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THE USE OF RELATIVE VELOCITY CORRECTIONS IN CARRIER ALIGNMENTS

BY

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PRELIMINARY DATA

This is an informal report and is transmitted for information only. The data presented are tentative and subject to later revision.

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FOREWORD

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ABSTRACT

The accuracy of an alignment of an inertial navigation system on a moving ship depends on the accuracy of the reference velocity available. The sensor obtaining this reference velocity is almost always located away from the navigation system being aligned suggesting that a relative velocity correction would be helpful. This is true when the sensor sees inertial velocity. When the source is an EM log which does not see inertial velocity, a relative velocity correction can either improve or degrade the reference velocity depending on the location of the source and the nature of the turn. With the appropriate data on how a ship turns it might be possible to find an empirical equation that could be used to correct EM log velocities to inertial velocities regardless of the nature of turn. This report gives the equations for making a relative velocity correction when the sensor is an inertial source, and also discusses the envelope of conditions under which these same corrections are helpful when the sensor is an EM log.

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INTRODUCTION

When aligning the inertial navigation system of an aircraft located on the deck of an aircraft carrier, the velocity of the carrier must be known. The sensor measuring this reference velocity is usually at an entirely different location on the carrier than is the system being aligned. To the extent that the inertial velocity at the point where the aircraft is located differs from the velocity measured by the sensor, the reference velocity is in error. The question arises as to whether various reference velocities can be corrected to be the inertial velocity at the aircraft.

It would be a simple matter to make the required corrections if the sensor measured inertial velocity. The only additional information needed would be the lateral and longitudinal separations of the sensor and aircraft and the continuous measurement of the turning rate of the ship. It would not be possible to make the required correction when the reference velocity source is an EM log, however, unless it would be possible to have a continuous measurement of skid angle. Equations for the corrections needed when the sensor is an inertial source and when the sensor is an EM log are given. Also, a discussion is given of the conditions under which the use of the required corrections for an inertial source is helpful even when the sensor is an EM log.

A. CASE OF INERTIAL VELOCITY SENSOR

If the alignment takes place while the ship is straight steaming, then all the points on the ship have the same inertial velocity and no corrections to the reference velocity would be needed. When the ship is turning, however, the inertial velocity at any point on the ship is the vector sum of the velocity of the ship along its path and the tangential velocity of the point due to the rotational motion of the ship. Since the translational velocity for each point on the ship is the same, the differences in the inertial velocity between the two points is due to the rotational motion of the ship.

Assuming that the origin of the ship's coordinate system is the pivoting center of the ship's rotational motion, we can see by looking at Figure 1 that the relative velocity of the point (X_2, Y_2) with respect to the point (X_1, Y_1) is

$$\Delta\bar{V} = \dot{\bar{R}}_2 - \dot{\bar{R}}_1 + \dot{\bar{\psi}} \times (\bar{R}_2 - \bar{R}_1) \quad (1)$$

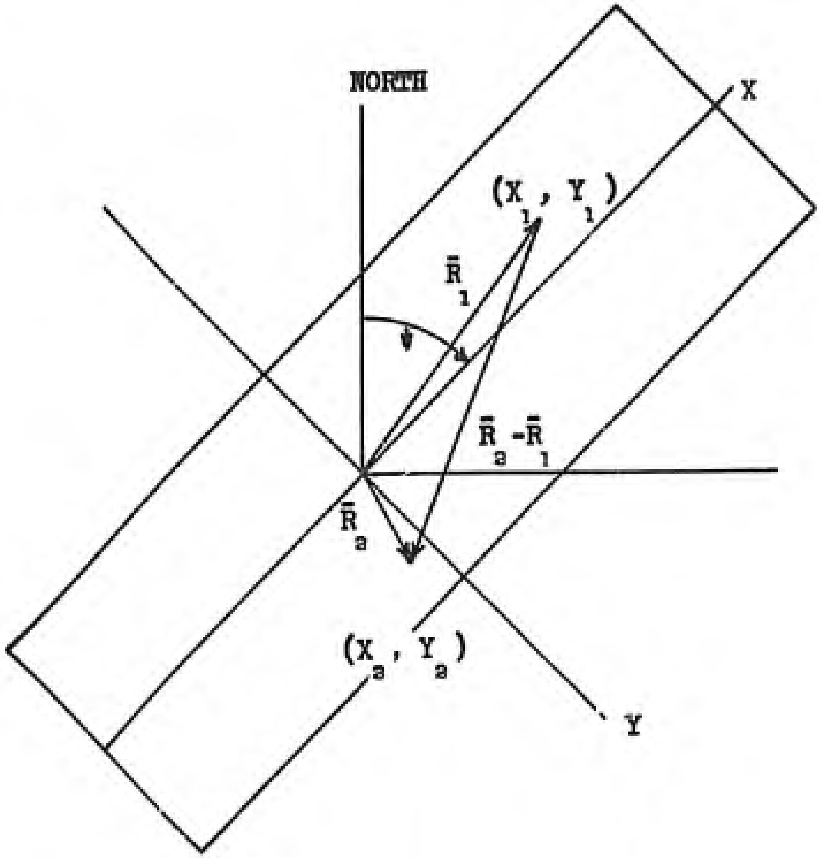

FIGURE 1.

where

$$\bar{R}_1 = X_1 \bar{i} + Y_1 \bar{j} \tag{2}$$

$$\bar{R}_2 = X_2 \bar{i} + Y_2 \bar{j} \tag{3}$$

$$\bar{\dot{\psi}} = \dot{\psi} \bar{k} \quad . \tag{4}$$

Substituting equations (2), (3) and (4) into equation (1) and further assuming that the aircraft is not moving around on the deck of the ship $(\dot{X}_1 = \dot{X}_2 = \dot{Y}_1 = \dot{Y}_2 = 0)$, we have

$$\Delta\bar{V} = - \dot{\psi}(Y_2 - Y_1)\bar{i} + \dot{\psi}(X_2 - X_1) \bar{j} \quad . \tag{5}$$

The transformation from the ship's coordinate system to the navigation coordinate system can be carried out with the relationships

$$\bar{i} = \cos\psi\, \bar{\ell} + \sin\psi\, \bar{m} \tag{6}$$

$$\bar{j} = -\sin\psi\, \bar{\ell} + \cos\psi\, \bar{m} \tag{7}$$

yielding,

$$\Delta\bar{V} = -\dot{\psi}[(Y_2 - Y_1)\cos\psi + (X_2 - X_1)\sin\psi]\bar{\ell}$$

$$+\dot{\psi}[(X_2 - X_1)\cos\psi - (Y_2 - Y_1)\sin\psi]\bar{m}\ . \tag{8}$$

If the velocity source is at (X_1, Y_1) and the system being aligned is at (X_2, Y_2), then the corrections to north and east velocities should be

$$C(V_N) = -\dot{\psi}[(Y_2 - Y_1)\cos\psi + (X_2 - X_1)\sin\psi] \tag{9}$$

$$C(V_E) = \dot{\psi}[(X_2 - X_1)\cos\psi - (Y_2 - Y_1)\sin\psi]\ . \tag{10}$$

These relative velocity corrections for the case when the velocity sensor is an inertial source will be called R.V.C. corrections hereafter. It can be noted from these equations that the R.V.C. corrections are independent of the location of the pivoting center of the ship depending only on the lateral and longitudinal separations.

According to the only information available for the U.S.S. Enterprise , shown in Figure 2, the appropriate R.V.C. corrections for the N5H tests would have been

$$C(V_N) = 324\,\dot{\psi}\sin\psi - 52\,\dot{\psi}\cos\psi \tag{11}$$

$$C(V_E) = -324\,\dot{\psi}\cos\psi - 52\,\dot{\psi}\sin\psi \tag{12}$$

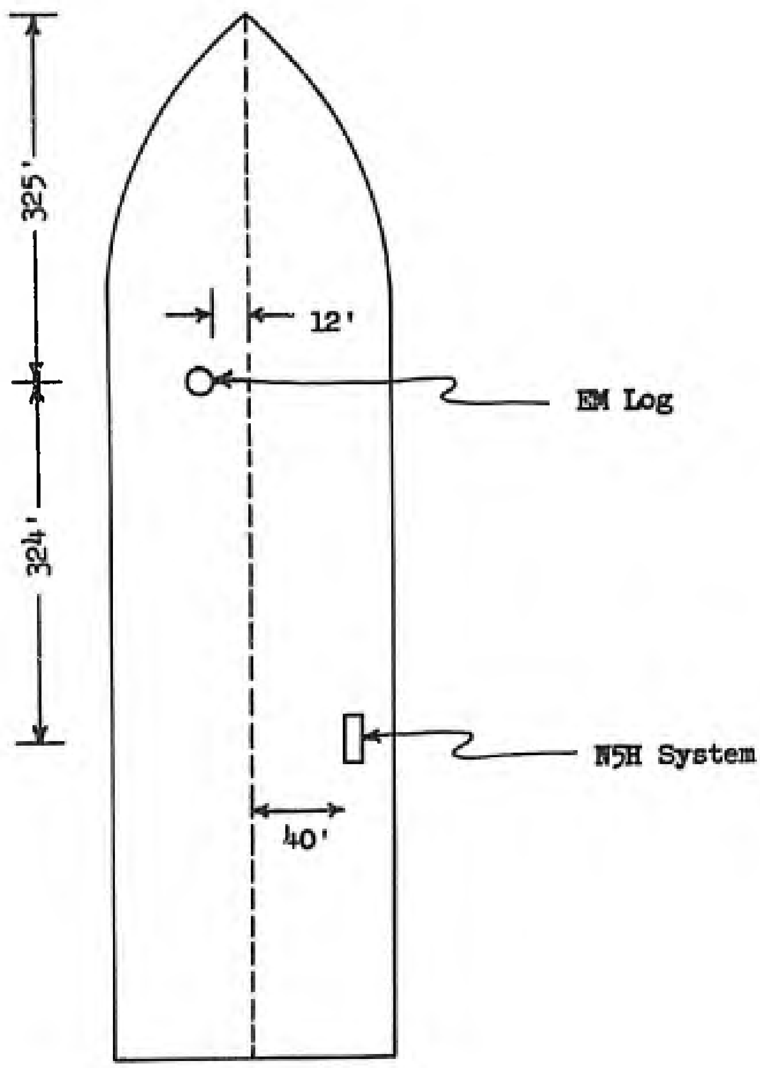

FIGURE 2.

Since an EM log sensor does not measure inertial velocity, these R.V.C. corrections are of varying help depending on the nature of the turn.

B. CASE OF NON-INERTIAL VELOCITY SOURCE

Assuming that an EM log measures only the longitudinal velocity of the ship, we can determine the conditions under which an R.V.C. to the EM log information would be helpful. This is, of course, neglecting the effect of turbulence and other sources of error in an EM log sensor.

From Figure 3, it can be seen that the inertial velocity vector, $\bar{V}_1$, at the EM log's location, (X_1, Y_1), is related to the translational velocity vector of the ship, $\bar{V}$, by

$$\bar{V}_1 = \bar{V} + \overline{X_1 \dot{\psi}} - \overline{Y_1 \dot{\psi}} \quad . \tag{13}$$

Similarily, at the N5H's location, (X_2, Y_2)

$$\bar{V}_2 = \bar{V} + \overline{X_2 \dot{\psi}} - \overline{Y_2 \dot{\psi}} \quad . \tag{14}$$

Equations (13) and (14) can also be expressed as

$$\bar{V}_1 = (V \cos \beta - Y_1 \dot{\psi})\bar{i} + (V \sin \beta + X_1 \dot{\psi})\bar{j} \tag{15}$$

$$\bar{V}_2 = (V \cos \beta - Y_2 \dot{\psi})\bar{i} + (V \sin \beta + X_2 \dot{\psi})\bar{j} \quad . \tag{16}$$

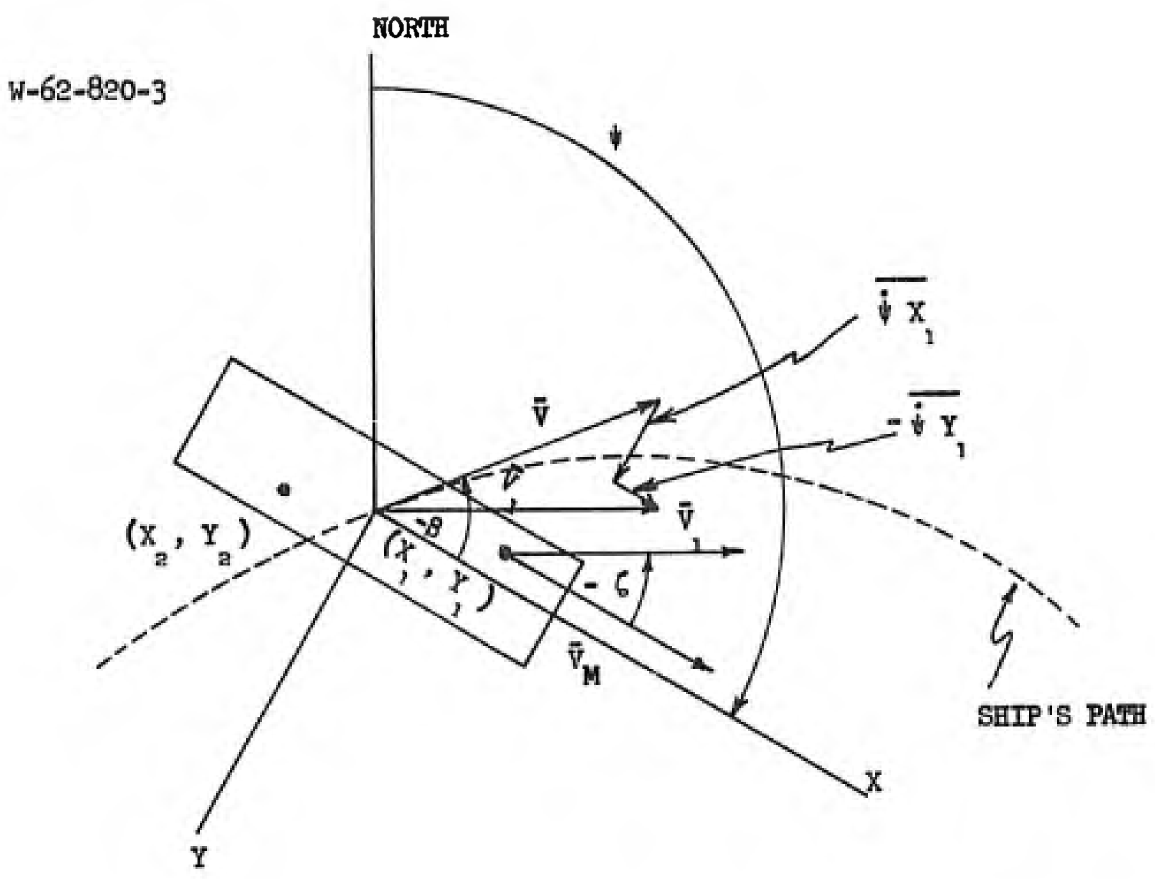

FIGURE 3.

In the case illustrated in Figure 3, β is negative and $\dot\psi$ is positive.

The EM log sees only the longitudinal component of $\bar V_1$, so that the measured velocity, $\bar V_M$, is

$$\bar V_M = (V \cos \beta - Y_1 \dot\psi)\bar i \, . \tag{17}$$

It is clear that the correction needed to compensate $\bar V_M$ to get $\bar V_2$ is not a simple R.V.C. The required correction in this case is

$$\overline{\Delta V}_M = \bar V_2 - \bar V_M \tag{18}$$

or

$$\overline{\Delta V}_M = [\ddot{\bar{V}}_2 - \bar{V}_1] + [\bar{V}_1 - \bar{V}_M] \qquad (19)$$

Obtaining the first term by use of equations (13) and (14) and the second term by use of equation (15) and (17), we have

$$\overline{\Delta V}_M = - \dot{\psi} (Y_2 - Y_1)\bar{i} + \dot{\psi}(X_2 - X_1)\bar{j} + (V \sin \beta + X_1 \dot{\psi})\bar{j} \ . \qquad (20)$$

Using the transformation given in equations (6) and (7), the corrections to north and east velocity in this case are

$$C(V_N) = - \dot{\psi}[(Y_2 - Y_1) \cos \psi + (X_2 - X_1)\sin \psi] - (V \sin \beta + X_1 \dot{\psi}) \sin \psi \qquad (21)$$

$$C(V_E) = \dot{\psi}[(X_2 - X_1) \cos \psi - (Y_2 - Y_1) \sin \psi] + (V \sin \beta + X_1 \dot{\psi}) \cos \psi \ . \qquad (22)$$

It can be noted from equations (21) and (22) that the corrections for this case are not independent of pivoting center's location. Also, if there is no skidding and both $(Y_2 - Y_1)$ and X_2 are small, as was true for the N5H tests aboard the U.S.S. Enterprise, then the EM log information need not be corrected.

Equations (21) and (22) show that R.V.C. corrections would be the most beneficial if

$$V \sin \beta + X_1 \dot{\psi} = \quad . \qquad (23)$$

This condition is satisfied when $\bar{V}_1$ and $\bar{V}_M$ are parallel. The angle between $\bar{V}_M$ and $\bar{V}_1$, ζ , is given in general by

$$\sin \zeta = \frac{|\bar{V}_M \times \bar{V}_1|}{|\bar{V}_M||\bar{V}_1|} \tag{24}$$

Substituting equations (15) and (17) and simplifying, we obtain

$$\sin \zeta = \frac{V \sin \beta + X_1 \dot{\psi}}{|\bar{V}_1|} \tag{25}$$

From equation (25) it is easy to see that the condition (23) holds when $\bar{V}_M$ and $\bar{V}_1$ are parallel. This condition is illustrated in Figure 4. Another way of looking at this condition is that the tangent

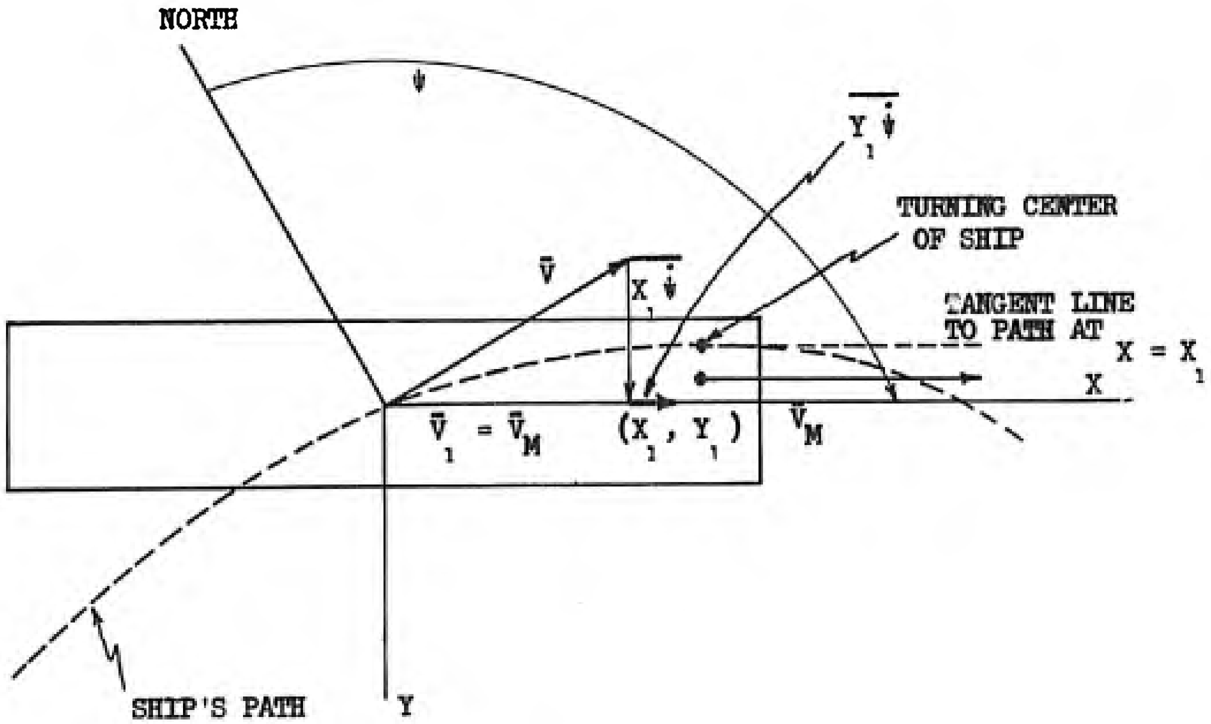

FIGURE 4.

line to the ship's path (if $\dot{\psi}$ constant) at $X = X_1$ is parallel to $\bar{V}_M$. This point on the ship's path is sometimes called the turning center of the ship. In the case shown, the turning center of ship is inside the ship.

Also, from equation (20) we can see that when an R.V.C. type correction is made to an EM log velocity, the remaining error in the reference velocity is just

$$- \Delta V_M^* = - (V \sin \beta + X_1 \dot{\psi})\bar{j} \tag{26}$$

Equation (20) can be rewritten to show that the error in the reference velocity without making an R.V.C. type correction is

$$- \overline{\Delta V}_M = \dot{\psi} (Y_2 - Y_1)\bar{i} - (V \sin \beta + X_2 \dot{\psi})\bar{j} \tag{27}$$

An R.V.C. correction, then, is certainly of advantage whenever both of the following inequalities are satisfied

$$|(V \sin \beta + X_1 \dot{\psi})| < | V \sin \beta + X_2 \dot{\psi} | \tag{28}$$

$$0 < | \dot{\psi} (Y_2 - Y_1) | \tag{29}$$

Equation (29) indicates that it is always helpful to make the R.V.C. correction needed due to lateral separation. Since for the N5H tests on the U.S.S. Enterprise, the location of the van was such that X_2 was probably very small, we will discuss the inequality

$$| V \sin \beta + X_1 \dot{\psi} | < | V \sin \beta | \ . \tag{30}$$

We shall consider only clockwise turns $(\dot{\psi} > 0)$, since this will cover all types of cases. If, in addition to our making a clockwise turn, we have both a negative skid angle and the EM log aft on the ship, or both a positive skid angle and the EM log forward on the ship, then the inequality (3) cannot be satisfied and an R.V.C. correction will actually increase the error in the reference velocity. As shown in Figure 2, $X_1 > 0$ on the U.S.S. Enterprise. Also, it seems more likely that a ship would skid with a negative skid angle when turning clockwise (this is the situation shown in Figures 3 and 4). And so, the inequality (3) not being satisfied due to β and X_1 both being of same sign would seem unlikely.

If, in addition to our making a clockwise turn, we have both a negative skid angle and the EM log forward on the ship, or both a positive skid angle and the EM log aft on the ship, then the inequality (30) may or may not be satisfied. The condition under which it can be satisfied and would help is

$$| X_1 \dot{\psi} | < 2 | V \sin \beta | \tag{31}$$

and the condition under which it cannot be satisfied and would hurt is

$$| X_1 \dot{\psi} | \geq 2 | V \sin \beta | \tag{32}$$

A completely analogous set of conclusions can be drawn for counter clockwise turns.

The limiting case is illustrated in Figure 5. If the velocity, V, and the skid angle, β, were as shown in Figure 5, but the turning rate, $\dot{\psi}$, or the forward position of the EM log, X_1, were increased, then $| X \dot{\psi} | \geq 2 | V \sin \beta |$ and the reference velocity would be degraded by R.V.C. However, with the turning rate shown, or any lower rate, and the same EM log location, then $| X \dot{\psi} | < 2 | V \sin \beta |$ and an R.V.C. correction would improve the reference velocity. Likewise, with the EM log location shown, or any location closer to the pivoting center of the ship, and the same turning rate, an R.V.C. correction would improve the reference velocity.

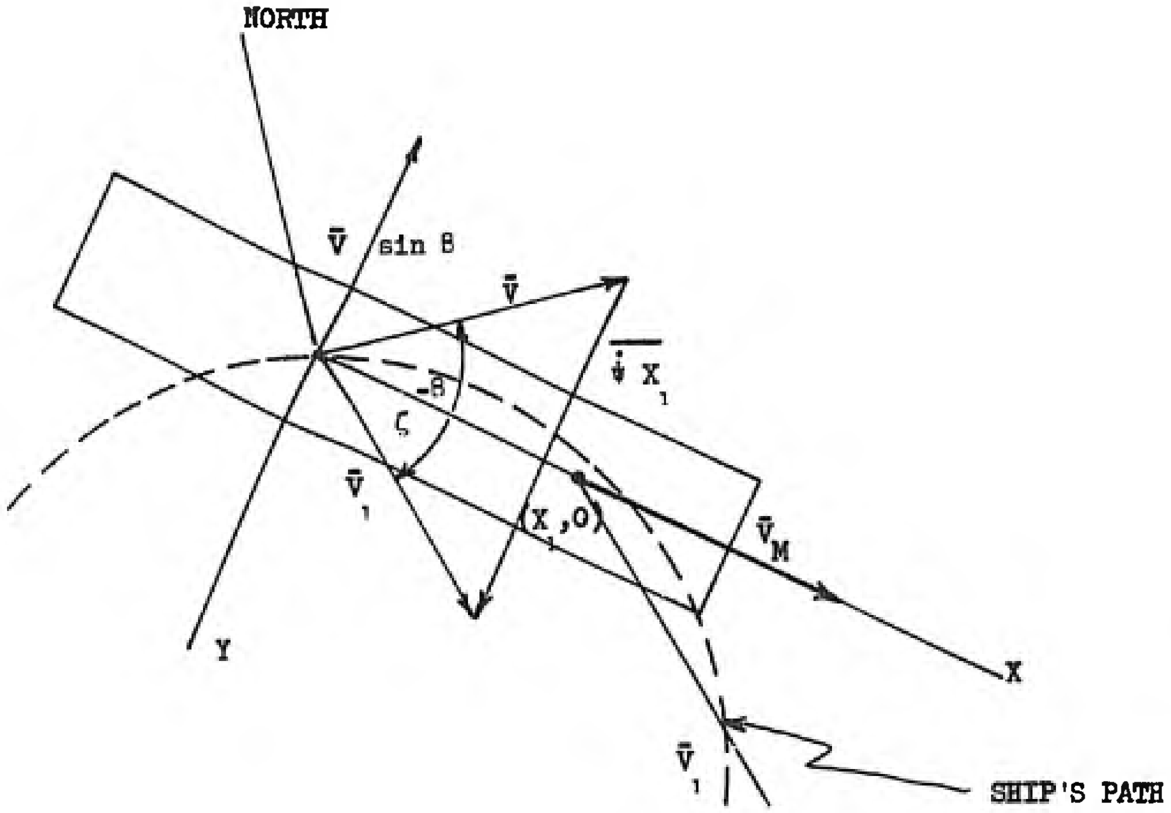
NORTH
$\bar{V}$ sin θ
$\bar{V}$
$\overline{\dot{\psi} X_1}$
-θ
ζ
$\bar{V}_1$
(X_1, O)
$\bar{V}_M$
Y
X
$\bar{V}_1$
SHIP'S PATH

FIGURE 5.

If on the other hand, the position of the EM log, $(X_1, 0)$ and the turning rate, $\dot{\psi}$, were the same as shown in Figure 5, but the velocity, V, or the skid angle, β, were decreased, then $|X_1 \dot{\psi}| \geq 2 |V \sin \beta|$ and again the reference velocity would be degraded by using R.V.C. However, with the velocity shown, or any higher velocity, and the same skid angle, then $|X_1 \dot{\psi}| < 2 |V \sin \beta|$ and an R.V.C. correction would be desirable. Also, another set of conditions under which an R.V.C. correction would be desirable, is when all variables are as shown, except skid angle is increased.

As a matter of interest, the limiting values of the skid angle for a turn of the type shown in Figure 5 ($\dot{\psi} > 0$, $\beta < 0$) were plotted as a function of the turning rate for various translational velocities. These plots, shown in Figure 6 are based on the assumption that the EM log's position is (324 , 0), which is approximately true for the U.S.S. Enterprise. The combinations of turning rate and skid angle which are below a particular velocity curve are cases where R.V.C. corrections would help. Similarly combinations of turning rate and skid angle which are above a particular velocity curve are cases where R.V.C. corrections would hurt. For example if the turning rate were $.5^{\circ}$/sec and the velocity were 20 knots, then an R.V.C. correction would help if the skid angle was less than -2.4°, or greater in magnitude than 2.4°. Also, an R.V.C. correction would hurt if the skid angle was between 0° and -2.4°.

Examination of Figure 6 indicates that for a particular turning rate, the lower the velocity the greater the magnitude of the skid angle must be if an R.V.C. correction is to be helpful. To the extent that there would be more skidding at high velocities it would appear that at higher velocities an R.V.C. correction is probably always helpful, but at lower velocities the use of an R.V.C. correction becomes of doubtful helpfulness.

Another way of writing equation (20) for $\overline{\Delta V}_M$ places in evidence the distance which $\dot{\psi}$ would have to be multiplied by in order to have to correct compensation for the EM velocity. Factoring $\dot{\psi}$ out in equation (20), we get

$$\Delta \bar{V}_M = \dot{\psi} \left[-(Y_2 - Y_1)\bar{i} + \left(\frac{V \sin \beta}{\dot{\psi}} + X_2 \right) \bar{j} \right] \tag{33}$$

but,

$$\dot{\psi} = \frac{V}{\rho} \tag{34}$$

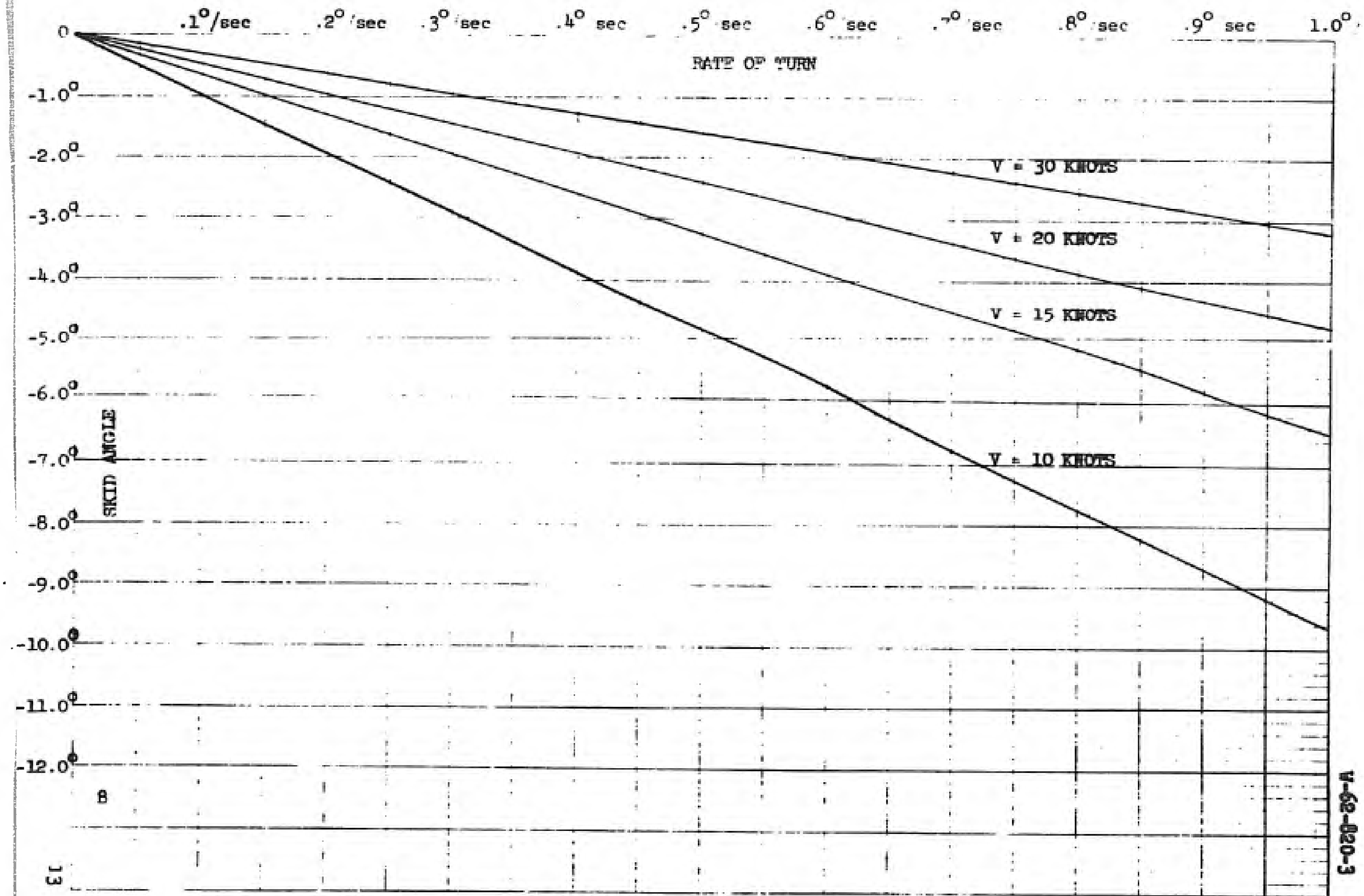

FIGURE 6.

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where ρ is the radius of curvature of the ship's path. Substituting equation (34) into equation (33), we have

$$\overline{\Delta V}_M = \dot{\psi}\,[-(Y_2 - Y_1)\bar{i} + (\rho \sin \beta + X_2)\bar{j}] \tag{35}$$

As stated before, in the case of the N5H tests on the U.S.S. Enterprise both $(Y_2 - Y_1)$ and X_2 are fairly small and so it would be approximately true that

$$\overline{\Delta V}_M \cong [\rho \sin \beta]\,\dot{\psi}\,\bar{j} \tag{36}$$

The geometrical significance of this distance, can be seen in Figure 7.

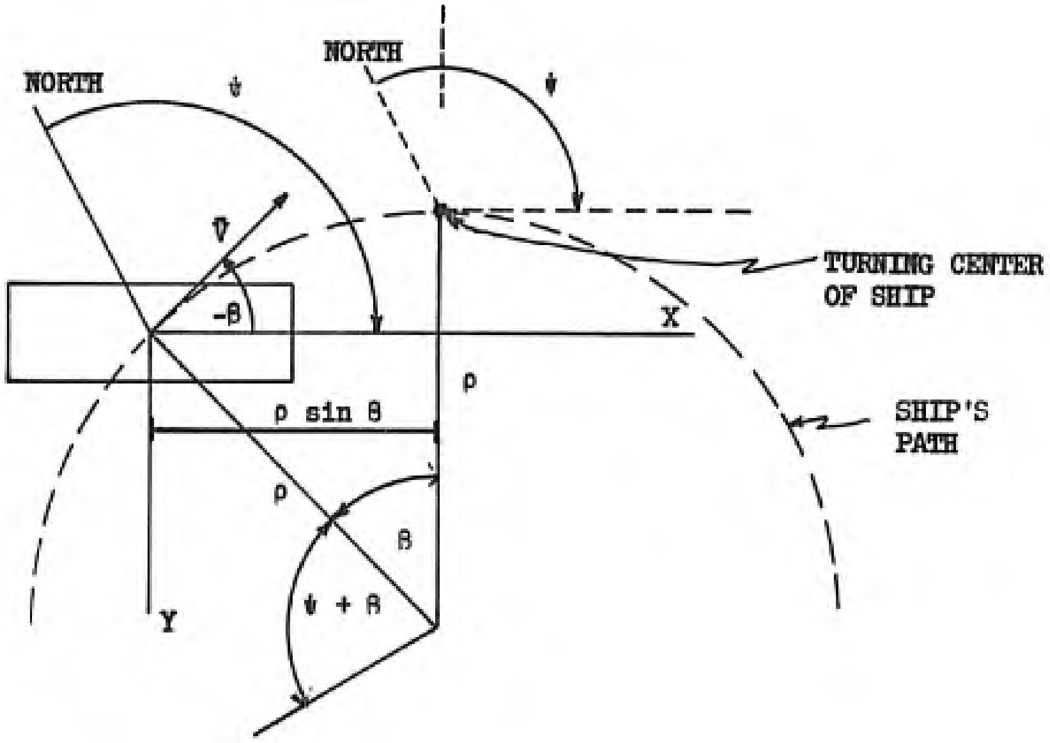

FIGURE 7.

From Figure 7 it can be seen that $[\rho \sin \beta]$ is the X coordinate of the turning center of the ship. The case shown in Figure 7 is one for which the turning center is outside of the ship.

If there tends to be a greater skid angle at higher velocities, then there is little hope of finding a constant value of $[\rho \sin \beta]$ to be used for all turns since for a particular turning rate ρ is also directly proportional to the velocity. On the other hand, if information were obtainable on just how a ship with various velocities and turning rates did skid, it might be possible to find an empirical equation for $\sin \beta$. The correction for an EM log reference velocity would then be

$$\overline{\Delta V}_m \cong \left[\frac{V}{\dot{\psi}} f(V, \dot{\psi})\right] \dot{\psi} \tag{37}$$

Since measured velocity and turning rate are both available on the ship, an equation of this type could make actual corrections needed rather than an R.V.C. correction which is of varying help depending on the nature of the turn.

CONCLUSIONS

It is apparent that the location of an EM log reference velocity sensor with respect to the navigation system being aligned is critical if one is considering making relative velocity corrections. However, in the case of the N5H tests aboard the U.S.S. Enterprise (CVA(N)-65) the EM log sensor was located favorably.

To the extent that skid angle is directly proportional to the velocity of the ship (for the same turning rate), it appears that the higher the velocity of the ship, the more certain it is that a relative velocity correction would improve the reference velocity (Figure 6.).

It would be desirable, if possible, to have experimental data on just how a ship does skid for various combinations of velocity and turning rate. With this kind of data it would be possible, using the criterion developed, to determine when an R.V.C. correction would help, or maybe even develop an empirical function of V and $\dot{\psi}$ giving the distance $\dot{\psi}$ must be multiplied by to yield a good estimate of the velocity correction needed for each particular turn.