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A SIMPLE AERODYNAMIC RULE FOR HYPERSONIC SMALL DISTURBANCE FLOWS

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FOREWORD

This technical report was prepared by D. J. Harney, Lt Col, USAF, of the Flight Mechanics Division, Air Force Flight Dynamics Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio. The work was performed under Project No. 1426, "Experimental Simulation of Flight Mechanics," Task No. 142604, "Theory of Dynamic Simulation of Flight Environments."

The report covers analytical studies conducted from May 1967 through December 1967. The report was released by the author in January 1968.

This technical report has been reviewed and is approved.

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ABSTRACT

A revival of interest in perfect gas hypersonic small disturbance theory is brought about by the practical possibilities of hypersonic vehicles whose slenderness, combined with the high altitude environment, makes the frozen flow, perfect gas assumption reasonable. In many cases of experimental slender body aerodynamics the predominant effect of reacting real gases is associated not with the flow over a model but rather with nonequilibrium effects in the hypersonic nozzle of high energy wind tunnels. Primarily to eliminate problems associated with wide variations and mismatch of Mach number and additional difficulties in the accurate measurement of Mach number in high energy facilities a rule is developed for the pressure coefficient for hypersonic small disturbance flows in which neither the Mach number nor the hypersonic similarity parameter appears explicitly. The simple solutions which result, generalized to planar and axisymmetric flows by the tangent-wedge and tangent-cone theories, accurately approximate the inherent nonlinearities of hypersonic flow for $M\tau \geq 1$, a region where classical hypersonic similarity is of most importance. However, the need to invoke similarity is eliminated by the availability of practical solutions. The only requirements for correlating two different inviscid flow situations over similar bodies are that γ be known but need not necessarily be the same and that both satisfy the usual limitations of hypersonic small disturbance theory.

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LIST OF ILLUSTRATIONS

SYMBOLS

b	constant in modified pressure coefficient (see Equation 8)
c	constant (see Equation 16)
C_p	pressure coefficient, $\frac{p-p_\infty}{q_\infty}$
C'_p	modified pressure coefficient, $\frac{p-(1+b)\,p_\infty}{q_\infty}$
K	constant (see Equation 16)
M	free-stream Mach number
p	pressure, lb_f/ft^2
q	dynamic pressure, $1/2\ \rho u^2$, lb_f/ft^2
r^*	radius of axisymmetric nozzle throat
r_o	reference nozzle throat radius = 1 inch
R	specific gas constant for undissociated air = $1716 \frac{ft^2}{sec^2\text{-}^\circ R}$
S_o	wind tunnel reservoir entropy
u	gas velocity, ft/sec
x_a	mass fraction of atomic species
γ	ratio of specific heats
δ	local flow inclination angle, radians
δ_w	local inclination of surface to free stream, radians
θ	initial opening half angle of axisymmetric nozzle, radians
τ	maximum flow inclination angle of a body to the free stream, radians
ρ	gas density, $slugs/ft^3$

Subscripts

a atomic constituents of nonequilibrium air

o reservoir properties (except for the reference throat radius, r_o)

∞ free-stream conditions

SECTION I
INTRODUCTION

Hypersonic cruise and high lift-to-drag ratio reentry vehicles as well as slender ballistic vehicles will operate generally in a range of the hypersonic similarity parameter of $2 \leq M\tau \leq 6$ where M is the free-stream Mach number and τ may be taken to be the maximum angle of inclination of a body to the free stream. In this range of $M\tau$ the inviscid flow about the vehicle will be nearly frozen except for blunt noses and leading edges. This will be true even if one assumes equilibrium reacting flows. For example, equilibrium dissociation is negligible at $M\tau = 6$ and may be considered energetically important for $M\tau \geq 10$. In practical cases of flight at high altitude, relaxation in the excitation of vibrational and chemical modes of internal energy should insure that the flow is nearly frozen. Further, the same $M\tau$ arguments apply to blunt leading edges of configurations such as highly swept delta planforms so that the only effect of inviscid gas phase reactions will be confined to the limited downstream region influenced by a blunt nose. Thus, the assumption of a constant ratio of specific heats, γ, should be valid and we can benefit from earlier hypersonic studies which employ the small disturbance equations and the assumption of a perfect gas. However, for aerodynamic wind tunnel tests it may not always be permissible to assume that this constant, frozen specific heat ratio is the same as that associated with atmospheric flight. Although the results have a more general application it is this wind tunnel problem that has motivated this study.

The aerodynamic test of slender bodies in high energy wind tunnels is complicated by the nonequilibrium expansion in a hypersonic nozzle. It is found that the gas at the test section may have a γ which varies with changing test conditions. That is, although the gas may be frozen at a fixed γ about a model due to the low density and high velocity, the free-stream γ will vary as a result of the nonequilibrium expansion of a reacting gas. The gas composition and, hence, this variation of γ is predictable on the basis of the similarity of nonequilibrium air expansions in hypersonic nozzles given in Reference 1.

The results of Reference 1 show that after experiencing a nonequilibrium expansion in a hypersonic nozzle, the frozen gas composition of air at the test section of a wind tunnel is a function of a similitude parameter of the form

$$\Sigma = \frac{S_o}{R} + 0.4 \ln \frac{\tan\theta}{r^*/r_o} \qquad (1)$$

where S_o/R is the reservoir entropy, θ is the opening half-angle of the nozzle, r^* is the throat radius, and $r_o = 1$ inch is a reference throat radius. Using the air composition determined from nonequilibrium computer calculations, the frozen ratio of specific heats, γ, in terms of the mass fraction at atomic species, x_a, may be approximated by

$$\gamma = \frac{7}{5}\left[1 + \frac{8}{35} x_a - \frac{3}{35} x_a^2\right] \qquad (2)$$

In Figure 1 the frozen γ for high energy wind tunnels with air as the working fluid is presented vs the nozzle similarity parameter, Σ. For reservoir temperatures up to roughly 8000°K the free-stream γ is determined by knowing only the reservoir entropy and the geometrical parameters of the nozzle.

It is tacitly assumed in the above that molecular vibration as well as chemical reactions are frozen in the flow about a flight vehicle and a wind tunnel model and that variations in γ are due only to changes in the gas composition of the wind tunnel. The neglect of vibrational reactions is likely more realistic for the wind tunnel case where the frozen vibrational temperature of the free stream will generally exceed the equilibrium vibrational temperature in the shocked flow over a slender body. In the correlation of the wind tunnel test to a free flight situation the free flight γ of the shocked flow should be evaluated for vibrational excitation which could result in values significantly less than $\gamma = 1.4$. However, for the sake of generality it will be assumed in this analysis that for free flight $\gamma = 1.4$.

A more difficult problem associated with slender body simulation in high energy wind tunnels relates to reproducing a specified Mach number. In a perfect gas aerodynamic wind tunnel the gas composition remains essentially unchanged from reservoir to test section resulting in a test section Mach number which is defined largely by the area ratio of the nozzle. In this situation the Mach number is highly predictable and reproducible. For example, an independent variation of Reynolds number by varying the reservoir pressure generally will produce only small changes in the test Mach number. Quite a different situation is encountered with a high energy wind tunnel such as an arc tunnel where variations in the overall pressure or density level may produce large changes in the Mach number which is now strongly dependent on the amount of energy which freezes in vibrational, chemical, and electronic energy sinks. This variation is of no particular concern for the study of blunt bodies where the aerodynamics of the flow becomes essentially independent of Mach number for Mach numbers as low as 3 and 4. On the other hand, this variation in Mach number becomes particularly troublesome for slender body aerodynamics; even the inviscid flow retains a Mach number dependence throughout the hypersonic flight regime. It is in this case that hypersonic similitude has played an important role in reducing the complexity of the perfect gas hypersonic problem.

The similitude as introduced by Tsien (Reference 2) for isentropic small disturbance flows states that two flows will be similar so long as the product $M\tau$ is the same. The separate dependence on the free-stream Mach number, M, and the inclination of the body, τ, is thus reduced to a dependence on the single hypersonic similarity parameter. Subsequent analyses (References 3 and 4) have demonstrated that the similitude is valid for a more general class of flows with shock waves and vorticity. Van Dyke (Reference 5) extended the analysis into the supersonic regime to produce a combined hypersonic-supersonic similitude. Other more complex flows have been analyzed (References 6 and 7) which produce additional similarity parameters relating to slightly blunted noses and equilibrium and nonequilibrium chemical reactions. The complexity of analyzing the problem using the conventional similitude of the governing differential equations is demonstrated by Inger (Reference 7). For the case of a nonequilibrium free stream even the inviscid flow of a simple dissociating gas over a slightly blunted body is a function of no less than 14 independent variables and similarity parameters. If one adds viscous effects and the coupled reactions of air chemistry it appears that some sweeping but judicious simplifications are required to produce workable solutions. Here we retain the complications of a nonequilibrium airstream but assume that the slenderness of the vehicle and a tenuous atmosphere justify the simplification of frozen chemistry throughout the aerodynamically important flow field.

The other assumptions which are implied are the usual ones for hypersonic small disturbance theory which are, in terms of the local inclination angle, δ, rather than the maximum inclination, τ.

$$\left.\begin{array}{l} M >> 1 \\ \sin\delta << 1 \\ M\sin\delta \geq 1 \end{array}\right\} \qquad (3)$$

Since the local Mach number is not used in this analysis M is taken as the free-stream Mach number throughout.

While the first two of these assumptions are order of magnitude arguments, it is necessary to respect the equality sign of the third assumption since this analysis will be restricted to nonlinear hypersonic theory which will employ an empirical extrapolation to $M \sin \delta = 1$ but not appreciably below this value. Finally, it should be noted that the second assumption of Equation 3 is a comparatively weak condition since the results of small disturbance theory usually may be applied to conditions where $\sin \delta$ may be as great as one half.

The problem of slender body testing in high energy wind tunnels is further simplified for the inviscid flow field in the following section by eliminating the explicit dependence of test data on Mach number and on the hypersonic similarity parameter. While the Newtonian-like pressure coefficient which results, directly correlates wind tunnel data to free flight without a need for the prediction and control of Mach number and without a recourse to affine scaling according to the hypersonic similarity parameter, it does include the essential nonlinearities of hypersonic theory and hypersonic similarity within the small disturbance assumptions of Equation 3.

SECTION II

AN AERODYNAMIC RULE FOR SMALL DISTURBANCE FLOWS

1. GENERAL

Beginning with an analysis of wedge flow a simplified equation is developed which is theoretically justified only for $M \sin \delta >> 1$; the result is empirically extended to $M \sin \delta = 1$. The same functional form of the rule can be extended further to cone flow to produce a rule which is generally valid for planar and axisymmetric flows within the limits of tangent-wedge and tangent-cone theory. The importance of the rule in this analysis is associated with eliminating the need for classical hypersonic similarity through simple solutions which correct for the nonlinearities of hypersonic flow. The other advantage of the rule is the elimination of any explicit dependence on Mach number. While the solutions are generally applicable to flight and any type of hypersonic wind tunnel, the advantage of eliminating any explicit dependence on Mach number is directed primarily to the wide variation in test conditions of high energy wind tunnels which are strongly influenced by nonequilibrium reactions in the nozzle expansion.

2. PLANAR FLOWS

For planar flow, such as the flow over a wedge, Linnell (Reference 8) produced the following approximation within the framework of small disturbance theory

$$\frac{C_p}{\sin^2\delta} = \frac{\gamma+1}{2} + \left[\left(\frac{\gamma+1}{2}\right)^2 + \frac{4}{M^2\sin^2\delta}\right]^{1/2} \qquad (4)$$

where δ is the angle of inclination to the free stream. This expression is usually written in terms of δ rather than $\sin\delta$. Here, allowing for angles as high as 20 to 30 degrees, the angle is replaced by its sine so that the similarity parameter becomes $M\sin\delta$.

For $M\sin\delta$ large, say, appreciably greater than 2, Equation 4 simplifies further to

$$\frac{C_p}{\sin^2\delta} = (\gamma+1) + \frac{4}{(\gamma+1)M^2\sin^2\delta} \qquad (5)$$

This can be rearranged in the form

$$\frac{C_p}{\sin^2\delta} = (\gamma+1) + \frac{2\gamma}{\gamma+1}\,\frac{p_\infty}{q_\infty\sin^2\delta} \qquad (6)$$

Now although the variation of the coefficient $2\gamma/\gamma+1$ with γ is only slightly less than that of the leading term $(\gamma+1)$, the effect of this variation of the coefficient is diminished because its multiplier $p_\infty/q_\infty\sin^2\delta$ is much less than one within the assumptions already employed. Thus the dominant effect of γ is contained in the leading term $\gamma+1$ and the coefficient $2\gamma/\gamma+1$ which is of the order 1 may be assumed to be a constant, say, b. Equation 6 then becomes

$$\frac{C_p}{\sin^2\delta} = (\gamma+1) + \frac{b p_\infty}{q_\infty\sin^2\delta} \qquad (7)$$

This suggests the use of a modified pressure coefficient defined as

$$C_p' = \frac{p-(1+b)p_\infty}{q_\infty} = (\gamma+1)\sin^2\delta \qquad (8)$$

in which there is no explicit dependence on Mach number or on the hypersonic similarity parameter. In its simplest form we may take $b = 1$, which corresponds to the Newtonian limit of $\gamma = 1$. In Figure 2 a comparison is made of Equation 4 and Equation 8 (with $b = 1$) as well as with exact calculations for $M = 10$ to 30. Comparing on the basis of C_p, C_p' inverts to

$$C_p = C_p' + \frac{b p_\infty}{q_\infty} = C_p' + \frac{2b}{\gamma M^2} \qquad (9)$$

As is often the case, the simplified approximation remains valid over a wider than expected range. The simple equation

$$\frac{C_p'}{(\gamma+1)\sin^2\delta} = \frac{p - 2p_\infty}{(\gamma+1)\,q_\infty \sin^2\delta} = 1 \qquad (10)$$

is about as good as Equation 4 for $M \sin\delta \geq 2$. Equation 10 overestimates for $M \sin\delta < 2$ while both Equations 4 and 10 underestimate for large $M \sin\delta$.

Resorting now to empirical corrections, Equation 10 is easily modified for better accuracy over the full range of interest by taking $b = 3/4$ and selecting a value for the coefficient on the right side which better approximates the exact solutions for $M \sin\delta >> 1$. Thus, for planar flows we arrive at the following, still simple expression:

$$\frac{C_p'}{(\gamma+1)\sin^2\delta} = \frac{p - 1.75p_\infty}{(\gamma+1)\,q_\infty \sin^2\delta} = 1.015 \qquad (11)^*$$

If plotted on Figure 2, Equation 11 would be seen to be accurate to within about 1% of the exact calculations for $\gamma = 1.4$. Values from Equation 11 are shown on Figure 3, this time for a fixed Mach number and different values of γ to insure that the predominant effect of γ is included in the factor $(\gamma+1)$. Again the deviation from the exact calculations is generally of the order of 1% and at most approximately 2%.

Although the modified pressure coefficients, Equations 8 through 11, provide quite accurate approximations to the pressure coefficients in the hypersonic small disturbance regime they must fail for $M \sin\delta$ less than one and cannot be extended to include the combined hypersonic-supersonic similitude of Van Dyke. This can be seen by comparing Equations 4 and 5. For small $M \sin\delta$, Equation 4 reduces to the linear supersonic theory result for small disturbances

$$C_p = 2\,\frac{\delta}{M} \qquad (12)$$

while Equation 5, which is the basis of the modified pressure coefficient, retains its quadratic dependence on Mach number.

3. AXISYMMETRIC FLOWS

As a representative case of axisymmetric flow we may consider the flow over cones at zero angle of attack. For this case an analytically derived, closed form of solution such as Equation 4 does not exist. So many simplifying assumptions are required for an analytical treatment of cone flow that it seems more productive to search the exact numerical solutions directly for an approximate solution of the form of Equation 8. A useful presentation for this purpose is given by Bertram (Reference 9). The exact solutions for the pressure coefficient are plotted vs the reciprocal of the similitude parameter $M \sin\delta$ (where δ is now the half angle of the cone) over a range of $M \sin\delta$ from 1.67 to ∞ for $\gamma = 1.4$. These solutions,

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*The factors 3/4 and 1.015 are established separately and empirically, that is, the value of 1.015 does not follow from Equation 10 by selecting $b = 3/4$ instead of $b = 1$.

restricted here to a small disturbance range $5° \leq \delta \leq 20°$, are shown in Figure 4. A good match to the exact solution is provided by a value of b = 1/4 in the following form:

$$\frac{C_p'}{\sin^2 \delta} = \frac{p - 1.25 p_\infty}{q_\infty \sin^2 \delta} = 2.094 \tag{13}$$

Equation 13 is shown as the short-dash curve in Figure 4. It approximates the dependence on the hypersonic similitude parameter to within a fraction of 1%. Also shown in Figure 4 is a comparison of the hypersonic cone theory of Lees (Reference 10) which is also quite good -- generally within 1 to 2% of the exact solution for small δ.

So far, no consideration has been given to variations in γ for axisymmetric flow. This effect is notably less important for axisymmetric flow as compared to planar flow. For planar flow it has been shown that the essential effect of γ is given by the hypersonic limit, i.e., $C_p \sim (\gamma+1)$. This result is extended to axisymmetric flows in accordance with the hypersonic limit as given by Lees (Reference 10), i.e., $C_p \sim (\gamma+1)(\gamma+7)/(\gamma+3)^2$. Using a linear approximation for this relation in the neighborhood of γ = 1.4 which is accurate to 0.1% over the range $1.2 \leq \gamma \leq 1.6$ and which matches the value of the coefficient of Equation 13 we have for cone flow

$$C_p' = 1.972 (1 + 0.044 \gamma) \sin^2 \delta \tag{14}$$

where, again,

$$C_p' = \frac{p - 1.25 p_\infty}{q_\infty}$$

. STATEMENT OF THE RULE

Generalizing the wedge and cone flow results on the basis of tangent-wedge and tangent-cone theory, the aerodynamic rule for hypersonic small disturbance flow is stated as follows:

Define the pressure coefficient as

$$C_p' = \frac{p - (1 + b) p_\infty}{q_\infty} \tag{15}$$

where b is a constant equal to 3/4 for planar flows and 1/4 for axisymmetric flows. The solutions then are of the form

$$C_p' = K (1 + C\gamma) \sin^2 \delta \tag{16}$$

For planar flows c = 1 and for axisymmetric flows c = 0.044. For simple wedge flow K = 1.015, and for simple cone flow K = 1.972. Within the tangent-cone and tangent-wedge approximations these values of K should be useful for calculating pressure distributions on more complex shapes. For wind tunnel measurements on arbitrary shapes K, of course, is determined experimentally.

SECTION III

ANALYSIS AND SUMMARY

Although Equation 16 is as simple in form and in application as Newtonian limit theory, it implicitly contains the essential nonlinearities of hypersonic small disturbance theory and the results of hypersonic similarity within the small disturbance limitations as given by Equation 3. However, where such simple solutions are permissible there is no need for classical hypersonic similitude and associated complications such as affine scaling. The modified pressure coefficient at any point on a surface having a fixed inclination to the free stream varies only with free-stream composition, or γ, and this effect is given explicitly in the solutions; it is otherwise independent of all other variations of free-stream parameters which satisfy the small disturbance limitations.

The elimination of the requirement for similitude by simple solutions which approximate the nonlinearities of hypersonic flow is an obvious advantage. Not so obvious are the benefits of eliminating all explicit dependence of Mach number. Clearly, if one chooses, the rule may be rewritten by combining Equations 9, 15, and 16 in the form

$$C_p = K'(1 + C\gamma)\sin^2\delta + \frac{2b}{\gamma M^2} \tag{17}$$

Now, in comparing Equation 17 with Equations 15 and 16 it would appear that the absence of any explicit dependence on Mach number in Equations 15 and 16 is trivial, since, under the simplifying frozen flow assumption of this analysis, if γ can be accurately estimated the additional measurements of p_∞ and q_∞ define the frozen Mach number and there is no need to eliminate Mach number, per se. In fact, for a perfect gas hypersonic wind tunnel in which the Mach number is essentially fixed by the nozzle area ratio and in which data is obtained by varying other parameters of interest at a constant Mach number, Equation 17 may be of more interest. However, in a high energy wind tunnel in which wide excursions of Mach number result from changes in other variables of interest, Mach number becomes a poor choice as a parametric variable and the elimination of any explicit dependence on Mach number, as in Equations 15 and 16, becomes useful and, thus, not trivial. Even in this case, of course, the test Mach number can be calculated from γ, p_∞ and q_∞ and the data can be reduced in the form of Equation 17. This is done, however, only after the fact since the prediction and control of Mach number, as other parameters are changed in a high energy wind tunnel, is quite complex and always of questionable accuracy. The difficulty of accurate prediction and control of Mach number is associated with estimating the strong and complex variation of the free-stream temperature and speed of sound. This difficulty is eliminated by the hypersonic small disturbance rule which allows direct prediction of flight performance in terms of the modified pressure coefficient without the additional labor of calculating and correcting for mismatch and wide variations of Mach number.

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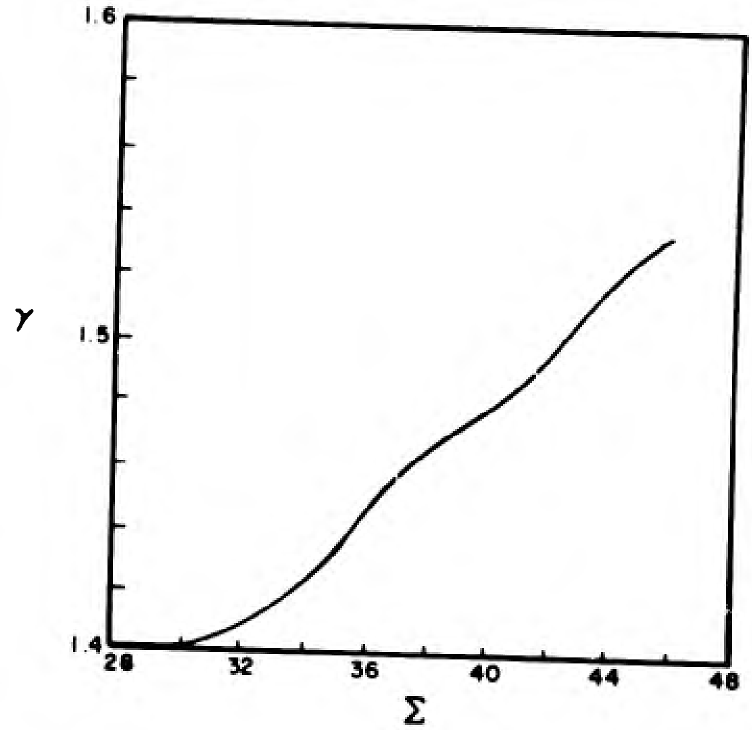

Figure 1. Frozen Ratio of Specific Heats vs the Nozzle Nonequilibrium Similarity Parameter for Air Expansions

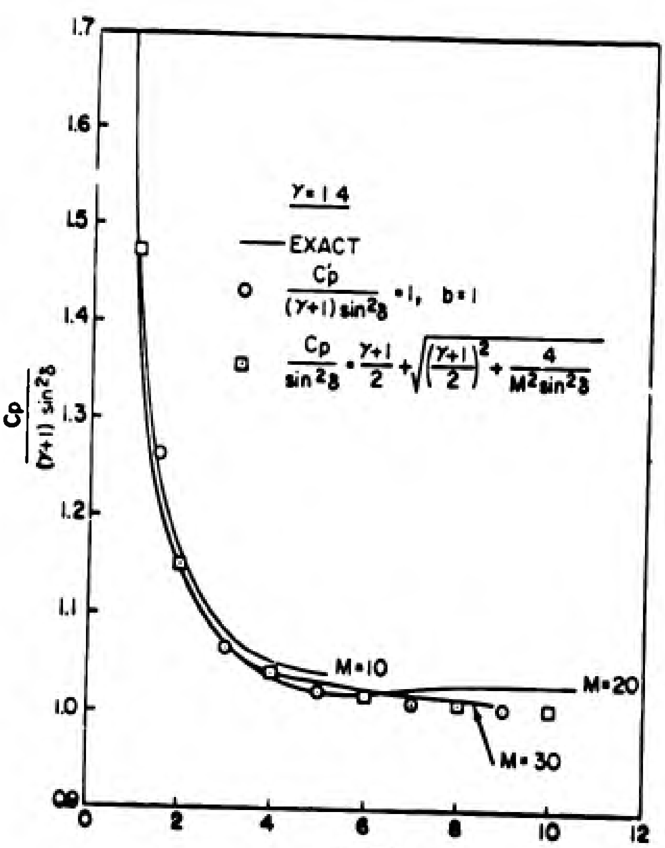

Figure 2. Comparison of Small Disturbance Approximations With Exact Wedge Flow Solutions (Perfect Gas, γ = 1.4)

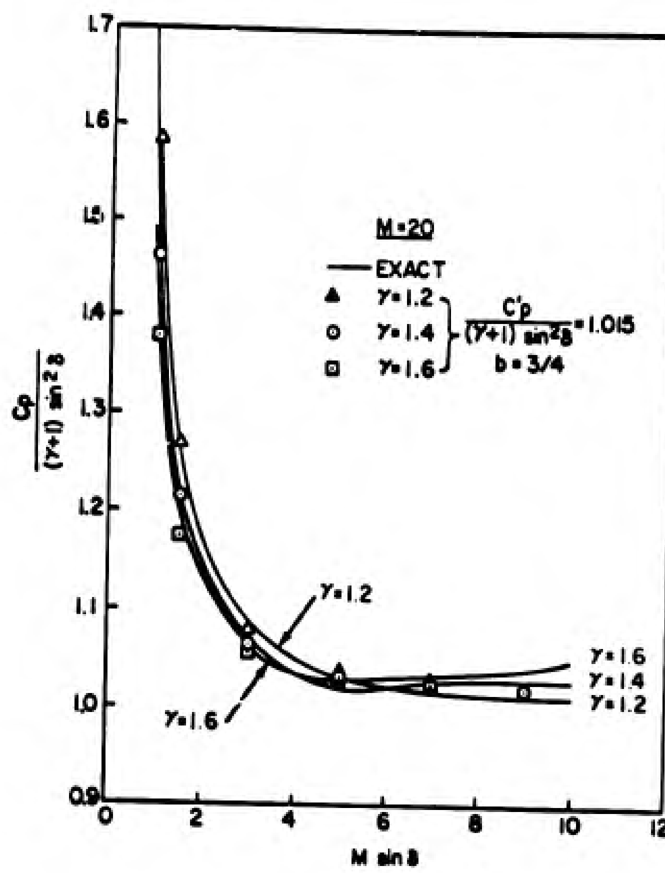

Figure 3. Comparison of the Hypersonic Small Disturbance Rule for Wedge Flow With Variable γ

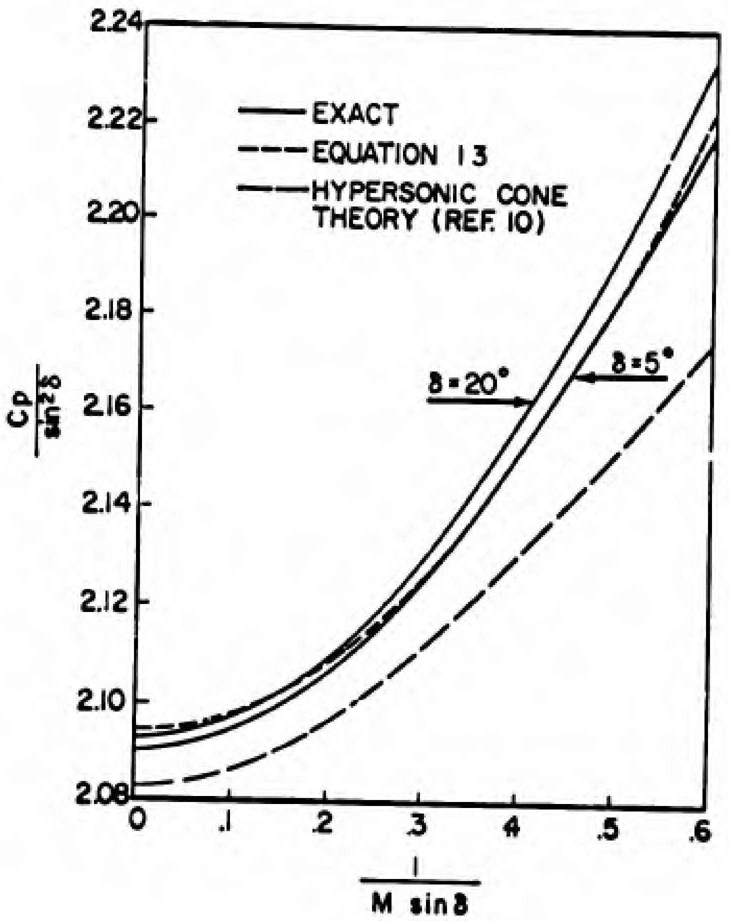

Figure 4. Comparison of Small Disturbance Approximations With Exact Cone Flow Solutions (Perfect Gas, γ = 1.4)

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13 ABSTRACT

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AFFDL-TR-68-4 June 1968

ERRATA - OCTOBER 1968

The following corrections are applicable to AFFDL-TR-68-4 "A Simple Aerodynamic Rule for Hypersonic Small Disturbance Flows" June 1968.

Although Equation 14 is numerically correct for $\gamma=1.4$, the γ-dependence is in error. That is,

$$\frac{(\gamma+1)(\gamma+7)}{(\gamma+3)^2} \sim (1+0.1\gamma)$$

rather than $(1+0.044\gamma)$. Thus, as a first approximation using the γ-dependence of the hypersonic limit, Equation 14 should read

$$C_p^{\prime} = 1.836(1+0.1\gamma)\sin^2\delta \qquad (14)$$

Further, the simplifying assumption that b in Equation 15 is a constant needs to be modified for cone flow since, as with planar flows,

$$b \sim \frac{2\gamma}{\gamma+1}$$

This γ-dependence, established by correlating numerical solutions of the Taylor-Maccoll equation, should be retained since it is no longer negligible when compared to the weak γ-dependence of Equation 14.

Again relying on a best fit to extensive numerical solutions of the Taylor-Maccoll equation the factors in Equations 15 and 16 should read

$$\left.\begin{aligned} b &= 0.371\left(\frac{\gamma}{\gamma+1}\right) \\ c &= 0.120 \\ K &= 1.800 \end{aligned}\right\} \text{ for cone flow}$$

and for cone flow Equation 17 becomes

$$C_p = 1.800(1+0.12\gamma)\sin^2\delta + \frac{.742}{(\gamma+1)M^2}$$

The error of this approximate equation is within 1% of the exact solution under the restrictions

$$M \geq 5.0$$

$$M\sin\delta \geq 1$$

$$\sin\delta \leq \tfrac{1}{2}$$

$$1.20 \leq \gamma \leq 1.67$$

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