -Ba(1 + \theta)x.$$

The common integral of this linear differential equation of the first order including the last term can be represented in the following form:

$$\zeta = e^{\int \frac{Bxdx}{\psi_0+k_1x-B_1x^2}} \left[C_1 - Ba(1 + \theta) \int e^{-\int \frac{Bxdx}{\psi_0+k_1x-B_1x^2}} xdx \right], \qquad (81)$$

where e is the base of natural logarithms, and C_1 is still an arbitrary constant.

We must introduce into the analysis Prof. Drozdov's function:

$$e^{\int \frac{Bxdx}{\psi_0+k_1x-B_1x^2}} = Z^{-\frac{B}{B_1}}.$$

Then the partial integral of the last equation with respect to the derivative $d\zeta/dx$ will be:

$$\zeta = Z^{-\frac{B}{B_1}} \left[C_1 - Ba(1 + \theta) \int_0^x Z^{\frac{B}{B_1}} xdx \right].$$

We shall now return to the desired path l of the projectile, and after substituting the obtained value of ζ, get:

$$l = Z^{-\frac{B}{B_1}} \left[C_1 - Ba(1+\theta) \int Z^{\frac{B}{B_1}} x dx \right] + a \frac{B\theta}{2} x^2 - l_\Delta.$$

Let us determine now the constant C_1 from the initial conditions, at which:

$$l = 0; \quad x = 0; \quad Z^{-\frac{B}{B_1}} = 1; \quad \int_0^x Zxdx = 0.$$

We have from the latter equation for the path of the projectile:

$$0 = 1(C_1 + 0) + 0 - l_\Delta,$$

whence

$$C_1 = l_\Delta.$$

Thus the desired path of the projectile is defined by the following expression:

$$l = Z^{-\frac{B}{B_1}} \left[l_\Delta - aB(1+\theta) \int_0^x Z^{\frac{B}{B_1}} x dx \right] + a \frac{B\theta}{2} x^2 - l_\Delta. \qquad (82)$$

2. PARTICULARS OF PROF. I.P. GRAVE'S SOLUTION

This method of solution was developed by Prof. I.P. Grave in order to perfect Bianchi's method, which was the first variant (in time) of the $l_{\psi_{av.}}$ method. In Bianchi's original equations the effect of the variation of $l_{\psi_{av.}}$ is discounted and in integrating he considers the quantity l_ψ as a certain incompletely determined constant. Bianchi divides the curve p, l into three segments, for which instead of l_ψ he takes l_Δ, $l_\Delta - \frac{1}{2}a$ and l_1, respectively, which corresponds to the following conditions:

$$\psi_{av.} = 0; \quad \psi_{av.} = 0.5; \quad \psi_{av.} = 1.$$

But in this case the curves p, l and v, l are not smooth; they have angular points corresponding to the beginning of the second and third segments.

In order to take into account the effect of the variation of l_ψ and obtain smooth p, l and v, l curves, Prof. I.P. Grave, in integrating the equation for the path of the projectile, considers l_ψ as a variable, defined by the average value of its derivative with respect to l. This average value of the derivative must be negative, because l_ψ decreases during the burning of the powder. Consequently:

$$\frac{dl_\psi}{dl} = -k,$$

whence, after integration, we obtain:

$$l_\psi = l_{\psi_0} - kl. \tag{83}$$

At the end of burning (when $\psi = 1$), we obtain from the above

$$l_1 = l_{\psi_0} - kl_K,$$

from which the constant k is determined:

$$k = \frac{l_{\psi_0} - l_1}{l_K} = \frac{l_{\psi_0} - l_1}{l_1} : \frac{l_K}{l_1}. \tag{84}$$

If we substitute the value of l_ψ obtained from equation (83) into the differential equation (78) for the projectile path, we will get:

$$\frac{dl}{dx} = \frac{Bx(l_{\psi_0} - kl + l)}{\psi - \frac{B\theta}{2}x^2},$$

whence, after separating the variables, we have:

$$\frac{dl}{l_{\psi_0} + (1 - k)l} = \frac{Bxdx}{\psi - \frac{B\theta}{2}x^2}.$$

Integrating this equation:

$$\frac{1}{1 - k} \ln \frac{l_{\psi_0} + (1 - k)l}{l_{\psi_0}} = \int_0^x \frac{Bxdx}{\psi - \frac{B\theta}{2}x^2}$$

and, consequently, the following must obtain:

$$(1 - k)\ l = l_{\psi_0} Z^{-(1-k)\frac{B}{B_1}} - l_{\psi_0}.$$

Using a second time equation (83), we obtain the following equation from the above:

$$l = l_{\psi_0} Z^{-(1-k)\frac{B}{B_1}} - l_{\psi}. \qquad (85)$$

A certain difficulty arises from the fact that in order to apply equation (85) it is necessary to know the constant k, for which, in turn, it is necessary to know l_K/l_1 [see equation (84)]; but the path l_K of the projectile at the end of the period is unknown beforehand.

In order to overcome this difficulty, a nomograph is given [see I.P. Grave, "VNUTRENNYAYA BALLISTIKA" (Internal Ballistics), Pyrodynamics, No. 1, p. 58] which enables one to determine the ratio l_K/l_1 if

$$\frac{l_{\psi_0}}{l_1} \text{ and } \frac{\Phi(x_K)}{H\kappa\lambda} = \frac{B}{B_1} \ln Z_K^{-1} = 2.303\ \frac{B}{B_1} \log Z_K^{-1}.$$

are known.

Having found from the conditions of loading:

$$\frac{l_0}{l_1} \text{ and } 2.303 \frac{B}{B_1} \log Z_K^{-1},$$

we can determine l_K/l_1 from this graph; this will enable us to calculate k from equation (84).

If it is necessary to find k more accurately, the obtained value of k may be rendered more exact by successive approximations. The value k_1 found from the graph is substituted into (85), $l_{K_1 2}/l_1$ is found in the second approximation, and a new value k_2 is then determined from (84) representing a second approximation, and so on, until two consecutive values of l_K/l_1 coinciding with the required degree of accuracy are obtained.

CHAPTER VI - SOLUTION OF THE FUNDAMENTAL PROBLEM OF INTERNAL BALLISTICS ON THE BASIS OF THE PHYSICAL LAW OF BURNING

(M.Ye. Serebriakov's Method)

As was shown in Part I of this text, the actual burning of powders deviates from the geometric law under the influence of a number of factors. An analysis obtained by the aid of the progressivity curves Γ, ψ and Γ, t has shown that certain anomalies actually occur even during the burning of powders of simple shapes: non-instantaneous ignition, accelerated burning of the outside layers (ballooning), etc. The burning law cannot be established at all on the basis of the geometric law for adulterated and porous powders used in pistol cartridges.

The actual burning law can be established only by burning powder in a test bomb at different loading densities and by obtaining pressure-time curves reflecting all the deviations and peculiarities of a given sample.

The variation in the intensity of gas formation Γ and $I = \int_0^t pdt$ as a function of ψ and t can be established from the obtained p, t curve.

Both graphs Γ, ψ and $\int_0^t pdt, \psi$ in conjunction with the fundamental p, t curve obtained from the bomb test enable us to solve the fundamental problem of pyrodynamics, i.e., to compute the gas-pressure and projectile velocity variation curves under conditions of actual burning of powder in the bore of a gun.

These graphs also enable us to establish the individual behavior of powder lots of different grades occasionally differing considerably as to their properties, which behavior could not be disclosed by any method other than by bomb tests.

There have been actual cases where powder lots of the same grade and the same manufacturer having identical chemical composition and dimensions produced a difference of 6-8% in the charging weights when fired at the same values of v_p and p_{max}.

Bomb tests had shown that the burning rate u_1 of these powders varied as much as 15-20%. This variation could not have been disclosed by any other means except the bomb test.

The fundamental problem of internal ballistics for adulterated or porous powders can be solved in exactly the same manner only on the basis of the experimental (physical) law of burning.

We are presenting below the method of solving the fundamental problem on the basis of the physical law of burning, when the burning rate law is $u = u_1 p$, which corresponds to the coincidence of the curves I, ψ or $\int pdt, \psi$ at various loading densities.

The basic assumption made here is that both in a bomb at different loading densities Δ and in a weapon with a variable space in the case of a continuously decreasing loading density, the value of $\int pdt$ is a single-valued function of ψ only, and does not depend on the loading density. This condition, which has been proved by bomb tests at different loading densities, is being extrapolated in the given case for considerably higher values of Δ in a weapon.

1. DERIVATION OF BASIC RELATIONSHIPS AS APPLIED TO THE PHYSICAL LAW OF BURNING.

The solution is based on applying the pressure curve obtained from bomb tests to the computation of curves depicting the gas pressure and velocity of the projectile in a weapon.

As the projectile moves through the bore and the initial air space becomes larger, the pressure will depend on the current value of the loading density $\Delta = \omega/(W_0 + sl)$.

We shall introduce the following designations:

Δ_1 - loading density of powder in test bomb;

Δ_0 - initial loading density in weapon.

P and τ - gas pressure and time corresponding to the given value of ψ at constant loading density Δ_1, at which the bomb test was conducted and at which the curve P, τ was obtained;

p and t - gas pressure and time corresponding to the same value of ψ when Δ is variable, which condition applies to a given disposition of the projectile in the bore of the barrel.

We shall designate the corresponding integral values as follows:

a) In bomb

$$\int_0^{\psi} Pd\tau = I$$

$$\int_0^{\psi_0} Pd\tau = I_0$$

$$\int_0^{1} Pd\tau = I_K$$

b) In weapon

$$\int_0^{\psi} pdt = i$$

$$\int_0^{\psi_0} pdt = i_0$$

$$\int_0^{1} pdt = i_K.$$

I is obtained from a table or graph as a function of ψ or τ, on the basis of bomb tests.

Inasmuch as the pressure impulse does not depend on Δ, we will have the equalities

$$\int_0^{\psi} p dt = \int_0^{\psi} P d\tau \quad \text{or } i = I \tag{86}$$

and, correspondingly,

$$I_0 = i_0; \quad I_K = i_K.$$

Differentiating (86), we get:

$$p dt = P d\tau. \tag{87}$$

Here $d\tau$ – an elementary time lapse during which the portion of change ψ burned up to a given instant under pressure P will receive the increment $d\psi$ when the powder is burned in a constant volume at a loading density Δ_1;

dt – time lapse during which the same portion ψ of the burned powder will receive the same increment $d\psi$ when the powder is burned in the gun barrel at pressure p at loading density Δ determined by the current disposition of the projectile in the bore of the barrel:

$$\Delta = \frac{\omega}{W_0 + sl}.$$

We shall consider henceforth the value of ψ as the independent variable.

2. DETERMINING THE PROJECTILE VELOCITY AS THE FUNCTION OF ψ

On the basis of the impulse theorem

$$\varphi m dv = spdt.$$

Integrating from the start of motion:

$$\varphi mv = s \int_{\psi_0}^{\psi} pdt = s(i - i_0),$$

where ψ_0 is the portion of the charge burned in the gun at the start of motion:

$$\psi_0 = \frac{\frac{1}{\Delta_0} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}}.$$

Determining v:

$$v = \frac{s}{\varphi m}(i - i_0) \tag{88}$$

or on the basis (86)

$$v = \frac{s}{\varphi m} \int_{\psi_0}^{\psi} Pd\tau = \frac{s}{\varphi m}(I - I_0). \tag{89}$$

The values of I and I_0 for ψ and ψ_0 are known from the bomb test, $s/\varphi m$ is known from the gun data. Thus the velocity of the projectile is determined from equation (89) as the function of ψ.

At the end of burning when $\psi = 1$

$$v_K = \frac{s}{\varphi m}(I_K - I_0). \qquad (90)$$

In contrast to the analogous formula in the case of the geometric law of burning, the value of I_K corresponds not to the average thickness of the powder but to the maximum thickness, which may considerably exceed the average thickness of powders having a variable thickness. A diagram of the pressure impulse of tabular powder usually obtained in bomb tests is offered in fig. 146.

The value of the impulse $I_1 = e_{1\,av.}/u_1$ corresponds to the burning of powder of average thickness; the value of $I_K > I_1$ corresponds to the burning of the thickest element of the charge.

Fig. 146 - Pressure Impulse of Tubular Powder

Therefore also v_K in the case of actual powder burning assumes a considerably greater value than in the case of the geometric law, but in that case the projectile will also traverse a considerably longer path at the end of powder burning, so that:

$$\underset{\text{(Physical law of burning)}}{l_K} > \underset{\text{(Geometric law of burning)}}{l_K}$$

3. DETERMINING THE PATH OF THE PROJECTILE AS A FUNCTION OF ψ

In the method outlined the relation between l and ψ is established by means of the auxiliary function L, ψ determined from the same bomb test at a loading density Δ_1, by the additional analysis of the test curve P, τ. And if the value of the pressure impulse I, ψ does not depend on the choice of Δ_1, then the function L, ψ depends on the value of Δ_1 chosen at the test.

As we shall see later, the function L has the dimensionality of the path and a definite physical meaning.

We will have from equation (88):

$$dl = vdt = \frac{s}{\varphi m}(i - i_0)dt,$$

where $(i - i_0)$ is a function of ψ.

Upon integrating, the path of the projectile will be determined by the formula

$$l = \frac{s}{\varphi m} \int_{\psi_0}^{\psi} (i - i_0)dt. \qquad (91)$$

Here the element dt corresponds to the element $d\psi$ when the powder is burned under conditions of variable volume (space) and depends on the value of pressure p at any given instant which is still unknown.

We can obtain an expression from the bomb test analogous to expression (91):

$$L_{\psi_0}^{\psi} = \frac{s}{\varphi m}\int_{\psi_0}^{\psi}(I - I_0)d\tau = \frac{s}{\varphi m}\int\int_{\psi_0}^{\psi} Pd\tau d\tau = \frac{s}{\varphi m}\cdot G. \qquad (92)$$

The value of L - function of ψ - is obtained by the second integration of curve $\int_{\psi_0}^{\psi} Pd\tau$ with respect to τ and by multiplying same by the coefficient $s/\varphi m$.

If l is the path traversed by the projectile at the instant the portion ψ of the charge is burned, then L is the path the projectile would have traversed if the pressure behind it developed according to the same law as in a bomb with a constant loading density Δ_1, at the instant the same portion of the charge ψ is burned.

L has a definite physical meaning. For example, it is obtained in practice in a bomb of considerable capacity with a free piston. Thus in a bomb of capacity $W_0 = 300\ cm^3$ the piston displacement (with the piston usually having a cross-sectional area of $s = 1\ cm^2$) is about 3 cm. The change in volume amounts to only 1%, and hence the piston is displaced by a pressure which increases in almost a constant volume.

The value of L as a function ψ is found from the bomb test using the procedure given in the table below.

It is necessary to establish the relation between l and L, and hence between l and ψ, because L is function of ψ.

Differentiating equations (91) and (92) and taking their ratio and reducing, we get:

$$dl = DL\frac{dt}{d\tau}. \tag{93}$$

From equation (87)

$$\frac{dt}{d\tau} = \frac{P}{p}, \tag{94}$$

whereas the ratio P/p is replaced by the ratio of the free volumes on the basis of the equation of state:

$$p = \frac{RT_1\omega_1\psi}{(W_\psi)} = \frac{f\Delta_1\psi}{1 - \frac{\Delta_1}{\delta} - \Delta_1\left(\alpha - \frac{1}{\delta}\right)\psi} = \frac{f\psi}{\frac{1}{\Delta_1} - \frac{1}{\delta} - \left(\alpha - \frac{1}{\delta}\right)\psi}.$$

The expression in the denominator represents the free specific gas volume at loading density Δ_1.

An analogous expression will obtain also in the formula for p. We shall replace them with average values, because the last term is small in comparison with th: first two, and, moreover, the ratios of the free volumes will enter hem all.

$$W_{\psi_{av.}} = W_0\left[1 - \frac{\Delta_1}{\delta} - \Delta_1\left(\alpha - \frac{1}{\delta}\right)\psi_{av.}\right] = W_0\left(1 - \frac{\alpha + \frac{1}{\delta}}{2}\Delta_1\right) =$$

$$= W_0(1 - \alpha'\Delta_1),$$

where $\alpha' = \dfrac{\alpha + \frac{1}{\delta}}{2}$.

We shall introduce the designations:

$$\frac{1}{\delta} + \left(\alpha - \frac{1}{\delta}\right) \cdot \frac{1}{2} = \frac{1}{2}\left(\frac{1}{\delta} + \alpha\right) = \alpha';$$

$$\frac{1}{\Delta_1} - \alpha' = a_1; \quad \frac{1}{\Delta_0} - \alpha' = a_0.$$

Then

$$p = \frac{f\psi}{a_1};$$

$$p = \frac{RT\omega\psi}{W_{\psi_{av.}} + sl} = \frac{f\Delta_0\psi}{1 - \alpha'\Delta_0 + \frac{l}{l_0}} \frac{T}{T_1} = \frac{f\psi}{a_0 + \frac{l}{l_0\Delta_0}} \frac{T}{T_1} =$$

$$= \frac{f\psi}{a_0\left(1 + \frac{l}{l_0 a_0 \Delta_0}\right)} \frac{T}{T_1},$$

but

$$l_0 a_0 \Delta_0 = l_0(1 - \alpha'\Delta_0) = l_{\psi_{av.}} = l_c.$$

For a gradually burned powder

$$\frac{T}{T_1} = \left(\frac{W_0 - \alpha'\omega}{W_0 - \alpha'\omega + sl}\right)^{\frac{\theta}{2}} = \left(\frac{a_0}{a_0 + \frac{l}{l_0\Delta_0}}\right)^{\frac{\theta}{2}} =$$

$$= \frac{1}{\left(1 + \frac{l}{l_c}\right)^{\frac{\theta}{2}}}.$$

The ratio P/p, following substitution of the proper expressions and simplification, will take on the form:

$$\frac{P}{p} = \frac{a_0}{a_1}\left(1 + \frac{l}{l_c}\right)^{1+\frac{\theta}{2}}. \tag{95}$$

Upon incorporating this expression in (94) and then in (93), we get:

$$dl = dL\,\frac{a_0}{a_1}\left(1 + \frac{l}{l_c}\right)^{k'}, \quad \text{where } k' = 1 + \frac{\theta}{2}.$$

Dividing the variables and integrating:

$$\int_0^l \frac{dl}{\left(1 + \frac{l}{l_c}\right)^{k'}} = \frac{a_0}{a_1}\int_{\psi_0}^{\psi} dL.$$

Designating $1 + l/l_c = x$, we get in the left side

$$\int_0^l \frac{dl}{\left(1 + \frac{l}{l_c}\right)^{k'}} = \frac{2}{\theta} l_c \left(1 - \frac{1}{x^{\frac{\theta}{2}}}\right);$$

and in the right side

$$\frac{a_0}{a_1} L_{\Psi_0}^{\Psi}.$$

We then obtain:

$$1 - \frac{1}{x^{\frac{\theta}{2}}} = \frac{\theta}{2} \frac{a_0}{a_1 l_c} L_{\Psi_0}^{\Psi} = B' L_{\Psi_0}^{\Psi}, \tag{96}$$

where

$$B' = \frac{\theta}{2} \frac{a_0}{a_1 l_c} = \frac{\theta}{2} \frac{\frac{1}{\Delta_0} - \alpha'}{\frac{1}{\Delta_1} - \alpha'} \frac{1}{l_0(1 - \alpha'\Delta_0)} = \frac{\theta}{2} \frac{1}{\Delta_0 l_0} \frac{1}{\frac{1}{\Delta_1} - \alpha'},$$

but

$$\Delta_0 l_0 = \frac{\omega}{s},$$

and

$$B' = \frac{\theta}{2}\,\frac{s}{\omega}\,\frac{1}{\frac{1}{\Delta_1} - \alpha'}. \tag{97}$$

Solving (96) for l, we get:

$$l = l_c\left[\frac{1}{(1 - B'L^{\Psi}_{\Psi_0})^{\frac{2}{\theta}}} - 1\right]. \tag{98}$$

As in the solution of the problem of internal ballistics by the average l_Ψ method, the value l_c - average for the entire burning process - can be replaced in this formula by the current value $l_{\Psi_{av.}}$ by means of the usual formula:

$$l_{\Psi_{av.}} = l_0\left[1 - \frac{\Delta}{\delta} - \Delta\left(\alpha - \frac{1}{\delta}\right)\frac{\Psi_0 + \Psi}{2}\right], \tag{99}$$

and then

$$l = l_{\Psi_{av.}}\left[\frac{1}{(1 - B'L^{\Psi}_{\Psi_0})^{\frac{2}{\theta}}} - 1\right] \tag{100}$$

Formula (100) gives the path l as a function of ψ by means of the auxiliary function $L_{\psi_0}^{\psi}$, determined in bomb tests at loading density $\Delta = \Delta_1$, which function reflects (depicts) the true burning law.

Comparing this formula with the analogous formula used in the method fo solution in which $l_\psi = l_{\psi_{av.}}$, we will note that in place of Prof. Drozdov's function Z_x^{-B/B_1} formula (100) contains the function $(1 - B'L_{\psi_0}^{\psi})^{-2/\theta}$. For the case where $\varkappa = 1$, $\lambda = 0$,

$$\frac{B}{B_1} = \frac{2}{\theta}.$$

Therefore the expression in parentheses $(1 - B'L_{\psi_0}^{\psi})$ has replaced in this solution the function Z_x in the case of the geometric law of burning.

Formulas (89) and (100) enable us to calculate and plot the projectile velocity curve as a function of path l.

Pressure p is found from the fundamental equation of pyrodynamics:

$$p = \frac{f\omega\psi - \frac{\theta}{2}\varphi m v^2}{s(l_\psi + l)}, \tag{101}$$

wherein the variable quantities as the ψ functions are already known.

To determine the maximum pressure p_m and the corresponding value ψ_m, we differentiate the equation (101) with respect to t:

$$\frac{dp}{dt} = \frac{p}{l_\psi + l}\left\{\frac{f\omega}{s}\Gamma\left[1 + \left(\alpha - \frac{1}{\delta}\right)\frac{p}{f}\right] - (1 + \theta)v\right\},$$

whereby $v = S/\varphi m(I - I_0)$ (89) and Γ are given in the table as a function of ψ.

Equating the expression in braces to zero and replacing v by its expression in (89), we get:

$$\frac{f\omega}{s}\Gamma_m\left[1 + \left(\alpha - \frac{1}{\delta}\right)\frac{p_m}{f}\right] - (1 + \theta)\frac{s}{\varphi m}(I_m - I_0) = 0,$$

whence

$$I_m - I_0 = \frac{f\omega\varphi m}{s^2(1 + \theta)}\left[1 + \left(\alpha - \frac{1}{\delta}\right)\frac{p_m}{f}\right]\cdot\Gamma_m. \qquad (102)$$

Denoting the factor of Γ_m by D, we get:

$$I_m - I_0 = D \cdot \Gamma_m.$$

The value of ψ_m is found as the point of intersection of curves $I - I_0$ and $D \cdot \Gamma$ as a function of ψ.

The point of intersection gives the values of $(I_m - I_0)$, ψ_m and Γ_m.

The diagram in fig. 147 clarifies the above.

It is not difficult to see that if $I_m - I_0$ and Γ_m are replaced by theoretical expressions in terms of z and x, on the basis of the geometric law, we will obtain the usual relationship for x_m.

At the end of burning when $\psi = 1$, we will have:

$$v_K = \frac{s}{\varphi m}(I_K - I_0);$$

$$l_K = l_c\left[\frac{1}{(1 - B'L'_{\psi_0})^{\frac{2}{\theta}}} - 1\right];$$

$$p_K = \frac{f\omega}{s}\,\frac{1 - \frac{v_K^2}{v_{\Pi p}^2}}{l_1 + l_K}.$$

The usual formulas apply to the second period:

$$p = p_K\left(\frac{l_1 + l_K}{l_1 + l}\right)^{1+\theta}; \qquad (103)$$

$$v = v_{\Pi p}\sqrt{1 - \left(\frac{l_1 + l_K}{l_1 + l}\right)^{\theta}\left(1 - \frac{v_K^2}{v_{\Pi p}^2}\right)}. \qquad (104)$$

GRAPHIC NOT REPRODUCIBLE

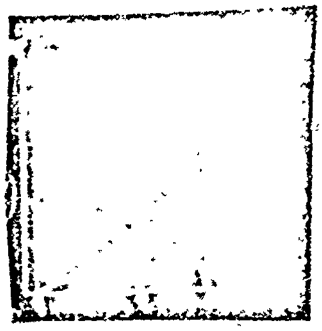

Fig. 147 - Determining ψ_m for Maximum Pressure.

4. GRAPHICAL CLARIFICATION OF THE METHOD OF SOLUTION

In order to solve the problem on the basis of the physical law of burning, it is first necessary to perform the ballistic analysis of the given powder. To do so, bomb tests are conducted at two loading densities Δ_1 and Δ_2, the ballistic characteristics - propellent force of powder f and covolume α - are determined, and also the test characteristic of the intensity of gas formation Γ, ψ and the impulse of pressure increase $\int_0^{\psi} P d\tau = I$ (fig. 148).

Knowing the loading conditions of the weapon, we determine

$$\psi_0 = \frac{\frac{1}{\Delta_0} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}};$$ from graph I, ψ we find the corresponding value of $I_0 = \int_0^{\psi_0} P d\tau$, and upon subtracting this value of I_0 from all the values of I, obtain the dependence of $I - I_0$ on ψ and τ (fig. 149). Integrating numerically the curve $I - I_0, \tau$ with respect to τ, we find the integral $\int_{\psi_0}^{\psi} (I - I_0) d\tau$ as a function of ψ.

Introducing the designation:

$$G_{\psi_0}^{\psi} = \int_{\psi_0}^{\psi}\int Pd\tau d\tau = \int_{\psi_0}^{\psi} (I - I_0)d\tau.$$

Multiplying $I - I_0$ and $G_{\psi_0}^{\psi}$ by $s/\varphi m$, we get

$$v = \frac{s}{\varphi m}(I - I_0); \tag{89}$$

$$L_{\psi_0}^{\psi} = \frac{s}{\varphi m} G_{\psi_0}^{\psi} = \frac{s}{\varphi m}\int_{\psi_0}^{\psi} (I - I_0)d\tau. \tag{92}$$

v does not depend on Δ (Δ_1 or Δ_2) and is a function of ψ only for all the loading densities. The function $L_{\psi_0}^{\psi}$, being a function of ψ. depends at the same time on Δ, because the time element $d\tau$ during which a definite portion of charge $d\psi$ is burned decreases with the increase of Δ.

Indeed, from the equality

$$\Gamma = \frac{d\psi}{Pd\tau}$$

it follows that

$$d\tau = \frac{d\psi}{\Gamma P},$$

where

$$p = \frac{f\Delta_1\psi}{1 - \frac{\Delta_1}{\delta} - \Delta_1\left(\alpha - \frac{1}{\delta}\right)\psi} \approx \frac{f\Delta_1\psi}{1 - \alpha'\Delta}$$

and hence $d\tau$ varies inversely with the change of Δ_1.

For this reason the curves $G^{\psi}_{\psi_0}$ and $L^{\psi}_{\psi_0}$ as a function of ψ will also be disposed the lower the greater Δ_1 when testing the powder in a bomb (see fig. 149).

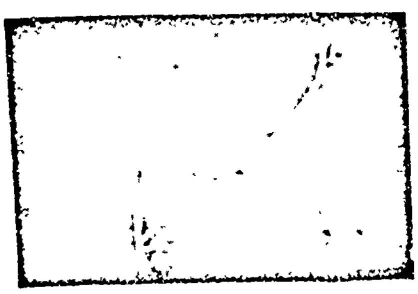

Fig. 148 - Basic Curves Γ, ψ and I, ψ.

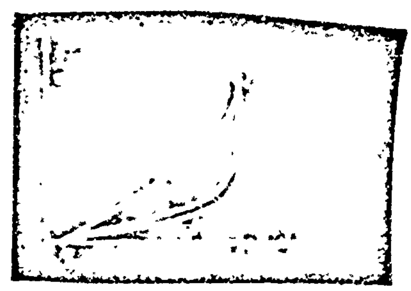

Fig. 149 - Auxiliary Curves for Determining the Elements of a Shot.

The arrangement of a table for analyzing bomb tests as a means of obtaining all the auxiliary functions is presented below; this table serves to clarify the graphs in figs. 148 and 149.

Procedure for Analyzing a Bomb Test at Loading Density Δ_1 as a Means of Obtaining the Basic Functions

$$G = \int_{\psi_0}^{\psi} (I - I_0)d\tau = \int\int_{\psi_0}^{\psi} Pd\tau d\tau \quad \text{and} \quad L_{\psi_0}^{\psi} = \frac{s}{\varphi m} G_{\psi_0}^{\psi}.$$

ψ_0 - from the preliminary period;

I_0 - according to curve I, ψ.

τ	P	ψ	$I = \int Pd\tau$	$I - I_0$	$I_{av.} - I_0$	$(I - I_0)_{av.}\Delta\tau = \Delta G$	$G_{\psi_0}^{\psi} = \Sigma(I - I_0)\Delta\tau$	$L = \frac{s}{\varphi m} G_{\psi_0}^{\psi}$
τ_B^0	5-7 P_B	0	0	–	–	–	–	–
τ^I	p^I	$\psi^I = \psi_0$	$I^I = I_0$	0			0	0
					$I_{av.}^{II} - I_0$	$(I_{av.}^{II} - I_0)\Delta\tau$		
τ^{II}	p^{II}	ψ^{II}	I^{II}	$I^{II} - I_0$			G^{II}	L^{II}
					$I_{av.}^{III} - I_0$	$(I_{av.}^{III} - I_0)\Delta\tau$	G^{III}	L^{III}
·II	p^{III}	ψ^{III}	I^{III}	$I^{III} - I_0$				
							G^{IV}	L^{IV}
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
τ_K	P_m	1	I_K	$I_K - I_0$	–	–	$G_K = G'_{\psi_0}$	$L_K = L'_{\psi_0}$

We shall construct, according to equations (89) and (92) for the same values of ψ, a curve showing the dependence of v on L when $\Delta = \Delta_1$ [fig. 150, curve v, $L(\Delta_1)$]. If the test were analyzed for $\Delta_2 > \Delta_1$, the relationship v, $L(\Delta_2)$ would obtain, which relationship curve has a larger slope angle and in which the shorter path of the projectile: $L_K(\Delta_2) < L_K(\Delta_1)$ corresponds to the end of powder burning. Both curves have a smaller curvature than the true v,l curve of the velocities of the projectile in the bore.

If we were to extrapolate the function $L_{\psi_0}^{\psi}$ for the initial loading density Δ_0 in the gun, we would have obtained curve v, $L(\Delta_0)$ (150), which at the start of motion has a common point of tangency with the true v,l curve. The latter is obtained when Δ decreases continuously, and as ψ and v increase curve v,l (heavy dotted line) gradually goes over from curve v, $L(\Delta_0)$ to curves corresponding to ever smaller Δ, which family of curves includes also curves v, $L(\Delta_2)$ and v, $L(\Delta_1)$.

This transition is the one given by the fundamental formula (100):

$$l = l_{\psi_{av.}} \left[(1 - B'L_{\psi_0}^{\psi})^{-\frac{2}{\theta}} - 1 \right]$$

together with formula (89):

$$v = \frac{s}{\varphi m}(I - I_0).$$

The difference between the method of solving the fundamental problem outlined above and other such methods lies in the fact that in deriving the dependence of l on ψ, use is made not of the fundamental equation of pyrodynamics, but, rather, of the equation of the state of powder gases for different positions of the projectile in the bore of the barrel.

In solving this problem use is made of the gas pressure curve P, τ obtained from bomb tests, which expresses the true burning law with all the deviations from the geometric law.

An analogous result can be obtained only by the numerical integration of Taylor's series or from finite differences.

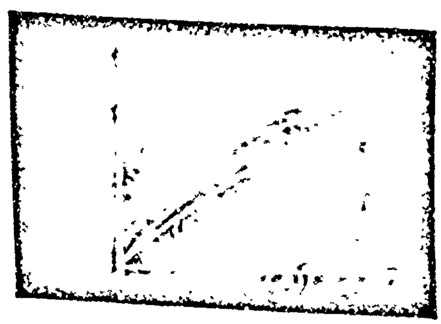

Fig. 150 - Relation Between Auxiliary Curves v, L and Actual Curve v, l.

The method outlined here permits the solution of the problem also in the case of the geometric law, by assuming the following theoretical relationship for Γ, ψ:

$$\Gamma = \frac{\varkappa\sigma}{I_K} = \frac{\varkappa}{I_K}\sqrt{1 + 4\,\frac{\lambda}{\varkappa}\,\psi} = \frac{1}{I_K}\sqrt{\varkappa^2 + 4\varkappa\lambda\psi}.$$

Therefore, this method is a more general one than the methods based on the geometric law of burning.(*)

(*) For a more detailed explanation see: Serebriakov, M.Ye. "FIZICHESKY ZAKON GORENIA VO VNUTRENNEY BALLISTIKE" (The Physical Law of Burning in Internal Ballistics). "OBORONGIZ" (State Publishers of Defense Literature) 1940.

5. ANALYSIS OF THE OBTAINED CURVES p, l AND v, l.

Analysis of curves p, l and v, l obtained on the basis of the physical law of burning indicates that:

Fig. 151 - Curves p, l and v, l Obtained on the Basis of the Physical and Geometric Laws of Burning.

Ф. З. Г. - physical law of burning.

Г. З. Г. - geometric law of burning.

1) Due to ballooning - accelerated burning of outer layers - the maximum pressure is attained earlier, and the pressure curve is disposed higher at the start than in the case of the geometric law;

2) The beginning and the first half of the velocity curve v, l obtained on the basis of the physical law, are disposed above the corresponding v, l curve in the case of the geometric law of burning; the curves merge at the end;

3) The same value of $p_{max.}$ is obtained at a smaller propellant force of powder than in the case of the geometric law;

4) Due to after-burning of the thicker elements of the charge,

the end of burning is transposed nearer the muzzle face, and the velocity v_K exceeds the theoretical value for the average powder thickness;

5) Due to the gradual decrease of the intensity of gas formation Γ, ψ at the end of burning, the transition of the pressure curve from the first period to the second proceeds without a jump and forms no turning points on the curve, as it does in the case of the geometric law.

The graph in fig. 151 clarifies the above.