ON THE IMPORTANCE OF DIFFERENT COMPONENTS IN A MULTICOMPONENT SYSTEM

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Technical Report No. 54

May 20, 1968

Contract N-onr-477(38)

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1. INTRODUCTION: Definitions and Notations

1.1 In a system whose performance depends on the performance of its components, some of these components may play a more important part than others. For example, if a system consists of n components in series, or of n components in parallel, one may be inclined to consider each component equally important for the performance of the system. In the system indicated in Figure 1, however, component c_1 would seem intuitively more important than $c_2, c_3, c_4, \ldots c_n$.

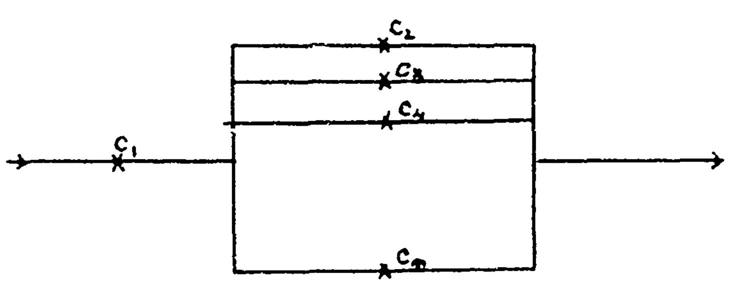

FIGURE 1

In the following we propose a quantitative definition of this notion of importance, and discuss some properties and applications of this concept.

1.2 We assume that every device, whether it is a single component or a system consisting of components, can be in one and only one of two states: it functions or it fails.

When a system consists of components $c_1, c_2, \ldots, c_n$, we can ascribe to each of them a binary indicator variable

$$x_i = \begin{cases} 1 & \text{when } c_i \text{ functions} \\ 0 & \text{when } c_i \text{ fails.} \end{cases}$$

Each n-tuple of 0's or 1's

$$(x_1, x_2, \ldots x_n) = \underline{x}$$

is called a <u>vector of component states</u> or in short a <u>state vector</u>. It can assume any one of the 2^n values represented by the vertices of the unit cube in n-dimensional space: (0,0,...,0), (1,0,...,0), (1,1,0,...,0), ... (1,1,...,1). We shall use the following notations:

$\underline{x} \leq \underline{y}$ when $x_i \leq y_i$ for $i=1,\ldots,n$

$\underline{x} = \underline{y}$ when $x_i = y_i$ for $i=1,\ldots,n$

$\underline{x} < \underline{y}$ when $\underline{x} \leq \underline{y}$ and $\underline{x} \neq \underline{y}$

$(0_k, \underline{x}) = (x_i, x_2, \ldots, x_{k-1}, 0, x_{k+1}, \ldots, x_n)$

$(1_k, \underline{x}) = (x_1, x_2, \ldots, x_{k-1}, 1, x_{k+1}, \ldots, x_n)$

$\underline{0} = (0,0,\ldots,0)$, $\underline{1} = (1,1,\ldots,1)$.

We ascribe to the system a binary indicator variable

$$u = \begin{cases} 1 & \text{when the system functions} \\ 0 & \text{when the system fails.} \end{cases}$$

When the design of a system is known, then the state vector $\underline{x}$ determines the state of the system so that

$$u = \Phi(\underline{x})$$

where Φ is a function with values 0 or 1. This function Φ is called the <u>structure function</u> of the system.

A structure function is called <u>coherent</u> [1] when it fulfills the conditions: $\Phi(\underline{0}) = 0$, $\Phi(\underline{x}) \leq \Phi(\underline{y})$ for $\underline{x} \leq \underline{y}$, and $\Phi(\underline{1}) = 1$. From now on we shall consider only coherent structure functions. One verifies immediately that $\Phi(\underline{x})$ can be represented for every $j = 1,2,\ldots,n$ in the form

$$(1.2.1) \quad \Phi(\underline{x}) = x_j[\Phi(1_j,\underline{x}) - \Phi(0_j,\underline{x})] + \Phi(0_j,\underline{x}) =$$

$$= x_j\delta_j(\underline{x}) + \mu_j(\underline{x})$$

where

$$(1.2.2) \quad \delta_j(\underline{x}) = \Phi(1_j,\underline{x}) - \Phi(0_j,\underline{x})$$

$$(1.2.3) \quad \mu_j(\underline{x}) = \Phi(0_j,\underline{x}),$$

and $\delta_j(\underline{x})$ as well as $\mu_j(\underline{x})$ do not depend on the state x_j of component c_j.

1.3 If the state of c_j is determined by chance, so that the value actually assumed by x_j is a binary random variable X_j with the probability distribution

$$(1.3.1) \qquad \begin{aligned} P\{X_j = 1\} &= p_j \\ && j=1,2,\ldots,n \\ P\{X_j = 0\} &= q_j = 1-p_j \end{aligned}$$

then p_j is called the <u>reliability</u> of c_j. In the following we shall assume that $X_1, X_2, \ldots, X_n$ are totally independent. The n-tuple of component reliabilities determines a point

$$(1.3.2) \qquad (p_1, p_2, \ldots, p_n) = \underline{p}$$

in the n-dimensional unit cube $(p_1,\ldots,p_n) : 0 \leq p_i \leq 1 = J_n$.

For a given structure function $\phi(\underline{x})$, the values of component reliabilities $(p_1,\ldots,p_n) = \underline{p}$ determine the probability that the system will function

$$(1.3.3) \qquad P\left\{\phi(\underline{x}) = 1 \mid \underline{p}\right\} = E[\phi(\underline{x}) \mid \underline{p}] = h_\phi(\underline{p}).$$

This function $h_\phi(\underline{p})$, defined on J_n, is the <u>reliability function</u> for ϕ.

There are situations when only the design of a system is known, i.e. $\phi(\underline{x})$ is given, but no information is available about the component reliabilities. We shall consider the relative importance of various components in such situations, and shall call it <u>structural importance</u>.

In other instances, both the structure function Φ and the component reliabilities $\underline{p}$ are known. The concept of importance which will be introduced for such situations will be referred to as reliability importance.

A third, substantially more complicated way of considering the importance of components will be briefly mentioned in Section 6.

2. STRUCTURAL IMPORTANCE

2.1 A component c_j is essential for structure Φ at the state vector (vertex of unit cube) $\underline{x}$ when

(2.1.1) $$\delta_j(\underline{x}) = \Phi(1_j,\underline{x}) - \Phi(0_j,\underline{x}) = 1;$$

c_j is essential at $\underline{x}$ for the functioning of Φ when

(2.1.2) $$(1-x_j)\delta_j(\underline{x}) = 1$$

and c_j is essential at $\underline{x}$ for the failure of Φ when

(2.1.3) $$x_j\delta_j(\underline{x}) = 1.$$

Clearly, if c_j is essential at $\underline{x}$ then it is either essential for functioning or for failure, depending on whether the vertex $\underline{x}$ has its coordinate x_j equal to 0 or to 1.

We define the <u>structural importance</u> of c_j <u>for the functioning</u> of ϕ as

(2.1.4) $$I_j(\phi,1) = 2^{-n} \sum_{(\underline{x})} (1-x_j)\delta_j(\underline{x})$$

where the sum is extended over all 2^n vertices of the unit cube (state vectors), and similarly the <u>structural importance of</u> c_j <u>for failure of</u> ϕ as

(2.1.5) $$I_j(\phi,0) = 2^{-n} \sum_{(\underline{x})} x_j\delta_j(\underline{x}).$$

Finally, the <u>structural importance</u> of c_j for ϕ is defined as

(2.1.6) $$I_j(\phi) = I_j(\phi,0) + I_j(\phi,1) = \sum_{(\underline{x})} \delta_j(\underline{x}).$$

One verifies that if c_j is essential at $\underline{x}$ for the functioning of ϕ then c_j is essential at $(1_j,\underline{x})$ for failure, and if c_j is essential at $\underline{x}$ for failure of ϕ then it is essential at $(0_j,\underline{x})$ for functioning. There is, therefore, a one-to-one correspondence between those vertices (state vectors) at which c_j is essential for functioning and those at which it is essential for failure, hence the number of either kind of vertices is the same and from (2.1.4), (2.1.5), (2.1.6) follows

$$(2.1.7) \qquad I_j(\phi,1) = I_j(\phi,0) = \frac{1}{2} I_j(\phi).$$

There is therefore no purpose in distinguishing between structural importance for functioning and for failure. We shall see, however, that for reliability importance a similar distinction is meaningful.

2.2 Examples

2.2.1 k-out-of-n structures.

A structure $\phi(\underline{x})$ with n components is called "k-out-of-n" when it functions whenever at least k of its components function. One verifies that for such ϕ

$$(2.2.1.1) \qquad I_j(\phi) = 2^{-n} \cdot 2\binom{n-1}{k-1}, \; j = 1,2,\ldots,n.$$

All components have the same structural importance, and this importance is greatest for $k = \frac{n}{2}$, if n is even, and for $k=[\frac{n}{2}]$ and $k = [\frac{n}{2}] + 1$ if n is odd. The importance of every component is smallest in the case of n components in series (n-out-of-n structure) and of n components in parallel (1-out-of-n structure, when $I_j(\phi) = 2^{-n} \cdot 2$.

2.2.2 k in series, in series with n-k in parallel.

Let

(2.2.2.1) $$\phi(\underline{x}) = x_1 x_2 \ldots x_k \cdot [1-(1-x_{k+1})\ldots(1-x_n)].$$

This structure may be represented by the diagram in Figure 2.

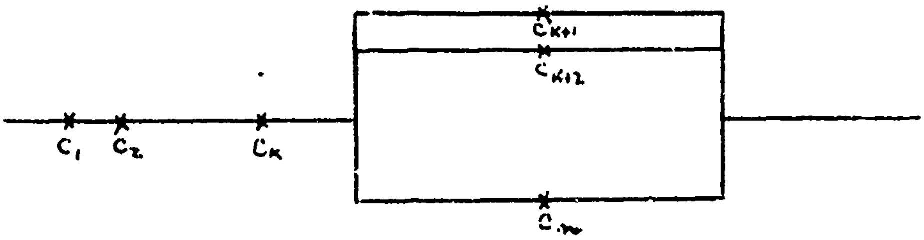

FIGURE 2.

One computes

(2.2.2.2) $$\delta_j = \prod_{\substack{r=1 \\ r\neq j}}^{k} x_r \, [1-\prod_{t=k+1}^{n} (1-x_t)] \qquad \text{for } j=1,\ldots,k$$

(2.2.2.3) $$\delta_j = \prod_{r=1}^{k} x_r \prod_{\substack{t=k+1 \\ t\neq j}}^{n} (1-x_t) \qquad \text{for } j=k+1,\ldots,n$$

hence

(2.2.2.4) $$I_j(\phi)=2.2^{-n}(2^{n-k}-1) = 2(2^{-k}-2^{-n}) \qquad \text{for } j=1,\ldots,k$$

(2.2.2.5) $$I_j(\phi) = 2^{n-+1} \qquad \text{for } j=k+1,\ldots,n.$$

We see that $c_1,\dots,c_k$ have each importance $2(2^{-k}-2^{-n})$, much greater than the importance 2.2^{-n} of each of $c_{k+1},\dots,c_n$. In particular, for $k=1$ we obtain the structure in Fig. 1 and have $I_1(\phi)=1-2^{-n+1}$, $I_2(\phi)=\dots=I_n(\phi)=2^{-n+1}$, which agrees with what one would intuitively expect.

3. RELIABILITY IMPORTANCE

3.1 From (1.2.1) and (1.3.4) one obtains immediately for the reliability function the expression

$$h_\phi \underline{p} = p_j E[\delta_j(\underline{X})] + E[\mu_j(\underline{X})] \tag{3.1.1}$$

for every $j=1,2,\dots,n$, and from (3.1.1) and (1.2.1) follows

$$\frac{\partial h_\phi(\underline{p})}{\partial p_j} = E[\delta_j(\underline{X})] = E\left[\frac{\phi(\underline{x})}{\partial x_j}\right], \quad \frac{\phi(\underline{x})}{x_j} \quad j=1,2,\dots,n. \tag{3.1.2}$$

One also proves by straightforward algebra [2] the identity

$$\operatorname{cov}[X_j,\phi(\underline{X})] = p_j(1-p_j)\, E[\delta_j(\underline{X})], \quad j=1,2,\dots,n. \tag{3.1.3}$$

3.2 We define the <u>reliability importance</u> of c_j <u>for the functioning of</u> ϕ as

(3.2.1) $R_j(\phi,1;p) = P\{\phi(\underline{X}) = 1 \mid X_j=1;\ \underline{p}\} - P\{\phi(\underline{X})=1;\ \underline{p}\}$

and, similarly, the <u>reliability importance</u> of c_j <u>for failure of</u> ϕ as

(3.2.2) $R_j(\phi,0;\underline{p}) = P\{\phi(\underline{X}) = 0 \mid X_j=0;\ \underline{p}\} - P\{\phi(\underline{X}) = 0;\ \underline{p}\}$

and the <u>reliability importance</u> of c_j for ϕ as

(3.2.3) $R_j(\phi;\underline{p}) = R_j(\phi,1;\underline{p}) + R_j(\phi,0;\underline{p})$.

The following identities will be frequently used:

(3.2.4) $R_j(\phi,1;\underline{p}) = (1-p_j)\dfrac{\partial h(\underline{p})}{\partial p_j} = E[(1-X_j)\ \delta_j(\underline{X})]$

(3.2.5) $R_j(\phi,0;\underline{p}) = p_j\ \dfrac{\partial h(\underline{p})}{\partial p_j} = E[X_j\delta_j(\underline{X})]$

(3.2.6) $R_j(\phi;\underline{p}) = \dfrac{\partial h(\underline{p})}{\partial p_j} = E[\delta_j(\underline{X})]$

Proof: using (3.1.3)

$$P\{\Phi(\underline{X}) = 1 | X_j = 1\} = \frac{P\{\Phi(\underline{X}) = X_j = 1\}}{P\{X_j = 1\}} = \frac{E[X_j \Phi(\underline{X})]}{p_j} =$$

$$= \frac{\mathrm{cov}[X_j, \Phi(\underline{X})] + E(X_j)E[\Phi(\underline{X})]}{p_j} = (1-p_j)E[\delta_j(\underline{X})] + E(\Phi(\underline{X})]$$

and from (3.2.1) and (1.3.4) one obtains (3.2.4). A similar argument yields (3.2.5), and (3.2.6) follows by adding (3.2.4) and (3.2.5).

3.3 If nothing is known about the reliabilities of the components and, for lack of better knowledge, it is assumed that all vertices $\underline{x}$ are equally probable i.e. each has probability 2^{-n}, then (3.2.4), (3.2.5), and (3.2.6) reduce to (2.1.4), (2.1.5) and (2.1.6), the corresponding structural importances.

3.4 Examples

3.4.1 k-out-of-n structures.

For a k-out-of-n structure we have $\delta_j(\underline{x}) = \Phi(1_j, \underline{x}) - \Phi(0_j, \underline{x}) = 1$ if and only if exactly k-1 of the n-1 components different from c_j function. Therefore,

$$(3.4.1.1)\quad R_j(\Phi;\underline{p}) = \Sigma\; p_{j_1} p_{j_2} \cdots p_{j_{k-1}} (1-p_{j_k})(1-p_{j_{k+1}}) \cdots (1-p_{j_{n-1}})$$

where the sum is extended over all permutations $(j_1, j_2, \ldots, j_{n-1})$ of the subscripts $(1,2,\ldots,j-1,j+1,\ldots,n)$.

3.4.2 Parallel components.

For $k=1$ one obtains from (3.4.1.1)

$$(3.4.2.1)\qquad R_j(\Phi,\underline{p}) = \prod_{i \neq j} (1-p_i) = \frac{\prod_{i=1}^{n} (1-p_i)}{1-p_j}$$

hence

$$(3.4.2.2)\quad R_j(\Phi,1,\underline{p}) = \prod_{i=1}^{n} (1-p_i)$$

and

$$(3.4.2.3)\quad R_j(\Phi,0,\underline{p}) = p_j \prod_{i \neq j}^{n} (1-p_j) = \frac{p_j}{1-p_j} \prod_{i=1}^{n} (1-p_i).$$

From (3.4.2.1) one sees that the component with the greatest reliability p_j has the highest reliability importance; from (3.4.2.2), that all components are equally important for functioning; and from (3.4.2.3) that the component with the greatest reliability p_j has also the highest reliability importance for failure.

3.4.3 Components in series

One obtains from (3.4.1.1) for $k=n$

$$(3.4.3.1) \quad R_j(\phi,\underline{p}) = \prod_{i\neq j} p_i = \frac{1}{p_i} \prod_{i=1} p_i$$

hence

$$(3.4.3.2) \quad R_j(\phi,1,\underline{p}) = \frac{1-p_j}{p_j} \prod_{i=1}^{n} p_i .$$

and

$$(3.4.3.3) \quad R_j(\phi,0,\underline{p}) = \prod_{i=1}^{n} p_i .$$

One sees that here all components have the same reliability importance for failure, and that the most reliable component has the smallest reliability importance and the smallest reliability importance for functioning.

3.4.4 k components in series, in series with n-k in parallel.

For the structure function (2.2.2.1) one computes

$$R_j(\phi,\underline{p}) = \frac{1}{p_j} \prod_{r=1}^{k} p_r \, [1- \prod_{t=k+1}^{n} (1-p_t)], \qquad \text{if } j=1,2,\ldots,k,$$

(3.4.4.1)

$$R_j(\phi,\underline{p}) = \prod_{r=1}^{k} p_r \cdot \frac{1}{p_j} \prod_{t=k+1}^{n} (1-p_t), \qquad \text{if } j=k+1,\ldots,n,$$

and corresponding expressions are immediately obtained for $R(\phi,1,\underline{p})$ and $R(\phi,0,\underline{p})$.

The special case $k=1$ which corresponds to Figure 1 yields for $j=1$

$$R_1(\phi,\underline{p}) = 1 - \prod_{t=2}^{n} (1-p_t)$$

$$R_1(\phi,1,\underline{p}) = p_1[1- \prod_{t=2}^{n} (1-p_t)]$$

$$R_1(\phi,0,\underline{p}) = (1-p_1)[1- \prod_{t=2}^{n} (1-p_t)]$$

and for $j=2,\ldots,n$

$$R_j(\phi,\underline{p}) = \frac{p_1}{p_j} \prod_{t=k+1}^{n} (1-p_t)$$

$$R_j(\phi;1,\underline{p}) = p_1 \frac{1-p_j}{p_j} \prod_{t=k+1}^{n} (1-p_t)$$

$$R_j(\phi,0,\underline{p}) = p_1 \prod_{t=k+1}^{n} (1-p_t).$$

4. STRUCTURES WITH MODULES

4.1 In designing multi-component systems one often proceeds step-by-step, first constructing a system of fewer components and then replacing some of these components by sub-systems, known as <u>modules</u>, each consisting of several components. Properties of coherent systems constructed of coherent modules have been studied a.o. in [3]. For our present purpose we shall use the following definitions:

Let

(4.1.1) $\phi(\underline{x}) = \phi(x_1,x_2,\ldots,x_n) = x_1\delta_{x_1}(\phi;\underline{x}) + \mu_{x_1}(\phi;\underline{x})$

and

(4.1.2) $\Psi(\underline{y}) = \Psi(y_1, y_2, \ldots, y_m)$

be two coherent structures. We shall say that the structure

$$\chi(y_1, y_2, \ldots, y_m, x_2, \ldots, x_n) = \Phi[\Psi(y_1, \ldots, y_m), x_2, \ldots, x_n] =$$

(4.1.3)

$$= \Phi[\Psi_1(\underline{y}), \underline{x}] = \Psi(\underline{y})\delta_{x_1}[\,\Phi;\underline{x}\,] + \mu_{x_1}(\Phi;\underline{x})]$$

was obtained by replacing component x_1 in $\Phi(\underline{x})$ by the module $\Psi(\underline{y})$.

4.2 From (4.1.3) one obtains

$$\delta_{y_1}(\chi; y_1, \ldots, y_m, x_2, \ldots, x_n) = \chi(1, y_2, \ldots, y_m, x_2, \ldots x_n) -$$

$$-\chi(0, y_2, \ldots, y_m, x_2, \ldots x_n) = [\Psi(1, \underline{y}) - \Psi(0_1, \underline{y})]\delta_{x_1}(\Phi;\underline{x}) =$$

$$= \delta_{y_1}(\Psi;\underline{y}) \cdot \delta_{x_1}(\Phi;\underline{x}).$$

From the so obtained identity

(4.2.1) $\delta_{y_1}(\ ) = \delta_{x_1}(\Phi;\underline{x})\delta_{y_1}(\Psi;\underline{y})$

and from (3.2.6) follows

$$(4.2.2)\quad R_{y_1}(\mathcal{X};y_1,\ldots,y_m,x_2,\ldots,x_n) = R_{x_1}(\Phi;\underline{x})\cdot R_{y_1}(\Upsilon;\underline{y}).$$

This "chain-rule" property (which could also have been obtained by the chain rule for differentiation using $E[\delta_j(\underline{x})] = \frac{\partial h(\underline{p})}{\partial p_j}$) makes it possible to compute the importance of each component of a module Υ for the entire system $\mathcal{X}$, and to repeat this step-by-step as modules are substituted for components. The computation of $R_{y_1}(\mathcal{X},1;y_1,\ldots,y_m,x_2,\ldots,x_n)$ and of $R_{y_1}(\mathcal{X},0;y_1,\ldots,y_m,x_2,\ldots,x_n)$ is then a simple matter, according to (3.2.4) and (3.2.5).

5. AN APPLICATION

If components with known reliabilities $(p_1,\ldots,p_n) = \underline{p}$ are available, and the known structure $\Phi(\underline{x})$ has the reliability $h(\underline{p}) = E[\Phi(\underline{X});\underline{p}]$, then the problem may arise to decide on which components additional research and development should be done to improve their reliabilities, so that the greatest gain is achieved in system reliability.

Let us assume that improving the reliability of c_j from p_j to $p_j + \Delta_j$ can be achieved at cost $\lambda_j(p_j) \cdot \Delta_j$, for $j=1,\ldots,n$. In practical situations $\lambda_j(p_j)$ will be an increasing function, such that $\lambda_j(0) = 0$, $\lambda_j(p) \underset{p \to 1}{\to} \infty$. The total cost of improving all components will be

(5.1) $$C(\underline{p},\underline{\Delta}) = \sum_{j=1}^{n} \lambda_j(p_j)\Delta_j,$$

and the gain in system reliability per unit of cost

(5.2) $$\frac{h_\phi(\underline{p} + \underline{\Delta}) - h_\phi(\underline{p})}{C(\underline{p},\underline{\Delta})} .$$

We shall look for the direction of steepest ascent of this gain, in the following sense:

Let

(5.3) $$\Delta_j = \alpha_j t, \; j=1,2,\ldots,n$$

with

(5.4) $$\sum_{j=1}^{n} \alpha_j^2 = 1.$$

We wish to determine the direction cosines $\alpha_1,\ldots,\alpha_j,\ldots,\alpha_n$ so that, for all Δ_j small, (5.2) is maximized. Since (5.2) now is

$$\frac{h_\phi(\underline{p}+\underline{\alpha}t) - h_\phi(\underline{p})}{t\sum_{j=1}^{n}\lambda_j(p_j)\alpha_j} \xrightarrow[t\to 0]{} \frac{\frac{d}{dt}h(\underline{p}+\underline{\alpha}t)|_{t=0}}{\sum_{j=1}^{n}\lambda_j(p_j)\alpha_j}$$

our problem becomes to maximize

$$\frac{\sum_{j=1}^{n}\frac{\partial h(\underline{p})}{\partial p_j}\alpha_j}{\sum_{j=1}^{n}\lambda_j(p_j)\alpha_j} = \frac{\sum_{j=1}^{n}R_j(\phi,\underline{p})\,\alpha_j}{\sum_{j=1}^{n}\lambda_j(p_j)\alpha_j}$$

Under the restriction (5.4). It can be shown that, except for degenerate cases, the maximum is attained by selecting that component c_{j_o} for which the importance-to-cost ratio $R_j(\phi,\underline{p})/\lambda_j(p_j)$ is maximum, and setting $\alpha_{j_o}=1$, $\alpha_j=0$ for $j\neq j_o$.

6. CONCLUDING REMARKS

6.1 We have considered situations where only the structure function of $\phi(\underline{x})$ of the system was known, and situations where also the reliabilities $\underline{p} = (p_1,p_2,\ldots,p_n)$ of the components were known, and for each of these situations we proposed a quantitative definition of importance of components. A third possibility should be considered, when the coherent structure ϕ is known and each component c_i has a life length T_i, with a known probability distribution $F_i(t) = P\{T_i \leq t\}$. Under these assumptions the system has a life-length [4] T, with a probability distribution $P\{T \leq t\} = F(t)$ which depends on ϕ and on all the $F_i(T)$, $i=1,2,\ldots,n$. Again, intuitively some of the components are more important than others for the life distribution $F(t)$, and their importance depends on their location within the structure as well as on all the life distributions. To our knowledge, a study of the problem arising in this context has not even been initiated.

6.2 Under some circumstances, it may be of interest to consider a property of components which one could call the Bays'ean importance. For example, when a complicated system fails, it may be of interest to make a guess which component has "caused" the failure, and for this purpose one may consider the quantities

$$P\{X_j = 0 \mid \phi(\underline{X}) = 0\} , \quad j=1,2,\ldots,n.$$

These quantities indicate how "important" the different components are for the failure of the system. The mathematics of these quantities seems to be quite straightforward,

The author wishes to express his appreciation to Dr. J.D. Esary for many helpful discussions.

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Security Classification

DOCUMENT CONTROL DATA - R & D

Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Laboratory of Statistical Research Department of Mathematics University of Washington, Seattle, Wash.	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE
On the Importance of Different Components in a Multicomponent System

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Technical Report No. 54, May 20, 1968

5. AUTHOR(S) (First name, middle initial, last name)
Z. W. Birnbaum

6. REPORT DATE	7a. TOTAL NO OF PAGES	7b. NO OF REFS
May 20, 1968	20	4

8a. CONTRACT OR GRANT NO. N-onr-477(38) b. PROJECT NO. NR 042038 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 54 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

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Qualified requesters may obtain copies from DDC

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	U. S. Navy Office of Naval Research Washington, D.C.

13. ABSTRACT

In a system whose functioning or failure depends on the functioning or failure of its components, some components may play a more important part than others. A quantitative definition of this notion of importance is proposed in the present paper for systems with coherent structures, assuming (1) that only the structure of the system is known, or (2) that also the reliabilities of all components are known. Some theoretical properties of the so defined concepts are discussed. and applications are presented to such problems as allocation of spare parts or appropriation of funds for improvement of component reliability.

DD FORM 1 NOV 65 1473 (PAGE 1)
S/N 0101-807-6811

Security Classification

A-31408

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Reliability						
Coherent Structures						
Essentiality						
Modules						