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CONVAIR (ASTRONAUTICS) DIVISION
GENERAL DYNAMICS CORPORATION

REPORT NO AE-60-0125
DATE 2 February 19
NO. OF PAGES 28

CONVAIR ASTRONAUTICS

CONVAIR DIVISION OF GENERAL DYNAMICS CORPORATION

RESPONSE OF A SINGLE DEGREE OF FREEDOM SYSTEM TO VARIOUS TRANSIENTS

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FOREWORD

Concern regarding the response of various structures to short pulses generated several questions as to the maximum deflection and the pulse duration at which pulse shape becomes important. In order to gain some knowledge of these problems analytical solutions were made for several cases without damping. Later, analog computer solutions, including damping were made.

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SUMMARY

The ratio of maximum deflection of a one degree of freedom system to the static deflection (corresponding to the peak force) is approximately independent of pulse shape (within ±5%) for effective pulse durations (impulse/peak force) less than about 20% of the free natural period of the system. The above deflection ratio equals unity when the effective pulse duration is about 17 to 18% of the free natural period of the system for an undamped system. For damped systems the duration for unity deflection ratio is somewhat greater.

THE PROBLEM

The deflection of an undamped one-degree-of-freedom system subjected to various types of transient forces has been calculated. Of particular interest was the maximum deflection and the way in which this maximum varies with the duration of applied transient force.

It is well known that, for short enough pulses, the motion is more a function of impulse rather than peak force and, in this case, the shape of the force-time pulse is of no consequence. The area of the force-time pulse determines the impulse and resulting motion.

As the pulse length increases, the shape of the pulse becomes important. There are four questions which come to mind and which are at least partly answered by the results in this memorandum.

1. At what pulse duration should shape be considered important?

2. At what pulse duration does the maximum deflection produced by the transient just equal the static deflection corresponding to the peak force?

3. What maximum deflections are obtained with various pulse shapes and durations?

4. What is the effect of damping on the foregoing questions?

ANALYTICAL SOLUTIONS

The body considered is shown in Figure 1 where m = mass, k = spring rate (assumed constant), F = force and x = displacement. Gravity is neglected.

For a limited number of relatively simple cases analytical solutions were obtained without the effects of damping.

Method of Solution

The system equation is

$$m\ddot{x} + kx + F = 0$$

and the solution from reference 1 is

$$dx = \frac{q\,dt}{p} \sin p(t_1 - t) \qquad p = \sqrt{\frac{k}{m}}, \quad q = \frac{F}{m}$$

where dx is the incremental displacement at time t_1 resulting from an impulse qdt at time, t, and $p = 2\pi/\tau$ where τ is the free natural period, (see Figure 2).

$$\tau = 2\pi\sqrt{\frac{m}{k}}$$

If one writes the static deflection corresponding to the peak force, F_p,

$$x_s = \frac{F_p}{k} = \frac{F_p}{m}\left(\frac{\tau}{2\pi}\right)^2$$

the solution becomes,

$$dx = \left\{\frac{F}{m} \frac{\tau}{2\pi} \sin \frac{2\pi}{\tau}(t_1 - t)\right\} dt$$

$$= \left\{\frac{-F_p}{m}\left(\frac{\tau}{2\pi}\right)^2 \frac{F}{F_p} \sin \frac{2\pi}{\tau}(t_1 - t)\right\} d\left(\frac{-2\pi}{\tau} t\right)$$

$$\frac{dx}{x_s} = \frac{-F}{F_p} \sin\left[\frac{2\pi}{\tau}(t_1 - t)\right] d\left(-\frac{2\pi}{\tau} t\right)$$

which can be directly integrated after substitution of a particular function for F/Fp which defines the pulse shape.

Where t_1 is within the pulse period, the integration is between 0 and t_1; otherwise, the integration is from 0 to t_p, i.e., the pulse duration.

Pulses Considered

Two pulse shapes were considered: rectangular and triangular, as shown in Figure 3. These show $q = F/m$ as a function of t/τ where the peak in each case is $q_p = F_p/m$.

Various durations are shown in Figure 3 for which solutions were obtained.

Results

The solutions are given below for x/x_s, and the results are shown on Figures 4 and 5.

Rectangular Pulse ($t_p/\tau = 1/8$)(Figure 4A)

$$\frac{x}{x_s} = \left[\frac{\sqrt{2}}{2} \sin 2\pi \frac{t_1}{\tau} - \left(1 - \frac{\sqrt{2}}{2}\right) \cos 2\pi \frac{t_1}{\tau} \right]_{t_1 > t_p}$$

Rectangular Pulse ($t_p/\tau = 1/4$)(Figure 4B)

$$\frac{x}{x_s} = \left[\sin 2\pi \frac{t_1}{\tau} - \cos 2\pi \frac{t_1}{\tau} \right]_{t_1 > t_p}$$

Rectangular Pulse ($t_p/\tau = 1/2$)(Figure 4C)

$$\frac{x}{x_s} = \left[1 - \cos 2\pi \frac{t_1}{\tau} \right]_{t_1 < t_p}$$

$$x_s = \left[-2 \cos 2\pi \frac{t_1}{\tau} \right]_{t_1 > t_p}$$

Rectangular Pulse ($t_p/\tau = 1$)(Figure 4D)

$$\frac{x}{x_s} = \left[1 - \cos 2\pi \frac{t_1}{\tau} \right]_{t_1 < t_p}$$

$$\frac{x}{x_s} = \left[0 \right]_{t_1 > t_p}$$

Triangular Pulse ($tp/\tau = 1/2$) (Figure 5A)

$$\frac{x}{x_s} = \left[\left(1-2\,\frac{t_1}{\tau}\right) - \cos 2\pi\frac{t_1}{\tau} + \frac{1}{\pi}\sin 2\pi\frac{t_1}{\tau}\right]_{t_1 < t_p}$$

$$\frac{x}{x_s} = \left[\frac{2}{\pi}\sin 2\pi\frac{t_1}{\tau} - \cos 2\pi\frac{t_1}{\tau}\right]_{t_1 > t_p}$$

Triangular Pulse ($tp/\tau = 1$) (Figure 5B)

$$\frac{x}{x_s} = \left[\left(1 - \frac{t_1}{\tau}\right) - \cos 2\pi\frac{t_1}{\tau} + \frac{1}{2\pi}\sin 2\pi\frac{t_1}{\tau}\right]_{t_1 < t_p}$$

Triangular Pulse ($tp/\tau = 2$) (Figure 5C)

$$\frac{x}{x_s} = \left[\left(1-\frac{t_1}{2\tau}\right) - \cos 2\pi\frac{t_1}{\tau} + \frac{1}{4\pi}\sin 2\pi\frac{t_1}{\tau}\right]_{t_1 < t_p}$$

This completes the analytical solutions which were calculated.

Maximum Deflection

If one selects the maximum values of x for any time at a given value of tp/τ and pulse shape, a plot can be made of x_{max}/x_s versus tp/τ.

This has been done; the result is shown on Figure 6 for the positive deflection and Figure 7 for the negative deflection.

Values of x_{max}/x_s versus tp/τ were obtained from Reference 2 for three other pulse shapes shown in Figure 8. In the nomenclature of this memo the results of Reference 2 are as follows:

Triangular whip

$$\frac{x_{max}}{x_s} = \left[\frac{2(1 - \cos\pi\, tp/\tau)}{\frac{t_p}{\tau}}\right]_{\frac{t_p}{\tau} < 0.5}$$

For $t_p > 0.5$ see Figure 4 of Reference 2.

Sine Whip

$$\frac{x_{max}}{x_s} = \frac{4\frac{t_p}{\tau}\cos\pi\frac{t_p}{\tau}}{1-4\left(\frac{t_p}{\tau}\right)^2} \qquad \frac{t_p}{\tau} < 0.5$$

For $\frac{t_p}{\tau} > 0.5$ see Figure 6 of Reference 2.

Shifted Cosine Whip

$$\frac{x_{max}}{x_s} = \frac{\sin\pi\, t_p/\tau}{1-\left(\frac{t_p}{\tau}\right)^2} \qquad \frac{t_p}{\tau} < 0.5$$

For $\frac{t_p}{\tau} > 0.5$ see Figure 8 of Reference 2.

These results are also plotted on Figure 6.

Solutions for x_{max}/x_s for rectangular pulses and for the shifted cosine pulse were also available from Reference 3 and agreed quite well with the results described above for these cases.

If one defines an effective pulse length

$$t_{pe} = \frac{\text{Impulse}}{\text{Peak force}}$$

then t_{pe} equals tp for rectangular pulses, 0.5 tp for triangular or shifted cosine pulses and 0.637 tp for sine pulses.

The values of the maximum deflection in terms of the effective pulse duration to free natural period ratio are shown on Figure 9. Notice that for the $t_{pe}/\tau < 0.2$ the pulse shape effect on deflection is less than ±5% of the mean value. For values of $t_{pe}/\tau > 0.2$ the pulse shape becomes important.

Conclusions from Analytical Solutions

1. The deflection of an undamped body is very little affected (±5% approximately) by pulse shape for effective pulse durations less than about 0.2 of the free natural period; the deflection is governed mainly by the impulse.

2. For rectangular pulses the maximum deflection of an undamped body just equals the static deflection (peak force) if the pulse duration equals 0.167 of the free natural period. For triangular pulses the corresponding ratio is 0.35. For the pulses considered, the maximum deflection equals the static deflection when the effective pulse duration equals $(0.17 \text{ to } 0.18)\tau$.

3. The maximum deflection to static deflection ratio of an undamped body for any pulse length has the following values:

   a. Rectangular pulse 2.0
   b. Triangular pulse $\rightarrow$ 2.0 as $tp/\tau \rightarrow \infty$
   c. Triangular whip 1.53
   d. Sine whip 1.77
   e. Shifted cosine whip 1.70

4. For non-abrupt, finite pulses (3c, d and e above) with $tp < \tau/3$ or $tp > 3\tau$ the maximum deflection to static deflection ratio does not exceed 1.20.

ANALOG COMPUTER SOLUTIONS

Method of Solution

In order to obtain knowledge of the effects of damping on the maximum deflection a series of solutions were made on the analog computer using the well-known loop having two integraters and two arbitrary constants, namely, spring constant and damping factor.

The value of the damping factors used were as follows; 0, 0.03, 0.1, 0.3 and 1.0 where the numbers refer to the fraction of critical damping.

Pulses Considered

The pulses considered were the rectangular pulse, triangular pulse, triangular whip, sine whip and a triangular ramp (linear rise followed by an abrupt drop to zero). These names have no special significance other than to distinguish the various cases in the discussion.

Results

The results of the analog computer solutions are shown on Figures 10 through 14. In each case, the pulse actually used is shown (at twice amplitude for clarity). In each case, the pulse was set and solutions were made for the various values of damping factor.

The peak values of the normalized deflection, x_{max}/x_s, were read from these curves and are listed in Table 1. The peak values are also plotted on Figures 15 through 18 as a function of effective pulse duration as previously defined. Each figure is for a particular value of damping factor and shows the results for the various pulse shapes. The results for a damping factor of 0.03 were not plotted because they were virtually identical to those for zero damping. The differences may be obtained from Table I.

There were several opportunities to check the analog results against the analytical results. In seven such checks, the values of x_{max}/x_s agreed within 1% on the average, the worst case (for a short pulse) being 4%. This probably could have been improved by slowing down the problem, but this was not considered important enough to warrant the effort.

Conclusions from Analog Computer Solutions

1. The maximum deflection of a body with or without damping is very little affected (±5% approximately) by pulse shape for effective

pulse durations less than about 0.2 of the free natural period; the deflection is governed mainly by the impulse.

2. The effective pulse duration at which the maximum deflection just equals the static deflection depends on the pulse shape and damping factor, the variation with pulse shape increasing with increasing damping factor.

3. The maximum deflection is reduced by damping.

REFERENCES

1. S. Timoshenko, Vibration Problems in Engineering, D. Van Nostrand, (1955).

2. J. T. Muller, Transients in Mechanical Systems, Bell System Technical Journal, 27 657-683 (Oct. 1948) (Also Bell System Monograph B-1601).

3. V. Rosa, Private communication, Convair-Astronautics Dynamics Group (Oct. 29, 1959).

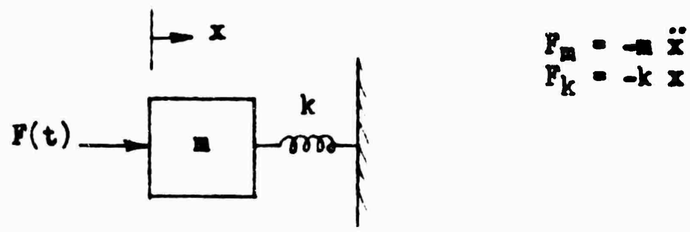

Figure 1 - One Degree of Freedom Body--No Damping

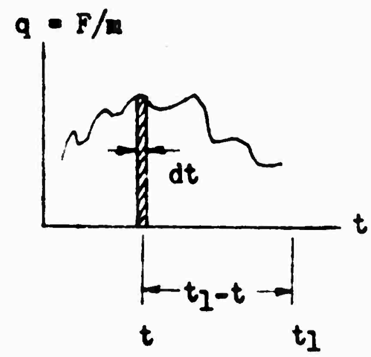

Figure 2 - Schematic Diagram

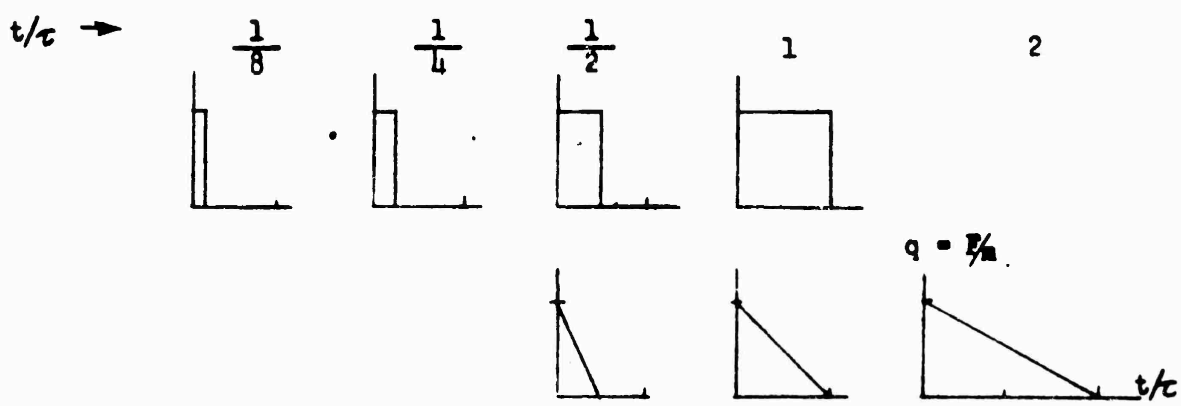

Figure 3 - Pulses Considered for Analysis
(peak q = q_p = Fp/m in all cases)

Figure 4
Response to Rectangular Pulses of Various Periods

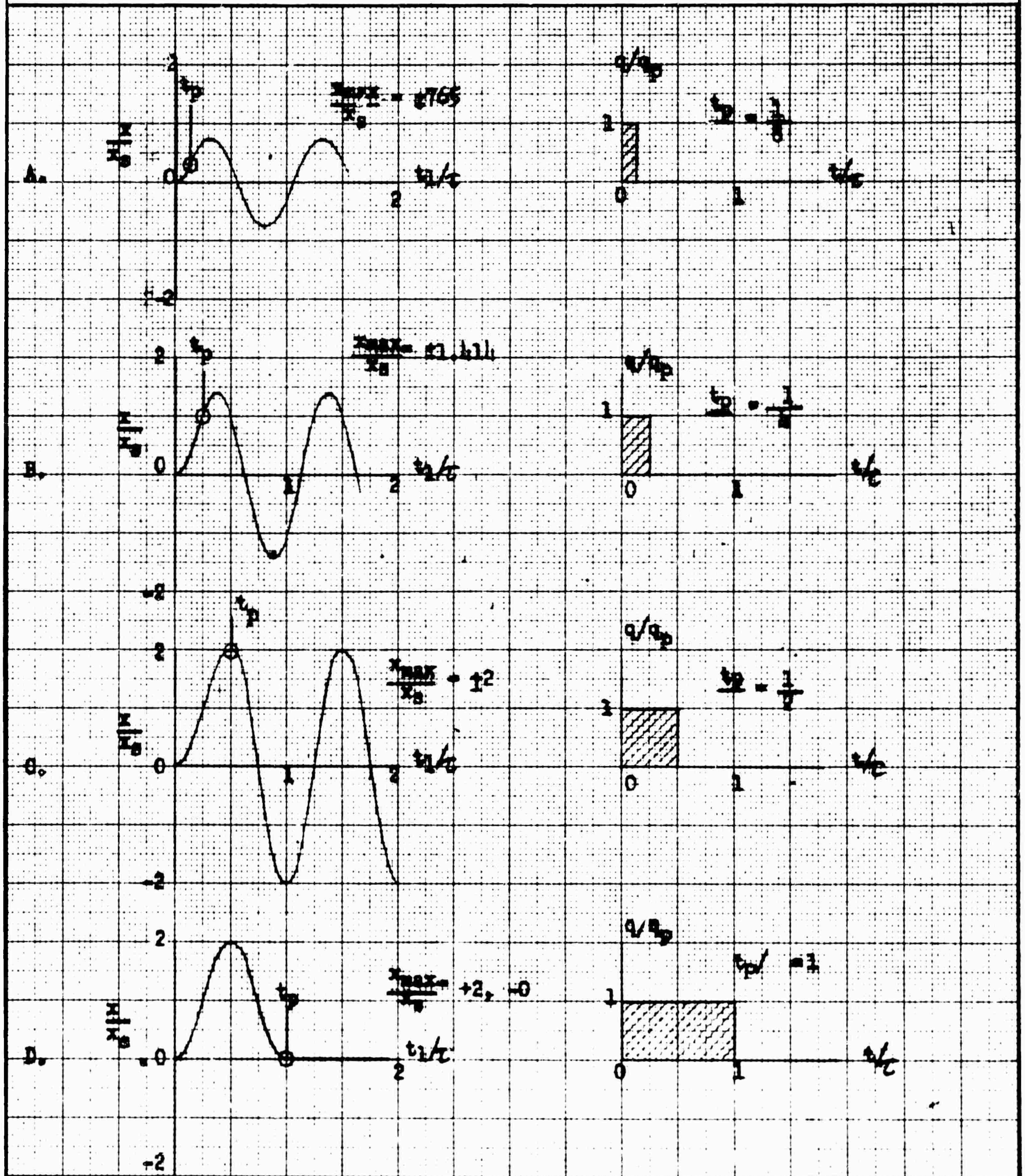

Figure 5
Response to Triangular Pulses of Various Durations

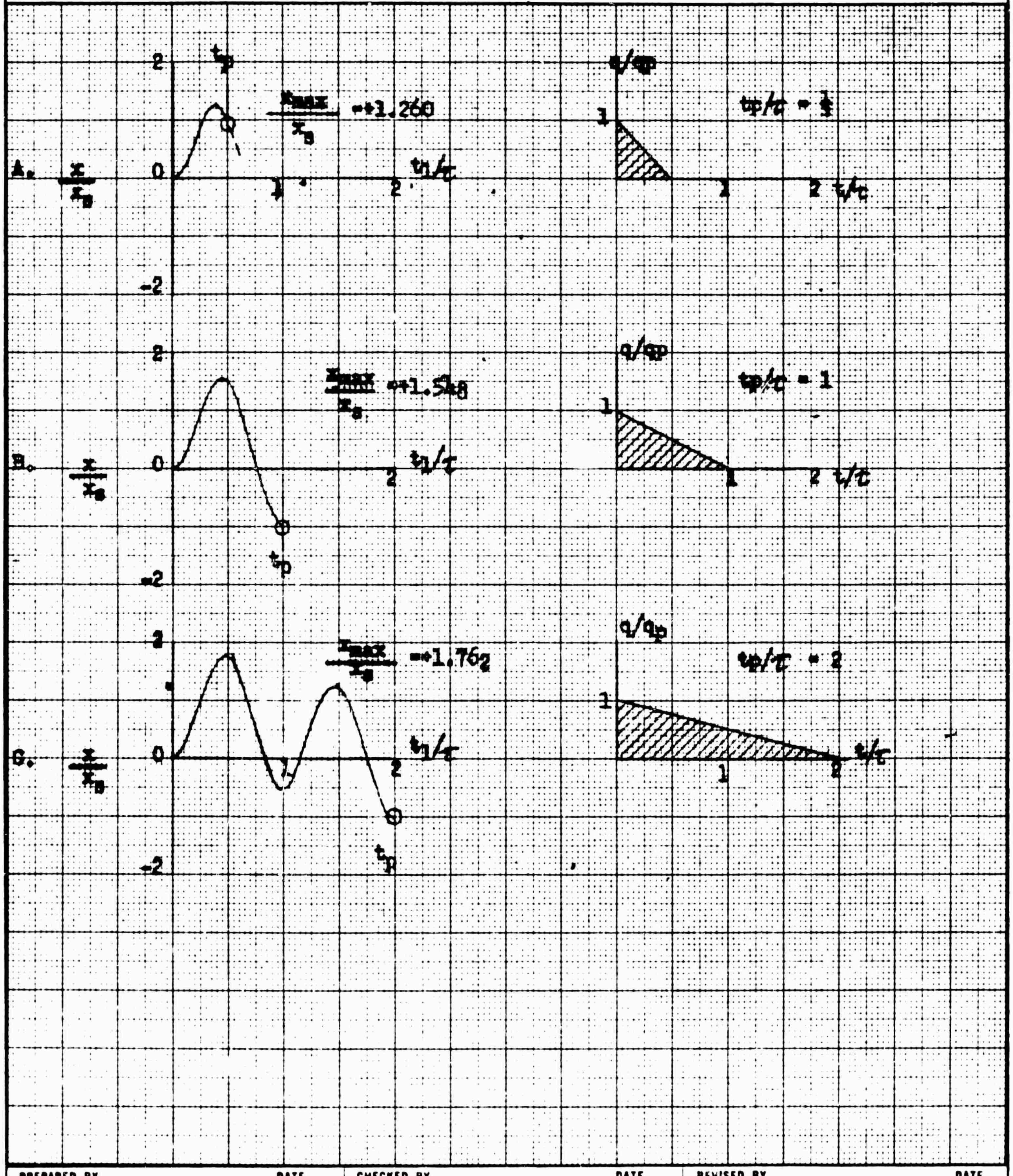

Figure 6
Maximum to Static Positive Deflection Ratio
for Various Pulse Shapes and Durations

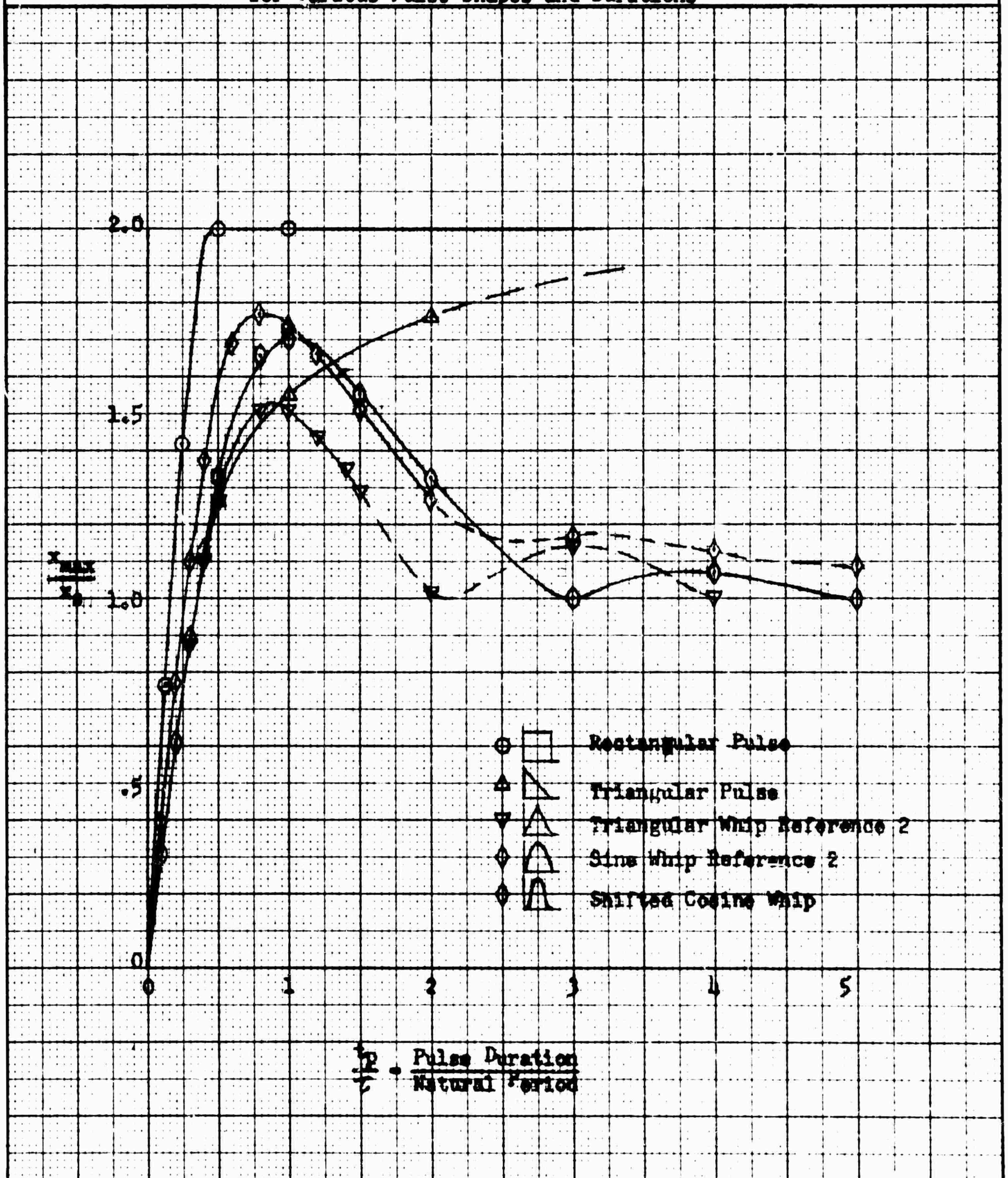

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Figure 7
Maximum to Static Negative Deflection Ratio
for Various Pulse Shapes and Durations

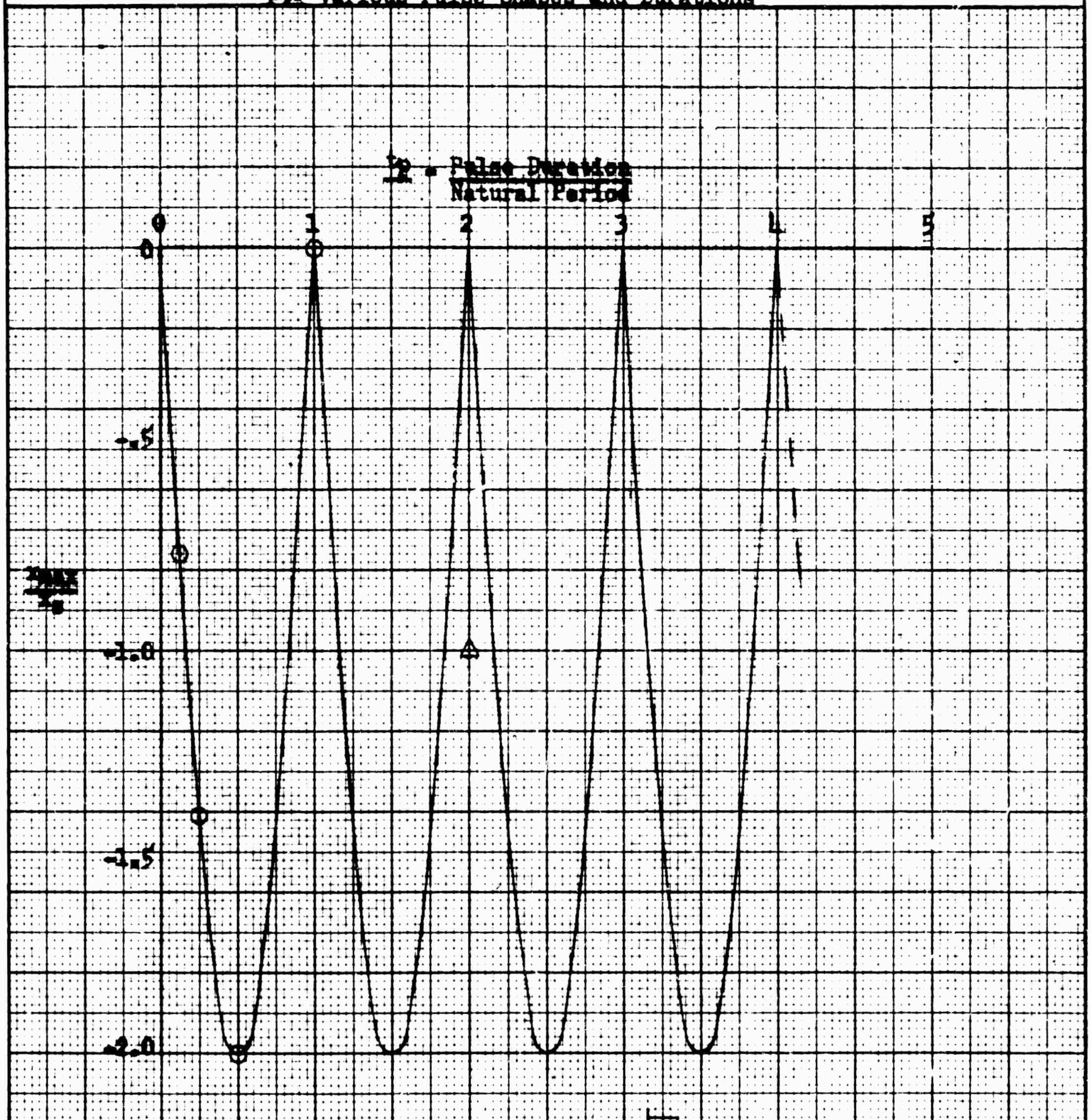

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Figure 8
Pulse Shapes (Whips) Used in Reference 2

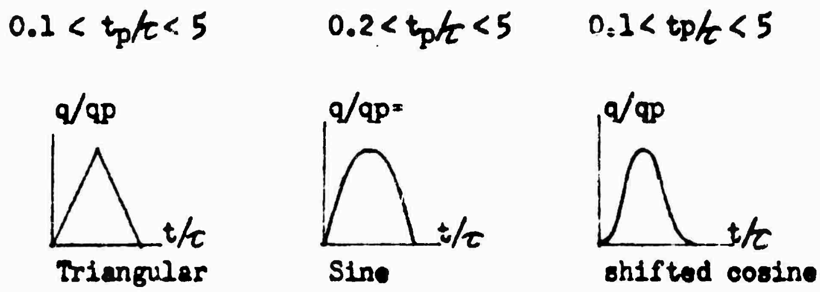

Figure 9

Maximum to Static Deflection Ratio for Various Pulse Shapes and Effective Durations (based on Impulse)

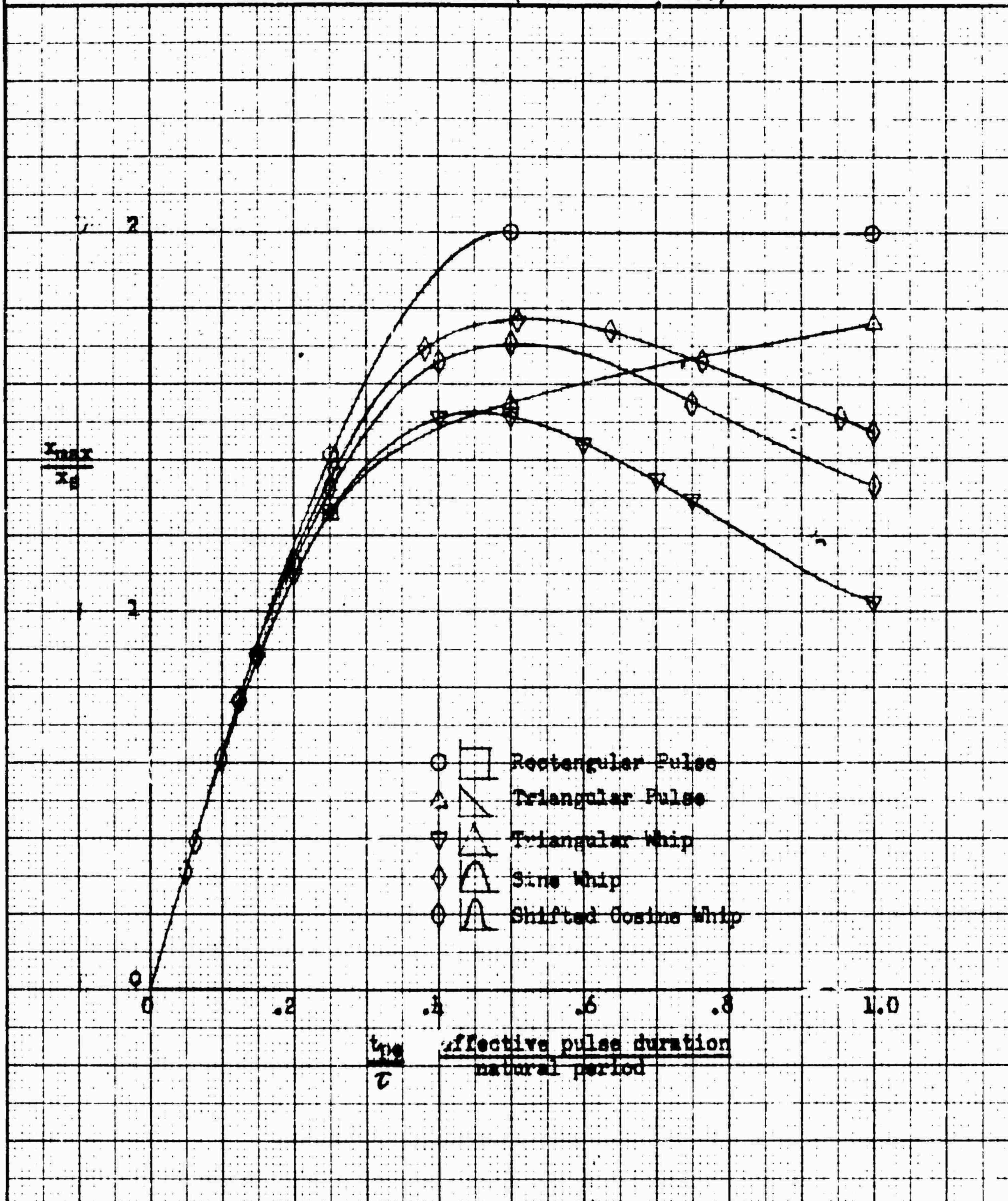

Figure 10
Deflection of a One Degree of Freedom
System Produced by a Rectangular Pulse

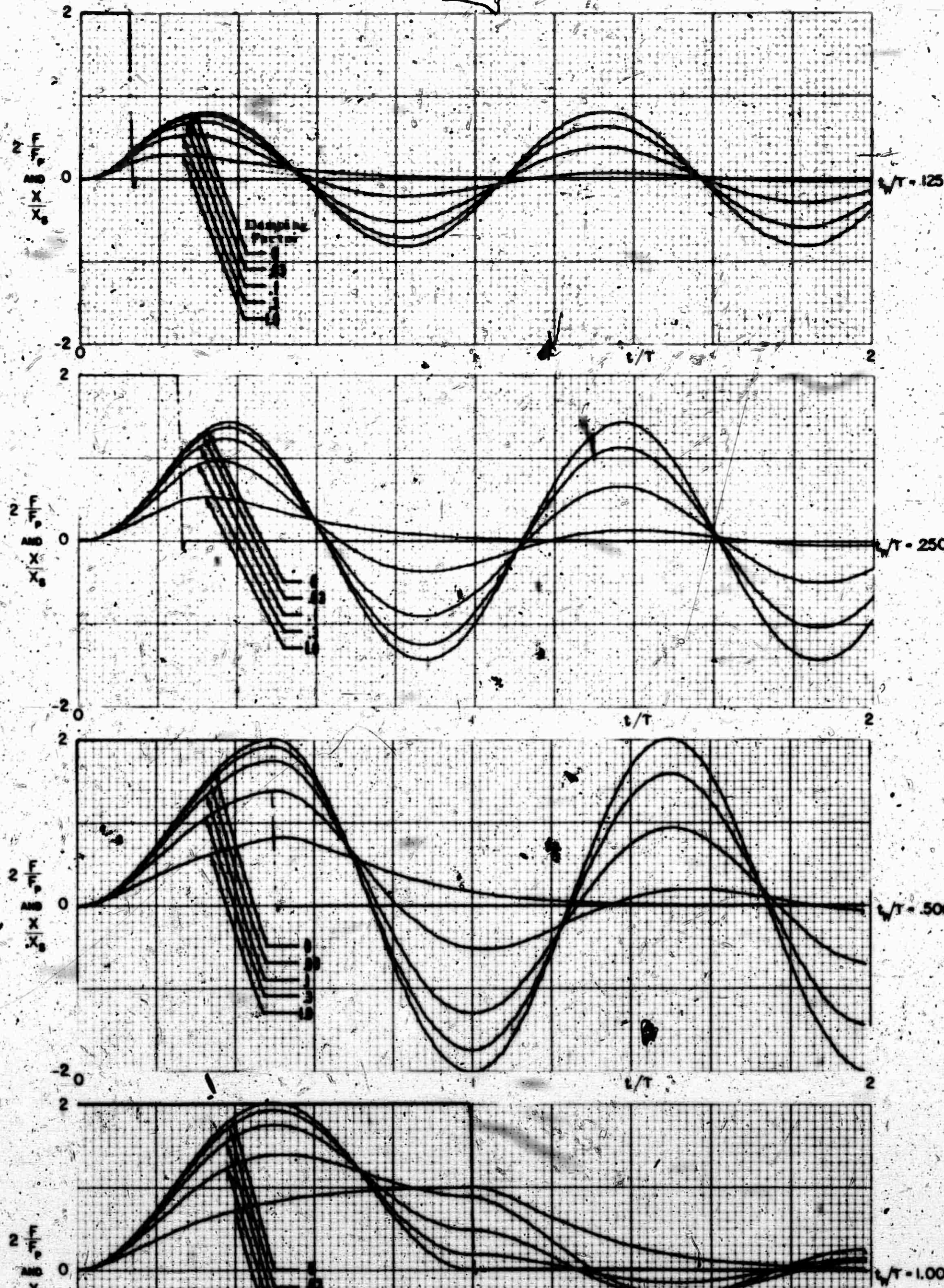

Figure 11
Deflection of a One Degree of Freedom System Produced by a Triangular Pulse

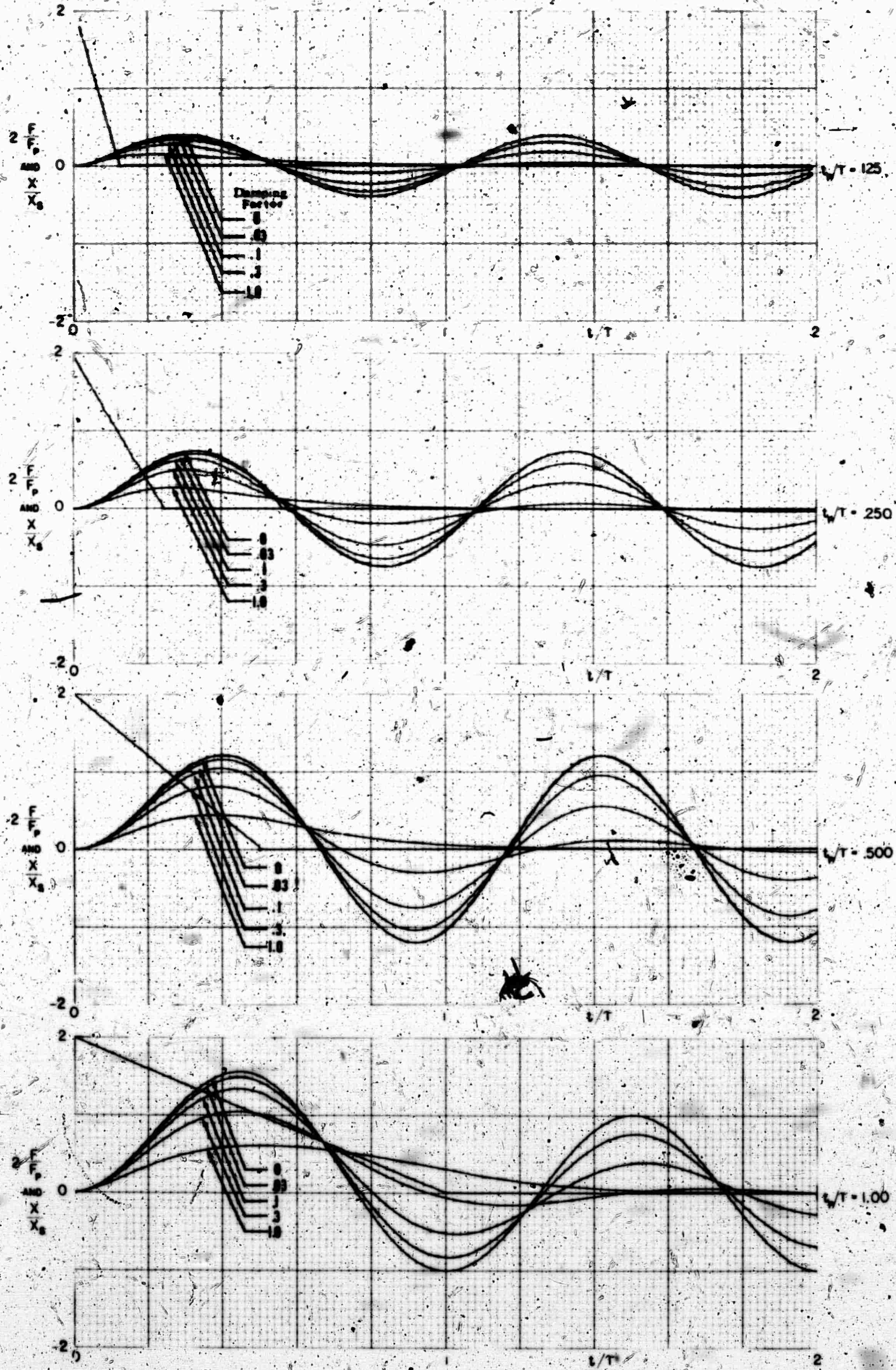

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Figure 13
Deflection of a One Degree of Freedom
System Produced by a Sine Whip

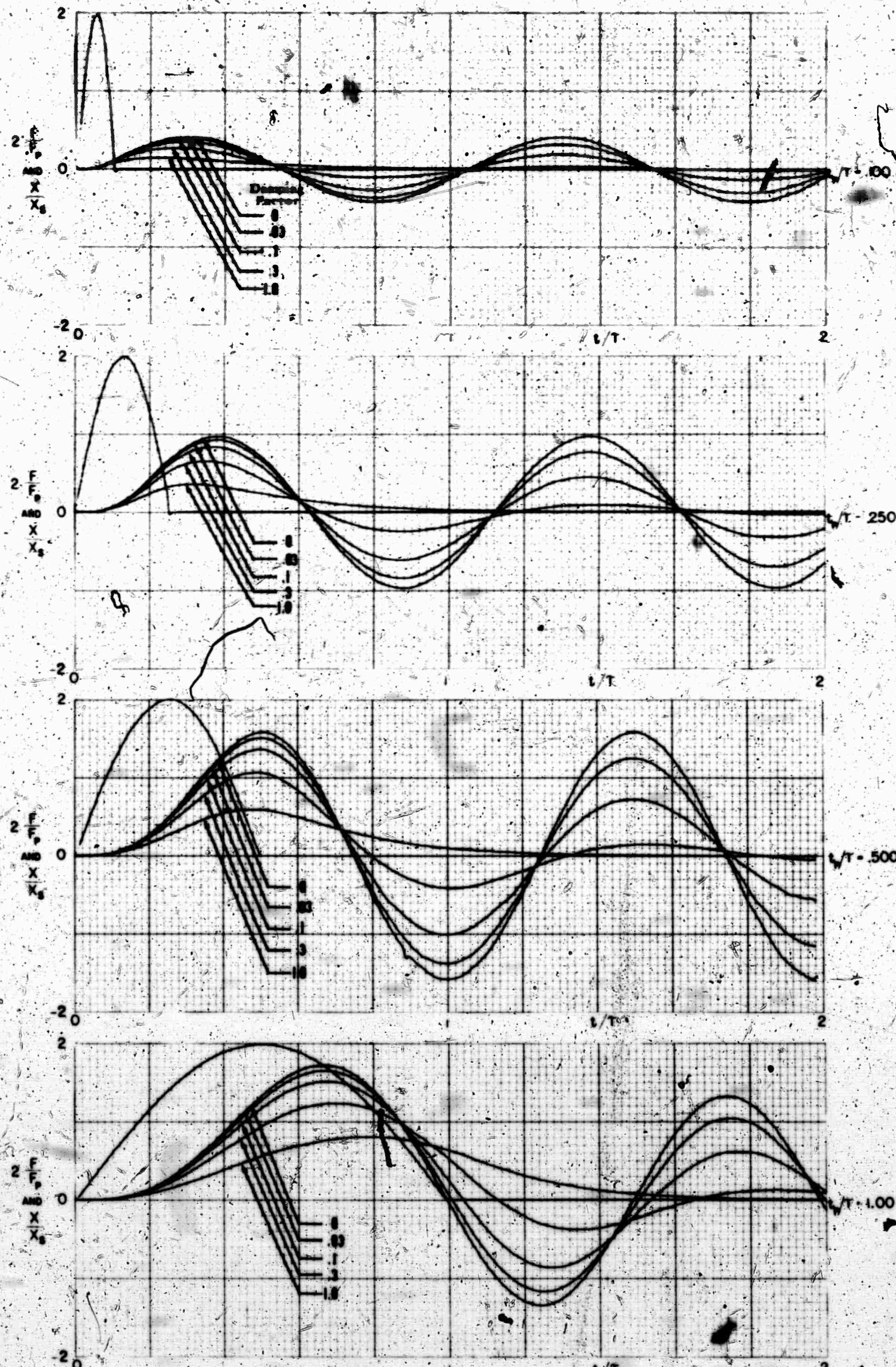

Figure 12
Deflection of a One Degree of Freedom
System Produced by a Triangular Whip

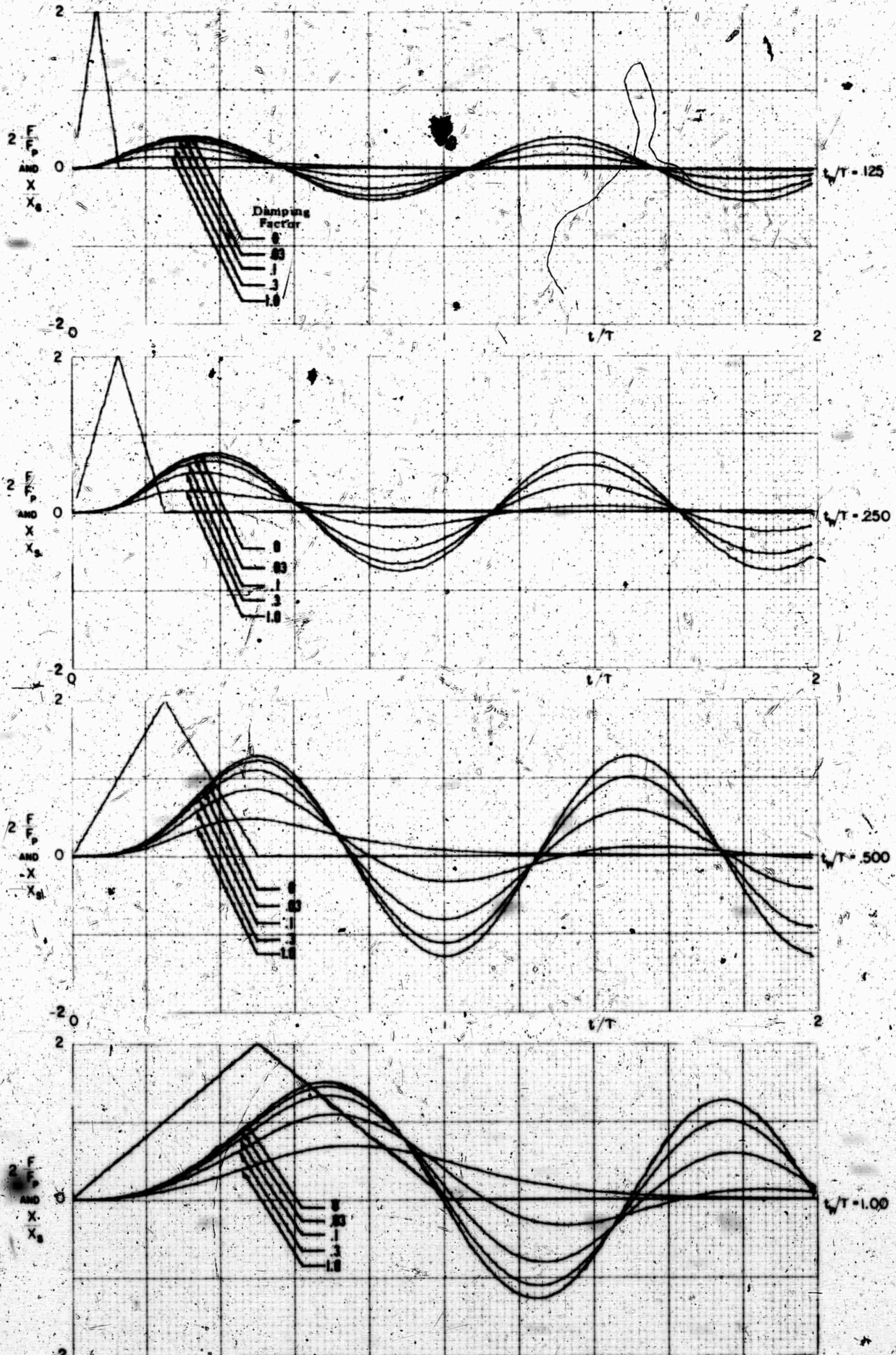

Figure 14
Deflection of a One Degree of Freedom
System Produced by a Triangular Ramp

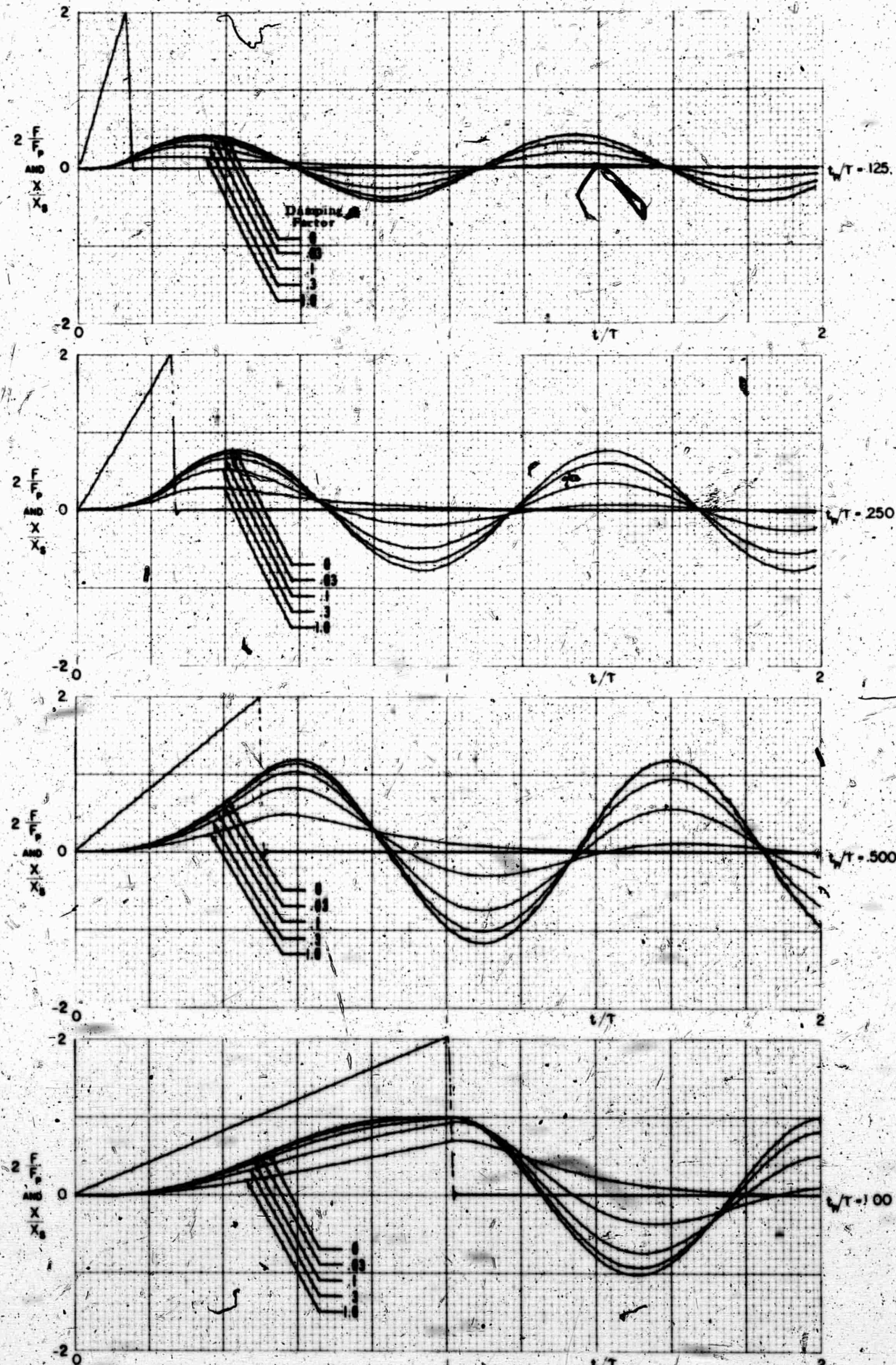

Figure 15
Maximum to Static Deflection Ratio versus Effective Pulse Duration - Damping Factor = 0

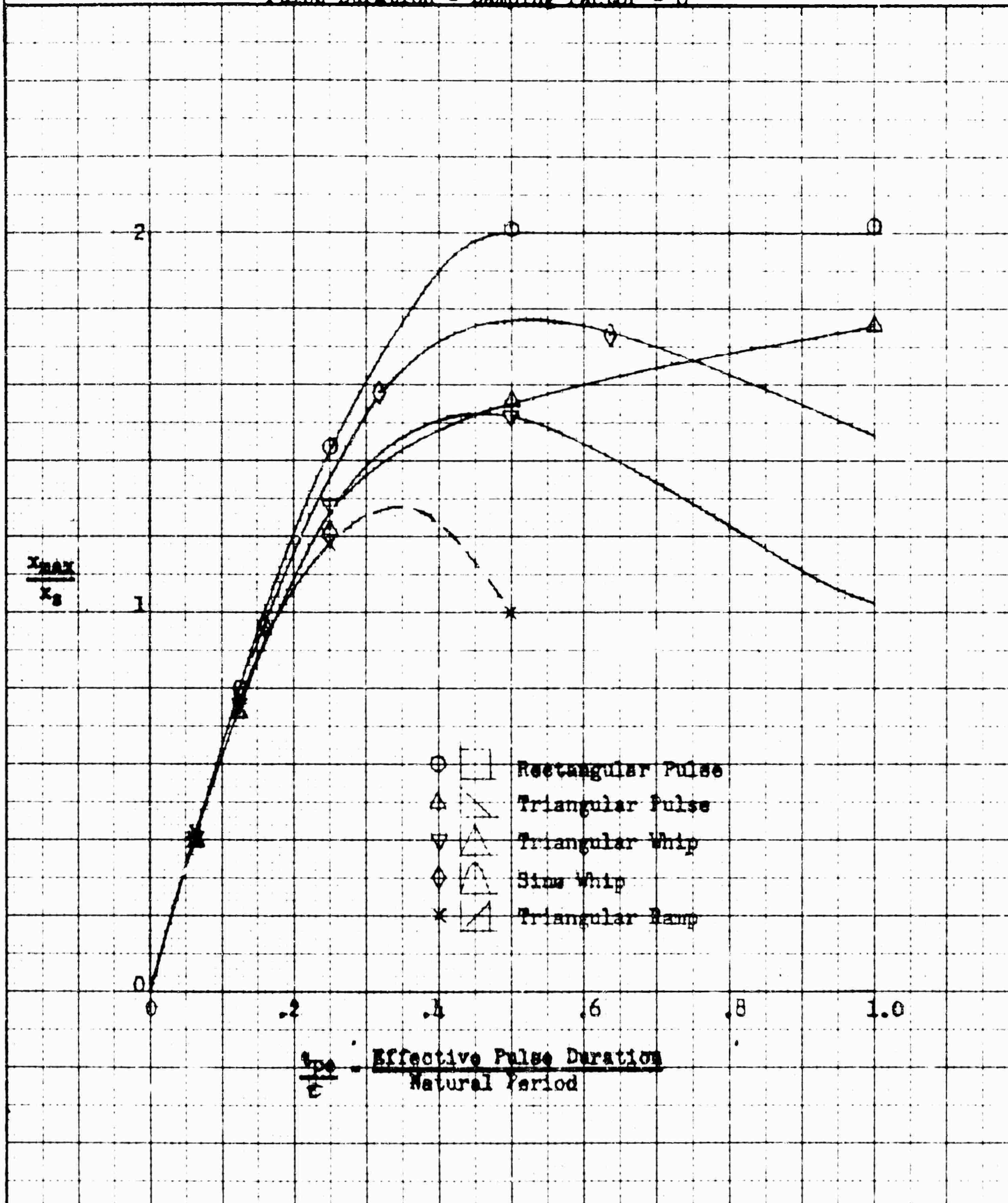

Figure 16
Maximum to Static Deflection Ratio Versus
Effective Pulse Duration - Damping Factor = 0.1

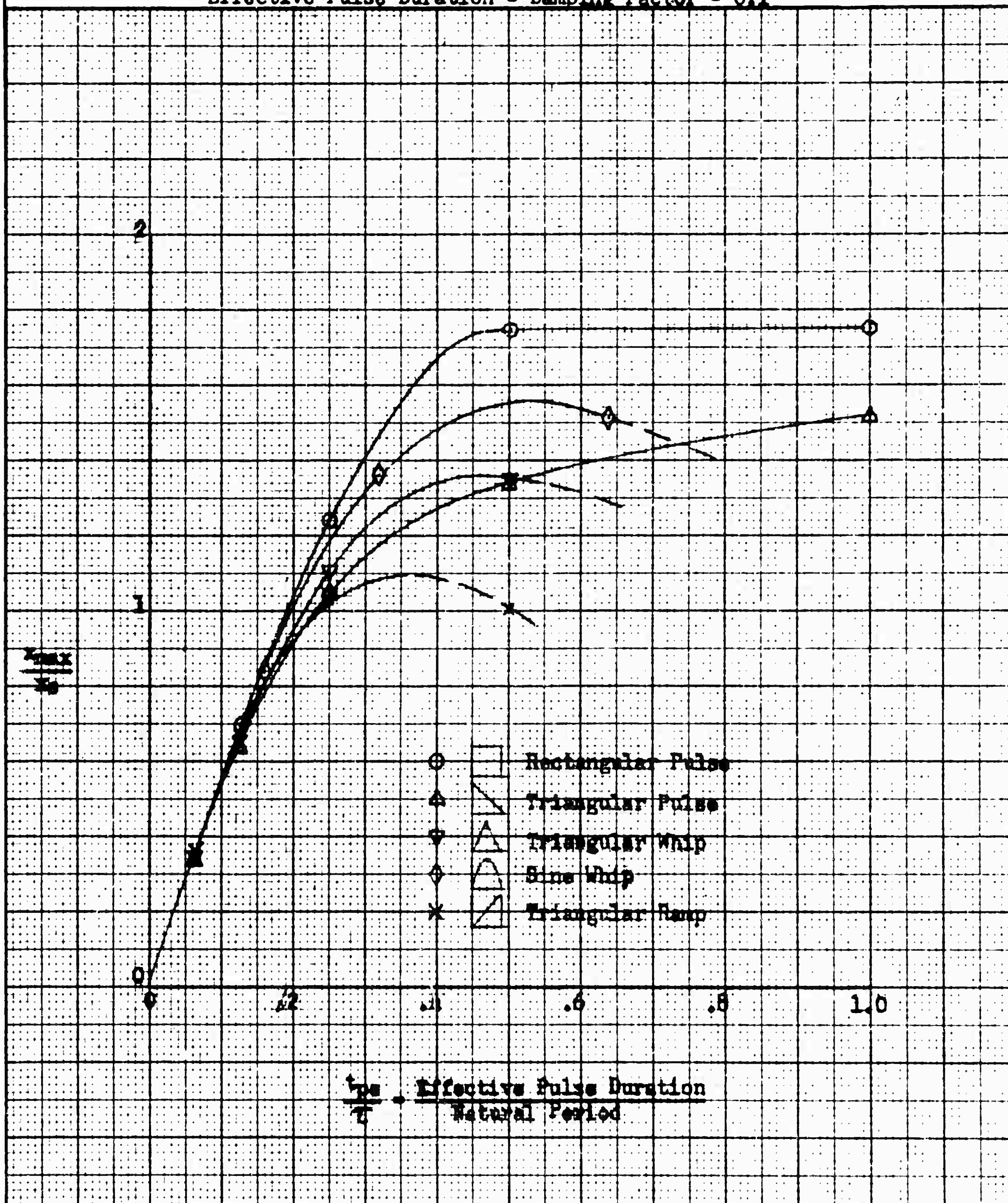

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Figure 17
Maximum to Static Deflection Ratio Versus
Effective Pulse Duration - Damping Factor = 0.3

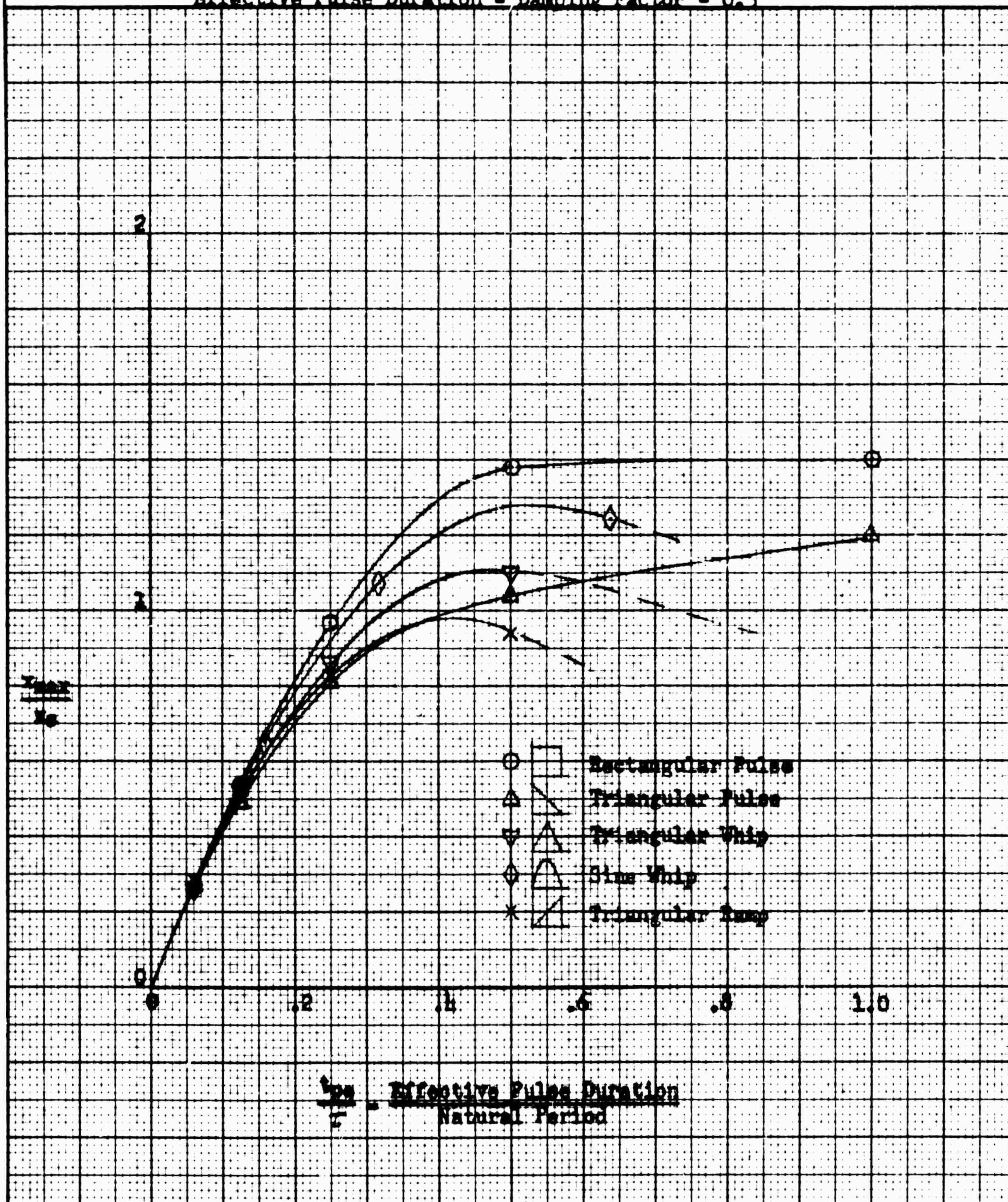

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Figure 18
Maximum to Static Deflection Ratio Versus
Effective Pulse Duration - Damping Factor = 1.0

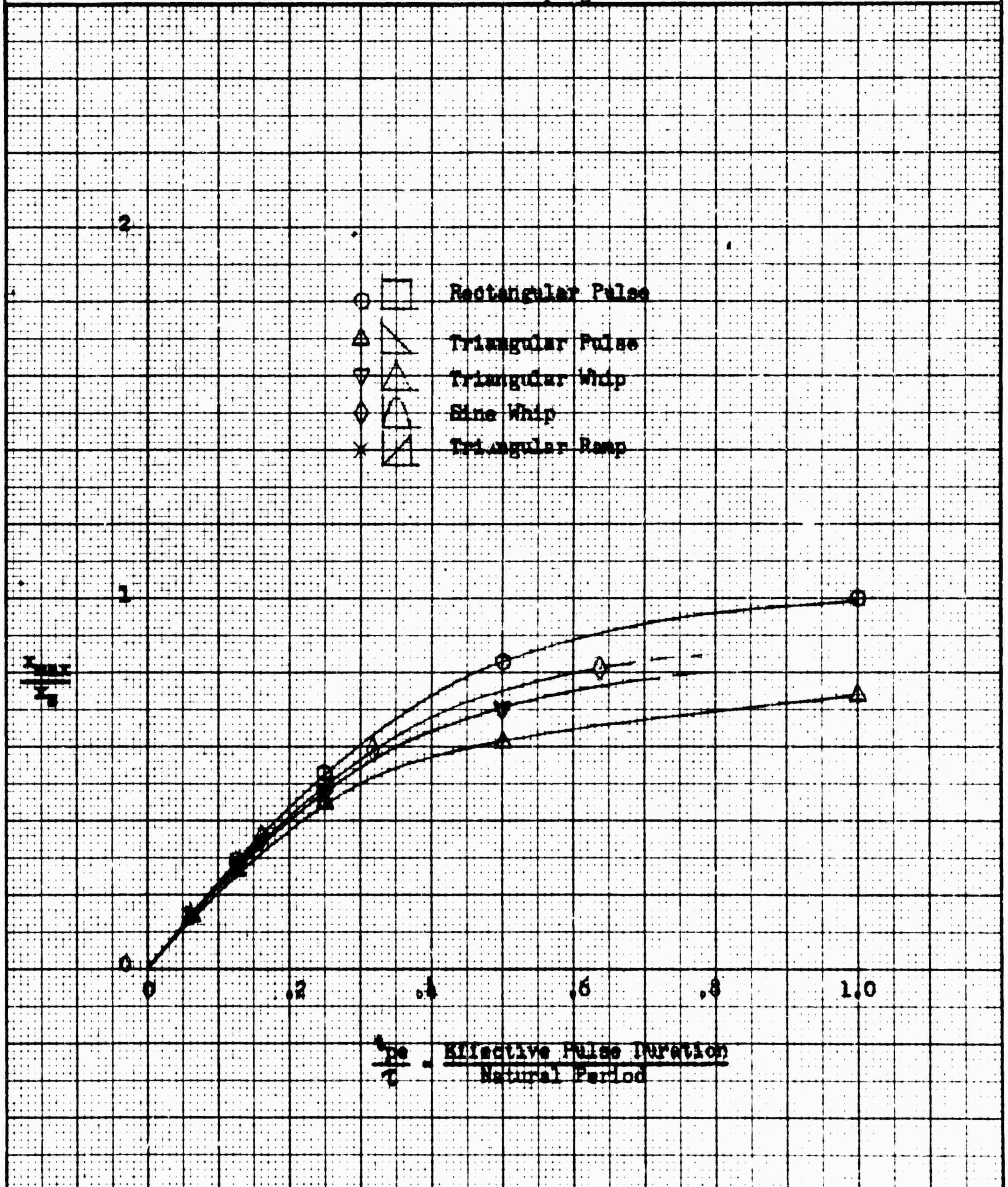

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Table I

Values of $\frac{x_{max}}{x_s}$ from Analog Computer Solutions

Pulse Shape	Damping Factor	.125	.25	.50	1.0	2.0
	0	+.80	+1.44	+2.01	+2.02	
		-.81	-1.44	-2.01	- 0	
	.03	+.77	+1.37	+1.92	+1.93	
		-.71	-1.26	-1.75	- .15	
	.1	+.70	+1.24	+1.75	+1.75	
		-.52	- .90	-1.30	- .33	
	.3	+.54	+ .97	+1.38	+1.40	
		-.21	- .36	- .53	- .32	
	1.0	+.29	+ .53	+ .83	+1.00	
		- 0	- 0	- 0	- 0	
	0	+.40	+ .74	+1.22	+1.56	+1.76
		-.40	- .75	-1.20	-1.00	--
	.03	+.38	+ .70	+1.16	+1.48	+1.68
		-.34	- .65	-1.04	- .83	--
	.1	+.34	+ .64	+1.05	+1.34	+1.52
		-.24	- .47	- .75	- .53	--
	.3	+.27	+ .49	+ .81	+1.04	+1.20
		-.09	- .20	- .30	- .16	--
	1.0	+.15	+ .27	+ .45	+ .61	+0.74
		- 0	- 0	- 0	- 0	- 0
	0	+.41	+ .76	+1.28	+1.52	
		-.41	- .76	-1.28	-1.27	
	.03	+.39	+ .73	+1.23	+1.46	
		-.35	- .66	-1.11	-1.10	
	.1	+.35	+ .66	+1.11	+1.35	
		-.26	- .47	- .81	- .80	
	.3	+.26	+ .52	+ .86	+1.10	
		-.10	- .19	- .32	- .33	
	1.0	+.15	+ .28	+ .48	+ .70	
		- 0	- 0	- 0		

Table I
(Continued)

Pulse Shape	Damping Factor	.125	.25	.50	1.0	2.0
	0	+.42	+ .77	+1.18	+1.00	
		-.43	- .77	-1.18	-1.02	
	.03	+.40	+ .74	+1.13	+1.00	
		-.37	- .67	-1.03	- .93	
	.1	+.36	+ .67	+1.03	+1.00	
		-.26	- .49	- .75	- .75	
	.3	+.28	+ .53	+ .82	+ .94	
		-.10	- .19	- .31	- .36	
	1.0	+.15	+ .29	+ .47	+ .70	
		- 0	- 0	- 0	0	
	0	+	+ .97	+1.58	+1.73	
			- .97	-1.58	-1.34	
	.03		+..93	+1.51	+1.65	
			- .84	-1.38	-1.16	
	.1		+ .84	+1.36	+1.51	
			- .61	-1.00	- .86	
	.3		+ .65	+1.07	+1.24	
			- .24	- .42	- .37	
	1.0		+ .36	+ .59	+ .81	
			- 0	- 0	0	

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