AD 673 799

TRANSCRITICAL DEFORMATION OF A CYLINDRICAL SHELL ON IMPACT

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Foreign Technology Division
Wright Patterson Air Force Base, Ohio

18 September 1967

FTD-HT-67-345

AD 673799

FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

TRANSCRITICAL DEFORMATION OF A CYLINDRICAL SHELL ON IMPACT

by

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FTD-HT-67-345

UNEDITED ROUGH DRAFT TRANSLATION

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English pages: 9

SOURCE: Prikladnaya Mekhanika (Applied Mechanics). Vol. 2, No. 10, 1966, pp. 29-35.

Translated under: Contract AF33(657)-16408

UR/0198-66-002-010 TP7001608

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ITIS INDEX CONTROL FORM

01 Acc Nr	68 Translation Nr	65 X Ref Acc Nr	76 Reel/Frame Nr
TP7001608	HT6700345	AP6035492	1881 1365

97 Header Clas	63 Clas	64 Control Markings	94 Expansion	40 Ctry Info
UNCL	UNCL, 0	0		UR

02 Ctry	03 Ref	04 Yr	05 Vol	06 Iss	07 B. Pg.	45 E. Pg.	10 Date
UR	0198	66	002	010	0029	0035	NONE

Transliterated Title
ZAKRITICHESKAYA DEFORMATSIYA TSILINDRICHESKOY OBOLOCHKI PRI UDARE

09 English Title
TRANSCRITICAL DEFORMATION OF A CYLINDRICAL SHELL ON IMPACT

43 Source
PRIKLADNAYA MEKHANIKA (RUSSIAN)

42 Author
BORISENKO, V. I.

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KLOKOVA, A. I.

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NONE

16 Co-Author
NONE

16 Co-Author
NONE

98 Document Location

47 Subject Codes
20

39 Topic Tags: cylindric shell structure, shell deformation, impact test, elastic deformation,

ABSTRACT : The axisymmetrical elastic deformation of a circular cylindrical shell under longitudinal impact is investigated by using a system of nonlinear equations with the propagation of elastic stress waves taken into account, and without any assumptions concerning the mode of buckling. One end of the shell is fixed, the other end is axially impacted by a rigid solid moving at a velocity V; the ratio m of the mass of the body to the mass of the shell is given. The analysis of the impact-deformation process in this shell is reduced to solving this nonlinear system with initial and boundary conditions by the method of finite differances, utilizing an explicit scheme whose convergence and stability was checked. The behavior of the shell was studied in the time interval in which the longitudinal compression wave propagates along the whole length of the shell, and the first reflected wave comes back. The results from calculating the normal displacements along the shell at various instants of both waves propagating, for the ratio m = 3.64 and nondimensional velocities V/a = 0.0005; 0.001; 0.002; and 0.004 (where a is the velocity of sound) are shown in diagrams and are examined. The qualitative aspect of the shell deformation, especially the formation of maximum local displacements during the passage of both the compression and the reflected waves as related to V is discussed and found to be in agreement with the A. Koppa phenomenological theory based on experimental results. Orig. art. has: 5 figures and 13 formulas. English translation: 9 pages.

TRANSCRITICAL DEFORMATION OF A CYLINDRICAL SHELL ON IMPACT

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On the basis of nonlinear equations which take account of shear and rotational inertia, the axially symmetric elastic deformation of a circular cylindrical shell in longitudinal impact is considered.

The deformation of the cylindrical shell in longitudinal impact is investigated on the basis of wave equations without resorting to complementary assumptions concerning the buckling configuration of the shell. It is assumed that one edge of the shell is supported while the other is subject to longitudinal axially symmetric impact by a rigid body moving at the velocity V_0.

Experimental studies of the process of loss of stability of a cylindrical shell on impact [5] showed that the wave nature of stress propagation in the shell markedly affects the nature of the loss of stability. In this connection, most of the studies of theory of this problem [1, 3] analyze the buckling process on the basis of systems of equations of parabolic type which make no allowance for the process of wave propagation in the shell. Moreover, the solution of the equations normally is associated with specified buckling configurations.

To investigate the process of the deformation of the shell on impact we will employ the equations derived by M.P. Galin [4]. For elastic deformation these equations are given in the monograph [7] on adding terms that take account of the geometric nonlinearity. The equations pertaining to the axially symmetric case are:

$$\begin{gathered}\frac{\partial T_{11}}{\partial x}-\frac{\partial}{\partial x}(\varphi Q_1)=\varrho h\frac{\partial^2 u}{\partial t^2};\\ \frac{\partial Q_1}{\partial x}+\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}T_{11}\right)-\frac{1}{R}T_{22}=\varrho h\frac{\partial^2 w}{\partial t^2};\\ \frac{\partial M_{11}}{\partial x}-Q_1=-\frac{\varrho h^3}{12}\frac{\partial^2\varphi}{\partial t^2}.\end{gathered}\qquad(1)$$

where the notation is the same as in [7].

The stress-strain ratios are as follows:

$$T_{11}=\frac{Eh}{1-\nu^2}\left[\frac{\partial u}{\partial x}+\frac{\nu}{R}w+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right];\qquad Q_1=hGk^2\left(\frac{\partial w}{\partial x}-\varphi\right);$$
$$T_{22}=\frac{Eh}{1-\nu^2}\left\{\frac{w}{R}+\nu\left[\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]\right\};\qquad M_{11}=-\frac{Eh^3}{12(1-\nu^2)}\frac{\partial\varphi}{\partial x}. \tag{2}$$

Equations (1) make it possible to investigate the propagation of longitudinal and transverse perturbations in the shell, which cannot be done by proceeding from the classical equations of the shell theory based on the Kirchhoff-Love hypothesis. These relations were derived [4] from general nonlinear equations of the theory of elasticity by expanding the displacements into a series with respect to the power of z (the z-axis is directed at right angles to the median surface of the shell) and preserving two terms of the series for tangential displacements and one term for normal displacement. There also exist other, more rigorous methods of reducing to two dimensions the three-dimensional problems of the theory of elasticity [5].

In the formula for stress Q_1 from (2) the coefficient $k^2 =$ $= 0.86$ is introduced in accordance with [8].

Following the substitution of Relations (2) in (1) we have the system of differential equations

$$\frac{\partial^2\tilde{u}}{\partial\tau^2}=\frac{\partial^2\tilde{u}}{\partial\alpha^2}+\left(\nu+\frac{\partial^2\tilde{w}}{\partial\alpha^2}-\delta\frac{\partial\varphi}{\partial\alpha}\right)\frac{\partial\tilde{w}}{\partial\alpha}+\delta\varphi\left(2\frac{\partial\varphi}{\partial\alpha}-\frac{\partial^2\tilde{w}}{\partial\alpha^2}\right);$$
$$\frac{\partial^2\tilde{w}}{\partial\tau^2}=\left(\delta+\frac{\partial\tilde{u}}{\partial\alpha}+\nu\tilde{w}\right)\frac{\partial^2\tilde{w}}{\partial\alpha^2}+\frac{\partial\tilde{w}}{\partial\alpha}\left(\frac{\partial^2\tilde{u}}{\partial\alpha^2}+\frac{\nu}{2}\frac{\partial\tilde{w}}{\partial\alpha}\right)-\tilde{w}-\delta\frac{\partial\varphi}{\partial\alpha}-\nu\frac{\partial\tilde{u}}{\partial\alpha}; \tag{3}$$
$$\frac{\partial^2\varphi}{\partial\tau^2}=\frac{\partial^2\varphi}{\partial\alpha^2}+\frac{\delta}{c^2}\left(\frac{\partial\tilde{w}}{\partial\alpha}-\varphi\right),$$

where $\alpha = x/R$, $\tau = at/R$, $\tilde{w} = w/R$ are a dimensionless coordinate, time and elastic displacement, respectively;

$$\tilde{u}=u/R;\qquad \delta=\frac{k^2(1-\nu)}{2}.$$

Assume that the edges of one side of the shell are rigidly fastened. Then the fastened side must satisfy the conditions:

$$\tilde{u}=\tilde{w}=\varphi=0,\qquad (\alpha=0). \tag{4}$$

The side subjected to the impact, on the other hand, must satisfy not only the two conditions

$$\tilde{w}=\varphi=0,\qquad (\alpha=\mu), \tag{5}$$

but also the condition that will obtain if the force of inertia acting on the striking body is equal to the longitudinal stress acting on the edge of the shell

$$M\frac{\partial^2 u}{\partial t^2} = -2\pi R T_{11}. \qquad (6)$$

In Eqs. (5) and (6) $\mu = l/R$ is the dimensionless length of the shell and M is the mass of the striking body.

Using Relation (2) for T_{11} and passing on to dimensionless coordinates and displacements, on taking account of (5) we have

$$\mu m \frac{\partial^2 \tilde{u}}{\partial \tau^2} = -\left[\frac{\partial \tilde{u}}{\partial \alpha} + \frac{1}{2}\left(\frac{\partial \tilde{w}}{\partial \alpha}\right)^2\right], \quad (\alpha = \mu). \qquad (7)$$

where m is the ratio of the mass of the striking body to the mass of the shell.

Considering that prior to the impact the shell was in a state of rest, we write the initial conditions as

$$\tilde{u} = 0; \quad 0 < \alpha < \mu, \quad (\tau = 0); \quad \frac{\partial \tilde{u}}{\partial \tau} = 0; \quad 0 < \alpha < \mu, \quad (\tau = 0);$$
$$\frac{\partial \tilde{u}}{\partial \tau} = -\frac{V_0}{a}; \quad \alpha = \mu, \quad (\tau = 0); \quad \tilde{w} = \frac{\partial \tilde{w}}{\partial \tau} = \varphi = \frac{\partial \varphi}{\partial \tau} = 0; \quad (\tau = 0). \qquad (8)$$

The investigation of the process of deformation of a rigidly attached cylindrical shell subjected to a longitudinal impact by a rigid body reduces to the solution of Eqs. (3) with initial (8) and boundary (4), (5), (7) conditions. In accordance with the Cauchy-Kovalevskaya theorem this problem has a solution that appears to be unique. Such a circumstance makes it possible to investigate the process of deformation of a shell lacking any initial irregularities. The solution will be sought in numerical form with the aid of the net method. We utilize the following difference relations:

$$\frac{\partial \psi}{\partial \beta} = \frac{\psi_{k+1,n} - \psi_{k-1,n}}{2\Delta};$$
$$\frac{\partial^2 \psi}{\partial \beta^2} = \frac{\psi_{k+1,n} - 2\psi_{k,n} + \psi_{k-1,n}}{\Delta^2}, \qquad (9)$$

where ψ is the differentiable function, β is an independent variable, Δ is a mesh of the net with respect to the variable β, and $\psi_{i,j}$ is the value of the function ψ at the net node numbered i, j.

In the region of the variables τ, α we isolate individual points — nodes of the net with a uniform mesh spacing, as is shown in Fig. 1. The mesh with respect to the variable α is denoted as l and the mesh with respect to the variable τ, as h. The subscript i, denoting the number of the vertical line (α = const) ranges in value from zero to k ($\alpha = \mu$). The subscript j, denoting the number of the horizontal line (τ = const) ranges in value from 0 to N, where N depends on the interval of time over which the solution is sought.

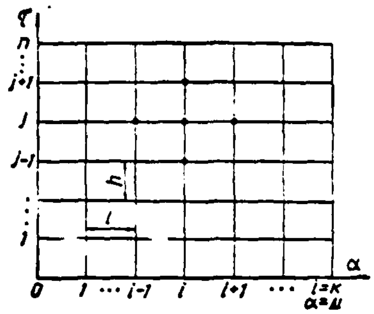

Fig. 1

With the aid of Relations (9) we rewrite System (3) as

$$\begin{aligned}
u_{i,j+1} &= 2\left(1-\frac{h^2}{l^2}\right)u_{i,j} - u_{i,j-1} + \frac{h^2}{l^2}(u_{i+1,j}+u_{i-1,j}) + \\
&+ \frac{h^2}{2l}(w_{i+1,j}-w_{i-1,j})\left[\nu + \frac{1}{l^2}(w_{i+1,j}-2w_{i,j}+w_{i-1,j}) - \right. \\
&\left. - \frac{\delta}{2l}(\varphi_{i+1,j}-\varphi_{i-1,j})\right] + \frac{\delta h^2}{l}\varphi_{i,j}\left[\varphi_{i+1,j}-\varphi_{i-1,j} - \right. \\
&\left. - \frac{1}{l}(w_{i+1,j}-2w_{i,j}+w_{i-1,j})\right]; \qquad (10) \\
w_{i,j+1} &= (2-h^2)w_{i,j} - w_{i,j-1} + \frac{h^2}{l^2}(w_{i+1,j}-2w_{i,j}+w_{i-1,j}) \times \\
&\times\left[\delta + \frac{1}{2l}(u_{i+1,j}-u_{i-1,j}) + \nu w_{i,j}\right] + \frac{h^2}{2l^2}(w_{i+1,j}-w_{i-1,j}) \times \\
&\times\left[\frac{1}{l}(u_{i+1,j}-2u_{i,j}+u_{i-1,j}) + \frac{\nu}{4}(w_{i+1,j}-w_{i-1,j})\right] - \\
&- \frac{\delta h^2}{2l}(\varphi_{i+1,j}-\varphi_{i-1,j}) - \frac{h^2\nu}{2l}(u_{i+1,j}-u_{i-1,j}); \\
\varphi_{i,j+1} &= 2\left(1-\frac{h^2}{l^2}-h^2\frac{\delta}{2c^2}\right)\varphi_{i,j} + \frac{h^2}{l^2}(\varphi_{i+1,j}-\varphi_{i-1,j}) - \\
&- \varphi_{i,j-1} + \frac{\delta h^2}{2lc^2}(w_{i+1,j}-w_{i-1,j}).
\end{aligned}$$

Thus the known values of the functions u, w and ψ in the $(j-1)$th and jth rows can be used to determine from Formulas (10) the values of these functions in the $(j+1)$th row. Such difference schemes have become termed explicit [2]. As is known, the advantage of explicit difference schemes over their implicit counterparts lies in the simplicity of the computational algorithm. However, explicit difference schemes do not always display computational stability. This stability largely depends on the selected ratio between the net meshes and their dimensions.

Hence, to select the ratios between the dimensions and meshes of the net, the linear parts of difference equations (10) were subjected to a numerical analysis on a digital computer with the aid of the so-called ε-scheme [2]. As a result, it turned out that the linear part of difference scheme (10) is unsuitable for computation, since the rounding-out error rapidly increases with transition from one row of the mesh to another. One of the reasons for this increase in error is the presence of high coefficients in the right-hand parts of Relations (10). To reduce these coefficients the substitution

$$\varphi_{i,j} \approx \frac{1}{2}(\varphi_{i,j+1} + \varphi_{i,j-1}). \tag{11}$$

was carried out in the third formula of (10).

Subsequent calculations on the basis of the ε-scheme showed that in the presence of ε the coefficients increase insignificantly with transition from one row of the nodes to the next. Further verification of the convergence and stability of the difference scheme consisted in that the necessary calculations were carried out on successively reducing the net meshes until the findings obtained before and after the reduction of the mesh coincided to specified degree of accuracy.

With the aid of initial conditions (8) we determine the values of the sought functions in the first two horizontal rows of the net (cf. Fig. 1)

$$\begin{aligned} &j=0;\ u_{i,0}=w_{i,0}=\varphi_{i,0}=0, \quad (i=0,1,\ldots,k);\\ &j=1;\ w_{i,1}=\varphi_{i,1}=0, \quad (i=0,1,\ldots,k);\\ &u_{i,1}=0,\ (i=0,1,\ldots,k-1);\ u_{k,1}=-\frac{V_0}{a}h. \end{aligned} \tag{12}$$

Boundary conditions (4), (5), (7) make it possible to locate the values of these functions at the vertical-line nodes $i = 0$ and $i = k$ (cf. Fig. 1)

$$\begin{aligned} &i=0;\ u_{0,j}=w_{0,j}=\varphi_{0,j}=0;\ i=k;\ w_{k,j}=\varphi_{k,j}=0;\\ &u_{k,j+1}=\left(2-\frac{h}{m\mu}\right)u_{k,j}-u_{k,j-1}+\frac{h}{m\mu}\left(u_{k-1,j}-\frac{w_{k-1,j}^2}{2h}\right). \end{aligned} \tag{13}$$

The difference scheme (10), (12), (13) was used to perform on a computer calculations whose results are illustrated in the diagrams.

The behavior of the shell was investigated over the time of the passage of the longitudinal compression wave, propagating after the impact from the struck end of the shell, and the first reflected compression wave spreading from the fastened end.

The values of the parameters entering into initial conditions (3), boundary conditions (7) and initial conditions (8) were taken as: $m = 3.64$; $\sigma^2 = 0.48 \cdot 10^{-6}$; $\mu = 2.4$; $V_0/a = 0.0005$, 0.001; 0.002, 0.004.

To verify the computational stability of the difference

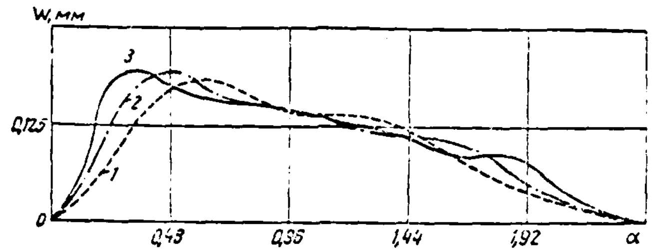

Fig. 2.

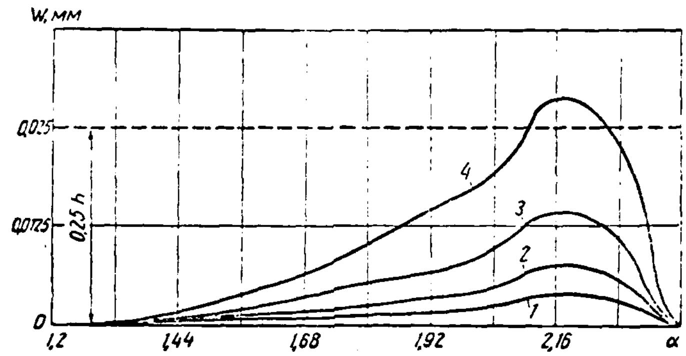

Fig. 3.

scheme, the decrease in the net mesh h with time was calculated. The initial mesh values were taken as $h = l = 0.03$. It turned out that a five-fold decrease in h results in an insignificant change in the unknown functions (less than 1%). This demonstrates the computational stability of the difference scheme employed.

To verify the convergence of the difference scheme, the decrease in the net meshes h and l was calculated. Figure 2 shows the values of w at the instant $\tau = 4.8$ (reflected compression wave reaches the originally struck side of the shell). These values were obtained for $V_0/a = 0.004$ with the following mesh values: $h = l = 0.12$ (curve 1), $h = l = 0.06$ (curve 2), $h = l =$ $= 0.03$ (curve 3).

As can be seen from the plot (cf. Fig. 2), the decrease in net meshes does not markedly change the picture of transverse deformation of the shell, which confirms the convergence of the difference scheme. It may therefore be assumed that the solution obtained on using a net with the meshes $h = l = 0.03$ reflects with sufficient accuracy the magnitude of the maximum flexure, the pattern of variation in flexure over the length of the shell and the process of its variation in time.

The plots in Figs. 3, 4, 5 present the flexure function w in relation to the coordinate α at the time instants when the longitudinal compression wave propagated as far as one-half of the shell's length (Fig. 3, $\tau = 1.2$) and throughout the shell (Fig. 4, $\tau = 2.4$) and at the time instant when the reflected compression wave reached the originally struck edge of the shell (Fig. 5, $\tau = 4.8$).

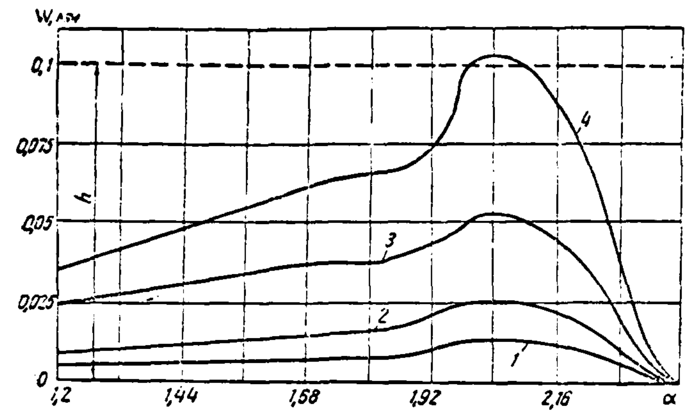

Fig. 4.

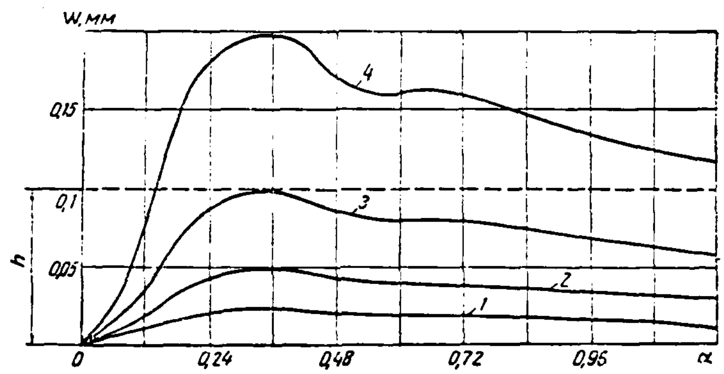

Fig. 5.

Curves 1, 2, 3, 4 correspond to the following values of the dimensionless velocities of the striking body: $V_0/a = 0.0005$, 0.001, 0.002, 0.004.

As can be seen from these plots, during the passage of the first longitudinal compression wave the greatest growth in transverse displacements w is observed in the neighborhood of the originally struck edge of the shell. Once the longitudinal wave gets reflected and begins to propagate from the supported side of the shell, however, the transverse deformations at this latter side grow at a faster rate. The local nature of intense transverse displacements is the more distinct the greater the velocity of the striking body is.

Thus, the qualitative picture of deformation of the shell following a longitudinal impact is as follows: as the longitudinal compression wave following the impact propagates throughout the shell, it brings transverse deformations in its wake, and the region of maximum transverse displacements encompasses the part of the shell adjoining the originally struck edge. This region becomes increasingly distinct with increase in the dimensionless velocity of the striking body.

Throughout the second stage of passage of the longitudinal compression wave transverse buckling decreases in the neighborhood of the originally struck edge and increases in the neighborhood of the supported edge. This occurs owing to the superposition of the reflected compression wave on the direct compression wave at the supported end and hence also owing to the attendant marked increase in longitudinal stresses at the supported end. Toward the instant when the reflected compression wave reaches the originally struck edge of the shell, a region of maximum transverse displacement occurs at the supported edge, by analogy with the picture that prevailed during the propagation of the direct compression wave.

The formation of local regions of maximum transverse displacements adjoining either one of the sides of the shell is in qualitative agreement with the postulates of the phenomenological theory of A. Koppa [6] based on experimental findings.

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