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BUCKLING OF CYLINDRICAL PANELS UNDER LATERAL PRESSURE.

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SUMMARY

The stability of simply-supported cylindrical panels under lateral pressure is investigated by linear theory. First, panels with classical simple supports are analysed with the usual Donnell 8th order equation. Numerical results are presented which confirm that panels may buckle at lower pressures than corresponding complete cylindrical shells. Then the effect of circumferential restraint along the straight edges is studied by analysis of a panel with SS4 ($u = v = 0$) boundary conditions and comparison with classical SS3 ($u = N_\phi = 0$) supports. The coupled Donnell equations are reduced to a set of algebraic equations and the eigenvalues are solved by an iterative technique. Circumferential restraint along the straight edges results in considerable stiffening under lateral pressure.

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LIST OF SYMBOLS

a	radius of cylindrical panel or shell.
A_i, B_i	coefficients in displacement functions Eqs. (16).
D	$Eh^3/12(1-\nu^2)$
E	modulus of elasticity
h	thickness of shell
L^*	length of panel
$L = (L^*/a)$	non-dimensional length of panel
m	number of longitudinal half waves
p	lateral pressure
r_i	roots of characteristic equations, Eq. (17).
SS3, SS4	"classical" and stiffest simple support boundary conditions (see Eqs. (5) and (11)).
t	number of circumferential half waves
t_{APP}	approximate value of t obtained from Eq. (9)
$t_{cont,min}$	t for λ_{min} in hypothetical shell with "continuous"
u, v, w	non-dimensional displacements; $u=(u^*/a)$, $v=(v^*/a)$, $w=(w^*/a)$
U, V, W	displacement components dependent on ϕ only
x^*, z^*, ϕ	axial, radial and circumferential coordinates (see Fig.
x	non-dimensional axial coordinate, $x = (x^*/a)$.
Z	$= (1-\nu^2)^{1/2}[(2\phi_0)^2(a/h)]$ Batdorf shell parameter for panel

β^2	$= 12(1-\nu^2)(a/h)^2$
λ	(pa/Eh) lateral pressure parameter
λ_{APP}	approximate value of λ obtained with Eq. (8)
λ_{CYL}	pressure parameter of corresponding complete cylindrical shell
ν	Poisson's ratio
ϕ	circumferential coordinate
ϕ_o	half central angle of panel

Subscripts following a comma indicate differentiation.

1. INTRODUCTION

The buckling of cylindrical panels under axial compression or torsion has been extensively studied (see for example [1-6]), whereas practically no attention has been given to the buckling under external pressure. In general, external pressure is critical for complete cylindrical shells, but there are structural configurations in which the instability of a cylindrical panel under external pressure may be crucial. An important example is local buckling in a stringer-reinforced cylindrical shell under lateral or hydrostatic pressure. This motivated a study of the instability of cylindrical panels under external pressure. The study is a linear analysis based on Donnell's equations [7]. In view of the importance of the in-plane boundary conditions along the curved edges of complete cylinders, brought out by recent work [8, 9, 10, 11 and 12], the influence of the in-plane boundary conditions along the straight edges of the panel is also investigated.

The analysis is written in non-dimensional form, the coordinates and physical displacements having been divided by the radius of the shell. u, v and w are the additional non-dimensional displacements during buckling. The notation and coordinate system employed are shown in Fig. 1. (The coordinate system of Fig. 1 is chosen for convenience in the analysis of Section 3).

2. PANEL ON CLASSICAL SIMPLE SUPPORTS

The usual 8th order Donnell equation of equilibrium for buckling under lateral pressure [2] may be written in non-dimensional form as

$$\nabla^8 w + 12(1-\nu^2)(\frac{a}{h})^2 w_{,xxxx} + \lambda\ 12(1-\nu^2)(\frac{a}{h})^2\ \nabla^4(w_{,\phi\phi}) = 0 \tag{1}$$

where λ is a pressure parameter defined as in [8] by

$$\lambda = \frac{pa}{Eh} \tag{2}$$

If a radial deflection function

$$w = A_{mn} \sin\left(\frac{m\pi x}{L}\right) \sin\left[\frac{t\pi(\phi+\phi_o)}{2\phi_o}\right] \tag{3}$$

is assumed, where L is the non-dimensional length of the panel defined by $L = (L^*/a)$ and t is the number of circumferential halfwaves, the classical simple support boundary conditions are fulfilled. These are for the radial displacement

$$\begin{aligned} &\text{at } x = 0, L && w = w_{,xx} = 0 \\ &\text{and at } \phi = \pm\phi_o && w = w_{,\phi\phi} = 0 \end{aligned} \tag{4}$$

The in-plane boundary conditions satisfied by Eqs. (3), on account of the relations between u and w and v and w implied by Eq. (1), see for example [2], are

$$\begin{aligned} &\text{at} \quad x = 0, L \qquad v = 0 \quad \text{and} \quad N_x = 0 \\ &\text{and at } \phi = \pm\phi_o \qquad u = 0 \quad \text{and} \quad N_\phi = 0 \end{aligned} \tag{5}$$

Substitution of the displacement function, Eq. (3) into Eq. (1) yields

$$\lambda = \left(\frac{\pi}{\phi_o}\right)^2 \left(\frac{h}{a}\right)^2 \frac{1}{12(1-\nu^2)} \left\{ \frac{1}{4t^2} \left[\left(\frac{2m\phi_o}{L}\right)^2 + t^2\right]^2 + \frac{48(1-\nu^2)\phi_o^4 (a/h)^2}{\pi^4 t^2 \left[1 + \left(\frac{tL}{2m\phi_o}\right)^2\right]^2} \right\} \tag{6}$$

The minimum value of λ is obtained from Eq. (6) if one sets $m = 1$. The physical meaning of $m = 1$ is that the panel will buckle in a single longitudinal half-wave. The integer value of t (the number of circumferential halfwaves) which makes λ a minimum has then to be found. Usually $(tL/2\phi_o) > 1$ or

$$t^2 (L/2\phi_o)^2 \gg 1 \tag{7}$$

With Eq. (7) direct minimization of Eq. (6) yields the following approximate formulae for λ_{min} and t_{min}

$$\lambda_{min} = \frac{\pi^2}{12(1-\nu^2)} \left(\frac{h}{2a\phi_o}\right)^2 \left(\sqrt{6} + \frac{2}{\sqrt{6}}\right) \frac{Z^{\frac{1}{2}}}{\pi(L/2\phi_o)} \approx 0.938 (h/a)^2 (2\phi_o L)^{-1} Z^{\frac{1}{2}}$$

$$= 0.938 \sqrt[4]{1-\nu^2}\, (h/a)^{1.5}/L \tag{8}$$

and

$$t^2_{min} = \frac{\sqrt{6}\left[\sqrt{1-\nu^2}\, (4\phi_o^2\, a/h)\right]^{1/2}}{\pi(L/2\phi_o)} = \frac{\sqrt{6}\, Z^{1/2}}{\pi(L/2\phi_o)} \approx 0.78\, (2\phi_o/L) Z^{1/2} \tag{9}$$

where

$$Z = \sqrt{1-\nu^2}\left[(2\phi_o)^2 a/h\right] \tag{10}$$

The critical pressure parameters were computed with Eq. (6) for two groups of typical panels. In one group (a/h) was varied, whereas in the other the central angle of the panel $2\phi_o$ was varied. Table I shows the variation of the pressure parameter λ with (a/h) and in Table 2 and Figs. 2 and 3 the dependence of λ on ϕ_o is presented. (Many additional points, not given in Table 2, were computed to draw Figs. 2 and 3). In the tables the approximate values of λ and t, obtained from Eqs. (9) and (8) are also given for comparison. The variation of λ with (a/h) is found to be similar to that in complete cylindrical shells. Table I also shows that, as for complete moderate length cylinders, Eq. (8) yields a conservative approximation for λ, that is very close when the panels are thin.

In Table 2 the values of λ for a complete cylinder (taking into account that t has to be an even integer) are also given. For (a/h) = 174, Table 2a, the t for minimum λ is 16. If the even integer value constraint on t were absent, however, the cylinder would buckle at a lower load with $14<t<16$. For certain central angles the panel can buckle into a number of half waves that correspond to this t, for "unrestrained" low λ in a complete cylinder. Hence the minimum λ values in Fig. 2 are below that of the corresponding complete cylinder, which requires periodicity of $(t/2)$, and the "lobes" of the curve cut the complete shell line. Or, in other words, the panel buckles at the buckling load corresponding to a hypothetical cylindrical shell with a "continuous" t, whenever its central angle is a multiple of $(\pi/t_{cont.min})$. At other values of central angle, the buckling load of the panel may be higher than that of the corresponding complete shell, and for narrow panels even appreciably higher. It may be noted that λ_{APP}, from Eq. (8), is a good approximation to a lower bound of the curve since (but for the approximation involved in neglecting unity compared to $(tL/2\phi_o)^2$) it represents a minimization of λ with respect to a continuous t. At the large central angles the second term is dominant in Eq. (6) and therefc e the neglect of unity in the denominator slightly increases λ_{APP}.

A similar behavior may be seen in Table 2b and Fig. 3 for (a/h) = 2000. Here the t for minimum λ of a complete cylinder is 30 but again at certain central angles the panels can buckle at lower pressures.

From the numerical results it appears that λ_{APP} of Eq. (8) is a suitable approximation for design of panels with SS3 type boundary conditions, except for narrow panels for which it may be too conservative.

3. EFFECT OF IN-PLANE BOUNDARY CONDITIONS

The influence of the in-plane boundary conditions along the curved edges of cylindrical panels will not differ noticeably from that found in complete cylinders under external pressure. Hence the curved edges need not be considered. Furthermore, in view of the results of Sobel [8] and Soong [12] the present study of cylindrical panels under lateral pressure deals only with two sets of boundary conditions. These are Eqs. (4) for the radial displacement and in the plane of the panel either "classical" simple supports, SS3 in the notation of [12], Eqs. (5), repeated here for clarity,

$$\text{at} \quad x = 0, L \qquad v = 0 \qquad N_x = 0$$

$$\text{at} \quad \phi = \pm \phi_o \qquad u = 0 \qquad N_\phi = 0$$

or axial restraints along the straight edges, SS4 in the notation of [12]

$$\begin{aligned} &\text{at} \quad x = 0, L \qquad v = 0 \qquad N_x = 0 \\ &\text{at} \quad \phi = \pm \phi_o \qquad u = 0 \qquad v = 0 \end{aligned} \tag{11}$$

It should be noted that the boundary conditions along the curved edges are identical in Eqs. (5) and (11).

The analysis follows that of [8]. The coupled Donnell stability equations are written in non-dimensional form

$$\frac{1}{\beta^2}\nabla^4 w + \frac{1}{(1-\nu^2)}(v_{,\phi} + w + \nu u_{,x}) + \lambda w_{,\phi\phi} = 0$$

$$u_{,xx} + \frac{(1-\nu)}{2} u_{,\phi\phi} + \frac{(1+\nu)}{2} v_{,x\phi} + \nu w_{,x} = 0 \tag{12}$$

$$v_{,\phi\phi} + \frac{(1-\nu)}{2} v_{,xx} + \frac{(1+\nu)}{2} u_{,x\phi} + w_{,\phi} = 0$$

where

$$\beta^2 = 12\,(1-\nu^2)(a/h)^2 \tag{13}$$

By separation of variables the partial differential equations (12) are reduced to three linear homogeneous ordinary differential equations. If the displacements are assumed in the following form

$$u(x,\phi) = U(\phi)\cos\left(\frac{\pi x}{L}\right)$$

$$v(x,\phi) = V(\phi)\sin\left(\frac{\pi x}{L}\right) \tag{14}$$

$$w(x,\phi) = W(\phi)\sin\left(\frac{\pi x}{L}\right)$$

they satisfy the "classical" simple support boundary conditions at the curved edges, ($x = 0, L$) and may be adjusted to fit any desired set of boundary conditions along the straight edges. Substitution of Eqs. (14) into Eqs. (12) yields

$$W^{IV} + [\lambda\beta^2 - 2(\frac{\pi}{L})^2]W'' + [(\frac{\pi}{L})^4 + \frac{\beta^2}{(1-\nu^2)}]W - \frac{\nu}{(1-\nu^2)}\beta^2(\frac{\pi}{L})U + \frac{\beta^2}{(1-\nu^2)}V' = 0$$

$$\frac{(1-\nu)}{2}U'' - (\frac{\pi}{L})^2 U + \frac{(1+\nu)}{2}(\frac{\pi}{L})V' + \nu(\frac{\pi}{L})W = 0$$

$$V'' - \frac{(1-\nu)}{2}(\frac{\pi}{L})^2 V - \frac{(1+\nu)}{2}(\frac{\pi}{L})U' + W' = 0 \qquad (15)$$

where $'$ represents differentiation with respect to ϕ.

The general solution of Eqs. (15) is

$$U = \sum_{i=1}^{4} \frac{(\frac{\pi}{L})[r_i + \nu(\frac{\pi}{L})^2]}{[r_i - (\frac{\pi}{L})^2]^2} \{A_i \sinh\sqrt{r_i}\phi + B_i \cosh\sqrt{r_i}\phi\}$$

$$V = \sum_{i=1}^{4} \frac{-\sqrt{r_i}[r_i - (2+\nu)(\frac{\pi}{L})^2]}{[r_i - (\frac{\pi}{L})^2]^2} \{A_i \cosh\sqrt{r_i}\phi + B_i \sinh\sqrt{r_i}\phi\}$$

$$W = \sum_{i=1}^{4} \{A_i \sinh\sqrt{r_i}\phi + B_i \cosh\sqrt{r_i}\phi \qquad (16)$$

and r_i (i = 1, ..., 4) are the roots of the characteristic equation

$$r^4 + [\lambda\beta^2 - 4(\frac{\pi}{L})^2]r^3 + [6(\frac{\pi}{L})^4 - 2\lambda\beta^2(\frac{\pi}{L})^2]r^2$$

$$+ [-4(\frac{\pi}{L})^6 + \lambda\beta^2(\frac{\pi}{L})^4]r + [(\frac{\pi}{L})^8 + \beta^2(\frac{\pi}{L})^4] = 0 \qquad (17)$$

The terms in Eqs. (16) that include the coefficients A_i represent an antisymmetric buckling mode,(with an even number of circumferential halfwaves), whereas those that include the coefficients B_i represent a symmetric mode.

After some manipulations the solution is expressed in real deflection functions with complex coefficients (details are given in [14]). Substitution of the general solution either in the boundary conditions SS3, Eqs (5),or SS4, Eqs. (11), along the straight edges of the panel then yields a set of 8 homogeneous algebraic equations with the complex coefficients as unknowns. The lowest eigenvalue of the determinant of the coefficients of these unknowns yields the critical pressure.

Since the boundary conditions at $\phi = \phi_o$ and $\phi = -\phi_o$ are identical, the 8th order determinant decomposes into a product of two 4th order determinants. One determinant includes only the antisymmetric terms of the displacements and the other the symmetric ones. The same 4th order determinant could have been obtained directly by separate consideration of the symmetric or antisymmetric mode at one edge only (for example at $\phi = \phi_o$). The lowest eigenvalue of both determinants have to be computed, and the lower of the two yields the buckling pressure and indicates the corresponding buckling mode.

Difficulties may arise in the computations, since one cannot readily express the characteristic roots r_i in terms of the pressure parameter λ. Hence an iterative technique is employed. An arbitrary value is assumed for λ, and with it solutions of the characteristic equation are obtained. These are then substituted in the stability determinants. If the determinant does not vanish, another value of λ is chosen and the procedure is repeated. Hence the value of the determinant may be plotted versus λ and the λ at which the determinant changes sign be found. Such a λ represents an eigenvalue of the determinant. Obviously, only the lowest eigenvalue is of interest in buckling calculations.

In the course of the calculations it was observed that all the stability determinants vanish at a λ that corresponds to the buckling pressure of a complete cylinder for which t, the number of half-waves in the circumferential direction, is treated as a continuous variable. For these values of λ, however, the characteristic equation has a multiple real root that causes the determinant to vanish automatically. One could then conclude erroneously that the solution is independent of the dimensions of the panel and of the boundary conditions. This conclusion would not be correct since in the case of multiple roots the solutions of Eqs. (16) are not independent and one has to look for new independent solutions. For example, such an independent solution for W of Eqs. (16) would be, see [15],

$$W_I = \frac{\partial(\sinh\sqrt{r_i}\,\phi)}{\partial r_i} = \frac{\phi}{2\sqrt{r_i}}\cosh\sqrt{r_i}\,\phi \tag{18}$$

where r_i is the multiple root. The eigenvalue is known, but now the stability determinant vanishes only for particular panel geometries corresponding to the waves of a complete cylinder. In general this solution is of no practical interest.

The procedure was programmed and numerical results were obtained for the two groups of panels considered in Section 2 with boundary conditions SS3 and SS4 along the straight edges. (Only a few cases of SS3 boundary conditions were actually calculated with this procedure as a check).

The critical pressure parameters for the cylindrical panels with SS3 and SS4 straight edges and their ratio are presented in Tables 3 and 4 and in Figs. 4 and 5. The considerable scatter of the computed points in Figs. 4 and 5 is caused by the difference in the phase of the ripples of the buckling curves compared. This difference is also indicated by the different dominant buckling modes in Tables 3 and 4.

If a mean curve is drawn in Fig. 3, a trend of a decrease in the effect of circumferential restraint with increasing (a/h) is observed. It should be remembered that the comparison is for different in-plane boundary conditions along the straight edges only, the curved edges being of the SS3 type for all panels. The stiffening due to circumferential restraint is about 50% for the thicker panels and falls to about 20% for the thinner ones. Comparison with the effect of axial restraint in complete cylindrical shells of similar Z, Fig. 2 of [12], shows there a stiffening of 30-40% with higher values of Z between 500 and 2000.

A mean curve drawn in Fig. 5 shows a more pronounced trend of increased stiffening for narrower panels. Such a trend could be expected on physical grounds. For large central angles, as the effect of the straight edges diminishes, the ratio tends to unity, whereas for narrow panels $2\phi_o < 45^o$

stiffening of more than 50% is indicated. The high (p_{SS4}/p_{SS3}) ratios predicted for narrow panels are however only approximate, since for such panels the pre-buckling stress cannot be represented satisfactorily anymore by the membrane stress, as assumed in the present analysis.

Furthermore, the behavior of a very narrow panel with $L >> 2\phi_o$ approaches that of an infinite curved panel for which snap buckling occurs. Hence the present analysis ceases to be applicable for very narrow panels.

The stiffening effect of circumferential restraint on the straight edges of curved panels is much larger for lateral pressure loading than for axial compression, as can be seen by comparison with a recent study on the effect of edge restraint on buckling of panels under axial compression [16]. There appreciable stiffening appears only in very narrow panels, whereas for $Z > 25$ the stiffening is less than 10%.

4. CONCLUSIONS

Cylindrical panels on classical simple supports (SS3 boundary conditions) may buckle at lateral pressures slightly below those of the corresponding complete cylindrical shells, since certain central angles permit a pattern that represents the hypothetical "continuous" t pattern of the complete shell which would be weaker due to the absence of the "even integer" restraint on t. For other central angles the panels may sustain higher buckling pressures than complete cylinders, and for narrow panels even appreciably higher. The approximate critical pressure parameters, that do not depend on the central angle, appear to be good approximations to a lower bound of the buckling curves.

The effect of circumferential restraint along the straight edge of the panel, found from comparison of SS4 and SS3 B.C.'s, decreases with increase in (a/h). The stiffening is about 50% for thicker panels, (a/h) = 150, and falls to about 20% for thinner ones, (a/h) = 2000. For (a/h) = 174 considerable stiffening, of more than 50%, is found in narrow panels, $2\phi_o < 45^o$, but the effect diminishes and the ratio (p_{SS4}/p_{SS3}) tends to unity for $2\phi_o > 150^o$. The stiffening effect of circumferential restraint along the straight edges of a panel is larger for lateral pressure than for axial compression.

REFERENCES

1) Timoshenko, S.P., and Gere, G.M., Theory of Elastic Stability Second Ed. McGraw-Hill Book Company Inc., 1961, pp.485-490.

2) Batdorf, S.B., " A Simplified Method of Elastic Stability Analysis for Thin Cylindrical Shells", NACA Report 874, 1947.

3) Leggett, D.M.A., " The Buckling of a Long Curved Panel under Axial Compression", R & M No. 1899, Ministry of Aviation, A.R.C., 1942.

4) Leggett, D.M.A., " The Initial Buckling of Slightly Curved Panels under Combined Shear and Compression" R & M No. 1972, Ministry of Aviation, A.R.C 1943.

5) Koiter, W.T., " Buckling and Post-Buckling Behavior of a Cylindrical Panel under Axial Compression" Report S. 476, National Luchtvaart-laboratorium, Amsterdam, Reports and Transactions, Vol. 20,1956.

6) Pope G.G., " On the Axial Compression of Long, Slightly Curved Panels", R & M. No. 3392, Ministry of Aviation, A.R.C., 1965.

7) Donnell, L.M., " Stability of Thin-Walled Tubes Under Torsion" NACA Report No. 479. 1933.

8) Sobel, L.H., "Effects of Boundary Conditions on the Stability of Cylinders Subject to Lateral and Axial Pressures", AIAA Journal, Vol. 2. No. 8, August 1964, pp. 1437-1440.

9) Singer, J., " The Effect of Axial Constraint on the Instability of Thin Circular Cylindrical Shells Under External Pressure", Journal of Applied Mechanics, Vol. 27, No. 4, December 1960, pp. 737-739.

REFERENCES (CONT'D)

10) Thielemann, W and Esslinger, M , "Einfluss der Randbedingungen auf die Beullast von Kreiszylinderschalen, " Der Stalbau, Vol 33, No. 12, December 1964, pp. 3 - 11.

11) Alfutov, N.A., " On the Dependence of the Upper Critical Pressure of a Cylindrical Shell on the Boundary Conditions for the Tangential Displacements", Theory of Shells and Plates, Proceedings of the 4th All-Union Conference on Shells and Plates, Erevan 1962, U.S.S.R., Israel Program for Scientific Translations, Jerusalem 1966, pp. 167-172.

12) Soong, T.C , " Buckling of Circular Cylindrical Shells Under External Pressure" SUDAER No. 228, Department of Aeronautics and Astronautics, Stanford University, April 1965.

13) Schnell, W., " Einfluss der Randverwölbung auf die Beulwerte von Zylinderschalen unter Manteldruck", Der Stahlbau Vol 34, No 6, June 1965, pp 187 - 190.

14) Meer, A., " Buckling of Cylindrical and Conical Panels Under External Pressure", M.Sc. Thesis, Technion, Israel Institute of Technology, Haifa, 1966. (In Hebrew).

15) Ince, E.L., Ordinary Differential Equations, Dover Publications 1956, p.136

16) Rehfield, L.W and Hallauer, W L , "Edge Restraint Effect on Buckling of Compressed Curved Panels", AIAA Journal, Vol. 6, No. 1, January 1968, pp. 187-189

TABLE I.

VARIATION OF λ WITH (a/h)

L = 1.57 $2\phi_o = 60^o$

a/h	174	233	308	500	1000	1500	2000
Z	182	244	321	523	1046	1569	2092
t_{APP} FROM EQ.(9)	2.64	2.84	3.05	3.45	4.10	4.54	4.88
t	3	3	3	3	4	5	5
$\lambda \times 10^4$	2.98	1.78	1.13	0.60	0.19	0.11	0.07
$\lambda_{APP} \times 10^4$ FROM EQ. (8)	2.54	1.64	1.08	0.52	0.18	0.10	0.07

TABLE 2

VARIATION OF CRITICAL PRESSURE PARAMETER WITH CENTRAL ANGLE OF PANEL

(a) $L = 1.57$ — $(a/h) = 174$

$2\phi_c$ (deg.)	10	15	20	22	25	30	35	40	46	55	60	70	77	90	112.5
t_{APP} from Eq. (9)	0.442	0.658	0.883	0.966	1.10	1.32	1.55	1.77	2.03	2.43	2.64	3.10	3.40	3.97	4.96
t_{min}	1	1	1	1	1	1	2	2	2	2	3	3	3	4	5
$\lambda \times 10^4$	10.0	4.70	2.96	2.67	2.80	4.13	3.57	2.96	2.59	3.12	2.98	2.55	2.72	2.54	2.53
$\lambda_{CYL} \times 10^4$	2.73														
$\lambda_{APP} \times 10^4$ from Eq. (8)	2.54														

(b) $L = 1.57$ — $(a/h) = 2000$

$2\phi_c$ (deg.)	10.	12.3	15	20	24.6	30	37.5	42	49.3	55	66	77	84	100	115
t_{APP} from Eq. (9)	0.812	1.00	1.22	1.63	2.04	2.44	3.05	3.41	4.01	4.46	5.36	6.25	6.86	8.10	9.34
t_{min}	1	1	1	2	2	3	3	3	4	5	5	6	7	8	9
$\lambda \times 10^6$	8.08	6.58	8.51	8.05	6.55	8.08	6.54	7.22	6.53	6.98	6.68	6.55	6.53	6.50	6.53
$\lambda_{CYL} \times 10^6$	6.69														
$\lambda_{APP} \times 10^6$ from Eq. (8)	6.52														

TABLE 3

VARIATION OF λ WITH (a/h)

FOR B.C. SS3 AND SS4

L = 1.57 $2\phi_o = 60^o$

a/h	174	233	308	500	1000	1500	2000
Z	182	244	321	523	1046	1569	2092
SS3 $\lambda_{cr} \times 10^4$	2.98	1.78	1.13	0.60	0.19	0.11	0.067
SS3 BUCKLING MODE	SYM	SYM	SYM	SYM	ANTISY	SYM	SYM
SS4 $\lambda_{cr} \times 10^4$	4.14	2.67	1.61	0.72	0.24	0.13	0.082
SS4 BUCKLING MODE	SYM	ANTISY	ANTISY	ANTISY	SYM	SYM	ANTISY
$\frac{\lambda_{SS4}}{\lambda_{SS3}}$	1.39	1.50	1.42	1.21	1.26	1.17	1.22

TABLE 4

VARIATION OF λ WITH $2\phi_o$

FOR B.C. SS3 AND SS4

L = 1.57 (a/h) = 174

		30	37.5	45	60	90	120	150
$2\phi_o$ DEG		30	37.5	45	60	90	120	150
Z		45.5	71 2	102	182	409	728	1140
SS3	$\lambda_{cr} \times 10^4$	4.13	3.24	2.73	2.98	2.73	2.74	2.80
	BUCKLING MODE	SYM	ANTISY	ANTISY	SYM	ANTISY	SYM	SYM
SS4	$\lambda_{cr} \times 10^4$	6.12	6.59	4.92	4.14	3.51	3 14	3.00
	BUCKLING MODE	ANTISY	ANTISY	SYM	SYM	SYM	ANTISY	SYM
$\frac{\lambda_{SS4}}{\lambda_{SS3}}$		1 48	2.04	1 80	1.39	1 28	1 14	1.07

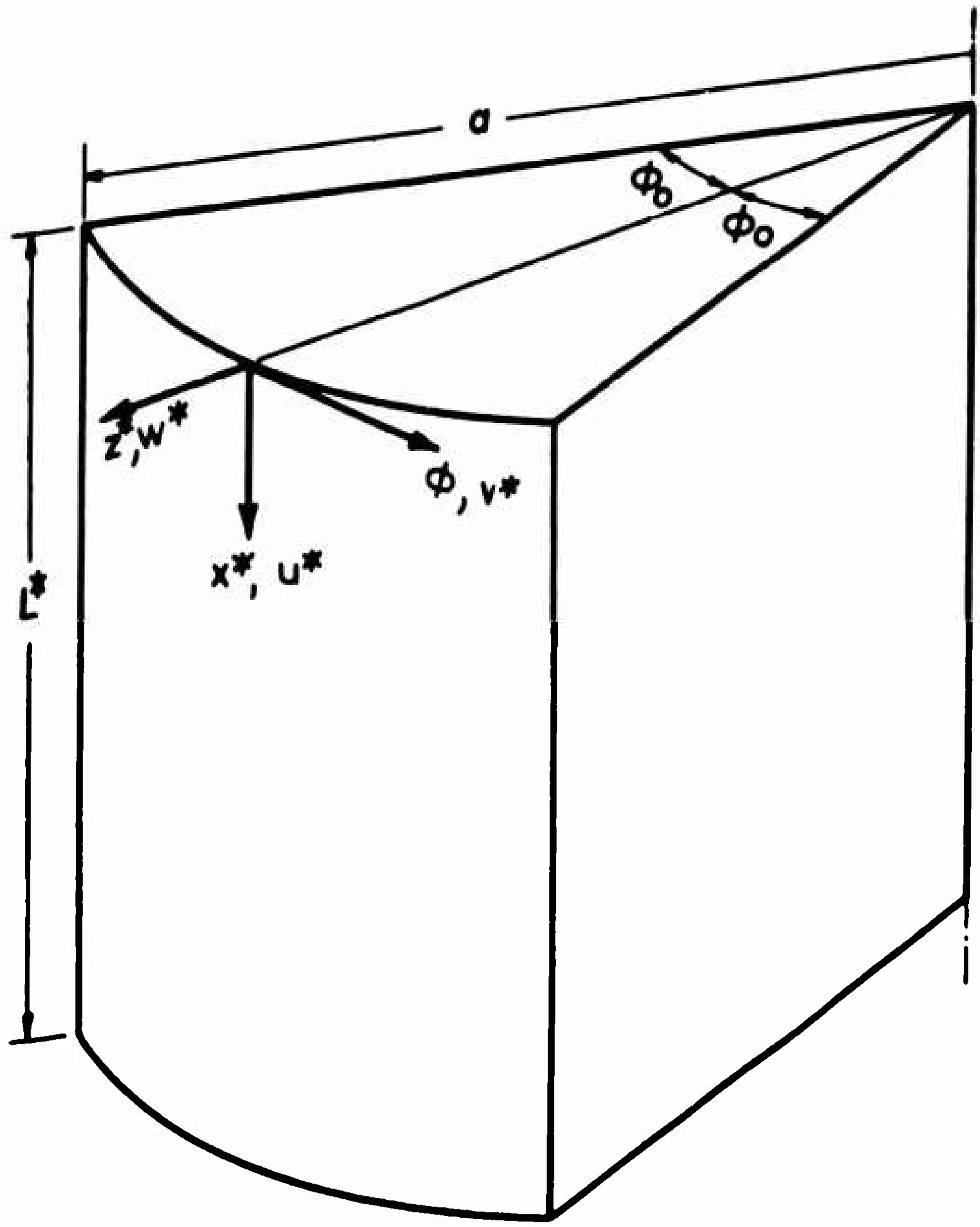

FIG. 1 NOTATION

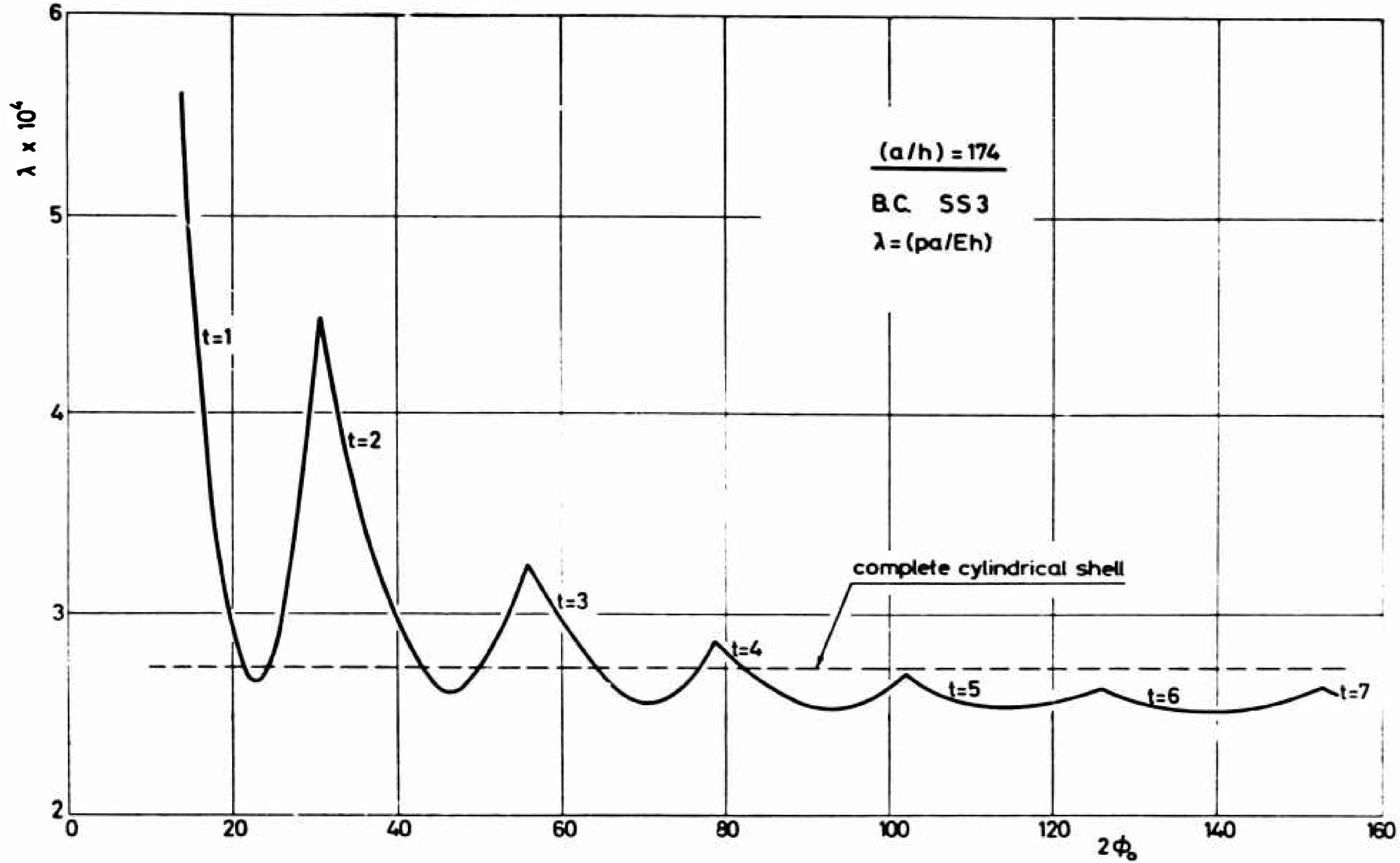

FIG. 2 VARIATION OF CRITICAL PRESSURE WITH CENTRAL ANGLE OF PANEL

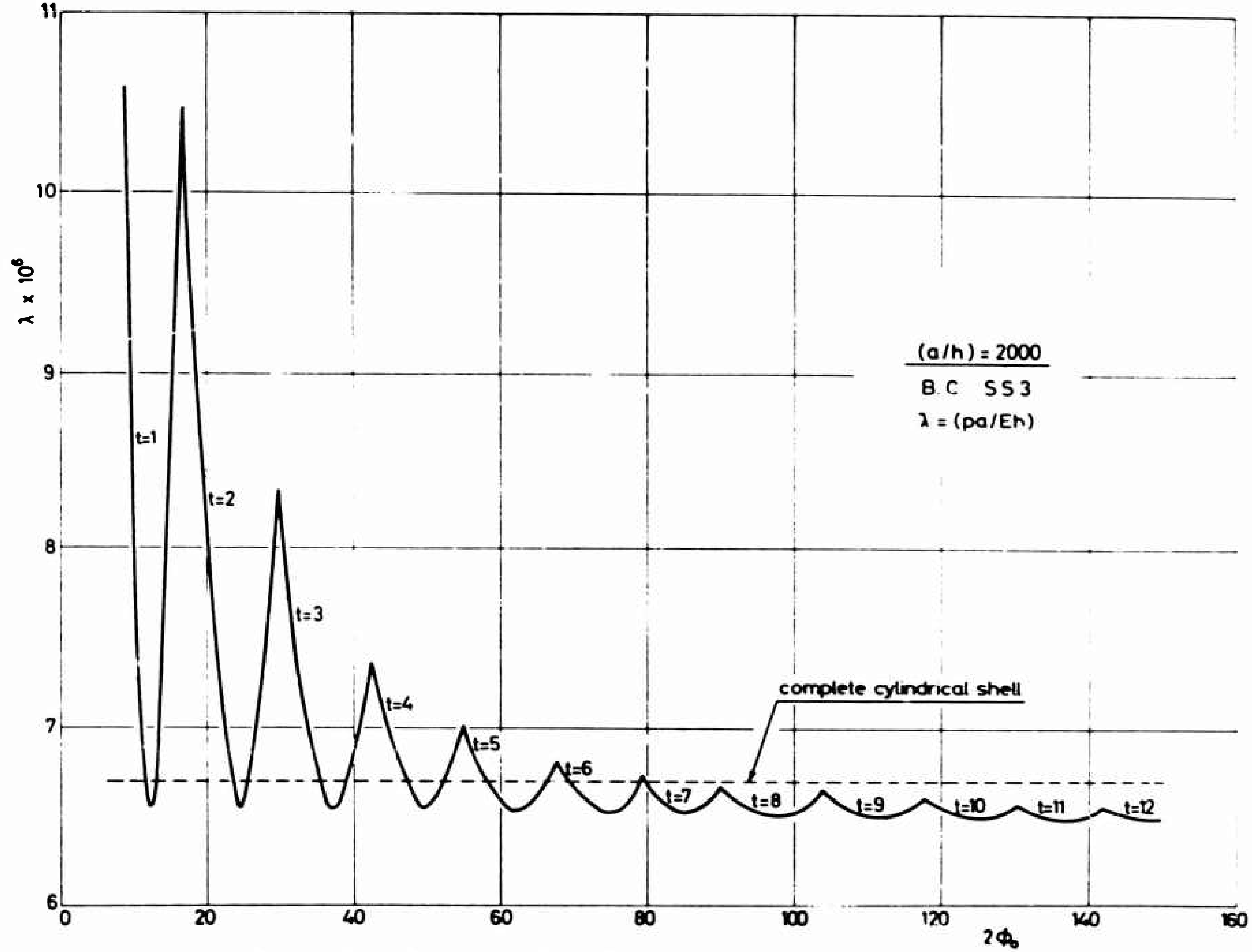

FIG. 3 VARIATION OF CRITICAL PRESSURE WITH CENTRAL ANGLE OF PANEL

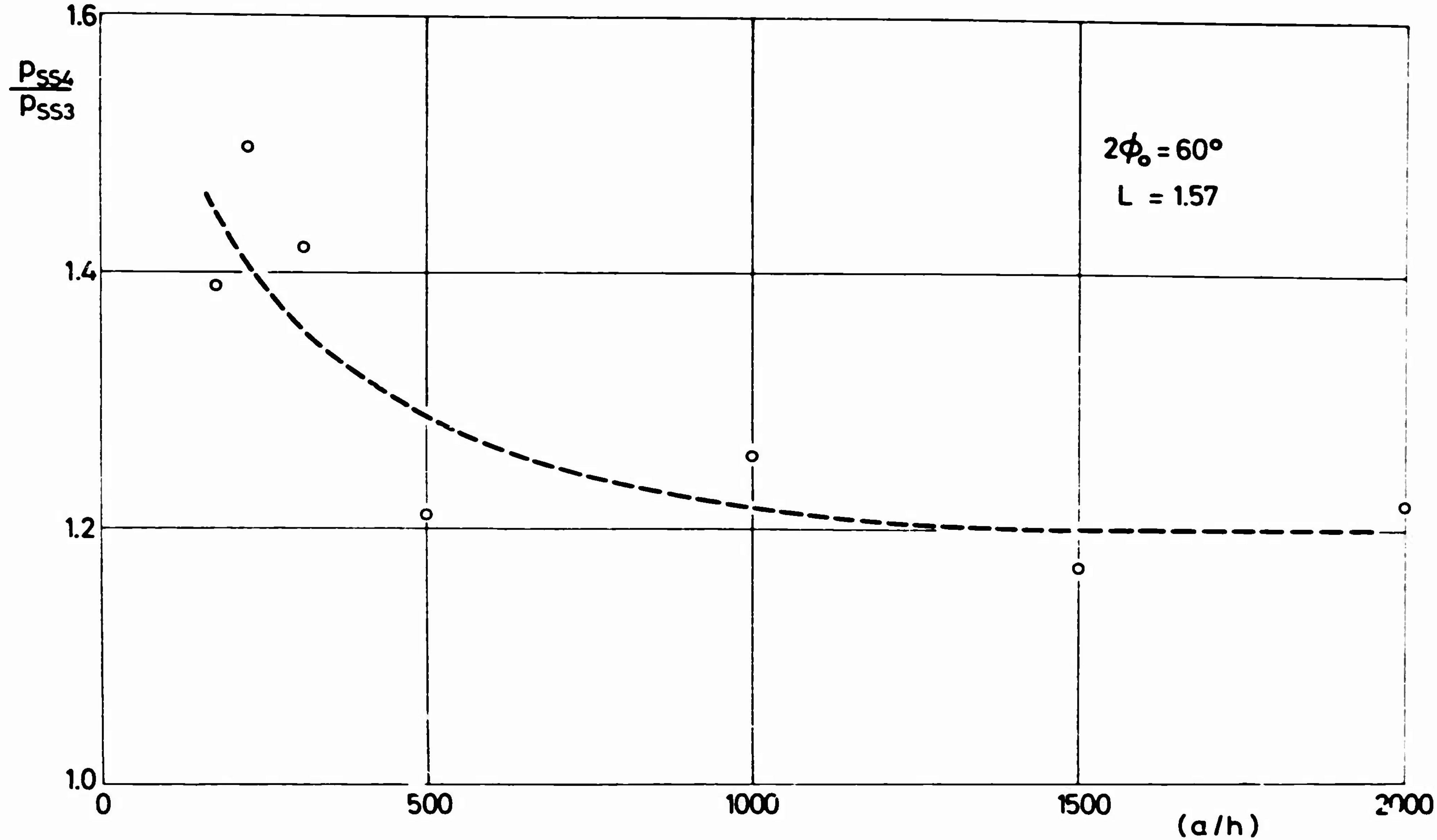

FIG. 4 VARIATION CF (p_{SS4}/p_{SS3}) WITH (a/h)

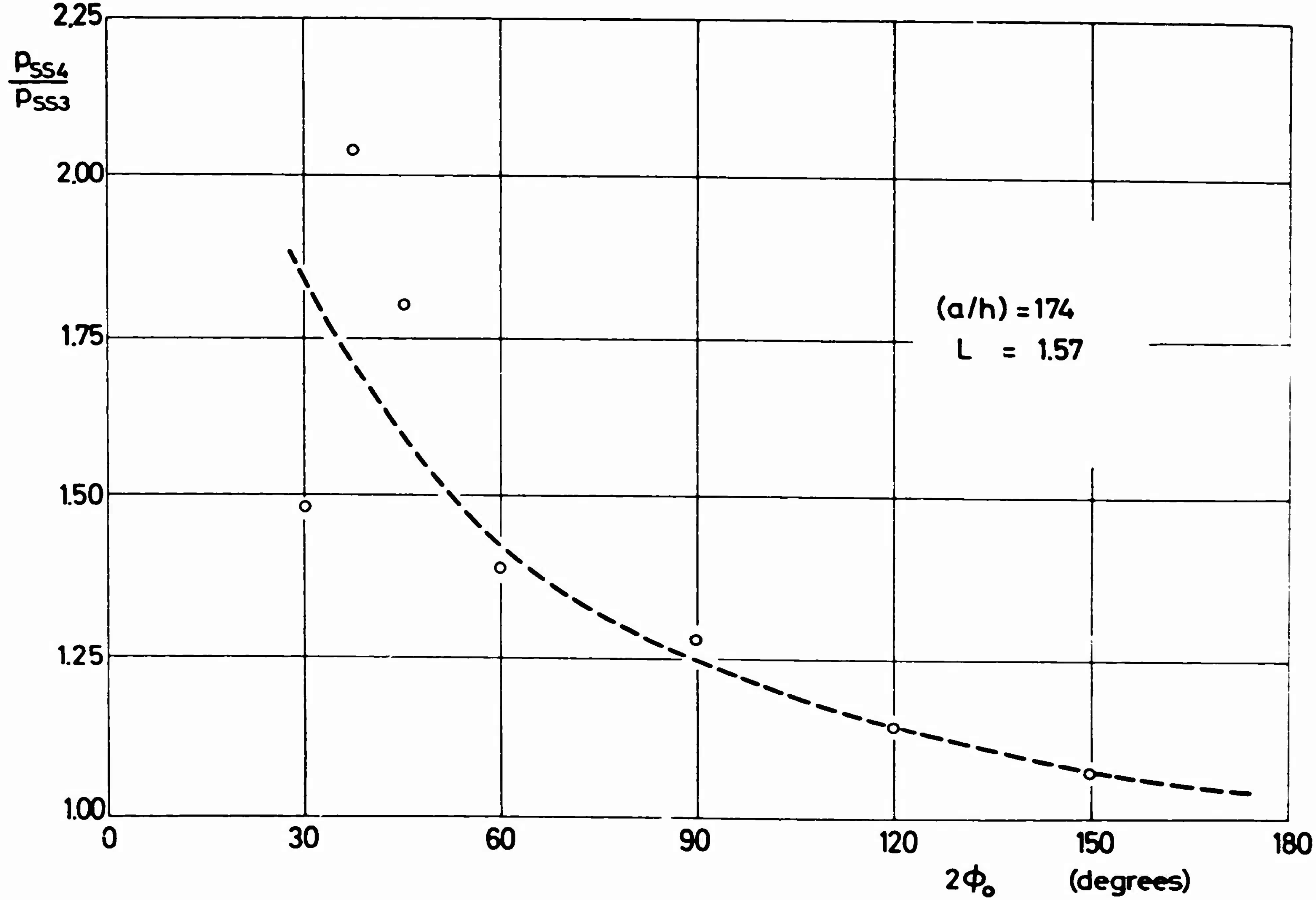

FIG. 5 VARIATION OF (p_{SS4}/p_{SS3}) WITH CENTRAL ANGLE OF PANEL

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13 ABSTRACT

The stability of simply-supported cylindrical panels under lateral pressure is investigated by linear theory. First, panels with classical simple supports are analysed with the usual Donnell 8th order equation. Numerical results are presented which confirm that panels may buckle at lower pressures than corresponding complete cylindrical shells. Then the effect of circumferential restraint along the straight edges is studied by analysis of a panel with SS4 ($u = v = o$) boundary conditions and comparison with classical SS3 ($u = N_{\phi} = 0$) supports. The coupled Donnell equations are reduced to a set of algebraic equations and the eigenvalues are solved by an iterative technique. Circumferential restraint along the straight edges results in considerable stiffening under lateral pressure.

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14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
1. Buckling Cylindrical Panels						
2. In-plane Boundary Conditions						

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