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# Noise Filtering Improvements

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#### ABSTRACT

Electrical noise filtering with augmented low pass resistance-capacitance networks provides reduced signal lags and steady-state biases as compared with those characteristic of the simple RC noise filter. These improvements apply, e.g., to airborne weapons control systems in which the displayed aim dot position errors must be minimal. Rate gyro output and system noise filtering, when required, is advantageously accomplished with the augmented RC low pass network.

#### PROBLEM STATUS

This is a final report on one phase of the problem; work is continuing on the basic problem.

#### AUTHORIZATION

NRL Problems D01-03 and R05-03  
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## NOISE FILTERING IMPROVEMENTS

### INTRODUCTION

Signal lags and steady-state biases characteristic of the usual noise-filter outputs are either markedly reduced or eliminated entirely by the use of augmented instead of simple resistance-capacitance low pass networks.

In modern airborne weapons-control-system radars, the display aim dot position is proportional to the product of target range and angular acceleration and is influenced by system noise. The signal proportional to angular acceleration is approximated at long ranges by a velocity step function. At closer ranges an acceleration step function is a better approximation of the time function.

When the usual RC low pass noise filter is used, the tracking error with either input function becomes greater, rather than settling to zero as steady state is approached.

This report presents the results of an investigation of transfer functions that provide essentially the same noise filtering but do not show such undesirable time-function tracking performance. Additionally, physical realizations in the form of RC networks that simultaneously provide noise filtering and reduced tracking error are presented. The advantages of these augmented RC low pass networks are shown by a comparison with the simple network performance.

### RESULTS

The conclusions of the study, supported by the detailed mathematical analysis presented in subsequent sections, may be portrayed graphically. Figure 1 shows the network configurations and the frequency-response plots for a simple and for an augmented RC low pass network. For identical bandpass corner frequencies, the augmented network provides slightly less filtering of white noise.

Figure 2 shows a normalized velocity step-function input signal and the corresponding network outputs for both the simple and the augmented RC low pass networks, while Fig. 3 shows a normalized acceleration step-function input signal and the corresponding outputs for both the simple and the augmented RC low pass networks.

These improved networks, although having twice as many network elements, are estimated to require perhaps only 50% more volume.

### SIMPLE LOW PASS TRANSFER FUNCTION

A single-section RC low pass network with the voltage transfer function, Fig. 4, is defined by

$$\frac{E_{out}}{E_{in}} = \frac{\tau_0}{s + \tau_0} \cdot \frac{1}{s\tau_0 + 1} \quad (1)$$

where

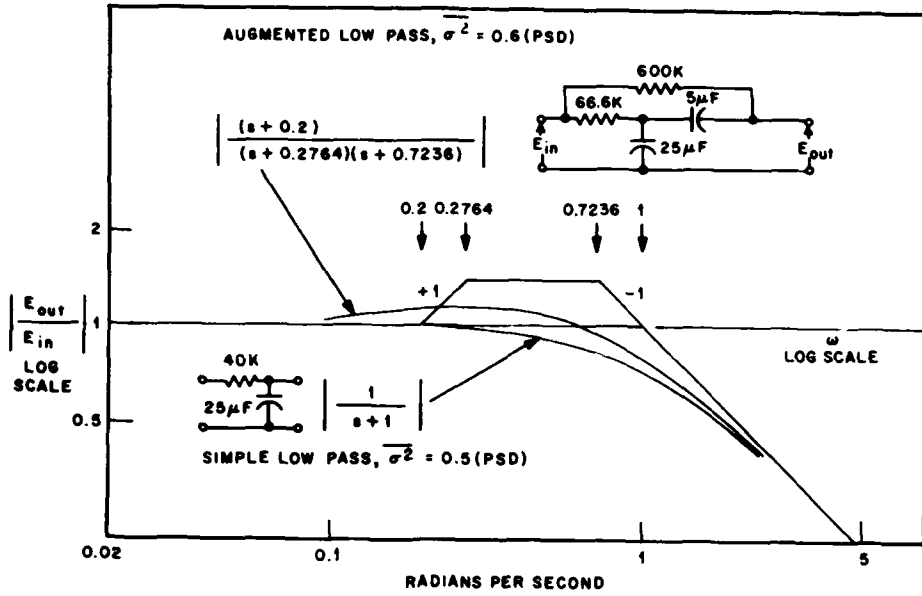


Fig. 1 - Simple and augmented low pass RC networks and frequency plots

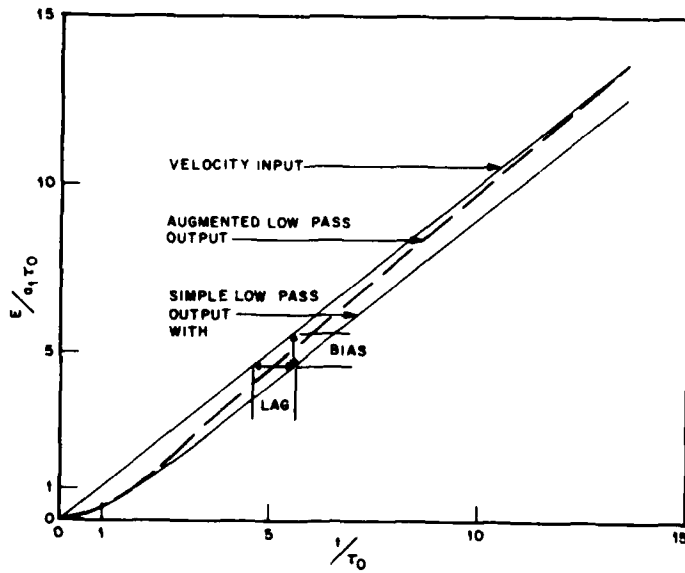


Fig. 2 - Velocity input tracking for simple and augmented low pass networks

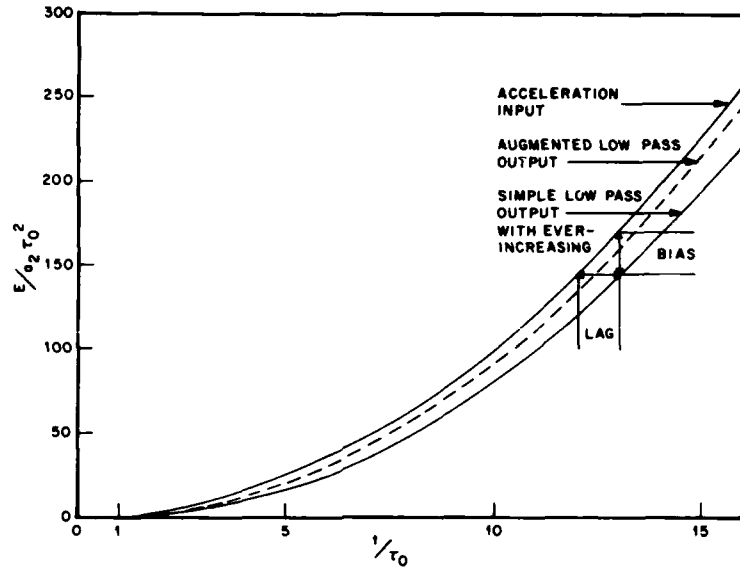


Fig. 3 - Acceleration input tracking for simple and augmented low pass networks

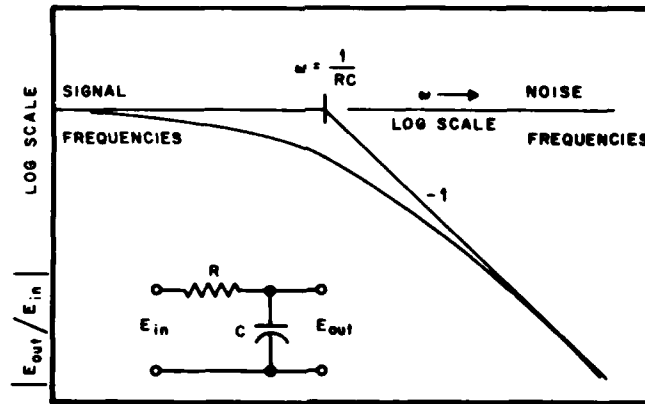


Fig. 4 - Single-section RC low pass steady-state frequency response

$$\tau_0 = \frac{1}{\omega_0} = RC. \quad (2)$$

For a white-noise input with a power spectral density (PSD), the mean-square noise output  $\overline{\sigma^2}$  is

$$\overline{\sigma^2} = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \frac{(\text{PSD}) \omega_0^2 ds}{(s + \omega_0)(-s + \omega_0)}. \quad (3)$$

Since  $\overline{\sigma^2} = \sum \text{residues} = \text{residues at } s = -\omega_0,$

$$\overline{\sigma^2} = \frac{(\text{PSD})(s + \omega_0) \omega_0^2}{(s + \omega_0)(-s + \omega_0)} \Big|_{s = -\omega_0} = \frac{(\text{PSD})}{2} \omega_0. \quad (4)$$

For  $\omega_0 = 1,$

$$\overline{\sigma^2} = (\text{PSD})(0.5). \quad (5)$$

Thus the low pass filter reduces the mean-square noise output by the factor 0.5. Since the purpose of the network in the system is to provide noise filtering, the above may be taken, in the absence of specific information regarding the noise-input spectrum, as a measure of the effectiveness of the network.

The information component in the signal (e.g., a rate gyro output) from which it is desired to selectively attenuate noise may be approximated by the time function

$$E_{in} = a_1 t + a_2 t^2. \quad (6)$$

Since we are studying a linear system, we may consider the two parts of Eq. (6) separately. For a velocity step input  $E_{in} = a_1 t$ , the simple low pass filter specified by Eq. (1) exhibits a tracking error  $\epsilon = E_{in} - E_{out}$  given by

$$\frac{\epsilon}{a_1 \tau_0} \Big|_{E_{in} = a_1 t} = 1 - e^{-t/\tau_0}. \quad (7)$$

For the acceleration step input  $E_{in} = a_2 t^2$ , we find

$$\frac{\epsilon}{a_2 \tau_0^2} \Big|_{E_{in} = a_2 t^2} = 2 \left[ \frac{t}{\tau_0} - \left( 1 - e^{-t/\tau_0} \right) \right]. \quad (8)$$

The normalized error functions given by Eqs. (7) and (8) are shown in Fig. 5.

#### AUGMENTED LOW PASS TRANSFER FUNCTION

Consider a voltage transfer function containing a simple zero and two simple poles on the negative real axis as follows:

$$\frac{E_{out}}{E_{in}} = \frac{\omega_b \omega_c}{\omega_a} \frac{(s + \omega_a)}{(s + \omega_b)(s + \omega_c)} \quad (9)$$

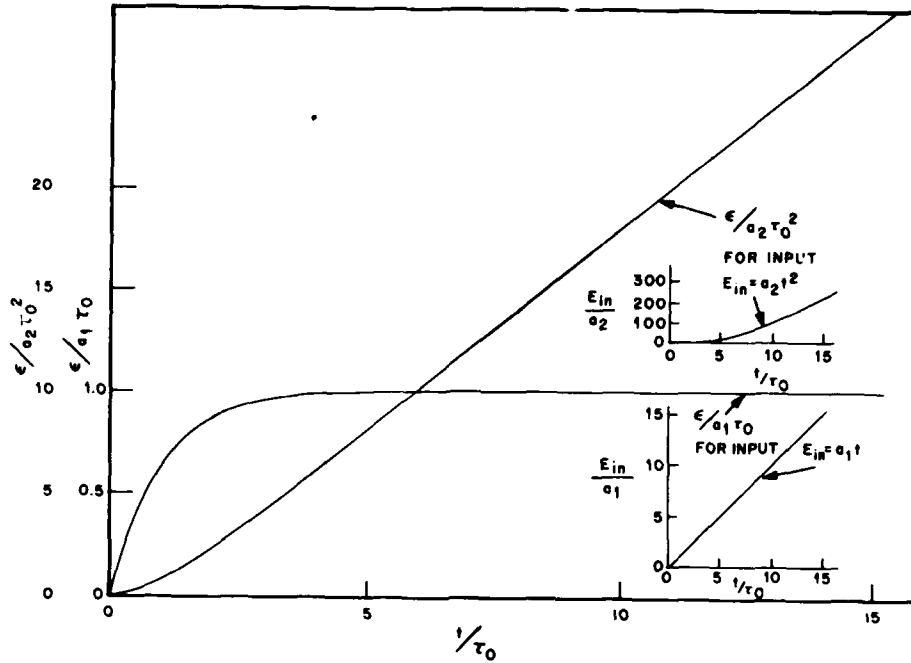


Fig. 5 - Input minus output tracking error  $\epsilon$  for inputs  $a_1 t$  and  $a_2 t^2$  to a simple low pass transfer function  $1/(s\tau_0 + 1)$

$$\frac{E_{out}}{E_{in}} = \frac{s\tau_a + 1}{(s\tau_b + 1)(s\tau_c + 1)} \tag{10}$$

where

$$\omega_a < \omega_b < \omega_c$$

and

$$\tau_a = \frac{1}{\omega_a}, \quad \tau_b = \frac{1}{\omega_b}, \quad \tau_c = \frac{1}{\omega_c}$$

To permit direct comparison with Eq. (1), the constraint

$$\frac{\tau_a}{\tau_b\tau_c} = \frac{1}{\tau_0} \tag{11}$$

is introduced. For an input  $E_{in} = a_1 t$ , we find, for the function of Eq. (10),

$$\frac{E_{out}}{a_1} = t - (\tau_b + \tau_c - \tau_a) - \frac{\tau_a - \tau_b}{\tau_b - \tau_c} \tau_b e^{-t/\tau_b} + \frac{\tau_a - \tau_c}{\tau_b - \tau_c} \tau_c e^{-t/\tau_c} \tag{12}$$

To eliminate the steady-state tracking error, we introduce the constraint

$$\tau_a = \tau_b + \tau_c \tag{13}$$



Then the normalized error function reduces to the two exponential terms as follows:

$$\frac{\epsilon}{a_1 \tau_0} \Big|_{E_{in} = a_1 t} = \frac{\tau_a - \tau_b}{\tau_b - \tau_c} \frac{\tau_a}{\tau_c} e^{-t/\tau_b} - \frac{\tau_a - \tau_c}{\tau_b - \tau_c} \frac{\tau_a}{\tau_b} e^{-t/\tau_c}. \quad (14)$$

With an input  $E_{in} = a_2 t^2$ , we find, for the function of Eq. (10),

$$\begin{aligned} \frac{E_{out}}{a_2} &= t^2 - 2(\tau_b + \tau_c - \tau_a)t - 2[\tau_b \tau_c - (\tau_b + \tau_c)(\tau_b + \tau_c - \tau_a)] \\ &\quad + 2 \frac{\tau_a - \tau_b}{\tau_b - \tau_c} \tau_b^2 e^{-t/\tau_b} - 2 \frac{\tau_a - \tau_c}{\tau_b - \tau_c} \tau_c^2 e^{-t/\tau_c}. \end{aligned} \quad (15)$$

When the constraints of Eqs. (11) and (13) are imposed, the system error becomes

$$\frac{\epsilon}{a_2 \tau_0^2} \Big|_{E_{in} = a_2 t^2} = 2 \left( \frac{\tau_a}{\tau_0} - \frac{\tau_a - \tau_b}{\tau_b - \tau_c} \frac{\tau_b^2}{\tau_0^2} e^{-t/\tau_b} + \frac{\tau_a - \tau_c}{\tau_b - \tau_c} \frac{\tau_c^2}{\tau_0^2} e^{-t/\tau_c} \right). \quad (16)$$

The mean-square noise output for a white-noise input is found for the augmented low pass function as follows:

$$\overline{\epsilon^2} = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \frac{(\text{PSD})(s + \omega_a)(-s + \omega_a) ds}{(s + \omega_b)(-s + \omega_b)(s + \omega_c)(-s + \omega_c)} \quad (17)$$

equals  $\Sigma$  residues at  $s = -\omega_b$  and  $s = -\omega_c$ ,

$$\begin{aligned} R_{-\omega_b} &= (\text{PSD}) \left[ \frac{(s + \omega_a)(-s + \omega_a)}{(s + \omega_b)(-s + \omega_b)(-s + \omega_c)} \right]_{s = -\omega_b} \\ &= (\text{PSD}) \frac{(\omega_a - \omega_b)(\omega_a + \omega_b)}{(\omega_b + \omega_b)(\omega_c - \omega_b)(\omega_c + \omega_b)}, \end{aligned} \quad (18)$$

and

$$\begin{aligned} R_{-\omega_c} &= (\text{PSD}) \left[ \frac{(s + \omega_a)(-s + \omega_a)}{(s + \omega_b)(-s + \omega_b)(-s + \omega_c)} \right]_{s = -\omega_c} \\ &= (\text{PSD}) \frac{(\omega_a - \omega_c)(\omega_a + \omega_c)}{(\omega_b - \omega_c)(\omega_b + \omega_c)(\omega_c + \omega_c)}. \end{aligned} \quad (19)$$

#### SPECIFIC EXAMPLE OF AUGMENTED FUNCTION

A specific example, consistent with  $\tau_0 = 1$  and the constraints of Eqs. (11) and (13), is represented by the set of values

$$\omega_a = 0.2, \quad \tau_a = 5, \tag{20a}$$

$$\omega_b = 0.2764, \quad \tau_b = 3.618, \tag{20b}$$

and

$$\omega_c = 0.7236, \quad \tau_c = 1.382, \tag{20c}$$

for which Eq. (14) becomes

$$\frac{f}{a_1 \tau_0} \Big|_{E_{in} = a_1 t} = 2.236 (e^{-t/3.618} - e^{-t/1.382}) \tag{21}$$

and Eq. (16) gives

$$\frac{f}{a_2 \tau_0^2} \Big|_{E_{in} = a_2 t^2} = 10 \cdot 6.18 e^{-0.724t} - 16.18 e^{-0.276t} \tag{22}$$

The error functions given by Eqs. (21) and (22) are plotted in Fig. 6.

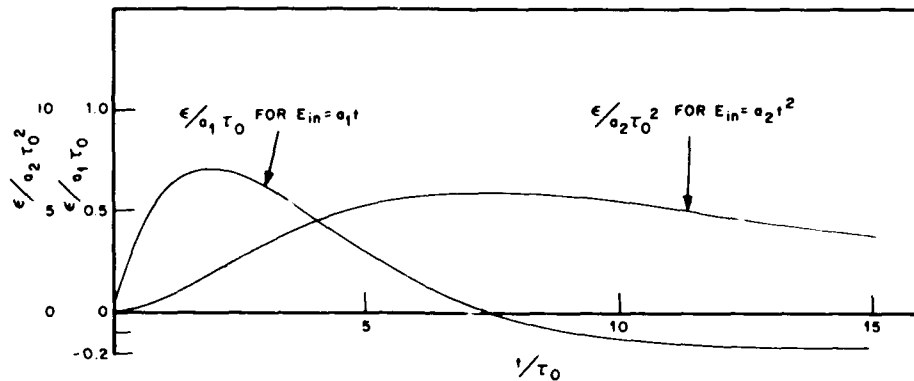


Fig. 6 - Tracking error for an augmented low pass transfer function

Returning to the calculation based upon a white-noise input, and using the values specified by Eqs. (20) in Eqs. (18) and (19),

$$R_{-\omega_b} = (\text{PSD})(-0.1472) \tag{23}$$

Similarly,

$$R_{-\omega_c} = (\text{PSD})(+0.7472) \tag{24}$$

Thus, for the augmented low pass function of Eq. (9) with an effective 0, -1 corner frequency of  $\omega_0 = 1$  and the particular time constants of Eqs. (20) we find that

$$\begin{aligned} \sqrt{2} & \text{ (PSD)} (-0.1472 + 0.7472j) \\ & \text{ (PSD)} (0.6) \end{aligned} \quad (25)$$

This value corresponds to a single-section low pass function with a corner frequency of 1.2.

Figure 7 provides a steady-state frequency response comparison of the simple low pass function and the augmented low pass function of Eq. (9) with values specified by Eqs. (20).

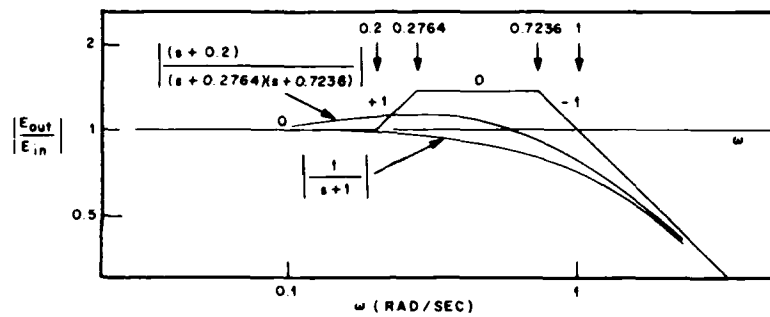


Fig. 7 - Simple low pass transfer function and the augmented low pass transfer function

#### PASSIVE REALIZATION OF AUGMENTED FUNCTION

Consider the network shown in Fig. 8 and its transfer function

$$\frac{E_{out}}{E_{in}} = \frac{s(ab + ac + de) + 1}{s^2 abde + s(ab + ac + de) + 1} \quad (26)$$

This is an augmented low pass function that exhibits *zero loss at dc*, i.e., no insertion loss into an open circuit, and zero steady-state error for a ramp input for any allowable combination of the network element parameters  $a$ ,  $b$ ,  $d$ , and  $e$ .

As a specific example, consider the corner frequencies of Fig. 7, i.e., those given by Eqs. (20). A possible set of network parameters is

$$aR = 1.3 \quad (27a)$$

$$bC = 5 \quad (27b)$$

$$dR = 3 \quad (27c)$$

and

$$eC = 1 \quad (27d)$$

These calculated values are shown, after scaling to an arbitrarily chosen impedance level, in the network of Fig. 9.

Fig. 8 - Augmented low pass RC network

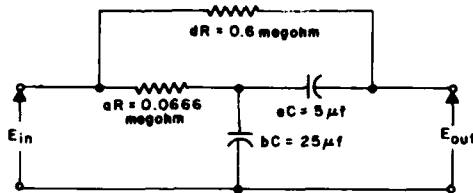
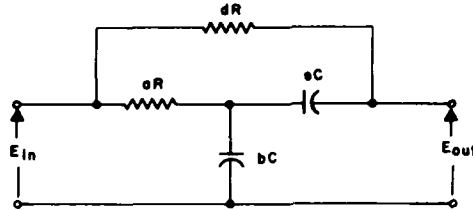


Fig. 9 - Network with the augmented low pass voltage transfer function shown in Fig. 7

**DOUBLY AUGMENTED LOW PASS TRANSFER FUNCTION**

Consider a voltage transfer function comprised of two simple zeros and three simple poles on the negative real frequency axis as follows:

$$\frac{E_{out}}{E_{in}} = \frac{\omega_3 \omega_4 \omega_5}{\omega_1 \omega_2} \frac{(s + \omega_1)(s + \omega_2)}{(s + \omega_3)(s + \omega_4)(s + \omega_5)} \tag{28}$$

$$= \frac{(s\tau_1 + 1)(s\tau_2 + 1)}{(s\tau_3 + 1)(s\tau_4 + 1)(s\tau_5 + 1)}, \tag{29}$$

where

$$\tau_1 = \frac{1}{\omega_1}, \quad \tau_2 = \frac{1}{\omega_2}, \quad \tau_3 = \frac{1}{\omega_3}, \quad \text{etc.} \tag{30}$$

To locate the function given by Eq. (28) at the same frequency as that of Eq. (1), set the constraint

$$\frac{\tau_1 \tau_2}{\tau_3 \tau_4 \tau_5} = \frac{1}{\tau_0}. \tag{31}$$

For an input  $E_{in} = a_1 t$ , we find, for the function of Eq. (29),

$$\begin{aligned} \frac{E_{out}}{a_1} = & t - (\tau_3 + \tau_4 + \tau_5 - \tau_1 - \tau_2) + \frac{(\tau_3 - \tau_1)(\tau_3 - \tau_2)}{(\tau_3 - \tau_4)(\tau_3 - \tau_5)} \tau_3 e^{-t/\tau_3} \\ & + \frac{(\tau_4 - \tau_1)(\tau_4 - \tau_2)}{(\tau_4 - \tau_3)(\tau_4 - \tau_5)} \tau_4 e^{-t/\tau_4} + \frac{(\tau_5 - \tau_1)(\tau_5 - \tau_2)}{(\tau_5 - \tau_3)(\tau_5 - \tau_4)} \tau_5 e^{-t/\tau_5}. \end{aligned} \tag{32}$$

To eliminate the steady-state tracking error, we introduce the constraint

$$\tau_1 + \tau_2 = \tau_3 + \tau_4 + \tau_5. \quad (33)$$

The tracking error then becomes

$$\begin{aligned} \frac{\epsilon}{a_1 \tau_0} \Big|_{E_{in} = a_1 t} &= - \left[ \frac{(\tau_3 - \tau_1)(\tau_3 - \tau_2) \tau_3}{(\tau_3 - \tau_4)(\tau_3 - \tau_5) \tau_0} e^{-t/\tau_3} \right. \\ &\quad \left. + \frac{(\tau_4 - \tau_1)(\tau_4 - \tau_2) \tau_4}{(\tau_4 - \tau_3)(\tau_4 - \tau_5) \tau_0} e^{-t/\tau_4} + \frac{(\tau_5 - \tau_1)(\tau_5 - \tau_2) \tau_5}{(\tau_5 - \tau_3)(\tau_5 - \tau_4) \tau_0} e^{-t/\tau_5} \right]. \quad (34) \end{aligned}$$

For an input  $E_{in} = a_2 t^2$ ,

$$\begin{aligned} \frac{E_{out}}{a_2} &= t^2 - 2(\tau_3 + \tau_4 + \tau_5 - \tau_1 - \tau_2)t \\ &\quad + 2[\tau_1 \tau_2 - (\tau_3 \tau_4 + \tau_4 \tau_5 + \tau_5 \tau_3) + (\tau_3 + \tau_4 + \tau_5)(\tau_3 + \tau_4 + \tau_5 - \tau_1 - \tau_2)] \\ &\quad - 2 \left[ \frac{(\tau_3 - \tau_1)(\tau_3 - \tau_2) \tau_4^2}{(\tau_4 - \tau_3)(\tau_4 - \tau_5)} e^{-t/\tau_4} + \frac{(\tau_5 - \tau_1)(\tau_5 - \tau_2) \tau_5^2}{(\tau_5 - \tau_3)(\tau_5 - \tau_4)} e^{-t/\tau_5} \right]. \quad (35) \end{aligned}$$

The term proportional to  $t$  is eliminated by the constraint specified by Eq. (33), and the constant term is reduced and, after introduction of Eq. (31), becomes equal to

$$-2(\tau_3 \tau_4 + \tau_4 \tau_5 + \tau_5 \tau_3 - \tau_1 \tau_2) = -2 \left\{ \tau_3 \tau_4 - \tau_5 \left[ \frac{\tau_3 \tau_4}{\tau_0} - (\tau_3 - \tau_4) \right] \right\}. \quad (36)$$

The constant term becomes zero for

$$\tau_5 = \frac{\tau_3 \tau_4 \tau_0}{\tau_3 \tau_4 - (\tau_3 + \tau_4) \tau_0}. \quad (37)$$

That is, for

$$\frac{1}{\tau_3} + \frac{1}{\tau_4} + \frac{1}{\tau_5} = \frac{1}{\tau_0} \quad (38)$$

or, in terms of frequencies, for

$$\omega_3 + \omega_4 + \omega_5 = \omega_0. \quad (39)$$

With both the constraints of Eq. (33) and Eq. (38) imposed,

$$\begin{aligned} \frac{\epsilon}{a_2 \tau_0^2} \Big|_{E_{in} = a_2 t^2} &= 2 \left[ \frac{(\tau_3 - \tau_1)(\tau_3 - \tau_2) \tau_3^2}{(\tau_3 - \tau_4)(\tau_3 - \tau_5) \tau_0^2} e^{-t/\tau_3} \right. \\ &\quad \left. + \frac{(\tau_4 - \tau_1)(\tau_4 - \tau_2) \tau_4^2}{(\tau_4 - \tau_3)(\tau_4 - \tau_5) \tau_0^2} e^{-t/\tau_4} + \frac{(\tau_5 - \tau_1)(\tau_5 - \tau_2) \tau_5^2}{(\tau_5 - \tau_3)(\tau_5 - \tau_4) \tau_0^2} e^{-t/\tau_5} \right]. \quad (40) \end{aligned}$$

If we require that Eqs. (31), (33), (38), and  $\tau_0 = 1$  hold simultaneously, we find the design formulas

$$(\tau_1, \tau_2) = \frac{(\tau_3 + \tau_4) + \frac{\tau_3 \tau_4}{\tau_3 \tau_4 - (\tau_3 + \tau_4)}}{2} \pm \sqrt{\left[ \frac{\tau_3 \tau_4 + \frac{\tau_3 \tau_4}{\tau_3 \tau_4 - (\tau_3 + \tau_4)}}{2} \right]^2 - \frac{\tau_3^2 \tau_4^2}{\tau_3 \tau_4 - (\tau_3 + \tau_4)}} \quad (41)$$

Figure 10 shows the asymptotic segment plot for the two extremes of (a) two poles coincident and (b) two zeros coincident. The illustration is for  $\omega_0 = 1$  and the  $\omega_3$ -pole at  $\omega = 0.1$ .

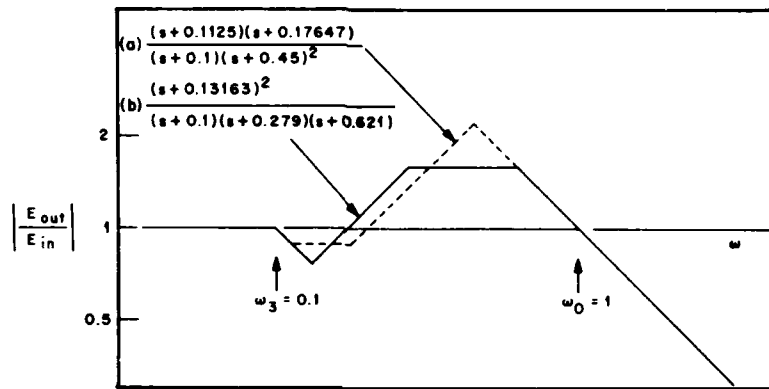


Fig. 10 - Extremes for the zero-steady-state error design of the doubly augmented transfer function

A specific example of a doubly augmented low pass transfer function exhibiting zero steady-state tracking error for an input  $E_{in} = a_1 t + a_2 t^2$  is provided by the following set of constants:

$$\tau_3 = 10, \quad \omega_3 = 0.1, \quad (42a)$$

$$\tau_4 = 3, \quad \omega_4 = 0.33335, \quad (42b)$$

$$\tau_5 = 1.76471, \quad \omega_5 = 0.56667, \quad (42c)$$

$$\tau_1 = 8.63054, \quad \omega_1 = 0.115868, \quad (42d)$$

and

$$\tau_2 = 6.13417, \quad \omega_2 = 0.163021, \quad (42e)$$

Equations (42) provide a 0, -1 effective corner frequency  $\omega_0 = 1$ .

For the set of values given by Eqs. (42), the tracking error functions given by Eqs. (34) and (40) become

$$\left. \frac{r}{a_1 \tau_0} \right|_{k_{in}, a_1 t} = -0.9184 e^{-0.1t} + 6.1225 e^{-0.3333t} - 5.2041 e^{-0.5667t} \quad (43)$$

and

$$\left. \frac{\epsilon}{a_2 \tau_0^2} \right|_{E_{in} = a_2 t^2} = +18.3674 e^{-0.1t} - 36.7438 e^{-0.3333t} + 18.3674 e^{-0.5667t} \quad (44)$$

Equations (43) and (44) are plotted in Fig. 11.

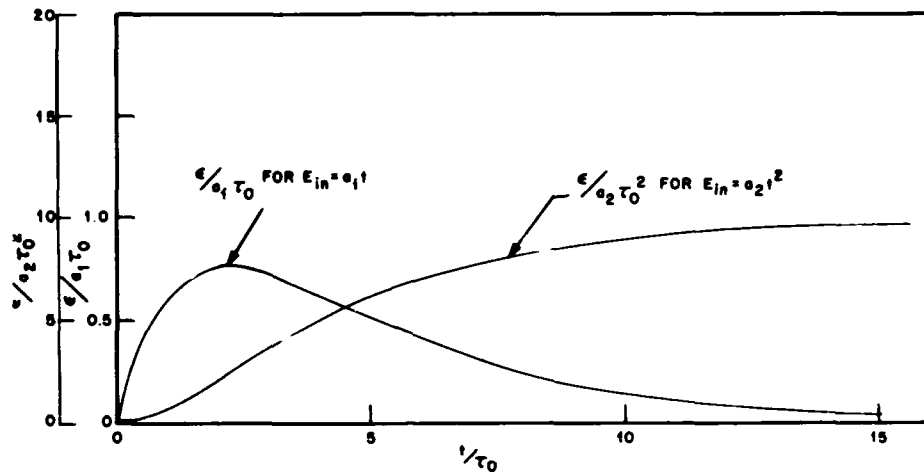


Fig. 11 - Tracking errors for a doubly augmented function

For a white-noise input the mean-square noise output for the doubly augmented low pass transfer function defined by Eqs. (28) and (42) was calculated to be

$$\begin{aligned} \overline{\sigma^2} &= (\text{PSD}) (0.009 - 0.583 + 1.224) \\ &= (\text{PSD}) (0.65) \end{aligned} \quad (45)$$

This value corresponds to a single-section low pass function with a corner frequency of  $\omega = 1.3$ .

Figure 12 shows the asymptotic segments for the steady-state frequency response of the specific example defined by Eqs. (42).

#### PASSIVE REALIZATION OF DOUBLY AUGMENTED FUNCTION

The transfer function of Eq. (29),

$$\frac{E_{out}}{E_{in}} = \frac{(s\tau_1 + 1)(s\tau_2 + 1)}{(s\tau_3 + 1)(s\tau_4 + 1)(s\tau_5 + 1)}$$

has been shown to exhibit zero steady-state tracking error with  $E_{in} = a_1 t + a_2 t^2$ , provided certain constraints are imposed. These may be specified by Eqs. (25), (27), and (32) and  $\tau_0 = 1$ .

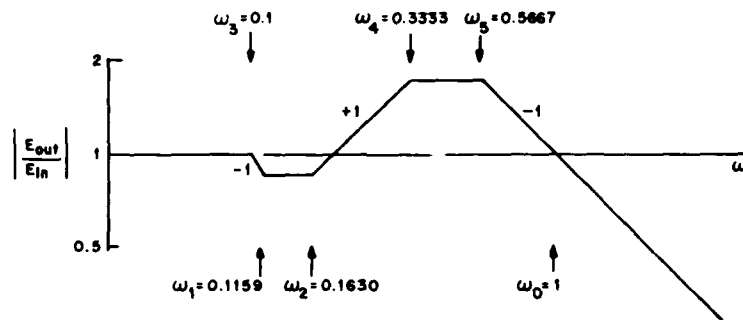


Fig. 12 - Asymptotic segment representation of the doubly augmented function

The network of Fig. 13 has the transfer function

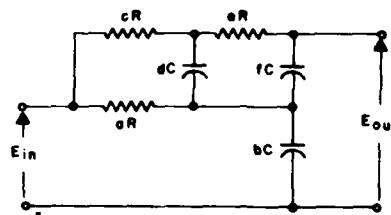
$$\frac{E_{out}}{E_{in}} = \frac{N(s)}{D(s)} \tag{46}$$

where

$$N(s) = (a + c)def s^2 C^2 R^2 + [ab + (a + c)(d + f) + ef]sCR + 1$$

$$D(s) = abcdef s^3 C^3 R^3 + [abcd + abf(c + e) + (a + c)def]s^2 C^2 R^2 + [ab + (a + c)(d + f) + ef]sCR + 1$$

Fig. 13 - Doubly augmented RC low pass network



Equation (46) shows that the passive RC network shown has the transfer function form and the multiplicative constant of unity desired. Also, for all values of the elements of the network, the desired  $\tau_1 + \tau_2 - \tau_3 + \tau_4 + \tau_5$  and consequent zero steady-state tracking error for an input  $E_{in} = a_1 t$  is obtained.

In the transfer-function study it was shown that zero steady-state error occurs for  $E_{in} = a_1 t + a_2 t^2$  if

$$\tau_1 + \tau_2 = \tau_3 + \tau_4 + \tau_5 \tag{47a}$$

$$\frac{1}{\tau_0} = \frac{1}{\tau_3} + \frac{1}{\tau_4} + \frac{1}{\tau_5} \tag{47b}$$



and

$$\tau_0 = 1. \quad (47c)$$

These imply a transfer function of the form

$$\frac{E_{out}}{E_{in}} = \frac{As^2 + Bs + 1}{As^3 + As^2 + Bs + 1}. \quad (48)$$

Analysis of Eq. (46) shows that these requirements cannot be met exactly by this network; i.e., a finite value for the steady-state tracking error with the input component  $E_{in} = a_2 t^2$  must be allowed. Additionally, it appears that any physical realization will involve many elements in the network and will be correspondingly less practical from an engineering viewpoint.

#### CONCLUSION

The best engineering answer to improvement in time-function tracking in the presence of a noise filtering requirement appears to be offered by the singly augmented low pass network.

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	ROLE	WT	ROLE	WT	ROLE	WT
Radar Airborne radar Weapons control systems Tracking errors Displays Rate gyro Noise filters Electric networks Network analysis RC networks Signal tracking Velocity step function Acceleration step function Steady state Signal lag Signal bias						