

MEMORANDUM
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AN AGGREGATE STOCKAGE POLICY FOR
EOQ ITEMS AT BASE LEVEL

John Y. Lu and Robin B. S. Brooks

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PREFACE

This is the initial report on RAND work in applying a system approach to the management of EOQ items. The mathematical model described in this Memorandum, however, is an extension of previous RAND work on the stockage problem ([7], [8], and [9]).

Most of the text is addressed primarily to the managers of Air Force Supply Systems. It discusses, in "philosophical" terms, the main features of the model and its usefulness as a management tool. For those interested in the technical details of the model and wanting to use the associated computer program, the necessary information is provided in the appendixes.

Future work is expected to continue in the following two areas:
1) To consider and to suggest solutions to various problems that may arise in implementing the results of this study; 2) to study means of improving the management of EOQ items at depot levels.

Robin Brooks is a consultant to The RAND Corporation.

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SUMMARY

A typical base supply account in the Air Force carries a large number of EOQ (economic order quantity) items. As a rule, unit prices of these items are quite small. This means that it is not practical to apply tight management control on an individual item basis, but it certainly does not mean that management of EOQ items may be neglected. This is because the total value of all EOQ items in the Air Force is not trivial, and more importantly, unless base supply is stocked with a proper mix of EOQ items (especially repair parts), the efficiency of base repair activities will drop and serious operational consequences are likely. What is needed, then, is a methodology that will enable base supply management to define its stockage objective clearly at the aggregate level, and to implement this objective by means of operating rules that are applicable to individual items.

This Memorandum suggests that it is useful to view the stockage objective of a system comprising many EOQ items as minimizing the average inventory investment cost, subject to constraints on the total expected number of backorders and replenishment order frequency per year. To achieve this objective, a mathematical model was developed. The model accepts two types of information as inputs: item data, consisting of unit price, past demand, and resupply time; and system data, consisting of future base activity level, variance to mean ratio of demand distribution, and constraints on the average backorder rate and order frequency per year.

Based on these inputs, the model computes a reorder point and order quantity for every item in the system in such a way as to satisfy the above objective.

Such a set of reorder points and order quantities is called an "efficient" policy. This Memorandum suggests that constraints on the backorder rate and order frequency be varied within the relevant range of the base operation and that the corresponding minimum-cost policies be derived. In this way, a set of alternative efficient policies (see Fig. 2, p. 6) can be presented to the base supply manager, instead of a single policy. He can then choose the one most appropriate for his operating conditions.

A comparative evaluation of the model and the current Air Force procedure (AFM 67-1) was carried out. The results indicate that the model offers significant improvement. The model uses a Bayesian approach for demand prediction. This appears to be a useful smoothing technique because, in general, it provides for a wider range of items having positive stock levels than the AFM 67-1 procedure can do for the same set of data. The model-computed policy is fairly sensitive to the average resupply time, but incorrect specification of the variance to mean ratio does not appear to cause an intolerable mis-allocation of resources.

The model has been programmed in FORTRAN IV for the IBM 7044 computer at RAND. It is therefore readily usable by other computers of comparable size.

ACKNOWLEDGMENTS

We wish to express our appreciation to Mrs. Gabriele Michels, who put together the original version of the computer program described in this document. We also thank Parrison Campbell for his interest and encouragement throughout the course of this study.

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I. INTRODUCTION

This Memorandum describes a technique for determining optimal operating rules to control an inventory system comprising a large number of so-called Economic Order Quantity (EOQ) items. An example of such a system is the base supply at an air base.

DEFINITION OF EOQ ITEMS

The Air Force defines an EOQ item as one that normally cannot be economically repaired by a field or depot maintenance activity. This includes consumable items, repair parts, components, tools, and hardware, on which accountability terminates upon issue and which are identified by ERRC codes of XB2 and XB3. These are items for which the cost incurred by the requisitioning process is significant relative to investment cost in inventory.

NEED FOR AGGREGATE APPROACH

Anyone who has some acquaintance with the Air Force Base Supply Systems knows that EOQ items typically have very low unit prices, but are multitudinous. Even in a relatively small base supply account, such as the one at Oxnard Air Force Base, the number of EOQ items exceeds 15,000. This means that it is not practical to apply close management control on an individual item basis; but it certainly does not mean that management of EOQ items may be neglected. Although unit prices may be small, the total dollar value of all EOQ items in the Air Force inventory is not trivial.* More important, unless base

*According to the "1138" report dated June 30, 1967, the assets of aircraft-related EOQ items (including some XF-3 items) at depot

supply is adequately stocked with a proper mix of EOQ items (especially repair parts) the efficiency of base repair activities will drop, with severe operational consequences.

What is needed, then, is a methodology that will enable base supply management to define its objectives at the aggregate level and to translate the objectives into a set of operating rules that are implementable at individual item levels.

OPERATING RULES FOR INDIVIDUAL ITEMS

Management of the inventory of an individual EOQ item is completely specified if we know when to replenish the inventory and how much to order for replenishment. In the language of inventory theory, this is known as determining the reorder point and economic order quantity. Figure 1 depicts the behavior of the inventory levels of an EOQ item as related to these two decision variables.

As the inventory position (the sum of on-hand and on-order quantities) reaches the reorder point, an order of a certain size will be placed for replenishment. Meanwhile, the net inventory (on hand minus backorders) will continue to diminish. After a certain time lapse, the item may even be in a backorder condition.* Finally, the replenishment stock will be received, bringing the inventory position above the reorder point, and the whole process will be repeated.

and base level were valued at \$1.337 and \$0.938 billion, respectively. In addition, there were nearly \$386 million of O&M funded EOQ items at bases.

*The term "backorder" is used here to mean due-outs from base supply to base maintenance. This is different from common Air Force usage, in which backorder means a due-in from the depot.

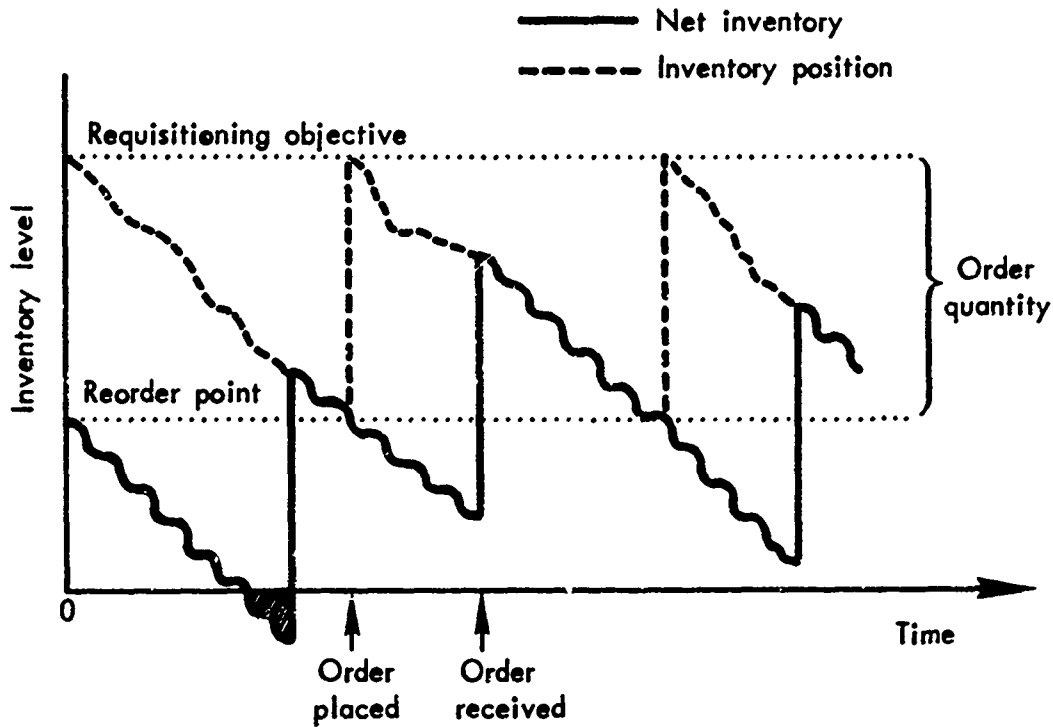


Fig. 1 -- Replenishment and depletion of inventory stock over time

For a specific item, varying the two decision variables will have the following effects: a lowering of the reorder point will result in a decrease in the average amount of stock on hand at the expense of exposing the item to a greater risk of stockout; raising the reorder point, of course, will have the reverse effects. If the order quantity is reduced, the average inventory will also decrease. This will be accompanied by an increased order frequency and a higher risk of stockout, because as the order frequency increases, the inventory position will hit the reorder point more frequently.

It is clear that even for inventory management of a single item, various kinds of trade-offs among inventory investment costs, order

frequency, and stockouts can be made by adjusting the decision variables in order to attain some desirable operational consequences. The next question is how these are all related to the aggregate approach mentioned above.

ATTAINING DESIRED AGGREGATE RESULTS

Any set of reorder points and economic order quantities^{*} will, for a specific inventory system, produce aggregate results that can be observed and measured. These results are usually stated in terms of investment in stock, ordering rate, and performance level. By investment in stock is meant the total dollar value of the average inventory of all items in the system. Ordering rate refers to the frequency of placing an order for replenishment per unit time. For measuring performance the total expected number of backorders (across all items) incurred per year is used. When this quantity is expressed as a percentage of total expected units demanded on base supply for the same time period, this percentage is here called the "backorder rate."

A supply manager uses these results to guide his efforts toward certain management goals. He would like (1) to keep his investment in stock as small as possible, (2) to order for replenishment as infrequently as possible, and (3) to keep the average backorder rate as low as possible. In most real situations, it is not possible to attain these objectives simultaneously because of limits on the resources available for operating an inventory system. A more realistic overall management goal is to provide the best possible service for a

* Sometimes this will be called a stockage policy.

predetermined ordering rate and investment or, alternatively, to operate the system to meet a specific service standard and ordering workload with a minimum stock investment. The former view of the overall inventory management goal is more commonly found in the industrial environment because ordering workload and investment are more tangible indicators of a system's performance than is the average backorder rate, which pertains to the quality of customer service. Furthermore, the immediate impact of varying the quality of customer service is hard to assess. In many military systems, on the other hand, the overriding goal is to attain a high level of customer service. In such a case, the latter view of the management goal may be more appropriate.

EMPHASIS OF THIS STUDY

The problem of determining the optimal reorder point and order quantity for a single item has been examined by several authors, such as Galliher et al. [10], and Yadley and Whitin [11].*

Their results for a single-item system are extended in this study to specify operational characteristics of a multi-item system. A mathematical model was developed to relate these characteristics to inventory operating rules. Optimization is then performed in the model to derive operating rules that will minimize one of the

*In Galliher's article, the steady state probabilities $p(i)$, where i is the difference between the requisitioning objective and net inventory position and is often referred to as the quantity in "routine resupply," were calculated under assumptions that demands arrive according to a compound Poisson distribution and all replenishment times are of constant length. Similar results are presented in [11], but assume that demands follow the Poisson distribution.

characteristics, subject to constraints that the remaining two system characteristics meet some prespecified standards. The resulting rules are called "efficient" rules.

This study emphasizes the way optimization results are to be presented to management for decision. Instead of presenting only a single set of efficient operating rules, the study suggests that optimization be performed throughout the entire spectrum of the three system characteristics, and that a resulting family of operating rules, each of which is efficient in the sense discussed above, be presented to management. This is illustrated in Fig. 2.

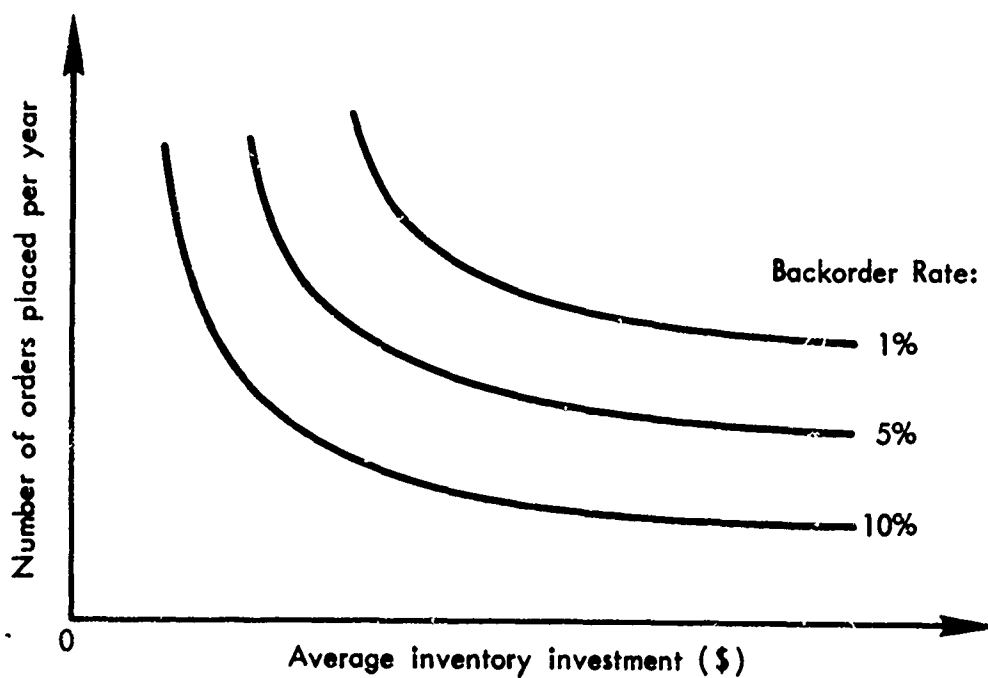


Fig. 2 -- Trade-off curves for a system of EOQ items

A point on the curves represents the system characteristics of a set of efficient operating rules. Each curve may be called an iso-backorder curve because it represents a family of efficient operating rules with the same backorder rate. Note that, along each curve, a reduction in the average inventory investment must be compensated for by an increase in the order frequency.

Since the policy implications of a number of alternative sets of efficient rules are clearly stated, management will have a better perspective in choosing a particular set of rules suitable to his operating conditions. It is contended here that this is preferable to obtaining decision rules by minimizing a total cost function, which is often based on arbitrary estimates of various cost parameters without any definite knowledge of the operational ramifications of the estimates.

The remainder of this Memorandum is organized as follows. Section II gives a general description of the important features of the model. Section III reports some numerical results based on sensitivity tests of the model, and a comparative evaluation of the model and the current Air Force procedure. Section IV gives the conclusions of the study and identifies problem areas that need to be studied if the results are to be used in the Air Force Supply System. The Appendixes contain technical details of the study. The model is described mathematically in Appendix A. Appendix B presents the associated computer program. The current Air Force procedure for managing EOQ items is summarized in Appendix C.

II. THE EOQ MODEL

A mathematical model of a multi-item inventory system was developed for this study. The model is an extension of two earlier RAND studies on inventory control systems. Several years ago, Ferguson and Fisher [9] studied stockage policies for medium- and low-cost parts. They formulated the long-run average cost per unit time for a system consisting of a large number of EOQ items and obtained minimum-cost policies for several alternative assumptions regarding the values of cost parameters. More recently, Feeney and Sherbrooke [8] proposed an aggregate approach to base stockage of reparable items. Two main features of their aggregate approach are: use of the Bayesian inference for predicting spares demand, which provides a better basis for stockage decisions on low-demand items; determination of stock levels that are optimum with respect to some aggregate measure of system performance within a constraint imposed on inventory investment.

The model described herein deals with a system of low-cost items at a single point of operations, as in the Ferguson-Fisher study, and is based on the philosophy of the Feeney-Sherbrooke aggregate approach. The primary difference between a system of reparable items and that of EOQ items is that the order quantity in the former system is always unity whereas in the latter system it is one of the decision variables to be determined, because ordering cost is not insignificant relative to item unit cost and, consequently, batch ordering may be desirable.

In the model, the criterion for stockage decisions is average inventory investment cost; or more precisely, the sum of the average stock level, priced at unit price, is minimized for any arbitrary

chosen level of supply effectiveness and order frequency. Supply effectiveness, or performance, of a given stockage policy is measured by the total expected number of backorders incurred per year. Order frequency refers to the number of orders placed for replenishment per year.

Ordinarily, the cost associated with order frequency is viewed as one of the factors in the long-run average inventory cost. It is felt, however, that this cost is qualitatively different from the investment cost associated with maintaining stock levels. It is therefore treated as a separate system characteristic to be observed and managed in the inventory control system considered here. The major advantage of formulating the model in the manner described above is that it avoids the problem of having to arbitrarily estimate holding, shortage, and order cost parameters. Instead, it explicitly relates a stockage policy to operational consequences that are much more understandable to the supply manager.

The model allows for two other alternative formulations: the total number of backorders, as well as order frequency, can be used as the function to be minimized subject to constraints imposed on the remaining two system characteristics.

The model consists of three closely related but logically distinct parts: (1) statistical forecast of demand for each item; (2) computation of the expected number of backorders as a function of stock levels, past demand, and predicted demand conditions; (3) optimization by which reorder points and order quantities are determined to meet specifications on supply effectiveness and order frequency at a minimum cost. These operations are stated in more detail in Appendix A, but it may be useful to give brief descriptions here.

DEMAND FORECASTING

It is well known that aircraft components fail at random; hence, demands for repair parts also follow some random patterns. The problem is how to describe and forecast these random patterns with sufficient accuracy so that they can be reflected in stock-leveling computations. One common approach is to assume that item demands can be represented by a probability distribution. The problem of demand forecasting is then reduced to that of estimating parameters of the assumed distribution. This is not so easy a task as it might appear on the surface, especially if a system contains a large number of items with very low demand rates and has a relatively short data collection period, as is the case in the Air Force Base Supply System.

The technique used to overcome this difficulty is a heuristic approach based on the Bayesian technique suggested by Feeney and Sherbrooke [7]. In this technique, past demands of all items are used to form a prior distribution of demand rates. The forecast of demand can then take the form of the conditional probability that the "true" demand rate, which can never be known exactly, is at any of a number of levels, given the observed number of demands. This provides a systematic means of assigning some probability to future demand outcomes even when past demand is zero, an action consistent with our prior notion of what the behavior of spares-demand is, i.e., many items with no demand in one period may show some activity in the next.

In the model, the form of prior distribution is assumed to be the lognormal distribution, i.e., the logarithms of the true item mean rates are assumed to be normally distributed. This is justified on

the basis of empirical studies (e.g., [3]). As to the distribution-in-time of demands, it is assumed to be the negative binomial distribution.* The statistical process that leads to units demanded per unit time period having a negative binomial distribution is as follows: the individual customers arrive with a Poisson distribution, but the number of units demanded per arrival follows a logarithmic distribution. The negative binomial distribution belongs to the family of compound Poisson distributions. This assumption is more flexible than the usual Poisson distribution, for it can provide for high variance to mean ratios if desired.

MEASURE OF SUPPLY EFFECTIVENESS

The effectiveness of a particular stockage policy is evaluated on the basis of the statistical forecast described above. In this model, we assume that demands not satisfied immediately are back-ordered, and the expected number of backorders per year is used as a measure of effectiveness, which is heuristically defined as follows:**

* One practical reason for assuming negative binomial instead of geometric Poisson as in [8] is that the computer code we have for calculating probability terms of the latter distribution will generate underflows for items with sufficiently high mean demand rates. Since one is apt to encounter more high-demand items when dealing with EOQ items, it seems desirable to use the negative binomial distribution to avoid the above difficulty.

** Strictly speaking, in order to compute this quantity, state probabilities of various number of units in routine resupply need to be computed. The probability of being out of stock is then derived from these state probabilities. Finally, the annual demand rate is multiplied by the probability of being out of stock. The exact formulation of the sort described above is rather complicated (e.g., see [10] and [11], pp. 181-191). Fortunately, it is seldom necessary in practice to compute the exact formulation, because it is not needed unless backorders cost very little, and in such cases there is no inventory problem.

Expected number of backorders per year = (expected number of backorders per order cycle) x (expected number of cycles per year).

For a system as a whole, the sum of individual item backorders is taken as an index of how well the entire system is operating under a given stockage policy. It should be noted that by using the arithmetic sum of backorders as a measure of system performance, we implicitly assume that each backorder carries an equal weight.

OPTIMIZATION

Mathematically, our problem is to find a reorder point and order quantity for each item that will satisfy some standards placed on two of the system characteristics while minimizing the remaining one. For such a constrained minimization problem, it is often computationally convenient to form a Lagrangian function if a mechanism is available for readily finding appropriate Lagrangian multipliers. For this purpose, the technique proposed in [2] for approximating the multipliers by linear programming was used.

The Lagrangian multipliers in the context of this inventory problem have meaningful economic interpretations. Suppose the problem is to minimize the average inventory investment subject to constraints imposed on system backorders and order frequency. The multipliers associated with these two constraints must have the dimensions of dollars per backorder and dollars per replenishment order, respectively. These are the amounts with which the investment may be reduced if the constraints are relaxed by one additional unit. The multipliers, therefore, can be considered as the imputed costs or shadow prices of a backorder and of placing a replenishment order.

The model described here has been programmed in FORTRAN IV, and consequently can be run on any large-scale computer. Usage of the program is discussed in Appendix B.

III. SOME NUMERICAL RESULTS

This section discusses two topics. The first relates to the effect on the model performance of changes in some key parameters, such as the average resupply time and the variance to mean ratio of item demands. The second topic is comparative evaluations of the model policy and the current Air Force procedures as stated in AFM 67-1. This gives a basis for estimating probable gains if the model is used for the management of EOQ items.

DATA USED IN THE SENSITIVITY ANALYSIS

The data consisted of one-year demand history on over 11,000 items applicable to the F-4C aircraft.* For each item, the unit price and the total demand for a twelve-month period were known. Table 1 presents a frequency distribution of items according to their annual demand.

Table 1

DISTRIBUTION OF ITEMS BY DEMAND

No. of Units Demand/Year	No. of Items	Relative Frequency (%)
0-10	8,164	73.6
11-20	1,083	9.8
21-30	572	5.2
31-40	348	3.1
41-50	244	2.2
51-60	191	1.7
61-70	160	1.4
71 + over	328	3.0
	<u>11,090</u>	<u>100.0</u>

*The data were obtained from AFLC Project PACER SORT.

Note the high frequency of low-demand items. Apparently, many low-unit-cost items are not necessarily fast-moving items.

EFFECT OF VARYING RESUPPLY TIME ON AVERAGE INVENTORY INVESTMENT

Sensitivity of the model performance to resupply time was studied. This provides information which is useful for deciding whether to have more inventory or shorter response time. The model was run for three different resupply times--20, 40, and 80 days. For all the runs, the replenishment order frequency was kept constant at 12,500 times per year but the backorder rates were kept at three levels--1, 5, and 10 percent. The results are summarized in Fig. 3.

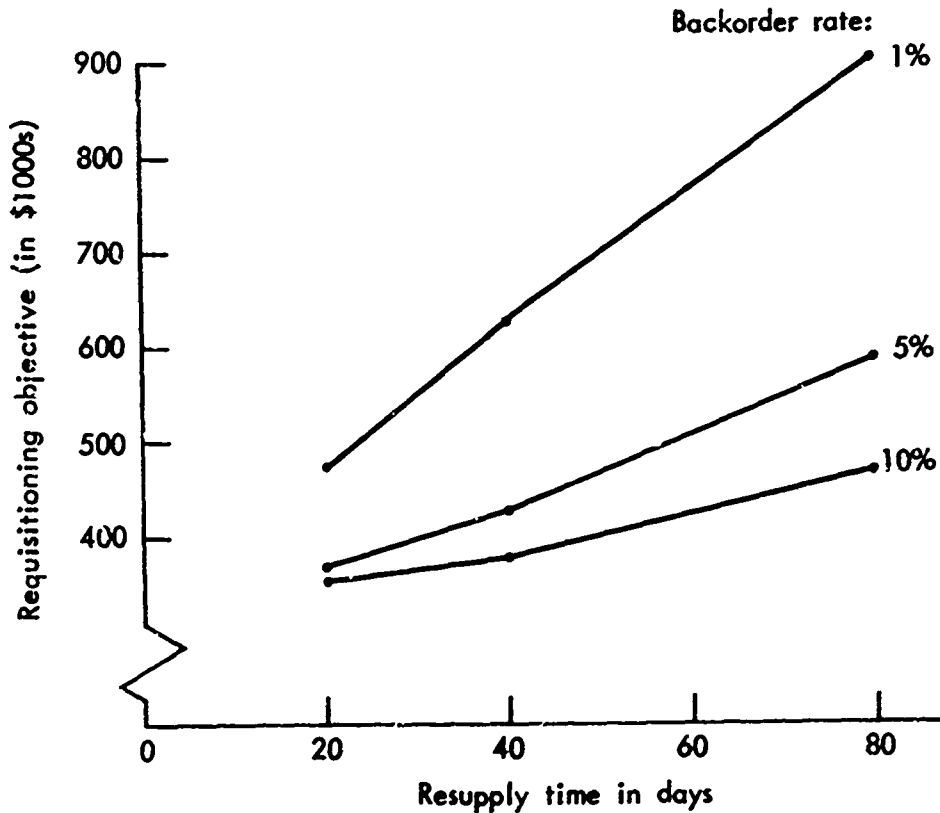


Fig. 3 -- Resupply time vs. inventory level of 11,090 EOQ items

The vertical axis is the requisitioning objective, which is the sum of reorder point and economic order quantity.* As the slopes of the curves reveal, the higher the performance level one wishes to maintain, the more pronounced is the impact of a longer resupply time. For instance, about an 89-percent increase is required in the inventory investment to compensate for the resupply time's changing from 20 days to 80 days, if the backorder rate is to be maintained at the one-percent level; an increase of only 33 percent is necessary if a 10-percent backorder rate is maintained.

Another way of viewing the impact of resupply time on model performance is to ask what the consequences are of incorrectly specifying the average resupply time. To answer this question, optimal policies were computed for resupply times of 20, 40, and 80 days. The backorder rates of these policies were then evaluated, under the pretense that the "true" resupply time was something other than what was assumed. In these calculations, the backorder rate was kept at 5 percent, and the number of replenishment orders per year was kept constant at 12,500. Results are given in Table 2.

Table 2

Assumed Resupply Time (Days)	True Resupply Time (in Days)		
	20	40	80
20	5.0%	11.8%	29.1%
40	2.0	5.0	15.2
80	0.8	1.7	5.0

*This is called a demand level in AFM 67-1.

From Table 2 it appears that incorrect specifications of the true resupply time are serious. For instance, if a computed policy is optimal for a 20-day resupply time but the actual resupply time turns out to be 80 days, the backorder rate will be six times as high.

SENSITIVITY OF BACKORDER RATE TO VARIANCE TO MEAN RATIO OF DEMAND DISTRIBUTION

The model assumes that the variance to mean ratio is a linear function of the item mean $\alpha + \beta \theta$, where θ is the item mean and α and β are constants. It was desired to know the sensitivity of backorder rate to various assumptions on the values of α and β . A policy was therefore computed that is optimal with respect to the assumption that $\alpha = 1.5$, $\beta = 0$; the backorder rates of this policy were then evaluated to see how much degradation would take place when $\alpha = 3$, $\beta = 0$ and $\alpha = 1.5$, $\beta = 0.5$. Results are presented in Table 3.

Table 3

SENSITIVITY OF BACKORDER RATE TO VARIANCE TO MEAN RATIO

Assumed Variance to Mean Ratio	True Variance to Mean Ratio		
	$\alpha = 1.5^a$ $\beta = 0$	$\alpha = 3.0$ $\beta = 0$	$\alpha = 1.5^b$ $\beta = 0.5$
$\alpha = 1.5, \rho = 0$	4.9%	5.5%	6.7%

^a To obtain this policy, the order frequency was kept at 12,500 and backorder rate at 5%, while the average inventory investment was minimized.

^b Since the average item mean per year was 9.84, the true variance to mean ratio would be 6.4.

From Table 3 it appears that system performance is not very susceptible to the variance to mean ratio. In other words, an incorrect

specification of this variance to mean ratio does not seem to result in an intolerable misallocation of resources.

COMPARISON OF THE EOQ MODEL WITH THE CURRENT AIR FORCE PROCEDURE

Simulation

This section summarizes results of a simulation study^{*} designed for evaluating the expected effects of the EOQ model and the current Air Force Policy of AFM 67-1.^{**}

The study was originally intended to base its comparative evaluation on historical supply data to be collected at an air base (Cam Ranh Bay); however, this idea was abandoned because of deficiencies in both the 1050-II base supply and depot supply systems. Instead, a random sample of 1000 EOQ items were selected, and synthetic demand histories were generated for each item.^{***} Subsequently, the stock levels computed by the EOQ model and AFM 67-1 procedure were tested against these demands for a one-year period. Two sets of reorder points and order quantities were computed by the model based on the same data used for the previously described sensitivity tests. In

* The study was carried by Mr. R. Alsedek of the Directorate of Maintenance, Ogden Air Materiel Area. Simulation results were documented in an unpublished paper, "A comparative Evaluation of Expected Effects of Alternative Inventory Policies Through Systems Simulation."

** The Air Force policy of AFM 67-1 is summarized in Appendix C.

*** The generation of demand history was based on the following assumptions: (1) item demand follows a geometric Poisson distribution; (2) the variance to mean ratio of these demand distributions is 1.5. The first assumption is at slight variance from the comparable assumption of the model. The model assumes item demand has a negative binomial distribution. This discrepancy did not pose a serious problem because the assumed variance to mean ratio was relatively low.

each of the runs, the average inventory investment was minimized with the following constraints on the remaining two system characteristics:

<u>Run</u>	<u>Backorder Rate</u>	<u>Order Frequency</u>
1	1%	30,000/year
2	15%	15,000/year

For the AFM 67-1 procedure, the order and shipping time of 30 days and the C factor of 1 were assumed. Results are given in Table 4.

Table 4

EOQ MODEL VS. AFM 67-1

Run	Average Inventory Investment	Orders Per Year	Backorder Rate
EOQ #1	\$18,622	3655	3%
EOQ #2	4,353	2875	14
AFM 67-1	28,025	1977	11

If we compare the characteristics of EOQ #1 with those of AFM 67-1, we note that the former policy requires a substantially smaller average inventory investment, but this advantage is offset by requiring more replenishment orders. However, the supply effectiveness of the model policy is definitely superior to that of the AFM 67-1 procedure. Between EOQ #2 and AFM 67-1, there is little basis to choose one policy over another in terms of the supply effectiveness. But in the other two characteristics, EOQ #2 is a little more attractive.

In summary, the simulation results seem to indicate that the EOQ model has a higher level of performance than the AFM 67-1 product for the same amount of resources; or stated differently, it seems to be able to achieve the same level of performance as AFM 67-1 but with less resources. The original author of this simulation attributes this superior performance of the model over the AFM 67-1 product to the fact that the former uses a Bayesian technique for demand forecasting and consequently is able to compute positive stock levels for more items than can the latter.

ANALYTIC COMPARISON

Another comparison of the two procedures was done analytically. A set of reorder points and order quantities was first computed according to the AFM 67-1 procedure as in the simulation, with the following modification. Every item is assigned the order quantity of at least one unit even though its daily demand rate might be too low to generate any order quantity or reorder point. This has the effect of broadening the range of items with some positive stock levels and should boost the effectiveness of the AFM 67-1 procedure. These reorder points and order quantities were then evaluated by the model and were found to have the following characteristics.

Dollar value of the demand levels....	\$2.456 million
Backorder rate.....	3.2%
Orders/year.....	22501

When the model was run with the same constraints as above on the average backorder rate and order frequency, it required a total stock investment of \$1.195 million.

Based on this calculation, a nearly 50-percent reduction in the investment seems feasible without sacrificing other aspects of system performance.

IV. CONCLUDING REMARKS

The EOQ model described appears to offer significant improvement over the AFM 67-1 procedure. Its aggregate approach to the stockage problem of EOQ items enables the base supply manager to translate given system-wide objectives into a set of operating rules that is implementable at item level. Its Bayesian procedures provide a basis for developing a smoothing technique for demand prediction. The policy computed by the model is fairly sensitive to the average re-supply time; however, it does not appear sensitive to the variance to mean ratio.

What are the potential uses of this model? The most straightforward application of the model is to take a group of EOQ items at base level, and to compute a reorder point and an order quantity for each item so as to meet certain specifications on the system performance. If these two model-computed policy variables can be made part of the 1050-II supply system, i.e., if they can be inserted into the 1050-II computer in place of the similar variables computed by the AFM 67-1 procedure, then the requisition and replenishment processes of the item's base stock can follow what is prescribed in AFM 67-1.

To make the above application feasible, some reasonable answers must be found to the following questions:

1. What group of EOQ items to select for application. There are O&M-funded EOQ items and AFLC-supplied EOQ items. From the standpoint of the Chief of Supply, the economics of managing these two different groups of items are different. He is probably more personally concerned with management of the first group of items because

they require him to obligate funds, and he will find the model useful for adjusting his stockage posture to meet various levels of fund availability. As to the management of AFLC-supplied items, it is desirable to have a two-echelon model in which the interaction of stocking at base versus depot levels is considered.

2. Another factor to consider in deciding the applicability of the model to a given group of items is the homogeneity, in some sense, of the items in the group. This relates to the question of essentiality. Since the model measures the performance level of a supply system by the total expected number of backorders, we must be sure it makes sense to add a backorder of one item to that of another. In other words, a group should be formed in such a way that it is reasonable to assume that every backorder, regardless of its item origin, carries the same weight. This means, for instance, one may want to apply the model separately to aircraft-related and missile-related items.

3. Whether the currently available base computer is capable of handling the model computation, and how often the entire EOQ account has to be recomputed to obtain the best performance on a continuing basis. Answers to these questions are not independent of each other. The computer program in the current configuration cannot be run on the 1050-II computer, primarily because it requires a large core storage. Hence, if the model is to be implemented as part of the 1050-II base supply system, a means must be found to reduce the computational task imposed on the base computer. This can be done, for instance, if a system-wide optimization can be performed by a large-scale computer (such as UNIVAC 1107) at a depot, and the appropriate

constants (Lagrange multipliers) can be sent to bases for optimizations by item.

The question of how often the model should be run cannot be answered without weighing the incremental cost of performing an additional computation as against the incremental gain to the base supply manager of having more up-to-date information for decision-making. This probably requires a fairly elaborate simulation of the base supply system. On the other hand, one can say, a fortiori, that daily releveling as practiced by the AFM 67-1 procedure is not only unnecessary but undesirable.

The foregoing may well have raised more problems than it answered questions. But these are problem areas where future research needs to be directed if a model such as this is to become a practical planning tool of the base supply manager.

Appendix A

MATHEMATICAL DESCRIPTION OF THE EOQ MODEL

This Appendix gives a mathematical description of the structure of the model, including a linear programming formulation of the original stockage problem.

Assumptions underlying the model have appeared either explicitly or implicitly in the discussion in Sec. II. They are summarized here for the convenience of the reader. The symbols to be used in the subsequent discussion are also defined.

ASSUMPTIONS

1. We are considering a system that manages many items. The system is the sole source of supply for users of these items; i.e., no lateral resupply is considered.

2. The operating rule for each item is to replenish its stock in batches under the simple procedure of placing replenishment orders of the same quantity when the sum of stock on hand and stock on order falls below a prespecified level. For this rule to be operative, a transaction reporting system is assumed to be in effect.

3. Demands for an individual item arrive at random with a stationary negative binomial probability distribution. These demands are independent of the demands of other items in the system.

4. All demands occurring during periods of stock depletion are backlogged.

5. All replenishment times of an item are of the same length. This means that two successive replenishment orders cannot arrive in reverse sequence.

6. Item unit cost is independent of order quantity, i.e., there is no price break due to a quantity discount.

THE MODEL

Three operational consequences interest us when we apply a particular stockage policy to manage EOQ items:

- o Average dollar investment in inventory
- o Backorders (across all items)
- o Order frequency

We wish to express these system characteristics as a function of the policy variables--reorder points and order quantities--and certain data on the items--unit cost, replenishment times, and observed demands. We will use the following notation:

r_i = the reorder point for item i

q_i = the order quantity of item i

π_i = unit price of item i

λ_i = the yearly demand rate for item i

d_i = the number of demands for item i during the past year

t_i = the replenishment time for item i

$p(\cdot; \mu)$ = a negative binomial probability distribution with mean μ .

Unfortunately, we do not know the value of λ_i . We do, however, know the quantity of demands d_i that actually occurred for the i^{th} item. We use the same technique as is used in [7] in order to get information about λ_i from d_i . We assume that λ_i is a random variable with some a priori distribution function. We then use Bayes's law together with d_i to obtain an a posteriori distribution function for λ_i (a conditional

distribution for λ_i conditioned on d_i). The prior and a posteriori functions for λ_i are obtained in the same fashion as that described in [7].

If λ_i were known, the expected investment in inventory for item i ($I_i(q_i, r_i)$), the expected number of shortages per year on item i ($B_i(r_i, q_i)$), and the expected number of orders per year ($O_i(r_i, q_i)$) are given below as in [11].

$$I_i(q_i, r_i) = \pi_i(q_i/2 + r_i - \lambda_i t_i) ,$$

$$B_i(r_i, q_i) = (\lambda_i/q_i) \sum_{u \geq r_i} (u - r_i) p(u ; \lambda_i t_i) ,$$

$$O_i(r_i, q_i) = \lambda_i/q_i .$$

And the system-wide policy characteristics are given by summing the above functions over all items i . Since λ_i is not known, we use the a posteriori distribution of λ_i to take the expectation of the above functions and denote the results by $\bar{I}_i(r_i, q_i)$, $\bar{B}_i(r_i, q_i)$, and $\bar{O}_i(r_i, q_i)$ respectively. Note that \bar{I}_i , \bar{B}_i , and \bar{O}_i are given by

$$\bar{I}_i(q_i, r_i) = \pi_i(q_i/2 + r_i - \bar{\lambda}_i t_i)$$

$$\bar{B}_i(q_i, r_i) = \frac{1}{q_i} S_i(r_i)$$

$$\bar{O}_i(q_i) = \bar{\lambda}_i/q_i ,$$

respectively, where $\bar{\lambda}_i$ is the a posteriori expectation of λ_i , and

$S_i(r_i)$ is the a posteriori expectation of $\lambda_i \sum_{\mu \geq r_i} (\mu - r_i) p(\mu ; \lambda_i t_i)$.

The optimization problem may now be formulated as follows: Find r_i, \dots, r_n and q_i, \dots, q_n so as to

$$\begin{aligned}
 & \text{Minimize} && \sum_i \bar{I}_i(r_i, q_i) \\
 (3) \quad & \text{Subject to} && \sum_i \bar{B}_i(r_i, q_i) \leq b \\
 & && \sum_i \bar{O}_i(r_i, q_i) \leq c .
 \end{aligned}$$

The above optimization problem is solved by the method of Lagrange multipliers [6]: we minimize a "Lagrangian"

$$(4) \quad \sum_i \bar{I}_i(r_i, q_i) + \beta \sum_i \bar{B}_i(r_i, q_i) + \omega \sum_i \bar{O}_i(r_i, q_i)$$

for certain values of the Lagrange multipliers β and ω . The values of β and ω which, when inserted in (4), tend to an approximate solution of (3) are found by the method suggested in [1] and [2]. We approximate (3) by the linear programming problem: Find nonnegative "weights" $w_{(r,q)}$ to attach to each policy $r = (r_1, \dots, r_n)$, $q = (q_1, \dots, q_n)$ so as to minimize the weighted average

$$\sum w_{r,q} \sum_i \bar{I}_i(r_i, q_i)$$

(5) Subject to

$$\sum w_{r,q} \sum_i \bar{B}_i(r_i, q_i) \leq b$$

$$\sum w_{r,q} \sum_i \bar{O}_i(r_i, q_i) \leq c$$

$$\sum w_{r,q} \leq 1 .$$

Here the outer sums are taken over all possible policies (r_1, \dots, r_n) ; (q_1, \dots, q_n) .

The dual variables associated with the first two constraints in (5) become the required "Lagrange multipliers" in (4).

In order to solve (5) we use the "simplex method using multipliers" [4].

This method requires repeated minimization of (4) for various values of β and ω .

In order to minimize (4) it suffices to minimize

$$(6) \quad L_i(r_i, q_i) = \bar{I}_i(r_i, q_i) + \beta \bar{B}_i(r_i, q_i) + \omega \bar{O}_i(q_i) .$$

In the discussion to follow, we will drop the item subscript i and write, e.g., $L(r, q)$ for $L_i(r_i, q_i)$.

Although the reorder point r and order quantity q must necessarily be integers, we do not, in fact, find integers r and q that minimize $L(r, q)$. Instead we find an integer $r \geq 0$ and a real number $q \geq 1$ that minimize $L(r, q)$. We then round q off to the nearest integer q' , say, and find an integer r' that minimizes $L(r', q')$.

We have two procedures for finding the integer $r \geq 0$ and real number $q \geq 1$. The first procedure does not always result in an r and q that minimize L . The second procedure always does. On the other hand, the first method is faster than the second.

In the first method we follow [11]. We first determine a value of q^0 from the well-known Wilson lot size formula,

$$q^0 = \sqrt{2\omega\bar{\lambda}/\pi} .$$

We then choose an integer r^0 so as to minimize $L(r^0, q^0)$. In general, for $t > 0$, we determine q^t by

$$(7) \quad q^t = \sqrt{\frac{2\beta S(r^{t-1}) + 2\omega\bar{\lambda}}{\pi}}$$

and then choose r^t so as to minimize $L(r^t, q^t)$. The process terminates when $q^t = q^{t-1}$. At this point we set $r = r^t$ and $q = q^t$. The formula (7) results from minimizing $L(r, q)$ with respect to q for fixed r . Thus the pair (r, q) will have the property

$$L(r, q) = \min_{r'} L(r', q)$$

and

$$L(r, q) = \min_{q'} L(r, q') .$$

But it will not necessarily be true that*

$$(8) \quad L(r, q) = \min_{r', q'} L(r', q') :$$

In order to obtain a value of q and r for which (8) does hold, we use the following procedure. Define $Q(r')$ as a function of r' by the formula

$$Q(r') = \sqrt{\frac{2\beta S(r') + 2\omega\bar{\lambda}}{\pi}} ,$$

choose r to minimize $L(r, Q(r))$, and then set $q = Q(r)$. This method requires more time than the first, but it is guaranteed to produce values of r and q for which (8) does hold. The way we have used these procedures is to use the first procedure until the linear programming problem (5) is apparently solved. After that we use the second procedure until the program is actually solved.

* This problem of nonconvexity was pointed out in [13].

Appendix B

THE COMPUTER PROGRAM

The model was programmed on the IBM 7044 computer. Since it is written in FORTRAN IV, it is readily adaptable to other large-scale computers. This Appendix contains descriptions of the structure of the program, input and output formats, and some notes on operating procedure of the program.*

1. Structure of the Program

In addition to a main routine, the program consists of a number of subroutines. We shall discuss the program structure by first presenting a flow chart of the main program and briefly describing the functions of subroutines.

A. Flow Chart of the Main Program

A flow chart of the program's main routine is presented in Fig. 4.

B. Functions of Subroutines

As can be seen in the following flow chart, the main routines call a number of subroutines, whose functions are as follows:

(1) AGGRE

This subroutine has two functions:

(a) To compute various aggregate statistics such as means and standard derivations of the following variables:

*The program may be made available on request.

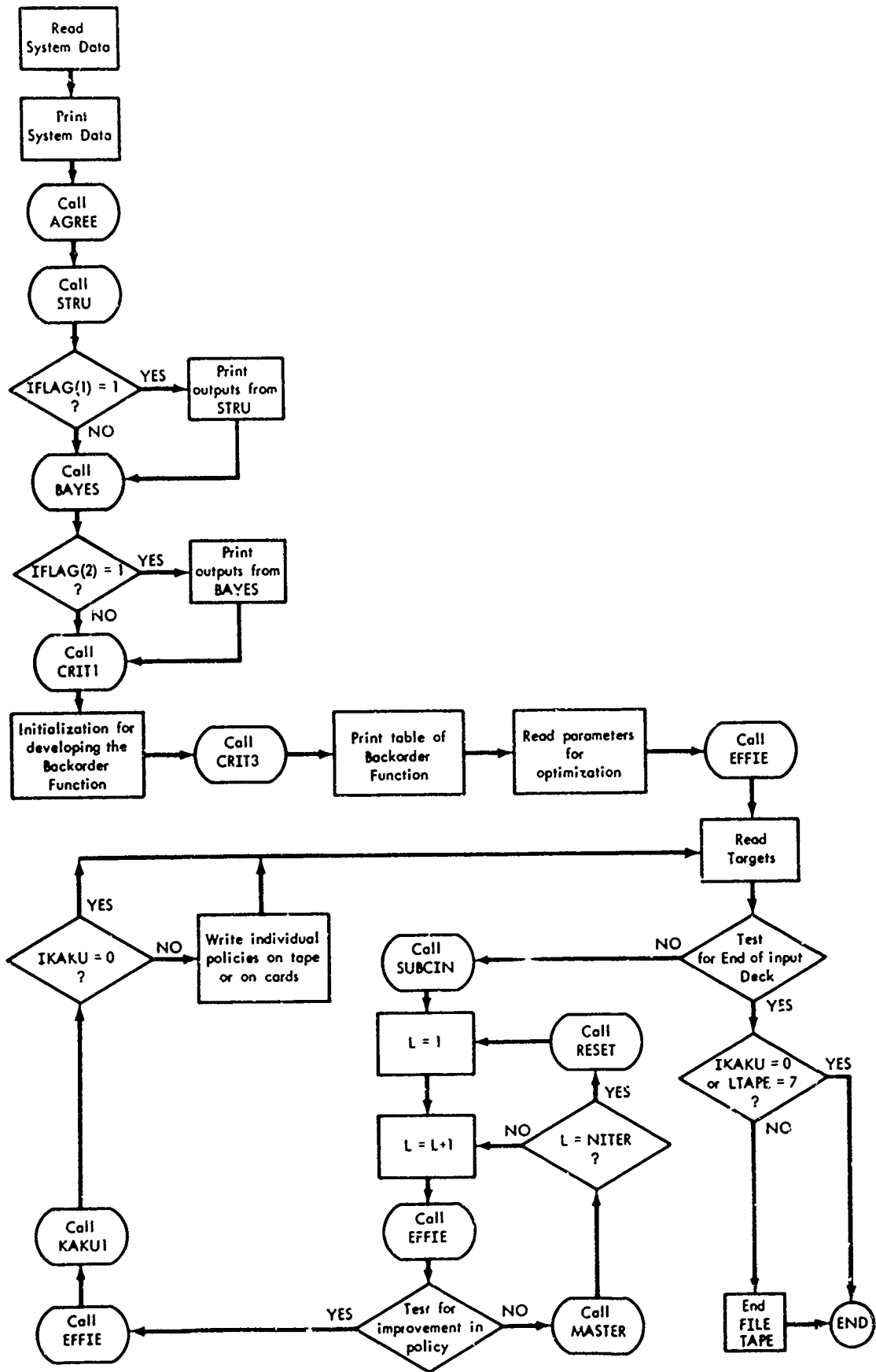


Fig. 4 -- Flow chart of the main program

- . Past demand during some fixed period
- . Logarithm of past demand
- . Logarithm of unit price

(b) To classify every item into one of the cells, each of which is uniquely identified by demand, unit price, and resupply time.

More specifically, the subroutine does the following:

- . All items with past demands in excess of NIU (see p. 36) are identified and then item characteristics are printed. No further analysis will be performed for these items within the program.
- . The remaining items are classified into two resupply time categories.
- . Within each category, a cost-demand matrix is provided. The maximum size of the matrix is 20 x 20, i.e., there can be at most 20 demand categories and 20 cost categories. Every item falling within the specified demand and cost dimensions is placed into an appropriate cell in its respective matrix.

The aggregation subroutine will produce a maximum of 800 cells and the optimization will be performed by other subroutines on these cells.*

(2) STRU

This subroutine is for estimating the parameters of the assumed prior distribution (lognormal) of mean item demands. The algorithm of Subroutine STRU has been described in [12], pp. 6-9.

(3) BAYES

Subroutine BAYES calculates the conditional probabilities of an item in various demand processes given its past demand. The interested reader is again referred to [12] for a detailed description of the algorithm.

* Since the number of EOQ items in most supply systems in the Air Force is large (over 10,000), some aggregation procedure is probably required to apply this program to "real world" problems.

(4) CRIT1 AND CRIT3

These two subroutines together calculate the conditional expectations of the following quantities for each demand category:

- . Mean demand in resupply time t_1
- . Mean demand in resupply time t_2
- . Mean demand in a one-year period
- . 1 - (cumulative probability distribution of demands in t_1).
- . 1 - (cumulative probability distribution of demands in t_2)
- . Number of units in backorder when resupply time is t_1
- . Number of units in backorder when resupply time is t_2

The outputs of these two subroutines are used to assess the effectiveness of a given stockage policy (a set of reorder points and order quantities).

(5) EFFIE

This subroutine will determine an efficient stockage policy with respect to a given price vector (i.e., a set of Lagrange multipliers).

(6) SUBCIN, MASTER AND RESET

The linear programming formulation of the original stockage problem is carried out by these three subroutines. Subroutine SUBCIN performs initializations of some relevant variables prior to the simplex iteration. The algorithm of the simplex method using multipliers (or the revised simplex method) is embedded in MASTER. And RESET will reinvert the basis after a certain specified number of iterations has been performed by MASTER.*

*The algorithm and general usage of these three subroutines will be discussed fully in a forthcoming publication by R.B.S. Brooks.

(7) KAKU1

Subroutine KAKU1 outputs results of the optimization.

2. Inputs

The program requires two types of data inputs: the individual item information and the system data. The system data include parameters that describe the entire aggregate of items and constants that control the running of the program.

A. Item information

The item information is prepared in the form of one-record-per-item with the following data on each item:

- (1) Federal stock number
- (2) Unit cost
- (3) Unit of issues
- (4) Resupply time code--the program allows two categories. The code should be either 1 or 2 corresponding to a specific resupply time.
- (5) Past demand over some fixed period of time

Item data must appear in the order indicated above. The length of each data field can be flexible because a format statement is read by the program as an input. The current version of the program will accept the item information written on a magnetic tape in a BCD form only. Since in most interesting applications of the program the number of items involved is very sizable, we feel that it is not practical to input the item information on cards.

It should be noted that the program expects the input tape to have an end-of-file mark.

B. System Data

The system data should be on cards. Various system input cards are described below in the proper order to be read by the program. In addition to describing each card and the variables it contains, the restrictions on the range of values that the variables may assume are indicated, whenever necessary. For the purpose of illustration, the input values that were used to obtain the sample output, which is discussed later in this appendix, are also presented.

CARD 1

<u>Columns</u>	<u>Variable Names</u>	<u>Formats</u>
1	IKAKU	I1
2	LTAPE	I1

IKAKU determines whether the computed optimum reorder points and order quantities are to be presented in the aggregate form only or to be presented on the individual item basis as well. When IKAKU = 0, the output will be given only in the aggregate form; if IKAKU \neq 0, each item will be assigned its proper optimum stockage policy.

If IKAKU \neq 0, LTAPE determines whether the individual output is to be written on a tape or to be punched in the card form. For the RAND computer installation, by setting LTAPE = 7 we can have the optimum reorder point and order quantity punched out in a one-card-per-item-basis. If the number of items involved is large, we would set LTAPE = 9 or any other available FORTRAN logical tape unit and write the result on the tape according to a specified BCD format.

CARD 2

<u>Column</u>	<u>Variable Name</u>	<u>Format</u>
1-48	DID	8A6

This variable is used for item data identification. It accepts any alpha-numeric description of 1 to 48 characters.

CARD 3

<u>Columns</u>	<u>Variable Names</u>	<u>Formats</u> *
1-10	NIU	I10
11-20	NT	I10
21-30	NR	I10
31-40	NTAPE	I10
41-50	MTAPE	I10

NIU is the maximum number of units demanded for an item over a base data period. For the current version of the program, the maximum allowable NIU is 200. In the example, we have NIU = 100.

NT is the number of response time categories. The program allows at most two such categories. In the example, NT = 1. NR is the maximum reorder point plus one. In the example, NR = 50; i.e., the highest reorder point that can be computed by the program is 49.

NTAPE is the FORTRAN logical unit name of the item input tape. It could be any system utility tape number.

MTAPE is the FORTRAN logical unit name of a binary utility tape used in the course of the program. Any available system utility tape can be used. Note that MTAPE need not be specified if IKAKU = 0.

* Unless otherwise indicated, all integer variables should be right-justified.

CARD 4

<u>Columns</u>	<u>Variable Names</u>	<u>Formats</u>
1	IFLAG(1)	I1
2	IFLAG(2)	I1
3	IFLAG(3)	I1
5-7	INTVL	I3

IFLAG(1), IFLAG(2), and IFLAG(3) are the names of variables used to control the printout of intermediate outputs.

IFLAG(1) = 1 → Output from subroutine STRU is printed.

IFLAG(1) ≠ 1 → Output suppressed.

IFLAG(2) = 1 → Output from subroutine BAYES is printed.

IFLAG(2) ≠ 1 → Output suppressed.

IFLAG(3) = 1 → Data for items with past demands in excess of NIU are printed.

IFLAG(3) ≠ 1 → Printout suppressed.

INTVL determines the range within each demand category in the aggregation table. The restriction on this variable is that

$$\frac{NIU}{INTVL} \leq 20 .$$

This is to conform to the dimension specification of the aggregation table. INTVL = 5 in the example.

CARD 5

<u>Column</u>	<u>Variable Name</u>	<u>Format</u>
1-72	FMT	12A6

FMT is the format description for the item data cards. It has the standard alpha-numeric form of a FORTRAN format statement.

CARD 6

<u>Columns</u>	<u>Variable Names</u>	<u>Formats</u>
1-10	ZZ	F10.6
11-14	DBASE	F4.1
15-17	THX	F3.2

ZZ is the value of a standard normal deviate used to develop the aggregation table. In the example $ZZ = 1.96$.

DBASE is the number of days in a base data period. THX is the multiplier which determines the future activity level as a fraction of the activity level in the base data period. DBASE = 360 and THX = 1 in the example.

CARD 7

<u>Columns</u>	<u>Variable Names</u>	<u>Formats</u>
1-5	B(1)	F5.2
6-10	B(2)	F5.2
:	:	:
46-50	B(10)	F5.2

CARD 8

<u>Columns</u>	<u>Variable Names</u>	<u>Formats</u>
1-5	B(11)	F5.2
6-10	B(12)	F5.2
:	:	:
41-45	B(19)	F5.2

The subscripted variables (B(i), $i = 1, 2, \dots, 19$), are the values of some standard normal deviates used to develop the aggregation

table. In the example they are set to 1.65, 1.28, 1.04, 0.84, 0.67, 0.52, 0.39, 0.25, 0.13, 0, -0.13, -0.25, -0.39, -0.52, -0.67, -0.84, -1.04, -1.28, -1.65.

CARD 9

<u>Columns</u>	<u>Variable Names</u>	<u>Formats</u>
1-10	XBAR	F10.5
11-20	SIGX	F10.5

XBAR is the initial estimate of the mean of the logarithms of unit prices and SIGX is the initial estimate of the standard deviation of the same variable. XBAR = 0.58 and SIGX = 2.45 were used in the example.

CARD 10

<u>Columns</u>	<u>Variable Names</u>	<u>Formats</u>
1-4	AL	F4.2
5-8	BE	F4.2
9-10	NK	I2
11-13	Z (1)	F3.1
14-16	Z (NK)	F3.1

AL and BE are the parameters for determining the relationship between the mean and variance of item demands. $\alpha = 1.5$ and $\beta = 0$ in the example. In this case α is interpreted as the variance to mean ratio.

NK is the number of discrete points used to approximate the prior distribution of item demands.

Z(1) and Z(NK) are the smallest and largest value of the standard normal deviate used to compute the prior distribution. They were set to -2 and 3 respectively in the example.

CARD 11

<u>Columns</u>	<u>Variable Names</u>	<u>Formats</u>
1-10	T(1)	F10.2
11-20	T(2)	F10.2

T(1) and T(2) are the response time, in days, of items in response time categories 1 and 2 respectively.

CARD 12

<u>Columns</u>	<u>Variable Names</u>	<u>Formats</u>
1-7	ROUND	F7.6
8-13	SCALEC	F6.2
14-19	SCALES	F6.2
20-25	SCALEO	F6.2
26	IHOLD	I1
27	ISHORT	I1
28	IORDR	I1
29-30	NITER	I2

ROUND, SCALEC, SCALES, and SCALEO are the values required by the program to control round-off errors that occur in iterations of the simplex method. In the example,

ROUND = 0.000005

SCALEC = 1000

SCALES = 1

SCALEO = 100

In a supply system of EOQ items, three system characteristics are of interest: the average investment in inventory, the average number of backordered units, and the total number of requisitions per unit time. In the model any one of the system characteristics can be minimized within the constraints imposed on the remaining two characteristics. IHOLD, ISHORT, and IORDR designate which system characteristics are to be minimized and which ones are to be the constraints for the minimization. Each of these variables can assume the value of either 1, 2, or 3. The value of 3 is associated with that system characteristic which is to be minimized. The value of either 1 or 2 means that the system characteristic is to be used as a constraint. IHOLD = 3, ISHORT = 1, IORDR = 2 in the example. This means that the average inventory is to be minimized subject to constraints on the number of backorders and order frequency per year.

NITER is the number of iterations in the simplex method before the basis is reinverted. We set NITER = 10 in the example.

CARD 13

<u>Column</u>	<u>Variable Name</u>	<u>Format</u>
1-72	CASEID	12A6

CASEID is the case identification card that can take any alpha-numeric characters. For instance, this can be used to indicate, as a part of output, which system characteristics have been minimized.

CARD 14

<u>Columns</u>	<u>Variable Names</u>	<u>Formats</u>
1-10	PRICE(IHOLD)	F10.5
11-20	PRICE(ISHORT)	F10.5
21-30	PRICE(IORDR)	F10.5

PRICE(IHOLD), PRICE(ISHORT), and PRICE(IORDR) are the initial estimates of Lagrange multipliers associated with the average inventory investment, backorders, and order frequency, respectively. Any positive values can be assigned to these variables.

CARD 15

<u>Columns</u>	<u>Variable Names</u>	<u>Formats</u>
1-8	TARGET(IHOLD)	F8.0
9-12	TARGET(ISHORT)	F4.3
13-18	TARGET(IORDR)	F6.0

These variables are the constraints placed on the three system characteristics. TARGET(IHOLD) is the average inventory investment in dollars. TARGET(ISHORT) is the average backorder rate in fraction. TARGET(IORDR) is the number of orders per year. For any run, we need to specify only two targets and leave unspecified the third one, which corresponds to the characteristics to be minimized. In the example, TARGET(IHOLD) = 0, TARGET(ISHORT) = 0.01, and TARGET(IORDR) = 12500.

CARD 16

<u>Column</u>	<u>Variable Name</u>	<u>Format</u>
1-72	RUNID	12A6

RUNID is the run identification for a specific set of targets. Any alpha-numeric characteristic can be used. We can have as many pairs of cards 15 and 16 as long as values assigned to TARGETS are consistent with the specifications of IHOLD, ISHORT, and IORDR.

A blank card following Card 16 signifies the end of a case and the program will then attempt to read another case card (Card 12) and the subsequent inputs (Cards 13-16). If there is no case card, the program execution will terminate.

3. Outputs

The outputs from a typical run of the program are presented in Figs. 5 through 9. The first part (Fig. 5), contains a data identification, which is followed by the format of each record on the item

```
PACER SORT DATA - RLD ITEMS EXCLUDED
VARIABLE FORMAT
(3A5,5X,F10.2,1X,A2,1X,11,15,11,F3.1,3I2,29X)
NIU  NT  NR  NTAPE  MTAPE  XBAR  SIGX  OBASE  THX  AL  BE  NK  Z(1)  Z(NK)  T(1)  T(2)
100  1  50   8      1   0.58  2.45  365.  1.00  1.5-0.0  10  -2.   3.   30.   0.

YBAR=  1.536713  SIGSOV=  1.498484  SIGY=  1.224126
XBAR=  0.576450  SIGCOX=  5.623470  SIGX=  2.371386
```

Fig. 5

data tape. It then recapitulates the basic inputs to the program. At this point, if the programmer desires, a detail printout of those items can be obtained, with past demand in excess of some specified maximum or with unit cost equal to zero. This printout is controlled by IFLAG(3) (see p. 38). Finally, some statistics concerning item demand and unit costs are presented.

YBR = mean of the logarithm of item past demand
SIGSQY = variance of the logarithm of item past demand
SIGY = standard deviation of the logarithm of item
past demand
XBR = mean of the logarithm of item unit cost
SIGSQX = variance of the logarithm of item unit cost
SIGX = standard deviation of the logarithm of item
unit cost

In the second part (Fig. 6), we have the aggregation table of items for each resupply time category. Along the top of the table, the boundaries of demand categories are displayed, e.g., the first column on the left is for items with demand between 100 and 96. Each row represents a different cost category. The boundaries of these cost categories are indicated in the policy table (Fig. 9). There are two numbers in each cell of the table. The upper figure refers to the number of items in that cell and the lower figure is the average of unit costs of those items in the cell. The total number of items in each demand and cost category is presented along the right-hand margin and bottom of the table. 1st MOMENT is the number of units demanded per item during the data base period, and 2nd MOMENT is the variance.

Printout of outputs from Subroutine STRU and BAYES will appear at this point if the appropriate values are assigned IFLAG(1) and IFLAG(2). (See p. 38.)

The third part (Fig. 7) contains tables of functions required for optimization. Each column represents a demand category. Along the top row of the table are mean demands of each category in a given response time. Mean demands per year are presented in the bottom row of the table.

AGGREGATION TABLE FOR ITEMS IN RESUPPLY CATEGORY 1

100.	95.	90.	85.	80.	75.	70.	65.	60.	55.	50.	45.	40.	35.	30.	25.	20.	15.	10.	5.	
1	1	8	0	2	0	2	0	1	4	0	1	3	8	7	15	11	29	53	306	444
221.00	725.00	0.00	0.00	*****	0.00	515.00	0.00	141.00	229.00	0.00	60.00	218.67	348.50	218.57	218.49	2.09	267.43	*****	*****	
0	0	1	1	2	2	3	0	2	1	0	4	7	3	16	15	24	37	95	416	629
0.00	0.00	73.00	44.00	45.50	75.00	69.33	0.00	81.90	100.00	0.00	72.00	61.86	56.67	62.88	62.67	72.93	63.36	67.71	66.67	
0	1	3	5	1	2	3	2	5	4	7	5	5	6	8	17	19	41	68	403	605
0.00	25.00	32.00	31.40	30.00	35.50	36.33	26.50	32.96	29.75	32.00	31.60	34.32	30.73	29.57	30.10	31.01	31.46	31.24	30.34	
0	1	0	2	2	0	2	2	2	3	3	3	3	5	13	8	13	28	59	463	611
0.00	20.00	0.00	22.20	20.00	0.00	18.50	16.50	16.00	16.00	16.05	17.33	18.00	15.62	16.44	17.75	17.52	18.34	17.41	17.50	
0	2	3	0	2	0	3	0	4	2	5	7	6	6	10	13	16	27	60	479	645
0.00	11.23	10.67	0.00	9.90	0.00	10.70	0.00	10.45	9.63	10.32	10.82	10.54	11.38	11.48	10.93	11.03	10.85	10.93	11.11	
1	3	0	0	2	2	1	3	1	4	4	5	6	6	13	6	13	39	47	370	522
8.03	7.57	0.00	0.00	7.52	6.25	7.50	7.48	8.15	7.92	7.61	7.80	7.29	7.83	7.57	7.75	7.61	7.68	7.52	7.83	
1	0	1	0	1	2	3	3	5	4	4	3	5	8	10	13	11	46	70	382	574
5.00	0.00	4.70	0.00	5.65	5.70	4.83	5.96	5.52	5.56	5.34	5.23	5.56	5.21	5.38	5.43	5.40	5.43	5.36	5.32	
1	1	3	3	5	1	3	2	1	4	2	3	5	8	15	10	19	31	58	349	524
3.50	4.15	5.45	4.10	3.74	4.35	4.08	4.40	3.50	4.41	4.17	3.55	3.90	3.72	3.96	4.01	3.91	3.96	3.85	3.95	
3	1	1	2	4	1	3	4	3	5	6	5	2	7	14	19	12	38	48	294	472
2.73	2.55	2.70	3.00	2.95	3.00	2.73	2.81	2.78	2.84	2.65	2.91	2.85	2.67	2.77	2.96	2.82	2.93	2.91	2.88	
2	2	1	2	3	3	4	3	0	5	9	3	8	11	9	15	8	43	55	336	522
2.15	2.25	2.20	2.17	2.19	2.32	2.12	2.13	0.00	2.02	2.01	2.10	2.09	2.07	2.14	2.08	2.10	2.08	2.05	2.07	
2	2	2	1	1	0	0	3	5	11	2	4	3	10	7	13	25	38	38	324	491
1.45	1.45	1.65	1.40	1.30	0.00	0.00	1.60	1.43	1.54	1.52	1.51	1.47	1.49	1.59	1.56	1.46	1.52	1.44	1.52	
2	0	6	1	3	4	1	3	3	1	7	3	9	13	15	21	18	52	54	338	534
1.17	0.80	1.05	1.15	1.03	1.04	1.00	1.03	1.04	1.00	1.06	1.03	1.09	1.10	1.10	1.04	1.09	1.07	1.07	1.08	
1	3	0	3	3	0	4	5	2	2	8	8	6	7	7	21	19	39	52	339	529
0.89	0.77	0.00	0.86	0.77	0.00	0.84	0.79	0.78	0.87	0.80	0.82	0.79	0.80	0.83	0.82	0.79	0.82	0.81	0.81	
2	2	3	0	5	3	5	2	3	5	14	8	3	7	11	23	18	27	49	316	506
0.53	0.54	0.59	0.00	0.56	0.58	0.59	0.57	0.58	0.56	0.55	0.54	0.58	0.55	0.61	0.56	0.60	0.56	0.57	0.57	
2	3	4	2	6	3	8	10	8	5	8	5	8	13	10	14	15	28	39	274	463
0.58	0.43	0.40	0.43	0.44	0.40	0.44	0.40	0.39	0.43	0.41	0.41	0.43	0.40	0.43	0.41	0.39	0.42	0.41	0.41	
4	1	8	2	9	7	10	4	6	4	6	7	10	16	14	16	32	34	46	359	595
0.29	0.25	0.29	0.27	0.27	0.27	0.26	0.30	0.29	0.27	0.26	0.27	0.26	0.28	0.28	0.27	0.27	0.29	0.28	0.28	
3	4	7	7	7	2	4	10	9	10	8	10	12	15	19	21	20	29	49	360	507
0.19	0.18	0.16	0.19	0.17	0.15	0.16	0.17	0.19	0.17	0.18	0.18	0.16	0.17	0.18	0.18	0.17	0.18	0.18	0.18	
8	5	3	8	11	13	8	9	15	11	11	12	17	18	18	28	27	51	57	430	761
0.10	0.09	0.10	0.10	0.09	0.11	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.11	0.10	0.10	0.10	0.10	0.11	0.10	
6	9	4	3	9	3	12	6	6	10	15	10	13	21	23	26	18	57	63	335	629
0.05	0.05	0.05	0.05	0.05	0.05	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	
5	3	3	5	9	4	6	3	11	6	9	7	10	19	6	15	16	21	16	235	407
0.01	0.02	0.01	0.02	0.02	0.02	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.03	0.02	0.02	0.02	0.02	0.02	
45	44	53	47	87	52	86	74	90	101	129	115	141	207	245	327	354	729	1056	7158	

NO. OF ITEMS = 11090 1ST MOMENT = 9.84 2ND MOMENT = 359.79

Fig. 6

RESPONSE TIME CATEGORY - 1

7.66	7.66	7.66	7.63	7.43	6.36	4.62	3.97	3.88	3.87	3.85	3.71	3.79	2.25	1.96	1.77	1.31	0.21	0.56	0.23
7.66	7.66	7.66	7.63	7.43	6.36	4.62	3.97	3.88	3.87	3.85	3.71	3.09	2.25	1.96	1.77	1.31	0.91	0.56	0.23
1.00	1.00	1.00	1.00	1.00	0.98	0.96	0.96	0.96	0.96	0.95	0.94	0.89	0.82	0.79	0.75	0.63	0.51	0.35	0.16
6.66	6.66	6.66	6.63	6.43	5.38	3.65	3.01	2.92	2.91	2.89	2.77	2.20	1.43	1.17	1.02	0.68	0.40	0.21	0.07
0.99	0.99	0.99	0.99	0.98	0.94	0.87	0.85	0.85	0.84	0.84	0.82	0.72	0.58	0.53	0.48	0.35	0.23	0.13	0.05
5.67	5.67	5.67	5.65	5.46	4.44	2.78	2.16	2.08	2.07	2.05	1.95	1.49	0.85	0.64	0.55	0.34	0.17	0.08	0.02
0.96	0.96	0.96	0.96	0.94	0.86	0.74	0.69	0.68	0.68	0.68	0.65	0.53	0.37	0.31	0.27	0.18	0.10	0.05	0.01
4.71	4.71	4.71	4.69	4.51	3.58	2.04	1.47	1.40	1.39	1.37	1.30	0.95	0.49	0.33	0.28	0.16	0.07	0.03	0.01
0.91	0.91	0.91	0.90	0.88	0.77	0.58	0.52	0.51	0.51	0.50	0.48	0.37	0.22	0.17	0.14	0.09	0.04	0.02	0.00
3.81	3.81	3.80	3.79	3.63	2.81	1.46	0.96	0.89	0.88	0.87	0.82	0.59	0.27	0.16	0.13	0.07	0.03	0.01	0.00
0.82	0.82	0.82	0.82	0.80	0.66	0.44	0.36	0.35	0.35	0.34	0.33	0.24	0.12	0.09	0.07	0.04	0.02	0.01	0.00
2.98	2.98	2.98	2.96	2.83	2.15	1.02	0.60	0.54	0.53	0.53	0.50	0.35	0.14	0.08	0.06	0.03	0.01	0.00	0.00
0.72	0.72	0.72	0.72	0.69	0.55	0.32	0.24	0.23	0.22	0.22	0.21	0.15	0.07	0.04	0.03	0.02	0.01	0.00	0.00
2.26	2.26	2.26	2.25	2.15	1.60	0.70	0.36	0.31	0.31	0.31	0.29	0.20	0.08	0.04	0.03	0.01	0.00	0.00	0.00
0.60	0.60	0.60	0.60	0.57	0.44	0.23	0.15	0.14	0.14	0.14	0.13	0.09	0.04	0.02	0.02	0.01	0.00	0.00	0.00
1.66	1.66	1.66	1.65	1.57	1.15	0.47	0.21	0.18	0.17	0.17	0.16	0.11	0.04	0.02	0.01	0.01	0.00	0.00	0.00
0.48	0.48	0.48	0.48	0.45	0.34	0.16	0.09	0.08	0.08	0.08	0.08	0.07	0.05	0.02	0.01	0.01	0.00	0.00	0.00
1.19	1.19	1.18	1.18	1.12	0.81	0.31	0.12	0.10	0.09	0.09	0.09	0.06	0.02	0.01	0.01	0.00	0.00	0.00	0.00
0.37	0.37	0.37	0.36	0.35	0.26	0.11	0.05	0.05	0.04	0.04	0.04	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00
0.82	0.82	0.82	0.81	0.77	0.6	0.20	0.07	0.05	0.05	0.05	0.05	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00
0.27	0.27	0.27	0.27	0.25	0.19	0.07	0.03	0.02	0.02	0.02	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.55	0.55	0.55	0.54	0.52	0.37	0.13	0.04	0.03	0.02	0.02	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.19	0.19	0.19	0.19	0.18	0.13	0.05	0.02	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.36	0.36	0.36	0.36	0.34	0.24	0.08	0.02	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.13	0.13	0.13	0.13	0.12	0.09	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.23	0.23	0.23	0.23	0.21	0.15	0.05	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.09	0.09	0.09	0.09	0.08	0.06	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.14	0.14	0.14	0.14	0.13	0.09	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.06	0.06	0.06	0.06	0.05	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.09	0.09	0.09	0.08	0.08	0.06	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.03	0.03	0.03	0.03	0.03	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.05	0.05	0.05	0.05	0.03	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.02	0.02	0.02	0.02	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.03	0.03	0.03	0.03	0.03	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.02	0.02	0.02	0.02	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
91.91	91.91	91.91	91.56	89.16	76.36	55.41	47.62	46.55	46.42	46.18	44.58	37.11	26.96	23.51	21.28	15.76	10.91	6.72	2.75

Fig. 7

The content of the main part of the table is to be read from top to bottom with the first row corresponding to the reorder point of 0 units, the second row to the reorder point of one unit, etc. Note that there are two lines of entries in each row. An upper entry is the number of units in a backorder condition for a given combination of resupply time, demand category and reorder point. A lower entry is 1 minus the cumulative probability distribution for the same combination of parameter values.

The last part contains optimization results. Initially, case and run identifications are given (Fig. 8), with a printout of the

```
CASE 1 ... AVERAGE INVENTORY INVESTMENT IS MINIMIZED
ROUND  SCALEC  SCALES  SCALED
.000005 1000.00   1.00   100.00
  IHOLD  ISHORT  IORDR  NITER
      3     1     2     10

SAMPLE RUN
AVE INVESTMENT=$      -0.  AVE SHORTAGE RATE=0.0100  NO OF ORDERS=12500.
```

Fig. 8

optimization parameters for verification. The policy table, (Fig. 9), is self-explanatory. Finally, the system characteristics of the policy and Lagrange multipliers (Shortage cost, Holding cost, and Ordering cost) used for optimization are given.

4. Operating Procedure

The program operates under the standard IBSYS monitor system. It may use as many as three tape units. As noted already, it reads item data from a tape. It may write some intermediate binary outputs

on a scratch tape and the final optimization results in the BCD form on another tape. Writing of these outputs is controlled by IKAKU and LTAPE. Since the program uses variable tape names, the actual tape assignment is done in the data deck.

To maximize the available case storage space, the program uses the UTV subroutine. (See IBM Systems Reference Library, Form C28-6318-3, p. 59.)

Appendix C

THE CURRENT AIR FORCE LEVELING TECHNIQUE FOR EOQ ITEMS

In this appendix, the base stock control policies for EOQ items as currently practiced by the Air Force are summarized and some critical commands are presented.

1. The current Air Force stock control procedure for EOQ items can be summarized as follows:*

a. For each item, the daily demand rate (DDR) is calculated by:

$$\text{DDR} = \frac{\text{Cumulative Recurring Demands}}{\text{Data Cumulation Period in Days}} \quad (1)$$

Some exceptions to the use of the above formula are:

1) No DDR will be calculated for an item unless it has at least two units of demand; 2) items that experienced their last activity more than a year ago will have zero DDR; 3) the data cumulation period will be 180 days for items with less than 180 days of demand experience; 4) no more than 365 days of demand experience will be kept.

The net effect of these adjustments is to produce a DDR somewhat smaller than the unadjusted value.

b. The order quantity is then calculated as follows:

$$\text{EOQ} = \frac{4.4 \sqrt{(\text{Unit Price}) \times (365 \times \text{DDR})}}{\text{Unit Price}} \quad (2)$$

Using a constant 4.4 in the above formula implies that a certain assumption has been made regarding the ratio of the cost per order to the cost of holding one unit of an item per year, the latter being expressed in terms of a percentage. Specifically, the assumed ratio is nearly 10.

* See Section A, "Releveling," AFM 67-1, Vol. II, Part Two.

c. The reorder point is calculated as the sum of the order and shipping time (O&ST) quantity and the safety level quantity:

$$\text{Reorder Point} = [(\text{O\&ST}) \times \text{DDR}] + c\sqrt{3 \times [(\text{O\&ST}) \times \text{DDR}]}, \quad (3)$$

where c is a variable factor in determining the safety level quantity. It is usually set to unity. As c is increased, the degree of support effectiveness will improve, but at the same time a higher inventory investment will be required.

d. The operating rule is to place an order equal to the quantity calculated by Eq. (2) when the sum of on-hand and on-order quantities reaches the reorder point derived by Eq. (3). This procedure is applied item by item. The virtue of the procedure is that it is simple to use. However, there are several shortcomings that offset this simplicity of usage, and make the procedure not completely adequate for stock control purposes.

2. Comments on the current policy.

a. **Conceptual Inconsistency.** First of all, there are some conceptual inconsistencies in the methodology used for deriving the EOQ and reorder point formulae described above. The EOQ formula is based on a model in which demand is assumed deterministic, i.e., the number of demands in any future period is known exactly; whereas, the reorder point formula is based on a model with random demand. In short, different assumptions are made regarding the nature of demand in determining the two decision variables. Actually these two decision variables are not independent, so their interaction should be considered in determining the quantities.

b. Demand Prediction. The current method for demand prediction is essentially a straightforward extrapolation of the past demand rate to a future time period. This method may be adequate for fast-moving items and for items with relatively small demand variability.

It is common knowledge that most recoverable items have a very low demand rate. Apparently the same situation exists among EOQ items. This can be seen in the table below.

Table 5

DISTRIBUTION OF ITEMS BY DEMAND,
OXNARD AIR FORCE BASE

Number of Units Demanded in 6-month period, July-December 1965	Number of Items
0	10,436
1-10	5,465
11-20	784
21-30	391
31-40	221
41-50	168
51-60	97
61-70	57
71-80	57
81-90	44
91-100	81
101-150	131
151-200	57
201 or more	<u>221</u>
Total	18,210

Data presented in Table 5 were obtained from Oxnard Air Force Base for the last half of 1965. The base's principal weapon system at that time was the F-101 interceptor. Note that even for EOQ items, 57 percent had no activity in a 6-month period. Furthermore, many of these zero-demand items are likely to experience demands in

the future. This means that the current procedure for calculating DDR is not an adequate means for anticipating future demands.

c. The current procedure might be called the item approach^{*} to stockage policy. In this approach, stockage decisions are made on each item without considering the collective impact these decisions have on the supply system as a whole. Hence, this approach is not adapted for solving such system problems as determining performance standards of the supply system, complying with budget restrictions, and adjusting stockage policies to changing demand conditions, resource availability or performance norms. In other words, it is difficult for supply management to respond to a change in the amount of funds available for stockage or to present a rational basis for requesting additional funds to meet a higher performance standard. Furthermore, as can be seen in Eq. (2), the item approach presumes that we know the ratio of ordering cost to investment cost. How much does it cost to place an order? How much money is tied up in stock? These cost figures are very difficult to estimate. Often the best that inventory managers can do is make an educated guess.

^{*}This term was first used in [8].

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10. ABSTRACT A description of a mathematical model for determining optimal operating rules for control of an inventory system comprising a large number of Economic Order Quantity (EOQ) items. The model accepts two types of information as inputs: item data, consisting of unit price, past demand, and resupply time; and system data, consisting of future base activity level, variance to mean ratio of demand distribution, and constraints on the average backorder rate and order frequency per year. Based on these inputs, the model computes a reorder point and order quantity for each item in the system. Constraints on the backorder rate and order frequency can be varied within the relevant range of the base operations and the corresponding minimum-cost policies derived. Alternative efficient policies can thus be presented to the base supply manager instead of a single policy and he can choose the one most appropriate for his operating conditions. () The model has been programmed in FORTRAN IV for the IBM 7044; it is readily adaptable to other computers of comparable size.		11. KEY WORDS Bases Logistics Inventory control Computer programs Models