USAAVLABS TECHNICAL REPORT 68-11

FEASIBILITY OF A
TOROIDAL STREAM ANGULAR RATE SENSOR

By

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April 1968

U. S. ARMY AVIATION MATERIEL LABORATORIES
FORT EUSTIS, VIRGINIA

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FEASIBILITY OF A
TOROIDAL STREAM ANGULAR RATE SENSOR

Final Report

by

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Prepared by

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ABSTRACT

It is theoretically feasible to build a toroidal stream angular rate
sensor (TSARS) based on the principle that a series of vortex rings is
deflected by the Coriolis force so that the rotational velocities
measured by a pair of symmetrically placed sensors are slightly
different. The sensors may be the input ports of a jet type of fluid
amplifier. It should be possible to detect angular rates as low as
0.05 deg/sec, with disturbance frequencies up to 10 cps. An uncertain
item is the precise reproducibility of the vortices that are generated.
An alternative design concept, having several advantages over the TSARS
as well as the jet type of angular rate sensor, is the vortex axis jet
angular rate sensor (VAJARS), in which a straight vortex is provided
along the axis of a large-diameter jet. Experimental studies are
recommended, with VAJARS considered to be more promising.

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LIST OF SYMBOLS

a	Coriolis acceleration, ft/sec^2
A	Flow cross-section carrying ions to electrode, ft^2
c	Concentration of ions, $number/ft^3$
f	Frequency of vortex generation, number/sec
g	Acceleration of gravity, ft/sec^2
L	Propagation distance, ft
n	Number of ions arriving at electrode in time Δt, dimensionless.
r	Distance from circular axis of vortex ring, ft
r_o	Radius of vortex core, ft
R	Radius of circular axis, ft
S	Spacing between successive vortex rings, ft
t	Time, sec
t_o	Time constant for vortex decay, sec
T	Propagation time, sec
v	Velocity due to vortex rotation, ft/sec
v_o	v for $r = r_0$, ft/sec
v_c	Fluid velocity along straight axis of vortex ring, ft/sec
x	Distance along axis of propagation, ft
y	Axis of rotation, Ω
z	Direction of Coriolis deflection
Z	Coriolis deflection, ft
ΔP	Pressure difference, lbs force/ft^2
Δt	Duration of signal pulse, sec

LIST OF SYMBOLS (cont'd)

δt Difference in release time for opposite parts of vortex ring, sec

Δv Velocity difference at detectors, ft/sec

Γ Circulation, ft^2/sec

Γ_o Initial value of Γ, ft^2/sec

ν Kinematic viscosity, ft^2/sec

Ω Angular rate, rad/sec

ρ Density of fluid, lb/ft^3

A. INTRODUCTION

This is the final report under contract DAAJ02-67-C-0043 which called for a six-month study (starting May 11, 1967) of the feasibility of using vortex rings in an angular rate sensor. The need for an angular rate sensor arises in connection with fluidic stability augmentation systems for helicopters and V/STOL aircraft.

As a guideline for the present study, it was assumed that variations in the angular rate at frequencies up to 10 cps have to be detected, and that the sensitivity sought for the rate sensor is 0.05 deg/sec (Reference 1).

B. DESCRIPTION OF TSARS CONCEPT

To describe the TSARS concept, it is convenient to consider the jet type of fluidic angular rate sensor, which works on the principle that a stream is deflected relative to the instrument when the instrument is subjected to an angular velocity (see Figure 1). The deflection is caused by the Coriolis acceleration experienced by each volume element of the stream due to its linear velocity in a rotating coordinate system. Its sensitivity is limited because, when the stream moves fast enough to generate a useable dynamic pressure at the pick-off, the deviation per unit of angular velocity becomes very small. Moreover, it is difficult to make a simple jet that remains a jet for more than about 10 diameters without excessive spreading and flattening of the velocity profile. The sensitivity of such a device can be increased by increasing the length of the jet, but the extent to which this can be done is limited by the maximum length of the jet beyond which substantial entrainment of stationary fluid occurs and the sharply defined boundary is destroyed (see Figure 2). A possible way to over - come this limitation is to use a series of toroidal vortex rings instead of a jet. The distance the vortex rings can be made to travel is much greater than the practically achievable length of a jet, because rotation stabilizes the boundary of the vortex ring.

A graphic example of the effect of toroidal rotation in stabilizing a boundary may be seen in the case of a smoke ring, which is a smoke-filled vortex ring in air (see Figure 3). The fluid in the smoke ring or vortex ring may be thought of as having two "axes". The first axis is a straight line, the axis of symmetry along which the smoke ring travels. The second axis is a circle. If the ring were cut and straightened out into a cylinder, this axis would become the axis of the cylinder. The fluid in the ring rotates about this second axis, and because of the stabilizing effect of this rotation, the smoke-filled air in the smoke ring does not mix with the surrounding air, even though the smoke ring is made to travel a large distance along a straight path.

The sensitivity of the toroidal stream angular rate sensor should be increased substantially over that of the plain jet rate sensor for another reason besides the long path made possible by the toroidal stream. In the jet type of fluidic angular rate sensor, the jet deflection results in a pressure difference between two receivers simply because of the dynamic pressure in the jet due to its velocity along its axis. Decreasing the jet velocity will increase the deflection caused by a given angular rate, but it will decrease the dynamic pressure difference caused by a given deflection, thus defeating the purpose of reducing the jet velocity. In the vortex ring, on the other hand, the pressure difference that may be detected will depend upon the speed of rotation of the fluid in the toroid, which can be
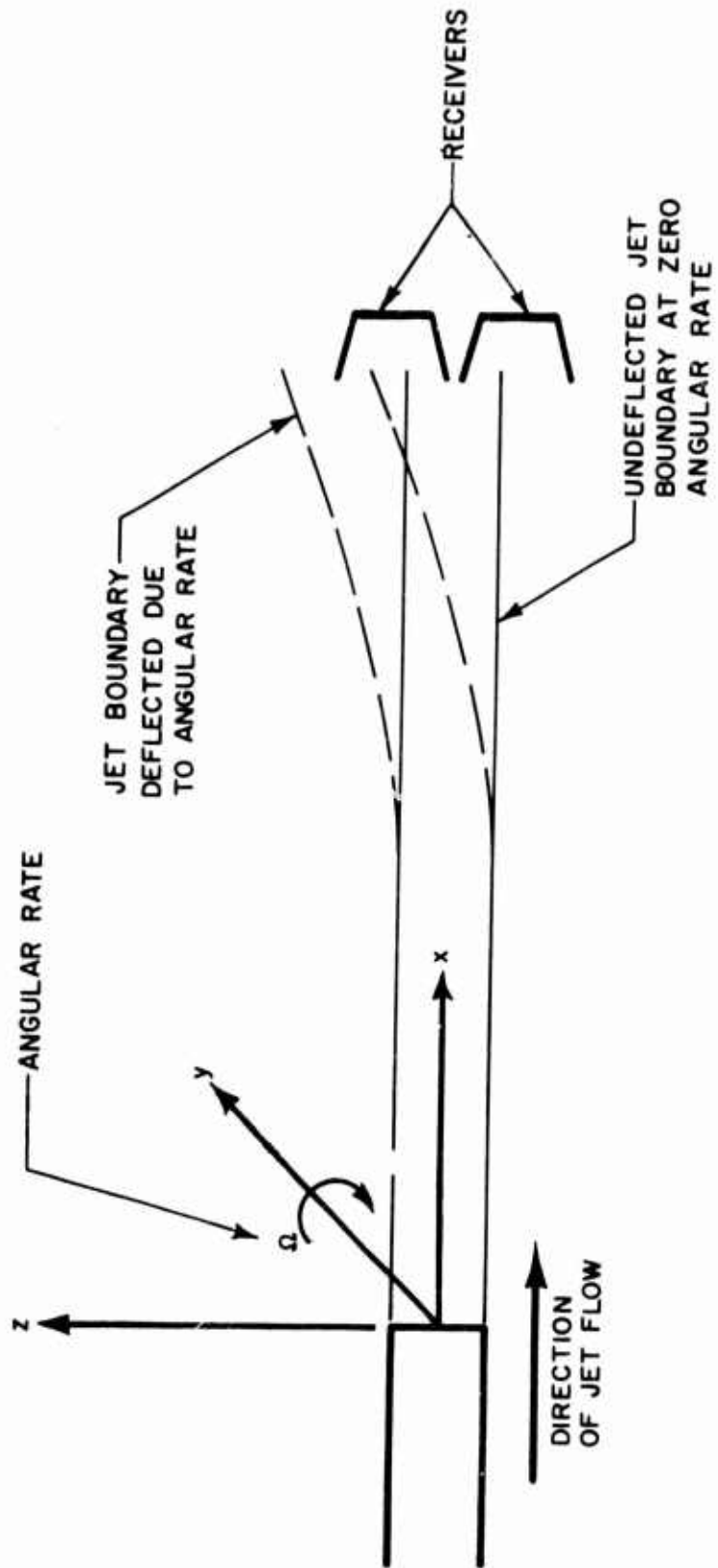
Figure 1. Principle of Jet Type of Fluidic Angular Rate Sensor

RECEIVERS

JET BOUNDARY DEFLECTED DUE TO ANGULAR RATE

UNDEFLECTED JET BOUNDARY AT ZERO ANGULAR RATE

ANGULAR RATE

DIRECTION OF JET FLOW

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ORDINARY NONROTATING JET

DIFFUSE BOUNDARY DUE TO ENTRAINMENT OF SURROUNDING FLUID

V = VELOCITY

R = DISTANCE FROM JET AXIS
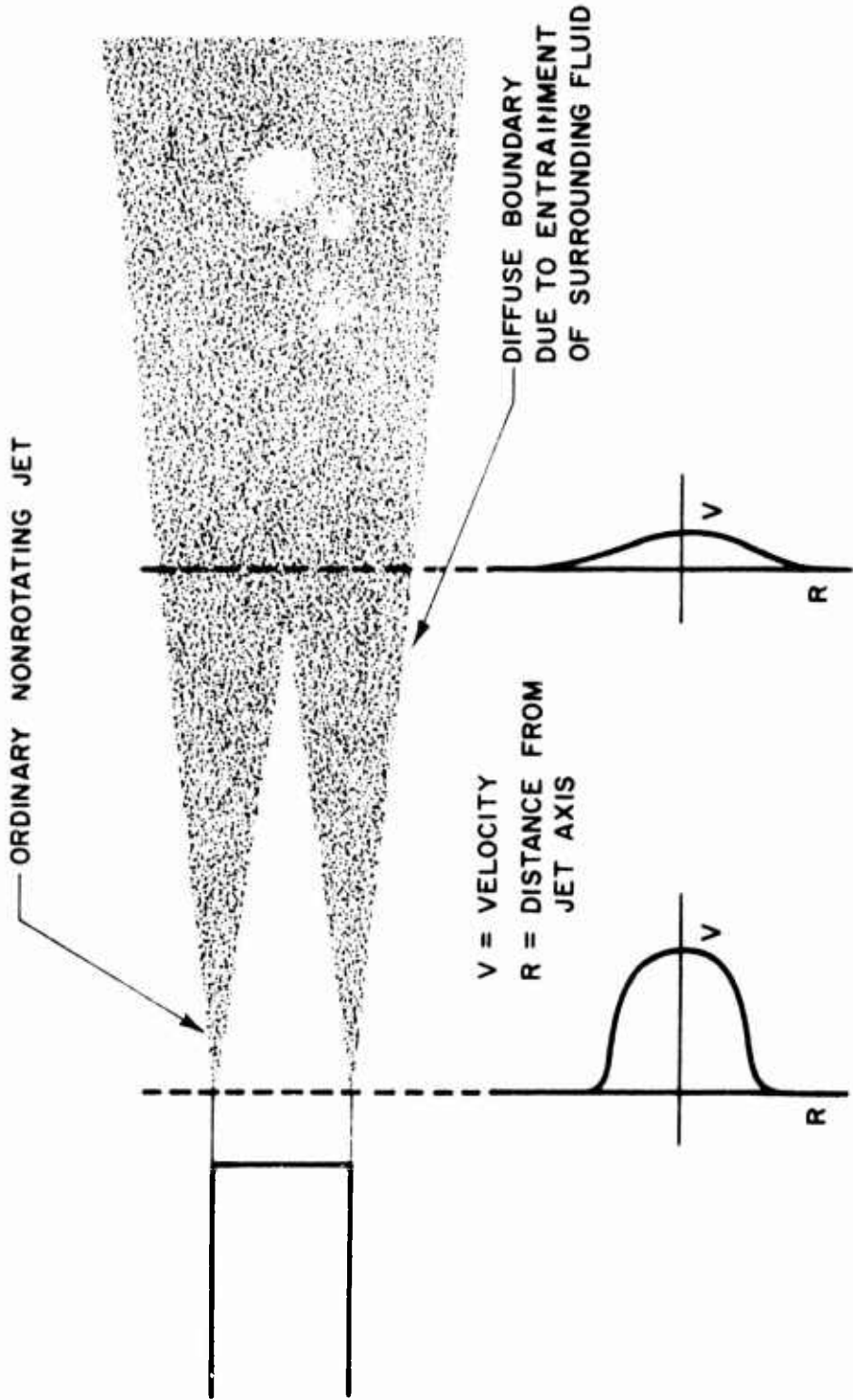
Figure 2. Limitation of Maximum Unstable Jet Length Due to Boundary Instability

SECOND AXIS, DIAMETER = D

FIRST AXIS

DIRECTION OF MOTION ALONG FIRST AXIS

BOUNDARY STABILIZED BY ROTATION

d = DIAMETER OF CROSS SECTION
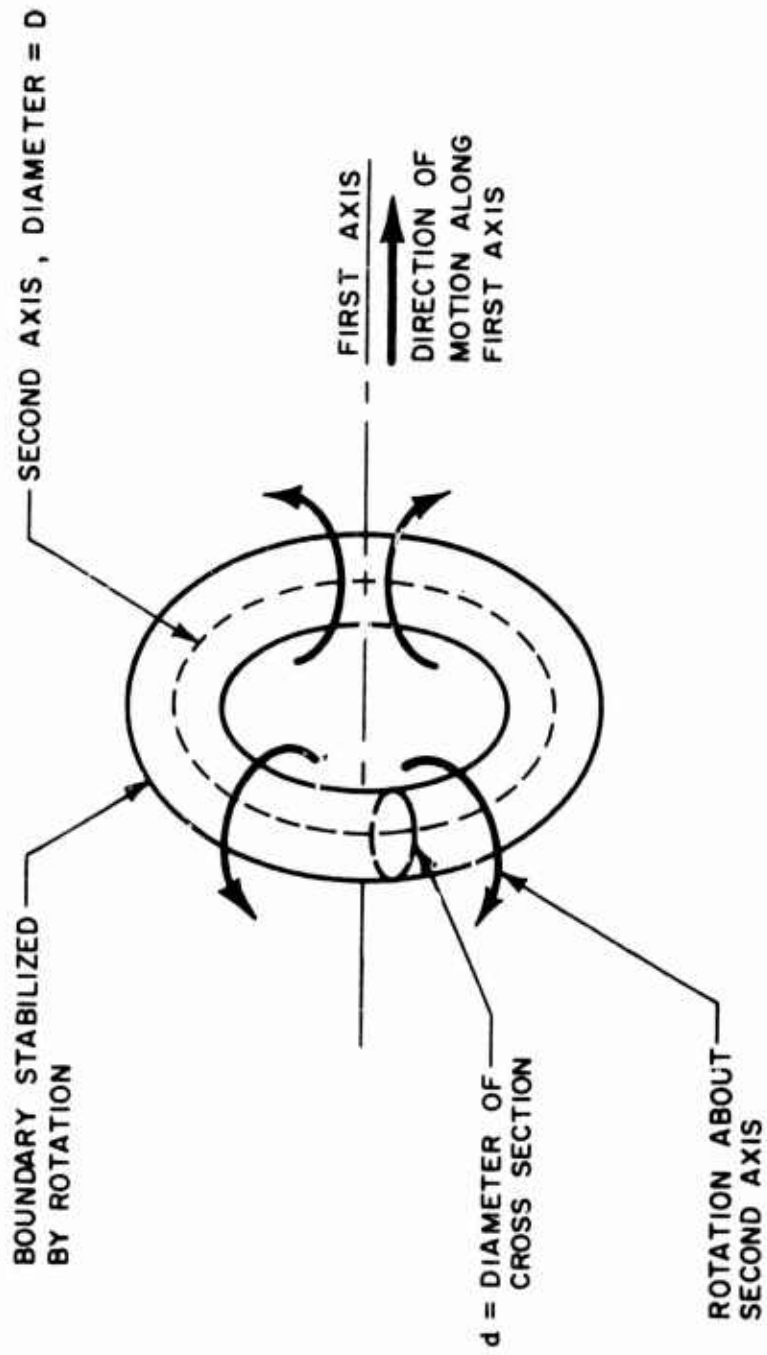
ROTATION ABOUT SECOND AXIS

Figure 3. Vortex Ring

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made very large, and will not depend appreciably upon the velocity of the toroid along its axis, which can be made very small. (For the toroidal vortex ring, the ratio of speed of rotation to velocity along the axis is related to r_o/R, the ratio of cross-section radius to radius of the second axis.)

This may be made clearer by considering the toroidal vortex ring as being aimed at a peg (see Figure 4). The axis of the peg coincides with the axis along which the ring normally travels. The peg has two pressure-sensing points, located on opposite sides. A small deviation of the ring axis from its nominal position will cause a pulse of pressure difference as the ring moves along the peg, the magnitude of the pulse being proportional to the product of the pressure gradient in the ring (large quantity if the rotation is rapid) times the deviation of the ring axis due to the angular rate being sensed (large quantity if velocity of ring along its axis is low). Thus, one may hope to achieve high sensitivity by combining rapid rotation, low speed along the axis, and large length of travel.
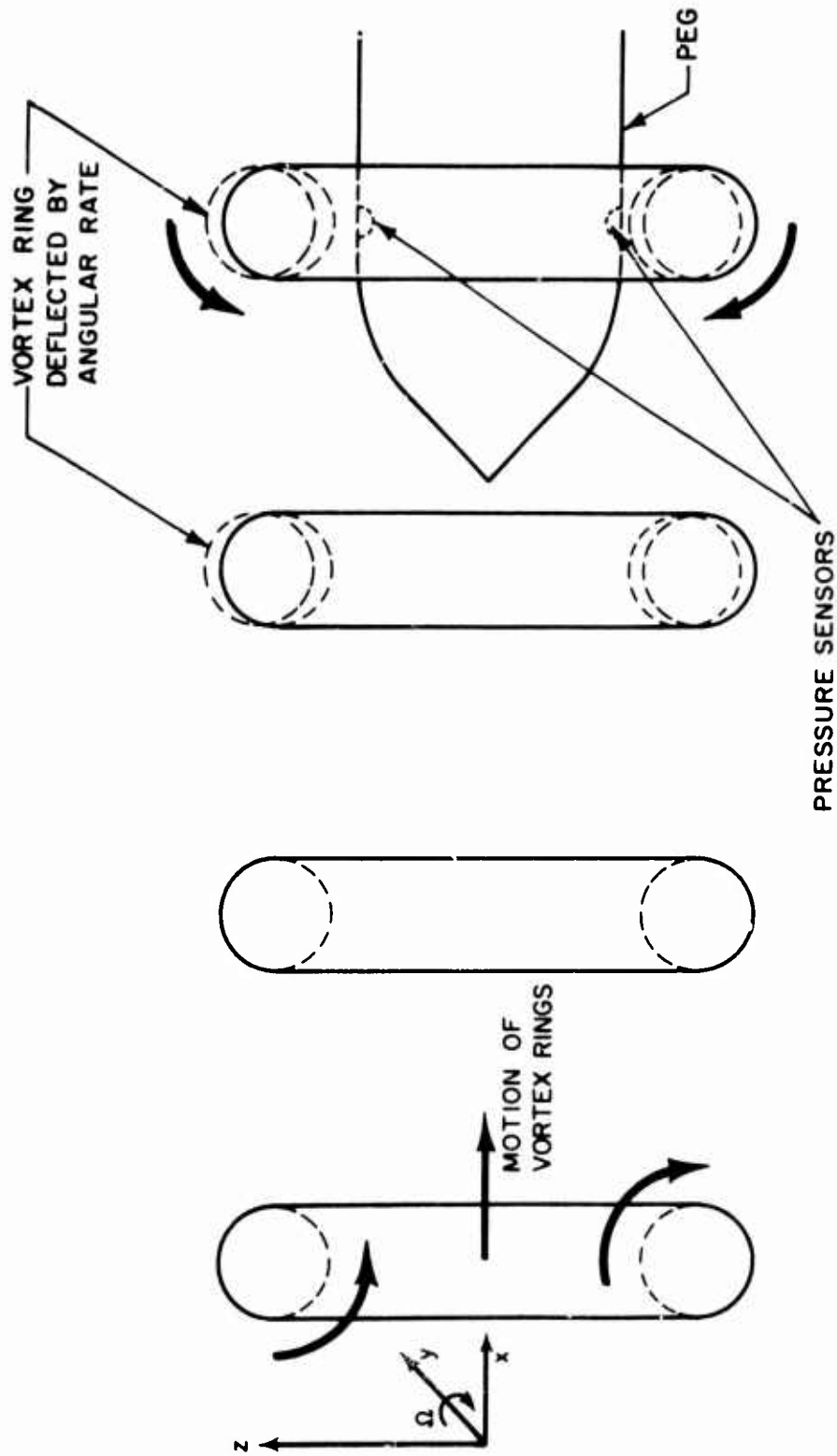
Figure 4. Use of Vortex Ring in Angular Rate Sensor

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C. CHOICE OF DESIGN PARAMETERS

Consider a vortex ring for which the circular axis has radius R and the radius of the vortex core is r_0, assumed to be small compared to R. Let r be the distance from the circular axis. The velocity distribution about the circular axis may be approximated by the velocity distribution about the single straight vortex, which may be divided ideally into a core region of radius r_0 in which the fluid rotates as a solid body, and a potential flow region outside the core region (References 2, 3, 4). This distribution is:

$$\text{For } r \leq r_o, \; v = v_o \, r/r_o$$

$$\text{For } r_o \leq r \ll R, \; v = v_o \, r_o/r$$

This ideal velocity distribution does not remain indefinitely, however, but is altered by the viscosity of the fluid, which has the effect of diffusing the core outward, decreasing the velocities within the core. For the case of zero initial core size, the exact solution for the velocity distribution about the straight vortex is (Reference 5):

$$v = \frac{\Gamma_o}{2\pi \, r} \, (1 - e^{-r^2/4\nu t})$$

where Γ_o is the initial value of Γ, ν is the kinematic viscosity, and Γ is the circulation,

$$\Gamma = 2\pi \, r \, v$$

Thus, for order-of-magnitude purposes, we may define an approximate time constant,

$$t_o = \frac{r_o^2}{4 \, \nu}$$

such that for $t \ll t_o$, decay of the vortex may be ignored, and for $t \gg t_o$, substantial decay has taken place.

The velocity of translation of the vortex ring along the linear axis, regarded as the x-axis, is (Reference 6):

$$\frac{dx}{dt} = \frac{\Gamma}{4\pi R} \left(\ln \frac{8R}{r_o} - \frac{1}{4} \right) \tag{1}$$

This, along with

$$\Gamma = 2\pi \; rv = 2\pi \; r_o \; v_o \tag{2}$$

yields

$$\frac{L}{T} = \frac{r_o v_o}{2R} \left(\ln \frac{8R}{r_o} - \frac{1}{4} \right) \tag{3}$$

where L is the propagation distance from the source of vortex rings and T is the propagation time.

If f is the frequency of vortex generation and S is the spacing between successive vortices,

$$S = \frac{L}{Tf} \tag{4}$$

To avoid interference between successive vortex rings, we require $S \gg 2 \; r_o$. In fact, to maintain stability of the stream of vortex rings, we impose the more stringent condition $S \geq R$.

If the angular velocity Ω about the y-axis is to be detected, it will produce a Coriolis acceleration in the z-direction given by

$$a = 2\Omega \; \frac{L}{T}$$

The Coriolis deflection will be

$$Z = \frac{1}{2} \; aT^2 = \Omega LT \tag{5}$$

In the neighborhood of the core of the vortex ring, the dynamic pressure, or reduction in static pressure below its stagnation value, is

$$\frac{\rho v^2}{2g}$$

The gradient of this quantity is

$$\frac{\rho}{g} \; v \; \frac{dv}{dr}$$

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The deflection Z will produce an increase in static pressure on one side of the peg at which the vortex ring is aimed, and a decrease in static pressure on the other side. The two sides of the peg are assumed to be fitted with static pressure sensors. In an arrangement in which the difference in static pressure between the two sides is detected, the pressure difference due to a small deflection is

$$\Delta P = \frac{2\rho}{g} \, v \, \frac{dv}{dr} \, Z$$

Since $rv = r_o v_o$, we obtain

$$\frac{g \Delta P}{2\rho} = v \, \frac{d}{dr} \, \left(\frac{r_o v_o}{r} \right) \, Z$$

$$\frac{g \Delta P}{2\rho} = \frac{v^2 Z}{r} \text{ (apart from sign)} \tag{6}$$

The smallest detectable deflection is related to the smallest detectable pressure difference by this equation, for any given v and r.

According to Reference 6, the fluid velocity along the x-axis (that is, at the center of the vortex ring) is

$$v_c = \frac{\Gamma}{2R} \tag{7}$$

so that with Equation (1),

$$\frac{1}{v_c} \, \frac{dx}{dt} = \frac{1}{2\pi} \left(\ell n \, \frac{8R}{r_o} - \frac{1}{4} \right) \tag{8}$$

The following table shows values of

$$\frac{1}{v_c} \, \frac{dx}{dt}$$

computed from Equation (8) for various values of R/r_o:

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$\dfrac{R}{r_o}$	$\dfrac{1}{v_c}\dfrac{dx}{dt}$
1	0.291
2	0.401
3	0.466
10	0.657
40	0.880
50	0.911
86	1.000
200	1.136

As a practical matter, we may assume that R/r_o is between 10 and 200.
This table then shows that $L/T (=dx/dt)$ may be fairly well approximated
by v_c.

With Equations (2) and (7), we then have

$$\frac{L}{T} = \pi v \frac{r}{R}$$

We assume that the peg at which the stream of vortex rings is aimed does
not disturb the vortex rings. We may select values of various parameters
in order to determine whether enough sensitivity is theoretically possible.
The following example is the result of some trial and error. Let
$r_o = 0.0025$ ft (about 1/32 in.) and suppose that $R = 40\ r_o = 0.1$ ft.
These values may be difficult to achieve, but they are not altogether
unreasonable. Suppose that $v_o = 100$ ft/sec. Then $v_c = 7.85$ ft/sec.

The table given above shows $dx/dt = 0.880\ v_c$. Thus, $L/T = 7.85 \times 0.880$
$= 6.9$ ft/sec. If we assume that $T = 0.05$ sec is a tolerable instrument
delay, we have $L = 0.345$ ft, which is not unreasonably large. If S, the
spacing between vortex rings, equals R, the frequency of generating vor-
tices is $6.9/0.1 = 69$ cps, which should be large enough for disturbances
at frequencies up to 10 cps to be resolved.

We may calculate the decay time constant t_o for air and water as
possible working fluids. The kinematic viscosities are, approximately,

$$v = 1.6 \times 10^{-4}\ ft^2/sec\ for\ air$$

and

$$v = 10^{-5}\ ft^2/sec\ \ for\ water$$

With

$$r_o = 0.0025\ ft$$

we obtain

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$$t_o = 0.01 \text{ sec} \quad \text{for air}$$

and

$$t_o = 0.15 \text{ sec} \quad \text{for water.}$$

We conclude that in time $T = 0.05$ sec, the vortex ring will decay very substantially in air, but will decay very little in water. For the purpose of the present study, we therefore assume that the fluid is water, although there are many other liquids for which t_o would be even longer.

The minimum detectable value of ΔP depends upon considerations of amplifi noise, bandwidth, and duration of the signal as the vortex ring passes the pressure sensors.

We envision an arrangement in which the pressure sensors that are on opposite sides of the peg at which the vortex ring is aimed are the control ports of a fluid amplifier. These control ports will be placed so that as the vortex ring passes, the ports will be at a distance r, slightly larger than r_o, from the circular axis of the vortex ring. We assume that they are oriented so as to respond to the static pressure of the fluid. As the vortex ring advances in the x direction, substantial decrease in the static pressure will occur at the ports when v is large, and this will be true for a range of positions of the vortex ring of approximately $\Delta x = r_o$. Consequently, the duration of the input signal is approximately

$$\frac{r_o}{dx/dt} = 0.36 \text{ millisecond}$$

To achieve the desired sensitivity of 0.05 deg/sec, or 0.87×10^{-3} radians/sec, we must detect the Coriolis deflection, Equation (5), $Z = 150 \times 10^{-6}$ ft $= 0.0018$ in. From Equations (2) and (6),

$$\Delta P = 2 \frac{v_o^2 Z}{r_o} \frac{\rho}{g} \left(\frac{r_o}{r}\right)^3$$

Since r is slightly more than r_o, we take $(r_o/r)^3 = 0.5$, approximately, and find that the signal to be detected is a pressure difference between the ports

$$\Delta P = 8 \text{ psi}$$

Information is lacking concerning the performance of fluid amplifiers at the signal frequencies needed to detect a 0.36 millisecond pulse. However, a jet deflection type of fluid amplifier using water as the working fluid has been reported (Reference 7) for which the threshold input signal

may be estimated from the reported data. With a supply pressure of 5 psig, the maximum output pressure is 60 percent of this value, or 3 psi. If we assume the output port pressure at zero signal to be about half the supply pressure, or 2.5 psi, the output difference in push-pull operation will be approximately $2(3.0-2.5) = 1.0$ psi. The gain of the amplifier is approximately 2.5, so that the maximum input pressure difference is about 0.4 psi. Noise is discussed in general terms in Reference 7, where it is stated that an improvement by a factor of 3 can be achieved over that observed. The observed noise level is not stated explicitly, but presumably it is less than one-third of the maximum input signal. With the indicated improvement factor of 3, we infer that the noise need not be more than 0.1 times the maximum input signal, or 0.04 psi. The measured gain was fairly constant at frequencies up to 200 cps.

An amplifier capable of responding only to low frequencies will respond to an input pulse of short duration as though it were of longer duration but lower amplitude. An input signal of 8 psi with a duration of 0.36 millisecond will have the same effect on such an amplifier as an input signal of 0.04 psi with a duration of 0.072 sec. Since the fluid amplifier described in Reference 7 is capable of responding to shorter signals than this, we conclude that a jet deflection type of fluid amplifier can be used in an entirely fluidic system to detect the Coriolis deflection of the vortex ring.

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D. CHOICE OF DETECTORS

In a completely fluidic system, it is reasonable to use the control ports of a fluid amplifier to detect the decrease in static pressure in the vortex ring, as described previously. It may not be necessary to have a completely fluidic system, however, and it is therefore worth considering other detectors. In effect, a pressure difference detector really detects the velocity difference at opposite sides of the vortex ring due to the deflection Z. Thus, the deflection measurement may be accomplished by measuring the velocity difference by any other means. Two methods will now be briefly discussed.

First, hotwire anemometry may be used. Reference 8 discusses the application of a commercial hotwire anemometer to the measurement of turbulence in an airstream. Using a bandwidth of 40,000 cps, a turbulence intensity of 0.8×10^{-3} was measured in a stream having a velocity of 20 meters/sec. Thus, velocity fluctuations having an rms value of 0.053 ft/sec were measured with a very wide bandwidth. Reference 8 also describes coated hot-film probes for measurements in liquids. The thin quartz coating does not affect the frequency response below 50,000 cps, and experimental data points are plotted with velocity deviations from a smooth curve that are very small compared to 1 ft/sec.

On the other hand, the velocity difference that has to be detected is

$$\Delta v = 2 \frac{dv}{dr} \ Z = 2 \frac{v_o}{r_o} \ Z \ \left(\frac{r_o}{r}\right)^2$$

Taking $(r_o/r)^2 = 0.5$, approximately, we find that $\Delta v = 6.0$ ft/sec. Thus, hotwire anemometry has more than enough sensitivity and bandwidth to detect the deflection Z.

A second method of detecting Δv is to use an electrolytic cell reaction. Reference 9 describes the use of a cell containing potassium iodide and iodine in aqueous solution, with a trace of citric acid to stabilize the solution. When a voltage difference is applied between a pair of electrodes, the voltage difference being less than the minimum needed to electrolyze the water, the reactions that take place are:

At the cathode (negative electrode):

$$I_o^- + 2e \rightarrow 3I^-$$
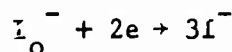
At the anode (positive electrode):

$$3I^- - 2e \rightarrow I_3^-$$
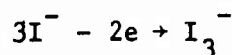
The I^- ions are much more mobile than the I_3^- ions. Consequently, the rate of reaction is governed entirely by the rate at which I_3^- ions are transported by convection and diffusion to the cathode. The technique of using the cell to measure velocity is to use a very small cathode located at the point where the velocity is to be measured. The container for the solution may conveniently be used as the anode. The current that flows in the external circuit is proportional to the rate at which I_3^- ions are brought to the cathode and, except for velocities that are much less than those of interest here, the transport is entirely convective. The time delay for the current to reach a value corresponding to the flow velocity, immediately following a velocity change, is on the order of microseconds, so that for our present purposes, response is instantaneous.

The concentrations of potassium iodide and iodine may be adjusted over a wide range to yield a convenient value for the electric current corresponding to a given velocity. We may estimate the electrical noise inherent in the method by considering the number n of I_3^- ions arriving at the cathode in time Δt, the duration of the signal pulse. Let A be the flow cross-section that contributes ions to the electrode, and let c be the concentration of ions in number per unit volume. Then,

$$n = Avc \, \Delta t$$

For a 0.1 normal solution, $c = 2.0 \times 10^{21}$ per ft^3. Taking $A = 10^{-6} \, ft^2$, $v = 1$ ft/sec, and $\Delta t = 10^{-4}$ sec, we find $n = 2 \times 10^{11}$. The random fluctuations in n are approximately $\sqrt{n}$, or 4.5×10^5, which is negligible compared to n. We conclude that the noise inherent in this method of detecting Δv, due to random arrival of ions, is negligible.

E. GENERATION OF VORTEX RINGS

Only a limited amount of information concerning experimental generation
of vortex rings has been found (References 10, 11). When flow takes place
through an orifice, a region of shear exists in the boundary layer ad-
jacent to the solid surface, introducing vorticity into the flow field.
If the duration of the flow is finite, the vorticity is contained in,
and travels with, a finite volume of fluid that forms the core of the
vortex ring as it separates from the orifice. While shear is essential
for the introduction of vorticity into the vortex core, inertial effects
govern the structure of the potential flow regime outside the core.
Consequently, the shape of the orifice is important in determining the
vortex ring that is generated by a pulse of flow through the orifice.
Reference 10 reports, for example, that when a short cylindrical tube
of the same diameter was added to the orifice, external to the chamber
from which the air emerged, normal vortex rings were still produced;
but when the short cylinder was added to the inner face of the orifice,
so that it projected into the chamber, vortex rings were no longer pro-
duced.

It appears, therefore, that an experimental development of a TSARS would
require some empirical development of the vortex ring generator, in view
of the rather large value, 100 ft/sec, required for v_0. Such a develop-
ment would include the shape of the orifice, the shape of the boundaries
inside the chamber, the flow rate required as a function of time, and
methods of producing the required flow. Preferably, the flow would be
generated by purely fluidic means, such as pressure oscillations in a
resonating flow. Furthermore, the sensitivity to disturbances would
have to be investigated.

Disturbances arise from various sources, such as the disturbance, by the
series of vortex rings, of the "quiescent" fluid through which the vortex
ring travels on its way to the deflection sensors. As a result, the fac-
tors that determine the instant of separation of the vortex ring from
the orifice may vary randomly from point to point around the orifice, so
that the vortex ring may not be released at precisely the same instant
all around the orifice. As a result, the direction of propagation will
vary randomly about the nominal direction of the x-axis.

With $L = 0.345$ ft and $Z = 150 \times 10^{-6}$ ft for the minimum deflection to
be detected, a random deviation of the direction of propagation by
$Z/L = 0.43 \times 10^{-3}$ radians will produce a noise equal to the signal.
With $R = 0.1$ ft and a drift velocity of $L/T = 6.9$ ft/sec this directional
deviation will be produced by having a difference δt in the instants
at which diametrically opposite parts of the vortex ring are released,
where δt is given by

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$$\frac{\frac{L}{T} \delta t}{2R} = \text{angular deviation} = \frac{Z}{L}$$

$$\delta t = \frac{2RZT}{L^2} = 0.126 \times 10^{-6} \text{ sec}$$

Thus, extremely precise release of the vortex rings has to be attained. Whether successive vortex rings will be reproducible with sufficient precision is a crucial question, for which it appears that an answer cannot be given at present without an experimental study.

F. ALTERNATIVE NONPULSATING DESIGN

Figure 5 shows a suggested alternative design, the vortex axis jet angular rate sensor (VAJARS). It may be viewed as a jet rate sensor with an ordinary nonrotating jet modified by the addition of a vortex along its axis. Alternatively, it may be viewed as a jet of small diameter, rotating rapidly about its axis, and shielded from the quiescent fluid by a non-rotating jet of large diameter. The axial component of velocity is the same for both the nonrotating jet and the vortex core. Deflection of the vortex (due to rotation about an axis normal to the plane of the diagram) is detected by the connections to the fluid amplifier, or by any alternative nonfluidic means for measuring velocity, just as in the case of the TSARS.

The vortex core is made to rotate by means of a twisted vane in its supply tube. A screen or orifice plate at the exit of its supply tube causes a pressure drop that results in an increase in axial velocity as the vortex emerges. Conservation of mass then requires a contraction of the diameter of the vortex and consequently an increase in v_0 over the value it would have without the screen or orifice plate. The result is an extremely stable vortex that mixes very little with the surrounding fluid. This stabilizing effect has been used by the late Dr. F. O. Ringleb in generating very stable smoke lines for flow visualization in an air flow tunnel at the Navy Air Engineering Center located at the Philadelphia Naval Base.

Compared with the jet rate sensor, VAJARS has the advantage that the nonrotating jet can be made to have as large a diameter as is needed so that the length of travel is less than, say, eight jet diameters, thus ensuring that the jet is not destroyed before it reaches the deflection sensors. In the jet rate sensor, on the other hand, use of a large-diameter jet would result in low sensitivity in detecting the deflection. Furthermore, as in the case of the TSARS, the VAJARS has enhanced sensitivity because the velocity to be detected is the velocity of rotation, rather than the much smaller drift velocity.

Compared with the TSARS, the VAJARS has the advantage that all flows are steady, so that the problems of vortex ring generation, random variations in direction of propagation, and disturbances due to violently pulsating flow are all avoided. Provided L, T, v_o, and r_0 are the same, the theoretical sensitivity is the same as for TSARS.

VAJARS has two further advantages over TSARS. First, the axial velocity of the vortex is not closely related to the geometrical dimensions and v_o, as it is for a vortex ring. Consequently, design procedure should be simplified. Second, brief pulses do not have to be detected by the deflection sensors. Consequently, a much lower frequency response is sufficient for VAJARS, so that noise problems should be much less severe.
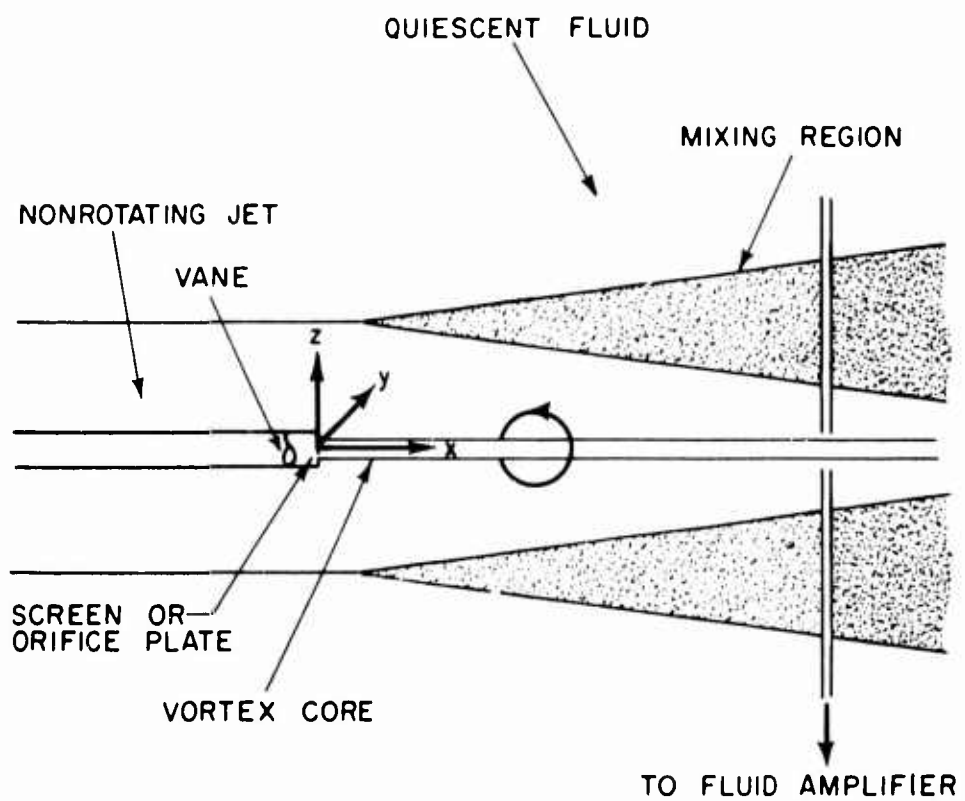
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Figure 5.   Principle of VAJARS

In view of these considerations, the arguments that show the feasibility
of a TSARS certainly also show the feasibility of a VAJARS, especially
in view of the fact that the VAJARS concept avoids the principal unre-
solved item in the demonstration of the feasibility of a TSARS; namely,
the problem of generating vortex rings.

G. RECOMMENDATIONS FOR EXPERIMENTAL STUDY

The TSARS and the VAJARS both appear to be suitable subjects for experimental study, but the VAJARS should be easier to develop. Consequently, it is recommended that experimental study of the VAJARS be considered first, with consideration of TSARS postponed.

The VAJARS concept requires experimental study of the following items:

1. Generation of vortex, including orifice plate design and required pressure drop.

2. Absorption and recirculation of jet fluid through a quieting circuit.

3. Steadiness of large-diameter jet and vortex.

4. Design and construction of suitable fluid amplifiers.

5. Demonstration of response to Coriolis effect by use of a simple rate table.

In studying these items, except for item 4, it is recommended that non-fluidic devices be used in the experimental technique. For example, following some simple calibration measurements, the iodine electrolytic cell method discussed earlier should be very useful as a tool in performing the study. Evidently, completion of items 4 and 5 would amount to construction and demonstration of a breadboard model.

If desired later on, the TSARS concept may be studied by means of experiments concerned with the following items:

1. Methods of generating reproducible vortex rings.

2. Fluidic generation of driving pressure.

3. Avoidance of reflection of vortex rings from the end of the chamber, to minimize disturbance to quiescent fluid.

4. Observation of variation from one vortex ring to another.

5. Design and construction of fluid amplifier.

Here, again, the iodine electrode reaction should provide a useful tool.

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REFERENCES

1. R. V. Burton, D. N. Carlson, R. L. Heinecke, R. R. St. John, FLUID STATE HYDRAULIC DAMPER, Honeywell Aeronautical Division, Minneapolis, Minnesota, USAAVLABS Technical Report 66-87, U. S. Army Aviation Materiel Laboratories, Fort Eustis, Virginia, February 1967.

2. Serge J. Zaroodny, REVISED THEORY OF VORTEX RINGS - A SIMPLIFIED REVIEW OF THE STATE-OF-THE-ART, Technical Memorandum No. 66-01, U.S. Army Limited War Laboratory, Aberdeen Proving Ground, Md. April 1966.

3. Zegmund O. Bleviss, THEORETICAL ANALYSIS OF LIGHT PLANE LANDING AND TAKE-OFF ACCIDENTS DUE TO ENCOUNTERING THE WAKES OF LARGE AIRPLANES, Report No. SM-18647, Douglas, Aircraft Div., Long Beach, California, December 1954.

4. E. L. Zwieback, TRAILING VORTICES OF JET TRANSPORT AIRCRAFT DURING TAKE-OFF AND LANDING, Final Report No. RD-64-3, for Federal Aviation Agency, Douglas, Aircraft Division, Long Beach, Calif., January 1964.

5. L. Prandtl, THE MECHANICS OF VISCOUS FLUIDS, Aerodynamic Theory, Vol. III, W. F. Durand, ed., Julius Springer, Berlin, 1935, pp. 68 and 69. Republished by Durand Reprinting Committee, Pasadena, Calif., 1943.

6. H. Lamb, HYDRODYNAMICS, Sixth Edition, 1932.

7. Final Report - INVESTIGATION OF WATER AND STEAM AS THE WORKING FLUIDS IN A FLUID AMPLIFIER SPEED GOVERNOR FOR TURBINE GENERATOR SETS, prepared for the Department of the Navy, Office of Naval Research, Power Branch, Code 429, prepared by General Electric Company, Research and Development Center, Schenectady, N. Y.

8. DISA Information, ELECTRONIC MEASUREMENTS OF MECHANICAL EVENTS, No. 3, January 1966, DISA Elektronik A/S, Herlev, Denmark.

9. P. M. W. Navé, TRANSIENT TRANSPORT PHENOMENA IN ELECTROLYTIC CELLS, Acta Physica Austriaca, Vol. 22, p. 259 (1966).

10. M. Sadron, CONTRIBUTION A L'ETUDE DE LA FORMATION ET DE LA PROPAGATION DES ANNEAUX DE TOURBILLON DANS L'AIR, J. de Physique et le Radium, 7. pp. 76-91, March 1926.

11. F. K. Elder, Jr , and N. deNaas, EXPERIMENTAL STUDY OF THE FORMATION OF A VORTEX RING AT THE OPEN END OF A CYLINDRICAL SHOCK TUBE, J. Appl. Physics, 23, p. 1065, October 1952.

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)*	2a. REPORT SECURITY CLASSIFICATION
The Franklin Institute Research Laboratories 20th and Race streets Philadelphia, Pennsylvania	UNCLASSIFIED
	2b. GROUP

3. REPORT TITLE

Feasibility of a Toroidal Stream Angular Rate Sensor

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Final Report

5. AUTHOR(S) *(First name, middle initial, last name)*

George P. Wachtell

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
April 1968	38	11

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
DAAJ02-67-C-0043	
b. PROJECT NO.	USAAVLABS Technical Report 68-11
c. Task No. 1F121401A14186	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	U.S. Army Aviation Materiel Labs. Fort Eustis, Virginia

13. ABSTRACT

It is theoretically feasible to build a toroidal stream angular rate sensor (TSARS) based on the principle that a series of vortex rings is deflected by the Coriolis force so that the rotational velocities measured by a pair of symmetrically placed sensors are slightly different. The sensors may be the input ports of a jet type of fluid amplifier. It should be possible to detect angular rates as low as 0.05 deg/sec, with disturbance frequencies up to 10 cps. An uncertain item is the precise reproducibility of the vortices that are generated. An alternative design concept, having several advantages over the TSARS as well as the jet type of angular rate sensor, is the vortex axis jet angular rate sensor (VAJARS), in which a straight vortex is provided along the axis of a large-diameter jet. Experimental studies are recommended, with VAJARS considered to be more promising.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Angular velocity measurement						
Fluidic devices						
Vortex rings						
Coriolis effect						
Vortex lines						
Rate sensor						
Helicopters						
V/stol Aircraft						
Stability augumentation						