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# INTERTEMPORAL BANK ASSET CHOICE WITH STOCHASTIC DEPENDENCE

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SRM 188

INTERTEMPORAL BANK ASSET

CHOICE

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## ERRATA

IN THE COVARIANCE MATRIX ON PAGE 11, THE ENTRIES "1" SHOULD READ "$\sigma_A^t$".

## 1. INTRODUCTION

This paper extends existing models of inter-temporal bank asset management (see [1], [5]) in the following respects:

(a) Bank customers are identified, with requirements that their demands for loan renewals be satisfied. Opportunities are provided for attracting new customers;

(b) feedback relationships between loans and deposits are introduced;

(c) costs of servicing loans with different degrees of risk are introduced explicitly;

(d) future deposits and loan repayments are expressed as jointly dependent random variables;

(e) the Federal Reserve Board's liquidity leverage suggestions are replaced by chance-constraints on meeting demands for loans. This leads to a policy of balancing maturities in the bond portfolio.

The format of the model is that of chance-constrained programming, with piecewise linear approximations to the non-linear constraints. A 5-period example, with parameterizing on the right hand side, is presented.

## 2. STRUCTURE OF THE MODEL

### (a) Loans

We assume that in each period $t$ $(t = 1, \ldots, T)$ the bank can allot its funds in six ways $(r = 1, \ldots, 6)$, apart from cash:

$x_{1t}$ = early maturity government bonds

$x_{2t}$ = late maturity government bonds

$x_{3t}$ = loans to new industrial customers

$x_{4t}$ = loans to new industrial customers, but more risky

$x_{5t}$ = renewals of existing industrial loans

$x_{6t}$ = personal loans

The opportunities for making new loans are constrained by the demand, which varies from period to period. Let

$d_{it}$ = demand for new loans in period $t$ $(i = 3, 4)$.

Then

$$0 \leq x_{it} \leq d_{it} \qquad i = 3, 4; \; t = 1, \ldots, T \ldots \quad (1)$$

All industrial loans are assumed repaid over two periods, and the bank, wishing to accommodate its customers, will meet all demands for renewal. The same applies to personal loans, a high proportion of which are to be renewed each period. We have not assumed a market limit on the amount of this type of loan which can be made. Let

$w_{it}$ = proportion of expiring loans to be renewed $(i = 5, 6)$

Then

$$x_{5t} = w_{5t} (x_{3t-n_2} + x_{4t-n_4} + x_{5t-n_5}) \quad t = 1, \ldots, T \quad \ldots \quad (2)$$

$$x_{6t} \geq w_{6t} \, x_{6t-n_6} \quad t = 1, \ldots, T \quad \ldots \quad (3)$$

$d_{it}$ and $w_{it}$ may be represented as parameters jointly influenced by the level of economic activity in each time period, and similarly with prevailing loan charges and bond prices, but for the present we shall take them as known constants.

(b) Service Costs

The cost of a loan is not simply a function of its size, but depends largely upon its riskiness, which determines the time taken to service it. An opportunity cost representation seems most appropriate. Let

$a_i$ = time taken to service loan type $i$ (per dollar per period) $\quad i = 1, \ldots, 6.$

$A_t$ = service time available in period $t$.

$$\sum_{i=1}^{6} a_i \, x_{it} \leq A_t \quad t = 1, \ldots, T \quad \ldots \quad (4)$$

(c) Deposits

$z_t$ = deposits brought by new customers attracted in in periods $i, \ldots, t$, $\quad z_o = 0$.

$\lambda_i$ = compensating balance with loan type $i$.

$$z_t = z_{t-1} + \sum_i \lambda_i x_{it} \qquad t = 1, \ldots, T. \quad \ldots \quad (5)$$

$\tilde{S}_t$ = other deposits in time $t$ -- a random variable.

Let $\tilde{s}_t = \tilde{S}_t - \tilde{S}_{t-1}$, and let $\tilde{s}_t$ be independently normally distributed:

$$\tilde{s}_t \sim N(\bar{s}_t, \theta_t).$$

(d) Cash Reserves

The bank is obliged to keep a minimum cash reserve against its deposits. Let

$\delta$ = minimum cash reserve (proportion).

$h_t$ = cash kept during period $t$.

$$h_t \geq \delta (S_t + z_t) \qquad t = 1, \ldots, T. \quad \ldots \quad (6)$$

(e) Returns

Let $n_i$ = duration of loan (or bond) type $i$

$m_{it}$ = proportion of loan $i$ to be repaid in $\tau^{th}$ period after loan made, where

$$\sum_{\tau=1}^{n_i} m_{i\tau} = 1.$$

$y_{it}$ = planned repayments loan type $i$ in period $t$.

We have

$$y_{it} = \sum_{\tau=1}^{n_i} m_{it} x_{it-\tau} \qquad i = 1, \ldots, 6; \; t = 1, \ldots, T \ldots (7)$$

For $i = 1$, we shall have $m_{i1} = 1$, hence $y_{1t} = x_{1t-1}$.

Let $\tilde{f}_{it}$ = random variable representing defaults and mistimed repayments of loan type i in period t, where the $f_{it}$ are assumed joint normally distributed with means $\bar{f}_{it}$ and covariance matrix $\sigma_{ij}^{(t)}$.

Then actual repayments in period t are given by

$$\sum_i (1 - f_{it}) y_{it} .$$

$\bar{f}_{it}$ is the expected default rate on loan i in period t, and a value of $\tilde{f}_{it}$ below $\bar{f}_{it}$ represents a late repayment, whereas $\tilde{f}_{it}$ greater than $\bar{f}_{it}$ represents a repayment before schedule. We shall take $\bar{f}_{it} = 0$, $\sigma_{it} = 0$ for government bonds.

(f) Budget Constraints

It will be convenient to first write the budget constraints using expected values.

$$x_{1t} + \sum_{i=2}^{6} x_{it} + h_t = \sum_{i=2}^{6} (1 - \bar{f}_{it}) y_{it} + x_{1t-1} + h_{t-1} + \bar{s}_t + z_{1t} - z_{t-1}$$

$$t = 1, \ldots, T \quad \ldots \quad (8a)$$

Investment in short-maturity (one period) government bonds $(x_{it})$ takes up the slack caused by the random nature of $f_{it}$ and $s_t$. In (8a), $x_{1t}$, $x_{1t-1}$ may be regarded as expected values.

By substituting successively for $x_{1t-1}, x_{1t-2}, \ldots$ we derive a cumulative form of (8a):

$$x_{1t} + \sum_{\tau=1}^{t} \sum_{i=2}^{6} x_{i\tau} + h_t = \sum_{\tau=1}^{t} \sum_{i=2}^{6} (1-\bar{f}_{it})\, y_{i\tau} + x_{1o} + h_o + \sum_{\tau=1}^{t} \bar{s}_\tau + z_t \qquad t=1,\ldots,T \quad \ldots \quad (8b)$$

$x_{1o}$, $h_o$ are known initially; recollect $z_o = 0$.

Now, we wish to ensure that with probability of at least $100\,\alpha\,\%$, the planned program of loans $(i = 3, \ldots 6)$ is feasible. Planned bond purchases $(i = 1,2)$ are secondary to this. Accordingly, we write

$$\text{Prob}\,\{ \sum_{i=3}^{6} x_{it} + h_t \le \sum_{i=2}^{6} (1-\tilde{f}_{it})\, y_{it} + x_{1t-1} + h_{t-1} + \tilde{s}_t + z_t - z_{t-1} \} \ge \alpha \qquad t = 1,\ldots,T \qquad (9a)$$

or in cumulative terms

$$\text{Prob}\,\{ \sum_{\tau=1}^{t-1} x_{2\tau} + \sum_{\tau=1}^{t} \sum_{i=3}^{6} x_{i\tau} + h_t \le \sum_{\tau=1}^{t} \sum_{i=2}^{6} (1-\tilde{f}_{i\tau})\, y_{i\tau} + x_{1o} + h_o + \sum_{\tau=1}^{t} \tilde{s}_\tau + z_t \} \ge \alpha \qquad t=1,\ldots,T \quad \ldots \quad (9b)$$

The deterministic equivalent for (9b) may be obtained in the usual way [ ]. Upon rewriting and introducing expected value terms, we have

$$\text{Prob}\,\{ \sum_{\tau=1}^{t} \sum_{i=2}^{6} (\tilde{f}_{i\tau} - \bar{f}_{i\tau})\, y_{i\tau} - \sum_{\tau=1}^{t} (\tilde{s}_\tau - \bar{s}_\tau) \le$$

$$\sum_{\tau=1}^{t} \sum_{i=2}^{6} (1-\bar{f}_{i\tau})\, y_{i\tau} + x_{1o} + h_o + \sum_{\tau=1}^{t} \bar{s}_\tau + z_t - \sum_{\tau=1}^{t-1} x_{2\tau} - \sum_{\tau=1}^{t} \sum_{i=3}^{6} x_{i\tau} \} \ge \alpha \qquad t = 1,\ldots,T \quad \ldots \quad (10).$$

But notice that the R. H. S. within (10) is precisely $x_{1t} + x_{2t}$, as defined by (8b). Upon substituting

$$F\left( \frac{x_{1t} + x_{2t}}{\text{s.d.}_t} \right) \ge \alpha$$

hence $$x_{1t} + x_{2t} \geq s.d._t \, F^{-1}(\alpha) \qquad t = 1, \ldots, T \quad \ldots \qquad (11)$$

where F is the cumulative N(0,1) distribution function, and $s.d._t$ is the standard deviation of the l.h.s. term of (10). (11) ensures that government bonds form a "cushion" against having to cut down on planned loans if returns plus deposits are lower than expected. With $\alpha = 0.95$, a cushion greater than 1.645 standard deviations of returns plus deposits is provided.*

Notice the relationship between variability of loans and deposits ($s.d._t$) and the confidence level ($\alpha$). An increase in one has exactly the same effect as an appropriate increase in the other. We may expect the optimal confidence level to be inversely related to the variability of returns and deposits, for as the latter increases, the cost of maintaining a given confidence level will in general increase.

The form of $s.d._t$, and piecewise linear approximations to (11) will be discussed in section 4.

(g) Objective Function

We take as the objective function the maximization of returns on loans, less interest paid on deposits, over a T-period horizon. Let

$r_{it}$ = per dollar return on loan (bond) type i made in period t, cumulated up to maturity of loan (bond) or horizon, whichever earlier.

*Our constraint seems to embody the conclusions of Hodgman [4]: "Therefore, the task of management is to anticipate those deposit drains which, while exceeding the amounts that can be met by borrowing at the Federal Reserve Bank, have a good probability of occurrence without involving the entire banking system and thus forcing a liberalization of central bank policy. The size of such drains determines the 'rock bottom' for the investment portfolio." p. 74

$c_t$ = interest paid on deposits in period t.

The objective function is then

$$\text{Max} \sum_{t=1}^{T} \sum_{i} r_{it} x_{it} - \sum_{t=1}^{T} c_t z_t \qquad \ldots \qquad (12)$$

Some form of horizon constraint may also be included, but we have not done so here.

## 3. SUMMARY OF THE MODEL

$$\text{Max} \sum_{t=1}^{T} \sum_{i} r_{it}\, x_{it} - \sum_{t=1}^{T} c_t\, z_t$$

Subject to

$$0 \le x_{it} \le d_{it} \qquad i = 3, 4$$

$$x_{5t} = w_{5t}\left(x_{3\,t-n_3} + x_{4\,t-n_4} + x_{5\,t-n_5}\right)$$

$$x_{6t} \ge w_{6t}\, x_{6\,t-n_6}$$

$$\sum_i a_i\, x_{it} \le A_t$$

$$z_t = z_{t-1} + \sum_i \lambda_i x_{it}$$

$$h_t \ge \delta\left(\bar{S}_t + z_t\right)$$

$$y_{it} = \sum_{\tau=1}^{n_i} m_{i\tau}\, x_{i\,t-\tau}$$

$$\sum_i x_{it} + h_t = \sum_i \left(1 - \bar{F}_{it}\right) y_{it} + h_{t-1} + \bar{S}_t + z_t - z_{t-1}$$

$$x_{1t} + x_{2t} \ge F^{-1}(\alpha)\ \text{s.d.}_t$$

## 4. PIECEWISE - LINEAR APPROXIMATIONS

Hartley and Hocking [3] discuss a method of replacing the non-linear constraint

$$f(x) \leq b \qquad \ldots (13)$$

by a set of linear restrictions formed by taking the tangent planes to the surface at a set of points $x^*$. This yields the set of linear inequalities.

$$f(x) + \sum_{j=1}^{n} \frac{\partial f(x^*)}{\partial x_j} \cdot (x_j - x_j^*) \leq b$$

If $f(x) \leq b$ defines a convex set, the polyhedral space defined by the inequalities closes down upon the original constraint region as the $x^*$ net becomes finer.

Hartley and Hocking develop a pricing algorithm for generating the $x^*$, but in the relatively simple case which we have, with only four non-linear constraints, it is easier to make finer approximations in successive runs, as the optimal region is gradually located. We also make additional simplifications for computational convenience.

We have to approximate the set of non-linear constraints

$$x_{1t} + x_{2t} \geq s.d._t \; F^{-1}(\alpha) \qquad t = 1, \ldots, T \qquad \ldots (11)$$

Suppose that $\tilde{r}_{it}$ and $\tilde{s}_t$ were distributed independently;

then

$$(s.d._t)^2 = \sum_{\tau=1}^{t} \sum_{i=2}^{6} \sigma_{i\tau}^2 y_{i\tau}^2 + \sum_{\tau=1}^{t} \omega_\tau^2$$

We shall assume, however, that in each period repayments on all industrial loans ($\iota$ = 3,4,5) are completely correlated, and independent of repayments on personal loans ($\iota$ = 6). Both are independent of deposits.

The covariance matrix is thus

$$\sigma_{ij}^{(t)} = \begin{bmatrix} \sigma_A^t & 1 & 1 & 0 \\ 1 & \sigma_A^t & 1 & 0 \\ 1 & 1 & \sigma_A^t & 0 \\ 0 & 0 & 0 & \sigma_B^t \end{bmatrix} \qquad i,j = 2,\ldots,6$$

Where $\sigma_A^{\ t}$ is the variance of returns on industrial loans, $\sigma_B^{\ t}$ on personal loans. We shall further assume, for computational convenience, that $\sigma_A^{\ t} = \sigma_B^{\ t} = \sigma$. The variance on government bonds (i =1,2) is assumed zero. In this case

$$(s.d_t)^2 =$$

$$\sigma^2 \sum_{\tau=1}^{t} \left[ \left( \sum_{i=3}^{5} y_{i\tau} \right)^2 + y_{6\tau}^{\ 2} \right] + \sum_{\tau=1}^{t} \theta_\tau^{\ 2} \qquad t=1,..,T \qquad \ldots (15)$$

Represent the R.H.S. by $\sigma^2 I_t + II_t$, where $II_t = \sum_{t=1}^{t} \theta_t^2$ is a known constant.

(11) reads

$$x_{1t} + x_{2t} \geq F^{-1}(\alpha) \sqrt{\sigma^2 I_t + II_t} \qquad \ldots\ldots\ldots (16)$$

It will now be convenient to split the s.d.$_t$ term and obtain the slightly more restrictive representation of (16)

$$x_{1t} + x_{2t} \geq F^{-1}(\alpha)\, \sigma \sqrt{I_t} + F^{-1}(\alpha) \sqrt{II_t} \qquad .\ .\ (17)$$

Introducing set of spacer variables $u_t$, we obtain

$$x_{1t} + x_{2t} \geq F^{-1}(\alpha)\, \sigma\, u_t + F^{-1}(\alpha) \sqrt{II_t}$$

$$u_t^{\ 2} \geq I_t$$

that is:

$$x_{1t} + x_{2t} \geq F^{-1}(\alpha)\, \sigma.\, u_t + F^{-1}(\alpha) \sqrt{\sum_{\tau=1}^{t} \theta_\tau^{\ 2}} \qquad \ldots \qquad (18)$$

$$u_t^{\ 2} \geq \sum_{\tau=1}^{t} \left[ \left( \sum_{i=3}^{5} y_{i\tau} \right)^2 + y_{6\tau}^{\ 2} \right] \qquad \ldots\ldots \qquad (19)$$

(18) is of course a linear constraint; (19) remains to be approximated. Write

$$y_{At} = \sum_{i=3}^{5} y_{it} \qquad \text{and } y_{Bt} = y_{6t} \qquad (20)$$

We now have a constraint of the form of (13)

$$f(y_{At}, y_{Bt}, u_t) = \sqrt{\sum_{t=3}^{t} (y_{At}^2 + y_{Bt}^2)} - u_t \leq 0 \quad \ldots (21)$$

We achieve the approximation by making a set of assumptions about the proportions $\{ P_{At}, P_{Bt} \}$ in which the $\{y_{At}, y_{Bt}\}$ be.

For any such set of proportions, the point defined by

$$y_{A\tau} = \frac{P_{A\tau} \cdot u_t}{\sqrt{\sum_{\tau=1}^{t} P_{A\tau}^2 + P_{B\tau}^2}} , \quad y_{B\tau} = \frac{P_{B\tau} \cdot u_t}{\sqrt{\sum_{\tau=1}^{t} P_{A\tau}^2 + P_{B\tau}^2}} ,$$

lies on the surface of (21) (regarding $u_t$ as constant for the moment). Derivatives of f are given by

$$\frac{\partial f}{\partial y_{A\tau}} = \frac{y_{A\tau}}{u_t} , \quad \frac{\partial f}{\partial y_{B\tau}} = \frac{y_{B\tau}}{u_t} , \quad \frac{\partial f}{\partial u_t} = -1 ,$$

which, evaluated at the chosen point, are

$$\frac{P_{A\tau}}{\sqrt{\sum_{\tau=1}^{t} P_{A\tau}^2 + P_{B\tau}^2}} , \quad \frac{P_{B\tau}}{\sqrt{\sum_{\tau=1}^{t} P_{A\tau}^2 + P_{B\tau}^2}} , \quad -1$$

Substituting these into (14) yields.

$$\sum_{\tau=1}^{t} \left( \frac{P_{A\tau} \cdot y_{A\tau}}{\sqrt{\sum_{\tau=1}^{t} P_{A\tau}^2 + P_{B\tau}^2}} + \frac{P_{B\tau} \, y_{B\tau}}{\sqrt{\sum_{\tau=1}^{t} P_{A\tau}^2 + P_{B\tau}^2}} \right) \leq u_t \qquad t = 1, \ldots, T \quad \ldots (22)$$

(22) is of course a set of linear inequalities, and each set $(P_{At}, P_{Bt})$ we choose will generate such a set. The non-linear constraint set (19) can therefore be approximated as closely as desired by appropriate choice of sets $(P_{At}, P_{Bt})$.

In the following example, it did not seem unreasonable to take $P_{At} = P_A$, $P_{Bt} = P_B$ for $\forall_t$, as a first choice, because (i) ratios of industrial to personal loans might be expected to remain roughly constant, and (ii) the amount of loans in one period would be roughly equal to that in another.

The first set of ratios used were 3:1, 4:1, 5:1.

After the first run, the set 3:1 was tight so the sets 4:1, 5:1 were replaced by 2.5:1, 3.5:1, and the set 3:1 still remained tight. These approximations were used to give the present results.

## 5. AN EXAMPLE

A number of prototype problems have been run, and we reproduce below a 5 period example with two different forecasts of deposits and loan demands, one involving an initial increase in deposits followed by a decrease, and the other with the opposite pattern. The parameters might refer to anything from thousands to hundreds of thousands of dollars.

(a) The parameters

| t | $d_{3t}$ I | $d_{3t}$ II | $d_{4t}$ I | $d_{4t}$ II | $\bar{s}_t$ I | $\bar{s}_t$ II | $\theta_t$ |
|---|---|---|---|---|---|---|---|
| 1 | 500 | 50 | 200 | 100 | 400 | 400 | 0 |
| 2 | 600 | 0 | 800 | 100 | 2000 | -200 | 50 |
| 3 | 300 | 0 | 500 | 200 | 100 | 0 | 100 |
| 4 | 0 | 250 | 200 | 250 | -1000 | 200 | 200 |
| 5 | 0 | 600 | 200 | 600 | -2000 | 600 | 400 |

| i | $a_i$ | $\lambda_i$ | $\sigma_{it} = \sigma_i$ | $m_{i1}$ | $m_{i2}$ | $m_{i3}$ | $\bar{f}_{it}$ | % Return per period, t | $x_{io}$ | $x_{i-1}$ | $x_{i-2}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 3 | 1200 | - | - |
| 2 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 5 | 1000 | 1000 | 1000 |
| 3 | 2 | 0.2 | 0.3 | 0.75 | 0.25 | 0 | 0 | 6.5 | 400 | 400 | - |
| 4 | 3 | 0.2 | 0.3 | 0.75 | 0.25 | 0 | 0 | 7 | 0 | 0 | - |
| 5 | 2 | 0.2 | 0.3 | 0.75 | 0.25 | 0 | 0 | 6.5 | 2400 | 2400 | - |
| 6 | 1 | 0 | 0.3 | 1 | 0 | 0 | 0 | 5.5 | 1500 | - | - |

| $r_{it}$: % Cumulative Return | | | | | | |
|---|---|---|---|---|---|---|
| $t \backslash i$ | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 3 | 15 | 13 | 14 | 13 | 5.5 |
| 2 | 3 | 15 | 13 | 14 | 13 | 5.5 |
| 3 | 3 | 15 | 13 | 14 | 13 | 5.5 |
| 4 | 3 | 10 | 13 | 14 | 13 | 5.5 |
| 5 | 3 | 5 | 6.5 | 7 | 6.5 | 5.5 |

| For all t: | |
|---|---|
| $w_{5t} = 0.9$ | $\delta = 0.08$ |
| $w_{6t} = 0.9$ | $\alpha = 0.99$ |
| $A_t = 8500$ | |
| $C_t = 3\%$ | $h_o = 720$ |
| $\sigma_{it} = 0.3$ | |

(b) <u>Results</u>

Projected Loans and Investments, Forecast I

| Period\Loan | $h_t$ | $x_{1t}$ | $x_{2t}$ | $x_{3t}$ | $x_{4t}$ | $x_{5t}$ | $x_{6t}$ | $z_t$ | $\bar{s}_t$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 804 | 0 | 2250 | 500 | 200 | 2520 | 1350 | 644 | 400 |
| 2 | 1019 | 1311 | 2593 | 600 | 348 | 2520 | 1215 | 1337 | 2000 |
| 3 | 1084 | 1114 | 3263 | 300 | 317 | 2927 | 1094 | 2047 | 100 |
| 4 | 1057 | 2341 | 3048 | 0 | 200 | 3153 | 984 | 2717 | -1000 |
| 5 | 952 | 0 | 6518 | 0 | 200 | 3222 | 886 | 3402 | -2000 |

Projected Loans and Investments, Forecast II

| Period\Loan | $h_t$ | $x_{1t}$ | $x_{2t}$ | $x_{3t}$ | $x_{4t}$ | $x_{5t}$ | $x_{6t}$ | $z_t$ | $\bar{s}_t$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 795 | 1135 | 1564 | 50 | 100 | 2520 | 1350 | 534 | 400 |
| 2 | 821 | 1255 | 1929 | 0 | 100 | 2520 | 1215 | 1058 | -200 |
| 3 | 863 | 984 | 2939 | 0 | 200 | 2427 | 1094 | 1583 | 0 |
| 4 | 925 | 1032 | 3667 | 250 | 250 | 2382 | 984 | 2160 | 200 |
| 5 | 1029 | 0 | 5728 | 519 | 600 | 2388 | 886 | 2861 | 600 |

It may be useful to translate the above into Balance Sheet figures.

For period t, outstanding loans are aggregated as follows:

Government bonds maturing in one period = $x_t + x_{2t-2}$

Government bonds maturing in two period = $x_{2t-1}$

Government bonds maturing in three period = $x_{2t}$

Industrial loans = $\sum_{i=3}^{5} x_{it} + 0.25 \sum_{i=3}^{5} x_{it-1}$

Personal loans = $x_{4t}$

Projected Balance Sheets, Forecast I

| Asset \| Period | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Cash | 720 | 804 | 1019 | 1084 | 1057 | 952 |
| Government Bonds maturing in 1 period | 2280 | 1000 | 2311 | 3364 | 4934 | 3263 |
| Government Bonds maturing in 2 periods | 1000 | 1000 | 2250 | 2593 | 3263 | 3048 |
| Government Bonds maturing in 3 periods | 1000 | 2250 | 2593 | 3263 | 3048 | 6518 |
| Industrial Loans | 3500 | 3920 | 4273 | 4411 | 4191 | 4277 |
| Personal Loans | 1500 | 1350 | 1215 | 1094 | 984 | 886 |
| Total | 10,000 | 10,324 | 13,661 | 15,809 | 17,477 | 18,944 |

Projected Balance Sheets, Forecast II

| Asset \| Period | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Cash | 720 | 795 | 821 | 863 | 925 | 1029 |
| Government bonds maturing in 1 period | 2280 | 2135 | 2255 | 2548 | 2961 | 2939 |
| Government bonds maturing in 2 periods | 1000 | 1000 | 1564 | 1929 | 2939 | 3667 |
| Government bonds maturing in 3 periods | 1000 | 1564 | 1929 | 2939 | 3667 | 5728 |
| Industrial Loans | 3500 | 3370 | 3275 | 3284 | 3602 | 4384 |
| Personal Loans | 1500 | 1350 | 1215 | 1094 | 984 | 886 |
| Total | 10,000 | 10,214 | 11,059 | 12,657 | 15,078 | 18,633 |

## 6. DISCUSSION

The following are some of the interesting features of the solution. (I or II in parentheses refers to forecast.) (i) Although autonomous deposits at times decrease, the bank continues to expand its asset base by attracting new customers.

(ii) In some periods, the bank can attract all the new customers it desires. In other periods it would be willing to pay more for them. Using information provided by the dual variables, we see that increasing $d_{31}$ (I) by one dollar would bring a return of 0.206, or an average of 4.1% per period.

(iii) We have set the return on risky new industrial loans ($i = 4$) to be higher than that on safer new industrial loans ($i = 3$), and in periods when staff for servicing loans is adequate, the former are preferred. But in periods 2 (I) for example, when staff are fully occupied the marginal value of a less risky customer (averaged over the horizon) is nearly 1.4% whereas that of the risky customer is zero.

(iv) The costs of servicing customers can cause a shift to government bonds, which are relatively simple to purchase. In period 2 (I) long term government securities are preferred to risky new industrial customers, despite the fact that the latter yield a higher return and bring in deposits.

(v) The shadow price of the service time constraint enables us to evaluate the worth of hiring extra staff to service more difficult loans. Under forecast I, extra staff hired for the whole of the 5-year period would bring in an average return of 2-1/4 %, (but 0 % under forecast 2), which should be set against the cost of hiring them.

(vi) The obligation to renew existing loans is never onerous in the present example, but in other examples we have run opportunity costs have been incurred in some periods.

(vii) The value of an extra unit of capital, raised in the first period (I), averages 5.4%. If we had included in the model capital leverage constraints, as in Chambers and Charnes [ 1 ], the value would presumably have been considerably higher.

(viii) The worth of a dollar of autonomous deposits in the first period only (I), may be calculated to be 5%. This takes into account the 8 % cash balance that must be held.

(ix) The lack of a terminal constraint is apparent in that purchases of short term government securities are allowed to be zero in the last period, but it is doubtful whether this significantly affects first period decisions, which are what the bank will be most concerned about.

(x) Observe that the bank maintains a balanced portfolio of government bonds (with respect to maturity dates). In particular, there is a steady inflow of maturing securities in each period. This provides a safeguard for the bank against a downturn in deposits or laggardly loan repayments. In case there is a liquidity crisis affecting the whole banking system, the spread of maturities, in particular the presence of short-dated ones, minimises the chances of losses as bond prices are depressed. The bank is also enabled to purchase longer maturities with monies becoming available if a steady inflow in the immediate future has been planned. In general, such securities, if held to maturity, are more profitable.

The examiners' criteria of the F.R.B. imply a somewhat more conservative policy than the above. It may be that they are designed to protect customers and bankers against risks of bankruptcy, rather than as a useful guide for liquidity policy in a time of more stable economic conditions. In the latter case the idea of a portfolio of steadily maturing loans may be more appropriate, and it is hoped that the techniques employed in the present paper will be useful in developing such a policy

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[1] Chambers, D. and A. Charnes, "Intertemporal Analysis and Optimization of Bank Portfolios," Management Science, Vol. 7, No. 4, July 1961.

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13. ABSTRACT


This paper extends existing models of inter-temporal bank asset management in the following respects:

(a) Bank customers are identified, with requirements that their demands for loan renewals be satisfied. Opportunities are provided for attracting new customers;
(b) feedback relationships between loans and deposits are introduced;
(c) costs of servicing loans with different degrees of risk are introduced explicitly;
(d) future deposits and loan repayments are expressed as jointly dependent random variables;
(e) the Federal Reserve Board's liquidity leverage suggestions are replaced by chance-constraints on meeting demands for loans. This leads to a policy of balancing maturities in the bond portfolio.

The format of the model is that of chance-constrained programming, with piecewise linear approximations to the non-linear constraints. A 5-period example, with parameterizing on the right hand side, is presented.


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| Chance-constraints | | | | | | |
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