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A COMPUTER PROGRAM FOR PHYSICAL-
OPTICS SCATTERING BY CONVEX
CONDUCTING TARGETS - 2430-7



J.H. Richmond

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ABSTRACT

This report describes a digital computer program which uses the physical optics approximation to calculate the scattering properties of convex perfectly conducting targets with arbitrary shape. The target shape is described in terms of the coordinates of a large number of points on the surface.

The program handles bistatic as well as backscattering problems. The input data specify the frequency, the incidence angles (θ_i, ϕ_i) and the scattering angles (θ_s, ϕ_s). In the CW case, the output data give the complex elements in the scattering matrix.

The program also handles the pulse case where the incident waveform has a finite number of cycles.

Graphs are included to illustrate typical results for the following target shapes: sphere, spheroids, ogive, and cone.

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A COMPUTER PROGRAM FOR PHYSICAL-OPTICS SCATTERING BY CONVEX CONDUCTING TARGETS

I. INTRODUCTION

Although the physical optics approximation for scattering by perfectly conducting targets has well-known limitations, it does provide rapid and efficient calculations which, in many cases, are reasonably accurate when the target is large in comparison with the wavelength.

In this report we present a digital computer program which uses the physical optics formulation to determine the scattering properties of perfectly conducting convex targets with arbitrary shape. Graphs are included to illustrate typical results for several target shapes.

The program is written in the Fortran IV language.

II. TARGET DESCRIPTION

The target shape is described in terms of the cartesian coordinates (x, y, z) of a large number of points on the surface. These points are selected on the intersections of the target surface and the planes $z = z_1, z = z_2, \text{ etc.}$ The coordinate origin is located in the interior of the target, and the surface is approximated by triangular facets with vertices at the given points. It is assumed that these points cover the surface with a density such that each facet is small in comparison with the wavelength. Experience indicates that reasonably good accuracy can be obtained with as few as 20 points per square wavelength on the target surface. The scattering data converge to the physical optics solution with approximately 80 points per square wavelength. Any further increase in the number of points will simply raise the computational time.

When used with an IBM 7094 computer, the program will handle up to 1300 points. Magnetic tape or disk storage could be utilized to extend this capability.

The meter is used as the unit of length for the input data.

Punched cards are used for input data with the computer program. Figure 1 illustrates suitable input data for a prolate spheroidal target with major and minor axis lengths of 2 meters and 1 meter, respectively. The first input data card gives an integer N which specifies the number of

planes $z = z_1, z = z_2, \dots, z = z_N$ employed to describe the target. The first line in Fig. 1 indicates that 30 planes are used for the spheroid. This first card is followed by N cards which list the z coordinates $ZI(I)$ of these planes and the number of points $NP(I)$ on each plane. These cards must be ordered with increasing values of the z coordinate. Thus, in Fig. 1, card 2 indicates that the first plane has $z = -1$ and there is only one point on this plane. Card 3 shows that the second plane is at $z = -0.98525$ and has 12 points. Finally, card 31 shows that the last plane is at $z = 1$ and has one point.

The remaining data cards (through 297) list the x and y coordinates of all the points on the target surface. These coordinates are given in pairs (x, y), with three pairs per card. For example, card 32 in Fig. 1 shows that the point on the first plane ($z = -1$) is at (0, 0). The next four cards (33-36) specify 12 points on the second plane, and card 297 locates the point on the last plane at (0, 0). The last cards (298-299) are described elsewhere in this report.

In the computer program, the coordinates x_k and y_k of point k on the plane $z = z_i$ are denoted by $F(K)$ and $G(K)$ or $FP(K)$ and $GP(K)$. The input data cards must give these points in an ordered manner, progressing in the clockwise direction (for a distant observer located on the negative z axis) around the contour on the plane $z = z_i$. Furthermore, the first given point on the plane $z = z_i$ should be the one nearest the first given point on the preceding plane.

The planes $z = z_i$ usually have unequal spacing. The spacing between these planes, and the number of points on each contour, should be designed to cover the target with a fairly uniform number of points per unit surface area.

The next section develops the physical-optics scattering equations used in the computer program.

III. THE PHYSICAL OPTICS FORMULATION

If the transmitting antenna is at a great distance from the target, it will illuminate the target with an incident field which is essentially a plane wave. In the CW case we let the time dependence $e^{j\omega t}$ be understood and represent the incident electric field intensity as follows:

$$(1) \quad \underline{E}^i = (\hat{\theta}_i E_\theta^i + \hat{\phi}_i E_\phi^i) e^{jk \hat{r}_i \cdot \underline{r}}$$

30						001
-1.00000	1					002
-0.98525	12					003
-0.94750	12					004
-0.89425	16					005
-0.83100	21					006
-0.76175	24					007
-0.68800	27					008
(CARDS 9 THROUGH 24 ARE NOT SHOWN)						
0.68725	27					025
0.76100	24					026
0.83025	21					027
0.89350	16					028
0.94674	12					029
0.98449	12					030
1.00000	1					031
0.	0.					032
0.08556	0.	0.07410	0.04278	0.04278	0.07410	033
0.00000	0.08556	-0.04278	0.07410	-0.07410	0.04278	034
-0.08556	0.00000	-0.07410	-0.04278	-0.04278	-0.07410	035
-0.00000	-0.08556	0.04278	-0.07410	0.07410	-0.04278	036
0.15988	0.	0.13846	0.07994	0.07994	0.13846	037
0.00000	0.15988	-0.07994	0.13846	-0.13846	0.07994	038
-0.15988	0.00000	-0.13846	-0.07994	-0.07994	-0.13846	039
-0.00000	-0.15988	0.07994	-0.13846	0.13846	-0.07994	040
0.22378	0.	0.20675	0.08564	0.15824	0.15824	041
0.08564	0.20675	0.00000	0.22378	-0.08564	0.20675	042
-0.15824	0.15824	-0.20675	0.08564	-0.22378	0.00000	043
-0.20675	-0.08564	-0.15824	-0.15824	-0.08564	-0.20675	044
-0.00000	-0.22378	0.08564	-0.20675	0.15824	-0.15824	045
0.20675	-0.08564					046
(CARDS 47 THROUGH 288 ARE NOT SHOWN)						
0.16099	0.	0.13942	0.08050	0.08050	0.13942	289
0.00000	0.16099	-0.08050	0.13942	-0.13942	0.08050	290
-0.16099	0.00000	-0.13942	-0.08050	-0.08050	-0.13942	291
-0.00000	-0.16099	0.08050	-0.13942	0.13942	-0.08050	292
0.08771	0.	0.07596	0.04385	0.04385	0.07596	293
0.00000	0.08771	-0.04385	0.07596	-0.07596	0.04385	294
-0.08771	0.00000	-0.07596	-0.04385	-0.04385	-0.07596	295
-0.00000	-0.08771	0.04385	-0.07596	0.07596	-0.04385	296
0.	0.					297
20.0	.0	20.0	.0			298
300.0	20	3				299

Fig. 1. Typical input data for the computer program.

where (r_i, θ_i, ϕ_i) are the spherical coordinates of the transmitting antenna, $(\hat{r}_i, \hat{\theta}_i, \hat{\phi}_i)$ are the corresponding unit vectors, *

$$(2) \quad k = 2\pi/\lambda,$$

λ denotes the wavelength, and E_θ^i and E_ϕ^i are complex constants. An arbitrary point on the target surface is assigned the coordinates (r, θ, ϕ) , the unit vectors $(\hat{r}, \hat{\theta}, \hat{\phi})$, and the position vector

$$(3) \quad \underline{r} = r \hat{r}$$

Finally, the receiving antenna is assigned the coordinates (r_s, θ_s, ϕ_s) and the unit vectors $(\hat{r}_s, \hat{\theta}_s, \hat{\phi}_s)$. Thus θ_i and ϕ_i specify the incidence angles and θ_s and ϕ_s are the scattering angles. The magnetic field intensity of the incident plane wave is given by

$$(4) \quad \underline{H}^i = (\hat{\theta}_i E_\phi^i - \hat{\phi}_i E_\theta^i) \frac{e^{jk \hat{r}_i \cdot \underline{r}}}{\eta}$$

where

$$(5) \quad \eta = \sqrt{\mu/\epsilon}.$$

The current density induced on the illuminated portion of the target surface is approximated as follows:

$$(6) \quad \underline{J} = 2\hat{n} \times \underline{H}^i$$

* The unit vectors in the spherical system are related to those in the rectangular system by $\hat{r} = (\hat{x}x + \hat{y}y + \hat{z}z)/r$, $\hat{\phi} = -\hat{x} \sin \phi + \hat{y} \cos \phi$ and $\hat{r} \times \hat{\theta} = \hat{\phi}$.

where \hat{n} denotes the outward unit normal vector on the surface. The vector potential for the distant scattered field is given by

$$(7) \quad \underline{A} = \frac{\mu}{4\pi r_s} e^{-jk r_s} \int \int \underline{J} e^{jk \hat{r}_s \cdot \underline{r}} ds .$$

At a great distance from the target, the scattered field is

$$(8) \quad \underline{E}^s = -j\omega \underline{A} = -\frac{j\omega\mu}{2\pi r_s} e^{-jk r_s} \int \int \hat{n} \times \underline{H}^i e^{jk \hat{r}_s \cdot \underline{r}} ds .$$

From Eqs. (4) and (8),

$$(9) \quad \underline{E}^s = \frac{e^{-jk r_s}}{r_s} (\hat{\theta}_i E_\phi^i - \hat{\phi}_i E_\theta^i) \times \underline{S}$$

where

$$(10) \quad \underline{S} = (j/\lambda) \int \int \hat{n} e^{jk(\hat{r}_i + \hat{r}_s) \cdot \underline{r}} ds .$$

The distant scattered field is represented by

$$(11) \quad \underline{E}^s = (\hat{\theta}_s E_\theta^s + \hat{\phi}_s E_\phi^s) \frac{e^{-jk r_s}}{r_s}$$

where E_θ^s and E_ϕ^s denote complex constants. From Eqs. (9) and (11) and the following vector identity,

$$(12) \quad \underline{A} \cdot (\underline{B} \times \underline{C}) = (\underline{A} \times \underline{B}) \cdot \underline{C} ,$$

it is found that

$$(13) \quad E_\theta^s = (E_\theta^i \hat{\phi}_i \times \hat{\theta}_s + E_\phi^i \hat{\theta}_s \times \hat{\theta}_i) \cdot \underline{S}$$

and

$$(14) \quad \underline{E}_{\phi}^s = (\underline{E}_{\theta}^i \hat{\phi}_i \times \hat{\phi}_s + \underline{E}_{\phi}^i \hat{\phi}_s \times \hat{\theta}_i) \cdot \underline{S} \quad .$$

It is convenient to define the CW scattering matrix as follows:

$$(15) \quad \begin{pmatrix} \underline{E}_{\theta}^s \\ \underline{E}_{\phi}^s \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} \underline{E}_{\theta}^i \\ \underline{E}_{\phi}^i \end{pmatrix}$$

From Eqs. (13) through (15), the complex elements in the scattering matrix are given by

$$(16) \quad S_{11} = (\hat{\phi}_i \times \hat{\theta}_s) \cdot \underline{S}$$

$$(17) \quad S_{12} = (\hat{\theta}_s \times \hat{\phi}_i) \cdot \underline{S}$$

$$(18) \quad S_{21} = (\hat{\phi}_i \times \hat{\phi}_s) \cdot \underline{S}$$

$$(19) \quad S_{22} = (\hat{\phi}_s \times \hat{\theta}_i) \cdot \underline{S} \quad .$$

It is convenient to define a "vector area function" $\underline{A}(w)$ as follows:

$$(20) \quad \underline{S} = (j/\lambda) \int \underline{A}(w) e^{jkw} dw \quad ,$$

where

$$(21) \quad w = (\hat{r}_i + \hat{r}_s) \cdot \underline{r}$$

It may be seen from Eq. (21) that w represents one of the coordinates of a point on the target surface, in a rectangular coordinate system that is rotated in space with respect to the (x, y, z) system. The w axis is coplanar with \hat{r}_i and \hat{r}_s and bisects the angle between these unit vectors. The unit of length for the w coordinate generally differs from that for $x, y,$ and z .

For any point (x, y, z) on the target surface,

$$(22) \quad w = (\sin \theta_i \cos \phi_i + \sin \theta_s \cos \phi_s)x + (\sin \theta_i \sin \phi_i + \sin \theta_s \sin \phi_s)y \\ + (\cos \theta_i + \cos \theta_s)z \quad .$$

Once the vector area function has been calculated, \underline{S} can be determined efficiently from Eq. (20) by numerical integration. Equation (10) would take more computation time since it involves a surface integral instead of a line integral.

Once \underline{S} has been calculated, the elements in the scattering matrix are determined as follows:

$$(23) \quad S_{11} = S_x X_{11} + S_y Y_{11} + S_z Z_{11}$$

$$(24) \quad S_{12} = S_x X_{12} + S_y Y_{12} + S_z Z_{12}$$

$$(25) \quad S_{21} = S_z Z_{21}$$

$$(26) \quad S_{22} = S_x X_{22} + S_y Y_{22} + S_z Z_{22}$$

where

$$(27) \quad \underline{S} = \hat{x} S_x + \hat{y} S_y + \hat{z} S_z$$

$$(28) \quad X_{11} = - \cos \phi_i \sin \theta_s$$

$$(29) \quad Y_{11} = - \sin \phi_i \sin \theta_s$$

$$(30) \quad Z_{11} = - (\sin \phi_i \sin \phi_s + \cos \phi_i \cos \phi_s) \cos \theta_s$$

$$(31) \quad X_{12} = \cos \theta_i \sin \phi_i \sin \theta_s - \sin \theta_i \cos \theta_s \sin \phi_s$$

$$(32) \quad Y_{12} = \sin \theta_i \cos \theta_s \cos \phi_s - \cos \theta_i \cos \phi_i \sin \theta_s$$

$$(33) \quad Z_{12} = (\sin \phi_i \cos \phi_s - \cos \phi_i \sin \phi_s) \cos \theta_i \cos \theta_s$$

$$(34) \quad Z_{21} = \cos \phi_i \sin \phi_s - \sin \phi_i \cos \phi_s$$

$$(35) \quad X_{22} = - \sin \theta_i \cos \phi_s$$

$$(36) \quad Y_{22} = - \sin \theta_i \sin \phi_s$$

$$(37) \quad Z_{22} = - (\cos \phi_i \cos \phi_s + \sin \phi_i \sin \phi_s) \cos \theta_i \quad .$$

In the pulse case, the incident wave is considered to have M complete cycles and a square modulation envelope. M is assumed to be an integer, and the incident field is expressed by

$$(38) \quad \underline{E}^i = (\hat{\theta}_i E_{\theta}^i + \hat{\phi}_i E_{\phi}^i) \sin (\omega t + k \hat{r}_i \cdot \underline{r}) \\ \cdot [u(\omega t + k \hat{r}_i \cdot \underline{r}) - u(\omega t + k \hat{r}_i \cdot \underline{r} - \omega \tau)]$$

where E_{θ}^i and E_{ϕ}^i are real constants, $u(x)$ denotes the unit step function, f is the carrier frequency,

$$(39) \quad \omega = 2\pi f \quad ,$$

$$(40) \quad T = 1/f \quad ,$$

and

$$(41) \quad \tau = MT \quad .$$

The scattered field is given by

$$(42) \quad \underline{E}^s(r_s, t) = [\hat{\theta}_s E_{\theta}^s(t - r_s/c) + \hat{\phi}_s E_{\phi}^s(t - r_s/c)] \frac{1}{r_s} ,$$

where c denotes the speed of light in free space .

It is convenient to write the following matrix equation

$$(43) \quad \begin{pmatrix} E_{\theta}^s(t) \\ E_{\phi}^s(t) \end{pmatrix} = \begin{pmatrix} F_{11}(t) & F_{12}(t) \\ F_{21}(t) & F_{22}(t) \end{pmatrix} \begin{pmatrix} E_{\theta}^i \\ E_{\phi}^i \end{pmatrix} .$$

The pulse response of the target is thus defined with four functions of time given by

$$(44) \quad F_{11}(t) = (\hat{\phi}_i \times \hat{\theta}_s) \cdot \underline{F}(t)$$

$$(45) \quad F_{12}(t) = (\hat{\theta}_s \times \hat{\theta}_i) \cdot \underline{F}(t)$$

$$(46) \quad F_{21}(t) = (\hat{\phi}_i \times \hat{\phi}_s) \cdot \underline{F}(t)$$

$$(47) \quad F_{22}(t) = (\hat{\phi}_s \times \hat{\theta}_i) \cdot \underline{F}(t)$$

where

$$(48) \quad \underline{F}(t) = \frac{1}{\lambda} \int \int \hat{n} \cos(\omega t + kw) \cdot [u(\omega t + kw) - u(\omega t + kw - \omega \tau)] ds$$

Equation (48) can also be written as follows

$$(49) \quad \underline{F}(t) = \frac{1}{\lambda} \int \underline{A}(w) \cos(\omega t + kw) [u(\omega t + kw) - u(\omega t + kw - \omega \tau)] dw .$$

Equation (49) is employed in the computer program since it permits more rapid calculations than Eq. (48).

The scattering waveforms $F_{11}(t)$, $F_{12}(t)$, $F_{21}(t)$ and $F_{22}(t)$ are calculated at a discrete set of equally spaced points in time. The spacing between these points is determined by an integer L , included in the input data, as follows

$$(50) \quad \Delta T = T/L \quad .$$

The first and last points coincide with the initiation and the termination of the pulse response.

It must be noted that the above formulation for the pulse response assumes the incident plane wave has linear polarization. However, it appears that the more general situation could be programmed quite readily.

The physical-optics scattering program is described in some detail in the following sections.

IV. APPROXIMATING THE TARGET WITH A POLYHEDRON

Figure 2 shows the first 91 statements or cards in the computer program. Card 26 reads the integer N which specifies the number of planes used in describing the target surface. Card 29 reads the z coordinate $ZI(I)$ for each of these planes and the number of points $NP(I)$ on each plane. The dimension statements reserve core storage for 200 planes, 500 points on each plane, and a total of 1300 points.

Card 37 reads the x and y coordinates of the points on the first plane, and card 48 reads the coordinates of the remaining points.

In addition to reading the target description, this first section of the program sets up a polyhedral approximation for the target. This polyhedron has many triangular facets, with vertices at the given points. If each facet is small in comparison with the wavelength, the scattering properties of the polyhedron will be nearly the same as for the real target of interest.

\$EXECUTE	IBJOB	001
\$IBJOB	MAP	002
\$IBFTC	PHYSOP NODECK	003
21	FORMAT(1X,DE15.7)	004
22	FORMAT(1X,8F15.7)	005
23	FORMAT(1H0)	006
24	FORMAT(1X,10F12.4)	007
25	FORMAT(8F9.3)	008
26	FORMAT(7F10.5)	009
27	FORMAT(7I10)	010
28	FORMAT(1H1)	011
29	FORMAT(1F10.5,6I10)	012
31	FORMAT(1X,7H TH1 =,F10.4,7H PHI =,F10.4,7H THS =,F10.4,	013
	27H PHS =,F10.4/)	014
32	FORMAT(1X,5HFMC =,F10.4,8H WAVE =,F15.7/)	015
33	FORMAT(1X,5HS11 =,2F15.7,7H S12 =,2F15.7,7H S21 =,2F15.7/	016
	27H S22 =,2F15.7/)	017
34	FORMAT(1X,6HEA11 =,F15.7,8H EA12 =,F15.7,8H EA21 =,F15.7,8H EA2	018
	22 =,F15.7/)	019
	DIMENSION AX(500),AY(500),AZ(500)	020
	DIMENSION X(2600),Y(2600),Z(2600),XN(2600),YN(2600),ZN(2600)	021
	DIMENSION F(500),G(500),FP(500),GP(500),ZI(200),NP(200)	022
	COMPLEX SX,SY,SZ,S11,S12,S21,S22,QA,QB,QC,QD,QK,QL	023
	DATA TP,FP1,RAD/6,2831853,12,56637,0,01745329/	024
1	J=0	025
	READ(5,27)N	026
	WRITE(6,27)N	027
	DO 60 I=1,N	028
60	READ(5,29)ZI(I),NP(I)	029
	FN=N	030
	M=NP(1)	031
	MM=M+1	032
	ZA=ZI(1)	033
	NC=(M+2)/3	034
	DO 80 I=1,NC	035
	K=3*I-2	036
80	READ(5,26)F(K),G(K),F(K+1),G(K+1),F(K+2),G(K+2)	037
	F(MM)=F(1)	038
	G(MM)=G(1)	039
	IF(M.EQ.1) MM=1	040
	DO 150 11=2,N	041
	MP=NP(11)	042
	MPP=MP+1	043
	ZC=ZI(11)	044
	NC=(MP+2)/3	045
	DO 90 I=1,NC	046
	K=3*I-2	047
90	READ(5,26)FP(K),GP(K),FP(K+1),GP(K+1),FP(K+2),GP(K+2)	048
	FP(MPP)=FP(1)	049
	GP(MPP)=GP(1)	050
	IF(MP.EQ.1) MPP=1	051
	L=1	052
	LP=1	053
95	XA=F(L)	054
	XC=FP(LP)	055
	YA=G(L)	056
	YC=GP(LP)	057
	J=J+1	058
	IF(L.EQ.MM) GO TO 98	059
	IF(LP.EQ.MPP) GO TO 100	060
	RA=(FP(LP+1)-F(L))**2+(GP(LP+1)-G(L))**2	061

Fig. 2. First section of the computer program.

	RB=(FP(LP)-F(L+1))**2+(GP(LP)-G(L+1))**2	062
	IF(RA.GT.RB) GO TO 100	063
98	LP=LP+1	064
	XB=FP(LP)	065
	YB=GP(LP)	066
	ZB=ZC	067
	GO TO 110	068
100	L=L+1	069
	XB=F(L)	070
	YB=G(L)	071
	ZB=ZA	072
110	X(J)=(XA+XB+XC)/3.	073
	Y(J)=(YA+YB+YC)/3.	074
	Z(J)=(ZA+ZB+ZC)/3.	075
	XN(J)=((YB-YA)*(ZC-ZA)-(ZB-ZA)*(YC-YA))*0.5	076
	YN(J)=((ZB-ZA)*(XC-XA)-(XB-XA)*(ZC-ZA))*0.5	077
	ZN(J)=((XB-XA)*(YC-YA)-(YB-YA)*(XC-XA))*0.5	078
	IF(L.LT.MM.OR.LP.LT.MPP) GO TO 95	079
	DO 120 I=1,MPP	080
	F(I)=FP(I)	081
120	G(I)=GP(I)	082
	M=MP	083
	MM=MPP	084
	ZA=ZC	085
150	CONTINUE	086
	NT=J	087
	DZ=.0	088
	DO 155 I=2,N	089
	DEL=Z1(I)-Z1(I-1)	090
155	IF(DEL.GT.DZ)DZ=DEL	091

Fig. 2. First section of the computer program. (cont.)

Since each facet is assumed to be small, the only parameters that must be determined and stored for each one are its vector area and the coordinates of its midpoint.

Each triangle is assigned an index number denoted by J . The coordinates of a point at the center of triangle J are denoted by $X(J)$, $Y(J)$ and $Z(J)$. The cartesian components of the vector area (using the outward normal direction) of triangle J are $XN(J)$, $YN(J)$ and $ZN(J)$. The dimension statements reserve core storage for these parameters for 2600 triangular facets. The number of triangles is approximately twice the number of points on the target surface, and is denoted by NT .

The technique for fitting a polyhedron to a given array of points is illustrated in Fig. 3. The computer first processes all the triangular facets on the first "zone" of the surface (i.e., the portion of

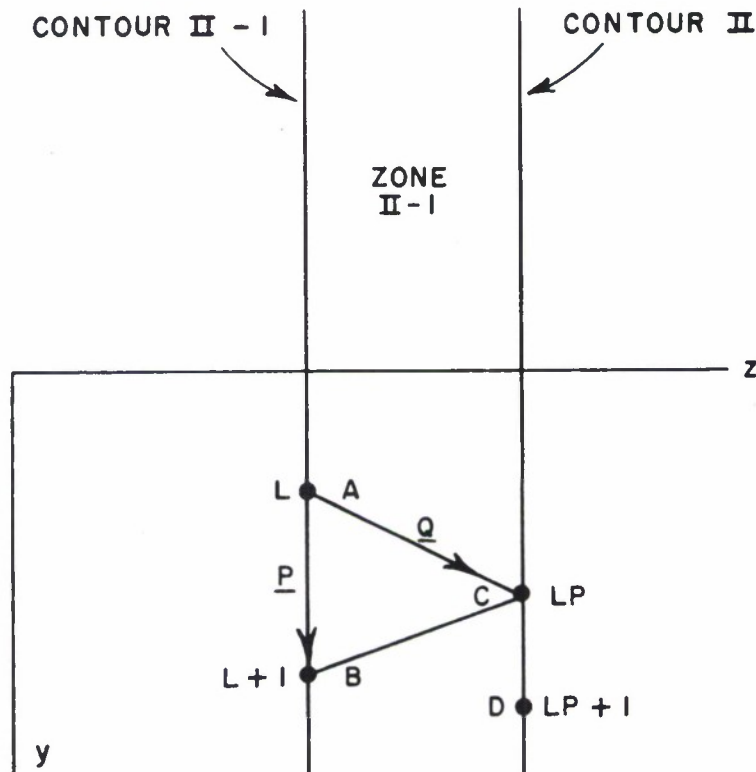


Fig. 3. A typical triangular facet on the polyhedron.

the surface bounded by the first two planes $z = z_1$ and $z = z_2$). It then proceeds to the next zone. In the situation illustrated in Fig. 3, the computer is identifying and processing the triangles in Zone II-1. The x and y coordinates of the points on contour II-1 are denoted by F and G, and those on contour II by FP and GP. The first triangle on this zone has one vertex at the point $x = F(1)$, $y = G(1)$ and $z = ZI(II-1)$. Another vertex is at $x = FP(1)$, $y = GP(1)$ and $z = ZI(II)$. The third vertex of this first triangle is at $(F(2), G(2))$ or $(FP(2), GP(2))$, which ever yields the "most compact" triangle.

When the computer has completed the processing of a given triangle in a given zone, two vertices on the next triangle are predetermined since they coincide with two of the vertices on the last completed triangle. In Fig. 3, these two vertices are indicated by points A and C. Point A is the L-th point on contour II-1 and its coordinates are $XA=F(L)$, $YA=G(L)$ and $ZA=ZI(II-1)$. Point C is the LP-th point on contour II and its coordinates are $XC = FP(LP)$, $YC = GP(LP)$ and $ZC = ZI(II)$. To establish the third point on the triangle, the computer compares the lengths of the lines AD and BC and selects the shorter one to form one side of the triangle. As shown in Fig. 3, B denotes the (L+1)-th point on contour II-1, and D is the (LP+1)-th point on contour II.

For the situation shown in Fig. 3, BC is shorter than AD. Therefore, B is selected as the third vertex of the triangle and assigned the following coordinates: $XB = F(L+1)$, $YB = G(L+1)$ and $ZB = ZA$. The coordinates of the midpoint of this triangle are calculated in cards 73, 74 and 75. The vector area of the triangle is given by $(\underline{P} \times \underline{Q})/2$, where the vector \underline{P} extends from A to B and the vector \underline{Q} extends from A to C. This calculation is arranged by cards 76, 77 and 78 to determine the rectangular components of the vector area.

Having completed the processing of triangle J, the computer proceeds with the next triangle in a similar fashion.

When the computer finishes the first section of the program, it has in storage a complete list of the midpoint coordinates and the vector areas of all of the triangular facets on the polyhedron.

V. COMPUTING THE VECTOR AREA FUNCTION

The second section of the computer program reads the coordinates (θ_i, ϕ_i) of the transmitting antenna and (θ_s, ϕ_s) of the receiving antenna

and calculates the corresponding vector area function $\underline{A}(w)$ for the target. This part of the program is shown in Fig. 4.

Card 92 reads $\theta_i, \phi_i, \theta_s$ and ϕ_s . These angles are denoted by THI, PHI, THS and PHS. They must be given in degrees (rather than radians) in the input data. For example, card 298 in Fig. 1 assigns a value of 20 degrees for θ_i and θ_s , and zero degrees for ϕ_i and ϕ_s .

In the DO LOOP in cards 115 through 137, the computer scans the w coordinates for all the illuminated facets on the polyhedron to determine the maximum and minimum values of w on the illuminated portion of the target surface. These are designated WMAX and WMIN. WX, WY and WZ represent the coefficients of x, y and z in Eq. (22).

Cards 138 through 147 calculate the coefficients X_{11}, Y_{11} , etc., defined by Eqs. (28) through (37).

The rectangular components of the vector area function $\underline{A}(w)$ are denoted by AX(K), AY(K) and AZ(K). The program is designed to calculate several values of AX, AY and AZ, representing samples of the functions for uniformly spaced points on the w axis. The dimension statements allow for a maximum of 500 sampling points. The area function is nonzero only for values of w between WMIN and WMAX. Therefore, this portion of the w axis is divided into KX segments and the sampling points are selected at the centers of these segments. Thus, AX(1), AY(1) and AZ(1) represent the components of $\underline{A}(w)$ at the center of the first segment (i.e., the segment which begins at WMIN). Likewise, AX(KX), AY(KX) and AZ(KX) represent the components of $\underline{A}(w)$ at the center of the last segment (which terminates at WMAX). The length of each segment (and the spacing between the sampling points) is denoted by DEL. WI and WF denote the values of w at the first and last sampling points.

If the number of sampling points is too small, the area function will not be represented adequately and one cannot expect accurate scattering data. Therefore, the program always calculates the largest number of samples consistent with the input data. To obtain a more detailed area function for a given target, one simply supplies a more detailed description of the target via the input data.

The following expression for the vector area function can be obtained by comparing Eqs. (10) and (20):

$$(51) \quad \underline{A}(w) = \frac{d}{dw} \int \int \hat{A} \, ds \quad .$$

The significance of Eq. (51) can be clarified by considering the equivalent finite-difference approximation:

$$(52) \quad \underline{A}(K) = \frac{1}{DEL} \int \int_{S_k} \hat{n} \, ds \quad ,$$

where $\underline{A}(K)$ represents the area function at the point $w = w_k$ and S_k denotes a narrow zone on the illuminated portion of the target surface bounded by the planes $w = w_k - DEL/2$ and $w = w_k + DEL/2$.

For the polyhedron with small facets, Eq. (52) takes the following form:

$$(53) \quad \underline{A}(K) = \sum \frac{\hat{x} XN(I) + \hat{y} YN(I) + \hat{z} ZN(I)}{DEL}$$

The summation indicated in Eq. (53) is programmed in cards 162 through 199 in Fig. 4. In Eq. (53) it is understood that the summation extends only over the illuminated facets with w coordinates in the range $w_k \pm DEL/2$.

A simple shadowing test is programmed in cards 163 and 164. RDTN represents the quantity $\hat{A}_i \cdot \hat{n}$. RDTN is calculated for each facet on the polyhedron. The facet is considered illuminated or shadowed according as RDTN is positive or negative. This test is, of course, adequate only for convex targets.

Figure 5 shows the vector area components $AX(w)$ and $AZ(w)$ for a prolate spheroid. $AY(w)$ is zero for this problem. The input data in Fig. 1 were used for these calculations.

Although independent data are not available for comparison with the curves in Fig. 5, there are two indications that these curves are reliable. First, these area functions lead to accurate physical-optics scattering data. Second, the area functions calculated in the same manner show close agreement with known results for the following cases: backscatter and bistatic scattering from spheres and axial backscatter from spheroids, ogives and cones.

2	READ(5,26)THI,PHI,THS,PHS	092
	WRITE(6,31)THI,PHI,THS,PHS	093
	STHI=SIN(RAD*THI)	094
	CTHI=COS(RAD*THI)	095
	STHS=SIN(RAD*THS)	096
	CTHS=COS(RAD*THS)	097
	SPHI=SIN(RAD*PHI)	098
	CPHI=COS(RAD*PHI)	099
	SPHS=SIN(RAD*PHS)	100
	CPHS=COS(RAD*PHS)	101
	SSCS=STHS*CPHS	102
	SSSS=STHS*SPHS	103
	SICI=STHI*CPHI	104
	SISI=STHI*SPHI	105
	WX=SICI+SSCS	106
	WY=SISI+SSSS	107
	WZ=CTHI+CTHS	108
	DEL=DZ*SQRT(WX**2+WY**2+WZ**2)	109
	WMAX=-1000000.	110
	WMIN=1000000.	111
	K=2	112
	ZA=ZI(1)	113
	ZC=ZI(2)	114
	DO 200 I=1,NT	115
	RDTN=XN(1)*SICI+YN(1)*SISI+ZN(1)*CTHI	116
	IF(RDTN.LE.0.) GO TO 200	117
	ZNI=ZN(1)	118
	ZB=Z(1)	119
	IF(ZC.GE.ZB)GO TO 170	120
165	K=K+1	121
	ZA=ZI(K-1)	122
	ZC=ZI(K)	123
	IF(ZC.LT.ZB)GO TO 165	124
170	DEN=X(1)*XN(1)+Y(1)*YN(1)	125
	D=DEN+ZB*ZNI	126
	F=(D-ZA*ZNI)/DEN	127
	WA=X(1)*F*WX+Y(1)*F*WY+ZA*WZ	128
	F=(D-ZC*ZNI)/DEN	129
	WC=X(1)*F*WX+Y(1)*F*WY+ZC*WZ	130
	IF(WC.GE.WA)GO TO 175	131
	WW=WA	132
	WA=WC	133
	WC=WW	134
175	IF(WC.GT.WMAX)WMAX=WC	135
	IF(WA.LT.WMIN)WMIN=WA	136
200	CONTINUE	137
	X11=-CPHI*STHS	138
	Y11=-SPHI*STHS	139
	Z11=- (SPHI*SPHS+CPHI*CPHS)*CTHS	140
	X12=CTHI*SPHI*STHS-STHI*CTHS*SPHS	141
	Y12=STHI*CTHS*CPHS-CTHI*CPHI*STHS	142
	Z12=(-CPHI*SPHS+SPHI*CPHS)*CTHI*CTHS	143
	Z21=-SPHI*CPHS+CPHI*SPHS	144
	X22=-STHI*CPHS	145
	Y22=-STHI*SPHS	146
	Z22=- (CPHI*CPHS+SPHI*SPHS)*CTHI	147
	DO 205 K=1,500	148
	AX(K)=0.0	149
	AY(K)=0.0	150
205	AZ(K)=0.0	151
	KX=(WMAX-WMIN)/DEL	152

Fig. 4. Second section of the computer program.

FK=KX	153
DEL=(WMAX-WMIN)/FK	154
WRITE(6,22)DZ,DEL,WMIN,WMAX	155
DELT=DEL/2.	156
WI=WMIN+DELT	157
WF=WMAX-DELT	158
K=2	159
ZA=Z1(1)	160
ZC=Z1(2)	161
DO 230 I=1,NT	162
RDTN=XN(I)*SICI+YN(I)*SISI+ZN(I)*CTHI	163
IF(RDTN.LE.0.) GO TO 230	164
ZNI=ZN(I)	165
ZB=Z(I)	166
IF(ZC.GE.ZB)GO TO 215	167
210 K=K+1	168
ZA=Z1(K-1)	169
ZC=Z1(K)	170
IF(ZC.LT.ZB)GO TO 210	171
215 DEN=X(I)*XN(I)+Y(I)*YN(I)	172
D=DEN+ZB*ZNI	173
F=(D-ZA*ZNI)/DEN	174
WA=X(I)*F*WX+Y(I)*F*WY+ZA*WZ	175
F=(D-ZC*ZNI)/DEN	176
WC=X(I)*F*WX+Y(I)*F*WY+ZC*WZ	177
IF(WC.GE.WA)GO TO 218	178
WW=WA	179
WA=WC	180
WC=WW	181
218 KA=(WA-WI)/DEL+1.5	182
KC=(WC-WI)/DEL+1.5	183
IF(KA.NE.KC)GO TO 220	184
AX(KA)=AX(KA)+XN(I)	185
AY(KA)=AY(KA)+YN(I)	186
AZ(KA)=AZ(KA)+ZN(I)	187
GO TO 230	188
220 FKA=KA-1	189
W=FKA*DEL+WI+DELT	190
D=(W-WA)/(WC-WA)	191
AX(KA)=AX(KA)+XN(I)*D	192
AY(KA)=AY(KA)+YN(I)*D	193
AZ(KA)=AZ(KA)+ZN(I)*D	194
F=1.-D	195
AX(KC)=AX(KC)+XN(I)*F	196
AY(KC)=AY(KC)+YN(I)*F	197
AZ(KC)=AZ(KC)+ZN(I)*F	198
230 CONTINUE	199
W=WI	200
DO 235 K=1,KX	201
AX(K)=AX(K)/DEL	202
AY(K)=AY(K)/DEL	203
AZ(K)=AZ(K)/DEL	204
FK=K	205
WRITE(6,22) FK,W,AX(K),AY(K),AZ(K)	206
235 W=W+DEL	207
AX(KX+1)=2.*AX(KX)-AX(KX-1)	208
AY(KX+1)=2.*AY(KX)-AY(KX-1)	209
AZ(KX+1)=2.*AZ(KX)-AZ(KX-1)	210
WRITE(6,23)	211

Fig. 4. Second section of the computer program. (cont.)

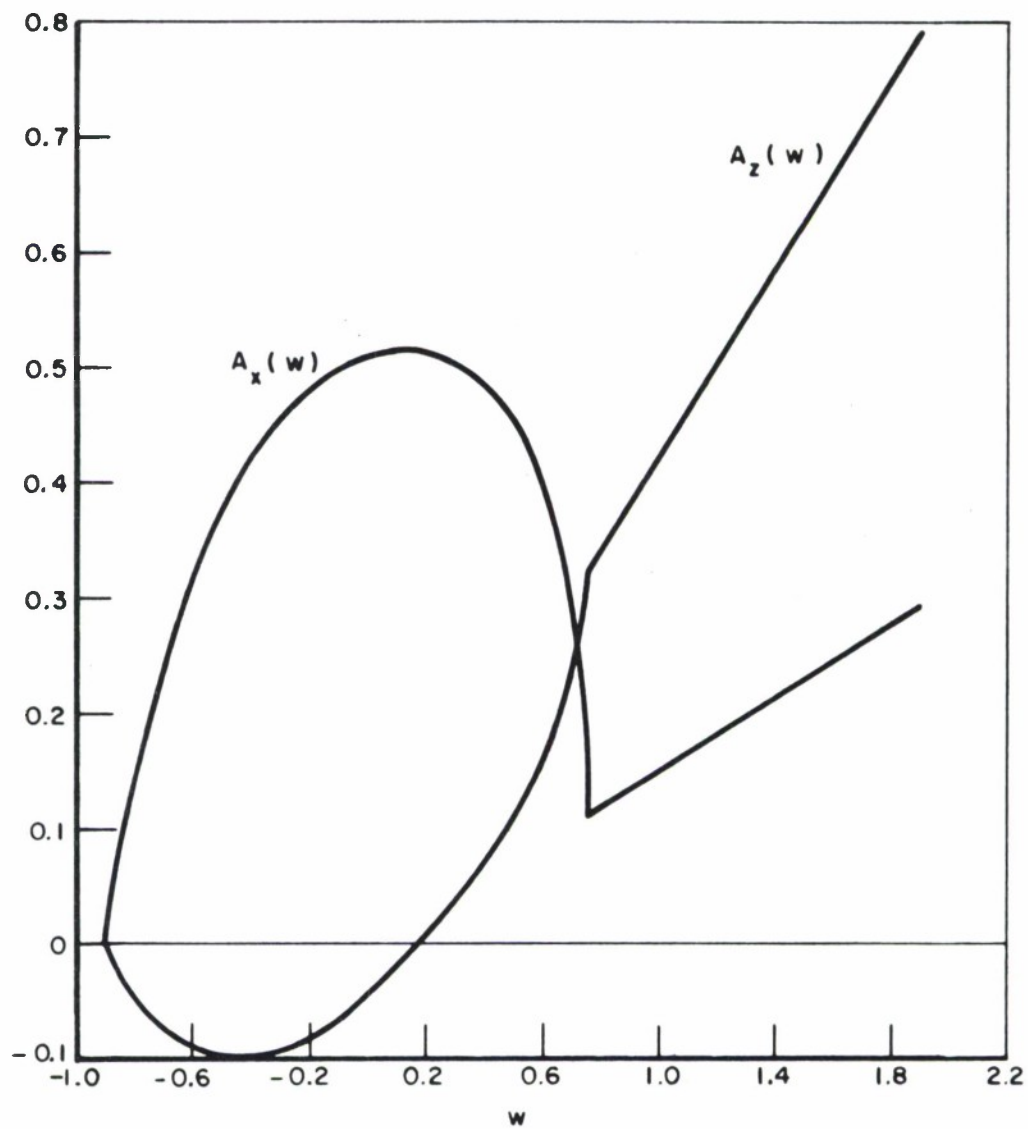


Fig. 5. Calculated area functions for a prolate spheroid using input data from Fig. 1.

VI. COMPUTING THE CW SCATTERING MATRIX

The third section of the computer program, shown in Fig. 6, reads the frequency in megahertz and calculates the CW scattering matrix and the corresponding echo areas. In addition to the frequency, card 212 also reads the integers L and M which are defined in the next section. L and M are not needed for the CW calculations and could just as well be read later at card 249. Suitable input data for card 212 are shown in card 299 in Fig. 1.

This section of the program calculates the vector \underline{S} defined by

$$(54) \quad \underline{S} = (j/\lambda) \int_{w_{\min}}^{w_{\max}} \underline{A}(w) e^{jkw} dw \quad .$$

The integral in Eq. (54) is evaluated with the "piecewise linear method" described in Reference 1. We integrate over segment k on the w axis and then sum on k. Segment k has length DEL and is centered at $w = w_k$. On segment k a straight-line approximation is used for the area function:*

$$(55) \quad \underline{A}(w) = \underline{a}_k + \underline{b}_k w \quad .$$

By fitting Eq. (55) to the stored samples $\underline{A}(K)$ at w_k and $\underline{A}(K+1)$ at w_{k+1} , we find that

$$(56) \quad \underline{a}_k = [\underline{A}(K) w_{k+1} - \underline{A}(K+1) w_k] / \text{DEL}$$

and

$$(57) \quad \underline{b}_k = [\underline{A}(K+1) - \underline{A}(K)] / \text{DEL} \quad .$$

* When k is used as a subscript it represents the index of a segment on the w axis and is denoted in the program by K. When used otherwise, as in Eqs. (54) and (59), k is defined by Eq. (2) and is denoted in the program by TPL.

The integration in Eq. (54) can be performed analytically when $\underline{A}(w)$ is given by Eq. (55). This leads to the following expression

$$(58) \quad \underline{S} = \frac{1}{2\pi DG} \sum_{k=1}^{KX} [QK \underline{A}(K) + QL \underline{A}(K+1)] ,$$

where

$$(59) \quad DG = k \text{ DEL} ,$$

$$(60) \quad QK = (GL - j) QD - QC ,$$

$$(61) \quad QL = QC - (GK - j) QD ,$$

$$(62) \quad QC = GB e^{j GB} - GA e^{j GA} ,$$

$$(63) \quad QD = e^{j GB} - e^{j GA} ,$$

$$(64) \quad GA = k(w_k - \text{DEL}/2) ,$$

$$(65) \quad GB = k(w_k + \text{DEL}/2) ,$$

$$(66) \quad GK = k w_k ,$$

and

$$(67) \quad GL = k w_{k+1} .$$

The summation in Eq. (58) is programmed in cards 223 through 236. The rectangular components of \underline{S} are denoted by SX, SY and SZ.

Equations (23) - (26) are programmed in cards 238 - 241 to calculate the complex elements in the scattering matrix. This scattering matrix is the principal output of this section of the program. With it available, one can use Eq. (15) to calculate the scattering properties of the target for any given incident polarization.

The remaining output from this section does not necessarily have any significance except in special cases. In backscatter problems, however, EA11 represents the echo area in square meters. Furthermore, EA11 and EA22 represent the E-plane and H-plane echo areas when $\phi_i = \phi_s$ and the target has axial symmetry with respect to the z axis.

Using the techniques described above and the input data shown in Fig. 1, the computer yields the following CW scattering data for the prolate spheroid at 300 MHz:

$$S_{11} = S_{22} = -0.1191 + j 0.0637$$

$$S_{12} = S_{21} = 0$$

$$EA_{11} = EA_{22} = 0.229$$

$$EA_{12} = EA_{21} = 0$$

Although the piecewise-linear integration technique is efficient and successful for most problems, it is not appropriate in forward scattering problems (where $w = 0$ everywhere on the target) or in specular scattering from a flat region on the target (where w is constant over a significant portion of the surface). Therefore, the computer output data are not reliable in these cases.

VII . COMPUTING THE PULSE RESPONSE

The pulse scattering problem is treated in the fourth and last section of the program shown in Fig. 7. The entire program is ready for the computer when the four sections are put together (with the card numbers running sequentially from 1 through 345) and backed up with suitable input data (such as cards 1 through 299 in Fig. 1).

To explain the pulse calculations, it is convenient to write Eq. (49) in the following form

3	READ(5,29)FMC,L,M	212
	WAVE=299.79/FMC	213
	WRITE(6,32) FMC,WAVE	214
	TPL=TP/WAVE	215
	SX=(.0,.0)	216
	SY=(.0,.0)	217
	SZ=(.0,.0)	218
	GK=TPL*W1	219
	GA=TPL*WMIN	220
	DG=TPL*DEL	221
	QA=(MPLX(COS(GA),SIN(GA)))	222
	DO 240 K=1,KX	223
	GB=GA+DG	224
	QB=(MPLX(COS(GB),SIN(GB)))	225
	GL=GK+DG	226
	QC=GB*QB-GA*QA	227
	QD=QB-QA	228
	QK=QD*GL-QC-(.0,1.)*QD	229
	QL=QC-QD*GK+(.0,1.)*QD	230
	SX=SX+QK*AX(K)+QL*AX(K+1)	231
	SY=SY+QK*AY(K)+QL*AY(K+1)	232
	SZ=SZ+QK*AZ(K)+QL*AZ(K+1)	233
	GA=GB	234
	GK=GL	235
240	QA=QB	236
	DEN=TP*DG	237
	S11=(SX*X11+SY*Y11+SZ*Z11)/DEN	238
	S12=(SX*X12+SY*Y12+SZ*Z12)/DEN	239
	S21=SZ*Z21/DEN	240
	S22=(SX*X22+SY*Y22+SZ*Z22)/DEN	241
	WRITE(6,33) S11,S12,S21,S22	242
	EA11=(CABS(S11)**2)*FP1	243
	EA12=(CABS(S12)**2)*FP1	244
	EA21=(CABS(S21)**2)*FP1	245
	EA22=(CABS(S22)**2)*FP1	246
	WRITE(6,34) EA11,EA12,EA21,EA22	247
	WRITE(6,23)	248

Fig. 6. Third section of the computer program.

$$(68) \quad \underline{F}(t) = \frac{1}{2\pi} \left[\cos \omega t \int_{x_a}^{y_a} \underline{A}(g) \cos g \, dg - \sin \omega t \int_{x_a}^{y_a} \underline{A}(g) \sin g \, dg \right] ,$$

where

$$(69) \quad g = kw \quad ,$$

$$(70) \quad x_a = GN \text{ or } -\omega t \text{ (whichever is larger)} \quad ,$$

$$(71) \quad y_a = GX \text{ or } \omega \tau - \omega t \text{ (whichever is smaller)} \quad ,$$

$$(72) \quad GN = k \text{ WMIN},$$

and

$$(73) \quad GX = k \text{ WMAX} \quad .$$

The area function $\underline{A}(g)$ is the same as $\underline{A}(w)$, and it has already been calculated and stored in AX(K), AY(K) and AZ(K).

The integer L determines the number of points per cycle to be calculated on the scattering waveforms. Card 254 calculates the size of the increments in ωt as follows:

$$(74) \quad DWT = 2\pi/L \quad .$$

The integer M specifies the number of complete cycles in the incident waveform. The pulse width of the incident waveform, as measured in radians and observed at an arbitrary point in space, is calculated by card 253 as follows:

$$(75) \quad \omega \tau = WTAU = 2\pi M \quad .$$

The scattering function $\underline{F}(t)$ is nonzero only in the following interval:
 $-GX < \omega t < \omega\tau - GN$.

It is convenient to assign symbols to the integrals in Eq. (68) as follows:

$$(76) \quad \underline{U}(t) = \int_{x_a}^{y_a} \underline{A}(g) \cos g \, dg,$$

and

$$(77) \quad \underline{V}(t) = \int_{x_a}^{y_a} \underline{A}(g) \sin g \, dg .$$

These are functions of time by virtue of the time dependent limits x_a and y_a . In the program the rectangular components of $\underline{U}(t)$ are denoted by UX, UY and UZ, and the components of $\underline{V}(t)$ are VX, VY and VZ. WT, SWT and CWT denote ωt , $\sin \omega t$ and $\cos \omega t$.

$\underline{U}(t)$ and $\underline{V}(t)$ are identically zero for all values of ωt less than or equal to $-GX$. Thus, the computer begins the pulse-response calculations by setting $\omega t = -GX$ (card 261), initializing $\underline{U}(t)$ and $\underline{V}(t)$ to zero (cards 269 - 274) and writing out the first data point on the scattering waveform (card 268). At this point in time, the integration limits are found from Eqs. (70) and (71) to be $x_a = y_a = GX$.

Suppose the computer has just completed the calculation of the scattering data for a given value of ωt , has incremented ωt in card 277, and is preparing to calculate the next point on the scattering waveform. The first step in the procedure is to determine the integration limits x_a and y_a , denoted by XA and YA. This is accomplished in cards 281 - 292 in accordance with Eqs. (70) and (71). These cards also determine the indices KA and LA of the segments on the g axis which contain the points $g = XA$ and $g = YA$, respectively.

The integrals $\underline{U}(t')$ and $\underline{V}(t')$ currently in storage have limits now denoted by XB and YB by virtue of cards 338 and 339. Since XA and YA are equal to or less than XB and YB, respectively,

$$(78) \quad \underline{U}(t) = \underline{U}(t') + \int_{XA}^{XB} \underline{A}(g) \cos g \, dg - \int_{YA}^{YB} \underline{A}(g) \cos g \, dg$$

and

$$(79) \quad \underline{V}(t) = \underline{V}(t') + \int_{XA}^{XB} \underline{A}(g) \sin g \, dg - \int_{YA}^{YB} \underline{A}(g) \sin g \, dg \quad .$$

Calculations based on Eqs. (78) and (79) are, of course, far more efficient than Eqs. (76) and (77).

Equations (78) and (79) are programmed in cards 294 - 329. The piecewise-linear technique is employed to evaluate the integrals in these equations. The integrals with limits XA and XB are evaluated in the first pass through the DO LOOP that begins with card 299, and those with limits YA and YB are calculated in the second pass. In each pass, ZA and ZB denote the limits of integration. Initially I denotes the index number of the g-axis segment containing the point $g = ZA$ and G is the value of g at the center of this segment. $\underline{A}(g)$ is treated as a linear function over this segment in the same manner as in Eqs. (55) through (57). GP denotes the value of g at the center of segment I + 1, and HA and HB are the limits of the subintegral on segment I. When the integration on segment I is finished, the computer moves on to the next segment (unless ZA and ZB lie on the same segment). The following equations are used in cards 310 - 319 to evaluate the subintegral on segment I:

$$(80) \quad \int_{h_a}^{h_b} \underline{A}(g) \cos g \, dg = \frac{1}{DG} [CI \underline{A}(I) + CII \underline{A}(I+1)] \quad ,$$

$$(81) \quad \int_{h_a}^{h_b} \underline{A}(g) \sin g \, dg = \frac{1}{DG} [DI \underline{A}(I) + DII \underline{A}(I+1)] \quad ,$$

$$(82) \quad CI = (h_a - g_{i+1}) \sin h_a + (g_{i+1} - h_b) \sin h_b + \cos h_a - \cos h_b \quad ,$$

$$(83) \quad CII = (g_i - h_a) \sin h_a + (h_b - g_i) \sin h_b + \cos h_b - \cos h_a \quad ,$$

$$(84) \quad DI = (g_{i+1} - h_a) \cos h_a + (h_b - g_{i+1}) \cos h_b + \sin h_a - \sin h_b \quad ,$$

and

$$(85) \quad DII = (h_a - g_i) \cosh h_a + (g_i - h_b) \cosh h_b + \sinh h_b - \sinh h_a .$$

DG is defined in Eq. (59). In the program, g_i , g_{i+1} , h_a and h_b are denoted by G, GP, HA and HB. SGN is positive in the first pass through this integration loop and negative in the second pass, in accordance with the plus and minus signs in Eqs. (78) and (79).

When the vectors $\underline{U}(t)$ and $\underline{V}(t)$ have been calculated, Eq. (68) is used to determine $\underline{F}(t)$. The rectangular components of $\underline{F}(t)$ are denoted by FX, FY and FZ in cards 330-332. Finally cards 333 - 336 calculate the scattering functions defined by Eqs. (43) through (47), and card 337 arranges the writeout of these data on the scattering waveforms for the pulse situation.

The results obtained in this manner (using the input data in Fig. 1) for a prolate spheroid are illustrated in Fig. 8.

VIII. CONCLUSIONS

A digital computer program is described which uses the physical optics approximation for scattering by convex perfectly conducting targets with arbitrary shape. The target size and shape are specified by input data giving the coordinates of many points on the surface. This program handles bistatic and backscatter problems. The input data specify the frequency, the incidence angles (θ_i, ϕ_i) and the scattering angles (θ_s, ϕ_s). In the CW case, the output data give the complex elements in the scattering matrix. Typical results are shown in the Appendix for scattering from spheres, spheroids, ogives and cones.

This scattering program also handles the pulse case where the incident wave has a finite number of cycles. For this case, the following additional input data are required: the number of cycles in the incident wavetrain, and the number of points per cycle to be computed for the scattering waveforms. The output data give the four waveforms associated with the four elements in the scattering matrix.

Simple modifications can be made in the program, if desired, to increment the frequency, the incidence angles or the scattering angles. This is convenient for generating various types of scattering curves such as those in the Appendix.

4	CONTINUE	249
	LM=L*M	250
	FL=L	251
	FM=M	252
	WTAU=TP*FM	253
	DWT=TP/FL	254
	IF(LM.EQ.0)GO TO 300	255
	DGT=TPL*DELT	256
	G1=TPL*WI	257
	GX=TPL*WMAX	258
	GN=TPL*WMIN	259
	DEN=TP*DG	260
	WTA=-GX	261
	WTB=WTAU-GN	262
	WT=WTA	263
	F11=0.0	264
	F12=0.0	265
	F21=0.0	266
	F22=0.0	267
	WRITE(6,22) WT,F11,F12,F21,F22	268
	UX=.0	269
	UY=.0	270
	UZ=.0	271
	VX=.0	272
	VY=.0	273
	VZ=.0	274
	XB=GX	275
	YB=GX	276
260	WT=WT+DWT	277
	IF(WT.GT.WTB) GO TO 300	278
	SWT=SIN(WT)	279
	CWT=COS(WT)	280
	XA=GN	281
	KA=1	282
	XX=-WT	283
	IF(XX.LE.XA) GO TO 265	284
	XA=XX	285
	KA=(XA-G1)/DG+1.5	286
265	YA=GX	287
	LA=KX	288
	YY=WTAU-WT	289
	IF(YY.GE.YA) GO TO 268	290
	YA=YY	291
	LA=(YA-G1)/DG+1.5	292
268	SGN=1.	293
	ZA=XA	294
	ZB=XB	295
	I=KA	296
	F=I-1	297
	G=F*DG+G1	298
	DO 290 K=1,2	299
	IF(ZA.GE.ZB) GO TO 285	300
270	GP=G+DG	301
	HA=G-DGT	302
	HB=G+DGT	303
	IF(HA.LT.ZA) HA=ZA	304
	IF(HB.GT.ZB) HB=ZB	305
	SA=SIN(HA)	306
	SB=SIN(HB)	307
	CA=COS(HA)	308
	CB=COS(HB)	309
	C1=(SA*(HA-GP)+SB*(GP-HB))+CA-CB)*SGN	310

Fig. 7. Last section of the computer program.

	C11=(SA*(G-HA)+SB*(HB-G)+CB-CA)*SGN	J11
	UX=UX+C1*AX(I)+C11*AX(I+1)	312
	UY=UY+C1*AY(I)+C11*AY(I+1)	313
	UZ=UZ+C1*AZ(I)+C11*AZ(I+1)	314
	D1=(SA-SB+CA*(GP-HA)+CB*(HB-GP))*SGN	315
	D11=(SB-SA+CA*(HA-G)+CB*(G-HB))*SGN	316
	VX=VX+D1*AX(I)+D11*AX(I+1)	317
	VY=VY+D1*AY(I)+D11*AY(I+1)	318
	VZ=VZ+D1*AZ(I)+D11*AZ(I+1)	319
	IF(HB*GE*ZB) GO TO 285	320
	I=I+1	321
	G=GP	322
	GO TO 270	323
285	ZA=YA	324
	ZB=YB	325
	I=LA	326
	F=I-1	327
	G=F*DG+GI	328
290	SGN=-1.	329
	FX=(UX*CWT-VX*SWT)/DEN	330
	FY=(UY*CWT-VY*SWT)/DEN	331
	FZ=(UZ*CWT-VZ*SWT)/DEN	332
	F11=FX*X11+FY*Y11+FZ*Z11	333
	F12=FX*X12+FY*Y12+FZ*Z12	334
	F21=FZ*Z21	335
	F22=FX*X22+FY*Y22+FZ*Z22	336
	WRITE(6,22) WT,F11,F12,F21,F22	337
	XB=XA	338
	YB=YA	339
	GO TO 260	340
300	CONTINUE	341
	WRITE(6,23)	342
	STOP	343
	END	344
\$DATA		345

Fig. 7. Last section of the computer program. (cont.)

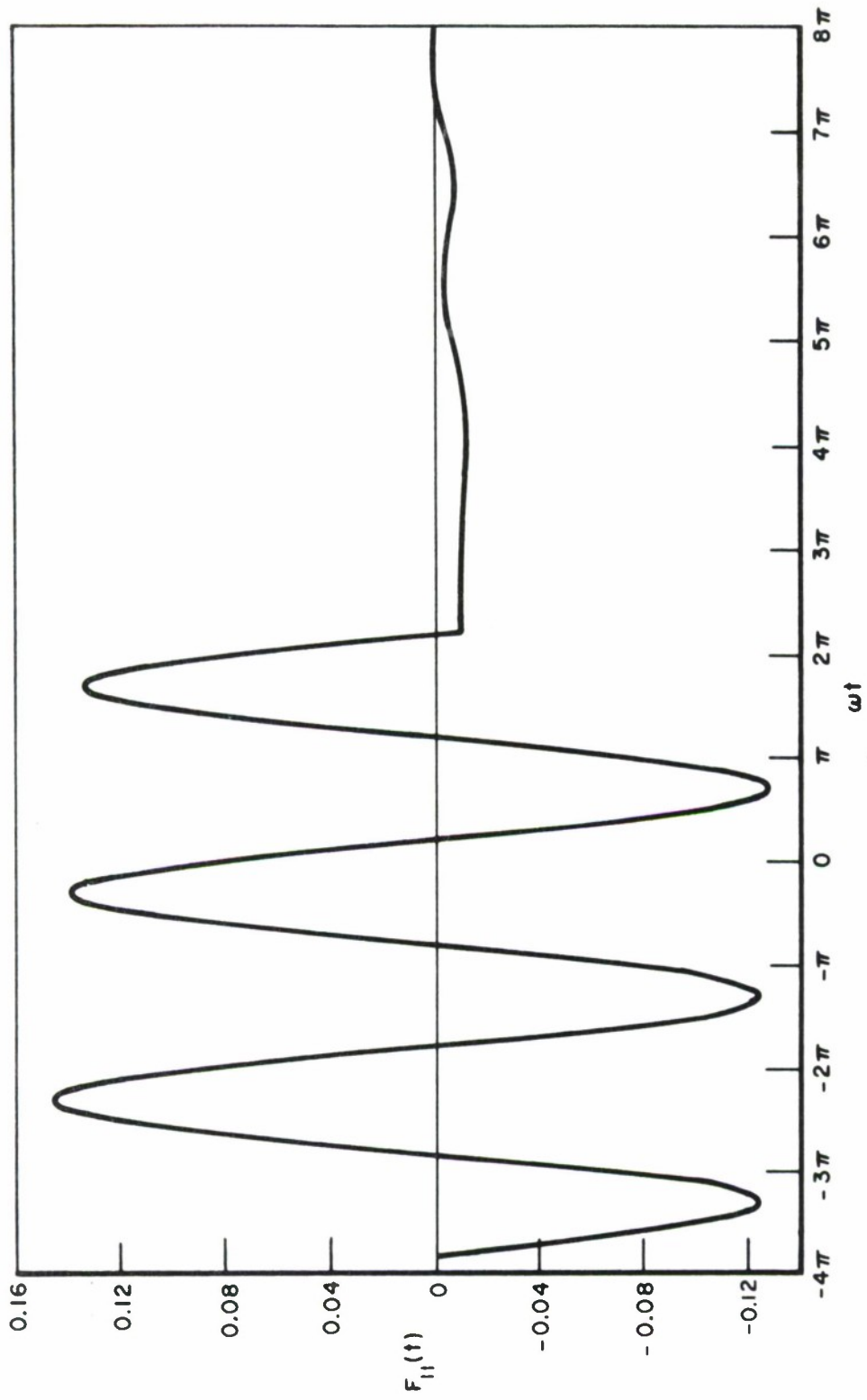


Fig. 8. Calculated pulse response for prolate spheroid, using input data from Fig. 1.

REFERENCES

1. Richmond, J.H., "The Numerical Evaluation of Radiation Integrals," IRE Transactions, Vol. AP-9, (July 1961), pp. 358-360.
2. Siegel, K.M., et al, "Studies in Radar Cross-Sections VIII - Theoretical Cross-Sections as a Function of Separation Angle Between Transmitter and Receiver at Small Wavelengths," UMM-115, Willow Run Research Center, University of Michigan, (October 1953).

APPENDIX
NUMERICAL RESULTS

This Appendix presents the results obtained with the physical-optics computer program for several target shapes. Table I lists the number of planes used to describe each target, the total number of points on the target, and the time required for an IBM 7094 to process the target description data and set up the polyhedral approximation. The last column lists the additional computer time needed to calculate the area function and the CW scattering matrix.

TABLE I
STATISTICS FOR THE TEST CASES

<u>Target Shape</u>	<u>Planes</u>	<u>Points</u>	<u>Time for Polyhedron</u>	<u>Area Function and CW Matrix</u>
Sphere	30	1083	8.6 sec	1.6 sec
Prolate Spheroid	30	774	6.4	1.3
Oblate Spheroid	30	1257	9.9	1.9
Ogive	50	1008	8.2	1.8
Cone	35	1176	9.8	3.3

The dimensions of the targets are specified in Figs. 9 through 13.

It is found from Table I that the computation time for processing the target description data and setting up the polyhedron increases linearly with the number of points on the target surface. The computer handles about 125 points (to establish 250 facets) per second, regardless of the shape of the target.

In generating the area function and the CW scattering matrix, the computer spends very little time on the shadowed facets. Thus, the computation time for this work increases linearly with the number of illuminated facets on the target. This explains why the computer spent twice as much time on the cone (3.3 seconds) as on the sphere (1.6 seconds) as shown in Table I. This sphere has approximately 2000 facets, and half

of them are illuminated. The cone also has approximately 2000 facets, and all of them are illuminated for axial incidence.*

In calculating the area function, the computer handles about 670 illuminated facets per second.

To use the computer program most efficiently, one would calculate a significant amount of scattering data for one target before passing on to the next one. Furthermore, to minimize the area-function calculations, one would calculate the scattering data for many different frequencies before changing θ_i , ϕ_i , θ_s or ϕ_s . When these incidence and scattering angles are held constant, the computer generates CW scattering data for 20 different frequencies in one second for the oblate spheroid in Table I, and for 10 frequencies per second for the cone. For the sphere, it generates CW and pulse-response data for a new frequency in one second (with three cycles in the incident wavetrain and 20 calculated points per cycle in the scattering waveform).

Figures 9 through 13 display the numerical results for scattering by several different target shapes. The dots represent the output data from the computer program described herein. The input data listed the coordinates of a large number of points (Table I) distributed almost uniformly over the target surface. The source of the physical-optics data (the solid curves in Figs. 9 - 13) is described a little farther on in this Appendix.

Figures 9a through 13a show curves of backscatter echo area (in square meters) versus frequency for axial incidence. For each of these targets, it may be noted that the computer output agrees closely with the physical optics solution for all frequencies up to 600 MHz. (This simply indicates that the computer program does a good job in generating physical optics data. It is not implied that the physical optics formulation always gives a good approximation to the exact scattering data.) To obtain equally good results at higher frequencies, it would be necessary to include more points in the input data description of the target.

* For all the targets listed in Table I except the cone, the input data points covered the whole surface and the computer set up a closed polyhedron. For the cone, however, the input data did not cover the flat plate at the base. Since the base is shadowed for all cases shown in Fig. 13, the computed data apply to a cone with a circular disk at the base.

Although the computer program has not been adequately tested for the most general target shapes and bistatic situations for which it is designed, it is believed that it will give accurate physical-optics data except for concave targets, forward scattering, and specular scattering from a large flat subsurface. The program could be modified without much difficulty to remove the last two of these exceptions.

All of the targets considered in this Appendix have axial symmetry with respect to the z axis. To check the accuracy of the computer output data shown in Figs. 9a - 13a, closed-form expressions were derived for the vector area function for each of these targets for the backscatter situations with axial incidence. Letting $\theta_i = \phi_i = \theta_s = \phi_s = 0$, we find from Eq. (22) that $w = 2z$. From symmetry considerations, $A_x(w) = A_y(w) = 0$. For targets having axial symmetry with respect to the z axis, it is convenient to describe the target shape with an equation of the form $\rho = \rho(z)$ where (ρ, ϕ, z) represent the coordinates in the cylindrical system. Then it is easy to show, with the aid of Eq. (51), that

$$(84) \quad A_z(w) = -\pi \rho \rho' ,$$

where ρ' represents the derivative of $\rho(z)$ with respect to z . Equation (84) applies over the illuminated region of the target where ρ' is negative, and $A_z(w) = 0$ over the shadowed region where ρ' is positive.

All the targets considered in this Appendix can be represented with the following equation:

$$(85) \quad F(\rho, z) = A_1 \rho^2 + A_2 z^2 + A_3 \rho z + A_4 z + A_5 \rho + A_6 = 0 .$$

From Eqs. (84) and (85), the area function for axial backscatter is given in closed form by

$$(86) \quad A_z = \pi \rho (2A_2 z + A_3 \rho + A_4) / (2A_1 \rho + A_3 z + A_5) .$$

For spheres, prolate spheroids and oblate spheroids,

$$(87) \quad A_z = \frac{\pi z B^2}{A^2}$$

where A and B denote the semimajor and semiminor axes, respectively. For the ogive,

$$(88) \quad A_z = \frac{\pi \rho z}{\sqrt{B^2 - z^2}},$$

where B is the radius of the circular arc that generates the ogive.

For the cone,

$$(89) \quad A_z = \pi z \tan^2 \alpha,$$

where α denotes the half-angle of the cone.

For axial backscatter from a target with axial symmetry, it is found from Eq. (54) that $S_x = S_y = 0$ and

$$(90) \quad S_z = (2j/\lambda) \int A_z e^{2jkz} dz.$$

The integral in Eq. (90) can be evaluated to obtain closed-form expressions for the scattering matrix and the echo area for the cone and the spheroids. The resulting data are shown by the solid curves with the label "physical optics" in Figs. 9a, 10a, 11a and 13a. For the ogive in Fig. 12a, the "physical optics" curve was obtained from Eqs. (88) and (90) with numerical integration.

Figure 9b shows backscatter results for the sphere as a function of the aspect angle. For a true sphere, the echo area is of course independent of aspect angle. Thus, the variations in the computer output data arise from the polyhedral approximation.

Figures 10b - 13b show the backscatter results as a function of the aspect angle ($\theta_i = \theta_s = \theta$) for the spheroids, ogive and cone. The solid curves with the label "physical optics" were obtained with another computer program which is designed especially for backscattering from targets with axial symmetry. The input data for this auxiliary program describes the target shape by assigning numerical values to the coefficients

in Eq. (85). The angles ϕ_i and ϕ_s are assumed to be zero, the cylindrical coordinates (ρ, ϕ, z) are employed for points on the target surface, and

$$(91) \quad w = 2(\rho \sin \theta \cos \phi + z \cos \theta).$$

It is found that $S_y = 0$,

$$(92) \quad \underline{S} = (j/\lambda) \int \int (\hat{x} \cos \phi - \hat{z} \rho') \rho e^{jk w} d\phi dz ,$$

$$(93) \quad S_{11} = S_{22} = -S_x \sin \theta - S_z \cos \theta ,$$

and $S_{12} = S_{21} = 0$. The auxiliary program uses numerical integration to evaluate \underline{S} in Eq. (92).

Bistatic scattering results are shown in Figs. 9c - 13c and 9d - 13d. These curves apply for axial incidence with $\theta_i = \phi_i = \phi_s = 0$, so that

$$(94) \quad w = \rho \sin \theta_s \cos \phi + z(1 + \cos \theta_s) ,$$

$$(95) \quad S_x = -k \int \rho J_1(k \rho \sin \theta_s) e^{jkz(1 + \cos \theta_s)} dz ,$$

$$(96) \quad S_y = S_{12} = S_{21} = 0 ,$$

$$(97) \quad S_z = -jk \int \rho \rho' J_0(k \rho \sin \theta_s) e^{jkz(1 + \cos \theta_s)} dz ,$$

$$(98) \quad S_{11} = -S_x \sin \theta_s - S_z \cos \theta_s ,$$

$$(99) \quad S_{22} = -S_z ,$$

and J_0 and J_1 represent Bessel functions.

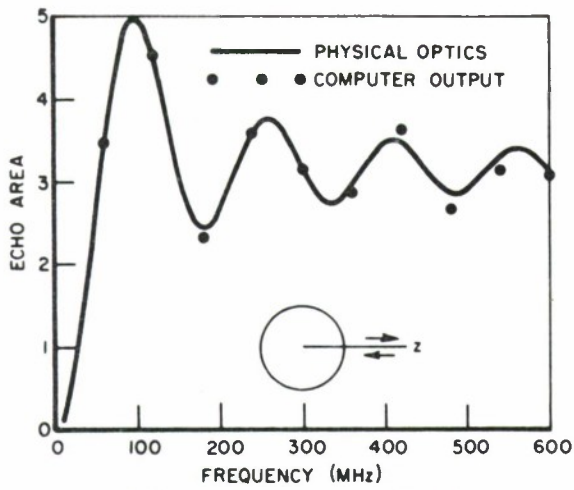
Equations (94) - (99) apply only for axial incidence on a target having axial symmetry. (Similar expressions are given in Reference 2.) These equations were used to develop another auxiliary computer program to generate the E-plane and H-plane curves with the label "physical optics". This program also used the coefficients in Eq. (85) as input data for the target shape, and evaluated S_x and S_z with numerical integration.

The bistatic curves in Figs. 9 - 13 agree with the physical-optics echo area expression for the forward-scattering situation:

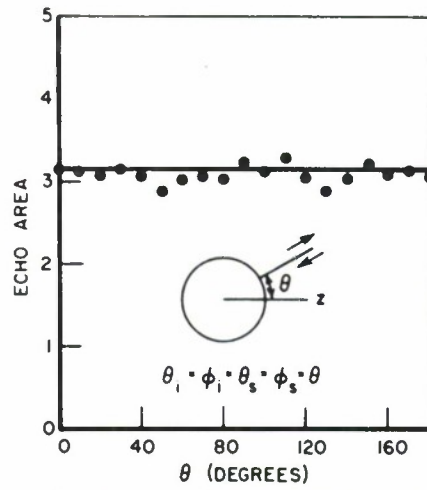
$$(100) \quad \sigma = 4\pi A^2 / \lambda^2 \quad ,$$

where A represents the area of the target projected along the line of sight between the transmitting and receiving antennas. In the forward-scattering situation (and also for backscattering), the physical optics echo area shows no polarization dependence even for a target with arbitrary shape. Equation (100) is not restricted to symmetrical targets or axial incidence.

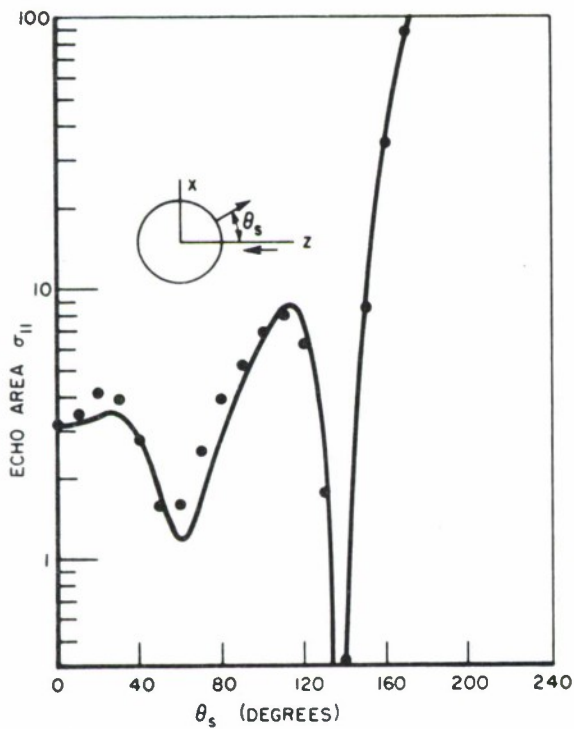
Though all the data in Figs. 9 - 13 were obtained with the aid of various digital computer programs based on the physical-optics concepts, the close agreement between the dots (with the label "computer output") and the curves (with the label "physical optics") is significant for two reasons. First, the general program uses a target description technique which differs greatly from that used in the specialized programs. Furthermore, some of the specialized programs used closed-form expressions and others were based on equations involving line integrals or surface integrals, whereas the general program handles each problem in the same manner (setting up the polyhedron, calculating the vector area function, and integrating the area function to obtain the scattering matrix). Having demonstrated that this general procedure gives correct physical-optics data for a variety of target shapes and bistatic and backscatter situations, we have confidence that the program will perform equally well with other targets and situations.



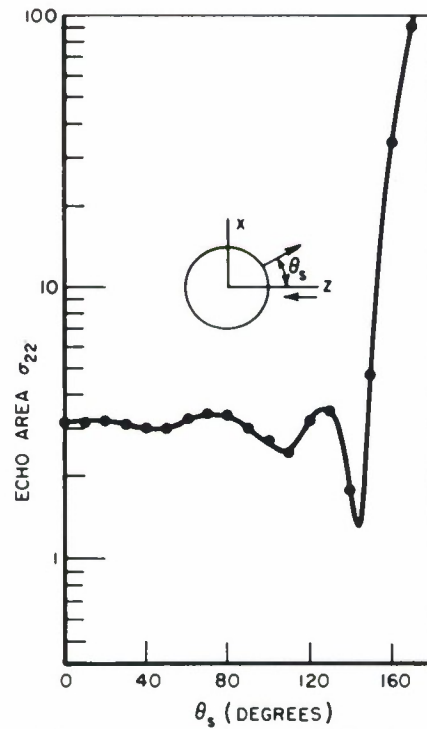
(a) BACKSCATTER vs. FREQUENCY



(b) BACKSCATTER vs. ASPECT AT 300 MHz

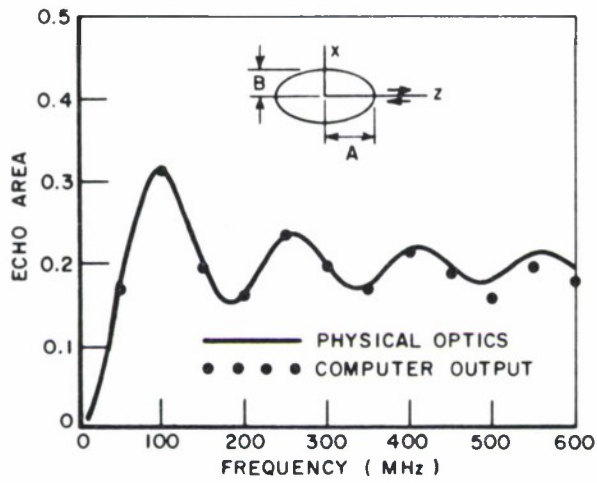


(c) E-PLANE BISTATIC SCATTERING AT 300MHz

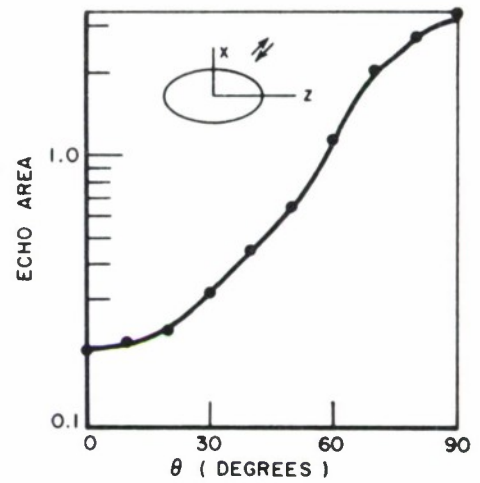


(d) H-PLANE BISTATIC SCATTERING AT 300 MHz

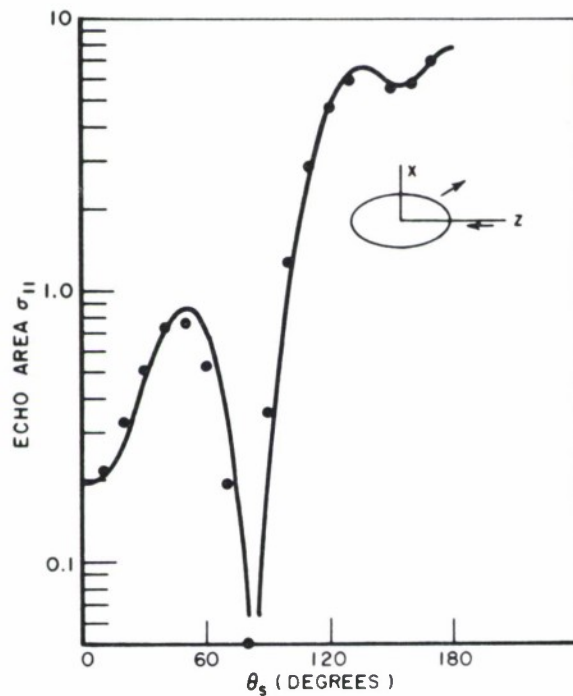
Fig. 9. Scattering data for a sphere with a radius of one meter.



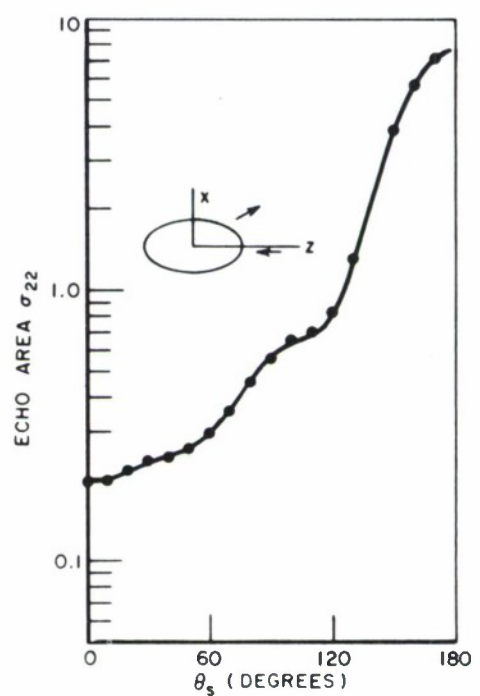
(a) BACKSCATTER vs. FREQUENCY FOR AXIAL INCIDENCE



(b) BACKSCATTER vs. θ AT 300 MHz



(c) E-PLANE BISTATIC SCATTERING AT 300 MHz FOR AXIAL INCIDENCE



(d) H-PLANE BISTATIC SCATTERING AT 300 MHz FOR AXIAL INCIDENCE

Fig. 10. Scattering data for a prolate spheroid with $A = 1$ m and $B = 0.5$ m.

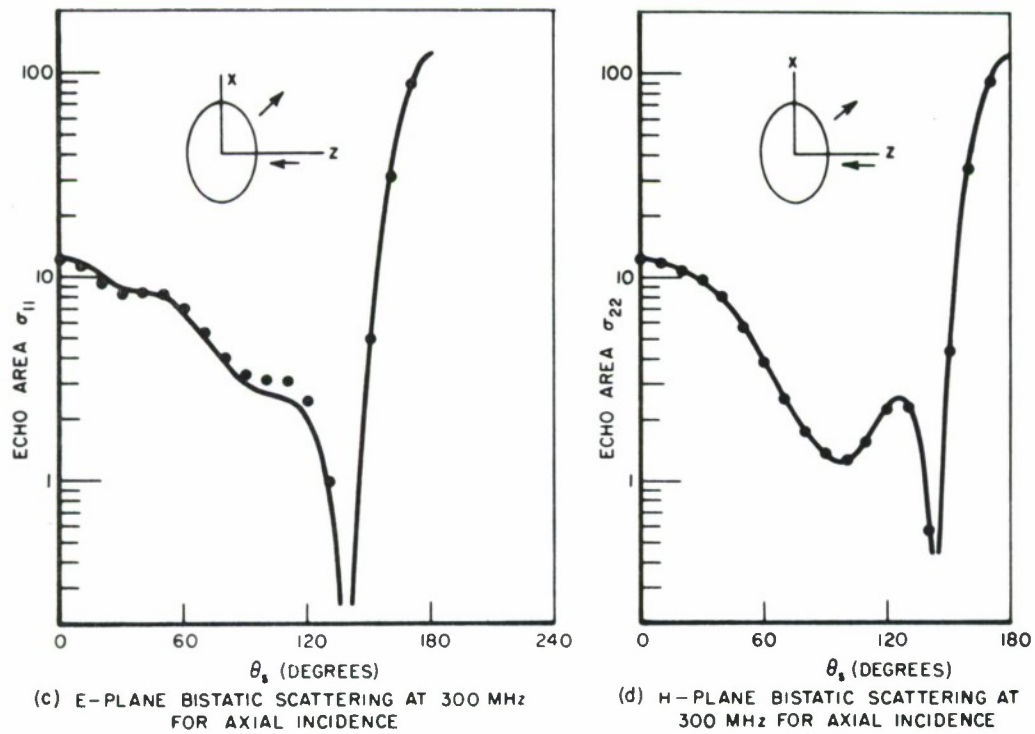
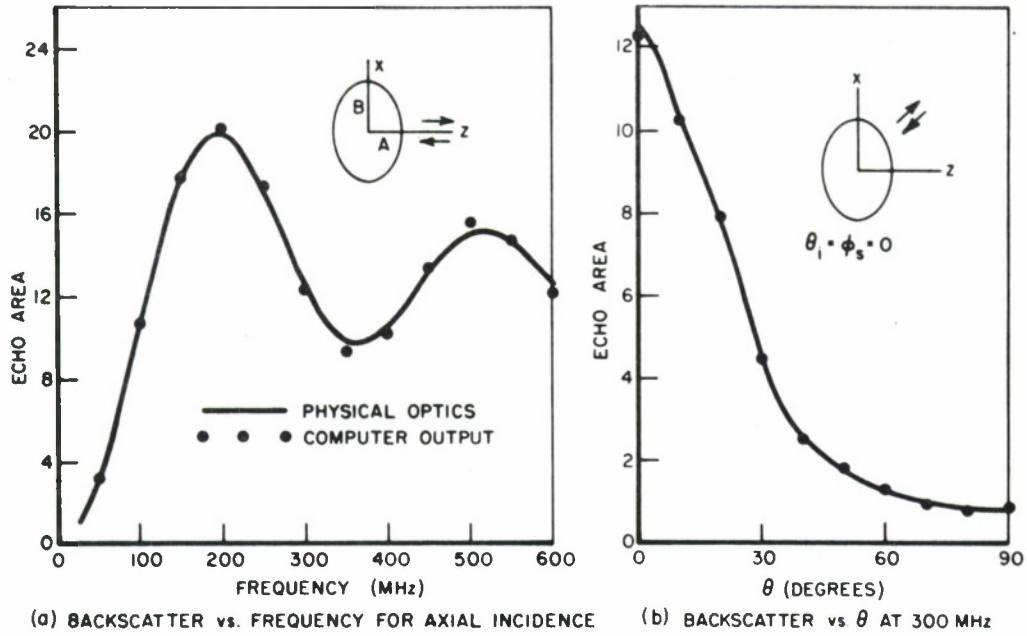
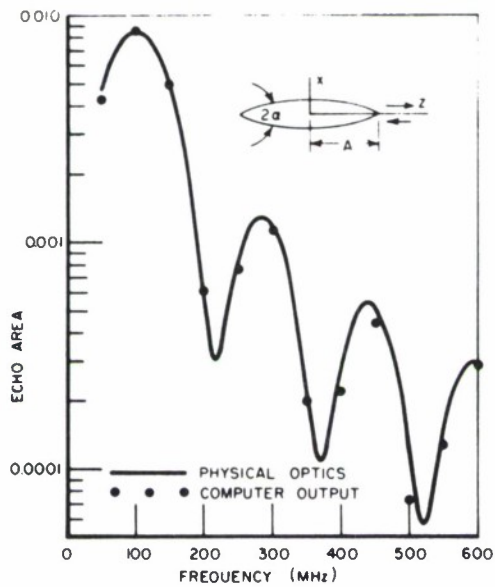
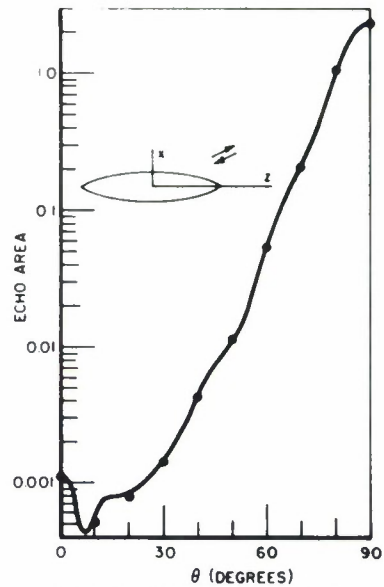


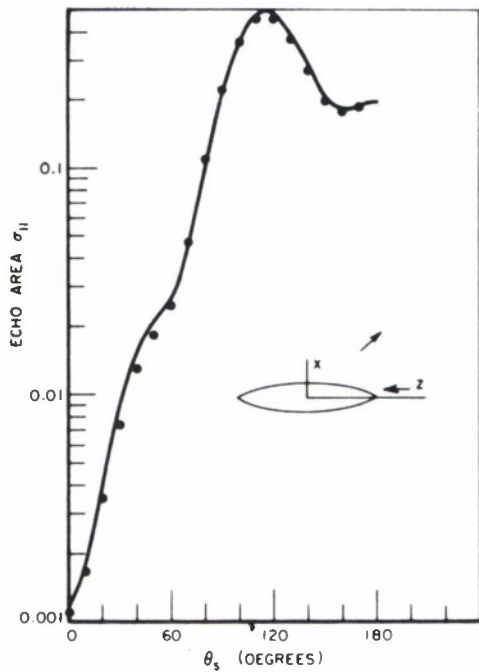
Fig. 11. Scattering data for an oblate spheroid with $A = 0.5$ m and $B = 1$ m.



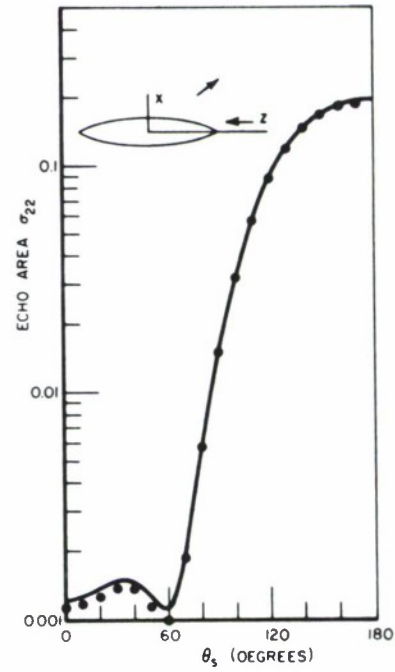
(a) BACKSCATTER vs. FREQUENCY FOR AXIAL INCIDENCE



(b) BACKSCATTER vs θ AT 300 MHz

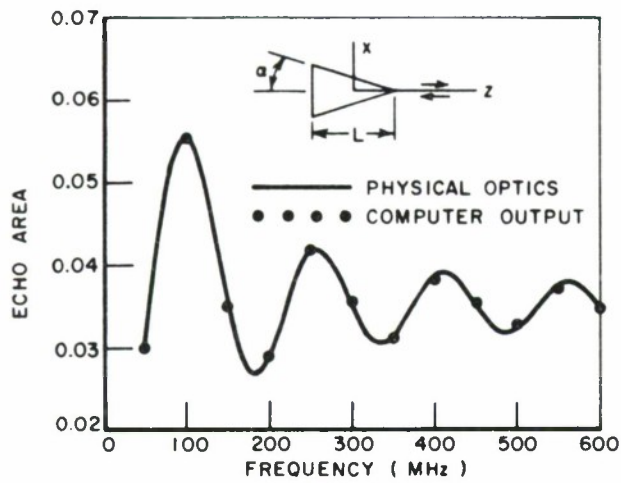


(c) E-PLANE BISTATIC SCATTERING AT 300 MHz FOR AXIAL INCIDENCE

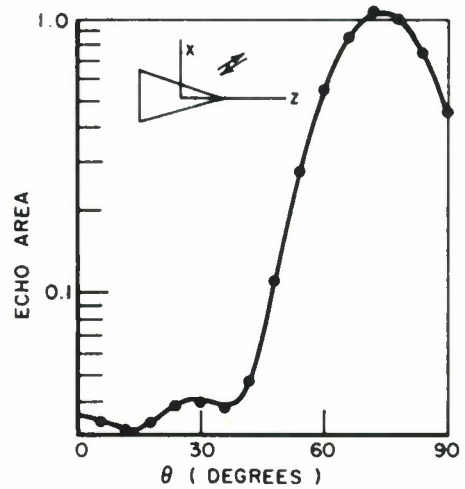


(d) H-PLANE BISTATIC SCATTERING AT 300 MHz FOR AXIAL INCIDENCE

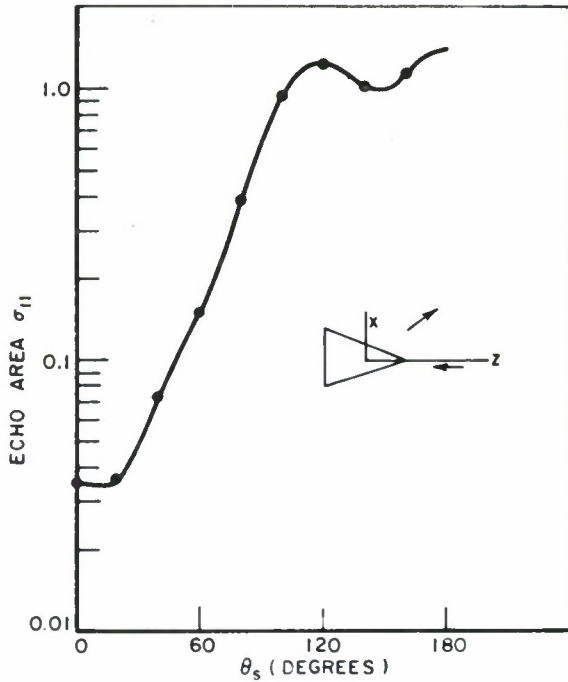
Fig. 12. Scattering data for an ogive with $A = 1$ m and $\alpha = 22.5^\circ$.



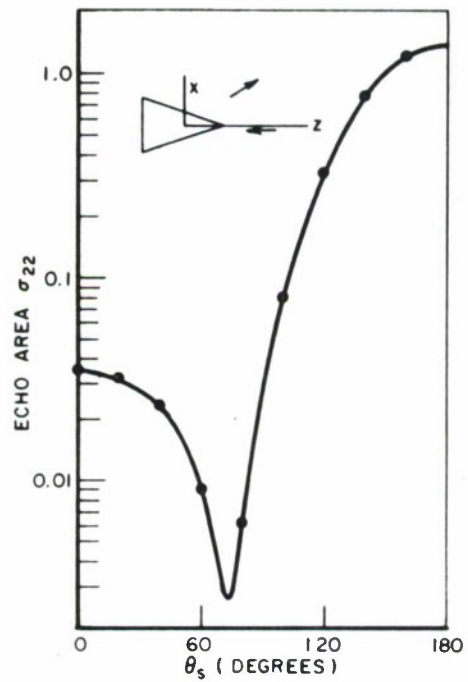
(a) BACKSCATTER vs FREQUENCY FOR AXIAL INCIDENCE



(b) BACKSCATTER vs θ AT 300 MHz



(c) E- PLANE BISTATIC SCATTERING AT 300 MHz FOR AXIAL INCIDENCE



(d) H- PLANE BISTATIC SCATTERING AT 300 MHz FOR AXIAL INCIDENCE

Fig. 13. Scattering data for a cone with $L = 1$ m and $\alpha = 18^\circ$.

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13. ABSTRACT <p>This report describes a digital computer program which uses the physical optics approximation to calculate the scattering properties of convex perfectly conducting targets with arbitrary shape. The target shape is described in terms of the coordinates of a large number of points on the surface.</p> <p>The program handles bistatic as well as backscattering problems. The input data specify the frequency, the incidence angles (θ_i, ϕ_i) and the scattering angles (θ_s, ϕ_s). In the CW case, the output data give the complex elements in the scattering matrix.</p> <p>The program also handles the pulse case where the incident waveform has a finite number of cycles.</p> <p>Graphs are included to illustrate typical results for the following target shapes: sphere, spheroids, ogive, and cone.</p>		

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