A COMPUTER PROGRAM FOR PHYSICAL-OPTICS SCATTERING BY CONVEX CONDUCTING TARGETS - 2430-7

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DEPUTY FOR SURVEILLANCE AND CONTROL SYSTEMS
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

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ABSTRACT

This report describes a digital computer program which uses the physical optics approximation to calculate the scattering properties of convex perfectly conducting targets with arbitrary shape. The target shape is described in terms of the coordinates of a large number of points on the surface.

The program handles bistatic as well as backscattering problems. The input data specify the frequency, the incidence angles ($\theta_i, \phi_i$) and the scattering angles ($\theta_s, \phi_s$). In the CW case, the output data give the complex elements in the scattering matrix.

The program also handles the pulse case where the incident waveform has a finite number of cycles.

Graphs are included to illustrate typical results for the following target shapes: sphere, spheroids, ogive, and cone.
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I. INTRODUCTION

Although the physical optics approximation for scattering by perfectly conducting targets has well-known limitations, it does provide rapid and efficient calculations which, in many cases, are reasonably accurate when the target is large in comparison with the wavelength.

In this report we present a digital computer program which uses the physical optics formulation to determine the scattering properties of perfectly conducting convex targets with arbitrary shape. Graphs are included to illustrate typical results for several target shapes.

The program is written in the Fortran IV language.

II. TARGET DESCRIPTION

The target shape is described in terms of the cartesian coordinates $(x, y, z)$ of a large number of points on the surface. These points are selected on the intersections of the target surface and the planes $z = z_1, z = z_2$, etc. The coordinate origin is located in the interior of the target, and the surface is approximated by triangular facets with vertices at the given points. It is assumed that these points cover the surface with a density such that each facet is small in comparison with the wavelength. Experience indicates that reasonably good accuracy can be obtained with as few as 20 points per square wavelength on the target surface. The scattering data converge to the physical optics solution with approximately 80 points per square wavelength. Any further increase in the number of points will simply raise the computational time.

When used with an IBM 7094 computer, the program will handle up to 1300 points. Magnetic tape or disk storage could be utilized to extend this capability.

The meter is used as the unit of length for the input data.

Punched cards are used for input data with the computer program. Figure 1 illustrates suitable input data for a prolate spheroidal target with major and minor axis lengths of 2 meters and 1 meter, respectively. The first input data card gives an integer $N$ which specifies the number of
planes \( z = z_1, z = z_2, \ldots z = z_N \) employed to describe the target. The first line in Fig. 1 indicates that 30 planes are used for the spheroid. This first card is followed by \( N \) cards which list the \( z \) coordinates \( Z(I) \) of these planes and the number of points \( NP(I) \) on each plane. These cards must be ordered with increasing values of the \( z \) coordinate. Thus, in Fig. 1, card 2 indicates that the first plane has \( z = -1 \) and there is only one point on this plane. Card 3 shows that the second plane is at \( z = -0.98525 \) and has 12 points. Finally, card 31 shows that the last plane is at \( z = 1 \) and has one point.

The remaining data cards (through 297) list the \( x \) and \( y \) coordinates of all the points on the target surface. These coordinates are given in pairs \((x, y)\), with three pairs per card. For example, card 32 in Fig. 1 shows that the point on the first plane \((z = -1)\) is at \((0, 0)\). The next four cards (33-36) specify 12 points on the second plane, and card 297 locates the point on the last plane at \((0, 0)\). The last cards (298-299) are described elsewhere in this report.

In the computer program, the coordinates \( x_k \) and \( y_k \) of point \( k \) on the plane \( z = z_i \) are denoted by \( F(K) \) and \( G(K) \) or \( FP(K) \) and \( GP(K) \). The input data cards must give these points in an ordered manner, progressing in the clockwise direction (for a distant observer located on the negative \( z \) axis) around the contour on the plane \( z = z_i \). Furthermore, the first given point on the plane \( z = z_i \) should be the one nearest the first given point on the preceding plane.

The planes \( z = z_i \) usually have unequal spacing. The spacing between these planes, and the number of points on each contour, should be designed to cover the target with a fairly uniform number of points per unit surface area.

The next section develops the physical-optics scattering equations used in the computer program.

III. THE PHYSICAL OPTICS FORMULATION

If the transmitting antenna is at a great distance from the target, it will illuminate the target with an incident field which is essentially a plane wave. In the CW case we let the time dependence \( e^{j\omega t} \) be understood and represent the incident electric field intensity as follows:

\[
E^i = (\hat{\theta_i} E_{0\hat{\theta}} + \hat{\varphi_i} E_{0\hat{\varphi}}) e^{jk \hat{r}_i \cdot r}
\]
### Fig. 1. Typical input data for the computer program.

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<table>
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<tr>
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<td>300</td>
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Note: Cards 9 through 24 and cards 47 through 288 are not shown in the table.
where \((r_i, \theta_i, \phi_i)\) are the spherical coordinates of the transmitting antenna, \((\hat{R}_i, \hat{\theta}_i, \hat{\phi}_i)\) are the corresponding unit vectors,*

\[ k = \frac{2\pi}{\lambda}, \]

\(\lambda\) denotes the wavelength, and \(E_i^\theta\) and \(E_i^\phi\) are complex constants. An arbitrary point on the target surface is assigned the coordinates \((r, \theta, \phi)\), the unit vectors \((\hat{R}, \hat{\theta}, \hat{\phi})\), and the position vector

\[ r = r \hat{R} \]

Finally, the receiving antenna is assigned the coordinates \((r_s, \theta_s, \phi_s)\) and the unit vectors \((\hat{R}_s, \hat{\theta}_s, \hat{\phi}_s)\). Thus \(\theta_i\) and \(\phi_i\) specify the incidence angles and \(\theta_s\) and \(\phi_s\) are the scattering angles. The magnetic field intensity of the incident plane wave is given by

\[ \vec{H}_i = (\hat{R}_i E_i^\theta - \hat{\theta}_i E_i^\phi) \frac{e^{jk r_i \cdot \hat{r}}}{\eta} \]

where

\[ \eta = \frac{\sqrt{\mu}}{\epsilon}. \]

The current density induced on the illuminated portion of the target surface is approximated as follows:

\[ \vec{J} = 2\hat{n} \times \vec{H}_i \]

* The unit vectors in the spherical system are related to those in the rectangular system by \(\vec{R} = (\hat{x} + \hat{y} + \hat{z})/r\), \(\hat{\theta} = \hat{x} \sin \phi + \hat{y} \cos \phi\) and \(\hat{\phi} = \hat{z}\).
where \( \hat{n} \) denotes the outward unit normal vector on the surface. The vector potential for the distant scattered field is given by

\[
A = \frac{\mu}{4\pi r_s} e^{-jkr_s} \int \int \hat{n} e^{jkr_s} \cdot r \, ds.
\]

At a great distance from the target, the scattered field is

\[
E_s = -j\omega A = -\frac{j\omega \mu}{2\pi r_s} e^{-jkr_s} \int \int \hat{n} \times H^i e^{jkr_s} \cdot r \, ds.
\]

From Eqs. (4) and (8),

\[
E_s = \frac{e^{-jkr_s}}{r_s} \left( \hat{\theta}_i E_\phi^i - \hat{\phi}_i E_\theta^i \right) \times S
\]

where

\[
S = (j/\lambda) \int \int \hat{n} e^{jk(r_i + r_s)} \cdot r \, ds.
\]

The distant scattered field is represented by

\[
E_s = (\hat{\theta}_s E_\theta^s + \hat{\phi}_s E_\phi^s) \frac{e^{-jkr_s}}{r_s}
\]

where \( E_\theta^s \) and \( E_\phi^s \) denote complex constants. From Eqs. (9) and (11) and the following vector identity,

\[
A \cdot (B \times C) = (A \times B) \cdot C,
\]

it is found that

\[
E_\theta^s = (E_\theta^i \hat{\theta}_i \times \hat{\theta}_s + E_\phi^i \hat{\phi}_s \times \hat{\phi}_i) \cdot S
\]

and
\begin{align}
E_s^\phi &= (E^i_\theta \phi_i \times \phi_s + E^i_\phi \phi_s \times \phi_i) \cdot S \\
\end{align}

It is convenient to define the CW scattering matrix as follows:

\begin{align}
\begin{pmatrix}
E^S_\theta \\
E^S_\phi
\end{pmatrix}
= \begin{pmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{pmatrix}
\begin{pmatrix}
E^i_\theta \\
E^i_\phi
\end{pmatrix}
\end{align}

From Eqs. (13) through (15), the complex elements in the scattering matrix are given by

\begin{align}
S_{11} &= (\phi_i \times \phi_s) \cdot S \\
S_{12} &= (\phi_s \times \phi_i) \cdot S \\
S_{21} &= (\phi_i \times \phi_s) \cdot S \\
S_{22} &= (\phi_s \times \phi_i) \cdot S
\end{align}

It is convenient to define a "vector area function" \( A(w) \) as follows:

\begin{align}
S = (j/\lambda) \int A(w) e^{jkw} \, dw,
\end{align}

where

\begin{align}
w &= (\hat{r}_i + \hat{r}_s) \cdot \mathbf{r}
\end{align}

It may be seen from Eq. (21) that \( w \) represents one of the coordinates of a point on the target surface, in a rectangular coordinate system that is rotated in space with respect to the \((x, y, z)\) system. The \( w \) axis is co-planar with \( \hat{r}_i \) and \( \hat{r}_s \) and bisects the angle between these unit vectors. The unit of length for the \( w \) coordinate generally differs from that for \( x, y, \) and \( z \).
For any point \((x, y, z)\) on the target surface,

\[
(22) \quad w = (\sin \theta_i \cos \phi_i + \sin \theta_s \cos \phi_s)x + (\sin \theta_i \sin \phi_i + \sin \theta_s \sin \phi_s)y + (\cos \theta_i + \cos \theta_s)z.
\]

Once the vector area function has been calculated, \(S\) can be determined efficiently from Eq. (20) by numerical integration. Equation (10) would take more computation time since it involves a surface integral instead of a line integral.

Once \(S\) has been calculated, the elements in the scattering matrix are determined as follows:

\[
(23) \quad S_{11} = S_x X_{11} + S_y Y_{11} + S_z Z_{11}
\]

\[
(24) \quad S_{12} = S_x X_{12} + S_y Y_{12} + S_z Z_{12}
\]

\[
(25) \quad S_{21} = S_z Z_{21}
\]

\[
(26) \quad S_{22} = S_x X_{22} + S_y Y_{22} + S_z Z_{22}
\]

where

\[
(27) \quad S = \hat{x} S_x + \hat{y} S_y + \hat{z} S_z
\]

\[
(28) \quad X_{11} = - \cos \phi_i \sin \theta_s
\]

\[
(29) \quad Y_{11} = - \sin \psi_i \sin \theta_s
\]

\[
(30) \quad Z_{11} = - (\sin \phi_i \sin \phi_s + \cos \phi_i \cos \phi_s) \cos \theta_s
\]

\[
(31) \quad X_{12} = \cos \theta_i \sin \phi_i \sin \theta_s - \sin \theta_i \cos \theta_s \sin \phi_s
\]
\( Y_{12} = \sin \theta_i \cos \theta_s \cos \phi_s - \cos \theta_i \cos \phi_i \sin \theta_s \)

\( Z_{12} = (\sin \phi_i \cos \phi_s - \cos \phi_i \sin \phi_s) \cos \theta_i \cos \theta_s \)

\( Z_{21} = \cos \phi_i \sin \phi_s - \sin \phi_i \cos \phi_s \)

\( X_{22} = -\sin \theta_i \cos \phi_s \)

\( Y_{22} = -\sin \theta_i \sin \phi_s \)

\( Z_{22} = -(\cos \phi_i \cos \phi_s + \sin \phi_i \sin \phi_s) \cos \theta_i \)

In the pulse case, the incident wave is considered to have \( M \) complete cycles and a square modulation envelope. \( M \) is assumed to be an integer, and the incident field is expressed by

\[
E^i = (\hat{E}_\theta^i \hat{E}_\phi^i + \tilde{E}_\theta^i \tilde{E}_\phi^i) \sin (\omega t + k \hat{r}_i \cdot \hat{r}) \\
\cdot [u(\omega t + k \hat{r}_i \cdot \hat{r}) - u(\omega t + k \hat{r}_i \cdot \hat{r} - \omega \tau)]
\]

where \( E^i_\theta \) and \( E^i_\phi \) are real constants, \( u(x) \) denotes the unit step function, \( f \) is the carrier frequency,

\( \omega = 2\pi f \)

\( T = 1/f \)

and

\( \tau = MT \)
The scattered field is given by

\[
E_s(r_s, t) = \left[ \hat{\theta}_s \ E_{\theta}^s(t - r_s/c) + \hat{\phi}_s \ E_{\phi}^s(t - r_s/c) \right] \frac{1}{r_s},
\]

where \( c \) denotes the speed of light in free space.

It is convenient to write the following matrix equation

\[
\begin{pmatrix}
E_{\theta}^s(t) \\
E_{\phi}^s(t)
\end{pmatrix}
= \begin{pmatrix}
F_{11}(t) & F_{12}(t) \\
F_{21}(t) & F_{22}(t)
\end{pmatrix}
\begin{pmatrix}
E_{\theta}^i \\
E_{\phi}^i
\end{pmatrix}
\]

The pulse response of the target is thus defined with four functions of time given by

\[
F_{11}(t) = (\phi_i \times \hat{\theta}_s) \cdot \overline{F}(t)
\]
\[
F_{12}(t) = (\hat{\theta}_s \times \hat{\phi}_i) \cdot \overline{F}(t)
\]
\[
F_{21}(t) = (\hat{\phi}_i \times \hat{\theta}_s) \cdot \overline{F}(t)
\]
\[
F_{22}(t) = (\hat{\phi}_s \times \hat{\phi}_s) \cdot \overline{F}(t)
\]

where

\[
\overline{F}(t) = \frac{1}{\lambda} \int \int \hat{n} \cos(\omega t + kw)
\]
\[
\cdot \left[ u(\omega t + kw) - u(\omega t + kw - \omega_T) \right] \ ds
\]

Equation (48) can also be written as follows

\[
\overline{F}(t) = \frac{1}{\lambda} \int \ A(w) \cos(\omega t + kw) \left[ u(\omega t + kw) - u(\omega t + kw - \omega_T) \right] \ dw.
\]
Equation (49) is employed in the computer program since it permits more rapid calculations than Eq. (48).

The scattering waveforms $F_{11}(t)$, $F_{12}(t)$, $F_{21}(t)$ and $F_{22}(t)$ are calculated at a discrete set of equally spaced points in time. The spacing between these points is determined by an integer $L$, included in the input data, as follows

\begin{equation}
\Delta T = \frac{T}{L}.
\end{equation}

The first and last points coincide with the initiation and the termination of the pulse response.

It must be noted that the above formulation for the pulse response assumes the incident plane wave has linear polarization. However, it appears that the more general situation could be programmed quite readily.

The physical-optics scattering program is described in some detail in the following sections.

IV. APPROXIMATING THE TARGET WITH A POLYHEDRON

Figure 2 shows the first 91 statements or cards in the computer program. Card 26 reads the integer $N$ which specifies the number of planes used in describing the target surface. Card 29 reads the $z$ coordinate $Z_I(I)$ for each of these planes and the number of points $NP(I)$ on each plane. The dimension statements reserve core storage for 200 planes, 500 points on each plane, and a total of 1300 points.

Card 37 reads the $x$ and $y$ coordinates of the points on the first plane, and card 48 reads the coordinates of the remaining points.

In addition to reading the target description, this first section of the program sets up a polyhedral approximation for the target. This polyhedron has many triangular facets, with vertices at the given points. If each facet is small in comparison with the wavelength, the scattering properties of the polyhedron will be nearly the same as for the real target of interest.
*EXECUTE  IBMJOB
$IBMJOB  MAP
$IBFTC  PHYSOP  NODECK

21 FORMAT(1X*E15.7)
22 FORMAT(1X*F15.7)
23 FORMAT(1HU)
24 FORMAT(1X*10F12.4)
25 FORMAT(8F9.3)
26 FORMAT(7F10.5)
27 FORMAT(7I10)
28 FORMAT(1H0)
29 FORMAT(7F10.5)
30 FORMAT(7I10)
31 FORMAT(1X*47H THI =*F10.4*7H PHI =*F10.4*7H THS =*F10.4*7H)
32 FORMAT(1X*5HFMC =*F10.4*8H WAVE =*F15.7*)
33 FORMAT(1X*HS111 =*2F15.7*7H S12 =*2F15.7*7H S21 =*2F15.7*7H)
34 FORMAT(1X*6HEA11 =*F15.7*8H EA12 =*F15.7*8H EA21 =*F15.7*8H EA2
2 =*F15.7*/)

DIMENSION AX(500),AY(500),AZ(500)
DIMENSION X(2600),Y(2600),Z(2600),XN(2600),YN(2600),ZN(2600)
DIMENSION F(500),G(500),FP(500),GP(500),ZI(200),NP(200)
COMPLEX SX,SY,SZ,SI1,S12,S21,S22,UA,QP,OC,QO,QK
DATA TP,FPI ,RA0/6,2831d53. 12.56637. ,01745-3,29/
J=0
READ(5,27)N
WRITE(6,27)N
DO 60 I =1,N
60 READ(5,29)ZI(I),NP(I)
FN=N
M=NP(I)
MM=M+1
ZA=ZI(I)
NC=(M+2)/3
DO 80 I =1,NC
K=3*I-2
80 READ(5,26)F(K),G(K),F(K+1),G(K+1),F(K+2),G(K+2)
F(MM)=F(1)
G(MM)=G(1)
IF(M.EQ.1) MM=1
DO 150 I=2,N
MP=NP(I)
MPP=MP+1
ZC=ZI(I)
NC=(MP+2)/3
DO 90 I=1,NC
K=3*I-2
90 READ(5,26)FP(K),GP(K),FP(K+1),GP(K+1),FP(K+2),GP(K+2)
FP(MPP)=FP(1)
GP(MPP)=GP(1)
IF(MP.EQ.1) MPP=1
L=1
LP=1
95 XA*F(L)
XC*FP(LP)
YA*G(L)
YC*GP(LP)
J=J+1
IF(L.EQ.MM) GO TO 98
IF(LP.EQ.MPP) GO TO 160
RA=(FP(LP+1)-F(L))*1/2+(GP(LP+1)-G(L))*1/2

Fig. 2. First section of the computer program.
RB = (FP(LP) - F(L+1))**2 + (GP(LP) - G(L+1))**2
IF(RA.GT.Rd) GO TO 100

98 LP=LP+1
XB=FP(LP)
YB=GP(LP)
ZB=ZC
GO TO 110

100 L=L+1
XB=F(L)
YB=G(L)
ZB=ZA

110 X(J)=(XA+XB+XC)/3
Y(J)=(YA+YB+YC)/3
Z(J)=(ZA+ZB+ZC)/3

XN(J)=((YB-YA)*(ZC-ZA)-(ZB-ZA)*(YC-YA))*S
YN(J)=((ZB-ZA)*(XC-XA)-(XB-XA)*(ZC-ZA))*5
ZN(J)=((XB-XA)*(YC-YA)-(YD-YA)*(XC-XA))*S

IF(L.LT.MM.OR.LP.LT.MPP) GO TO 95
DO 120 1=1*MPP
F(I)=FP(I)
G(I)=GP(I)

120 M=MP
MM=MPP
ZA=ZC

150 CONTINUE

NT=J
DZ=0
DO 155 I=2,N
DEL=ZI(I)-ZI(I-1)

155 IF(DEL.GT.DZ) DZ=DEL

Fig. 2. First section of the computer program. (cont.)
Since each facet is assumed to be small, the only parameters that must be determined and stored for each one are its vector area and the coordinates of its midpoint.

Each triangle is assigned an index number denoted by J. The coordinates of a point at the center of triangle J are denoted by X(J), Y(J) and Z(J). The cartesian components of the vector area (using the outward normal direction) of triangle J are XN(J), YN(J) and ZN(J). The dimension statements reserve core storage for these parameters for 2600 triangular facets. The number of triangles is approximately twice the number of points on the target surface, and is denoted by NT.

The technique for fitting a polyhedron to a given array of points is illustrated in Fig. 3. The computer first processes all the triangular facets on the first "zone" of the surface (i.e., the portion of

Fig. 3. A typical triangular facet on the polyhedron.
the surface bounded by the first two planes \( z = z_1 \) and \( z = z_2 \). It then proceeds to the next zone. In the situation illustrated in Fig. 3, the computer is identifying and processing the triangles in Zone II-1. The \( x \) and \( y \) coordinates of the points on contour II-1 are denoted by \( F \) and \( G \), and those on contour II by \( FP \) and \( GP \). The first triangle on this zone has one vertex at the point \( x = F(1), y = G(1) \) and \( z = ZI(II-1) \). Another vertex is at \( x = FP(1), y = GP(1) \) and \( z = ZI(II) \). The third vertex of this first triangle is at \((F(2), G(2))\) or \((FP(2), GP(2))\), whichever yields the "most compact" triangle.

When the computer has completed the processing of a given triangle in a given zone, two vertices on the next triangle are predetermined since they coincide with two of the vertices on the last completed triangle. In Fig. 3, these two vertices are indicated by points A and C. Point A is the \( L \)-th point on contour II-1 and its coordinates are \( XA = F(L), YA = G(L) \) and \( ZA = ZI(II-1) \). Point C is the \( LP \)-th point on contour II and its coordinates are \( XC = FP(LP), YC = GP(LP) \) and \( ZC = ZI(II) \). To establish the third point on the triangle, the computer compares the lengths of the lines AD and BC and selects the shorter one to form one side of the triangle. As shown in Fig. 3, B denotes the \((L+1)\)-th point on contour II-1, and D is the \((LP+1)\)-th point on contour II.

For the situation shown in Fig. 3, BC is shorter than AD. Therefore, B is selected as the third vertex of the triangle and assigned the following coordinates: \( XB = F(L+1), YB = G(L+1) \) and \( ZB = ZA \). The coordinates of the midpoint of this triangle are calculated in cards 73, 74 and 75. The vector area of the triangle is given by \((P \times Q)/2\), where the vector \( P \) extends from A to B and the vector \( Q \) extends from A to C. This calculation is arranged by cards 76, 77 and 78 to determine the rectangular components of the vector area.

Having completed the processing of triangle J, the computer proceeds with the next triangle in a similar fashion.

When the computer finishes the first section of the program, it has in storage a complete list of the midpoint coordinates and the vector areas of all of the triangular facets on the polyhedron.

V. COMPUTING THE VECTOR AREA FUNCTION

The second section of the computer program reads the coordinates \((\theta_1, \phi_1)\) of the transmitting antenna and \((\theta_s, \phi_s)\) of the receiving antenna
and calculates the corresponding vector area function $A(w)$ for the target. This part of the program is shown in Fig. 4.

Card 92 reads $\theta_i, \phi_i, \theta_s$ and $\phi_s$. These angles are denoted by THI, PHI, THS and PHS. They must be given in degrees (rather than radians) in the input data. For example, card 298 in Fig. 1 assigns a value of 20 degrees for $\theta_i$ and $\theta_s$, and zero degrees for $\phi_i$ and $\phi_s$.

In the DO LOOP in cards 115 through 137, the computer scans the $w$ coordinates for all the illuminated facets on the polyhedron to determine the maximum and minimum values of $w$ on the illuminated portion of the target surface. These are designated WMAX and WMIN. WX, WY and WZ represent the coefficients of $x$, $y$ and $z$ in Eq. (22).

Cards 138 through 147 calculate the coefficients $X_{11}$, $Y_{11}$, etc., defined by Eqs. (28) through (37).

The rectangular components of the vector area function $A(w)$ are denoted by AX(K), AY(K) and AZ(K). The program is designed to calculate several values of AX, AY and AZ, representing samples of the functions for uniformly spaced points on the $w$ axis. The dimension statements allow for a maximum of 500 sampling points. The area function is nonzero only for values of $w$ between WMIN and WMAX. Therefore, this portion of the $w$ axis is divided into KX segments and the sampling points are selected at the centers of these segments. Thus, AX(1), AY(1) and AZ(1) represent the components of $A(w)$ at the center of the first segment (i.e., the segment which begins at WMIN). Likewise, AX(KX), AY(KX) and AZ(KX) represent the components of $A(w)$ at the center of the last segment (which terminates at WMAX). The length of each segment (and the spacing between the sampling points) is denoted by DEL. WI and WF denote the values of $w$ at the first and last sampling points.

If the number of sampling points is too small, the area function will not be represented adequately and one cannot expect accurate scattering data. Therefore, the program always calculates the largest number of samples consistent with the input data. To obtain a more detailed area function for a given target, one simply supplies a more detailed description of the target via the input data.

The following expression for the vector area function can be obtained by comparing Eqs. (10) and (20):
\( A(w) = \frac{d}{dw} \int \int \hat{A} \, ds \).

The significance of Eq. (51) can be clarified by considering the equivalent finite-difference approximation:

\( A(K) = \frac{1}{\text{DEL}} \int \int_{S_k} \hat{n} \, ds \),

where \( A(K) \) represents the area function at the point \( w = w_k \) and \( S_k \) denotes a narrow zone on the illuminated portion of the target surface bounded by the planes \( w = w_k - \text{DEL}/2 \) and \( w = w_k + \text{DEL}/2 \).

For the polyhedron with small facets, Eq. (52) takes the following form:

\( A(K) = \sum \frac{\hat{A} \cdot XN(I) + \hat{Y} \cdot YN(I) + \hat{Z} \cdot ZN(I)}{\text{DEL}} \)

The summation indicated in Eq. (53) is programmed in cards 162 through 199 in Fig. 4. In Eq. (53) it is understood that the summation extends only over the illuminated facets with \( w \) coordinates in the range \( w_k \pm \text{DEL}/2 \).

A simple shadowing test is programmed in cards 163 and 164. \( \text{RDTN} \) represents the quantity \( \hat{A}_I \cdot \hat{A} \). \( \text{RDTN} \) is calculated for each facet on the polyhedron. The facet is considered illuminated or shadowed according as \( \text{RDTN} \) is positive or negative. This test is, of course, adequate only for convex targets.

Figure 5 shows the vector area components \( AX(w) \) and \( AZ(w) \) for a prolate spheroid. \( AY(w) \) is zero for this problem. The input data in Fig. 1 were used for these calculations.

Although independent data are not available for comparison with the curves in Fig. 5, there are two indications that these curves are reliable. First, these area functions lead to accurate physical-optics scattering data. Second, the area functions calculated in the same manner show close agreement with known results for the following cases: backscatter and bistatic scattering from spheres and axial backscatter from spheroids, ogives and cones.
READ(5,26)THI,PHI,THS,PHS
WRITE(6,31)THI,PHI,THS,PHS
STHI=SIN(RAD*THI)
CTHI=COS(RAD*THI)
STHS=SIN(RAD*THS)
CTHS=COS(RAD*THS)
SPHI=SIN(RAD*PHI)
CPHI=COS(RAD*PHI)
SPHS=SIN(RAD*PHS)
CPHS=COS(RAD*PHS)
SSCS=STHS*CPHS
SSSS=STHS*SPHS
SICI=STHI*CPHI
ESISI=STHI*SPHI
WX=SICI*SSCS
WY=SISI+SSSS
WZ=CTHI+CTHS
DEL=OZ*SORT(WX**2+WY**2+WZ**2)
WMAX=-1000000.
WMIN=1000000.
K=2
ZA=Z(1)
ZC=Z(2)
DO 200 I=1,NT
RDTN=X(I)*SICI+YN(I)*ESISI+ZN(I)*CTHI
IF(RDTN.LE.0.) GO TO 200
ZNI=ZN(I)
IF(ZC.GE.ZU)GO TO 170
165 K=K+1
ZA=Z(K+1)
ZC=Z(K)
IF(ZC.LT.ZB)GO TO 165
170 DEN=X(I)*XN(I)+Y(I)*YN(I)
D=DEN+ZB*ZNI
F=(D-ZA*ZNI)/DEN
WA=X(I)*F+WX+Y(I)*F+YW+ZA*WZ
F=(D-ZC*ZNI)/DEN
WC=X(I)*F+WX+Y(I)*F+YW+ZC*WZ
IF(WC.GE.WA)GO TO 175
WW=W
WA=WC
WC=WW
175 IF(WC.GT.WMAX)WMAX=WC
IF(WA.LT.WMIN)WMIN=WA
200 CONTINUE
XI=-CPHI*STHS
YI=-SPHI*STHS
ZI=(-(SPHI*SPHS+CPHI*CPHS)*CTHS
X12=CTHI*SPHI+STHS=STHI*CTHS*SPHS
Y12=STHI*CTHS*CPHS=CTHI*CPHI*STHS
Z12=(-CPHI*SPHS+SPHI*CPHS)*CTHI*CTHS
Z2I=-(SPHI*CPHS+CPHI*SPHS
X22=STHI*CPHS
Y22=STHI*SPHS
Z22=(CPHI*CPHS+SPHI*SPHS)*CTHI
DO 205 K=1,500
AX(K)=O.0
AY(K)=0.0
205 AZ(K)=O.0
KX=(WMAX-WMIN)/DEL

Fig. 4. Second section of the computer program.
FK = KX
DEL = (WMAX - WMIN) / FK
WRITE (6, 22) DZ + DEL, WMIN, WMAX
DEL = DEL / 2
WI = WMIN + DEL
WF = WMAX - DEL
K = 2
ZA = ZI(1)
ZC = ZI(2)
DO 230 I = 1, NT
RDTN = XN(I) * SIC1 + YN(I) * S1SI + ZN(I) * CTMI
IF (RDTN .LE. 0.) GO TO 230
ZNI = ZN(I)
ZB = Z(1)
IF (ZC .GE. ZD) GO TO 215
210 K = K + 1
ZA = ZI(K - 1)
ZC = ZI(K)
IF (ZC .LT. ZB) GO TO 210
215 DEN = X(I) * XN(I) + Y(I) * YN(I)
D = DEN + ZB * ZNI
F = (D - ZA * ZNI) / DEN
WA = X(I) * F + XN(I) * F * W + ZA * W
F = (D - ZC * ZNI) / DEN
WC = X(I) * F + XN(I) * F * W + ZC * W
IF (WC .GE. WA) GO TO 218
Wt* = WA
*A = WC
WC = WW
218 KA = (WA - WI) / DEL + 1.5
KC = (WC - WI) / DEL + 1.5
IF (KA .NE. KC) GO TO 220
AX(KA) = AX(KA) + XN(I) * D
AY(KA) = AY(KA) + YN(I) * D
AZ(KA) = AZ(KA) + ZN(I) * D
F = 1.0 - D
AX(KC) = AX(KC) + XN(I) * F
AY(KC) = AY(KC) + YN(I) * F
AZ(KC) = AZ(KC) + ZN(I) * F
220 FKA = KA - 1
W = FKA * DEL + WI + DEL
D = (W - WA) / (WC - WA)
AX(KA) = AX(KA) + XN(I) * D
AY(KA) = AY(KA) + YN(I) * D
AZ(KA) = AZ(KA) + ZN(I) * D
F = 1.0 - D
AX(KC) = AX(KC) + XN(I) * F
AY(KC) = AY(KC) + YN(I) * F
AZ(KC) = AZ(KC) + ZN(I) * F
230 CONTINUE
w = w + 1
DO 235 K = 1, XX
AX(K) = AX(K) / DEL
AY(K) = AY(K) / DEL
AZ(K) = AZ(K) / DEL
FK = K
WRITE (6, 22) FK, w, AX(K), AY(K), AZ(K)
235 w = w + DEL
AX(KX + 1) = 2 * AX(KX) - AX(KX - 1)
AY(KX + 1) = 2 * AY(KX) - AY(KX - 1)
AZ(KX + 1) = 2 * AZ(KX) - AZ(KX - 1)
WRITE (6, 23)

Fig. 4. Second section of the computer program. (cont.)
Fig. 5. Calculated area functions for a prolate spheroid using input data from Fig. 1.
VI. COMPUTING THE CW SCATTERING MATRIX

The third section of the computer program, shown in Fig. 6, reads
the frequency in megahertz and calculates the CW scattering matrix and
the corresponding echo areas. In addition to the frequency, card 212
also reads the integers L and M which are defined in the next section.
L and M are not needed for the CW calculations and could just as well
be read later at card 249. Suitable input data for card 212 are shown
in card 299 in Fig. 1.

This section of the program calculates the vector $S$ defined by

\[
S = (j/\lambda) \int_{w_{\text{min}}}^{w_{\text{max}}} A(w) e^{jk w} dw.
\]

The integral in Eq. (54) is evaluated with the "piecewise linear method"
described in Reference 1. We integrate over segment $k$ on the $w$ axis
and then sum on $k$. Segment $k$ has length $\Delta L$ and is centered at $w = w_k$. On segment $k$ a straight-line approximation is used for the area function:

\[
A(w) = a_k + b_k w
\]

By fitting Eq. (55) to the stored samples $A(K)$ at $w_k$ and $A(K+1)$ at $w_{k+1}$, we find that

\[
a_k = \frac{[A(K) w_{k+1} - A(K+1) w_k]}{\Delta L}
\]

and

\[
b_k = \frac{[A(K+1) - A(K)]}{\Delta L}
\]

* When $k$ is used as a subscript it represents the index of a segment on
the $w$ axis and is denoted in the program by $K$. When used otherwise, as
in Eqs. (54) and (59), $k$ is defined by Eq. (2) and is denoted in the program
by $TPL$. 

20
The integration in Eq. (54) can be performed analytically when $A(w)$ is given by Eq. (55). This leads to the following expression

\begin{equation}
\mathcal{S} = \frac{1}{2\pi D_G} \sum_{k=1}^{K_X} \left[ Q_K A(K) + Q_L A(K+1) \right],
\end{equation}

where

\begin{align*}
D_G &= k \, \text{DEL}, \\
Q_K &= (G_L - j) \, Q_D - Q_C, \\
Q_L &= Q_C - (G_K - j) \, Q_D, \\
Q_C &= G_B \, e^{j \frac{G_B}{2}} - G_A \, e^{j \frac{G_A}{2}}, \\
Q_D &= e^{j \frac{G_B}{2}} - e^{j \frac{G_A}{2}}, \\
G_A &= k(w_k - \text{DEL}/2), \\
G_B &= k(w_k + \text{DEL}/2), \\
G_K &= k \, w_k, \\
\text{and} \\
G_L &= k \, w_{k+1}.
\end{align*}

The summation in Eq. (58) is programmed in cards 223 through 236. The rectangular components of $\mathcal{S}$ are denoted by $S_X$, $S_Y$ and $S_Z$. 

21
Equations (23) - (26) are programmed in cards 238 - 241 to calculate the complex elements in the scattering matrix. This scattering matrix is the principal output of this section of the program. With it available, one can use Eq. (15) to calculate the scattering properties of the target for any given incident polarization.

The remaining output from this section does not necessarily have any significance except in special cases. In backscatter problems, however, $E_{A11}$ represents the echo area in square meters. Furthermore, $E_{A11}$ and $E_{A22}$ represent the E-plane and H-plane echo areas when $\phi_1 = \phi_6$ and the target has axial symmetry with respect to the z axis.

Using the techniques described above and the input data shown in Fig. 1, the computer yields the following CW scattering data for the prolate spheroid at 300 MHz:

\[
S_{11} = S_{22} = -0.1191 + j 0.0637
\]

\[
S_{12} = S_{21} = 0
\]

\[
E_{A11} = E_{A22} = 0.229
\]

\[
E_{A12} = E_{A21} = 0
\]

Although the piecewise-linear integration technique is efficient and successful for most problems, it is not appropriate in forward scattering problems (where $w = 0$ everywhere on the target) or in specular scattering from a flat region on the target (where $w$ is constant over a significant portion of the surface). Therefore, the computer output data are not reliable in these cases.

VII . COMPUTING THE PULSE RESPONSE

The pulse scattering problem is treated in the fourth and last section of the program shown in Fig. 7. The entire program is ready for the computer when the four sections are put together (with the card numbers running sequentially from 1 through 345) and backed up with suitable input data (such as cards 1 through 299 in Fig. 1).

To explain the pulse calculations, it is convenient to write Eq. (49) in the following form:
READ(5,29)FMC,L,M
AVE=299.79/FMC
WRITE(6,32)FMC,WAVE
TPL=TP/WAVE
SX=(0.0,0)
SY=(0.0,0)
SZ=(0.0,0)
GK=TPL*W1
GA=TPL*WMIN
DG=TPL*DEL
QA=CMPLX(COS(GA),SIN(GA))
DO 240 K=1,KX
GB=GA+DG
QB=CMPLX(COS(GB),SIN(GB))
GL=GK+DG
QC=GB*QB-GA*QA
QD=QB-QA
OK=QC*GL-GC-(0,1)*QD
QL=QC-OD*GK+(0,1)*QD
SX=OK*AX(K)+QL*AX(K+1)
SY=OK*AY(K)+QL*AY(K+1)
SZ=OK*AZ(K)+QL*AZ(K+1)
GA=GB
GK=GL
240 QA=QB
DEN=TP*DG
S11=(SX*X11+SY*Y11+SZ*Z11)/DEN
S12=(SX*X12+SY*Y12+SZ*Z12)/DEN
S21=SZ*Z21/DEN
S22=(SX*X22+SY*Y22+SZ*Z22)/DEN
WRITE(6,33)S11,S12,S21,S22
EA11=(CABS(S11)**2)*FP1
EA12=(CABS(S12)**2)*FP1
EA21=(CABS(S21)**2)*FP1
EA22=(CABS(S22)**2)*FP1
WRITE(6,34)EA11,EA12,EA21,EA22
WRITE(6,23)

Fig. 6. Third section of the computer program.
(68) \[ F(t) = \frac{1}{2\pi} \left[ \cos \omega t \int_{x_a}^{y_a} A(g) \cos g \, dg - \sin \omega t \int_{x_a}^{y_a} A(g) \sin g \, dg \right], \]

where

(69) \[ g = kw, \]

(70) \[ x_a = GN \text{ or } -\omega t \text{ (whichever is larger)}, \]

(71) \[ y_a = GX \text{ or } \omega_T - \omega t \text{ (whichever is smaller)}, \]

(72) \[ GN = k \text{ WMIN}, \]

and

(73) \[ GX = k \text{ WMAX}. \]

The area function \( A(g) \) is the same as \( A(w) \), and it has already been calculated and stored in \( AX(K), AY(K) \text{ and } AZ(K) \).

The integer \( L \) determines the number of points per cycle to be calculated on the scattering waveforms. Card 254 calculates the size of the increments in \( \omega t \) as follows:

(74) \[ DWT = \frac{2\pi}{L}. \]

The integer \( M \) specifies the number of complete cycles in the incident waveform. The pulse width of the incident waveform, as measured in radians and observed at an arbitrary point in space, is calculated by card 253 as follows:

(75) \[ \omega_T = WTAU = 2\pi M. \]
The scattering function $F(t)$ is nonzero only in the following interval:
-\( \omega_T - GN \).

It is convenient to assign symbols to the integrals in Eq. (68) as follows:

\[
(76) \quad U(t) = \int_{x_a}^{y_a} A(g) \cos g \, dg,
\]
and

\[
(77) \quad V(t) = \int_{x_a}^{y_a} A(g) \sin g \, dg .
\]

These are functions of time by virtue of the time dependent limits \( x_a \) and \( y_a \). In the program the rectangular components of \( U(t) \) are denoted by \( UX, UY \) and \( UZ \), and the components of \( V(t) \) are \( VX, VY \) and \( VZ \).

\( WT, SWT \) and \( CWT \) denote \( \omega_T, \sin \omega_T \) and \( \cos \omega_T \).

\( U(t) \) and \( V(t) \) are identically zero for all values of \( \omega_T \) less than or equal to \(-GX\). Thus, the computer begins the pulse-response calculations by setting \( \omega_T = -GX \) (card 261), initializing \( U(t) \) and \( V(t) \) to zero (cards 269 - 274) and writing out the first data point on the scattering waveform (card 268). At this point in time, the integration limits are found from Eqs. (70) and (71) to be \( x_a = y_a = GX \).

Suppose the computer has just completed the calculation of the scattering data for a given value of \( \omega_T \), has incremented \( \omega_T \) in card 277, and is preparing to calculate the next point on the scattering waveform. The first step in the procedure is to determine the integration limits \( x_a \) and \( y_a \), denoted by \( XA \) and \( YA \). This is accomplished in cards 281 - 292 in accordance with Eqs. (70) and (71). These cards also determine the indices \( KA \) and \( LA \) of the segments on the \( g \) axis which contain the points \( g = XA \) and \( g = YA \), respectively.

The integrals \( U(t') \) and \( V(t') \) currently in storage have limits now denoted by \( XB \) and \( YB \) by virtue of cards 338 and 339. Since \( XA \) and \( YA \) are equal to or less than \( XB \) and \( YB \), respectively,

\[
(78) \quad U(t) = U(t') + \int_{XA}^{XB} A(g) \cos g \, dg - \int_{YA}^{YB} A(g) \cos g \, dg .
\]
and

\begin{align}
V(t) &= V(t') + \int_{X_A}^{X_B} A(g) \sin g \, dg - \int_{Y_A}^{Y_B} A(g) \sin g \, dg .
\end{align}

Calculations based on Eqs. (78) and (79) are, of course, far more efficient than Eqs. (76) and (77).

Equations (78) and (79) are programmed in cards 294 - 329. The piecewise-linear technique is employed to evaluate the integrals in these equations. The integrals with limits \(X_A\) and \(X_B\) are evaluated in the first pass through the DO LOOP that begins with card 299, and those with limits \(Y_A\) and \(Y_B\) are calculated in the second pass. In each pass, \(Z_A\) and \(Z_B\) denote the limits of integration. Initially \(I\) denotes the index number of the \(g\)-axis segment containing the point \(g = Z_A\) and \(G\) is the value of \(g\) at the center of this segment. \(A(g)\) is treated as a linear function over this segment in the same manner as in Eqs. (55) through (57). \(G_P\) denotes the value of \(g\) at the center of segment \(I + 1\), and \(H_A\) and \(H_B\) are the limits of the subintegral on segment \(I\). When the integration on segment \(I\) is finished, the computer moves on to the next segment (unless \(Z_A\) and \(Z_B\) lie on the same segment). The following equations are used in cards 310 - 319 to evaluate the subintegral on segment \(I\):

\begin{align}
\int_{h_a}^{h_b} A(g) \cos g \, dg &= \frac{1}{DG} \left[ CI A(I) + CII A(I+1) \right] , \\
\int_{h_a}^{h_b} A(g) \sin g \, dg &= \frac{1}{DG} \left[ DI A(I) + DII A(I+1) \right] ,
\end{align}

\begin{align}
CI &= (h_a - g_{i+1}) \sin h_a + (g_{i+1} - h_b) \sin h_b + \cos h_a - \cos h_b , \\
CII &= (g_i - h_a) \sin h_a + (h_b - g_i) \sin h_b + \cos h_b - \cos h_a , \\
DI &= (g_{i+1} - h_a) \cos h_a + (h_b - g_{i+1}) \cos h_b + \sin h_a - \sin h_b ,
\end{align}

and
DII = (ha - gi) cosha + (gi - hb) coshb + sinh b - sinh a.

DG is defined in Eq. (59). In the program, gi, gi+1, ha and hb are denoted by G, GP, HA and HB. SGN is positive in the first pass through this integration loop and negative in the second pass, in accordance with the plus and minus signs in Eqs. (78) and (79).

When the vectors U(t) and V(t) have been calculated, Eq. (68) is used to determine F(t). The rectangular components of F(t) are denoted by FX, FY and FZ in cards 330-332. Finally cards 333 - 336 calculate the scattering functions defined by Eqs. (43) through (47), and card 337 arranges the writeout of these data on the scattering waveforms for the pulse situation.

The results obtained in this manner (using the input data in Fig. 1) for a prolate spheroid are illustrated in Fig. 8.

VIII. CONCLUSIONS

A digital computer program is described which uses the physical optics approximation for scattering by convex perfectly conducting targets with arbitrary shape. The target size and shape are specified by input data giving the coordinates of many points on the surface. This program handles bistatic and backscatter problems. The input data specify the frequency, the incidence angles (θi, φi) and the scattering angles (θs, φs). In the CW case, the output data give the complex elements in the scattering matrix. Typical results are shown in the Appendix for scattering from spheres, spheroids, ogives and cones.

This scattering program also handles the pulse case where the incident wave has a finite number of cycles. For this case, the following additional input data are required: the number of cycles in the incident wavetrain, and the number of points per cycle to be computed for the scattering waveforms. The output data give the four waveforms associated with the four elements in the scattering matrix.

Simple modifications can be made in the program, if desired, to increment the frequency, the incidence angles or the scattering angles. This is convenient for generating various types of scattering curves such as those in the Appendix.
Fig. 7. Last section of the computer program.
Fig. 7. Last section of the computer program. (cont.)
Fig. 8. Calculated pulse response for prolate spheroid, using input data from Fig. 1.
REFERENCES


APPENDIX
NUMERICAL RESULTS

This Appendix presents the results obtained with the physical-optics computer program for several target shapes. Table I lists the number of planes used to describe each target, the total number of points on the target, and the time required for an IBM 7094 to process the target description data and set up the polyhedral approximation. The last column lists the additional computer time needed to calculate the area function and the CW scattering matrix.

<table>
<thead>
<tr>
<th>Target Shape</th>
<th>Planes</th>
<th>Points</th>
<th>Time for Polyhedron</th>
<th>Area Function and CW Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>30</td>
<td>1083</td>
<td>8.6 sec</td>
<td>1.6 sec</td>
</tr>
<tr>
<td>Prolate Spheroid</td>
<td>30</td>
<td>774</td>
<td>6.4</td>
<td>1.3</td>
</tr>
<tr>
<td>Oblate Spheriod</td>
<td>30</td>
<td>1257</td>
<td>9.9</td>
<td>1.9</td>
</tr>
<tr>
<td>Ogive</td>
<td>50</td>
<td>1008</td>
<td>8.2</td>
<td>1.8</td>
</tr>
<tr>
<td>Cone</td>
<td>35</td>
<td>1176</td>
<td>9.8</td>
<td>3.3</td>
</tr>
</tbody>
</table>

The dimensions of the targets are specified in Figs. 9 through 13.

It is found from Table I that the computation time for processing the target description data and setting up the polyhedron increases linearly with the number of points on the target surface. The computer handles about 125 points (to establish 250 facets) per second, regardless of the shape of the target.

In generating the area function and the CW scattering matrix, the computer spends very little time on the shadowed facets. Thus, the computation time for this work increases linearly with the number of illuminated facets on the target. This explains why the computer spent twice as much time on the cone (3.3 seconds) as on the sphere (1.6 seconds) as shown in Table I. This sphere has approximately 2000 facets, and half
of them are illuminated. The cone also has approximately 2000 facets, and all of them are illuminated for axial incidence.*

In calculating the area function, the computer handles about 670 illuminated facets per second.

To use the computer program most efficiently, one would calculate a significant amount of scattering data for one target before passing on to the next one. Furthermore, to minimize the area-function calculations, one would calculate the scattering data for many different frequencies before changing $\theta_i$, $\phi_i$, $\theta_s$ or $\phi_s$. When these incidence and scattering angles are held constant, the computer generates CW scattering data for 20 different frequencies in one second for the oblate spheroid in Table I, and for 10 frequencies per second for the cone. For the sphere, it generates CW and pulse-response data for a new frequency in one second (with three cycles in the incident wavetrain and 20 calculated points per cycle in the scattering waveform).

Figures 9 through 13 display the numerical results for scattering by several different target shapes. The dots represent the output data from the computer program described herein. The input data listed the coordinates of a large number of points (Table I) distributed almost uniformly over the target surface. The source of the physical-optics data (the solid curves in Figs. 9 - 13) is described a little farther on in this Appendix.

Figures 9a through 13a show curves of backscatter echo area (in square meters) versus frequency for axial incidence. For each of these targets, it may be noted that the computer output agrees closely with the physical optics solution for all frequencies up to 600 MHz. (This simply indicates that the computer program does a good job in generating physical optics data. It is not implied that the physical optics formulation always gives a good approximation to the exact scattering data.) To obtain equally good results at higher frequencies, it would be necessary to include more points in the input data description of the target.

* For all the targets listed in Table I except the cone, the input data points covered the whole surface and the computer set up a closed polyhedron. For the cone, however, the input data did not cover the flat plate at the base. Since the base is shadowed for all cases shown in Fig. 13, the computed data apply to a cone with a circular disk at the base.
Although the computer program has not been adequately tested for the most general target shapes and bistatic situations for which it is designed, it is believed that it will give accurate physical-optics data except for concave targets, forward scattering, and specular scattering from a large flat subsurface. The program could be modified without much difficulty to remove the last two of these exceptions.

All of the targets considered in this Appendix have axial symmetry with respect to the z axis. To check the accuracy of the computer output data shown in Figs. 9a - 13a, closed-form expressions were derived for the vector area function for each of these targets for the backscatter situations with axial incidence. Letting $\theta_i = \phi_i = \theta_s = \phi_s = 0$, we find from Eq. (22) that $w = 2z$. From symmetry considerations, $A_x(w) = A_y(w) = 0$. For targets having axial symmetry with respect to the z axis, it is convenient to describe the target shape with an equation of the form $\rho = \rho(z)$ where $(\rho, \phi, z)$ represent the coordinates in the cylindrical system. Then it is easy to show, with the aid of Eq. (51), that

\[
A_z(w) = -\pi \rho \rho',
\]

where $\rho'$ represents the derivative of $\rho(z)$ with respect to $z$. Equation (84) applies over the illuminated region of the target where $\rho'$ is negative, and $A_z(w) = 0$ over the shadowed region where $\rho'$ is positive.

All the targets considered in this Appendix can be represented with the following equation:

\[
F(\rho, z) = A_1 \rho^2 + A_2 z^2 + A_3 \rho z + A_4 z + A_s \rho + A_6 = 0.
\]

From Eqs. (84) and (85), the area function for axial backscatter is given in closed form by

\[
A_z = \pi \rho \left(2A_2 z + A_3 \rho + A_4 \right)/(2A_1 \rho + A_3 z + A_5).
\]

For spheres, prolate spheroids and oblate spheroids,
where $A$ and $B$ denote the semimajor and semiminor axes, respectively. For the ogive,

$$(88) \quad A_z = \frac{\pi \rho z}{\sqrt{B^2 - z^2}} ,$$

where $B$ is the radius of the circular arc that generates the ogive.

For the cone,

$$(89) \quad A_z = \pi z \tan^2 \alpha ,$$

where $\alpha$ denotes the half-angle of the cone.

For axial backscatter from a target with axial symmetry, it is found from Eq. (54) that $S_x = S_y = 0$ and

$$(90) \quad S_z = (2j/\lambda) \int A_z e^{2jkz} \, dz .$$

The integral in Eq. (90) can be evaluated to obtain closed-form expressions for the scattering matrix and the echo area for the cone and the spheroids. The resulting data are shown by the solid curves with the label "physical optics" in Figs. 9a, 10a, 11a and 13a. For the ogive in Fig. 12a, the "physical optics" curve was obtained from Eqs. (88) and (90) with numerical integration.

Figure 9b shows backscatter results for the sphere as a function of the aspect angle. For a true sphere, the echo area is of course independent of aspect angle. Thus, the variations in the computer output data arise from the polyhedral approximation.

Figures 10b - 13b show the backscatter results as a function of the aspect angle ($\theta_i = \theta_s = \theta$) for the spheroids, ogive and cone. The solid curves with the label "physical optics" were obtained with another computer program which is designed especially for backscattering from targets with axial symmetry. The input data for this auxiliary program describes the target shape by assigning numerical values to the coefficients.
in Eq. (85). The angles $\phi_i$ and $\phi_s$ are assumed to be zero, the cylindrical coordinates $(\rho, \phi, z)$ are employed for points on the target surface, and

\begin{equation}
(91) \quad w = 2(\rho \sin \theta \cos \phi + z \cos \theta).
\end{equation}

It is found that $S_y = 0$,

\begin{equation}
(92) \quad S = (j/\lambda) \int \int (\hat{x} \cos \phi - \hat{z} \rho') \rho \ e^{jkw} \ d\phi \ dz,
\end{equation}

\begin{equation}
(93) \quad S_{11} = S_{22} = - S_x \sin \theta - S_z \cos \theta,
\end{equation}

and $S_{12} = S_{21} = 0$. The auxiliary program uses numerical integration to evaluate $S$ in Eq. (92).

Bistatic scattering results are shown in Figs. 9c - 13c and 9d - 13d. These curves apply for axial incidence with $\theta_i = \phi_i = \phi_s = 0$, so that

\begin{equation}
(94) \quad w = \rho \sin \theta_s \cos \phi + z(1 + \cos \theta_s),
\end{equation}

\begin{equation}
(95) \quad S_x = - k \int \rho J_1(k \rho \sin \theta_s) e^{jkz(1+\cos \theta_s)} \ dz,
\end{equation}

\begin{equation}
(96) \quad S_y = S_{12} = S_{21} = 0,
\end{equation}

\begin{equation}
(97) \quad S_z = - jk \int \rho \rho' J_0(k \rho \sin \theta_s) e^{jkz(1+\cos \theta_s)} \ dz,
\end{equation}

\begin{equation}
(98) \quad S_{11} = - S_x \sin \theta_s - S_z \cos \theta_s,
\end{equation}

\begin{equation}
(99) \quad S_{22} = - S_z,
\end{equation}

and $J_0$ and $J_1$ represent Bessel functions.
Equations (94) - (99) apply only for axial incidence on a target having axial symmetry. (Similar expressions are given in Reference 2.) These equations were used to develop another auxiliary computer program to generate the E-plane and H-plane curves with the label "physical optics". This program also used the coefficients in Eq. (85) as input data for the target shape, and evaluated $S_x$ and $S_z$ with numerical integration.

The bistatic curves in Figs. 9 - 13 agree with the physical-optics echo area expression for the forward-scattering situation:

\[
\sigma = \frac{4\pi A^2}{\lambda^2},
\]

where $A$ represents the area of the target projected along the line of sight between the transmitting and receiving antennas. In the forward-scattering situation (and also for backscattering), the physical optics echo area shows no polarization dependence even for a target with arbitrary shape. Equation (100) is not restricted to symmetrical targets or axial incidence.

Though all the data in Figs. 9 - 13 were obtained with the aid of various digital computer programs based on the physical-optics concepts, the close agreement between the dots (with the label "computer output") and the curves (with the label "physical optics") is significant for two reasons. First, the general program uses a target description technique which differs greatly from that used in the specialized programs. Furthermore, some of the specialized programs used closed-form expressions and others were based on equations involving line integrals or surface integrals, whereas the general program handles each problem in the same manner (setting up the polyhedron, calculating the vector area function, and integrating the area function to obtain the scattering matrix). Having demonstrated that this general procedure gives correct physical-optics data for a variety of target shapes and bistatic and backscatter situations, we have confidence that the program will perform equally well with other targets and situations.
Fig. 9. Scattering data for a sphere with a radius of one meter.
Fig. 10. Scattering data for a prolate spheroid with $A = 1\, \text{m}$ and $B = 0.5\, \text{m}$. 
Fig. 11. Scattering data for an oblate spheroid with $A = 0.5$ m and $B = 1$ m.
Fig. 12. Scattering data for an ogive with $A = 1\text{ m and } \alpha = 22.5^\circ$. 


Fig. 13. Scattering data for a cone with $L = 1$ m and $\alpha = 18^\circ$. 
A Computer Program for Physical-Optics Scattering by Convex Conducting Targets

This report describes a digital computer program which uses the physical optics approximation to calculate the scattering properties of convex perfectly conducting targets with arbitrary shape. The target shape is described in terms of the coordinates of a large number of points on the surface.

The program handles bistatic as well as backscattering problems. The input data specify the frequency, the incidence angles \((\theta_i, \phi_i)\) and the scattering angles \((\theta_s, \phi_s)\). In the CW case, the output data give the complex elements in the scattering matrix.

The program also handles the pulse case where the incident waveform has a finite number of cycles.

Graphs are included to illustrate typical results for the following target shapes: sphere, spheroids, ogive, and cone.
14. KEY WORDS

Radar cross section
Backscatter
Bistatic scattering
Physical optics
Digital computation
Pulse response

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