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THE RESISTANCE OF A FLAT PLATE LOCATED NORMAL
TO THE FLOW OF GREATLY RAREFIED GAS

By

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THE RESISTANCE OF A FLAT PLATE LOCATED NORMAL TO THE FLOW OF GREATLY RAREFIED GAS

By: V. A. Perepukhov

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ABSTRACT: A method is presented for calculating the aerodynamic properties of a flat plate of any given shape at arbitrary values of the accommodation coefficient in a rarefied gas flow. First-time collisions of free-stream and reflected molecules are considered, with the effect of attenuation of the reflected flow on free-stream flow taken into account. The presence of the arbitrary accommodation coefficient means that the velocity of a reflected molecule cannot be neglected at the time of collision with a free-stream molecule, and this leads to a complex collision integral. The method is based on calculating the function of the effect of molecules reflected from an elementary surface upon the aerodynamic characteristics of the other elementary surface. A detailed scheme for computer calculation by means of the Monte Carlo method was developed. The results of numerical calculations of the aerodynamic characteristics of a square plate for various values of the accommodation coefficient $\alpha_A^* = 1, 1/2, 1/32,$ and $1/128$ are presented as an illustrative example. Original article has: 7 figures, 33 formulas, and 3 tables.
English translation: 14 pages.

U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

* ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

FOLLOWING ARE THE CORRESPONDING RUSSIAN AND ENGLISH DESIGNATIONS OF THE TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}
rot	curl
lg	log

THE RESISTANCE OF A FLAT PLATE LOCATED NORMAL TO THE FLOW OF GREATLY RAREFIED GAS

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Given in the work is a method of calculation of aerodynamic properties of a plate of arbitrary form with an arbitrary value of the coefficient of accommodation in the flow of greatly rarefied gas, taking into account first collisions between molecules of reflected and incident flows. As an example calculations of the aerodynamic properties of a square are given.

1. Introduction. In works [1-8] there was investigated the streamline flow of bodies of different form possessing a coefficient of accommodation $\alpha_A^* = 0$ in the range of the so-called hyperthermal theory, taking into account first intermolecular collisions. Let us recall the basic positions of this theory.

At very large macroscopic speeds of flows of greatly rarefied gas, the characteristic speed in the flow is the mean thermal velocity of the molecules $V_T = \sqrt{2RT_\infty}$ and not the speed of sound, as it was in the hypersonic theory. Therefore, instead of the dimensionless number M the new number $S_\infty = U_\infty/\sqrt{2RT_\infty}$ is introduced.

Let us assume that molecules of the flow incident on the body possess macroscopic velocity U_∞, much greater than the mean thermal velocity, i.e., $S_\infty \gg 1$. Not introducing gross errors into further calculations, it is possible to assume that the incident flow consists of molecules moving in parallel to each other at macroscopic speed U_∞ (such an assumption is acceptable when $S_\infty \geq 5$).

Let us assume that the temperature of the body is low because of heat removal so that $T_w \simeq T_\infty$; since the coefficient of accommodation $\alpha_A^* \approx 0$ the speed of the reflected molecules has an order of thermal velocity of molecules of incident flow, which permits disregarding the latter at the time of collision of the molecule of incident flow with the reflected speed as compared to the speed U_∞.

In works carried out earlier there is proposed the following method of calculation of aerodynamic properties of a body. On the surface of the body an element of the surface dF was selected, and there was calculated a change of some molecular criterion arriving on the element dF because of collisions of molecules of incident flow with molecules reflected from the whole surface of the body

$$\dot{\Pi}_{jdF} = \Pi_{0jdF} + \dot{\Pi}_{(+)jdF} - \Pi'_{(-)jdF},$$

where Π_{jdF} — flow of the j-th molecular sign on element dF, Π_{0j} — flow of the j-th molecular criterion on element dF during freely molecular streamline flow, $\Pi_{(+)j}$ — flow of the j-th molecular sign arriving on the element dF due to collision of molecules of incident flow with the reflected, $\Pi_{(-)j}$ — flow of j-th molecular sign which because of collisions of molecules of incident flow with the reflected does not fall on the element dF.

In this work the solution of another problem with the help of a calculation different from the preceding method is proposed.

2. Formulation of problem. Let us consider the streamline flow of a flat plate of arbitrary form in the plan possessing an arbitrary

coefficient of accommodation α_A^* and arbitrary temperature T_w and a flow of greatly rarefied gas.

The law of reflection is best to select on the basis of the experiment, but since up to now there has been no reliable experimental data then we will select the classical law of reflection, diffuse the Maxwellian function of distribution of reflected molecules and temperature T_0 (i.e., with energy E_0):

$$f_0 = n_0\left(\frac{h_0}{\pi}\right)^{3/2} e^{-h_0 v_0^2}, \quad h_0 = \frac{m}{2kT_0}.$$

The presence of the arbitrary coefficient of accommodation leads to the fact that at the moment of collision of the molecule of incident flow with the reflected speed the latter cannot be neglected, which in its turn leads to the complication of the integral of collisions. Thus as earlier, we will assume that the molecules are solid balls with a diameter σ.

Of all the forms of possible collisions of molecules among themselves belonging to different flows, we will consider only the collisions, of molecules of incident flow with reflected flow, considering the effect of damping of the reflected flow on the incident.

Taking into account those difficulties which are found in the solution of the preceding problems and the laboriousness of the method of calculation, in this problems we will use a somewhat different method founded on the calculation of the influence function for some molecular criterion of one element of the surface on another [6].

Let us examine the small, as compared to the surface of the body, element of the surface dF_1 and calculate what is influence of molecules reflected from this element on aerodynamic properties of another element of the surface dF_j. By knowing the influence function of element dF_i on the remaining elements of the plane, one can determine (in virtue of linearity of the problem) the influence of all elements entering into the plate of a given configuration on the element of plate dF_i, and then by adding the flow of the corresponding molecular sign on the whole plate we will obtain the total flow of some molecular sign for the whole surface of the body.

3. Diagram of solution of the problem on a computer. We will solve this problem with the application of the Monte-Carlo method, which in preceding works was used for the calculation of the integral of collision.

Let us fix in space near the element dF_i an arbitrary point $A(X, Y, Z)$, then select an velocity vector arbitrary in magnitude of reflected molecules v_0; its direction is determined by coordinates of the element and point $A(X, Y, Z)$.

Let us assign further the impact parameters Ψ and ε (Fig. 1) with respect to the direction of speed $g = U_\infty - v_0$ and determine by it the directing cosines of the line of centers at the time of collision of the reflected molecule with the molecule of incident flow (in the system of coordinates connected with the element dF_i). Then the quantity of collisions in the element of volume at point A is such that the reflected molecules possess speed in the element dV, and the line of centers, found in the solid angle $d\Psi$, will be equal to

$$K_1 = \sigma^2 n_\infty n_0 \left(\frac{h_0}{\pi}\right)^{3/2} e^{-h_0 v_0^2} g v_0^2 e^{-\frac{r}{v_0} \pi \sigma^2 n_\infty g} \times$$
$$\times \frac{d\tau}{r^2} \cos\mu \, dv \, d\mu \, d\beta \sin\Psi \cos\Psi \, d\Psi \, d\varepsilon \, dX \, dY \, dZ,$$

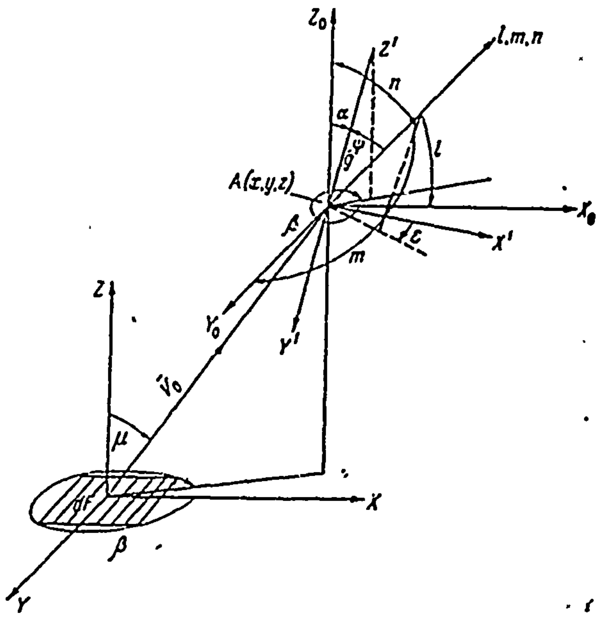


Fig. 1.

where $g^2 = v_0^2 + U_\infty^2 + 2v_0U_\infty \cos\mu$

$$X = r \sin\mu \cos\beta,\ Y = r \sin\mu \sin\beta,\ Z = r \cos\mu.$$

The expression for n_0 can be obtained from the law of preservation of the number particles on the surface of the plate

$$n_0 = 2n_\infty U_\infty (\pi h_0)^{1/2}.$$

The direction cosines of the line of centers with respect to the system of coordinates connected with element dF_i can be written thus

$$\begin{aligned} l(x) &= \cos\Psi \cos\beta \sin\alpha + \sin\Psi \cos\alpha \cos\beta \cos\varepsilon - \sin\Psi \sin\varepsilon \sin\beta; \\ m(y) &= \cos\Psi \sin\beta \sin\alpha + \sin\Psi \cos\alpha \sin\beta \cos\varepsilon + \sin\Psi \sin\varepsilon \cos\beta; \\ n(z) &= \cos\Psi \cos\alpha - \sin\alpha \sin\Psi \cos\varepsilon, \end{aligned}$$

where

$$\cos\alpha = \frac{v_z + U_\infty}{g}, \quad \cos\beta = \frac{X}{\sqrt{X^2 + Y^2}}.$$

The variables of problem have the following range of variation:

$$0 \leqslant \Psi \leqslant \pi/2,\ 0 \leqslant \mu \leqslant \pi/2,\ 0 \leqslant \varepsilon \leqslant 2\pi,$$
$$0 \leqslant \beta \leqslant 2\pi,\ 0 < r < \infty,\ 0 < v_0 < \infty.$$

In the expression for the number of collisions the recording used for the solid angle $r^{-2}dF \cos \mu$ is accurate, strictly speaking, when $\sqrt{dF} \ll r$, i.e., with a tendency $r \to 0$ dF it should approach zero faster. However, in our case dF is the fixed magnitude and with a tendency $r \to 0$ the solid angle is equal to ∞. In order that this not be true, it is possible, not exceeding the bounds of accuracy of the method, to record the solid angle in the following form:

$$d\Omega = f(r) \cos \mu, \quad \text{where} \quad f(r) = \begin{cases} 2\pi \left(\frac{1}{r^2} > 2\pi\right) \\ \frac{1}{r^2} \left(\frac{1}{r^2} < 2\pi\right) \end{cases}.$$

All the variables of the problem for every collision can be selected equiprobable within limits of a change of each variable.

Above it was stated that for every element of surface dF_j it is necessary to determine two forms of flow of the n-th molecular sign with the index (+) and (-). Flows with the (+) index correspond to flows arriving on the element dF_j due to collision of the reflected and incident molecules at an arbitrarily selected point of space (X_m, Y_m, Z_m). Flows with the (-) index correspond to those flows of molecular signs which do not attain the element dF_j due to collisions of reflected and incident molecules at points (X_m, Y_m, Z_m), i.e., at points located above the element dF_j.

If we want to determine the flow of some molecular sign arriving on element dF_j as a result of dispersion of molecules of incident flow on reflected flow from the element dF_j, we should multiply the number of collisions K_i by the appropriate weight $R_{n(\pm)}$. For the flow of mass, the flow of normal pulse, and the energy flow expressions for $R_{n(+)}$ and $R_{n(-)}$ have the following form:

$$R_{1(+)} = m, \quad R_{2(+)} = mv_z^{(1,2)}, \quad R_{3(+)} = \frac{m}{2} v^{2(1,2)},$$
$$R_{1(-)} = m, \quad R_{2(-)} = mU_\infty, \quad R_{3(-)} = \frac{m}{2} U_\infty^2,$$

where the indices 1, 2, 3 below correspond to the above-mentioned flows, and indices (1, 2) above correspond to two molecules appearing as a result of the collision. It is possible to obtain an expression for speeds of molecules after collision depending upon the value of these speeds up to the collision and impact parameters

$$v_x^{(1)} = v_0 \sin\mu\cos\beta - l\omega, \quad v_y^{(1)} = v_0 \sin\mu\sin\beta - m\omega, \quad v_z^{(1)} = v_0\cos\mu - n\omega,$$
$$v_x^{(2)} = l\omega, \quad v_y^{(2)} = m\omega, \quad v_z^{(2)} = -U_\infty + n\omega,$$
$$\omega = lv_0\sin\mu\cos\beta + mv_0\sin\mu\sin\beta + n(U_\infty + v_0\cos\mu),$$

l, m, n are direction cosines of the line of centers at the time of collision in the system of coordinates connected with the element of dF_i.

After the collision of the trajectory of the molecules one can determine the direction cosines in the system of coordinates with the center at point $A(X_m, Y_m, Z_m)$:

$$\lambda_x^{(1)} = \frac{v_x^{(1)}}{\sqrt{v_x^{2(1)} + v_y^{2(1)} + v_z^{2(1)}}}, \qquad \lambda_x^{(2)} = \frac{v_x^{(2)}}{\sqrt{v_x^{2(2)} + v_y^{2(2)} + v_z^{2(2)}}},$$
$$\nu_y^{(1)} = \frac{v_y^{(1)}}{\sqrt{v_x^{2(1)} + v_y^{2(1)} + v_z^{2(1)}}}, \qquad \nu_y^{(2)} = \frac{v_y^{(2)}}{\sqrt{v_x^{2(2)} + v_y^{2(2)} + v_z^{2(2)}}},$$
$$\tau_z^{(1)} = \frac{v_z^{(1)}}{\sqrt{v_x^{2(1)} + v_y^{2(1)} + v_z^{2(1)}}}, \qquad \tau_z^{(2)} = \frac{v_z^{(2)}}{\sqrt{v_x^{2(2)} + v_y^{2(2)} + v_z^{2(2)}}},$$

if $v_z^{(2)} < 0$ or $v_z^{(1)} < 0$, then

$$X_1 = X_m + \frac{\lambda_x^{(1)}}{\tau_z^{(1)}} Z_m, \quad X_2 = X_m + \frac{\lambda_x^{(2)}}{\tau_z^{(2)}} Z_m,$$
$$Y_1 = Y_m + \frac{\nu_y^{(1)}}{\tau_z^{(1)}} Z_m, \quad Y_2 = Y_m + \frac{\nu_y^{(2)}}{\tau_z^{(2)}} Z_m,$$

i.e., after the given collision the molecule will arrive in elements dF_1 and dF_2 with the corresponding speeds $v^{(1)}$ and $v^{(2)}$.

Thus it is possible to construct completely the form of the influence function with weight R_n of element dF_i on any element of plane dF_j, after which the calculation of integral aerodynamic properties is not difficult.

4. Results of calculation. A calculation was produced for the influence functions corresponding to the particle flux, to the flux of the normal pulse, and to the energy flux. With this the influence was determined of the square element on a certain quantity of other elements, so that as a result of the calculation it became to determine the aerodynamic properties of a flat plate arbitrary form inscribed into square (11 × 11).

A calculation was produced for the following values of the coefficient $\alpha_A^* = 1, 1/2, 1/32, 1/128$, where

$$\alpha_A^* = 8RT_w/U_\infty^2.$$

The corresponding influence functions with a different value of α_A^* are given in Tables 1-3. Results of the calculation when $\alpha_A^* = 1/128$ practically do not differ from results of the calculation when $\alpha_A^* = 1/32$. In order to obtain a correction for some flow of molecular sign arriving on element dF_j it is necessary to add the influence functions for the corresponding criteria

$$\Delta\Phi_i^{(k)} = dF_i \sum_j W_{ij}^{(k)} dF_j.$$

Here the Knudsen number $Kn_\infty = (2\pi\sigma^2 n_\infty)^{-1}$, since the characteristic dimension is equal to 1 (unit of length is the side of the unit square element).

$$W_{ij}^{(1)} = mn_\infty U_\infty \frac{2}{Kn_\infty} \Delta N_{ij}, \quad W_{ij}^{(2)} = \frac{mn_\infty U_\infty^2}{2} \frac{2}{Kn_\infty} \Delta P_{ij},$$
$$W_{ij}^{(3)} = \frac{mn_\infty U_\infty^3}{2} \frac{2}{Kn_\infty} \Delta E_{ij},$$

which corresponds to the fluxes of molecules, normal pulse, and energy.

Table 1

$\overset{\bullet}{\alpha}_A = 1$

Number of cell	$\Delta N = N(-) - N(+)$	$\Delta P = P(-) - P(+)$	$\Delta E = E(-) - E(+)$
0; 0	0,4355	1,2817	0,6213
1; 0	0,1148	0,4091	0,1511
1; 1	0,0316	0,2031	0,1121
2; 0	0,0403	0,1582	0,0311
2; 1	0,0236	0,1202	0,0122
2; 2	0,0305	0,0581	0,0150
3; 0	0,0035	0,0769	0,0133
3; 1	0,0031	0,0651	0,0134
3; 2	0,0011	0,0552	0,0125
3; 3	0,0012	0,0173	0,0113
4; 0	0,0161	0,0159	0,0121
4; 1	0,0055	0,0151	0,0115
4; 2	0,0032	0,0385	0,0012
4; 3	0,0025	0,0276	0,0051
4; 4	−0,0052	0,0112	0,0001
5; 0	0,0005	0,0277	0,0012
5; 1	0,0001	0,0258	−0,0012
5; 2	−0,0027	0,0185	−0,0014
5; 3	−0,0068	0,0072	−0,0012
5; 4	−0,0055	0,0157	−0,0007
5; 5	−0,0036	0,0038	−0,0011
6; 0	−0,0052	0,0031	−0,0005
6; 1	−0,0077	0,0033	−0,0005
6; 2	−0,0064	0,0054	−0,0015
6; 3	−0,0044	0,0085	−0,0018
6; 4	−0,0030	0,0023	−0,0013
6; 5	−0,0060	0,0017	−0,0031
6; 6	−0,0053	0,0028	−0,0021
7; 0	−0,0035	0,0103	−0,0010
7; 1	−0,0036	0,0033	−0,0011
7; 2	−0,0043	0,0059	−0,0015
7; 3	−0,0058	0,0075	−0,0023
7; 4	−0,0076	0,0026	−0,0034
7; 5	−0,0013	0,0025	−0,0018
7; 6	−0,0035	0,0027	−0,0009
7; 7	−0,0116	0,0012	−0,0016
8; 0	−0,0179	0,0026	−0,0013
8; 1	−0,0176	0,0025	−0,0019
8; 2	−0,0065	0,0027	−0,0021
8; 3	−0,0012	0,0025	−0,0019
8; 4	−0,0032	0,0031	−0,0012
8; 5	−0,0039	0,0023	−0,0018
8; 6	−0,0050	0,0003	−0,0026
8; 7	−0,0057	−0,0008	−0,0021
8; 8	−0,0056	−0,0017	−0,0032
9; 0	−0,0031	0,0031	−0,0011
9; 1	−0,0032	0,0029	−0,0011
9; 2	−0,0035	0,0027	−0,0010
9; 3	−0,0012	0,0022	−0,0007
9; 4	−0,0017	0,0013	−0,0016
9; 5	−0,0034	0,0001	−0,0012
9; 6	−0,0050	−0,0014	−0,0023
9; 7	−0,0055	−0,0016	−0,0031
9; 8	−0,0015	−0,0013	−0,0025
9; 9	−0,0033	−0,0002	−0,0025
10; 0	−0,0031	0,0050	−0,0036
10; 1	−0,0050	0,0007	−0,0035
10; 2	−0,0033	0,0034	−0,0048
10; 3	−0,0036	−0,0033	−0,0015
10; 4	−0,0050	−0,0013	−0,0025
10; 5	−0,0050	−0,0019	−0,013
10; 6	−0,0051	−0,0014	−0,0028
10; 7	−0,0013	−0,0053	−0,0025
10; 8	−0,0032	−0,0001	−0,0023
10; 9	−0,0035	−0,0002	−0,0027
10; 10	−0,0041	−0,0003	−0,0019

Table 2

$\overset{\bullet}{\alpha}_A = 1/2$

Number of cell	$\Delta N = N(-) - N(+)$	$\Delta P = P(-) - P(+)$	$\Delta E = E(-) - E(+)$
0; 0	0,4062	1,6258	0,7106
1; 0	0,0073	0,3153	0,1231
2; 0	0,0224	0,2253	0,0675
2; 1	0,0113	0,1052	0,0531
2; 2	0,0101	0,1451	0,0122
3; 0	−0,0032	0,1352	0,0288
3; 1	−0,0013	0,1281	0,0241
3; 2	−0,0052	0,1011	0,0205
3; 3	−0,0051	0,0313	0,0112
4; 0	−0,0050	0,0352	0,0125
4; 1	−0,0049	0,0323	0,0123
4; 2	−0,0062	0,0721	0,0135
4; 3	−0,0060	0,0591	0,0072
4; 4	−0,0063	0,0163	0,0061
5; 0	−0,0060	0,0530	0,0071
5; 1	−0,0061	0,0552	0,0065
5; 2	−0,0062	0,0413	0,0052
5; 3	−0,0061	0,0312	0,0050
5; 4	−0,0071	0,0351	0,0019
5; 5	−0,0070	0,0270	0,0005
6; 0	−0,0073	0,0121	0,0110
6; 1	−0,0065	0,0123	0,0090
6; 2	−0,0067	0,0112	0,0019
6; 3	−0,0069	0,0382	0,0016
6; 4	−0,0071	0,0331	0,0001
6; 5	−0,0074	0,0251	−0,0102
6; 6	−0,0076	0,0212	−0,0096
7; 0	−0,0072	0,0161	0,0101
7; 1	−0,0071	0,0202	0,0142
7; 2	−0,0072	0,0271	0,0350
7; 3	−0,0073	0,0252	0,0121
7; 4	−0,0075	0,0221	−0,0005
7; 5	−0,0076	0,0191	−0,0016
7; 6	−0,0077	0,0162	−0,0007
7; 7	−0,0078	0,0152	−0,0012
8; 0	−0,0073	0,0125	−0,0005
8; 1	−0,0075	0,0102	−0,0005
8; 2	−0,0076	0,0103	−0,0006
8; 3	−0,0076	0,0175	−0,0016
8; 4	−0,0077	0,0161	−0,0007
8; 5	−0,0079	0,0152	−0,0054
8; 6	−0,0079	0,0153	−0,0012
8; 7	−0,0079	0,0131	−0,0011
8; 8	−0,0078	0,0121	−0,0013
9; 0	−0,0077	0,0122	−0,0006
9; 1	−0,0076	0,0162	−0,0006
9; 2	−0,0078	0,0153	−0,0005
9; 3	−0,0077	0,0153	−0,0009
9; 4	−0,0077	0,0141	−0,0009
9; 5	−0,0075	0,0132	−0,0011
9; 6	−0,0076	0,0121	−0,0012
9; 7	−0,0074	0,0112	−0,0013
9; 8	−0,0072	0,0091	−0,0015
9; 9	−0,0073	0,0081	−0,0015
10; 0	−0,0077	0,0075	−0,0012
10; 1	−0,0061	0,0131	−0,0012
10; 2	−0,0059	0,0121	−0,0010
10; 3	−0,0065	0,0122	−0,0011
10; 4	−0,0033	0,0105	−0,0010
10; 5	−0,0064	0,0122	−0,0009
10; 6	−0,0062	0,0121	−0,0009
10; 7	−0,0063	0,0021	−0,0008
10; 8	−0,0054	0,0072	−0,0007
10; 9	−0,0055	0,0061	−0,0008
10; 10	−0,0063	0,0032	−0,0012

Table 3

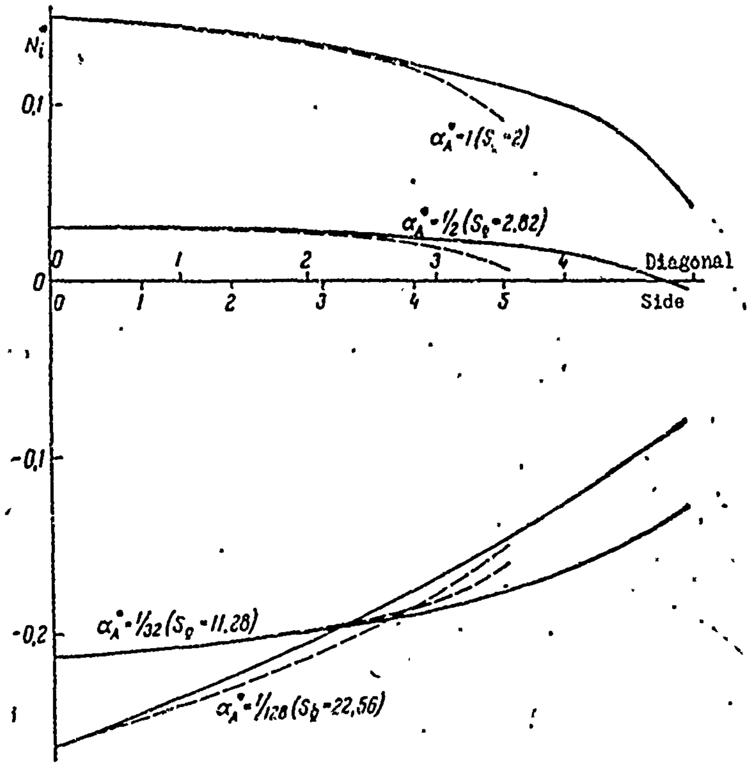
$\overset{\bullet}{\alpha}_A = 1/32$

Number of cell	$\Delta N = N(-) - N(+)$	$\Delta P = P(-) - P(+)$	$\Delta E = E(-) - E(+)$
0; 0	0,0810	2,0301	1,4112
1; 0	−0,4210	0,6323	0,3263
1; 1	−0,3231	0,5234	0,1821
2; 0	−0,2033	0,2050	0,1013
2; 1	−0,1521	0,1362	0,1036
2; 2	−0,0821	0,2002	0,1025
3; 0	−0,0310	0,2123	0,1023
3; 1	−0,0302	0,1970	0,1014
3; 2	−0,0871	0,1434	0,0721
3; 3	−0,0215	0,0721	0,0262
4; 0	−0,0336	0,0832	0,0332
4; 1	−0,0333	0,0754	0,0297
4; 2	−0,0334	0,0621	0,0224
4; 3	−0,0302	0,0123	0,0113
4; 4	−0,0812	0,0219	0,0162
5; 0	−0,0235	0,0123	0,0143
5; 1	−0,0334	0,0358	0,0134
5; 2	−0,0372	0,0305	0,0099
5; 3	−0,0776	0,0216	0,0013
5; 4	−0,0572	0,0149	0,0015
5; 5	−0,0321	0,0098	0,0014
6; 0	−0,0751	0,0102	0,0032
6; 1	−0,0702	0,0181	0,0026
6; 2	−0,0601	0,0156	0 0017
6; 3	−0,0430	0,0121	0,0012
6; 4	−0,0371	0,0163	−0,0001
6; 5	−0,0465	−0,0162	−0,0006
6; 6	−0,0621	−0,0133	−0,0007
7; 0	−0,0322	0,0101	−0,0008
7; 1	−0,0324	0,0088	−0,0003
7; 2	−0,0311	0,0056	−0,0007
7; 3	−0,0572	−0,0014	−0,0004
7; 4	−0,0663	−0,0121	−0,0121
7; 5	−0,0612	−0,0132	−0,0096
7; 6	−0,0531	−0,0138	−0,0098
7; 7	−0,0475	−0,0146	−0,0121
8; 0	−0,0652	−0,0121	−0,0001
8; 1	−0,0661	−0,0122	−0,0121
8; 2	−0,0655	−0,0125	−0,0004
8; 3	−0,0613	−0,0132	−0,0005
8; 4	−0,0704	−0,0133	−0,0056
8; 5	−0,0482	−0,0130	−0,0121
8; 6	−0,0475	−0,0152	−0,0102
8; 7	−0,0364	−0,0153	−0,0075
8; 8	−0,0282	−0,0118	−0,0024
9; 0	−0,0513	−0,0134	−0,0009
9; 1	−0,0505	−0,0135	−0,0008
9; 2	−0,0492	−0,0137	−0,0008
9; 3	−0,0474	−0,0141	−0,0101
9; 4	−0,0472	−0,0145	−0,0102
9; 5	−0,0125	−0,0137	−0,0096
9; 6	−0,0332	−0,0092	−0,0058
9; 7	−0,0275	−0,0012	−0,0021
9; 8	−0,0321	−0,0013	−0,0001
9; 9	−0,0324	−0,0022	−0,0002
10; 0	−0,0475	−0,0146	−0,0110
10; 1	−0,0462	−0,0145	−0,0101
10; 2	−0,0463	−0,0141	−0,0099
10; 3	−0,0421	−0,0129	−0,0092
10; 4	−0,0355	−0,0091	−0,0062
10; 5	−0,0321	−0,0026	−0,0031
10; 6	−0,0315	−0,0041	−0,0009
10; 7	−0,0272	−0,0007	−0,0001
10; 8	−0,0272	−0,0029	−0,0003
10; 9	−0,0242	−0,0028	−0,0002
10; 10	−0,0184	−0,0018	−0,0001

The flow of the k-th molecular criterion arriving on element dF_i can be written in the following form:

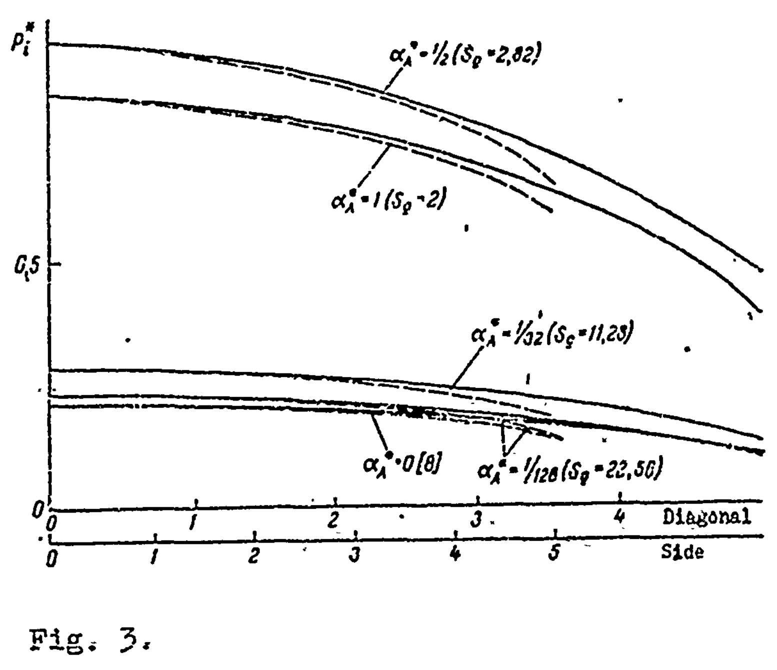
$$\Phi_i^{(k)} = \Phi_{c.м}^{(k)} - \Delta\Phi_i^{(k)},$$

where $\Phi_{c.м}^{(k)}$ — is the value of the corresponding flow of molecular criterion in freely molecular conditions of streamline flow. For the flow of normal pulse and energy flow it is necessary to consider



N_i^*
0,1
$\alpha_A^* = 1 (S_\ell = 2)$
$\alpha_A^* = 1/2 (S_\ell = 2,82)$
Diagonal
Side
-0,1
-0,2
$\alpha_A^* = 1/32 (S_\ell = 11,28)$
$\alpha_A^* = 1/128 (S_\ell = 22,56)$

Fig. 2.



P_i^*
$\alpha_A^* = 1/2 (S_\ell = 2,82)$
$\alpha_A^* = 1 (S_\ell = 2)$
0,5
$\alpha_A^* = 1/32 (S_\ell = 11,28)$
$\alpha_A^* = 0 [8]$
$\alpha_A^* = 1/128 (S_\ell = 22,56)$
Diagonal
Side

Fig. 3.

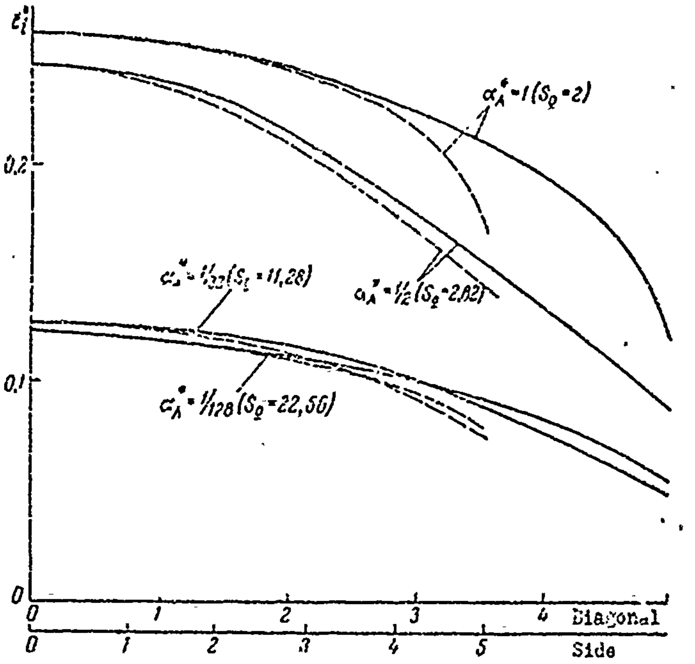


Fig. 4.

the reactive value of the corresponding flows.

As an example there is produced a calculation of the aerodynamic properties of a square plate with side d. For an appraisal of the accuracy of the method a comparison is made with the calculation produced in [8] when $M_\infty \to \infty$ $(S_0 \to \infty)$.

Figure 2 gives curves (solid line corresponds to the distribution along the diagonal, dashed line – along the side) for correction to the flow particles

$$N_i^* = N_i \, \mathrm{Kn}_\infty / n_\infty U_\infty S_0.$$

Figures 3-4 give curves for corrections to the flow of the normal pulse and to the energy flow without taking into account the reactive flow

$$P_i^* = \frac{P_i \, \mathrm{Kn}_\infty}{{}^1\!/_2 m n_\infty U_\infty^2 S_0}, \quad E_i^* = \frac{E_i \, \mathrm{Kn}_\infty}{{}^1\!/_2 m n_\infty U_\infty^3 S_0}.$$

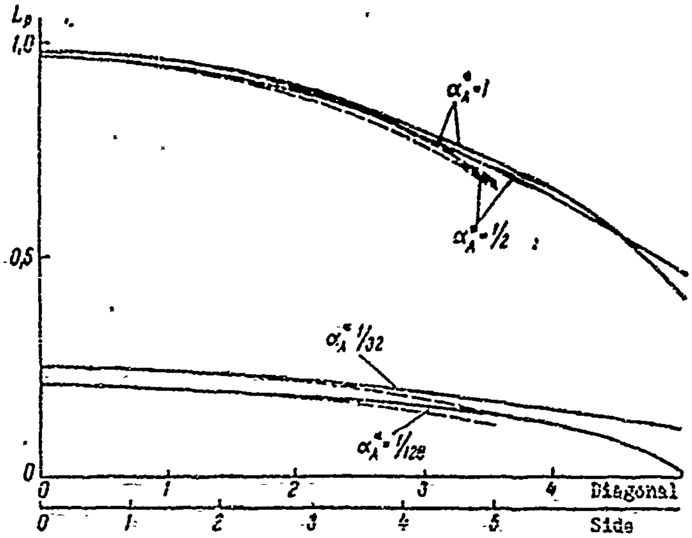


Fig. 5.

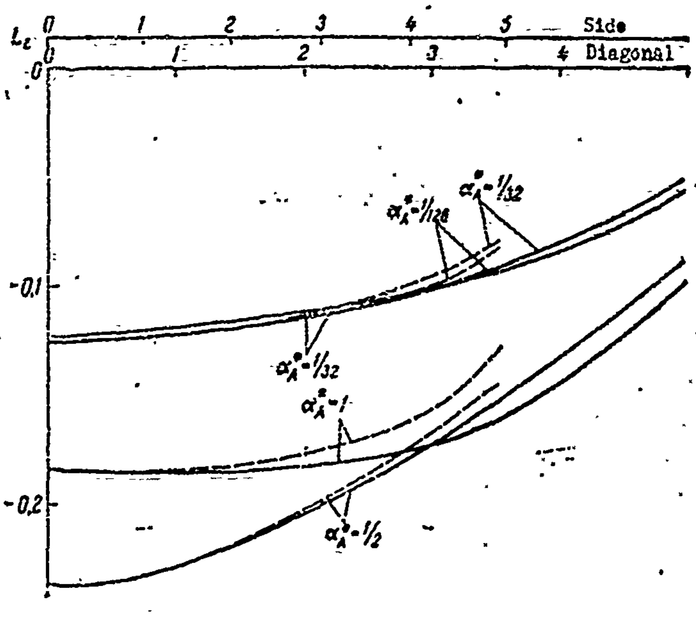


Fig. 6.

Taking into account the jet stream the expression for the normal pulse is recorded in the following form:

$$\overline{P}_{N(i)} = \frac{mn_\infty U_\infty^2}{2}\left[2 + \frac{\sqrt{\pi}}{S_0} - \frac{S_0}{\mathrm{Kn}_\infty}\left(N_i\frac{\sqrt{\pi}}{S_0} + P_i\right) + P_i N_i\left(\frac{S_0}{\mathrm{Kn}_\infty}\right)^2\right],$$

and for the energy flow

$$\overline{E}_i = \frac{mn_\infty U_\infty^3}{2}\left[1 - \frac{2}{S_0^2} + \frac{S_0}{\mathrm{Kn}_\infty}\left(\frac{2}{S_0^2}N_i - E_i\right) + E_i N_i\left(\frac{S_0}{\mathrm{Kn}_\infty}\right)^2\right];$$

the corresponding curves are given in Figs. 5 and 6 where

$$L_p = N_i\frac{\sqrt{\pi}}{S_0} + P_i \quad \text{and} \quad L_E = \frac{2}{S_0^2}N_i - E_i.$$

For the total fluxes of the mass, normal pulse, and energy the corresponding expressions, taking into account the jet stream (disregarding terms $(S_0/Kn_\infty)^2$ and of higher order) are obtained

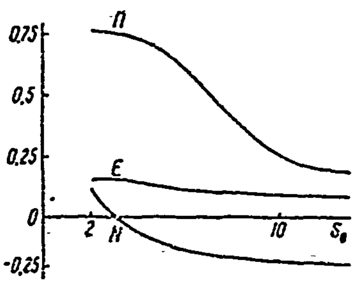


Fig. 7.

$$C_N = 1 - \frac{S_0}{\mathrm{Kn}_\infty}\sum_i N_i$$

(the total flow of the mass is referred to $mn_\infty U_\infty F$),

$$C_x = 2 + \frac{\sqrt{\pi}}{S_0} - \frac{S_0}{\mathrm{Kn}_\infty}\left(\sum_i N_i\frac{\sqrt{\pi}}{S_0} + \sum_i P_i\right)$$

(the total flow of the normal pulse is referred to $1/2\ mn_\infty U_\infty^2 F$),

$$C_E = 1 - \frac{2}{S_0^2} + \frac{S_0}{\mathrm{Kn}_\infty}\left(\frac{2}{S_0^2}\sum_i N_i - \sum_i E_i\right)$$

(the total flow is referred to $1/2\ mn_\infty U_\infty^3 F$). Figure 7 gives curves plotted depending upon S_0 for

$$N = \sum_i N_i, \quad \Pi = \sum_i N_i\frac{\sqrt{\pi}}{S_0} + \sum_i P_i \quad \text{and} \quad E = \frac{2}{S_0^2}\sum_i N_i - \sum_i E_i,$$

$$S_0 = \frac{U_\infty}{\sqrt{2RT_0}}, \quad \mathrm{Kn}_\infty = [2\pi\sigma^2 n_\infty d]^{-1}.$$

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26 July 1964

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