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BASIC LAWS OF TURBULENT MIXING IN THE GROUND LAYER OF THE ATMOSPHERE

by

A. S. Monin and A. M. Obukhov

The article contains an analysis of the processes of mixing in a turbulent atmosphere, based on systematic application of the methods of the theory of similitude. Empirical data on the distribution of wind velocity under various conditions of temperature stratification are generalized and a method is proposed for computing the austausch characteristics on the basis of measuring wind velocity and temperature gradient.

INTRODUCTION

The questions of the physics of the ground layer have occupied a considerable place in meteorological investigations during the past 10-15 years. The laws of the processes in the ground layer are of interest not only to agrometeorology, which studies the effect of a "meteorological medium" on the growth of vegetation, but they also have a general geophysical significance, since the dynamic interaction of the atmosphere and the substrate, the "feeding" of the atmosphere by moisture and heat, is realized through the ground layer.

A large amount of research in the field of ground-layer physics has been done at the Main Geophysical Observatory; the works of S. A. Saposhnikova [1], D. L. Laikhtman and A. F. Chudnovskii [2], M. I. Budyko [3] and M. T. Timofeev [4] are well known to Soviet meteorologists.

This research has provided valuable observational data on the distribution of wind, temperature and humidity in the ground layer, and a number of specific propositions have been drawn up on the methodology for computing turbulent austausch characteristics (Budyko, Laikhtman).

In this regard there are still a number of debatable questions in the theory of ground-layer mixing. The simplest system of the "logarithmic boundary layer," borrowed from technical aerodynamics, describes quite well the phenomena in a neutrally stratified atmosphere, and is supported by much empirical data. However, this system is insufficient for describing processes in a real atmosphere where the temperature inhomogeneity is an essential factor influencing the development of turbulence. This latter fact (the temperature inhomogeneity) determines the specific nature of the problem of atmospheric turbulence as applied to ground-layer physics.

The works of Laikhtman [5] and Budyko [3], as well as those of a number of foreign researchers (Sverdrup, Rossby and Montgomery; see, e.g., [6]) have been devoted to computing the influence of temperature stratification on turbulent exchange. The individual results of these works contradict one another; in many respects the physical sense of the initial hypotheses is still not clear. Thus, e.g., Budyko proposes that the atmospheric stratification be considered within the frameworks of the simplest system of the logarithmic boundary layer, formally replacing the Karman "universal constant" by a variable parameter, a function of stratification. In Budyko's system, the basic characteristic of the substrate, roughness, is also a function of meteorological conditions. The purely formal nature of these relations is one of the shortcomings of Budyko's system. It should also be noted that the observed profiles of wind distribution with height regularly deviate from the logarithmic law during stratification conditions which differ from neutral equilibrium.

Laikhtman proposes a more complete method of approximating wind and temperature profiles (an exponential law with a variable exponent), which makes it possible to discern the nature of deviations from the logarithmic law under various conditions of atmospheric stratification. However, Laikhtman's system contains too many free parameters which have to be determined in each individual case. This creates difficulties familiar in determining these parameters from empirical data and decreases the computational accuracy.

These critical remarks by no means are meant to detract from the value of the results obtained by Budyko and Laikhtman when solving individual problems; however, they indicate the necessity of developing the theory further and making the initial physical hypotheses more exact.

When analyzing the highly complex phenomena of ground-layer turbulence, where the temperature factors play an essential role, it is expedient to use the methods of the theory of similitude which are widely used in applied aerodynamics and thermal physics, and are the generally accepted method of investigation in this area.

In 1943, A. M. Obukhov attempted to apply methods of the theory of similitude to problems of ground-layer physics [7]. The results obtained in this work were subsequently developed by A. S. Monin [8]. The theory developed in [7] and [8] evidently give a satisfactory qualitative description of the processes.

Furthermore, the data used in [7] to determine the numerical parameters in the proposed systems were not sufficiently reliable (the critical Richardson number was mistakenly assumed to be 1/11, on the basis of Sverdrup's data), which made it impossible to make direct use of the formulas obtained in this work in actual computations.

The present work gives an analysis of the processes of turbulent mixing in the ground layer of the atmosphere on the basis of a systematic application of the methods of the theory of similitude, and the values of the numerical parameters are more exactly defined by using a sufficiently large amount of empirical data on gradient observations, obtained from the expeditions of the Main Geophysical Observatory and the Geophysical Institute of the Academy of Sciences of the USSR. On this basis, working formulas were obtained for computing the basic characteristics of the ground layer, viz., turbulent heat transfer, friction, the austausch coefficient, and moisture flux, from gradient measurement data. The computational method is illustrated by specific examples.

1. THE LOGARITHMIC BOUNDARY LAYER

When analyzing the processes in the ground layer of the atmosphere on a theoretical basis, we will proceed from the generally accepted system of a current above an infinitely rough surface whose horizontal properties are assumed to be quite uniform. The averaged characteristics of the current in this system are a function only of the vertical coordinate z. The most important characteristics are the momentum, heat, and humidity fluxes.

The momentum flux can be treated as turbulent friction stress. Instead of turbulent friction

$$\tau = -\overline{\rho u'w'} \qquad (1)$$

where u' and w' are the pulsations of the horizontal and vertical wind velocity components, ρ is air density, and the bar indicates averaging, it is convenient to examine the dynamic velocity

$$v_* = \sqrt{\frac{\tau}{\rho}} \qquad (2)$$

Within the confines of the ground layer, τ and the turbulent heat flux q can be considered to be practically independent of height z.

The condition that fluxes τ and q are constant (within the given tolerance) can serve to determine the actual concept of the ground layer. Let us attempt to give an approximate estimate of the height of the ground layer on the basis of possible changes in τ. We will proceed from the averaged equations of hydromechanics in a Coriolis force field. The corresponding equation for the x-coordinate (wind-velocity direction at the earth's surface) in a quasistationary case has the following form:

$$\frac{\partial \overline{u'w'}}{\partial z} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x} + l\overline{v} \tag{3}$$

where $\partial\overline{p}/\partial x$ is the pressure gradient, l the Coriolis parameter and $\overline{v}$ the component of averaged wind velocity along the y-axis.

Let us integrate both sides of the equation with respect to height within the limits of a layer of thickness H and estimate the right-hand side:

$$\frac{\tau(0) - \tau(H)}{\rho} = \int_0^H \left|\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x} - l\overline{v}\right| dz < \int_0^H \frac{1}{\rho}\left|\frac{\partial \overline{p}}{\partial x}\right| dz \tag{4}$$

The discarding of term $l\overline{v}$ leads to a strengthening of the inequality, since the Coriolis force partially compensates the effect of the pressure gradient. Introducing the dynamic velocity v_* and the geostrophic wind velocity $v_g = (1/\rho l)|\partial\overline{p}/\partial x|$, we can write the resultant inequality in the following form:

$$v_*^2(0) - v_*^2(H) < Hlv_g \tag{5}$$

Let us define H such that the relative change of v_*^2 in a layer of thickness H does not exceed the tolerance α, i.e.,

$$\frac{v_*^2(0) - v_*^2(H)}{v_*^2(0)} \leqslant \alpha \tag{6}$$

On the strength of inequality (5) it suffices that

$$H < \frac{\alpha v_*^2(0)}{lv_g} \tag{7}$$

in order that (6) be fulfilled. The ratio of friction velocity to geostrophic wind velocity can be estimated to be a value of the order of 0.05:

$$\frac{v_*}{v_g} \sim 0.05$$

from which it follows that

$$H \leqslant 2.5 \cdot 10^{-3}\,\alpha\frac{v_g}{l}$$

When $v_g \sim 10$ m/sec and $\ell = 10^{-4} sec^{-1}$ we get

$$H \sim \alpha \cdot 250 \text{ m.}$$

With a tolerance $\alpha = 20\%$ we get the estimate of the height of the ground layer which we seek:

$$H = 50 \text{ m.}$$

Within the limits of this layer, v_* can be considered practically constant and the effect of the Coriolis force (rotation of wind with height) can be neglected. The estimate obtained agrees quite well with observations.

Under conditions of neutral stratification the processes of turbulent mixing in the ground layer can be described by the system of the logarithmic boundary layer. The corresponding laws have been studied in detail in experimental aerodynamics, and are widely used in meteorology.

Let us bear in mind the derivation of the logarithmic law of wind distribution on the basis of the hypotheses of similitude. Let us assume that for values of $z \gg h_1$, where h_1 is the height of the grass (the characteristic scale of the micro-inhomogeneities of the substrate), the statistical characteristics for relative movements in a stream are invariant with respect to transformations of similitude $x' = kx$, $y' = ky$, $z' = kz$, $t' = kt$. In these transformations the half-space $z > 0$ converges, while the equations of motion remain constant. This factor is the theoretical basis for the accepted hypothesis of similitude. Let us also note that the natural scale of velocity $v_* = \sqrt{\tau/\rho}$ remains invariant with respect to the indicated transformations. Let us examine the stationary regime and establish a ratio of the difference of the averaged velocities at two levels z_2 and z_1 to the dynamic velocity v_*. The corresponding non-dimensional magnitude is a function of z_1 and z_2 and, on the strength of the assumption of the self-similitude of the current, can be a function only of the ratio z_2/z_1:

$$\frac{\bar{v}(z_2) - \bar{v}(z_1)}{v_*} = f(z_2/z_1). \qquad (8)$$

Let us determine the form of function $f(\zeta)$. Evidently for all three heights $z_3 > z_2 > z_1$,

$$\bar{u}(z_3) - \bar{u}(z_1) = \bar{u}(z_3) - \bar{u}(z_2) + \bar{u}(z_2) - \bar{u}(z_1) \quad (9)$$

and along with this,

$$\frac{z_3}{z_1} = \frac{z_3}{z_2} \cdot \frac{z_2}{z_1} \quad (10)$$

From this it follows that function f satisfies the functional equation

$$f(\zeta_1 \cdot \zeta_2) = f(\zeta_1) + f(\zeta_2),$$
$$(\zeta_1 = z_2/z_1,\ \zeta_2 = z_3/z_2) \quad (11)$$

The logarithmic function $f(\zeta) = C \ln \zeta$ is the only solution of this functional equation. Assuming $C = 1/\varkappa$, we get

$$\frac{\bar{v}(z_2) - \bar{v}(z_1)}{v_*} = \frac{1}{\varkappa} \ln \frac{z_2}{z_1} \quad (12)$$

where $\varkappa$ is the familiar Karman constant. According to empirical data, $\varkappa \approx 0.4$. Equation (12) can be written in the usual differential form, examining the infinitely close values z_1 and z_2:

$$\frac{d\bar{v}}{dz} = \frac{v_*}{\varkappa z} \quad (13)$$

Equations (12) and (13) do not contain characteristics of a particular substrate but can pertain to any substrate, if the condition $z_1,\ z_2 \gg h_1$ is fulfilled*). Then, too, formula (13) defines only changes in mean wind velocity with height. The properties of the substrate must be considered in order to determine the absolute value of $\bar{v}(z)$.

Now let us assume that observations of wind velocity are conducted at a definite height H above some definite substrate. Let us assume that we can conduct independent measurements of the turbulent friction and, accordingly, in each individual case we can determine $v_* = \sqrt{\tau/\rho}$. The value v_* can be deter-

*) Determination of the values of height z in formula (13) involves a certain arbitrariness in the choice of the starting point for the computation (within the limits of the height of the grass h_1). However, when $z \gg h_1$, this indefiniteness is of no significant value.

mined, e.g., from thermoanemometer observations of pulsations u' and w', or summarily, on the basis of measurement of the drag intensity at the earth's surface. This latter method is used in practice when studying turbulent motion in tubes. Sheppard [9] attempted to use the dynamometric method of measuring τ under atmospheric conditions.

A comparison of a number of observations of $\bar{v}(H)$ and τ allows us to determine the relationship between these magnitudes. Aerodynamic experiments teach us that with large Reynolds' numbers and surface "roughness" the dependence of τ on $\bar{v}$ is of a quadratic nature, from which it follows that

$$v_* = \gamma(H)\ \bar{v}(H) \tag{14}$$

where $\gamma(H)$ is a non-dimensional coefficient which is a function of the properties of the substrate. At a fixed height H the "drag coefficient" $\gamma(H)$ can serve as an objective characteristic of the properties of the substrate with respect to its dynamic influence on the current. However, use of $\gamma(H)$ has the disadvantage that a specific observation height must be selected. The dependence of $\gamma(H)$ on the observation height H can be easily established by substituting $\bar{v}(H) = v_*/\gamma(H)$ in formula (12). For any two heights H_1, $H_2 \gg h_1$ we will have

$$\frac{1}{\gamma(H_2)} - \frac{1}{\gamma(H_1)} = \frac{1}{\varkappa} \ln \frac{H_2}{H_1} \tag{15}$$

From (15) it follows that, in particular, $\gamma(H)$ decreases with height. Taking the antilogarithms and combining the magnitudes which contain H_1 and H_2 respectively, we get

$$H_1\ [\exp - \varkappa/\gamma(H_1)] = H_2\ [\exp - \varkappa/\gamma(H_2)] = h_0 \tag{16}$$

i.e., a magnitude which is not a function of height. Thus the magnitude h_0, which has length, is determined only by the properties of the substrate; it is called "dynamic roughness." Let us express the drag coefficient $\gamma(z)$ by h_0:

$$\gamma(z) = \frac{\varkappa}{\ln \frac{z}{h_0}} \tag{17}$$

whence on the basis of (14) we get the desired wind velocity distribution:

$$v(z) = \frac{v_*}{\varkappa} \ln \frac{z}{h_0} \tag{18}$$

The method given above for introducing the concept of roughness of the substrate has the advantage that it depends exclusively on the properties of the current at rather great heights, where there are sufficient grounds for using the universal laws of developed turbulence. In most cases, however, we do not have at our disposal the means for making direct measurements of τ (and, accordingly, $\gamma(H)$), and in this regard, when making practical determinations of the characteristics of dynamic roughness, we must use the properties of the wind profile which can be determined directly from observations. When dealing with a mature vegetation cover, additional difficulties arise in connection with choosing the start of computation of z. A number of authors (Paeschke [10], Konstantinov [11]) recommend the use of a certain arbitrary level z_1 for the start of height computations; this level lies between the soil and the top of the grass h_1. This level can be called the height of the displacement layer.

The concept of "displacement height" z_1 can be introduced into the general system as follows. Equation (13) describes the asymptotic regularities occurring when $z \gg h_1$, and in this region it is insensitive to slight changes in the starting point for computation of z (within the limits of the top of the grass h_1). Let us now examine the region of values of z which, although they exceed h_1, are nevertheless comparable with it, and are such that the ratio h_1/z can be treated as a first-order value. To be specific, we will compute z from ground level. In this case, a numerical correction factor $f(h_1/z)$ should be introduced into formula (13); this describes the deviation from the automodular regime, connected with the direct effect of the grass:

$$\frac{d\bar{v}}{dz} = \frac{v_*}{\kappa z} f(h_1/z) \tag{19}$$

Evidently, when $z \to \infty$, formula (19) should convert into (13), from which it follows that $f(0) = 1$. Expanding function f in series, we get

$$\frac{d\bar{v}}{dz} = \frac{v_*}{\kappa z} [1 + \alpha(h_1/z) + \beta(h_1/z)^2 + \ldots] \tag{20}$$

Let us now introduce a new starting point for computations of z, assuming $z = z' + z_1$, where z' is comparable with h_1, and rewrite the equation with respect to the new variables

$$\frac{d\bar{v}}{dz'} = \frac{v_*}{\kappa z'} [1 + (\alpha - \frac{z_1}{h_1}) (h_1/z') + \beta'(h_1/z')^2 + \ldots] \tag{21}$$

Let us select z_1 such that in expansion (21) the first-order term reverts to zero. With a correspondir. choice of z_1 with an accuracy up to the second-order terms, we get

$$\frac{d\overline{v}}{dz} = \frac{v_*}{\kappa(z - z_1)} \tag{22}$$

Thus, the height of the displacement layer can be defined as the height of some arbitrary level of computation, using which we get the best approximation of the wind profile by the logarithmic law in a layer situated above the grass layer. Let us note that the physical determination given above of dynamic roughness h_0 is insensitive to a substitution of $z - z_1$ for z (since $H \gg h_1$); however, in the final formula for the wind velocity profile we should calculate the height from the level of the displacement layer, i.e., replace z by $z - z_1$:

$$\overline{v}(z) = \frac{v_*}{\kappa} \ln \frac{z - z_1}{h_0} \tag{23}$$

The characteristics of the substrate, z_1 and h_0, can be determined empirically on the basis of measurements of wind profile in the layer above the grass level, under conditions close to equilibrium. To increase the computational accuracy we should use data averaged for a group of analogous cases.

Let us use, as an example, values of z_1 and h_0 according to Paeschke's work [10] (table 1):

Table 1

Characteristics of the substrate

	z_1, cm	h_0, cm
Snow surface	3	0.5
Airport	10	2.5
Sugar beet plantation	45	6.5
Wheat field	130	5

Some data on the question of choosing the initial level z_1 can be found in an article by A. R. Konstantinov [11]. It is worth noting that the dynamic roughness of a wheat field is less than that of a sugar beet plantation, although the grass is three times higher in the first case. In the case of a low grass stand (steppe) the value of z_1 does not play an essential role, and when computing h_0 and v_* from observations made at heights of more than 1 meter, we can consider formally that $z_1 = 0$, i.e., we can compute the height directly from the ground.

In further sections of this work, when examining the effect of stratification, we will consider that height is calculated from some arbitrary level (the "displacement layer"), not limiting it in any way, while the dynamic roughness h_0 will be computed by some given characteristic of the substrate which is independent of meteorological conditions.

2. BASIC CHARACTERISTICS OF THE TURBULENT REGIME IN AN INHOMOGENEOUS TEMPERATURE MEDIUM

One of the most important practical characteristics of the turbulent regime in the ground layer of the atmosphere is the vertical turbulent heat flux:

$$q = c_p \rho \overline{w'T'} \tag{24}$$

where c_p is the specific heat of the air at constant pressure, ρ is density, w' and T' are, respectively, the pulsations of the vertical wind velocity component and of temperature, caused by the passage of turbulent elements through a given point, and the bar indicates averaging. The magnitude q is the average amount of heat carried by turbulent pulsations across a unit area per unit time. We have sufficient grounds for considering that for all intents and purposes the turbulent heat flux q in the ground layer under stationary conditions is not a function of height*). Instead of q we may also use the "temperature flux"

$$\frac{q}{c_p \rho} = \overline{w'T'} \tag{25}$$

The magnitude of the turbulent heat flux q can be determined directly experimentally, on the basis of electronic measurements of the pulsations of temperature T' and of the vertical wind velocity component w'. Modern technology has shown that such measurements are possible, in principle [12, 13]. Nevertheless, in practice one must still use indirect methods to determine q, based on simpler gradient measurements. To interpret these measurements correctly, one must investigate the connection between the characteristics of turbulence q and

*) Here we are digressing from an examination of radiational energy fluxes. Strictly speaking, the total flux $q + q_1$ is not a function of height; here q_1 is the radiation flux. Then, too, in the ground layer, changes in the radiation flux q_1 can hardly be considered essential. This question, however, should be the subject of special investigations.

v_* and the distribution of mean wind velocity and temperature. When solving this problem we will follow the methods of the theory of similitude and attempt to establish a system with a minimum number of parameters which describe the turbulent regime in an inhomogeneous temperature medium.

The inhomogeneities of the temperature field, being of a systematic nature (change of mean temperature with height), exert a definite influence on the general turbulent regime (the effect of Archimedean forces). Provided that the temperature pulsations are slight compared with the mean temperature of the layer T_0, the equations for the dynamics of an inhomogeneous temperature medium can be written in the following form:

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p_1}{\partial x}$$

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p_1}{\partial y}$$

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p_1}{\partial z} + \frac{g}{T_0}T_1 \qquad (26)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{dT_1}{dt} = 0$$

In this system, p_1 and T_1 indicate deviations from the standard values. The simplifications made when deriving the system of equations are: neglect of the Coriolis force and the radiation influx of heat, and also the linearization of the standard statistical distribution of pressure and temperature. This latter indicates that changes in density due to pressure changes are neglected, and it assumes that the deviations of density and temperature from the standard values are proportional (L. D. Landau and E. M. Lifshits [14, chapter 5]). These simplifications, used in the convection theory, allow us to describe the Archimedean force by the component $(g/T_0)T_1$. Thus, the equation contains a dimensional constant g/T_0, which we should consider in the future when establishing the criteria of similitude.

Let us note that we cannot linearize the equations of velocity variations, since in this case turbulence would be lost. In addition, in the equations, the terms containing viscosity and heat conductivity would be omitted*). It is

*) Under conditions of a developed turbulent regime, these terms must be con-

natural to assume that changes in mean velocity and temperature with height can be expressed by coordinate z, parameter g/T_o, and the "external parameters" v_* and q, while the corresponding equations can be written in non-dimensional form, since they do not contain other dimensional constants. This proposition is the basic hypothesis of the theory of similitude, formulated in the first section of the present work, generalized for the case of an inhomogeneous temperature medium.

The hypothesis of the theory of similitude which we used agrees with equations (26) and is equivalent to the proposition that the system of equations (26) together with the conditions

$$\overline{w'T'} = \frac{q}{c_p\rho} = \text{const.}$$
$$-\overline{\rho u'w'} = \tau = \text{const.} \qquad (27)$$

are an analogue of the boundary conditions and define the statistical characteristics of the turbulent regime unequivocally. Thus, the three parameters g/T_o, v_*, and $q/c_p\rho$ can be considered the definitive characteristics of the turbulence of the ground layer (in the layer above the grass). From these parameters we can establish unequivocally (with an accuracy of the numerical coefficients) the scale of length L and temperature T_*, which can be written in the following form:

$$L = -\frac{v_*^3}{\varkappa \frac{g}{T_o} \frac{q}{c_p\rho}}, \quad T_* = -\frac{1}{\varkappa v_*} \frac{q}{c_p\rho} \qquad (28)$$

It is natural to use dynamic velocity v_* as the characteristic velocity scale. The minus sign and the Karman constant $\varkappa$ are introduced for the sake of convenience. The signs of L and T_* are determined by the nature of the stratification. With stable stratification the turbulent heat flux is directed downward, $q < 0$, and correspondingly $L > 0$ and $T_* > 0$. With unstable stratification, on the other hand, $q > 0$, $L < 0$ and $T_* < 0$. Thus, we must visualize two qualitatively different regimes, corresponding to the cases $q < 0$ and $q > 0$. These regimes should unite as conditions of neutral stratification ($q = 0$) are approached.

Let us examine the non-dimensional magnitudes $\frac{\varkappa z}{v_*}\frac{d\overline{v}}{dz}$ and $\frac{z}{T_*}\frac{d\overline{T}}{dz}$ (from now on,

sidered only when investigating the very fine details of the microstructure of the wind and temperature field. The vertical transport of momentum and heat is caused by the inhomogeneities of some "mean scale," for which the direct influence of viscosity and heat conductivity are rather slight.

the bar which indicates averaging will be omitted). These non-dimensional characteristics of the averaged field of velocities and temperatures should be definite functions of the "external parameters" and of coordinate z. The only non-dimensional combination which we can make from $q/c_p\rho$, v_*, g/T_0 and z is z/L, from which it follows that

$$\frac{\varkappa z}{v_*}\frac{dv}{dz} = \varphi_1(z/L) \tag{29}$$

$$\frac{z}{T_*}\frac{dT}{dz} = \varphi_2(z/L) \tag{30}$$

or

$$\frac{dv}{dz} = \frac{v_*}{\varkappa z}\varphi_1(z/L) \tag{29'}$$

and

$$\frac{dT}{dz} = \frac{T_*}{z}\varphi_2(z/L) \tag{30'}$$

where T_* and L are determined by formula (28).

Let us introduce the concept of the austausch coefficient. Let us assume formally that

$$\tau = \rho K\frac{dv}{dz}$$
$$q = -c_p\rho K_T\frac{dT}{dz} \tag{31}$$

and call the dynamic austausch coefficient and the coefficient of turbulent heat conductivity K and K_T respectively. Introducing the magnitudes $v_* = \sqrt{\tau/\rho}$ and $T_* = -\frac{1}{\varkappa v_*}\frac{q}{c_p\rho}$ in place of τ and q, and using equations (29) and (30), we get

$$K = \frac{\varkappa v_* z}{\varphi_1(z/L)}, \quad K_T = \frac{\varkappa v_o z}{\varphi_2(z/L)} \tag{32}$$

Now let us examine the hypothesis, shared by a majority of meteorologists, that within the limits of meteorological observations we can consider that $K = K_T$ *),

*) Generally speaking, $K > K_T$, since the effect of pressure pulsations, as well as mixing, can be expressed in a momentum exchange. However, as of now we have

from which it follows that

$$\varphi_1(z/L) = \varphi_2(z/L) = \varphi(z/L) \qquad (33)$$

The similitude of the temperature and wind profiles follows directly from the accepted hypothesis that $K = K_T$. Dividing (30) by (29) we get

$$\frac{dT}{dv} = -\frac{q}{c_p\tau} = \frac{\varkappa T_*}{v_*} \qquad (34)$$

and, accordingly, for any heights H_1 and H_2

$$T(H_2) - T(H_1) = \frac{\varkappa T_*}{v_*}\left[\bar{v}(H_2) - \bar{v}(H_1)\right] \qquad (35)$$

Thus, the ratio of the difference of mean temperatures at two levels H_1 and H_2 to the difference in velocities at the same heights is not a function of choice of heights H_1 and H_2, but is determined entirely by external conditions - the ratio of the turbulent heat flux q to turbulent drag resistance τ.

Let us now show that the non-dimensional factor $\varphi(z/L)$, where $L = -\dfrac{v_*^3}{\varkappa \frac{g}{T_o}\frac{q}{c_p\rho}}$, is directly connected with the Richardson number at a given level. Substituting the values dv/dz and dT/dz, determined from formulas (29) and (30), in the expression for the Richardson number*)

$$Ri = \frac{g}{T_o}\frac{\frac{dT}{dz}}{(dv/dz)^2} \qquad (36)$$

we got

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no conv[illegible] evidence that this difference is essential. The theory developed in the present work can be generalized for the case $\varkappa_T/\varkappa = \alpha \neq 1$, if we replace T by $T\alpha$ in all instances.

*) It follows that T should indicate potential temperature, since T does not change with vertical shifts of the turbulent elements (the state of the latter can be considered adiabatic). In the ground layer the numerical values of potential and molecular temperature are very close. With the large temperature gradients usually observed in the ground layer, the difference between the gradients of potential and molecular temperature are inconsequential; however, in states close to isotherny, this difference is significant.

$$Ri = \frac{g\varkappa^2}{T_0} \frac{T_* z}{v_*^2 \varphi(z/L)} \qquad (37)$$

or, using the determination of the scale of L (28),

$$Ri = \frac{z}{L} \cdot \frac{1}{\varphi(z/L)} \qquad (38)$$

from which it follows that the dependence of the Richardson number on height is defined by a single parameter - the scale of L.

Comparing formula (32) for the austausch coefficient with the expression for the Richardson number, we get an important relationship between the austausch coefficient, the scale of L and the Richardson number:

$$K = \varkappa v_* L \cdot Ri \qquad (39)$$

Let us explain the physical sense of "the scale of L." Under any conditions of stratification we have

$$\frac{dv}{dz} = \frac{v_*}{\varkappa z} \varphi(z/L) \qquad (40)$$

Let us fix value z and decrease magnitude q infinitely, approaching the conditions of neutral stratification, which corresponds to infinite growth of the scale of L (with respect to absolute magnitude). Obviously, within this range, we should obtain formula (22), from which it follows that

$$\varphi(0) = 1$$

Under given external conditions characterized by magnitudes v_* and q and the corresponding magnitude of L, in the region of values of z which are quite small compared to L, $\varphi(z/L)$ will be quite close to unity. This indicates that austausch conditions with $z \ll L$ differ little from austausch conditions in a neutrally stratified atmosphere and, accordingly, turbulence is caused mainly by purely dynamic factors. Thus, the scale of L, first introduced by Obukhov [7], is an important physical characteristic of the state of the ground layer and can be called the height of the substrate of dynamic turbulence. On the strength of the fact that $\varphi(0) = 1$ and formula (38), when $z \to 0$, we get

$$\frac{1}{L} = (\partial Ri/\partial z)_{z=0} \tag{41}$$

This formula can serve as the basis for determining the scale of L from empirical data (from the wind and temperature profiles).

The function $\varphi(z/L)$ should, in the general case, be determined from the aggregate of empirical data. It should be noted that the data available at present are insufficient to determine function φ reliably in a sufficiently wide range of changes of the argument z/L. However, a number of important problems can be solved for the case $z/L < 1$, where we can limit ourselves to the first terms of function φ expanded in series. This case requires special examination.

3. DETERMINATION OF THE TURBULENCE CHARACTERISTICS FROM DATA ON GRADIENT MEASUREMENTS

In the case $|z/L| < 1$ we can limit ourselves to the first terms of the function $\varphi(z/L)$ expanded in - power series and assume

$$\varphi(z/L) = 1 + \beta\frac{z}{L} \tag{42}$$

where β is some universal constant which can be determined on the basis of empirical data. From formulas (29), (30) and (42), integrating with respect to z, we get

$$v(z) = \frac{v_*}{\varkappa}\left[\ln(z/h_0) + \beta(z/L)\right]$$
$$T(z) - T(h_0) = T_*\left[\ln(z/h_0) + \beta(z/L)\right] \tag{43}$$

Here we replaced the components $\beta[(z-h_0)/L]$ by $\beta(z/L)$, with the intention of using formula (43) only when $z \gg h_0$.

Let us note that analogous formulas can be used to describe the profiles of the concentration of any passive substance in the ground layer of the atmosphere. For example, with a stationary turbulent regime and the absence of phase conversions of humidity in the atmosphere, the vertical moisture flux ("rate of evaporation") $E = \rho\overline{w'Q'}$ (Q is specific humidity) can be considered independent of height and, analogously to (30), we can set

$$\frac{dQ}{dz} = \frac{Q_*}{z}\varphi(z/L), \quad Q_* = -\frac{1}{\varkappa v_*}\frac{E}{\rho} \tag{44}$$

whence

$$Q(z) - Q(h_0) = Q[\ln (z/h_0) + \beta (z/L)] \tag{45}$$

Finally, the expression for the austausch coefficient $K = \varkappa v_* L \cdot Ri$ in approximation (42), according to formula (38), will have the form

$$K(z) = \frac{\varkappa v_* z}{1 + \beta \frac{z}{L}} \tag{46}$$

With equilibrium stratification ($|L| = \infty$) we get, from (43), the usual logarithmic formulas for wind and temperature distribution with height. Non-equilibrium of stratification is described in (43) by the component $\beta (z/L)$ and leads to regular distortions of the logarithmic law. With unstable stratification ($L < 0$), intense turbulent mixing leads to equalization of wind velocity in different layers of the atmosphere, so that the wind velocity should increase with height more slowly than in the case of neutral stratification, i.e., $\beta (z/L)$ should be less than zero. Accordingly, $\beta > 0$.

Formulas (43) for $v(z)$ and $T(z)$ are in good quantitative (and, with correct selection of the parameters, also qualitative) agreement with the observed profiles of wind velocity and temperature in the ground layer. Actual measurements confirm the presence and nature of regular deviations of the logarithmic law in the wind and temperature distribution with height, indicated by formulas (43). This can be seen, e.g., from the data of table 2, which shows wind profiles averaged by groups with an identical stability parameter $S = \frac{g}{T_0} \frac{\Delta T}{v^2}$ (taken from data of the Main Geophysical Observatory expeditions of 1945 [15], 1947 [16] and 1950 [17] and the expedition of the Geophysical Institute of the Academy of Sciences of the USSR in 1951 [18]). The form of profiles $v(z)$ and $T(z)$, in agreement with formulas (43), is given in figure 1. Figures 2 and 3 give the averaged profiles of wind velocity and temperature obtained by the 1951 expedition of the Geophysical Institute of the Academy of Sciences of the USSR*).

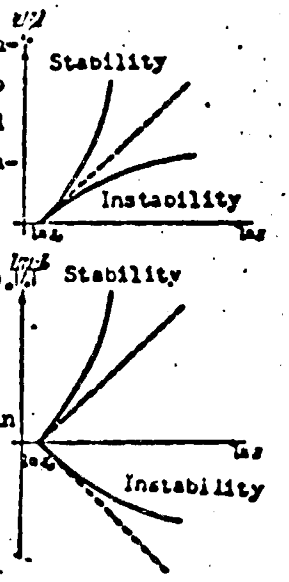

Figure 1. Nature of the wind and temperature profiles.

*) The straight dashed lines in figures 1, 2 and 3 correspond to the logarithmic

Tablo 2

Wind profiles, determined by groups with an identical stability parameter S.

1945 Main Geophysical Observatory expedition

z_0 = 0.2 cm

Ident no.	Number of profiles	100 S	Wind speed (m/sec) at height (m)						[illegible]	[illegible]	[illegible]
			[illegible]	[illegible]	1.0	[illegible]	[illegible]	[illegible]			
—2	18	—3,47	0,55	0,74	0,91	1,21	1,72	2,40	0,13	0,26	0,03
—1	6	—0,22	1,21	1,19	1,76	2,10	2,78	3,80	0,25	0,27	1,0
0	30	0,25	2,41	2,90	3,25	3,60	4,10	4,56	0,30	0,01	46,2
1	21	0,56	1,69	2,00	2,26	2,43	2,68	2,85	0,36	—0,07	—8,2
2	10	1,49	1,32	1,58	1,70	1,82	1,97	2,02	0,27	—0,11	—5,6

1947 Main Geophysical Observatory expedition

z_0 = 0.5 cm

Ident no.	Number of profiles	100 S	Wind speed (m/sec) at height (m)						[illegible]	[illegible]	[illegible]
			[illegible]	1.0	2.0	[illegible]	[illegible]	[illegible]			
—4	3	—5,73	0,74	0,91	1,06	1,40	1,61	1,86	0,16	0,23	2,0
—3	13	—0,91	1,09	1,32	1,48	1,76	1,85	2,09	0,25	0,04	16,2
—2	9	—0,37	1,21	1,50	1,67	1,93	2,03	2,03	0,28	0,002	300,0
—1	13	—0,18	1,68	2,00	2,22	2,57	2,73	2,92	0,37	—0,01	—54,5
0	22	0	1,90	2,24	2,51	2,84	2,90	3,26	0,42	—0,02	—30,0
1	37	0,09	3,36	3,93	4,41	4,85	5,14	5,57	0,75	—0,06	—11,1
2	41	0,26	2,66	3,15	3,50	3,80	3,98	4,18	0,60	—0,07	—9,1
3	34	0,44	2,45	2,91	3,18	3,34	3,63	3,64	0,56	—0,07	—8,3
4	19	0,57	2,24	2,61	2,86	3,12	3,16	3,26	0,50	—0,08	—7,0
5	26	0,74	2,02	2,36	2,60	2,81	2,82	3,00	0,45	—0,08	—5,4
6	14	0,55	1,85	2,13	2,31	2,56	2,60	2,75	0,41	—0,09	—6,3
7	9	1,47	1,32	1,53	1,64	1,63	1,60	1,89	0,29	—0,11	—5,7

profile. The numbers -1, -2, ... , +3 near the curves in figures 2 and 3 correspond to the identification number of the group of profiles in table 2.

Table 2 (conclusion)

1950 Main Geophysical Observatory expedition
z_0 = 0.8 cm

Ident no.	Number of profiles	100 S	Wind speed (m/sec) at height (m) 0,5	1,0	2,0	4,0	8,0	16,0	[illegible]	[illegible]	[illegible]
—6	19	—8,42	0,54	0,65	0,80	1,00	1,31	1,60	0,12	0,46	1,3
—5	9	—1,82	0,89	1,01	1,22	1,50	1,91	2,30	0,20	0,29	2,1
—4	16	—1,18	1,06	1,25	1,45	1,76	2,16	2,58	0,25	0,18	3,3
—3	15	—0,36	1,32	1,79	2,01	2,31	2,71	3,23	0,36	0,07	8,4
—2	17	—0,21	1,80	2,12	2,42	2,78	3,19	3,66	0,43	0,06	10,3
—1	11	—0,13	2,02	2,35	2,70	3,06	3,50	4,00	0,49	0,04	14,3
0	25	—0,03	2,76	3,21	3,69	4,14	4,58	5,00	0,66	—0,01	—60,7
1	27	—0,09	3,35	3,90	4,78	5,40	5,52	6,03	0,73	—0,01	—63,7
2	25	0,14	2,48	2,90	3,33	3,91	4,10	4,40	0,60	—0,10	—20,0
3	29	0,22	2,40	2,80	3,20	3,55	3,88	4,15	0,58	—0,04	—17,1
4	26	01,26	2,56	2,86	3,25	3,00	3,58	4,10	0,60	—0,05	—11,5
5	28	0,29	2,50	2,75	3,10	3,45	3,80	4,10	0,57	—0,03	—12,4
6	33	0,36	2,28	2,50	2,81	3,16	3,16	3,42	0,32	—0,04	—15,0
7	116	00,46	2,01	2,36	2,69	2,98	3,20	3,40	0,50	—0,05	—11,5
8	18	0,66	1,68	1,95	2,19	2,40	2,62	2,80	0,41	—0,05	—12,5
9	22	0,25	1,41	1,65	1,88	2,01	2,20	2,31	0,35	—0,0[illegible]	—10,2
10	15	1,22	1,25	1,11	1,61	1,76	1,97	2,00	0,30	—0,06	—9,2
11	19	1,83	1,00	1,11	1,30	1,47	1,53	1,56	1,56	—0,08	—7,7
12	15	4,06	0,73	0,81	0,91	1,02	1,06	1,10	1,18	—0,09	—4,6

1951 expedition of the Geophysical Institute of the Academy of Sciences of the USSR
z_0 = 1 cm

Ident no.	Number of profiles	100 S	Wind speed (m/sec) at height (m) 0,5	1,0	2,0	4,0	8,0	16,0	[illegible]	[illegible]	[illegible]
—2	6	—0,64	1,32	1,65	1,91	2,31	2,80	3,91	0,33	0,32	1,8
—1	11	—0,21	1,83	2,14	2,55	3,05	3,71	4,62	0,45	0,20	3,0
0	10	0,02	2,97	3,53	4,01	4,72	5,20	5,66	0,76	0,02	24,0
1	7	0,17	4,01	4,61	5,25	5,80	6,41	6,88	1,01	—0,03	—18,2
2	10	0,36	3,09	3,60	4,01	4,45	4,77	5,07	0,78	—0,06	—10,3
3	8	0,81	2,21	2,55	2,86	3,10	3,28	3,45	0,54	—0,06	—7,8

Halstead [19] proposed that the influence of stratification be computed by introducing correction factors into the logarithmic formulas, analogous to (43), but without analyzing the coefficients from the point of view of the theory of similitude.

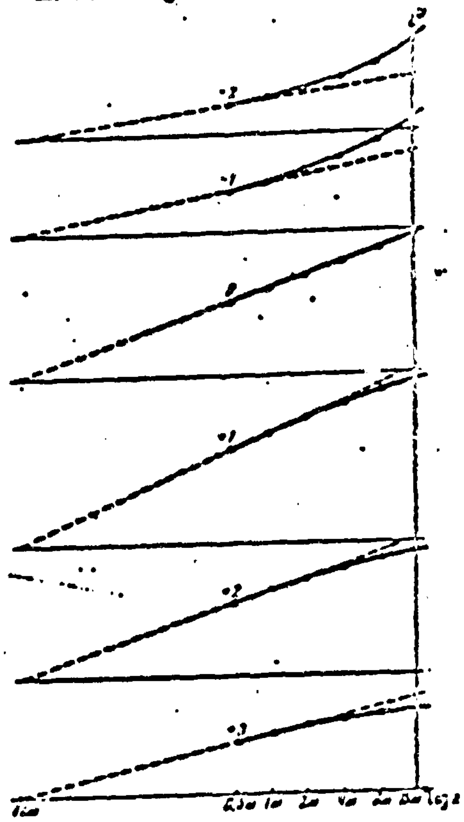

Figure 2. Averaged wind profiles according to empirical data.

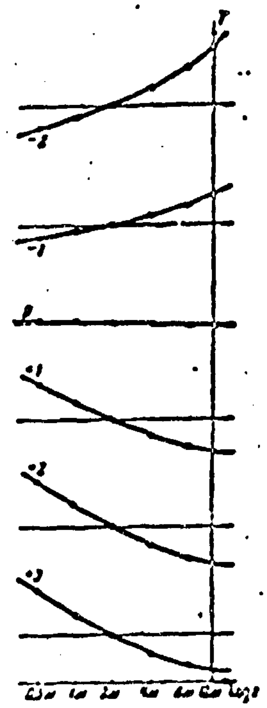

Figure 3. Averaged temperature profiles according to empirical data.

Approximating the measured wind and temperature profiles by formulas (43), we can determine the turbulence characteristics from gradient measurement data. In practice, during such an approximation we must first determine the starting point for computing height z_1 - the thickness of the displacement layer. The magnitude z_1 can be determined experimentally, so that on the graph with the logarithmic scale the wind profiles, corresponding to cases of equilibrium stratification (i.e., actually, to cases of isothermy) would be depicted by straight lines with respect to height. Extrapolating the obtained rectilinear wind profile graph to zero velocity, we get the value of the roughness height h_0.

The magnitude h_0 and the parameters $v_*/\varkappa$ and β/L which enter into formulas (43) can be most accurately determined by using the least-squares method to process the wind profiles measured at the same station, generally speaking, under various conditions of stratification. Thus, assuming

$$v_i(z) = A_i(\gamma + \log z) + C_i z$$

for each profile, where i is the number of the profile, and selecting A_i, γ, and C_i because of the requirement that the sum of the squares of the deviations be minimum,

$$\Delta^2 = \sum_{i,k} [A_i(\gamma + \log z_k) + C_i z_k - v_i(z_k)]^2 \qquad (47)$$

we got for each profile

$$\frac{v_*}{\varkappa} = \frac{A_i}{\ln 10}, \quad \frac{\beta}{L} = \frac{C_i}{A_i} \ln 10$$

and we got a common roughness height $h_0 = 10^{-\gamma}$ for all profiles.

Having determined β/L for each profile by the indicated method, knowing h_0, and computing the value of the stability parameter $S = \frac{g}{T_0} \frac{T(z_1) - T(z_3)}{v^2(z_2)}$ (where, e.g., $z_1 = 0.5$ m, $z_2 = 1$ m, $z_3 = 2$ m), we can determine β, using the formula

$$S = \frac{1}{\beta} \left[\frac{\beta}{L} \frac{\ln z_1/z_3}{\left(\ln \frac{z_2}{h_0}\right)^2} \frac{1 + \frac{\beta}{L} \frac{z_1 - z_3}{\ln z_1/z_3}}{\left(1 + \frac{\beta}{L} \frac{z_2}{\ln z_2/h_0}\right)^2} \right] = \frac{1}{\beta} F(\beta/L) \qquad (48)$$

which follows from (43). The number β can be determined as the regression coefficient of values of $F(\beta/L)$, computed from the previously calculated β/L, for the computed values of S. The regression coefficient β, computed from the data of the four expeditions listed in table 2, is 0.62; the accuracy in determining β in this case is probably not more than 10%. A determination of β from the data of just one Main Geophysical Observatory expedition [16] yielded a value of 0.57.

Using formulas (43) we can compute the drag velocity v_*, as well as the turbulence characteristic which has the most practical interest, i.e., the heat flux q, using the results of wind velocity and temperature measurements at only two heights. For example, let $z_1 = H/2$, $z_2 = H$, and $z_3 = 2H$, and let us assume that the values $T_1 = T(z_1)$, $T_3 = T(z_3)$, and $v_2 = v(z_2)$ m/sec have been measured. Then from (43) we get

$$v_* = \frac{\kappa v_2}{\ln\frac{z_2}{h_0}\left(1 + \frac{\beta}{\ln\frac{z_2}{h_0}} \cdot \frac{H}{L}\right)} = \frac{0.19 v_2}{\log\frac{H}{h_0}} \cdot \frac{1}{1 + \frac{0.26}{\log\frac{H}{h_0}} \cdot \frac{H}{L}} \ \frac{m}{sec}$$

$$q = -\frac{c_p \rho \kappa v_*(T_2 - T_1)}{\ln\frac{z_2}{z_1}\left(1 + \beta\frac{z_2 - z_1}{H \ln\frac{z_2}{z_1}} \cdot \frac{H}{L}\right)} = -0.58 \frac{v_*(T_2 - T_1)}{1 + 0.65\frac{H}{L}} \ \frac{cal}{cm^2 min} \qquad (49)$$

Here we used the value $\kappa = 0.43$ for the Karman constant. The magnitude H/L is determined from relationship (48), which assumes the form

$$\frac{L}{H} = -\frac{0.26}{\log\frac{H}{h_0}} + \frac{1}{2B}\left[1 + \sqrt{1 + 4B\left(0.65 + \frac{0.26}{\log\frac{H}{h_0}}\right)}\right] \qquad (50)$$

where $B = 0.107\, H (\log h_0/H)^2 \cdot \frac{T_2 - T_1}{v_2^2}$, while H and v_2 are expressed in meters and m/sec, respectively.

The influence of stratification on the magnitudes v_* and q is expressed by the appearance of the components with H/L in the denominators of formulas (49). As a rule, the correction for stratification appears to be slight ($H/L \sim 10^{-1}$), which is natural, since turbulence in the lower part of the ground layer is determined mainly by dynamic factors.

Formulas (49) and (50) can be useful when mass-processing the gradient measurement data. In specific cases, with fixed h_0 and H, these formulas acquire a relatively simple form. For example, when $h_0 = 1$ cm and $H = 1$ m, we have

$$B = 0.43\frac{T_2 - T_1}{v_2^2}; \quad L = -0.13 + \frac{1}{2B}\left(1 + \sqrt{1 + 2.1 B}\right) \ m; \qquad (51)$$

$$v_* = \frac{0.095 v_2}{1 + \frac{0.13}{L}} \ \frac{m}{sec}; \quad q = 0.58\frac{v_*(T_2 - T_1)}{1 + \frac{0.65}{L}} \ \frac{cal}{cm^2 min}$$

Examples of computations of the turbulent heat flux q (from data of the Geophysical Institute of the Academy of Sciences of the USSR expedition of 1951) are given in [18]. Computations using specific data show that the scale of L is usually of the order of 10 m, and it approaches 3-4 m only in specific cases with great instability or abrupt inversions. In cases close to isothermy, L reaches values of several tens of meters. The drag velocity v_* is about 8% of the wind velocity at 8 m with unstable stratification, and about 5% with stable stratification. In Kazakhstan, in summer, the turbulent heat flux q reaches 0.25 - 0.35 cal/cm²min on hot sunny days, while it is of the order of 0.06 cal/cm²min at night.

Considering that some researchers use the formulas proposed by Budyko [3] and Laikhtman [5] when determining the turbulence characteristics from gradient measurement data, let us derive the relationship between the scale of L and the basic parameters of the Budyko and Laikhtman formulas. Budyko approximates the wind profiles by the logarithmic law:

$$v(z) = \frac{v_*}{\varkappa m} \ln \frac{mz}{h_0} \qquad (52)$$

where m is a parameter which is a function of atmospheric stratification (with neutral stratification, m reverts to unity). Equating the expressions for $v(z_2)/v(z_1)$, computed from formulas (43) and (52), we get the ratio

$$\frac{\beta}{L} = \frac{\ln m \cdot \ln (z_2/z_1)}{z_1 \ln \frac{z_2}{h_0} - z_2 \ln \frac{z_1}{h_0} + (z_1 - z_2) \ln m} \qquad (53)$$

Passing to the limit when $z_2 \to z_1 = H$, we get

$$\beta \frac{H}{L} = \frac{\ln m}{1 - \ln m + \ln \frac{h_0}{H}} \qquad (54)$$

Laikhtman approximates the wind profiles by the exponential law:

$$v(z) = v(z_1) \frac{z^\delta - h_0^\delta}{z_1^\delta - h_0^\delta} \qquad (55)$$

where δ is a parameter which is a function of atmospheric stratification (with neutral stratification, δ reverts to zero). Equating the expressions for $v(z_2)/v(z_1)$, computed from formulas (43) and (55), we get the relationship

$$\frac{\beta}{L} = \frac{(z_2^\delta - h_0^\delta) \ln \frac{z_1}{h_0} - (z_1^\delta - h_0^\delta) \ln \frac{z_2}{h_0}}{z_2 (z_1^\delta - h_0^\delta) - z_1 (z_2^\delta - h_0^\delta)} \qquad (56)$$

Passing to the limit when $z_2 \to z_1 = H$, we get

$$\beta \frac{H}{L} = \frac{\delta \ln \frac{H}{h_o} - [1 - (h_o/H)^{\delta}]}{1 - \delta - (h_o/H)^{\delta}} \tag{57}$$

Taking advantage of the fact that the value of δ is insignificantly small, and expanding the righthand side of (57) in series according to the δ-exponents, we get the approximation

$$\beta \frac{H}{L} \approx - \frac{\delta}{2} \frac{\ln^2 \frac{h_o}{H}}{\ln \frac{H}{h_o} + 1} \tag{58}$$

4. ASYMPTOTIC FORMULAS FOR THE UNIVERSAL FUNCTION

From formulas (30) and (40) it follows that in a stationary turbulent ground layer, the wind and temperature profiles can be described using one universal function of z/L. Thus, integrating (30) with respect to z and setting $f(\xi) = \int^{\xi} \frac{\varphi(\xi)\, d\xi}{\xi}$, we get

$$v(z) = \frac{v_*}{\varkappa} [f(z/L) - f(h_o/L)]$$
$$T(z) = T(h_o) + T_* [f(z/L) - f(h_o/L)] \tag{59}$$

In the present section we will investigate the form of the universal function $f(z/L)$ taken as a whole.

Since $\varphi(\xi) \to 1$ when $\xi \to 0$, with small z/L the function $f(z/L)$ is of an asymptotically logarithmic nature:

$$f(z/L) \sim \ln \left| \frac{z}{L} \right| + \text{const.} \quad \text{when} \left| \frac{z}{L} \right| \ll 1 \tag{60}$$

With large z/L the asymptotic behavior of function $f(z/L)$ will differ in cases of unstable ($L < 0$) or stable ($L > 0$) stratification, since in these cases there are actually two qualitatively different regimes of turbulent motions.

To analyze the case of unstable stratification, first let us examine the limiting case of purely thermal turbulence (with no wind). In this case, due to the lack of an averaged wind, the friction stress, on an average, will be zero

($v_* = 0$), while the turbulence regime is characterized by only the parameters q and g/T_0 (the turbulence receives its energy exclusively from the instability energy, and therefore is a function only of the degree of instability, characterized by the heat flux $q > 0$ and of the magnitude of the Archimedean forces, characterized by the parameter g/T_0).

We cannot form the scale of length from the parameters q and g/T_0; therefore, the regime of purely thermal turbulence is automodular , i.e., all its characteristics are combinations of q, g/T_0, and z. From the concepts of dimensions we get

$$T(z) = T_{\infty} + \frac{C}{\varkappa^{4/3}} (q/c_p\rho)^{2/3} (gz/T_0)^{-1/3} \tag{61}$$

where C is the non-dimensional (universal) constant, the factor $\varkappa^{-4/3}$ is introduced for convenience, and T_{∞} is a constant which has a temperature dimension.

From (61) it is evident that with an increase in height the distribution of temperature approaches isothermy*). This is natural, since in the case of unstable stratification at great heights, large turbulent elements develop (whose dimensions are limited only by the distance to the earth's surface), bringing about very intense mixing of the air, which leads to an equalization of the temperature profile.

From (61) it follows that the austausch coefficient

$$K = \frac{-q}{c_p\rho \frac{dT}{dz}} = \frac{3}{C} (q/c_p\rho)^{1/3} (g/T_0)^{1/3} (\varkappa z)^{4/3} \tag{62}$$

rapidly increases with height, which is explained by the augmentation of the turbulent elements with an increase in height and the simultaneous increase in the intensity of the pulsations**).

Formally, formula (61) can be written

$$\frac{T(z) - T(h_0)}{T_*} = C (z/L)^{-1/3} - C (h_0/L)^{-1/3} \tag{63}$$

---

*) In (61) we are speaking of the approach to "potential isothermy" with an increase in height (see footnote on page 14).

**) The concepts here presented on the regime of purely thermal turbulence agree with the system proposed by A. A. Skvortsov [20], with the sole difference that Skvortsov introduces a concept of the discrete spectrum of the scales of turbulent formations, while in the system presented here, the spectrum of the scales is assumed to be continuous.

so that in the case of purely thermal turbulence, the universal function $f(z/L)$ (determined with an accuracy of the constant component) has the form $f(z/L) = C(z/L)^{-1/3} + \text{const.}$

The case of purely thermal turbulence can be derived from the general case of unstable stratification by passage to the limit with $v_* \to 0$. Here $L \to 0$ and $z/L \to -\infty$. Therefore the asymptotic behavior of the universal function $f(z/L)$ is determined by the relationship

$$f(z/L) \sim C(z/L)^{-1/3} + \text{const.} \quad \text{when } \frac{z}{L} \ll -1. \tag{64}$$

This result indicates that at great heights $z \gg |L|$ (in the ground layer) the turbulent regime, in the case of unstable stratification, is determined mainly by thermal factors (the wind profile is smoothed, and turbulence receives its energy mainly from the energy of turbulent instability, not from the energy of average motion).

An explanation of the asymptotic behavior of the function $f(z/L)$ when $z \gg L$, in the case of stable stratification, requires that we introduce additional concepts. Turbulence degenerates in the limiting case of abrupt inversion with a vanishingly weak wind. The existence of large turbulent elements becomes impossible in the case of stable stratification (since they must expend too much energy on opposing the Archimedean forces), and turbulence can exist only in the form of small eddies. Large waves cannot lose stability, which is natural from the point of view of the theory of stability. In this case turbulent exchange between different atmospheric layers is hampered and turbulence takes on a local character; at rather high altitudes $z \gg L$ (or, to put it another way, with strong stability, i.e., at low heights $L > 0$) the turbulence characteristics evidently cannot be functions of the distance z to the substrate. This pertains, in particular, to the mixing coefficient K and, accordingly, also to the Richardson number Ri.

Thus, we may consider that in the case of stable stratification with an increase in height z (or, with an increase in stability, i.e., a decrease in L), the coefficient of mixing K and the Richardson number Ri tend toward certain constant values. This is natural, since with an increase in stability, K evidently cannot increase, while Ri cannot decrease. Accordingly, there is a (universal) value R of the Richardson number, which is such that when $z/L \gg 1$,

$$Ri \sim R = \text{const}, \quad K \sim \varkappa v_* L \cdot R \tag{65}$$

The limiting value of R evidently cannot be greater than the critical value Ri_{cr}, but since, asymptotically, $K \neq 0$, i.e., turbulence does not completely degenerate, R should be less than Ri_{cr}. The limiting value obtained will be called the stationary Richardson number.

From (65) it follows that when $z/L \gg 1$, $f'(\zeta)$ should approximate $1/R$, or

$$f(z/L) \approx \frac{1}{R}\frac{z}{L} + \text{const.} \qquad (66)$$

Here we have

$$v(z) \sim -\frac{1}{R}\frac{g}{T_0}\frac{q}{c_p\rho}\frac{z}{v_*^2} + \text{const.} \qquad (67)$$

$$T(z) \sim \frac{1}{R}\frac{g}{T_0}(q/c_p\rho)^2\frac{z}{v_*^4} + \text{const.} \qquad (68)$$

Our formulas (60), (64) and (66) show the behavior of function $f(\zeta)$ when $|\zeta| \ll 1$, $\zeta \ll -1$ and $\zeta \gg 1$, respectively.

For an empirical determination of the universal function $f(\zeta)$ in a sufficiently broad range of changes in the parameter ζ, using the data of the four expeditions, given in table 2, and determining v_* and L (when $\beta = 0.6$) for each wind profile, we construct the empirical universal function

$$\frac{\varkappa}{v_*}[v(z) - v(|L|/2)] = f(z/L) - f(\pm 1/2)$$

where the plus sign corresponds to stable stratification, and the minus sign to unstable stratification.

The empirical points obtained are plotted on the graph in figure 4. The graph gives convincing evidence of the suitability of the hypotheses of similitude used in the present work; these hypotheses reduce to the existence of a single universal function $f(z/L)$. The empirical points lie along smooth curves with a very small scatter, despite the inaccuracies of the wind measurements and the computation of L and v_* by the approximation methods shown above. Some scatter of the points is noted only in highly stable cases. The drawing shows the limiting behavior of the curve quite well for the case of high stability (approaching a linear profile) and high instability (approaching a constant).

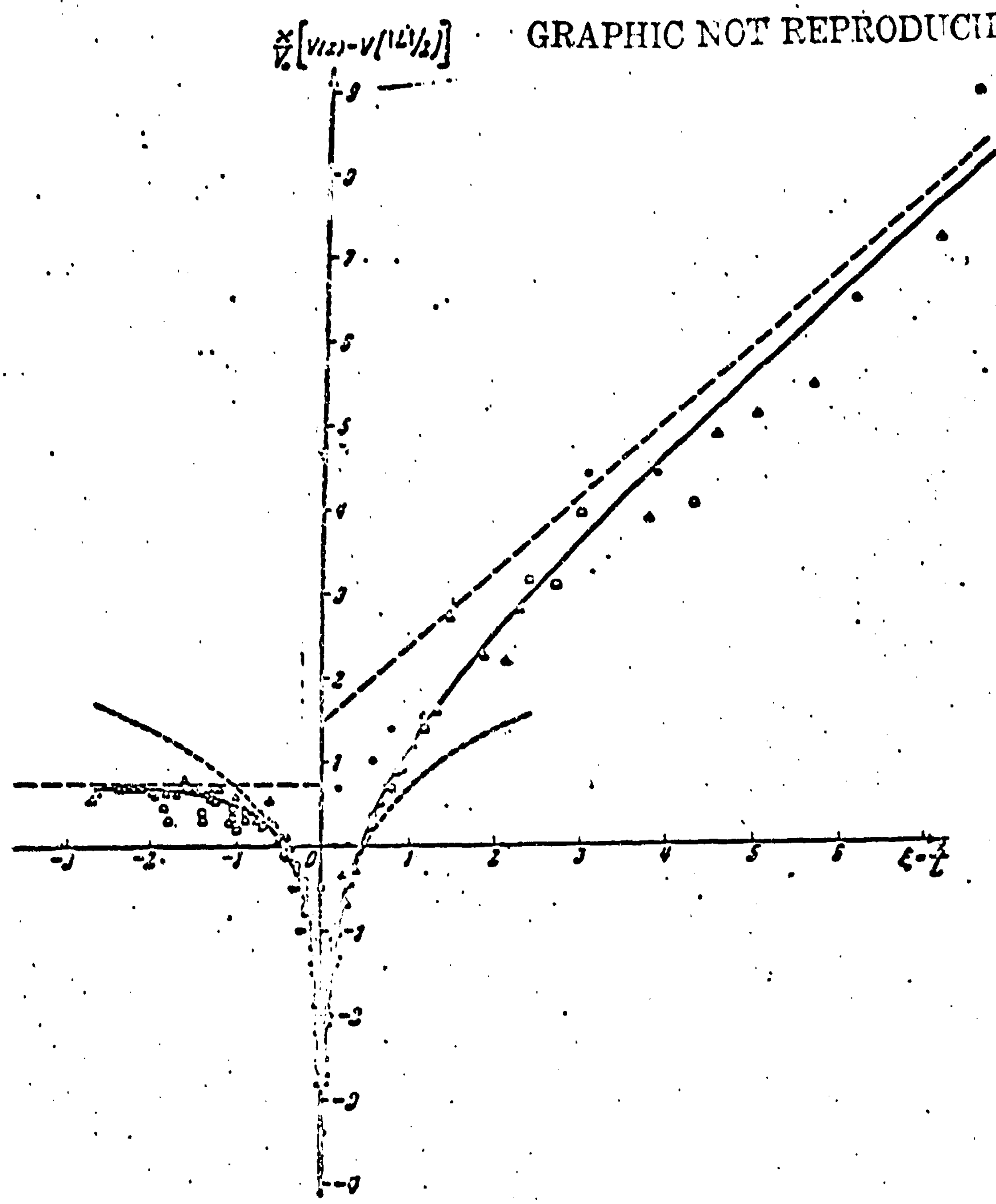

Figure 4. Wind velocity distribution in non-dimensional coordinates

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