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**ERROR ANALYSIS OF THE RECOVERY OF DATA
ACQUIRED BY A VOLTAGE-TO-FREQUENCY DATA
ACQUISITION SYSTEM FOR SINUSOIDAL, STEP, AND
RAMP INPUTS**



**G. L. Williams
ARO, Inc.**

August 1968

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Tennessee Space Institute.

FOREWORD

The work reported herein was sponsored by the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC).

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This technical report has been reviewed and is approved.

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ABSTRACT

The errors generated by the recovery of data acquired by a voltage-to-frequency (V/F) data acquisition system are theoretically analyzed, using a digital computer. The input waveforms considered are the sinusoid, the step function, and the ramp function. In each case, the well known "sampling theorem" of C. E. Shannon, viz, two samples per period of the smallest period present in the signal are sufficient for recovery of data with perfect fidelity, will be shown to be inapplicable as a criterion for accurate and reliable recovery of data acquired by a V/F system. The errors are of two categories: One is the error from linear interpolation recovery, and the other is from the fact that data points do not always coincide with the input. The sinusoid analysis reveals that a sampling rate of 11.4 summing intervals per period is required to assure 5 percent or less error. The step function analysis shows that the errors are independent of sampling rate; however, its response time is approximately equal to the summing interval. The analysis of the ramp function indicated that the ratio of ramp rise time to summing interval must be five or greater to assure 5 percent or less error. These criteria are unique to each case, and in the case of the sinusoid, are many times larger than the criterion of the "sampling theorem."

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NOMENCLATURE

E	Error at a data point
e	Error at any point using linear interpolation

f	Frequency, Hz
$\text{Int} ()$	Integer part of
K	Synchronization parameter for step and ramp input
K_v	Gain of V/F converter (pulses/sec-v)
m	Summing intervals per period of sinusoid
N	Output counts of V/F converter in τ seconds
n	Number of arbitrary summing interval
T	Period of sinusoidal input, sec
T_D	Time delay, sec
t	Time
V	Voltage
α	Argument of Eqs. (11) and (21)
β	Ratio of ramp rise time to summing interval
$\Delta ()$	Change in
μ_1	Step function
μ_2	Ramp function
τ	Length of summing interval, sec
ψ	Synchronization parameter for sinusoidal input

SUBSCRIPTS

i	Input
max	Maximum value
min	Minimum value
n	Value in summing interval number n
o	Output

SECTION I INTRODUCTION

Data acquisition relies heavily on the process of sampling and representing a continuous time function in terms of discrete time samples. Numerous methods and systems to accomplish this are in existence and have their own uniquenesses. However, all of these methods and systems have a common basis that justifies the representation of a continuous variable by discrete samples, and that is the sampling theorem.

The sampling theorem, though known and used much earlier, is widely attributed to C. E. Shannon (Ref. 1). It may be stated as follows:

" $2/T$ samples per second suffice to represent perfectly and permit perfect recovery of a time function provided that the time function contains no periodic components of period less than T seconds."

However, the data recovery from an integrating system such as the voltage-to-frequency data acquisition system when used as a sampling system, i. e., used to recover the actual waveshape, does not have a direct application through the sampling theorem.

This investigation is a derivation of the number of integrating intervals required to accurately reconstruct an input using linear interpolation of the data points obtained from an integrating data acquisition system. Assuming that an error also exists at the data points, the analysis will include both a determination of data point error and linear interpolation error. Error will be the absolute value of the difference of the input and the representative output at a given time as a percentage of the maximum magnitude of the input. This is a modification of the definition of deviation as defined in Ref. 2, page 209.

Three ideal signals will be treated theoretically: sinusoid, step, and ramp. The digital computer will be used to simulate this type of data recovery, and the maximum error for various parameter configurations will be determined. From this, the limits of the parameters for accurate (5 percent) recovery can be determined.

Experimental verification of the error analysis for the sinusoid was performed using the VIDAR[®] voltage-to-frequency data acquisition system in the Large Rocket Facility at AEDC.

SECTION II VOLTAGE-TO-FREQUENCY CONVERSION METHOD

The method of data acquisition that is to be treated in this study is a voltage-to-frequency (V/F) conversion. The block diagram in Fig. 1 depicts the setup of such a system.

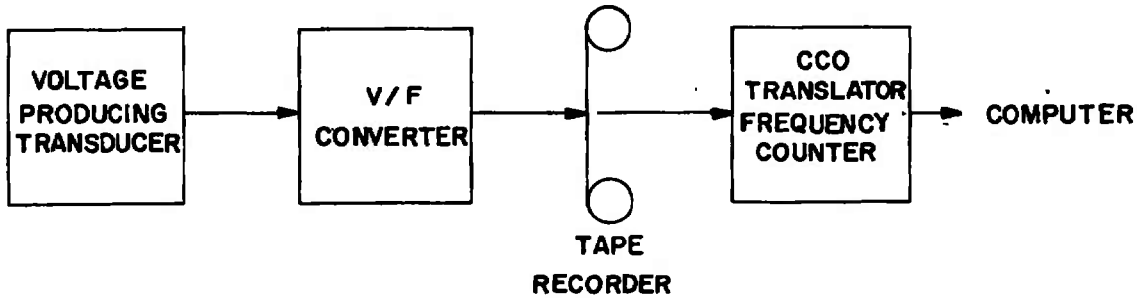


Fig. 1 Block Diagram of LRF V/F Acquisition and Recovery System

The V/F converter develops output pulses at a rate precisely proportional to the input voltage. The output pulses are recorded on magnetic tape for later data reduction. The recorded signals are recovered by means of a translator, which works like a gated frequency counter that counts the pulses during a time interval, τ . The counts contained in a summing interval are converted to a representative voltage, $V_{\tau n}$, by a digital computer using the calibration of the V/F converter and summing interval of the translator.

To illustrate: Let the V/F converter be adjusted so that the output is N_0 counts in τ seconds with the input shorted. Then apply a calibration voltage of V_c volts to the input of the V/F converter; let N_c be the counts in τ seconds of the output. For a number of counts in the n th interval of τ for an arbitrary input of $V_i(t)$ defined as $N_{\tau n}$, the following expression defines $V_{\tau n}$:

$$V_{\tau n} = \frac{N_{\tau n} - N_0}{N_c - N_0} V_c \quad (1)$$

But, because the V/F data recovery technique is an integrating method,

$$N_{\tau n} - N_0 \equiv K_v \int_{n\tau}^{(n+1)\tau} V_i(t) dt \quad (2a)$$

and

$$N_c - N_0 \equiv \int_0^{\tau} K_v V_c dt = K_v V_c \tau \quad (2b)$$

where K_V is the gain of V/F converter with units of pulses per second-volt.

Substituting the definitions of Eqs. (2a) and (2b) into Eq. (1), it becomes:

$$V_{\tau_n} = \frac{1}{\tau} \int_{n\tau}^{(n+1)\tau} V_i(t) dt \quad (3)$$

Equation (3) will be used to theoretically simulate the V/F conversion method of data acquisition and analyze the errors resulting from taking the value of V_{τ_n} as a data point that occurs at the midpoint of the summing interval from $n\tau$ to $(n+1)\tau$ and the points linearly connected to represent the input waveform. Figure 2 illustrates this type of data acquisition and waveform reconstruction.

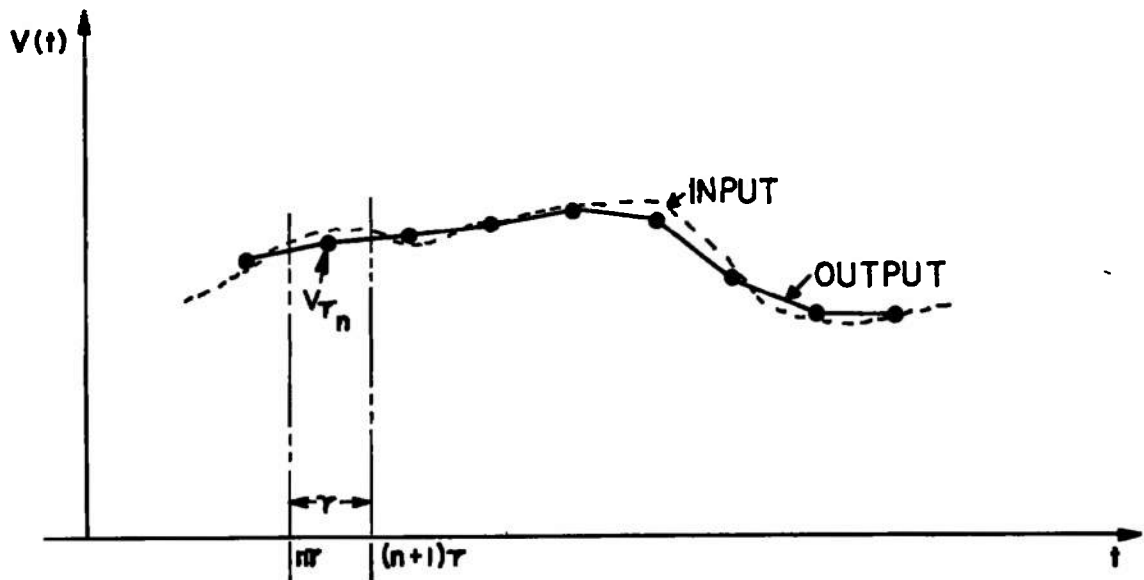


Fig. 2 Example Illustrating the Summing Interval Method of Data Representation

Since the value V_{τ_n} is an integral, its value and the value of $V_i(t)$ at $t = \tau_n = \frac{2n+1}{2}\tau$, the time at the midpoint of the n th interval, are not always equal; therefore, the error analysis also investigates the data point error. To distinguish the two, the lower case letter, e , designates the error from linear interpolation and the upper case letter, E , is used to designate the error at data points.

SECTION III SINUSOIDAL INPUT

For the sinusoidal analysis, let $V_i(t) = \sin \frac{2\pi t}{T}$, which is normalized for ease of application and does not affect the actual error analysis.

Substituting this function in Eq. (3), the expression for the representative output becomes:

$$\begin{aligned} V_{rn} &= \frac{1}{r} \int_{nr}^{(n+1)r} \sin \frac{2\pi t}{T} dt \\ &= -\frac{T}{2\pi r} \left[\cos \frac{2\pi t}{T} \right]_{nr}^{(n+1)r} \\ &= -\frac{T}{2\pi r} \left[\cos \frac{2\pi(n+1)r}{T} - \cos \frac{2\pi nr}{T} \right] \end{aligned} \quad (4)$$

Using the identity from Ref. 3, page 18 for the difference of two cosine functions, Eq. (4) becomes:

$$V_{rn} = \frac{T}{\pi r} \left[\sin \frac{(2n+1)\pi r}{T} \cdot \sin \frac{\pi r}{T} \right] \quad (5)$$

Letting the ratio T/r , which is the number of summing intervals per period, be designated with the letter m , the expression in Eq. (5) becomes:

$$V_{rn} = \frac{m}{\pi} \left[\sin \frac{(2n+1)\pi}{m} \cdot \sin \frac{\pi}{m} \right] \quad (6)$$

3.1 LINEAR INTERPOLATION ERROR ANALYSIS OF SINUSOIDAL INPUT

An expression for e consists of n equations; therefore, the determination of a maximum value is a tremendous task to perform manually. Consequently, a digital computer program was written to determine the errors for a sinusoidal input as m is varied from 1 to 25.0 in increments of 0.1. The program was designed to search the number of cycles that include a whole number of summing intervals. The reason for this approach to the e_{\max} determination is that, after this number of cycles, the errors become repetitive. By incorporating this test, the e_{\max} determination is expedited. The program is listed in Table I-I (Appendix).

The solutions for e_{\max} versus m are plotted in Fig. 3. The plot depicts an irregular relationship, and the most significant fluctuations are in the range of $m = 1.0$ to 3.0 . Therefore, little faith can be placed in the application of the basic sampling theorem of two summing intervals per period to the V/F system. Another significant factor is that, when the sampling rate is close to frequency components of the data that have prominent magnitude, the errors greatly reduce the accuracy of the data recovery.

As an example of the use of Fig. 3, consider an input voltage signal that contains a 50-Hz component. What error would be introduced in the data reduction of this component if τ were 10 msec? What is the error for $\tau = 2$ msec? For $\tau = 10$ msec,

$$m = \frac{10^3}{(.50)(10)} = 2$$

From Fig. 3, $e_{\max} = 44$ percent.

For $\tau = 2$ msec,

$$m = \frac{10^3}{(.50)(2)} = 10$$

From Fig. 3, $e_{\max} = 6$ percent.

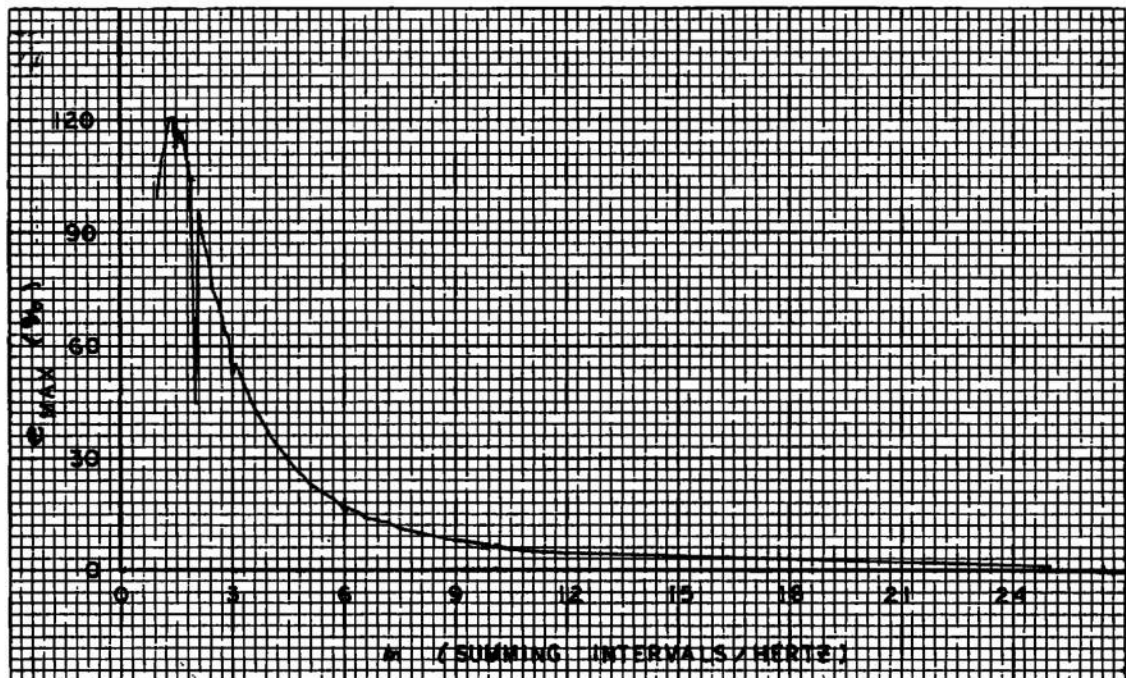
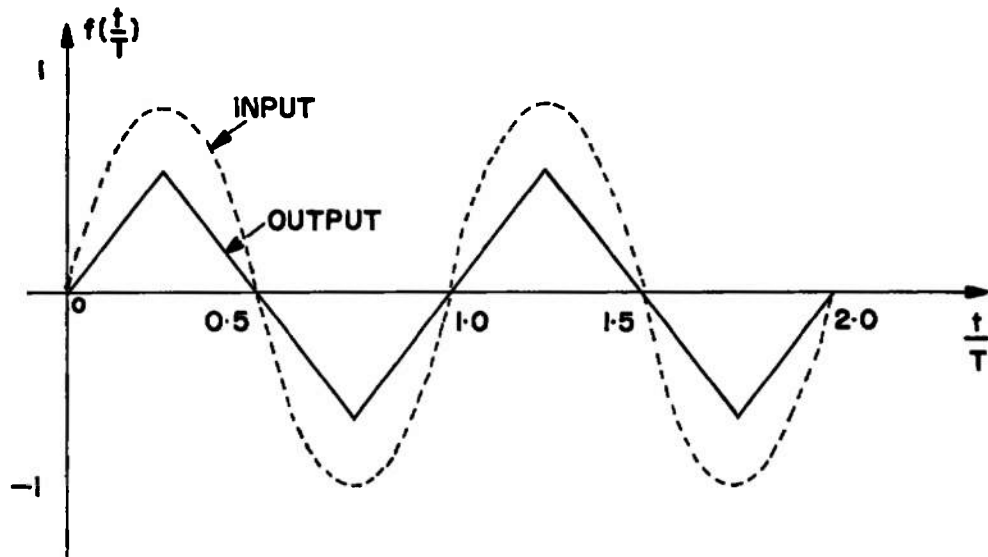


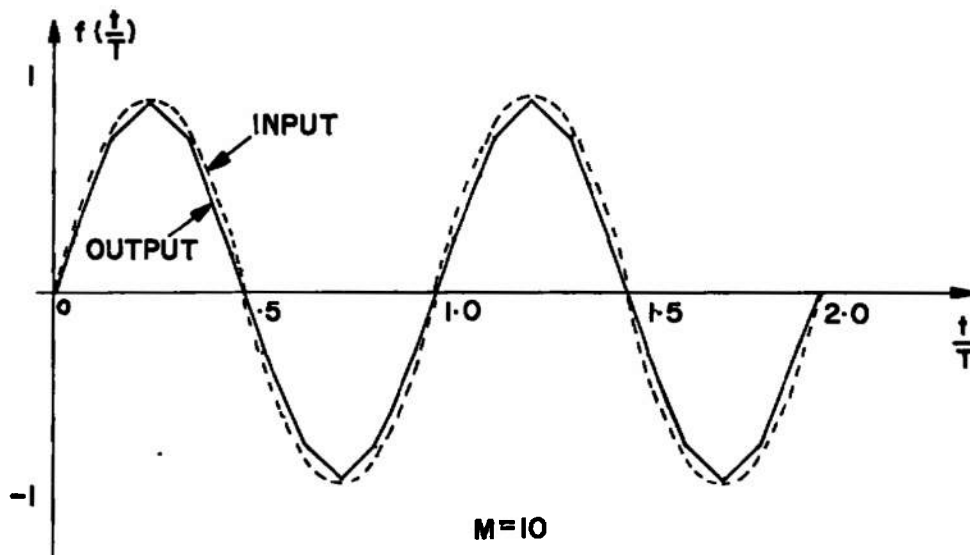
Fig.3 Plot of m versus e_{\max} for Sinusoidal Input

Figure 4 depicts the input-output relationship for these two summing intervals for the 50-Hz component.

In order that e_{\max} be 5 percent or less for a sinusoidal input, m must be greater than 11.4 and at $m = 2$, $e_{\max} = 46$ percent. This indicates a large error at the rate given by the sampling theorem.



$M=2$



$M=10$

Fig. 4 Input-Output Waveforms for Sample Problem

3.2 DATA POINT ERROR ANALYSIS OF SINUSOIDAL INPUT

The error analysis for E_{\max} is much simpler than for e_{\max} . The data point error is defined by this relationship:

$$E = 100 |V_i(t_n) - V_{r_n}| \quad (7)$$

since $|V_i(t)|_{\max}$ is equal to one.

Combining Eqs. (6) and (7) and substituting the normalized sinusoid with the definition of m included, the expression for E becomes:

$$E = 100 \left| \sin \frac{(2n+1)\pi}{m} \left(1 - \frac{m}{\pi} \sin \frac{\pi}{m} \right) \right| \quad (8)$$

Even though the error is expressible in one equation, many calculations are required to find E_{\max} . Therefore, the program used to find e_{\max} also has an E_{\max} option. These solutions are depicted in Fig. 5.

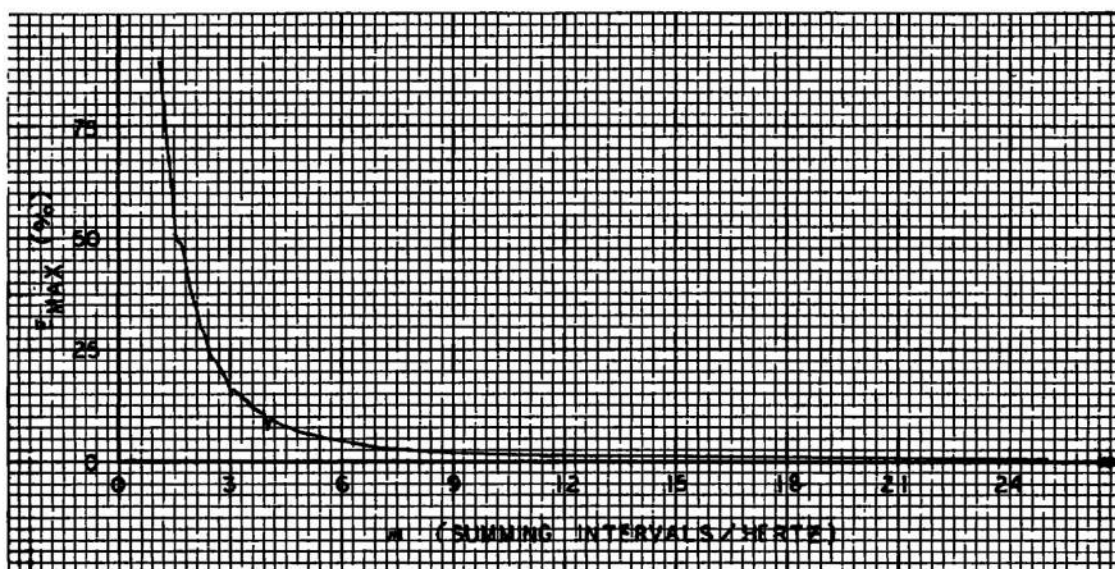


Fig. 5 Plot of m versus E_{\max} for Sinusoidal Input

This plot of E_{\max} versus m is a smoother curve than the one for e_{\max} versus m . The minimum value of m for all data points to be within 5 percent is 5.7, and at the basic sampling rate given by the sampling theorem (two summing intervals/period), the data point is in error at a maximum by 36+ percent.

3.3 SYNCHRONIZATION EFFECTS ON ERRORS OF SINUSOIDAL INPUT

The analyses of the sinusoid for e_{\max} and E_{\max} have made the assumption that the sinusoid and the $n = 0$ interval both started at a time when the value of the sinusoid was zero and had a positive slope. This section deals with the effects of synchronization upon e_{\max} and E_{\max} .

The input signal must be expressed with a synchronization parameter, ψ . The input then is:

$$V_i(t) = \sin\left(\frac{2\pi t}{T} + \psi\right)$$

Substituting this expression for $V_i(t)$ into Eq. (3):

$$\begin{aligned} V_{r_n} &= \frac{1}{T} \int_{nr}^{(n+1)r} \sin\left[\frac{2\pi}{T}t + \psi\right] dt \\ &= -\frac{T}{2\pi r} \cos\left[\frac{2\pi}{T}t + \psi\right]_{nr}^{(n+1)r} \\ &= -\frac{T}{2\pi r} \cos\left[\frac{2\pi(n+1)r}{T} + \psi\right] + \cos\left[\frac{2\pi nr}{T} + \psi\right] \\ &= \frac{T}{\pi r} \sin\left[(2n+1)\frac{\pi r}{T} + \psi\right] \cdot \sin\frac{\pi r}{T} \end{aligned} \quad (9)$$

Using the definition of m , Eq. (9) becomes:

$$V_{r_n} = \frac{m}{\pi} \sin\left[\frac{(2n+1)\pi}{m} + \psi\right] \cdot \sin\frac{\pi}{m} \quad (10)$$

Since many solutions of Eq. (10) are required to determine the synchronization effects, a program was written to vary m from 1 to 25.0 in increments of 0.5 and to vary ψ from 0 to 180 deg in increments of 1 deg. The solutions for the E_{\max} option are in Table I and the solutions for the e_{\max} option are in Table II. The program is listed in Table I-II (Appendix).

TABLE I
E_{max} SOLUTIONS OF SYNCHRONIZATION EFFECTS ON SINUSOIDAL INPUTS

DATA POINT ERROR ANALYSIS					
M	PSI	LARGEST E(MAX)	PSI	SMALLEST E(MAX)	DELTA E(MAX)
1.00	90.00	100.00	0.0	0.00	100.00
1.50	30.00	58.65	0.0	50.79	7.86
2.00	0.0	36.34	90.00	0.00	36.34
2.50	18.00	24.32	72.00	23.13	1.19
3.00	30.00	17.30	0.0	14.98	2.32
3.50	90.00	12.90	0.0	12.57	0.32
4.00	45.00	9.97	0.0	7.05	2.92
4.50	10.00	7.93	0.0	7.81	0.12
5.00	18.00	6.45	0.0	6.14	0.32
5.50	41.00	5.35	0.0	5.30	0.05
6.00	0.0	4.51	30.00	3.90	0.60
6.50	90.00	3.85	0.0	3.82	0.03
7.00	90.00	3.32	0.0	3.24	0.08
7.50	6.00	2.90	60.00	2.89	0.02
8.00	22.00	2.55	90.00	2.36	0.19
8.50	16.00	2.26	180.00	2.25	0.01
9.00	10.00	2.02	0.0	1.99	0.03
9.50	52.00	1.81	0.0	1.81	0.01
10.00	0.0	1.64	18.00	1.56	0.08
10.50	73.00	1.49	120.00	1.48	0.00
11.00	8.00	1.35	180.00	1.34	0.01
11.50	43.00	1.24	0.0	1.24	0.00
12.00	15.00	1.14	0.0	1.10	0.04
12.50	11.00	1.05	0.0	1.05	0.00
13.00	76.00	0.97	0.0	0.96	0.01
13.50	10.00	0.90	0.0	0.90	0.00
14.00	103.00	0.84	90.00	0.82	0.02
14.50	34.00	0.78	0.0	0.78	0.00
15.00	6.00	0.73	0.0	0.73	0.00
15.50	32.00	0.68	180.00	0.68	0.00
16.00	11.00	0.64	45.00	0.63	0.01
16.50	101.00	0.60	0.0	0.60	0.00
17.00	16.00	0.57	0.0	0.57	0.00
17.50	85.00	0.54	108.00	0.54	0.00
18.00	0.0	0.51	10.00	0.50	0.01
18.50	17.00	0.48	107.00	0.48	0.00
19.00	71.00	0.46	180.00	0.45	0.00
19.50	67.00	0.43	60.00	0.43	0.00
20.00	9.00	0.41	0.0	0.41	0.00
20.50	11.00	0.39	180.00	0.39	0.00
21.00	4.00	0.37	180.00	0.37	0.00
21.50	23.00	0.36	21.00	0.36	0.00
22.00	33.00	0.34	90.00	0.34	0.00
22.50	2.00	0.32	12.00	0.32	0.00
23.00	59.00	0.31	0.0	0.31	0.00
23.50	6.00	0.30	46.00	0.30	0.00
24.00	8.00	0.29	180.00	0.28	0.00
24.50	46.00	0.27	136.00	0.27	0.00
25.00	4.00	0.26	108.00	0.26	0.00

TABLE II
 e_{\max} SOLUTIONS OF SYNCHRONIZATION EFFECTS ON SINUSOIDAL INPUTS

M	PSI	LARGEST F(MAX)	PSI	SMALLEST E(MAX)	DELTA E(MAX)
1.00	0.0	100.00	9.00	99.95	0.05
1.50	90.00	120.67	0.0	113.77	6.91
2.00	90.00	100.00	0.0	44.65	55.35
2.50	90.00	76.61	0.0	73.76	2.85
3.00	90.00	58.65	0.0	52.15	6.50
3.50	90.00	45.69	0.0	44.68	1.02
4.00	90.00	36.34	45.00	26.89	9.45
4.50	90.00	29.47	0.0	29.03	0.44
5.00	162.00	24.32	36.00	23.18	1.14
5.50	90.00	20.38	164.00	20.15	0.23
6.00	90.00	17.30	0.0	15.06	2.24
6.50	90.00	14.86	55.00	14.74	0.12
7.00	90.00	12.90	0.0	12.56	0.33
7.50	90.00	11.29	0.0	11.20	0.10
8.00	90.00	9.97	68.00	9.24	0.73
8.50	90.00	8.86	0.0	8.79	0.07
9.00	90.00	7.93	0.0	7.79	0.14
9.50	90.00	7.13	126.00	7.08	0.05
10.00	126.00	6.45	0.0	6.14	0.32
10.50	90.00	5.86	123.00	5.81	0.05
11.00	90.00	5.35	100.00	5.27	0.08
11.50	90.00	4.90	69.00	4.85	0.05
12.00	0.0	4.51	135.00	4.32	0.19
12.50	54.00	4.16	108.00	4.15	0.01
13.00	90.00	3.85	127.00	3.79	0.06
13.50	90.00	3.57	23.00	3.53	0.04
14.00	90.00	3.32	0.0	3.19	0.13
14.50	90.00	3.10	57.00	3.07	0.03
15.00	18.00	2.90	36.00	2.86	0.04
15.50	90.00	2.72	54.00	2.70	0.02
16.00	0.0	2.55	55.00	2.47	0.08
16.50	90.00	2.40	19.00	2.38	0.02
17.00	90.00	2.26	0.0	2.21	0.05
17.50	90.00	2.13	108.00	2.12	0.02
18.00	90.00	2.02	104.00	1.97	0.05
18.50	90.00	1.91	0.0	1.90	0.02
19.00	90.00	1.81	79.00	1.79	0.03
19.50	90.00	1.72	50.00	1.71	0.02
20.00	0.0	1.64	117.00	1.62	0.02
20.50	90.00	1.56	20.00	1.54	0.02
21.00	90.00	1.49	33.00	1.46	0.02
21.50	90.00	1.42	26.00	1.39	0.02
22.00	90.00	1.35	105.00	1.31	0.04
22.50	90.00	1.29	180.00	1.26	0.03
23.00	90.00	1.24	23.00	1.20	0.03
23.50	90.00	1.19	0.0	1.15	0.04
24.00	0.0	1.14	43.00	1.09	0.05
24.50	90.00	1.09	0.0	1.04	0.05
25.00	11.00	1.05	108.00	1.05	0.00

These solutions depict the cases in which the variations of E_{\max} or e_{\max} are the largest. The cases are the integer values of m and m with a decimal part of 0.5. All other values of m exhibit less variation in E_{\max} or e_{\max} as ψ varies.

From Table I,

for $m > 6$, $\Delta E_{\max} \leq 0.19$ percent

for $m > 11.4$, $\Delta E_{\max} \leq 0.04$ percent

for $m \geq 14.0$, $\Delta E_{\max} \leq 0.01$ percent

The $m > 11.4$ values yield accurate recovery.

From Table II,

for $m > 8$, $\Delta e_{\max} \leq 0.32$ percent

for $m > 11.4$, $\Delta e_{\max} \leq 0.19$ percent

for $m > 16$, $\Delta e_{\max} \leq 0.05$ percent

This clearly indicates that synchronization has negligible effect on the error at a summing interval-to-period ratio that is required to accurately recover the data.

3.4 EXPERIMENTAL CORRELATION OF THEORETICAL ERRORS

A verification, by actual errors encountered in the data recovery, of the theoretical treatment was desired; therefore, an experiment to determine actual errors was formulated. For the input waveform, a sinusoid was chosen. There were, basically, two reasons for this choice: (1) a sinusoid is easily obtainable from a sinusoidal generator, and (2) the frequency or period of cycle is a very important factor in the error of a signal.

The VIDAR Data Acquisition System was set up with a sinusoidal generator connected to the input of a channel of the V/F converter. The frequency output was recorded, using direct electronics, on a magnetic tape recorder. The recorded tape was translated using a summing interval of 2 msec, and the raw counts were printed out.

In order to get the negative values of the sinusoid, the VIDAR was adjusted so that for zero input, the output counts were at midscale.

Then a known voltage of 1.5 v was applied, and the VIDAR spanned to nearly full scale. The count levels of these adjustments were:

$$0\text{v Input} = 25,003 \text{ counts/sec}$$

$$1.5\text{v Input} = 47,500 \text{ counts/sec}$$

The signal from the sinusoid generator was set at 20 Hz and 2 v peak-to-peak which is easily accommodated in magnitude by the chosen calibration voltage.

Since the theoretical error analysis used m as the variable, m had to be determined for this case of the translation summing interval; m was defined as T/τ or $1/f\tau$; giving $m = 1/20 (0.002) = 25$.

Synchronization of output and input for actual error analysis proved to be a difficult problem; therefore, since for an m of 25, $e_{\max} = 1.05$ percent and $E_{\max} = 0.26$ percent, this linear interpolated curve was used as the reference or standard. Small m 's were obtained by taking two or more adjacent summing intervals at a time. A computer program, depicted in Table I-III (Appendix), was written to do the task of summing up adjacent summing intervals. The program limits the number of adjacent summing intervals to 13 since this gives an m of 1.92, which is below the level set forth by the sampling theorem. Table III is the solution for this program which has the option of determining either e_{\max} or E_{\max} .

As shown in Table III, the maximum errors determined by the theoretical approach were confirmed to be reasonably accurate; therefore, the general rule of thumb of 11.4 summing intervals per period for $e_{\max} \leq 5$ percent is a valid one. This also indicates that the theoretical simulation is a reasonably accurate error determination technique.

SECTION IV STEP INPUT

In the theoretical analysis of errors of the V/F converter data acquisition system in following a step input, the time reference of $t = 0$ is set at the start of the summing interval in which the step function occurs.

TABLE III
 e_{\max} and E_{\max} THEORETICAL AND ACTUAL SOLUTIONS FOR SINUSOIDAL INPUT

ACTUAL			THEORY		
H	E(MAX)		H	E(MAX)	
1.92	103.31	103.68	2.08	94.82	95.63
2.27	85.46	86.51	2.50	75.63	73.76
2.78	64.50	65.81	3.13	54.01	54.39
3.57	43.32	44.08	4.17	33.76	33.75
5.00	23.88	23.18	6.25	16.34	15.93
8.33	10.47	9.19	12.50	5.35	4.15

DATA POINT ERROR ANALYSIS							
H	ACTUAL		THEORY	H	ACTUAL		THEORY
	E(MAX)				E(MAX)		
1.92	38.67		38.83	2.08	32.33		33.75
2.27	23.57		23.88	2.50	22.85		23.13
2.78	20.31		19.96	3.13	15.35		15.93
3.57	13.47		12.38	4.17	8.57		9.19
5.00	8.00		6.14	6.25	4.29		4.15
8.33	4.76		2.35	12.50	0.01		1.05

The input waveform, a normalized step function, is denoted by the symbol:

$$\mu_1(t-a) = \begin{cases} 0 & t < a \\ 1 & t \geq a \end{cases} \quad (11)$$

For this analysis, the input is:

$$V_i(t) = \mu_1(t - Kr) \quad (12)$$

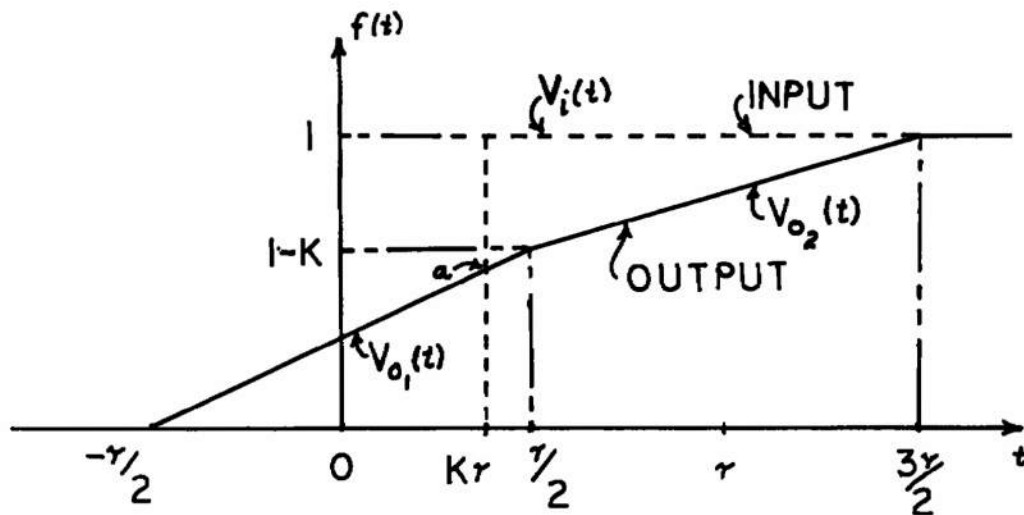
where $0 \leq K \leq 1$ is the shift of the step with respect to the summing interval reference.

Using Eqs. (3) and (12) to find the output, the particular interval for $n = 0$ is the only one of interest since in all intervals for $n < 0$, $V_{\tau n} = 0$, and in all intervals for $n > 0$, $V_{\tau n} = 1$.

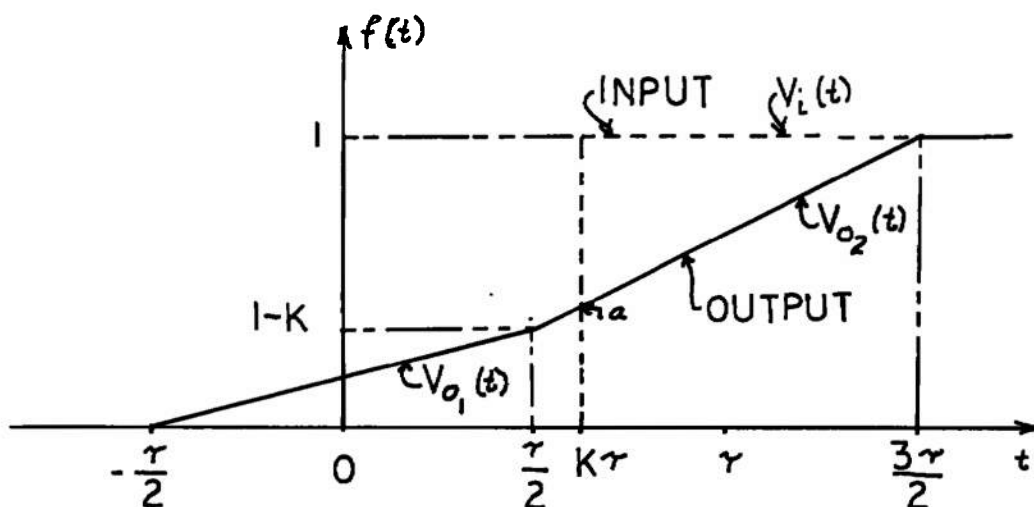
The interval for $n = 0$ has an output value of:

$$\begin{aligned} V_{\tau 0} &= \frac{1}{\tau} \int_0^{\tau} \mu_1(t - Kr) dt \\ &= \frac{1}{\tau} \int_{Kr}^{\tau} dt = \frac{1}{\tau} (\tau - Kr) \\ &= 1 - K \end{aligned} \quad (13)$$

The input-output relationship is depicted in Fig. 6.



$$0 \leq K \leq 1/2$$



$$1/2 \leq K \leq 1$$

Fig. 6 Input-Output Waveforms for Step Input

4.1 LINEAR INTERPOLATION ERROR ANALYSIS OF STEP INPUT

As shown in Fig. 6, the maximum error occurs at $t = K\tau$. Point a is used to designate this intersection of the input and output waveforms at $t = K\tau$. The output line from $t = -\frac{\tau}{2}$ to $\frac{\tau}{2}$ is designated $V_{o_1}(t)$, and the output line from $t = \frac{\tau}{2}$ to $\frac{3\tau}{2}$ is designated $V_{o_2}(t)$. For $0 \leq K \leq 1/2$, point a occurs on the output line $V_{o_1}(t)$. The equation for this line, which has a slope of $\frac{1}{\tau}(1-K)$ and passes through the point $(-\frac{\tau}{2}, 0)$, is:

$$V_{o_1}(t) = \frac{1}{\tau}(1-K)(t + \frac{\tau}{2})$$

Since $|V_1(t)|_{\max} = 1$,

$$\begin{aligned} e_{\max} &= 100 V_{o_1}(K\tau) \\ &= 100(1-K)(K + \frac{1}{2}) \\ &= 50(1-K-2K^2) \end{aligned} \tag{14}$$

for $0 \leq K \leq 1/2$.

For $1/2 \leq K \leq 1$, point a occurs on the output line $V_{o_2}(t)$. The equation for this line, which has a slope of K/τ and passes through the point $(\frac{3\tau}{2}, 1)$, is:

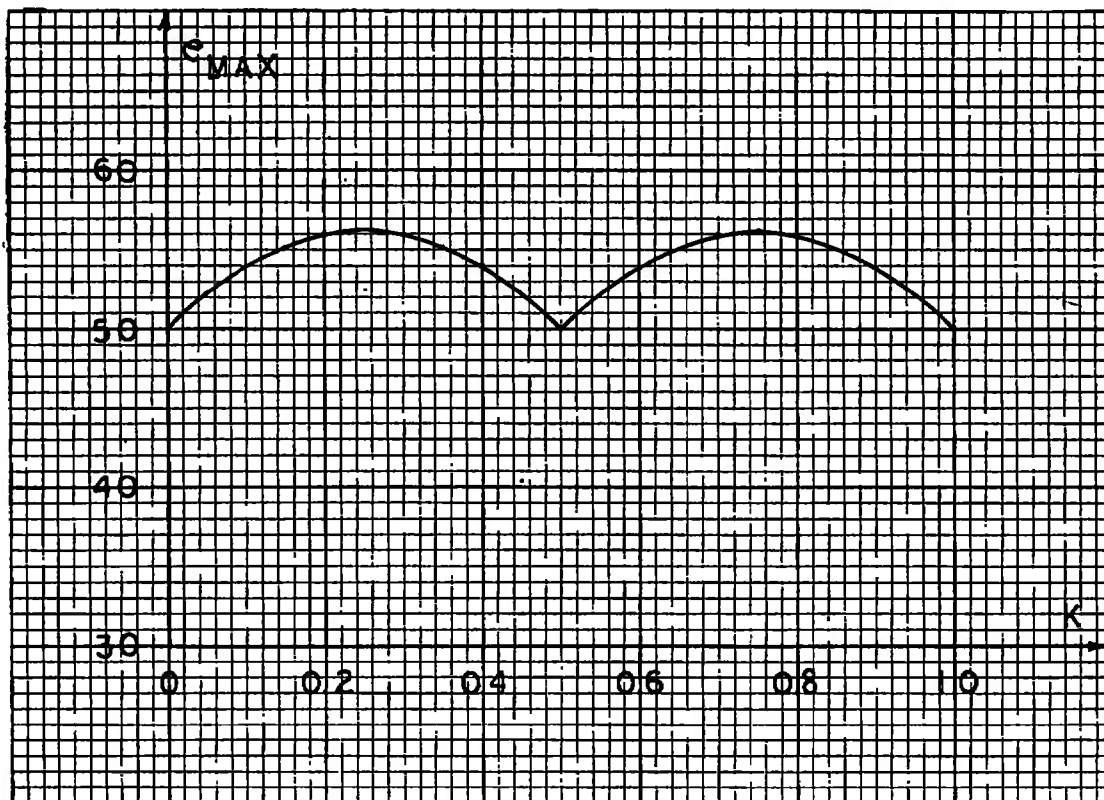
$$V_{o_2}(t) = \frac{K}{\tau}(t - \frac{3\tau}{2}) + 1$$

Likewise,

$$\begin{aligned} e_{\max} &= 100[1 - V_{o_2}(K\tau)] \\ &= -100K(K - 3/2) \\ &= 150K - 100K^2 \\ &= 50(3K - 2K^2) \end{aligned} \tag{15}$$

for $1/2 \leq K \leq 1$.

Figure 7 is a plot of e_{\max} versus K using Eqs. (14) and (15). As can be seen, e_{\max} varies from the lowest value of 50 percent at $K = 0$, $1/2$, and 1 to a highest value of 56.25 percent at $K = 1/4$ and $3/4$.

Fig. 7 Plot of K versus e_{\max} for Step Input

4.2 DATA POINT ERROR ANALYSIS OF STEP INPUT

For this input signal, all data point errors are 0 percent with the exception of the one for $n = 0$. This data point of interest occurs at time $t = \tau_0$ or 0.5.

Using Eq. (13) and the fact that $|v_i(t)|_{\max} = 1$, the E_{\max} for step input is:

$$E_{\max} = 100 |V_i(\tau_0) - (1 - K)| \quad (16)$$

However, $V_i(\tau_0)$ is dependent on the value of K in the following manner:

$$V_i(\tau_0) = \begin{cases} 1 & 0 \leq K \leq 0.5 \\ 0 & 0.5 \leq K \leq 1.0 \end{cases} \quad (17)$$

Combining Eqs. (16) and (17), the maximum error becomes:

$$E_{\max} = \begin{cases} 100K & 0 \leq K \leq 0.5 \\ 100(1 - K) & 0.5 \leq K \leq 1.0 \end{cases}$$

A plot of E_{\max} versus K is shown in Fig. 8. As was the case in the total error, the maximum data point error is independent of the summing interval length and is a function of K alone.

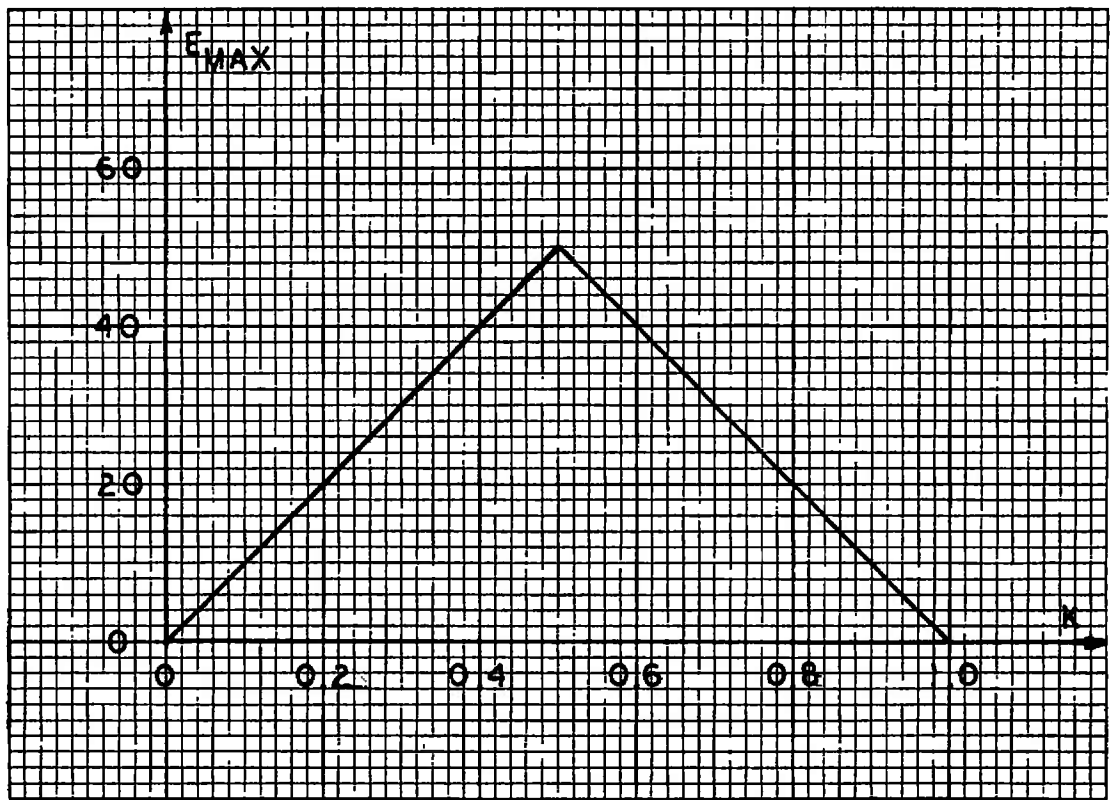


Fig. 8 Plot of K versus E_{\max} for Step Input

4.3 TIME DELAY ANALYSIS OF STEP INPUT

The determination of time delay, i. e., time for the response of the linear interpolated output to be within 5 percent error, was desired. The notation T_D is used for this parameter.

This determination must, like the error analysis, be divided into two parts. Likewise, the first part uses the output, $V_{O1}(t)$, in its determination. The ranges of K , however, are not the same. In using $V_{O1}(t)$, the following inequality must be satisfied:

$$1 - V_{O1}\left(\frac{T}{2}\right) \leq 0.05 \quad (18)$$

Since $V_{O1}\left(\frac{T}{2}\right) = 1 - K$, Eq. (18) defines the limits of K to be:

$$0 \leq K \leq 0.05$$

For a time delay of T_D , the actual time $t = T_D + K\tau$, and from the equation for $V_{O1}(t)$,

$$V_{O1}(T_D + K\tau) = 0.95 = \frac{1}{\tau} (1 - K) (T_D + K\tau + \frac{\tau}{2})$$

Solving for T_D ,

$$T_D = \frac{0.45 - 0.5K + K^2}{1 - K} \quad (19)$$

for $0 \leq K \leq 0.05$.

The second part, $0.05 \leq K \leq 1$, uses $V_{O2}(t)$ in the determination of T_D , with $t = T_D + K\tau$.

$$V_{O2}(T_D + K\tau) = 0.95 = \frac{K}{\tau} (T_D + K\tau - 1.5\tau) + 1$$

Solving for T_D ,

$$T_D = \frac{-0.05 + 1.5K - K^2}{K^2} \quad (20)$$

for $0.05 \leq K \leq 1$.

Using Eqs. (19) and (20), a plot of T_D versus K is depicted in Fig. 9. As shown in this plot, T_D varies from a minimum of 0.449τ to a maximum of 1.053τ .

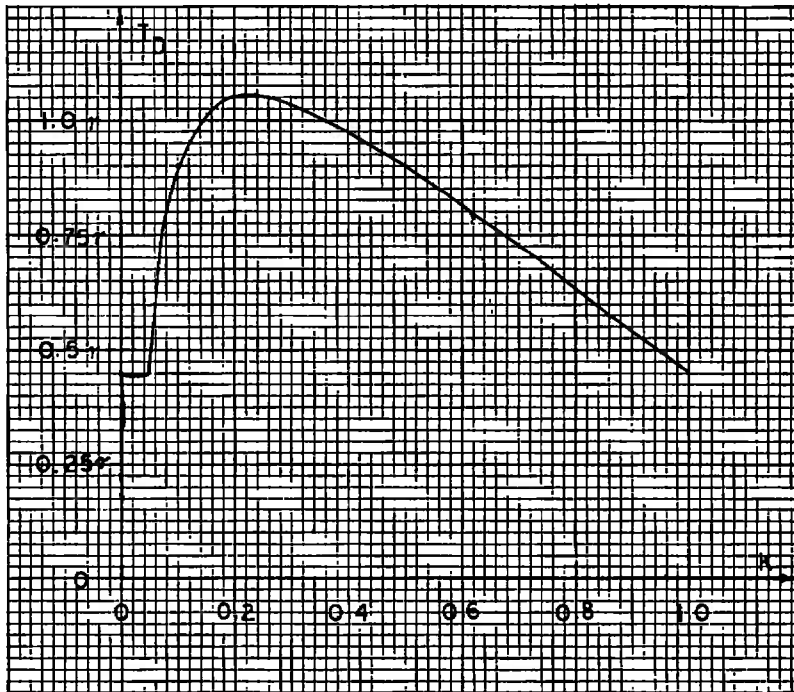


Fig. 9 Plot of K versus T_D for Step Input

The step response will always appear to be a ramp. For $K = 1/2$, the rise time of the ramp is 2τ sec, and for $K = 0$ or 1 , the rise time will be τ sec; therefore, to minimize this effect, τ must be made as small as possible. Also, the output will respond to within 5 percent error in a delay time, T_D , of approximately τ .

SECTION V RAMP INPUT

In order to theoretically analyze the error of the V/F converter data acquisition system in the data recovery of a ramp input, time, $t = 0$, is set at the start of the summing interval in which the ramp function begins.

The ramp function is defined as:

$$\mu_2(t - a) = \begin{cases} 0 & t < a \\ t - a & t \geq a \end{cases} \quad (21)$$

For a normalized ramp function which has a rise time of β summing intervals, the input function is expressed as:

$$V_i(t) = \frac{1}{\beta\tau} [\mu_2(t - K\tau) - \mu_2(t - (K + \beta)\tau)] \quad (22)$$

The value of V_{τ_0} is dependent upon the value of $K + \beta$. For $(K + \beta) \leq 1$, the combination of Eqs. (3) and (22) becomes:

$$\begin{aligned} V_{\tau_0} &= \frac{1}{\beta\tau^2} \int_0^\tau [\mu_2(t - K\tau) - \mu_2(t - (K + \beta)\tau)] dt \\ &= \frac{1}{\beta\tau^2} \int_{K\tau}^{(K + \beta)\tau} (t - K\tau) dt + \frac{1}{\tau} \int_{(K + \beta)\tau}^\tau dt \\ &= \frac{1}{\beta\tau^2} \left[\frac{1}{2}t^2 - K\tau t \right]_{K\tau}^{(K + \beta)\tau} + \frac{1}{\tau} \left[t \right]_{(K + \beta)\tau}^\tau \\ &= \frac{1}{\beta\tau^2} \left[\frac{1}{2}(K + \beta)^2\tau^2 - \frac{1}{2}K^2\tau^2 - K(K + \beta)\tau^2 + K^2\tau^2 \right] \\ &\quad + \frac{1}{\tau} [\tau - (K + \beta)\tau] \\ &= \frac{\beta}{2} + 1 - (K + \beta) \\ &= 1 - K - \frac{\beta}{2} \end{aligned} \quad (23)$$

for $0 \leq K + \beta \leq 1$.

For $(K + \beta) > 1$, V_{τ_0} is defined by the integral:

$$\begin{aligned} V_{\tau_0} &= \frac{1}{\beta r^2} \int_{Kr}^{\tau} (t - Kr) dt \\ &= \frac{1}{\beta r^2} \left[\frac{1}{2} t^2 - K\tau t \right]_{Kr}^{\tau} = \frac{1}{\beta r^2} \left[\frac{1}{2} \tau^2 - \frac{1}{2} K^2 \tau^2 - K\tau + K^2 \tau \right] \\ &= \frac{(1 - K)^2}{2\beta} \end{aligned} \quad (24)$$

for $(K + \beta) > 1$.

For all summing intervals up to, but not including, the one in which the ramp breaks to a zero slope, the value of the output is equal to the value of the input at the midpoint of the summing interval. That is:

$$V_{\tau_n} = V_i \left(\frac{2n+1}{2} \tau \right)$$

for $0 < n < \text{Int}(K + \beta)$.

For the interval $n = \text{Int}(K + \beta)$, the output value is:

$$\begin{aligned} V_{\tau_n} &= \frac{1}{\beta r^2} \int_{n\tau}^{(K+\beta)\tau} (t - Kr) dt + \frac{1}{\tau} \int_{(K+\beta)\tau}^{(n+1)\tau} dt \\ &= \frac{1}{\beta r^2} \left[\frac{1}{2} t^2 - K\tau t \right]_{n\tau}^{(K+\beta)\tau} + \frac{1}{\tau} \left[t \right]_{(K+\beta)\tau}^{(n+1)\tau} \\ &= \frac{1}{2} (K + \beta)^2 - \frac{K(K + \beta)}{\beta} - \frac{n^2}{2\beta} + \frac{Kn}{\beta} + (n + 1) - (K + \beta) \\ &= (n + 1) - K - \beta/2 - \frac{(n - K)^2}{2\beta} \end{aligned} \quad (25)$$

Figure 10 depicts the input-output relationships for a ramp function.

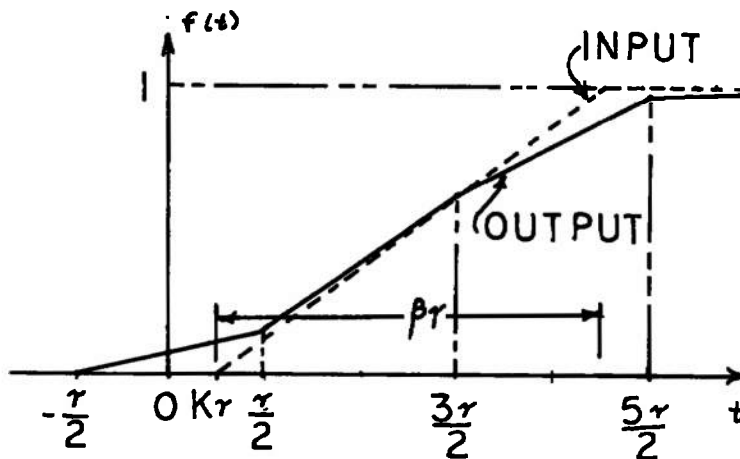


Fig. 10 Input-Output Waveforms for Ramp Input

5.1 LINEAR INTERPOLATION ERROR ANALYSIS OF RAMP INPUT

As shown in Fig. 10, the e_{\max} occurs at either $t = K\tau$ or $t = (K + \beta)\tau$; therefore, e_{\max} is equal to the maximum of these two values. An expression for the e_{\max} is so cumbersome to deal with manually that a digital computer program, depicted in Table I-IV (Appendix), was written to calculate the many solutions required to determine the minimum β for 5 percent or less error. The program calculates the β_{\min} for $e_{\max} \leq 5$ percent; then β is varied from 0.1 to β_{\min} in increments of 0.1 for $K = 0.0$ and 0.50 and the value of e_{\max} determined for each set of parameters. The family of curves that was calculated is depicted in Fig. 11. These two values of K are the extreme cases; all other values are in between. The plots in Fig. 11 indicate that a value of $\beta \geq 5.0$ assures a response within 5 percent.

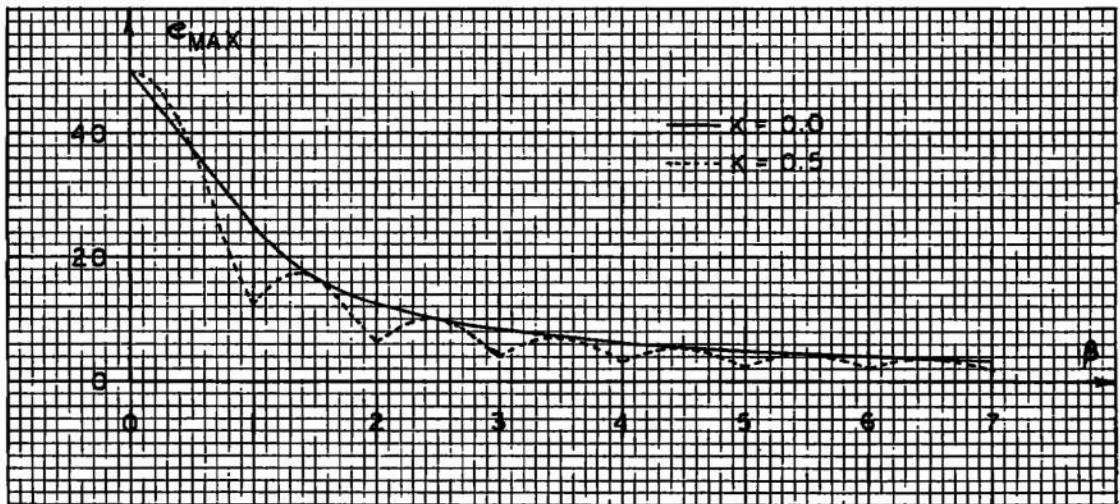


Fig. 11 Plot of e_{\max} versus β for Ramp Input with K Constant

5.2 DATA POINT ERROR ANALYSIS OF RAMP INPUT

The value of E_{\max} , as shown in Fig. 10, occurs at either $t_1 = 1/2\tau$ or $t_2 = [\text{Int}(K + \beta) + 1/2]\tau$. The expression for E_{\max} is the maximum of:

$$E_1 = 100 |V_i(1/2\tau) - V_{r_0}|$$

or

$$E_2 = 100 |V_i\left(\frac{2n+1}{2}\tau\right) - V_{r_n}|$$

where $n = \text{Int}(K + \beta)$.

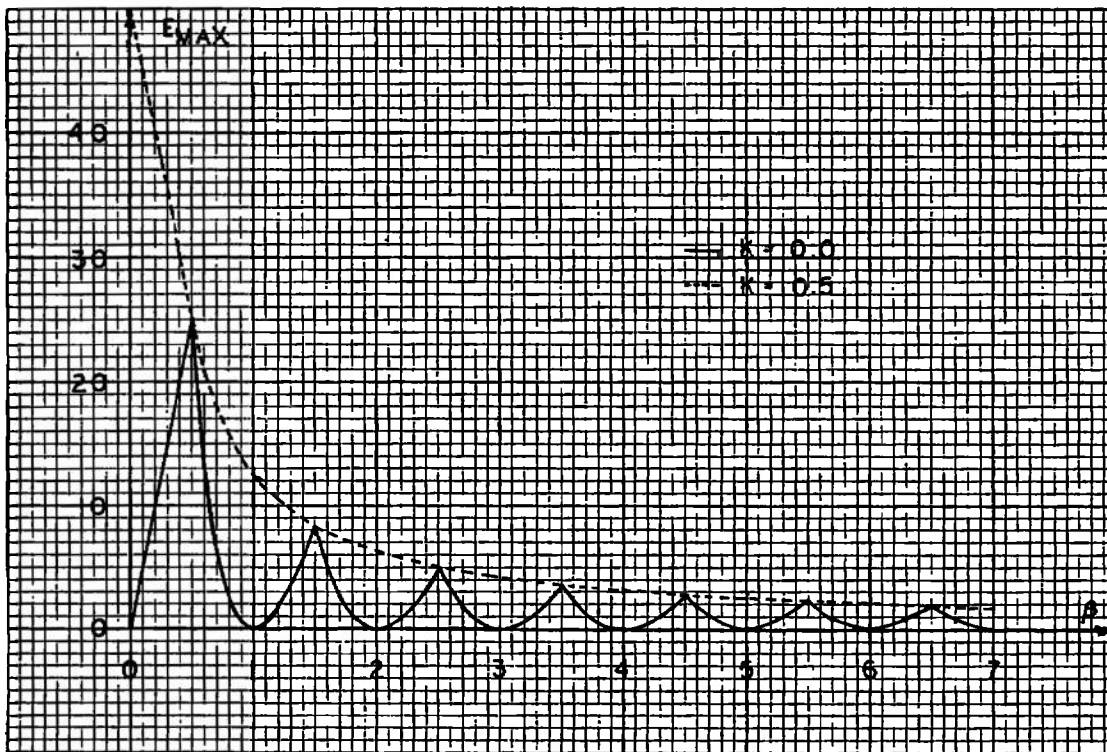


Fig. 12 Plot of E_{\max} versus β for Ramp Input with K Constant

The digital computer program that calculates e_{\max} was written for the E_{\max} option also. The value of β_{\min} for $E_{\max} \leq 5$ percent is first calculated; then β is varied from 0.1 to β_{\min} in increments of 0.1 for $K = 0.0$ and 0.5, since these errors are the extreme cases, and the value of E_{\max} determined for each set of parameters. Figure 12 depicts the family of curves that was calculated. These plots indicate that, for a value of $\beta \geq 2.5$, the data points are in error by 5 percent or less. To illustrate, for a summing interval of 2 msec, any ramp with a rise time of 5 msec or greater will be reconstructed with all data points within 5 percent error.

SECTION VI SUMMARY AND CONCLUSIONS

6.1 SUMMARY

A voltage-to-frequency data acquisition system was theoretically analyzed for errors that occur when it is used as a sampling system. Since the standard sampling theorem is not directly applicable, the

system was simulated on a digital computer to aid in the determination of parameter criteria for expected levels of accuracy for sinusoidal, step, and ramp inputs.

Parameter limits were determined for such parameters as m , summing intervals per period of a sinusoid, K , delay of step and ramp inputs with respect to start of summing interval, and β , ratio of ramp rise time to summing interval length.

An investigation of the effect of signal synchronization was accomplished for the sinusoid, and an experiment was performed, using a sinusoid, to determine the validity of the theoretical solutions.

6.2 CONCLUSIONS

The voltage-to-frequency conversion for data acquisition does not offer accurate recovery (5 percent error) of sinusoidal data until the ratio of summing interval to period is about 11.4, i. e., 11.4 summing intervals per period. This means that for a 2-msec summing interval, any frequency component above 43.9 Hz has a maximum error greater than 5 percent and, from Fig. 3, any component of 151.5 Hz or greater is in error at its maximum by more than 50 percent.

The voltage-to-frequency conversion system has an inherent error because all data points do not coincide with the input waveform. For the sinusoid, the ratio of summing intervals to period must be 12.8 for the data point to be within 1 percent of the input waveform. At the value of $m = 11.4$, e_{\max} was 4.95 percent and E_{\max} is 1.26 percent; therefore, a sizeable portion of the error is the result of the data point error.

The step input error is independent of the summing interval and is symmetrical with respect to $K = 0.5$ for both e_{\max} and E_{\max} . However, delay time, time required to bring the output to be within 5 percent of the input, is a function of K and τ , and as K varies, the time delay varies in the range from 0.45τ to 1.05τ .

Ramp inputs can be followed within 5 percent if the ratio of ramp rise time to summing interval is larger than five. This is to say that a 2-msec summing interval can accurately follow a ramp of rise time greater than 10 msec.

The ramp function has an unusual E_{\max} versus β for constant K relationship. At low and high values of K , the E_{\max} oscillates as a function of β . At midvalues of K , the relationship is nearly exponential.

If $\beta \geq 12.5$, all data points are within 1 percent of the input waveform. The reverse conditions exist in the relationship of e_{\max} versus β for constant K .

An additional error that may be significant at small summing interval lengths is resolution error from the low pulse counts that occur. However, errors from resolution were not included in this analysis.

REFERENCES

1. Mallinckrodt, A. J. "Aliasing Errors in Sampled Data Systems." Report 316 AGARD, NATO, Paris, 1961.
2. Tucker, G. K., and Willis, D. M. A Simplified Technique of Control System Engineering, Minneapolis-Honeywell Regulator Company, Brown Instrument Division, Philadelphia, 1962.
3. Burington, R. S., Compiler. Handbook of Mathematical Tables and Formulas. Handbook Publishers, Inc., Sandusky, Ohio, 1954.

**APPENDIX
COMPUTER PROGRAMS**

TABLE I-I
 COMPUTER PROGRAM TO FIND E_{\max} AND e_{\max} FOR A SINUSOIDAL INPUT
 AS m VARIES FROM 1.0 TO 25.0 IN INCREMENTS OF 0.1

```

L.0001 7JOB GO, TIME=10
L.0002      DIMENSION XMA(241), DIFMA(241)
L.0003 C      IND=0 (E(MAX) FOR DATA POINT)
L.0004 C      IND=1 (E(MAX) FOR LINEAR INTERPOLATION AT ANY POINT)
L.0005      READ(5,1) IND
L.0006 1      FORMAT(I10)
L.0007      PSI=0.
L.0008      DO 103 IM=10, 250
L.0009      I=IM-9
L.0010      XM=FLOAT(IM)/10.
L.0011      XMA(I)=XM
L.0012      DIFMA(I)=VEIMAX(XM, PSI, IND)
L.0013 103    CONTINUE
L.0014      WRITE(6,2)
L.0015 2      FORMAT('1')
L.0016      IF(IND)100,100,101
L.0017 100    WRITE(6,3)
L.0018 3      FORMAT(10X'DATA POINT ERROR ANALYSIS'/)
L.0019 101    WRITE(6,4) (XMA(J), DIFMA(J), J=1, 241)
L.0020 4      FORMAT(4(5X'M'4X'E(MAX)'4X)//4(2F8.2, 4X))
L.0021      STOP
L.0022      END
  
```

TABLE I-1 (Concluded)

```

L.0001      FUNCTION VENAX(XH,PSI,IND)
L.0002      DIMENSION FA(252),TA(252)
L.0003      PI=3.1415926
L.0004      PSI=PSI*PI/180.
L.0005      IF(XH-25.)99,99,98
L.0006  98   STOP 2525
L.0007  99   DO 110 L=10,100,10
L.0008      XP=0.001
L.0009      DO 109 I=1,2
L.0010      IXC=XH*FLOAT(L)+XP
L.0011      IF(ICX-(ICX/10)*10)103,103,110
L.0012  103  XP=XP+0.9
L.0013  109  CONTINUE
L.0014      GO TO 111
L.0015  110  CONTINUE
L.0016  111  NS=(ICX+25)/10
L.0017      ARG1=PI/XH
L.0018      EMAX=0.
L.0019      DO 100 N=1,NS
L.0020      XH=N-2
L.0021      ALPHA=2.*XH+1.
L.0022      TA(N)=ALPHA/XH/2.
L.0023      ARG2=ALPHA*ARG1+PSI
L.0024      FA(N)=SIN(ARG2)*SIN(ARG1)/ARG1
L.0025      ERR=ABS(SIN(ARG2)-FA(N))
L.0026      EMAX=AMAX1(ERR,EMAX)
L.0027  100  CONTINUE
L.0028      IF(IND)103,103,101
L.0029  101  EMAX=0.
L.0030      IXS=TA(NS)*100.+1.
L.0031      DO 102 IX=1,IXS
L.0032      X=FLOAT(IX)/100.
L.0033      TEST=TAB2(FA,TA,X,NS)
L.0034      DEV=ABS(SIN(2.*PI*X+PSI)-TEST)
L.0035      EMAX=AMAX1(EMAX,DEV)
L.0036  102  CONTINUE
L.0037  103  VENAX=EMAX*100.
L.0038      RETURN
L.0039      END

```

TABLE I-II
 COMPUTER PROGRAM TO FIND THE LARGEST, SMALLEST, AND DELTA
 E_{\max} OR e_{\max} as ψ VARIES FROM 0 TO 180 deg IN INCREMENTS OF
 1 deg AND m VARIES FROM 1 TO 25 IN INCREMENTS OF 0.5

```

0001      DIMENSION TA(252),FA(252)
0002      PI=3.1415926
0003      DO 906 ND=1,2
0004      IND=ND-1
0005      WRITE(6,1)
0006      1 FORMAT('1')
0007      IF(IND)902,902,903
0008      902 WRITE(6,2)
0009      2 FORMAT('/15X'DATA POINT ERROR ANALYSIS')
0010      903 WRITE(6,3)
0011      3 FORMAT('/23X'LARGEST'13X'SMALLEST      DELTA'/7X
      1'M'2X2(6X'PSI'5X'E(MAX)'),4X'E(MAX)')
0012      DO 906 IM=10,250,5
0013      XM=FLOAT(IM)/10.
0014      DO 99 I=1,252
0015      99 TA(I)=(2.*FLOAT(I-2)+1.)/XM/2.
0016      EMAOUT=0.
0017      EMOUT=150.
0018      DO 102 L=10,100,10
0019      XP=0.001
0020      DO 101 I=1,2
0021      IXC=XM*FLOAT(L)+XP
0022      IF(IXC-(IXC/10)*10)100,100,102
0023      100 XP=XP+.9
0024      101 CONTINUE
0025      GO TO 103
0026      102 CONTINUE
0027      103 NS=(IXC+25)/10
0028      ARG1=PI/XM
0029      DO 900 I=1,181
0030      CSI=I-1
0031      PSI=CSI*PI/180.
0032      EMAX=0.
0033      DO 104 N=1,NS
0034      ARG2=(2.*FLOAT(N-2)+1.)*ARG1+PSI
0035      FA(N)=SIN(ARG2)*SIN(ARG1)/ARG1
0036      ERR=ABS(SIN(ARG2)-FA(N))*100.
0037      EMAX=AMAX1(ERR,EMAX)
0038      104 CONTINUE
0039      IF(IND)107,107,105
0040      105 EMAX=0.
0041      IXS=TA(NS)*100.
0042      K=2
0043      DO 106 IX=1,IXS
0044      X=FLOAT(IX-1)/100.
0045      701 DIF=X-TA(K)
0046      IF(DIF)703,703,702
0047      702 K=K+1
0048      GO TO 701
0049      703 DEV=ABS(SIN(2.*PI*X+PSI)-DIF*(FA(K)-FA(K-1))
      1/(TA(K)-TA(K-1))-FA(K))*100.
0050      EMAX=AMAX1(DEV,EMAX)

```

TABLE I-II (Concluded)

0051	106 CONTINUE
0052	107 IF(EMADUT-EMAX)800,801,801
0053	800 EMADUT=EMAX
0054	PSIL=CSI
0055	801 IF(EMAX-EMIOUT)802,900,900
0056	802 EMIOUT=EMAX
0057	PSIS=CSI
0058	900 CONTINUE
0059	DIFMA=EMADUT-EMIOUT
0060	WRITE(6,4) XM,PSIL,EMADUT,PSIS,EMIOUT,DIFMA
0061	4 FORMAT(6F10.2)
0062	906 CONTINUE
0063	STOP
0064	END

TABLE I-III
COMPUTER PROGRAM TO COMPARE THEORETICAL AND ACTUAL
 e_{\max} or E_{\max} FOR SINUSOIDAL INPUT

```

L.0001 /JOB GO, TIME=10
L.0002 DIMENSION DATA(292), FA(292), TA(292), FB(140), TB(140),
L.0003 XI(12), EMAX(12), ETMAX(12)
L.0004 READ(5,1) IND, IZC, IUC, VU, DATA
L.0005 1 FORMAT(3I10, F10.3/(16F5.1))
L.0006 WRITE(6,2)
L.0007 2 FORMAT('1')
L.0008 TAU=0.002
L.0009 VKT=TAU*FLOAT(IUC-IZC)/VU
L.0010 ZF=FLOAT(IZC)*TAU
L.0011 VMAX=0.
L.0012 DO 100 N=1,292
L.0013 TA(N)=FLOAT(2*N-27)*TAU/2.
L.0014 FA(N)=(DATA(N)-ZF)/VKT
L.0015 100 VMAX=AMAX1(VMAX, ABS(FA(N)))
L.0016 DO 104 J=2,13
L.0017 XJ=15-J
L.0018 TAUJ=TAU*XJ
L.0019 VKTJ=VKT*XJ
L.0020 TZCJ=ZF*XJ
L.0021 XI(J-1)=0.05/TAUJ
L.0022 ETMAX(J-1)=VEMAX(XI(J-1), 0., IND)
L.0023 ERR=0.
L.0024 K=279/(15-J)+1
L.0025 DO 102 L=1, K
L.0026 DH=0.
L.0027 LL=1+(15-J)*(L-2)
L.0028 LU=LL+14-J
L.0029 DO 101 I=LL, LU
L.0030 101 DH=DH+DATA(I)
L.0031 FB(L)=(DH-TZCJ)/VKTJ
L.0032 TB(L)=FLOAT(2*L-3)*TAUJ/2.
L.0033 VV1=TAB2(FA, TA, TB(L), 292)
L.0034 ERR=AMAX1(ABS(VV1-FB(L)), ERR)
L.0035 102 CONTINUE
L.0036 IF(IND)105,105,106
L.0037 106 ERR=0.
L.0038 DO 103 I=1,1001
L.0039 TIME=FLOAT(I-1)*0.0005
L.0040 VV1=TAB2(FA, TA, TIME, 292)
L.0041 VV2=TAB2(FB, TB, TIME, K)
L.0042 103 ERR=AMAX1(ABS(VV1-VV2), ERR)
L.0043 105 EMAX(J-1)=ERR/VMAX*100.
L.0044 104 CONTINUE
L.0045 IF(IND)107,107,108
L.0046 107 WRITE(6,3)
L.0047 3 FORMAT(10X'DATA POINT ERROR ANALYSIS')
L.0048 108 WRITE(6,4) (XI(J), EMAX(J), ETMAX(J), J=1,12)
L.0049 4 FORMAT(2(20X'ACTUAL'2X'THEORY'4X)/2(15X'M'4X'E(MAX)'2X
L.0050 1'E(MAX)'4X))/2(10X'F8.2,4X))
L.0051 STOP
L.0052 END

```

TABLE I-IV
COMPUTER PROGRAM TO FIND e_{\max} OR E_{\max} FOR A CONSTANT K AS β
VARIES FROM 0.1 TO 12.0 IN INCREMENTS OF 0.1

```

L.0001 /JOB GO
L.0002 DIMENSION ERR(150),B(150)
L.0003 C IND=0 (E(MAX) FOR DATA POINT)
L.0004 C IND=1 (E(MAX) FOR LINEAR INTERPOLATION AT ANY POINT)
L.0005 DO 105 J=1,2
L.0006 IND=2-J
L.0007 WRITE(6,2)
L.0008 2.. FORMAT('1')
L.0009 IF(IND)97,97,98
L.0010 97 IP=2101362752
L.0011 GO TO 99
L.0012 98 IP=1077952576
L.0013 99 NB=120
L.0014 DO 105 I=1,4
L.0015 XK=FLOAT(I-1)/4.
L.0016 DO 104 IB=1,NB
L.0017 B(IB)=FLOAT(IB)*0.1
L.0018 ERR(IB)=EMAXR(B(IB),XK,IND)
L.0019 104 CONTINUE
L.0020 WRITE(6,3) IP,XK,(B(IB),ERR(IB),IB=1,NB)
L.0021 3 FORMAT('// ERROR ANALYSIS FOR BETA 'A1' WITH K = 'F5.2
L.0022 1//6(4X'BETA E(MAX)'4X)//6(2F8.2,4X))
L.0023 105 CONTINUE
L.0024 STOP
L.0025 END
L.0026 FUNCTION EMAXR(BETA,XK,IND)
L.0027 DIMENSION FN(23),TN(23)
L.0028 C IND=0 (E(MAX) FOR DATA POINT)
L.0029 C IND=1 (E(MAX) FOR LINEAR INTERPOLATION AT ANY POINT)
L.0030 RAMPF(T)=AMIN1(1.,AMAX1(T-XK,0.)/BETA)
L.0031 FN(1)=0.0
L.0032 TN(1)=-0.5
L.0033 DO 100 J=2,23
L.0034 TN(J)=TN(J-1)+1$
L.0035 100 FN(J)=1.0
L.0036 TS1=XK+BETA
L.0037 IS1=TS1
L.0038 TS2=IS1
L.0039 IS2=IS1+1
L.0040 TS3=TS2-XK
L.0041 FN(2)=1.-XK-BETA/2.
L.0042 IF(IS1-1)105,101,101
L.0043 101 FN(2)=(1.-XK)**2/BETA/2.
L.0044 IF(IS1-2)104,102,102
L.0045 102 DO 103 J=3,IS2
L.0046 103 FN(J)=RAMPF(TN(J))
L.0047 104 FN(IS2+1)=TS3+1.-(BETA+TS3**2/BETA)/2.
L.0048 105 IF(IND)106,106,107
L.0049 106 EM1=ABS(RAMPF(0.5)-FN(2))
L.0050 EM2=ABS(RAMPF(TS2+0.5)-FN(IS2+1))
L.0051 GO TO 108
L.0052 107 EM1=ABS(TAB2(FN,TN,XK,13))
L.0053 EM2=ABS(TAB2(FN,TN,TS1,13)-1.)
L.0054 108 EMAXR=AMAX1(EM1,EM2)*100.
L.0055 RETURN
L.0056 END

```

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13. ABSTRACT The errors generated by the recovery of data acquired by a voltage-to-frequency (V/F) data acquisition system are theoretically analyzed, using a digital computer. The input waveforms considered are the sinusoid, the step function, and the ramp function. In each case, the well known "sampling theorem" of C. E. Shannon, viz, two samples per period of the smallest period present in the signal are sufficient for recovery of data with perfect fidelity, will be shown to be inapplicable as a criterion for accurate and reliable recovery of data acquired by a V/F system. The errors are of two categories: One is the error from linear interpolation recovery, and the other is from the fact that data points do not always coincide with the input.			

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