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ABSTRACT

Stock rationing is the attempt to insure that scarce material goes where it can do the most good. Two forms of stock rationing are investigated for application at the NICP level.

A reserve for high priority requisitions is designed to channel scarce stock away from low priority users to high priority users. The value of maintaining a reserve is empirically shown. A mathematically optimum method for setting the size of the reserve level is developed. This and other methods are empirically compared. A simple rule for setting reserve levels is recommended which bases the level at any given time on the high priority demand expected during the time remaining before stock replenishment. It is shown how to implement this rule in a multi-priority system.

Reduced shipments can be made to economize on scarce stock. The use of such a procedure is investigated in the case where the requisitioner is ordering in economic quantities to replenish his own supplies rather than for immediate use. It is shown that proper application of the procedure provides a way of improving supply performance which is inexpensive relative to the costs of raising the safety level.

ACKNOWLEDGEMENT

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U.S. Army Aviation Command, St. Louis, Missouri, for supplying the data used in Part I of this report.

Consolidated Property Office, Ft Dix, N. J. for guidance and assistance in the collection of data used in Part II of this report.

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SUMMARY

1. PROBLEM AND BACKGROUND

The National Inventory Control Points (NICP's) do not always have adequate stock on hand to satisfy all demands for all items. Such a situation is a natural consequence of the existence of procurement lead time, and is characteristic of most supply systems. Procurement lead times mean that the NICP must buy ahead, based on an estimate of future demand. Inevitably, these estimates will sometimes be too low; while safety stocks are designed to protect against low estimates, it would be prohibitively expensive to carry safety stocks sufficiently large to protect against all possible changes or peaks in demand. The problem is compounded because procurement lead time itself cannot be estimated with complete precision.

Difficulties are likely to be most acute during critical periods. A decision to increase the commitment in Vietnam, for instance, can necessitate a reestimation of future demands virtually across an NICP's entire line of inventory. Large numbers of items which were previously above their reorder points, suddenly drop below them, as the reorder points are raised. Furthermore, industry cannot always expend capacity sufficiently in such times to meet normal procurement lead time. Mobilisation and contingency stocks, just like safety stocks during more normal periods, can be vital, but cannot be the complete solution.

The recognition that the NICP's must inevitably experience short supply on occasion suggests the need for stock rationing techniques. Stock rationing is the use of issue policy to mitigate the effects of short supply.

The most important current application of stock rationing in the Army Materiel Command is the use by all the NICP's of the Uniform Military Issue Priority System (UMIPS) to determine which backorders for an item should be cleared up first as stock becomes available. This is "an after the fact" application of stock rationing in that it is by nature not used until stock of an item has first been exhausted. It is prescribed in AR 725-50, the MILSTRIP regulation.

Use of stock rationing before supplies of an item have been completely exhausted has not received the attention it deserves.

2. PURPOSE AND OBJECTIVES

Part I of Study.

(a) Determine the validity of the high priority reserve concept:* a reserve level is established for an item, and if stock on hand falls below this level, issues to low priority requisitioners are temporarily suspended.

(b) Determine how a reserve concept should be applied, if it is valid.

Part II of Study.

(a) Develop and justify new techniques for stock rationing.

3. SCOPE AND METHODS

Part I of Study.

There were three phases to the investigation -

(a) Develop alternative policies for setting the size of the reserve level for high priority demand.

(b) Test the alternative policies, and in so doing evaluate the effectiveness of having a reserve at all.

(c) Apply the knowledge gained in phases (a) and (b) toward making specific recommendations.

The major part of the effort in phase (a) was devoted to development of a mathematically optimum procedure for setting the size of the reserve level. This effort was successful.

An alternative procedure was also developed which was not sophisticated, but did take into account the two most important factors for determining the size of the reserve level. The mathematical work has identified these to be: the amount of high priority demand expected before stock replenishment would return the item to a good supply position, and

*This concept is enunciated in AR 725-50.

the relative importance of high vs low priority demand.

In phase (b) a computer simulation was developed by which the effect of using the different reserve policies could be tested. Historical demand data on 235 items which was provided by U. S. Army Aviation Materiel Command was used as input to the simulation. A series of tests was conducted.

In phase (c) the relative importance of delays in satisfying different priorities of demand was considered. Reference was made to the MILSTRIP time standards for maximum allowable order and ship times for overseas requisitions of different priorities. This work permitted the formulation of quite specific recommendations based on the simulation results.

4. FINDINGS AND CONCLUSIONS

Part I of Study.

(a) A reserve level for high priority demand could advantageorsly be used by the National Inventory Control Points in the management of secondary items.

(b) A computer procedure to calculate the levels should be incorporated in the NAPALM* computer system for supply management subsequent to pilot test of the basic system in St. Louis.

(c) This procedure should follow the detailed guidelines developed in this report (Chapter I, section 10). The chief features of the proposed procedure are:

(1) The procedure is not sophisticated because the study showed a sophisticated procedure could lead only to a marginal improvement.

(2) Not only is a reserve level for high priority demand calculated (UMIPS priorities 1-8), but to afford additional protection to the highest priority requisitions a two level concept is utilized.

(3) The size of the reserve level is a fraction of the expected high priority demand for the item in the period prior to stock replenishment.

*National ADP Program for AMC Logistics Management

(4) As a consequence of (3), the reserve level is decreased as the time until an item will be back in a good supply position is decreased.

3.1 SCOPE AND METHODS

Part II of Study.

A requisitioner may not immediately need all the stock he requests from the NICP. Under some circumstances he is encouraged to order up to a year's supply of stock (in terms of his needs) in order to avoid the processing of a large number of requisitions for small amounts.

An investigation was made into whether the NICP could advantageously make reduced shipments to such requisitioners for items in short supply, i.e. ship less than the quantity specified on the requisition. In so doing the NICP could decrease the likelihood of running out of stock and incurring backorders. At the same time, however, it would force the requisitioner to submit another requisition for the item sooner than he would otherwise have to, and if the NICP was not in a good supply position by the time the requisitioner submitted another requisition, nothing would be gained by the extra effort.

A cost effectiveness analysis was made balancing expected reductions in NICP backorders through use of reduced shipments against increased requisition processing cost.* Basic to this analysis was demand data collected manually at Ft Dix, a typical NICP customer. Using this data and a rather simple simulation program, it was possible to see what the effect on Ft Dix's need to requisition would be if various reduced shipment policies were used by the NICP. Another major input to the cost effectiveness analysis was a study done by Harbridge House, Inc. on what Army requisition processing costs are.

These two inputs were sufficient to develop an approximate, but meaningful estimate of the cost of reducing backorders using a reduced shipment policy. In developing this estimate, the fact was taken into account that on occasion a reduced shipment policy would merely have the effect of "robbing Peter to pay Paul", i.e. the NICP would be unable to satisfy the original requisitioner when he reordered. Also, on occasion, a reduced shipment would prove to have been unnecessary; the NICP would not have run out of stock anyway. Improving supply

*As used here the term includes the processing of the physical shipment as well as the requisition.

performance by the use of reduced shipments was compared with the costs of raising safety levels to accomplish the same objective. The economic concept of marginal cost was used in determining the most economical reduced shipment policy. Finally estimates were developed of the potential for application of reduced shipments.

4.1 FINDINGS AND CONCLUSIONS

Part II of Study.

(a) The NICP's should utilize informations on their customers' replenishment cycles to make reduced shipments when economical for items in poor supply. Such a step would provide a cheap method for improving supply performance, albeit one with limited application.

(b) To provide the NICP with the necessary information, a simple numeric code should be placed in column 44 of the MILSTRIP requisition on replenishment actions, in place of the R currently used. This would not be costly to implement and the information could have several uses, e.g. in excess quantity checks.

(c) Determination of when a reduced shipment should be made, and how large it should be, should be made in accordance with a procedure similar to that developed in this study (Chapter II, section 9). This procedure can be automated.

(d) Subject to acceptance of the above conclusions the Inventory Research Office, Army Logistics Management Center, should collect and analyze additional data to make more definite the exact procedures to be followed.

(e) A study should be undertaken, regardless of whether the reduced shipment concept is implemented, to develop a method for eliminating double requisitioning; at present the requisitioner sometimes has to submit a high and a low priority requisition at the same time for the same item.

CHAPTER I

RESERVE LEVELS FOR HIGH PRIORITY REQUISITIONS

1.1 The Reserve Level Concept.

A reserve level is a device designed to insure satisfaction of high priority demand even on items in poor supply positions. It works as follows: when stock on hand falls below the reserve level, satisfaction of low priority demand is temporarily suspended to conserve the stock available for high priority needs. The magnitude of the reserve level can change with circumstances; its determination is part of the work of this paper.

The use of a reserve implies that the supplier is willing to sacrifice the satisfaction of a lower priority demand now to have material available for higher priority requisitions later. The supplier is willing to do this because the satisfaction of one is more important than the satisfaction of the other. The supplier is taking the chance that the potential high priority demand for which he is setting aside stock may not materialize, or not until shortly before the arrival of new stock would have permitted him to satisfy it anyway. Whether it is worthwhile for the supplier to take the chance depends on just what the probabilities are, and how much more important the high priority demand is than the lower priority demand.

The study did not attempt to directly estimate probabilities, but instead used simulation to determine the effect of different reserve policies on both high and low priority backorders. In evaluating these effects, weighting factors were used to express the relative importance of high priority demand.

Applicable weighting factors are considered in detail in section 8. The meaning of the term is best explained by illustration: a weight of 5 implies that a delay of 1 day in shipping a unit of stock needed to satisfy a high priority need is as harmful to the Army's immediate mission capability as a delay of 5 days in shipping a unit to satisfy a low priority need.

1.2 Alternative Policies Considered.

Three methods for setting the reserve level were considered.

Method 1: Set the level equal to zero i.e. have no reserve.

Method 2: Set the reserve as a fixed proportion of expected high priority demand during the TTR ("Time to Replenishment" or the time remaining before the arrival of new stock eliminates the short supply situation.

Method 3: Use a set of tables developed to give mathematically optimum reserve levels.

The basis for these tables is a mathematical model which is described in the appendix.

Both the second and third methods considered are time dependent; they take into account how long a period is left before the expected restoration of stock levels. Time-independent rules have been suggested e.g. to use the safety level as a reserve. If the NICP has no real idea of the TTR some form of time independent rule has appeal. Otherwise, use of a time-independent rule ignores a vital factor, namely how long will the shortage last.

The second method is not based on any explicit description of demand probabilities, while the third is. Since the description available may not be accurate, there is no certainty that the third method will give the best results. And there are considerations of simplicity, and of robustness, which would prevent adoption of the third method unless it gives significantly better results than the simple method. Robustness relates to the efficacy of a decision when the assumptions upon which the decision is made are not completely true, e.g. the correct weight for high priority is larger than estimated.

The work devoted to development of method 3 (see Appendix) proved that the reserve level should be independent of expected low priority demand, and that the reserve level should not necessarily be increased because of increased variability of demand. Method 2 consequently ignores both these factors.

1.3 The Simulation.

The major part of the research effort was devoted to the development and running of a simulation by which the effects of different reserve policies could be determined. The simulation was the experimental tool for the reserve level part of the project. This simulation used real demand histories to create realistic test cases. The source of this data was the U. S. Army Aviation Materiel Command (see Note 1 for how the histories were constructed). Data was provided for 323 low and medium dollar value items, but only 235 could be used because insufficient amounts of history were available on the others.

This is how the simulation worked: We would specify the number of days in the testing period, 84 for example. This would then be the initial "Time to Replenishment" (TTR) of the NICP stock levels. The program would set stock on hand at the beginning of the 84 days equal to expected high priority (Issue Group I*) damand for 84 days. Depending on the reserve policy being tested some or all of this stock might be considered as a reserve. Every review cycle**, the reserve level was changed in accordance with the reserve policy being tested and changing circumstance, e.g. the change in the TTR.

Demands meanwhile would be received for the item. These demands, their quantities, time of occurrence, and priority were taken from the AVCOM data. When a demand was received stock was issued if available; that is, if stock on hand were above zero for a high priority demand or above the reserve level for a low priority demand. Stock on hand changed appropriately when an issue was made.

If stock were not available for issue, the demand was backordered (if it could be only partly filled with available stock, the unfilled part would be backordered). If at the end of a review cycle, the reserve level was reduced and this freed some stock for issue to low-priority customers, this stock would be applied against any backorders outstanding, in the order in which they had arrived. To illustrate: if the reserve level were 10, stock on hand 9 and a low priority order was received, it would be backordered; later, the reserve level could drop to 6. If no high priority demand had occurred meanwhile, stock on hand would still be 9, and 3 units could be issued against low priority backorders. Note that the reserve level is computed without reference to stock on hand, but rather stock on hand is compared to the reserve level to determine whether to issue to low priority customers.

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*Issue Group I constitutes an important class of requisitions quantitatively, as well as qualitively. There was no difficulty in extending the reserve concept to multi-priority classification of demand (sections 8, 10)

**A review cycle of 2 weeks was used. In a special test a review cycle of 1 week was used with negligible effect on results.

Records were kept on all backorders which occured, also the number of low and high priority demands on backorder at any time as well as the quantities on backorder. These statistics were used to build up measures of performance.

The whole process described could be executed simultaneously for several different reserve policies with performance statistics recorded on each policy. At the end of the time period specified (e.g. 84 days), stock levels would be presumed restored. The process would then be repeated. In the simulation 336 days of history for each item were used. Hence, when there were 84 days in the test period, there could be 336/84 = 4 test cases per item.

1.4 Use Made of the Simulation.

Two series of tests were made, supplemented by various special tests. The first series consisted of three computer runs. In each run not only were several reserve rules tested, but results were obtained under varying assumptions about the relative importance of high priority vs low priority needs. High priority weighting factors of 2, 4 and 10 were considered in each computer run.

The three computer runs differed only in that initial TTR's of 56, 84 and 122 days were used respectively (the "initial TTR" was explained in the last section). By comparing the various methods for setting reserves discussed in section 2, the tests were designed to answer the questions: (a) Is a reserve useful? (b) Is a sophisticated method leading to a mathematically optimal level to be recommended? or (c) Does a simple method give us good results?

Actually 4 rules were tested.

Rule 1: Keep no reserves.

Rule 2: Set the reserve equal to the expected high priority demand which will be sustained prior to stock replenishment.

Rule 3: Set the reserve level equal to a fraction of expected high priority demand, the fraction depending on the weight assumed for high priority. For weights of 2, 4, and 10, fractions of .5, .75, and .9 were used respectively. These fractions actually have some theoretical significance*.

*It can be shown that if high priority demand occurs at a constant known rate, then the optimum reserve level equals D x (W-1)/W where
W is assumed weight for high priority
D is demand for high priority in the TTR.

Rule 4: Use the tables constructed by our mathematical model.

To anticipate, it was concluded that rationing could be useful, and that using a fraction of expected demand in the TTR was simple, yet would not give significantly inferior results than more sophisticated rules. The second series of simulation tests was used to determine the best fraction to recommend and how sensitive the recommendation was to different assumptions about the "correct" weight for high priority requisitions.

1.5 Performance Measures.

In evaluating test results, two measures of performance were used primarily. The first was the average dollar value of requisitions on backorder, weighted by priority. For instance, suppose that a requisition for 5 widgets was backordered for 10 days, that a widget costs $2.00, and that a weight for high priority of 4 was assumed. Then, if the requisition were low priority, the penalty would be 5 (widgets) x 10 (days) x $2.00 (value of a widget) = 100. If the requisition had been high priority, the penalty would be 100 x 4 = 400. Total penalty for all backordered requisitions is divided by the number of days in the simulation (336) to get the average value of requisitions on backorder, weighted by priority.

The other performance measure given much attention was the average number of requisitions on backorder, weighted by priority. For the requisition just cited we would have calculated a penalty of 1 (# of requisitions) x 10 (# of days) if the requisition were low priority, and 40 if it were high priority. Again after getting a total penalty figure we divide by 336.

In reporting results, backorders produced when no reserve is used are compared to backorders produced by each of the rationing rules. A percent of 100 means no difference in performance, while 95% means that with the rationing rule average backorders, calculated as we have described, are only 95% as large as they are without rationing.

The shortcoming of the first performance measure is that it gives more importance to the dollar value of a backorder in determining its significance than it should. On the other hand, the second measure, which is more consistent with other measures current employed, assumes in effect that all requisitions are equally important irrespective of how much they are for: yet it would be expected, for instance, that a low priority backorder for 5 widgets would be less important than one for 50 widgets. The first measure is considered "better", but is usefully complemented by the second (as in section 7).

1.6 Initial Findings.

The initial findings (see table 1) indicated that for an applicable weighting factor of 2 a reserve policy could not be justified; if the applicaable factor were 4, a reserve policy would lead to a small improvement in supply performance. For an applicable weight of 10, improvement would be substantial.

These results should be taken as being very conservative estimates of the improvement which could be expected. The effectiveness of a reserve policy will increase with the accuracy of the forecasts of high priority demand. One of the problems in running the simulation was that there are no statistics available which could be used as good indicators of what the error in forecasting would typically be in practice, nor could the forecasts of high priority demand which AVCOM might have made for the particular items in the study be feasibly reproduced. A simple forecasting scheme was used in the simulation, one which did not utilize problem factors for instance (see Note 1). Demand itself was showing the impact of Vietnam and was especially difficult to predict.

Large forecast errors were experienced. Quarterly forecasts had average errors close to 100%. Table 2 shows the improvement effected by rationing when very good forecasts were used, as explained in Note 1. Other results (see next section) also make rationing look better.

A look at table 1 reveals a tendency for the reserve rules to do better as the TTR increases. This is not an accident. Over longer periods of time the irregular fluctuations of demand became much less significant; therefore, better forecasts can be made, unless there are unanticipated shifts in the general level of demand.

Does a sophisticated rationing rule do much better than a simple rule? A comparison of the results given in table 1 indicates that the sophisticated rule, rule 4, gives only very modest overall improvement over rules 2 and 3; in some cases it actually gave poorer results. By themselves these results would be somewhat inconclusive, because rule 4 depends upon an explicit description of demand probabilities, and it is certain that the description of the probabilities used could be improved with additional research. However, the results are confirmed by a special experiment which was made.

A simulation was run with demand data we had generated ourselves, (see appendix). Because of the manner in which this demand data was

Table 1: First Test Series

	Weights 2	4	10
56 days per trial			
rule 2 vs no reserve	103.0%	100.4%	97.7%
rule 3 vs no reserve	100.9%	99.9%	97.7%
rule 4 vs no reserve	100.0%	99.7%	98.1%
84 days per trial			
rule 2 vs no reserve	103.1%	99.1%	94.6%
rule 3 vs no reserve	100.7%	99.0%	94.7%
rule 4 vs no reserve	100.1%	99.0%	93.4%
112 days per trial			
rule 2 vs no reserve	102.4%	98.4%	94.1%
rule 3 vs no reserve	100.6%	98.2%	94.4%
rule 4 vs no reserve	99.9%	98.4%	93.4%

Table 2: Effect of Very Good Forecasts

	Weights 2	4	10
84 days per trial			
rule 2 vs no reserve	103.1%	96.4%	88.0%
rule 3 vs no reserve	100.0%	96.1%	88.3%
rule 4 vs no reserve	99.5%	96.3%	86.0%

generated, we could be sure that rule 4 would be the optimum rule. Even in this case, however, the degree of improvement resulting from use of the sophisticated, best possible rule, compared to using the simple reserve rules, was relatively small. Most of the benefit of rationing can apparently be obtained by using a simple rule. This is true even though the specific reserve levels suggested by rule 4 are in certain cases quite different than those suggested by rules 2 and 3.

1.7 Final Simulation Results.

The main results of the tests* conducted to determine what fraction of expected high priority demand should be used as a reserve level are depicted in two graphs, with table 3 giving selected results from the graphs. Each curve on a graph is a performance curve, relative to maintaining no reserve at all. There are performance curves for setting the reserve level equal to .5, .75, 1.0, 1.25 and 1.5 times the expected demand for high priority needs in the TTR. The vertical axis of the graphs indicates the degree of improvement over maintaining no reserve, and the horizontal axis the applicable high priority weighting factor for which this improvement is realized. As in our earlier figures, a 90% performance figure means that the average penalty for backorders was 90% with a particular reserve rule of what it would have been with no reserve.

The difference between the two graphs is in the performance measure used. The performance measure for Graph I was that used previously based on average dollar value of requisitions on backorder weighted by priority. The performance measure for Graph II was based on number of requisitions on backorder, as described in section 5.

Table 3 abstracts from the graphs the answer to the following questions:

(1) How high must the weighting factor for high priority be before a given reserve level does better than no reserve (where does its performance curve cross the 100% line)?

(2) At what weight would a reserve level first do better than the next smaller reserve level (where do their lines first cross)?

*In these tests initial on hand was set equal to 1.5 times expected high priority demand. A low bias which was noted in the simulation forecasts was removed to avoid distortion. The initial TTR was 84 days.

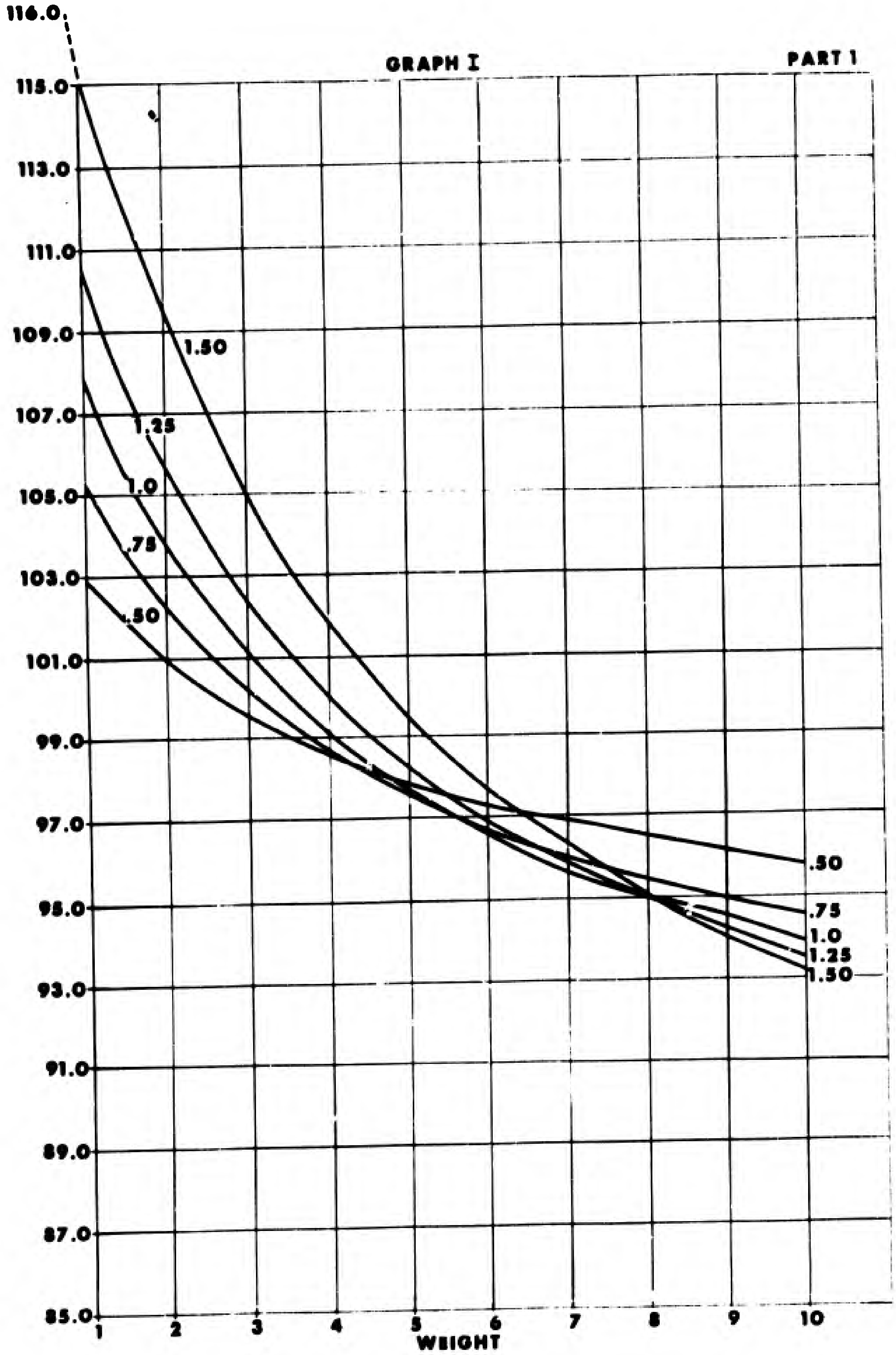
Table 3: Selected Results From Graphs

Graph I

reserve level fraction		.5	.75	1.0	1.25	1.50
(a) crosses 100% line		2.6	3.2	3.6	4.0	4.8
(b) crosses line of next lower factor			4.2	5.2	8.1	8.2
(c) improvement for weight						
	=2	100.9	102.1	103.7	105.6	109.4
	=5	97.8	97.6	97.7	98.3	99.5
	=10	95.9	94.7	94.0	93.6	93.2
	=100	93.3	90.6	88.7	87.2	84.4

Graph II

reserve level fraction		.5	.75	1.0	1.25	1.50
(a) crosses 100% line		1.0	1.2	1.4	1.6	2.4
(b) crosses line of next lower factor			2.6	3.2	4.8	8.2
(c) improvement for weight						
	=2	95.4	95.9	97.0	98.6	101.8
	=5	90.2	88.9	88.2	88.1	89.3
	=10	87.0	84.7	83.0	82.2	81.8
	=100	83.0	79.6	76.6	74.6	72.5



GRAPH I
PART 1
116.0
115.0
113.0
111.0
109.0
107.0
105.0
103.0
101.0
99.0
97.0
95.0
93.0
91.0
89.0
87.0
85.0
1.50
1.25
1.0
.75
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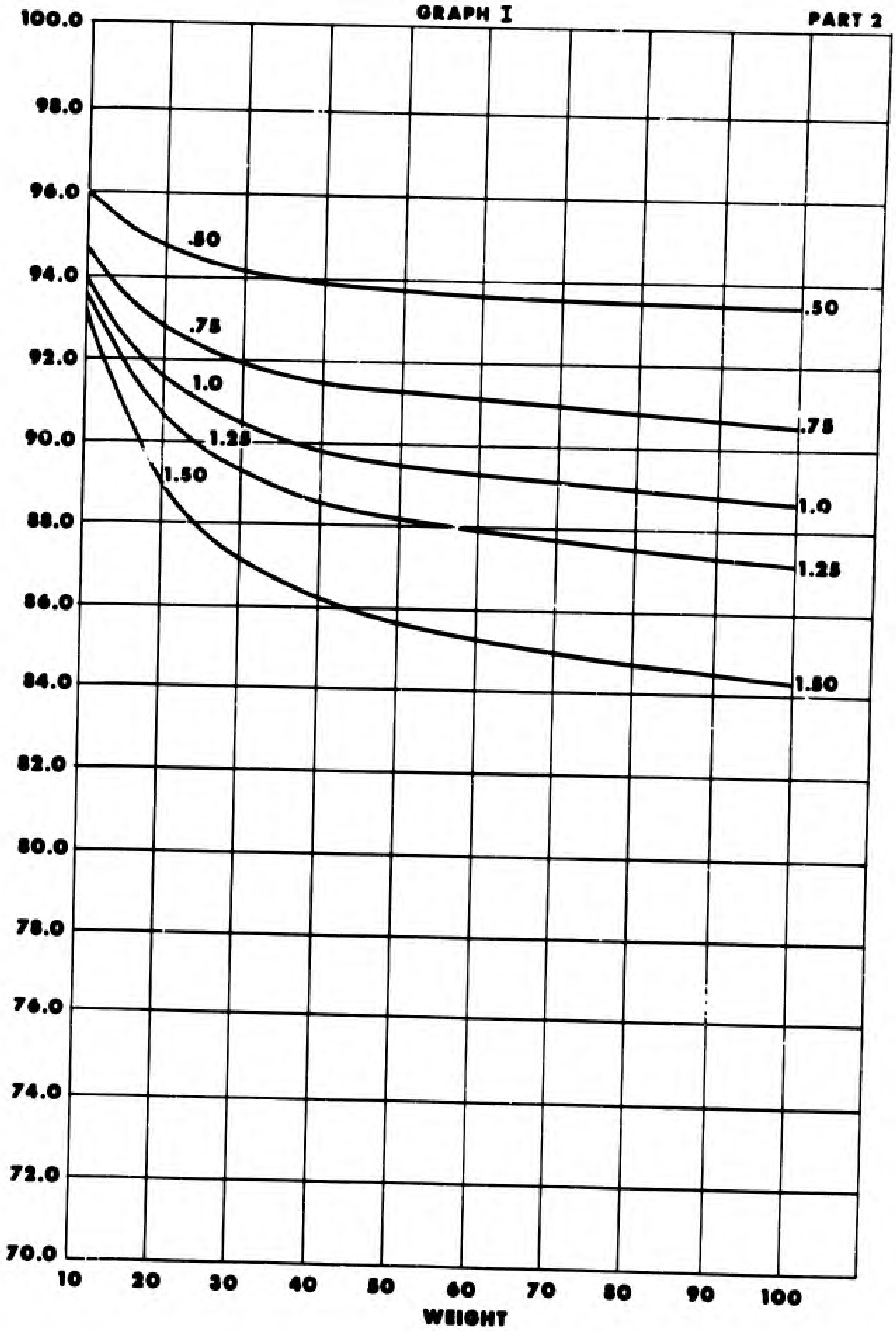
10
WEIGHT


GRAPH I
PART 2
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98.0
96.0
94.0
92.0
90.0
88.0
86.0
84.0
82.0
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78.0
76.0
74.0
72.0
70.0
.50
.75
1.0
1.25
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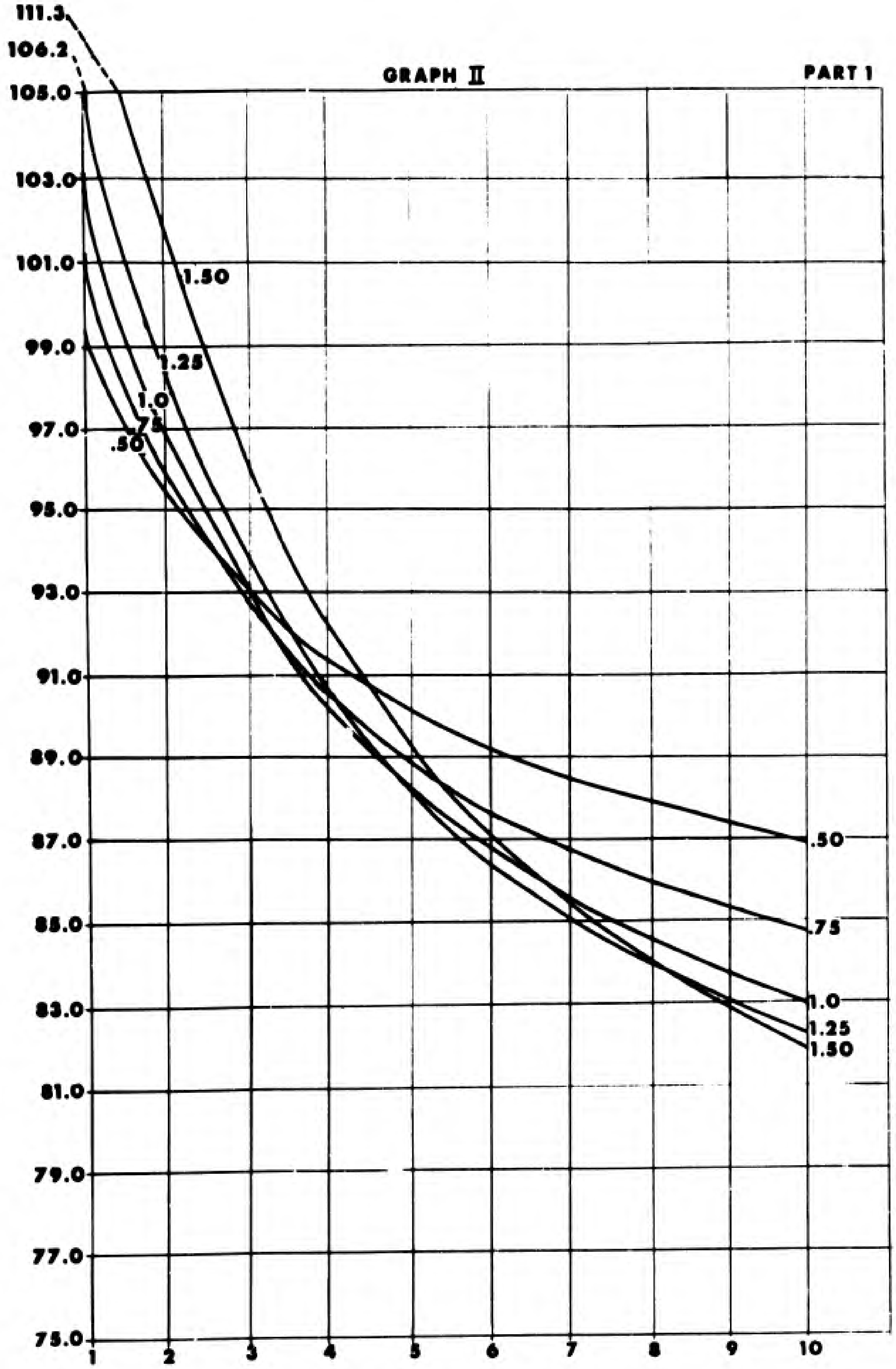
90
100
WEIGHT


GRAPH II
PART 1
111.3
106.2
105.0
103.0
101.0
99.0
97.0
95.0
93.0
91.0
89.0
87.0
85.0
83.0
81.0
79.0
77.0
75.0
1
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3
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5
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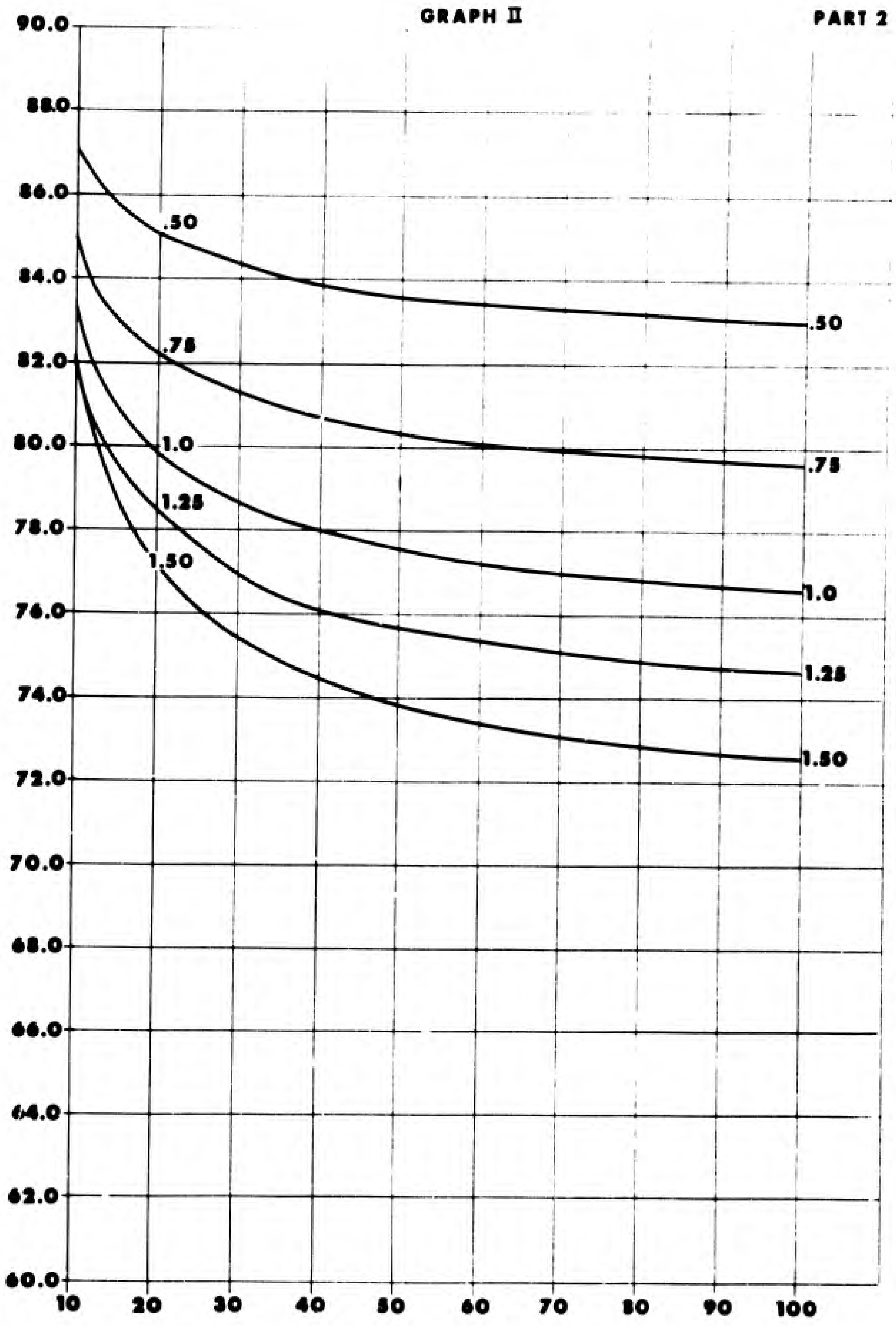
1.25
1.50



GRAPH II
PART 2
90.0
88.0
86.0
84.0
82.0
80.0
78.0
76.0
74.0
72.0
70.0
68.0
66.0
64.0
62.0
60.0
10
20
30
40
50
60
70
80
90
100
.50
.75
1.0
1.25
1.50

(3) How do the different levels compare at selected points: weights of 2, 5, 10, 100, and so on?

This examination suggested that first of all the usefulness of a reserve is not very sensitive to the exact level chosen. Consequently, it is difficult to make too strong a case in favor of one reserve level factor rather than another. As a corollary to this, it is evident that only a rough estimate of the weighting factor is needed, because choice of the exact magnitude of the reserve level is not critical. This is discussed again in the next section "Importance of High Priority Demand".

Rationing looks very much more effective in Graph II than in Graph I. There is no single explanation, but rather several factors, which are listed below. The first two factors tend to make Graph II results better than they should be; the third factor offers another indication that Graph I results are poorer than they should be.

a. The weights assumed for high priority demand no longer have the same interpretation. Low priority requisitions are typically larger than high priority requisitions (see note 8). Hence, even by weighting high and low priority requisitions equally, the implication is that the high priority need is more important since the low priority requisition of say 6 widgets on backorder is counted no more heavily than the high priority requisition for 4 widgets.

b. The rationing process will tend to keep larger low priority requisitions on backorder, accentuating the effect referred to in reason a.

c. Items with few requisitions would tend to be harder to forecast, and this would affect the dollar value based measure of performance more.

d. The performance measure based on requisitions gives more weight to items with requisitions whose average dollar value is relatively small. Such items, at least in the AVCOM data worked with, tend to have a smaller proportion of high priority demand. For high weights this increases the effectiveness of rationing.

If use of a reserve level is to be justified, the applicable high priority weighting factor must be at least 3. If the applicable weighting factor is believed to be between 3 and 10, reserve level fractions of either .50 or .75 are conservative choices. If it is believed that the weight for high priority should be at least 5 a reserve level fraction of 1.0 is recommended. Higher fractions do not lead to very much greater improvement even for high assumed weights. A result not shown in the table is that for a weight of 100, a reserve level factor of 2 was tested and gave a performance measure of 83.8% (cf to 88.7% for fraction of 1.0).

1.8 Importance of High Priority Demand.

The effectiveness of a reserve is not very sensitive to the exact magnitude of the reserve level. As a coroliary, only a rough estimate of the high priority weighting factor is needed, since the choice of levels based on this estimate will not be inappropriate for "true" weights different from the estimated weights. For example, while a reserve level fraction of .75 is optimum only if the weight is between 3 and 6, it can effectively be used for weights between 3 and 100.

To derive an estimate of the applicable weights the maximum overseas order and ship time allowances for troops support requisitions as specified in AR 725-50 were used. These are:

	Priority	Time	Implied Weights
Issue Group I	1-3	7 days	9.3
II	4-8	15 days	4.3
III	9-15	50 days	1.3
IV	16-20	65 days	1.0

The implied weights are derived as follows: if 7 days delay for Issue Group I is considered to be the equivalent of 65 days delay for Issue Group IV, this implies a weight of 65:7 and 65/7 is 9.3; 65/15 is 4.3 and 65/50 is 1.3.

Maximum ship times are influenced by the out of pocket costs for improving them, so no preciseness can be claimed for the implied weights.

There is one context in which the implied weights are not suitable for rationing purposes. Once stock issue to priorities 9-20 is suspended, a distinction should be made between priorities 4-6 and 7-8, which is not made in the time standards. The rationale is this: priorities 7 and 8 as defined in MILSTRIP represent requisitions from units with high urgency of need, but with low priority: typically units in CONUS which are being maintained in a state of operational readiness for deployment after D+30 or D+90. If stock is withheld from such requisitioners it may be issued to priority 1-6 requisitioners which is not undesirable. If it is not, it will still be available to the original priority 7-8 requisitioners within 7 days if the status of their units should increase.

These considerations led to a dual-level concept (see recommendations). Once the on hand stock has fallen so low that issue to priorities 9-20 is suspended, further protection is given to priorities 1-6 consistent with the following approximate weighting scheme.

Priority	Weight
1-3	9.3
4-6	6.0
7-8	2.0

1.9 Effect of a Reserve on Initial Fill

Initial fill statistics were kept as a secondary performance measure. They are presented in table 4 for the simulation run with TTR of 84 days. The actual initial fill achieved was highly dependent on the simulation mechanism for initially setting stock on hand and is not to be taken as indicative of initial fill achieved by the NICPs.

Two valid conclusions can be drawn from table 4, however. A reserve policy will generally decrease over-all initial fill while increasing initial fill for high priority requisitions. The degradation in total initial fill will be small.

TABLE 4

INITIAL FILL

Reserve Level Factor	Total	High Priority
.00	60.9%	80.2%
.50	61.0	83.3
.75	60.3	84.1
1.00	59.5	84.7

1.10 Recommendations.

a. There should be two reserve levels as per the following:

Level 1:

Purpose: To protect stock for priorities 1-8 against issue to priorities 9-20.

Computation: Expected demand in the TTR for priorities 1-3 + 1/2 expected demand for priorities 4-8.

Level 2:

Purpose: To protect stock for priorities 1-6 against issue to priorities 7-8.

Computation: 3/4 expected demand in the TTR for priorities 1-3 + 1/2 expected demand for priorities 4-6.

Most items will of course have stock on hand in excess of both reserve levels. As an item gets into trouble, stock on hand will fall below level 1 and issue to priorities 9-20 will be temporarily suspended. If the item gets into more serious trouble, stock will fall below level 2 and issue to priorities 7-8 will be suspended as well.

b. Reserve levels should be recomputed periodically, in particular as the time to stock replenishment decreases. Requisitions should not be put on backorder, or kept on backorder, on the basis of a reserve level which has not been updated in the previous 2 weeks.

c. In determining whether mobilization stocks can be counted in comparing stocks on hand to a reserve level, the following rule of thumb should be used: if the mobilization stocks are normally issueable to satisfy <u>all</u> priorities protected by the level, they can be counted.

d. In the absence of a better method, high priority demand can be forecasted by multiplying the projection of total demand by the proportion of high priority demand in the last 12 months.

e. Rationing should be automatic except for high dollar items, or long TTR's. In particular, rationing should not be initiated automatically if the reserve level exceeds the safety level. For all items, the item manager should be notified when stock on hand first falls to a reserve level.

1.11 Validity of the Results.

The validity of the results may be questioned from two viewpoints. Was the sample of items sufficiently large and representative? Did the simulation distort the real world in such a way as to give misleading results?

One rough check of the adequacy of the sample is to see whether by using just half the data, either half, we would have reached the same conclusions as we did using all of the data. For the final simulation run we looked at the output based on the first 112 of 235 items in the sample (it was inconvenient to look at 117 items or exactly half). Selected results were computed. The results were somewhat poorer than those based on all items, and a reserve fraction of .5 did relatively better compared to fractions of .75 and 1.0 than it did in the total sample (This of course means that results based on the second half of the items were better than those based on the total sample; the .5 factor did relatively worse).

None of the recommendations would be invalidated by the results based only on the first half of the items. Rationing did considerably better in reference to the requisition-based performance measure than in reference to the dollar based measure in both halves of the data.

All items in the sample had backorders at the time the selection process was made. Therefore, if it is true that it is certain items which are most prone to backorders, and that these have different demand characteristics than the others, we have avoided this problem. In the sample there were 194 items with annual demand less than $2500 and 41 with annual demand greater than $2500. There were no HDV items. We looked at results based on the two separate groups, and found that rationing was somewhat less effective with the higher dollar value items.

Was the simulation misleading? It was assumed that the time until stock replenishment was known. Even with material already on contract this is not known with certainty. Essentially, however, we are not so much interested in the date of stock replenishment as in the high priority demand that will occur by that date. In the typical case where there is a delay in receipt of material, the effect will be that our estimates of high priority demand in the TTR, and hence our reserve levels, will be too low. In view of the relative insensitivity of performance to the exact size of the reserve, this is not so serious.

In the simulation it was assumed that stock freed for issue to low priority requisitions by a drop in the needed reserve quantity could always be used to partially fill the most pressing low priority backorder if insufficient to fully satisfy it. In practice some other policy might be desirable to avoid too many shipments.

Was it proper to extrapolate from the results of the simulations to the final recommendations? The soundness of the multi-reserve concept and application follows analytically from the work done on the 2-priority case. It is true, however, that as the demand for which stock is being reserved becomes a large percent of total demand what a reserve level can accomplish decreases - at the extreme, a reserve for 100% of total demand is equivalent to having no reserve. Therefore, reserving for Issue Group II as well as I runs into the law of diminishing returns. On the other hand, it is to be hoped that the current quantitative importance of high priority demand will diminish.

The most serious question is whether taking account of "feedback", if it had been practical to do so, would have changed the conclusions. Most Issue Group IV requisitions are for stock replenishment; backordering then can induce subsequent high priority requisitions, which we call feedback.

It is difficult to estimate how much feedback occurs. While feedback from unsatisfied Issue Group IV requisition is one source of high priority demand, there are other important sources. Items which the NICP stocks are not stocked by all of its customers. When a lower level supplier does stock an item, it is quite possible for him to be out of stock temporarily although none of his requisitions are backordered. This happens because of great demand variability.

Statistically, it is apparent that feedback is not the major source of high priority demand. The NICP's maintain an average stock availability of 80% or higher. Issue Group IV demand accounts for far less than half of total demand. Backordering of these requisitions, therefore, could not account for most of the other requisitions. A look at the figures on the statistical importance of Issue Group IV demand given in section II of the next chapter reveals an interesting fact: for the items in the AVCOM sample Issue Group IV demand was slightly more significant than the figures obtained for all NICP's for the Jan - June 1966 time frame. Yet the AVCOM items were all items in trouble with backorders.

Given that some feedback will occur, what does this mean to rationing as considered in this chapter? It means that some improvement in troop support has been obtained at the cost of forcing the NICP customers to submit extra requisitions, the induced high priority requisitions.* Scarce stock is issued only to satisfy existing, important needs as they occur. This keeps stock from sitting in customer storage bins, but at the same time some of the cushion between NICP and final user is temporarily destroyed.

*There would be some reduction expected in low priority requisitions from monthly orderers.

CHAPTER II

REDUCED SHIPMENTS

2.1 Concept

Under the Economic Order Quantity policy (AR 711-16) and Economic Inventory Policy (AR 711-25) a requisitioner can request up to a year's supply of stock (in terms of his needs) from the NICP. The advantage in permitting this is that the number of requisitions which must be submitted during a year is decreased, thereby reducing the attendant costs to both the requisitioner and the NICP. One disadvantage, however, is that infrequent ordering contributes to the irregularity of NICP demand and so indirectly to backorders.

In respect to these policies there is a way the Army can have its cake and eat it too. It is proposed that the NICP be allowed to nullify the unhappy effect of the EOQ and EIP policies by being permitted to reduce requisition quantities when appropriate for customers ordering under these policies. Suppose the NICP were running short of stock, and that its TTR* were 3 months. If a requisitioner orders what for him is 6 months supply, it might make sense for the NICP to reduce the quantity to be shipped from 6 months to 3 months supply, using an appropriate status code to inform the requisitioner of the action. The decision to reduce ship, in most cases, could be made automatically by computer in accordance with specific decision rules.

Reduced shipments tend to increase the number of requisitions which must be processed. If a reduced shipment is to be justified, the backorders expected to be avoided by it must offset the increased costs of requisition processing. An analysis was done demonstrating that it was often much cheaper to avoid backorders by using a reduced shipment policy than by increasing safety levels. Various rules for determining the size of reduced shipments were considered.

For a reduced shipment policy to be implemented, information must be provided to the NICP on how many months supply each replenishment requisition represents. This information could possible be provided through the Army Field Stock Control System. Preferably, the requisitioner

*TTR as defined previously is the "Time to Replenishment" of the NICP's stock levels, perhaps through material already on contract.

would indicate on the MILSTRIP requisition card the number of months supply being ordered. This indication could be a simple numberical code placed in column 44 instead of the R, which would no longer be needed to indicate that the demand is a recurring demand.

2.2 Methodology.

The main thrust of the work done on reduced shipments was directed toward estimating the average cost which would be incurred per dollar reduction in average backorders. Since the cost of reducing backorders by increasing the safety level is readily estimated, a cost effectiveness comparison could then be made. A multi step procedure was needed.

First, using data collected at Ft Dix, it was experimentally determined how many extra requisitions would need to be processed for each $1000 of stock conserved by reduced shipments. Then this result was converted into a cost per dollar conserved using an estimate of the cost of requisition processing. The cost of requisition processing was calculated by updating and adjusting the costs found by Harbridge House, Inc. in a major study done in 1959.

Stock is conserved when it is withheld from customers who do not need it during the time the item is in short supply. It is conserved in the sense that more stock is made available for issue to other customers, without hurting the original requisitioner, whose next order is received when the item is back in a good supply position and can be satisfied in full.

The expected reduction in average backorders which will be realized for each dollar of stock conserved can be calculated. Hence, it was possible to convert the cost per dollar of stock conserved by the use of reduced shipments into a cost per dollar reduction in average backorders. This step made use of thecretical work done in connection with the "MIT model" for Variable Safety Levels and Economic Order Quantities.

The actual cost of reducing backorders by raising the safety level depends in a mathematically simple fashion on how well the supply system is performing, expressed in terms of item availability*. In general, it is less costly to effect improvement when the supply system is performing poorly. Since the cost of reducing backorders using reduced shipments does not depend on how well the supply system is performing, a cost

*Item availabilityis % of time item is in stock.

effectiveness comparison was done assuming that the system was operating at various performance levels.

In demonstrating the cost effectiveness of a reduced shipment policy, the formulation of specific rules for determining the size of reduced shipments was considered. Use was made of the marginal cost concept of economic analysis. In the work done the TTR was always taken to be 3 months. Prior to implementation of a reduced shipment policy, additional data needs to be collected, and additional calculations made so that the implementation procedure can be formulated in detail.

Some attention was given to estimating the potential for application of a reduced shipment policy.

2.3 Extra Requisitions Processed.

When a reduced shipment is made, there are 3 possible outcomes:

(a) The shipment satisfies the customer's needs until such times as the NICP's stock levels are replenished.

(b) The shipment proves inadequate in that the customer has to reorder before the end of the TTR. However, the customers' demand was such that even a full shipment would not have satisfied his needs through the TTR.

(c) The reduced shipment proves inadequate, but a full shipment would have satisfied the customer's needs through the TTR.

With outcome "a" stock is conserved and no extra requisitions are processed in the TTR. With outcome " c" any true conservation of stock is doubtful and an extra requisition must be processed. Outcome " b" is something of a standoff, although in general there is some advantage to having shipped more initially.

Experimental work was done in which the data collected at Ft Dix was used to estimate how likely each outcome was. Note 2 goes into some detail on the collection, validation, and use of the Ft Dix data. By recording when Ft Dix ordered various items, what it thought its demand was going to be when it ordered, and what the demand actually was, it was possible to determine how often full shipments were inadequate*, and how often reduced shipment would have been inadequate.

*It was assumed Ft Dix had just enough in stock when it ordered to cover demand during the order and shiptime, so that alternatively order and shiptime could be considered zero and Ft Dix on hand zero.

The probabilities for each outcome were calculated, assuming a TTR of 3 months, for various rules determining the size of reduced shipments. Probabilities were calculated separately for items for which Ft Dix orders 6 months supply and for items for which it orders 12 months supply.

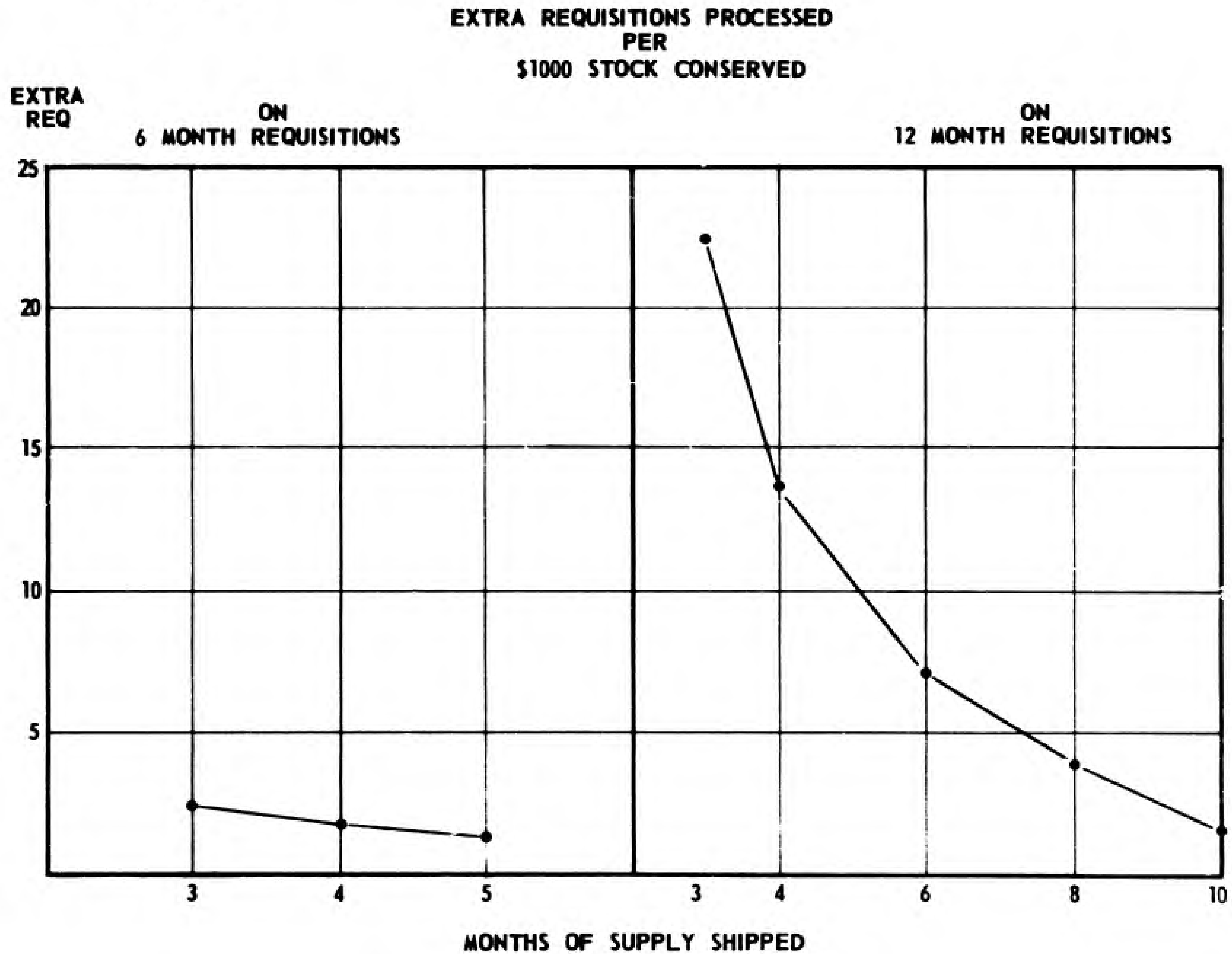
Using the probabilities, an initial estimate was made of stock conserved and extra requisitions processed under various reduced shipment policies. For this estimate outcome "b" was ignored. This initial estimate was then revised using an analysis of outcome b. Also the need to process induced high priority requisitions was taken into account (see section 4). Finally results were reduced to the form of extra requisitions processed for $1000 of stock conserved. A discussion of the methodology is given in note 3.

The results are depicted in Graph C. Each rule for determining the size of the reduced shipment specified shipping a different number of months supply, but in no case less than the customer's expected demand in the TTR. Two patterns are striking: (a) It is more costly to apply a reduced shipment policy to 12 month requisitioners than to 6 month requisitioners, and (b) it becomes progressively more costly as less is shipped. Both patterns are understandable.

The greater success of reduced shipments made on 6 month requisitions than those made on 12 month requisitions relates to the fact that under the economic ordering policy as given in AR 711-16, the 6 month requisitions must always be of a larger dollar value than 12 month requisitions. Data collected at Ft Dix indicated that the actual values averaged $289.53 and $55.47 respectively. Now suppose, for instance, a 3 month supply of stock is shipped to a requisitioner. The likelihood of its being inadequate is not affected very much by whether the customer was a 6 or 12 month requisitioner. The amount of stock which can be potentially conserved is affected, however. While 3/6 of $289.53 is $144.76, 9/12 (amount saved on12 month requisition) of $55.47 is only $41.60; for about the same risk of an extra requisition being processed, more stock can be conserved on the 6 month requisition.

The increasing costliness of smaller shipments is a manifestation of the "Law of Diminishing Returns". Reducing the shipment size by 2 months supply may increase the incidence of inadequate shipments only slightly, but reducing it by still 2 more months has a much greater effect and so on. This does not mean smaller reduced shipments should be ruled out. The point is that by shipping close to a full shipment not enough stock may be conserved on a particular item to avert sizeable backorders. The concept of marginal cost, which applies here, will be illustrated in section 8.

GRAPH C


EXTRA REQUISITIONS PROCESSED
PER
$1000 STOCK CONSERVED
EXTRA
REQ
ON
6 MONTH REQUISITIONS
ON
12 MONTH REQUISITIONS
25
20
15
10
5
3
4
5
3
4
6
8
10
MONTHS OF SUPPLY SHIPPED

2.4 Induced High Priority Requisitions.

As would be expected, demand on the Ft Dix supply system was highly irregular. This meant that demand in a given month would often not only bring the post to its reorder point, but could actually cause backorders. In such a situation, it is authorized by AR 725-50 to submit a higher priority requisition in addition to the Issue Group IV replenishment requisition. Using the Ft Dix data and a computer simulation it was possible to estimate how many follow up requisition to inadequate shipments would be accompanied by high priority requisitions. Including these resulted in an increase in the estimate of requisitions to be processed.

Induced high priority requisitions are a problem even without reduced shipments. In 11.8% of the cases where a 12 month shipment was made, the customer had the need to reorder within only 3 months. An estimated 68.4% of these reorders would presumably have been accompanied by high priority requisitions. If demand in a month was sufficient to drop the customer more than two months below his reorder point-that is, create backorders since under AR 711-16 reorder points are not greater than 2 months of supply-a high priority requisition was presumed to be sent. For 6 month shipments, 20.49% proved inadequate within 3 months and 39.6% of these would presumably have been accompanied by high priority requisitions.

The need, if MILSTRIP guidance is to be followed, to submit two requisitions at one time for the same item, one for stock replenishment, and a higher priority one to clear up backorders, is in fact experienced by Ft Dix. It led to recommendation e (section 9).

2.5 Costs of Requisition Processing.

In 1959 Harbridge House, Inc. did a study on the cost of requisition processing in connection with the formulation of the EOQ policy embodied in AR 711-16.* Data was collected for this study at Ft Devens, Ft Meade, Letterkenny Army Depot and Tobyhanna Depot. Costs included station requisitioning costs, depot requisition processing costs, depot shipping costs, depot to station transportation costs and station physical receipt costs.

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*Economic Inventory Policy Report #2, Harbridge House Inc., Boston, Mass. 1959; AD215 953

STATION ORDERING COSTS
(From 1959 Harbridge House Study)

Station Requisition Cost	$2.46
Depot Requisition Processing Cost	1.66
Depot Shipping Cost	3.28
Depot to Station Transportation Cost	.93
Station Physical Receipt Cost	.29
	$8.62

The Harbridge House result was modified in two ways to get the results used in this study.

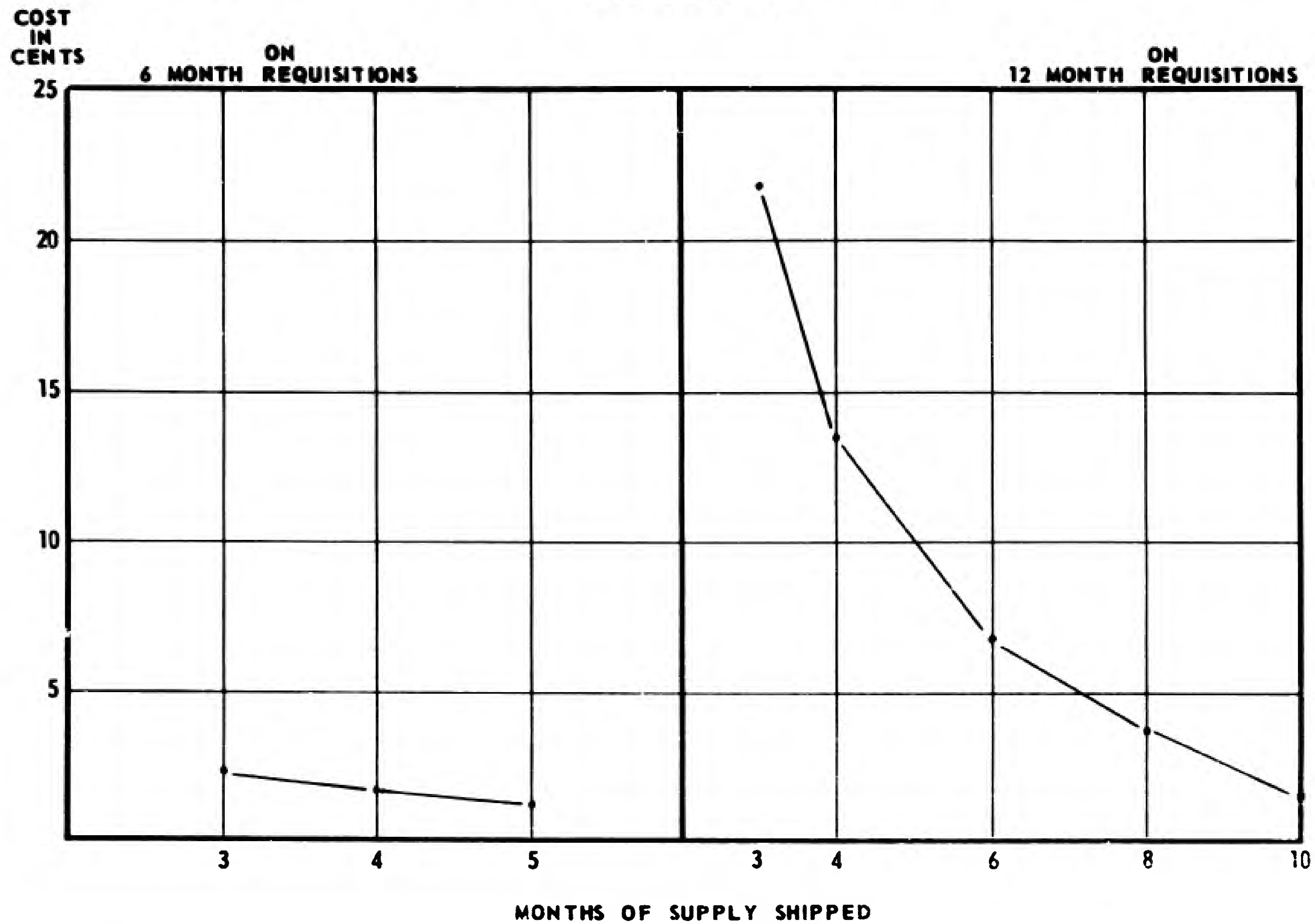
(a) Only 1/3 of transportation costs from depot to requisitioner were included (in total they averaged $.93). The amount of material transported is unaffected by the frequency of ordering. Less frequent ordering can reduce transportation costs through economies of scale, but it seems excessive, especially in view of modern techniques for consolidating shipments, to consider most transportation costs as being variable with ordering frequency and hence part of ordering costs. Shipment processing costs including physical makeup of the shipments, outgoing inspection, depot transportation division functions and station receiving costs are all included in full in requisition processing costs.

(b) The cost figure was corrected by the general increase in costs between 1959 and 1965 as indicated by the increase in post office costs. We considered using the comsumer price index or components of the wholesale price index for this purpose, but decided the post office had a function much closer to requisition processing than the types of goods and services covered in the two indices. Post office costs were adjusted for increased volume and type of mail handled, etc. (see note 4). Cost figures after 1965 were not available. The increase in cost between 1959 and 1965 is estimated to have been 23.9%.

The final cost figure calculated as described was $9.91. This admittedly was an approximate result. Using this result it is simple arithemetic to transform number of requisitions processed for $1000 of stock conserved to cost per dollar of stock conserved. This gives us Graph D.

GRAPH ?

COST PER DOLLAR OF STOCK CONSERVED



2.6 Reduction in NICP Backorders

Stock conserved through a reduced shipment policy will have a beneficial impact on both item availability and item backorders. The analysis concentrates on the impact of conserved stock on the average value of requisitions on backorder as the more significant measure.

When stock is conserved through a reduced shipment it is not known for certain of what value this will be. It is possible, for instance, that future NICP demand during the TTR will be much lower than expected so that the conserved stock is not needed at all. The expected value approach basic to much of statistics overcomes this problem in the sense that the "average" impact on backorders for conserved stock may be determined.

The mathematics for determining the expected reduction in average backorders per dollar of stock conserved is given in note 5. The expectation depends on the supply position of the items for which stock is conserved: stock on hand when reduced shipment is made, length of TTR, forecasted demand. Conserving stock is more beneficial when the stock on hand is low. It is envisioned that a reduced shipment policy would be utilized only when stock on hand was below or falling below expected demand in the TTR. In doing the analysis a stock position equal to expected demand was assumed.

The value of conserving stock depends to some extent on demand variability and hence on the price - annual demand characteristics of the item, since demand variability is related to these characteristics.*

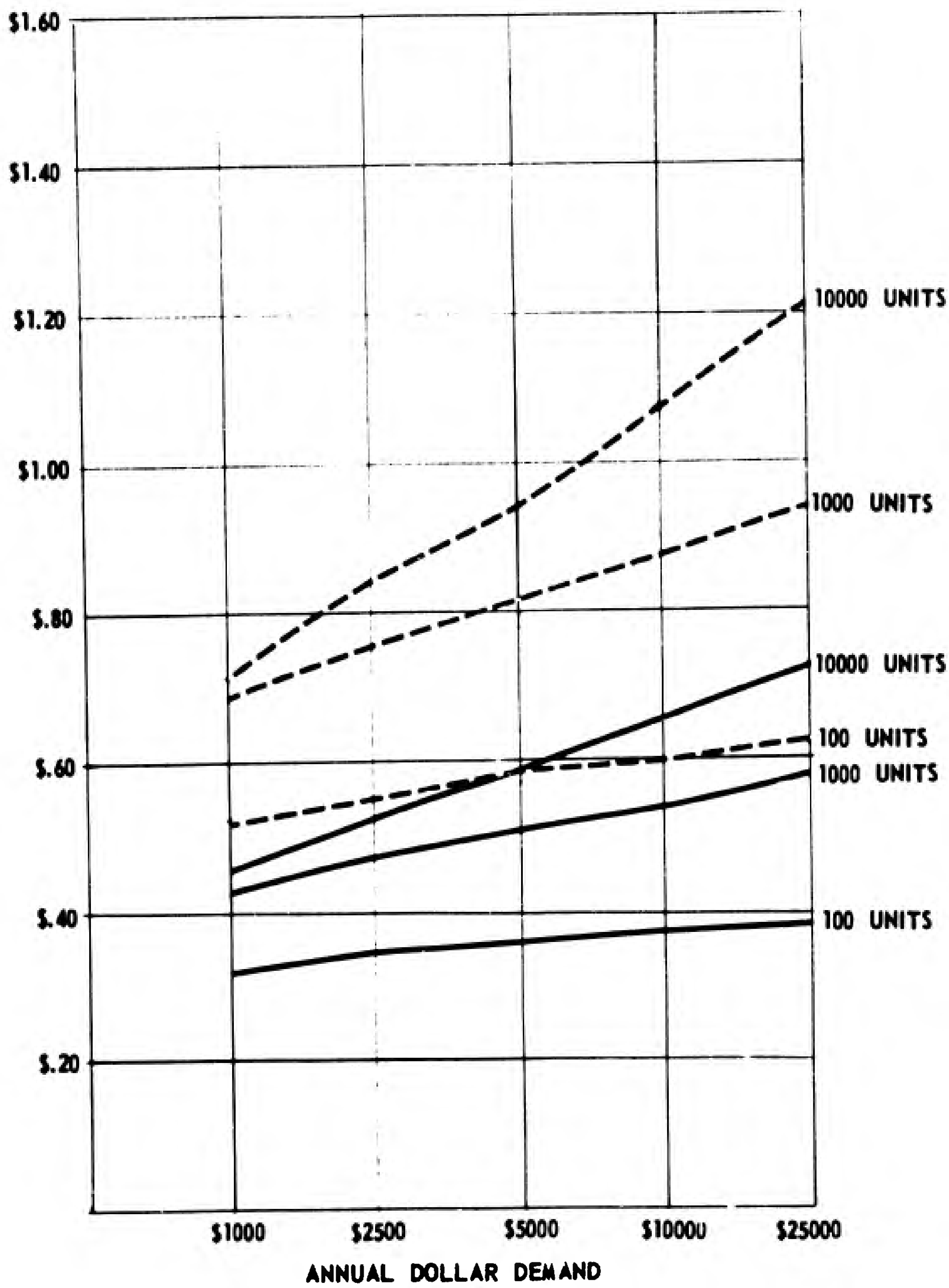
Combining the expected reduction in average yearly backorders for each dollar of stock conserved with the cost per dollar of stock conserved, gives us Graph E.

There are 2 sets of lines on the graph. Each line gives the cost per dollar reduction in annual backorders read from vertical axis, for items with annual dollar demand read from the horizontal axis and annual demand in units - 100, 1000, 10000-corresponding to the particular line. The higher cost set of lines are those which would result from shipping 8 months supply to 12 month requisitioners; the lower costs are those which would result from shipping 3 months supply to 6 month requisitioners.

*See Recommendation on Implementation of Massachusetts Institute of Technology Research in Secondary Item Supply Control, Office of Ordnance Research, March 1959; AD 268 372.

GRAPH E

COST PER DOLLAR REDUCTION
IN
AVERAGE YEARLY BACKORDERS



2.7 Cost Effectiveness of Reduced Shipments.

A standard method of reducing backorders is to increase the safety level. The cost of reducing backorders in this manner depends on the existing supply performance. As supply performance improves it becomes increasingly more costly to improve it further by use of increased safety levels. A mathematical relationship exists between an item's availability (% of time item is in stock) and the cost of decreasing its average backorders one dollar by raising the safety level.

$$\text{Cost/year} = \frac{A}{1-A} C_h$$

where A is the availability and C_h is the cost of holding a dollar's worth of inventory for a year (see note 6).

Typically availabilities of 85% and higher are striven for depending on the dollar value of the item. For low dollar value items it is 99% or higher. Average availability in Fiscal Year 1966 on all requisitions for stocked items Army Stock Fund and PEMA Secondary items inclusive, was 80%.* C_h is usually estimated to be 17%**.

Graph E-1 reproduces graph E, superimposing on it the costs of reducing backorders using increased safety levels, when the supply system is operating at 80%, 85% and 90% availability.

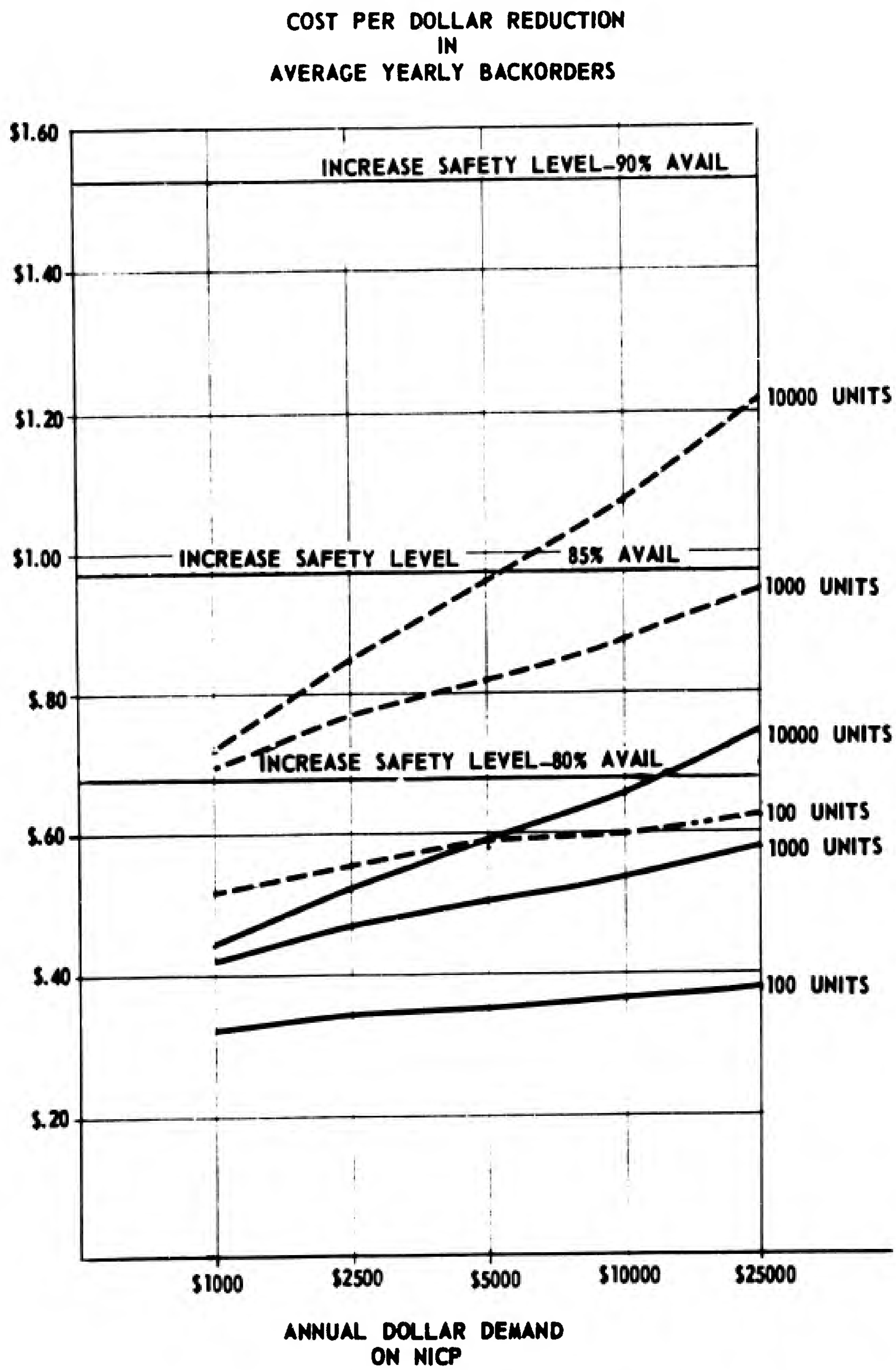
It is seen from the graph that reduced shipments to 6 month requisitioners are generally economical substitutes for raising safety levels even when the supply system is operating at 80% availability. At higher performance levels the cost effectiveness is striking: at 85% availability the cost of reducing backorders by reduced shipments averages about half the costs resulting from raising safety levels, while at 90% it is 1/3.

Reduced shipments to 12 month requisitioners appear from the graphs to be economical in general only at 85% availability and to make a considerable cost reduction possible at 90% availability. This is misleading, however, in that the costs are based on an average requisition value of $55.47 as stated in section 3.

*Taken from "Supply in Review", put out by Programs' Office, Directorate of Supply, USAMC.

**See FM 38-22 for instance.

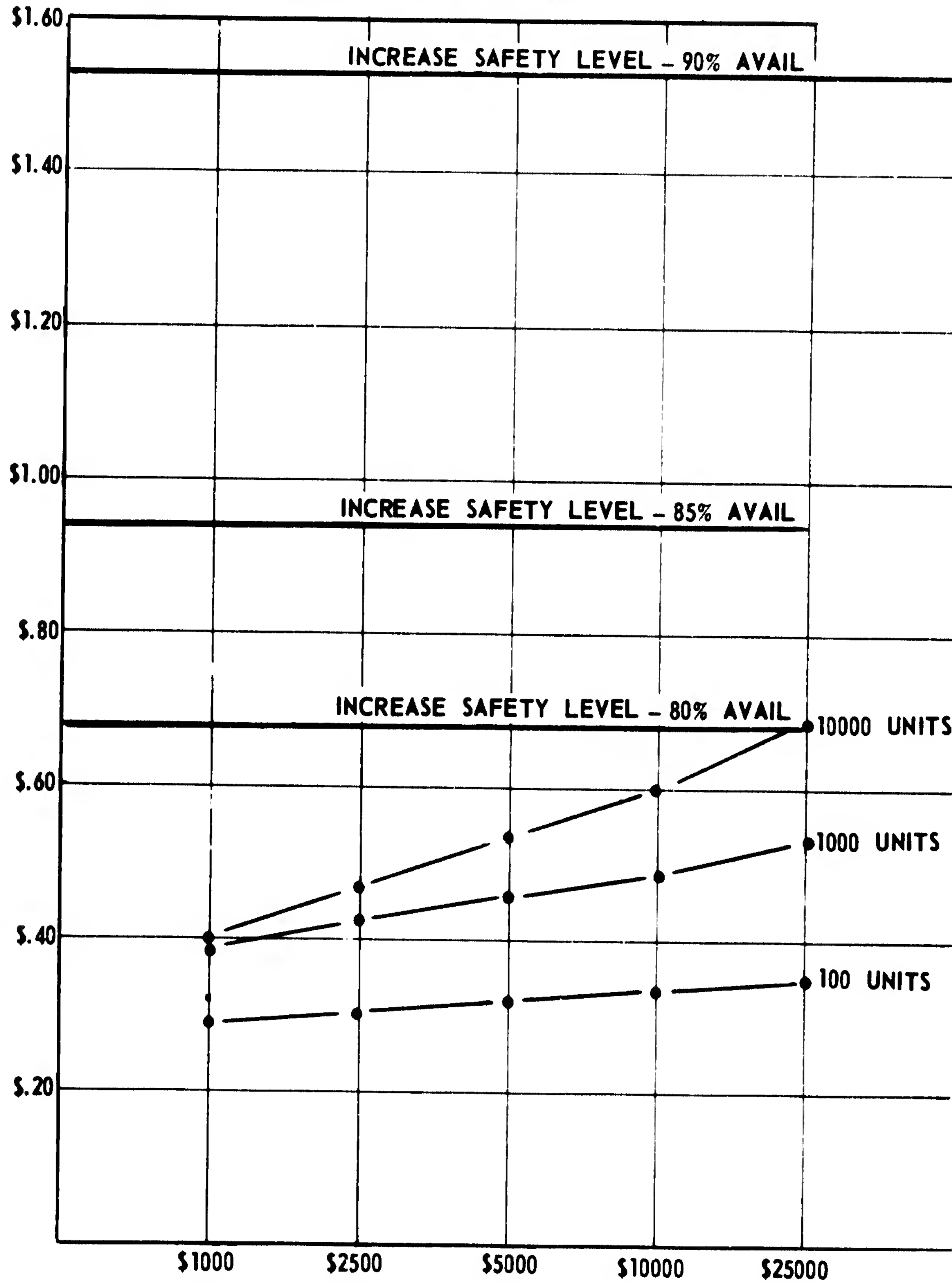
GRAPH E1



COST PER DOLLAR REDUCTION
IN
AVERAGE YEARLY BACKORDERS
$1.60
$1.40
$1.20
$1.00
$.80
$.60
$.40
$.20
INCREASE SAFETY LEVEL–90% AVAIL
INCREASE SAFETY LEVEL
85% AVAIL
INCREASE SAFETY LEVEL–80% AVAIL
10000 UNITS
1000 UNITS
10000 UNITS
100 UNITS
1000 UNITS
100 UNITS
$1000
$2500
$5000
$10000
$25000
ANNUAL DOLLAR DEMAND
ON NICP

GRAPH E2

COST PER DOLLAR REDUCTION
IN
AVERAGE YEARLY BACK ORDERS



ANNUAL DOLLAR DEMAND
ON NICP

While $55.47 may be a reasonable estimate of the value of 12 months requisitions from CONUS customers, for overseas requisitioners ordering under AR 711-25 the average value would certainly be much higher*. Moreover, a reduced shipment policy can be formulated in such a manner that reduced shipments are not made or are larger when the requisition is of small value, as will be seen in the next section. Graph E-2 depicts costs on 12 month requisitions averaging $100. Cost savings are then realized even at 80% availability.

2.8 Formulating the Reduced Shipment Policies.

Two concepts are necessary to the formulation of a reduced shipment policy: marginal cost of decreasing shipment size, and relationship between costs and requisition value. These concepts are embodied in Table 4. The table tells us, first of all, the cost per dollar reduction in average backorders for a typical item (annual dollar demand $5000, annual quantity demand of 1000 units) when reduced shipments of varying numbers of months are made (TTR is assumed to be 3 months), on requisitions of varying value.

Secondly, the table gives marginal costs (in parentheses). It shows for example, the additional cost incurred per additional dollar of backorder avoided when 4 months of supply are shipped instead of 6 months. This is important because it is the marginal cost which should be used in determining how many months supply to ship. Marginal cost indicates where the Law of Diminishing returns has reduced the benefit from further reductions in shipment size to the point where it becomes cheaper to reduce backorders by raising all safety levels.

Suppose we are working with availabilities of 80%, so the cost per dollar reduction in annual backorders with the safety level approach is $.96. Then a look at the marginal costs of Table 4 indicates that on 12 month requisitions it is never worthwhile to ship less than 6 months supply (the minimum marginal cost is $3.15) and that actually for requisitions worth less than $200 it is better to ship 8 or 12 months supply. A reasonable rule is to ship 8 months of supply, but in no event less than $65 on a reduced shipment. This rule prevents the making of unprofitable reduced shipments and specifies shipments which for most requisitions are close to optimum. More complicated rules would specify exactly when to ship 6, 8, or 10 months.

*Under AR 711-16 requisitioners must have annual demand for an item of less than $300 to order 12 months supply, while under AR 711-25 annual demand can be as high as $1000.

Table 4: Costs Per Dollar Reduction in Annual Backorders

On 6 Month Requisitions	Months of Supply Shipped				
requisitions value	3		4		5
$150	.96		.72		.50
		(1.64)		(.97)	
$200	.72		.54		.38
		(1.23)		(.73)	
$300	.48		.36		.25
		(.82)		(.49)	
$500	.29		.22		.15
		(.49)		(.29)	

On 12 Month Requisitions	Months of Supply Shipped						
requisition value	4		6		8		10
$25	6.50		3.32		1.82		.72
		(37.74)		(7.20)		(3.98)	
$50	3.25		1.66		.91		.36
		(18.87)		(3.60)		(1.49)	
$100	1.63		.83		.46		.18
		(9.44)		(1.80)		(.75)	
$200	.81		.42		.23		.09
		(4.72)		(.90)		(.37)	
$300	.54		.28		.15		.06
		(3.15)		(.60)		(.25)	

For 6 month requisitions a reasonable rule is to ship 3 months supply with a minimum amount of $125. Three months supply might also be shipped to all overseas requisitioners ordering under EIP for quantities in excess of $300.

Table 4 was based on an item with one set of demand-price charactetistics. However, graph D indicates that the results will hold good for items with quite different demand characteristics (note that the cost lines of the graph are reasonably flat and that the 1000 and 10,000 unit lines are close). For some items the rule could be modified by using different minimum shipment values.

The results of table 4 are based on an NICP stock position equal to expected demand in the TTR. If there is less stock on hand, each dollar of stock saved by a reduced shipment would have greater impact on average backorders as discussed in note 5. A sophisticated rule would specify smaller reduced shipments when the NICP stock position deteriorated further after the first resort to rationing.

2.9 Recommendations.

a. The NICP should utilize information on its customers' operating levels to automatically make reduced shipments when economical.

b. A numeric code in column 44 of the MILSTRIP requisition should be used on replenishment requisitions to indicate the requisitioner's operating level factor as defined in AR 711-16. This would also serve to identify the demand as recurring, currently indicated by "R" in column 44.

c. Tentatively, the reduced shipment policy would be formulated as follows:

(1) Reduced shipment would not be made when stock on hand exceeded expected demand in the TTR.

(2) For customers with 3 month or 6 month operating levels, the reduced shipment would be equal to enough stock to satisfy the customers expected needs for the number of months in the TTR.

(3) For customers with 12 month operating levels, the reduced shipment would be equal to enough stock to satisfy the customers expected needs for 8 months.

(4) The reduced shipment rule would be supplemented by specifying the minimum dollar values of reduced shipments.

d. Subject to approval of recommendations 1-3, Inventory Research Office, Army Logistics Management Center should be directed to:

(1) Arrange for the collection of data, like that collected at Ft Dix, at one more post, camp, and station and possibly at an overseas theatre control point.

(2) Do a more refined analysis (see next section) using the expanded data base, to make a final detailed formulation of the reduced shipment procedure.

e. A study should be undertaken on possible methods for eliminating the not infrequent need to submit two requisitions for the same item at one time (each of a different priority). This recommendation is not contingent upon acceptance of the other four.

2.10 Validity of Results.

Three considerations impinge on the validity of the results. In deriving the impact of conserved stock on backorders it was assumed that the NICP at the time it made a reduced shipment had enough stock on hand to satisfy expected demand, if not to absorb demand fluctuations. The impact increases strikingly if the NICP has less stock on hand. For a typical item -- annual demand of 1000 units worth $5000 -- the impact is almost twice as great if the NICP has only 2 months supply on hand instead of 3, as was assumed, when the TTR is 3 months.

The effects of successful reduced shipments (outcome a of section 3) on costs were ignored. They will result in an increase in requisitions processed after the end of the TTR, and a decrease in inventory costs, because the NICP's customers will have less stock on hand. Taking this into account increases the cost of reduced shipments as described in note 7, particularly for shipments in response to 12 month requisitions worth less than $150.

Finally, the methodology to handle outcome b used in the report could be improved as stated in note 3 and this also leads to an increase in the estimated cost of reduced shipments: of 13% for 3 month shipments to 6 month requisitioners, and of 66% for 8 month shipments to 12 month requisitioners.

The net effect of all these considerations is that in the final procedure recommended the minimum value of a reduced shipment to 12 month requisitioners will be perhaps as high as $100, and it may be recommended that reduced shipments not be initiated until the NICP has significantly less stock on hand than expected demand in the TTR. The relative benefits of reduced shipments reported would then be maintained.

No explicit consideration has been made of the costs of implementing and administering a reduced shipment policy because the costs are small, and there are beneficial by-products. Actual decision making would be done by computer at small cost. The requisitioner would need to provide on the MILSTRIP card an indication of the number of months supply being ordered. However, under AR 711-16 this information is essentially* available in the form of the operating level factor: the requisitioner need merely copy the factor in column 44 using a zero instead of a 12 for monthly shipments. Under AR 711-25 he uses either zero for monthly buy or 1 for yearly buy.

The effort spent in administering the reduced shipment policy would reap beneficial by-products. The information on customer operating levels supplied to the NICP would enable it to make one effective excess quantity check as prescribed in AMCR 725-4** and might possibly be used in demand forecasting. The surveillance of expected demand in the TTR relative to stock on hand gives the item manager advanced warning on items likely to be in trouble.

For the simulation run with the Ft Dix data there were 135 observations on 12 month orders and 88 on 6 month orders. This is a reasonable number, but more data should be collected before final implementation is made to assist in specific policy formulation.

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*"Essentially" because the requisitioner, if he is below his reorder warning point can order somewhat more than the amount indicated by his replenishment cycle.

**For instance, if a CONUS requisitioner who indicates a replenishment cycle of 12 months requests a quantity whose value is much more than $300, then there has been an error; it is possible to check monthly orderers by determining what the requisition quantity implies about their annual demand. Large orders are a major problem - see final report, USAMC Project #151-66S "NICP Forecasting Techniques".

2.11 Potential for Application of Reduced Shipments.

The potential for applications of reduced shipments is governed by the significance of material ordered under the EIP and EOQ policies. This, in turn, relates first to the significance of low priority stock replenishment demand (Issue Group IV, UMIPS) and then to the use made of these policies in ordering for stock replenishment.

The estimates which were readily obtainable indicate that the overall significance of material ordered under EIP and EOQ policies is rather small. Presumably, the current trend to increased use of high priority requisitions will be reversed in the future. In any event, the relatively small costs of administering a reduced shipment policy, stemming from the ability to automate it, and the beneficial by-products, mean that even a rather small potential for application of reduced shipments should not preclude their use as a highly effective secondary tool in achieving good supply performance. Moreover, the impact on particular items can be great.

In the period between Jan - June 1966 only 29.2% of total requisitions for stocked items at all the NICP's* were for issue group IV. This understates the importance of low priority demand in those months because of the tendency for low priority requisitions to be for larger amounts (see note 8). For instance, in the AVCOM data used for this project, 30.7% of the requisitions were for issue group IV. Yet on the average, low priority demand accounted for 41.0% of the total quantity demanded for each item** For items accounting for more than 20% of the total requisitions, low priority demand actually accounted for at least 60% of the total quantity demanded for each item.

*This statistic was compiled by the AMC Depot Data Center, Letterkenny Army Depot, from the CONUS Supply Performance Report, RCS AMSS14-118, put out by that organization.

**In computing the average each item was weighted by its activity, i.e. the number of requisitions for it. This removes the possibility of unimportant items causing a misleading result.

A breakdown of Ft Dix items by operating level, based on a random sample of 231 items revealed -

Operating Level	% of Total Items	% of Total Dollar Value
12 months	86.6%	15.5%
6 months	8.2%	15.3%
3 months	3.0%	16.0%
1 month	2.2%	53.2%

A letter to Supply and Maintenance Agency, COMZ, elicited the response that for all items stocked (this includes items ordered from non-Army organizations) 73.6% were ordered on a 1 month basis, and only 26.4% were ordered less frequently, on a 1 year basis. This was surprising.

Note 1. AVCOM Data

The data received from AVCOM consisted of issue history and a record of outstanding backorders. The date of the requisition was taken to be the date of the demand, be it issue or backorder. Use of this definition, which was necessary because better information was lacking, led to one problem. Since the simulation ran until 29 days before the cutoff date of the data, there was some chance a requisition might have been made which should have been included in the simulation, but which was not yet received or recorded at the NICP. An adjustment was made to compensate for this.

Although requisitions issued as several partial shipments would appear as several demands, they would all have the same date.

The first date for which an issue was recorded was taken as the initial date of demand history for each item. Items with less than 18 months of history were excluded.

The 18 months of demand history were divided into three 6 month periods for forecasting purposes. The high priority demand rate during periods 1 and 2 was projected forward as a forecast for period 3. The rate during periods 1 and 3 was used as a forecast for period 2. The demand data showed a marked acceleration in demand over the 18 months, in particular between period 1 and periods 2 and 3.

To get the good forecasts periods 1 and 2 were used to predict demand in period 2 by averaging. In addition the forecasts were corrected by one factor such that in aggregate forecasts of period 2 were correct (i.e. total value of demand forecastedforall items was correct) Period 3 was forecasted using demand in periods 1 and 3 using the same adjustment procedure.

Note 2. - Ft Dix Data.

Monthly demand experience for a sample of items was collected at Ft Dix. The sample included almost exclusively those items ordered from Army NICP's, as indicated by the stock classes to which they belonged (we looked at stock classes 1005-2510, 2520-2990, 5805-6665). Along with the demand experience, a record of when requisitions were submitted was kept, as was a record of the Ft Dix demand forecasts upon which the requisition quantities were based.

Each order, with accompanying demand data, constituted one observation. For instance, suppose Ft Dix requisitioned widgets on Jan 1966: then the forecast of demand made in Jan 1966 along with actual monthly demand for Feb 66 thru Jan 67 would comprise one observation.

Data was collected manually by two members of the Inventory Research Office, with the full cooperation of personnel of the Ft Dix Consolidated Property Office in about a week's work. Reference was made to DA Form 1300-2 records, which record orders and forecasts, and DA Form 1300-1 and DA Form 1296 records which give demands. There were difficulties in collecting the data. One difficulty was the adoption of the SALTI concept for many items during the period for which data was available. Under the SALTI concept, monthly demand records are not kept. Another difficulty was that for some items, older demand history was not readily available. Data was collected for as far back as 1961 to get some observations on items which later become SALTI items.

Some tests were made for homegeneity of data from different historical years. For instance, as an indication of the irregularity of demand, number of months with zero demand was examined for items with like operating levels observed in different years. There was nothing to indicate lack of homeogeneity so it was concluded the somewhat non-random nature of the sample had no ill effects.

To remove the effects of forecast bias in the sample, a stratified sampling plan was used: results would be obtained for observations in which Ft Dix forecasts were low, then for observations in which they were high and the results averaged irrespective of the number of observations in each group. Observations for which there were less than 10 months of demand history following the time of requisitioning were excluded, because it could not be determined which group to put them in. The majority of Ft Dix forecasts happened to be high.

To determine the probabilities referred to in the report it was merely a question of determining shipment size on the basis of the Ft Dix demand forecasts and the shipment rule being used, and then seeing whether the shipment would be exhausted during any month of the TTR.

Ft Dix data was also used to determine the average value of annual demand for items with the various operating levels, and the proportion of items which had each operating level. For these purposes a random sample of items was chosen: Ft Dix had the item records grouped in books and we looked at the first half of every second book for the appropriate stock classes. An item was not included in the sample if it was neither ordered nor reviewed in the year before we visited Ft Dix, or if, at the time of last review, assets exceeded requisition objective.

Note 3. Calculation of Extra Requisitions Processed per $1000 Stock Conserved.

Initial Analysis -

Let I_F be the % of time a full shipment is inadequate (outcome b)

Let I_R be the additional % of the time the reduced shipment made in accordance with the policy being tested is inadequate (outcome c)

Let P be the proportion of the requisition quantity not shipped under the reduced shipment policy.

Let V be the average value of a requisition.

Savings per 100 reduced shipments: $S=P \times V \times 100 \times (1 - I_F - I_R)$

Extra requisitions processed per 100 reduced shipments: $R=100 \times I_R$

Extra requisitions processed per $1000 of stock saved: $R \times 1000/S$

The initial analysis omits consideration of outcome b and of induced high priority requisitions. The approach described below for extending this analysis represents an improvement over that actually used, as explained in Chapter II, section 10, "Validity of Results". The impact on results and recommendations is described there.

No approach would be fully satisfactory. The motivation for using the approach described is that it is the most reasonable which could be developed short of expending a major effort to get "definitive" numbers. This would not be worthwhile as the major conclusions of Chapter II are based on very large differences between the cost effectiveness of the two approaches to improving supply performance considered, so that the exact magnitude of difference is not crucial.

Outcome b

Proper analysis of outcome b must consider what happens when the initial shipment proves inadequate and the customer submits his second requisition. A table may be developed:

Initial Shipment	Second Shipment		
	full	reduced	delayed
reduced	33-1/3%	33-1/3%	33-1/3%
full	50%	-	50%

This table expresses the estimate that if a reduced shipment is made initially, it is equally probable that the next shipment to the same customer will be full, reduced, or delayed (i.e. his requisition will be backordered until the end of the TTR).

The numbers in the table may be arranged:

Subcase 1: 33-1/3% of the second shipments are full regardless of what the first shipment is.

Subcase 2: 33-1/3% of the second shipments are delayed until the end of the TTR regardless of what the first shipment is.

Subcase 3: 33-1/3% of the time two reduced shipments are made instead of two full shipments or a full shipment and a delayed shipment.

If the character of the second shipment is not changed by the character of the first, the effect of having made a reduced shipment originally is small. Under subcase 1 there is some stock conserved. Under subcase 2 a reduced shipment results in "robbing Peter (the original requisitioner) to pay Paul (a customer who was satisfied only because less was shipped to the original requisitioner)"; backorders are temporarily postponed.

When a second reduced shipment is made to a customer, it will reflect the customers increased demand forecast based on the demand experience which made the first inadequate. Never-the-less, it can be inferred from the circumstances of outcome b that the customer will need to reorder still again much sooner than expected. The reduced shipment policy will result in an extra requisition being processed. 3 instead of 2.

The conclusion is that added to extra requisitions processed under a reduced shipment policy should be $100 \times I_F \times .333$.

Induced High Priority Requisitions.

The percent of follow up requisitions potentially accompanied by induced high priority requisitions was calculated as discussed in Chapter II, using a month by month simulation. Added to extra requisitions processed should be $I_R \times H$ where H is the percent calculated.

Note 4. Use of Post Office Costs.

Information is available in the Statistical Abstract of the United States (issued annually) on post office expenditures allocated to classes of mail, and on the volume of mail in each class. Hence, the cost per piece of mail could be evaluated for 1965 and 1959 and the ratio taken for each class of mail (figures were available on domestic 1st class, air mail, 2nd class, 3rd class, 4th class (parcel post), publications, and international mail). The ratios were weighted by 1965 expenditures for each class to give an overall measure of the increase in costs.

It was necessary to take ratios by class of mail so that a structural change in the type of mailing class used would not affect results (e.g. a tendency to use more expensive classes of mail). It was possible to break total cost down into, among other categories, rural and city delivery service and non-postal transportation. It was believed the first two might be excluded, and 1/3 of the last taken to make postal costs still more comparable to requisition processing costs. However, doing so had negligible effect and the results given omits this adjustment.

The table on the next page gives the detailed data.

Note 5. Reduction in Backorders vs Stock Conserved.

The concept of average quantity on backorder leads directly to the concept of backorder unit times or "bouts". A backorder for 10 units which exists for 1 month represents 10 bout's while if it is on backorder for 2 months it represents 20 bout's and so on. Average quantity on backorder over 12 months, for instance, equals the bout's which will be sustained during the 12 months divided by 12.

TABLE

POST OFFICE COSTS

	Apportioned Expenditures (millions)	Volume (millions)	Cost/letter or package	ratio to 1959
1959				
domestic air	152.5	1,368	$.1111	
1st class	1,303.3	32,274	.0404	
2nd class	372.5	7,099	.0524	
3rd class	677.7	16,978	.0399	
4th class (parcel post)	708.6	1,038	.6827	
publications	9.8	126	.0782	
int'l mail	105.8	553	.1913	
1965				
domestic air	197.8	1,629	$.1214	1.093
1st class	1,965.2	38,068	.0516	1.277
2nd class	498.9	8,600	.0580	1.107
3rd class	998.8	19,454	.0513	1.286
4th class (parcel post)	845.5	1,045	.8091	1.185
publications	24.8	281	.0826	1.056
int'l mail	157.5	579	.2720	1.422

average of ratios, weighted by 1965 expenditures: 1.239

Let B(n) be expected bout's over a period of T months, when stock on hand is initially n

Let G (y, T) be the probability that demand during the T months will be greater than or equal to the quantity y.

Let u be the expected value of demand over T months

$$B(n) = \frac{T}{u} \times \int_n^{\infty} (y-n)\, G(y, T)\, dy$$

If we take the derivative of B (n) with respect to n we get, using Leibnitz's rule,

$$\frac{d}{dn} B(n) = \frac{T}{U} \times \int_n^{\infty} -G(y, T)\, dy$$

Now suppose we wish to calculate the difference in expected bout's if we have on hand of n, or on hand of only $n - \Delta$. This equals $B(n) - B(n-\Delta)$, or approximately $\Delta \times \frac{d}{dn} B(n)$ for small Δ.

Hence, i.e. we wish to determine the value of conserving 1 unit of stock when we receive a demand and our on hand equals the expected demand in the TTR, we can evaluate the derivative for n = u, multiply by 1 and then divide by 12 to get the reduction in units of average yearly backorders. It follows mathematically that if we substitute one dollar for one unit we get the answer in dollars.

If we conserve stock when our on hand is not equal to u, but is actually lower, the value of doing so is greater. Each additional unit of stock conserved becomes more important as the stock on hand gets smaller. This is intuitively clear and can be seen mathematically by noting that the magnitude of the derivative increases as n decreases.

For demand which is normally distributed the derivative is easily evaluated using a property of the normal distribution.

Let σ^2 be the variance of demand

Let f (x) and L(x) be the density and loss function of the normal unit variate. L (x) = 1-F(x) where F (x) is the distribution function.

Let $x = \frac{y-u}{\sigma}$

$n' = \frac{n-u}{\sigma}$

$\frac{T}{U}\int_{n}^{\infty} -G(y)\,dy =$ by change of variable of integration

$\frac{T}{U}\int_{n'}^{\infty} -L(x)\,\sigma\,dx =$ by property of normal distribution*

$\frac{-\sigma T}{U}\left\{ f(n') - n' \cdot L(n') \right\}$;

for n = u, $n' = o$ and we get, using a table of the normal distribution

$\frac{-\sigma T}{U} \cdot f(o) = -\ .39894\ \frac{\sigma T}{U}$

Note 6. Availability and Cost of Reducing Backorders.

A formal proof of the relationship proceeds by using the expressions for the expected values of average stock on hand and average backorders; these expressions are cited in the reference footnote, for example.** Denoting the derivatives of these two expressions with respect to changes in the safety level by dH/ds and dB/ds respectively, it turns out

$$\frac{dH}{ds} = 1 + \frac{dB}{ds}$$

$- \frac{dB}{dS} = (1 - A)$, A being availability

Hence

$$\frac{dH/ds}{-dB/ds} = \frac{1-(1-A)}{1-A} = \frac{A}{1-A}$$

*see, for instance, "Analysis of Inventory Systems", G. Hadley and T. M. Whitin, Prentice Hall, Englewood Cliffs, N. J., 1963; Appendix 4, property 6).

**MIT Research in Secondary Item Supply Control: Recommendations on Implementation, AD 268372, p82.

Thus, the marginal increase in safety level per dollar reduction in average backorders is \$A/(1-A). The inventory cost is $C_H \times A/(1-A)$ where C_H is the inventory cost per dollar of inventory per year.

A heuristic proof of the result has been developed which is more meaningful. As does the formal proof, it implicitly uses the fact that expected value of stock on order does not depend on the safety level, but on procurement lead time only.*

By definition an extra dollar invested in safety level is not needed when the item would be in stock anyway and merely contributes to stock on hand at a cost of 1 x A (fraction of year item is in stock) x C_h (cost per year of holding a dollars extra stock on hand). If C_h is 20¢ and A is .75 the cost is 15¢.

When the item is out of stock the dollar invested in safety level has been issued and is serving to reduce backorders by \$1 for an average annual reduction of 1 x (1-A) where 1-A is the fraction of the year there is \$1 less stock on backorder. If 1-A equals .25, then there is a reduction of stock on backorder for .25 year of \$1, equivalent to a reduction in average annual stock on backorder of 25¢. Hence, the cost of reducing backorders 25¢ is 15¢ and cost per dollar is 15¢/.25 or $\frac{C_h \times A}{1-A}$

Note 7. Cost of Successful Reduced Shipments.

This note explains the approach developed for estimating the costs associated with successful reduced shipments, i.e. those corresponding to outcome a of section 3.

We can compute the total processing and inventory costs associated with order quantities of varying sizes. Suppose the NICP makes a 3 month shipment when a 6 month shipment is close to optimal on a strict cost basis. We can compare the processing and inventory costs associated with the 3 month shipment with 1/2 those associated with the 6 month shipment -- this represents the cost of the reduced shipment.

*Also, both proofs assume demand to be continuous. For items with small demand, this is not valid and a slight modification is needed.

For an item with annual demand of $580 -- this was found to b e average for items requisitioned semiannually -- the respective costs are -

6 month shipment

$$\frac{290}{2} \times .17^{*} \times \frac{6}{12} + 9.91 = 22.24$$

3 month shipment

$$\frac{145}{2} \times .17 \times \frac{3}{12} + 9.91 = 12.99$$

Difference:

$$12.99 - 1/2 \times 22.24 = \$1.87$$

Approximately 7 successful reduced shipments are needed to conserve $1000 worth of stock -- more precisely, 6.9 or 1000/145. Thus, the cost of the successful reduced shipments to 6 month requisitioners associated with conserving $1000 of stock is 6.9 x $1.87 or $12.90. This increases the costs calculated in the body of the report by 56%.

Similar analysis for shipping 8 months supply to 12 month requisitioners indicated that the costs of Chapter 3 are very much understated for requisitions worth less than $150. For requisitions worth $150 consideration of the costs of successful reduced shipments increases the costs calculated in Chapter 3 by 62%. For customers with annual demand of $200, it is actually more economical to ship 8 months than 12 months supply.

The analysis is based on an expected value approach which would normally be valid for what is being done. However, successful reduced shipments do not correspond to a random sample of customer orders -- they tend to correspond to cases where customers have overestimated their demand. By ignoring this, this analysis overstates the cost of reduced shipments.

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*As noted in section 3.7, inventory costs are estimated to be 17%. In the Harbridge House study costs were estimated as 15% for the post camps and stations, but certain methodological assumptions were made which explain this lower estimate. Inventory costs should be greater, if anything, in the field than at the NICP depots.

Note 8. Average Order Sizes.

We compared the average order sizes of Issue Groups I - III, treated as a whole, vs Issue Group IV and of Issue Group I vs Issue Group IV. For a given FSN we would determine which of the two groups being compared had the smaller average size and assign the group a "1" for the item. If the average order size for the other group was 2 - 1/2 times this size, it would be assigned a 2-1/2 for the FSN. To aggregate over the sample of items we used 3 methods: a simple average of all the numbers assigned, and weighted averages giving more weight first to items with the larger dollar value of demands, and then to items with the larger activity as reflected in number of requisitions (actually issues). The weighted averages were used so that items with only a few demands for small amounts, would not have much effect on the results. Some items were excluded from the sample because there were no requisitions in one of the groups.

The results are tabled below. The table indicates that it is for items whose requisitions are for large dollar amounts that average sizes tend to be least different between low and high priority.

ORDER SIZES

based on:	IG 4 vs IG 1	IG 4 vs (IG 1-3)
Straight Average	1.74	1.27
Weight Item by Activity	1.91	1.63
Weight Item by Dollar Value	1.28	1.12

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APPENDIX

INTRODUCTION

A supplier may distinguish between the demands he receives for an item; attaching greater importance to satisfying some categories of demand than others. The distinction may be based on customer characteristics, on the need for the item, and so on. Such a supplier must make a decision when stocks appear to be running low: should he conserve stock on hand by issuing only to high priority demands?

Section 1 describes a model we found useful in working with the problem as it is faced by the Army Materiel Command. Section 2 proves the existence, under a limited set of assumptions, of optimum reserve levels,[1] i.e. optimum stock levels at which to stop issuing to lower priority demands. The proof suggests an algorithm for calculating the levels. Section 3 describes some empirical results obtained on the usefulness of reserve levels and, in particular, of the algorithm developed. Section 4 gives the conclusions we drew from the work.

THE MODEL

Stock is in short supply, but at some known date in the future stock levels will be replenished. Before that time two types of demand must be satisfied, low priority and high priority. Failure to satisfy high priority demand is more serious.

When a demand is received, it is satisfied if there is issuable stock available. If it is a high priority demand, any stock on hand is issuable. If it is a low priority demand, any stock on hand in excess of the "scheduled" reserve level is issuable. Any demand not satisfied is backordered (if a demand can be partially satisfied, the unsatisfied part is backordered).

The scheduled reserve level is recomputed periodically, (e.g. weekly) as the time until stock replenishment decreases. During this time, however, no new demand forecasts are made. If the scheduled reserve level is decreased, it may free stock for issue against outstanding low priority backorders. If the scheduled reserve level were 10, stock on hand 9, and a low priority demand was received, it would be backordered.

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[1] Essentially the same results have been derived independently by Donald M. Topkis, Decision Studies Group, Palo Alto, California. The case where demands are lost rather than backordered is also treated by Mr. Topkis, and he worries about holding costs.

With the passage of time, the scheduled reserve could drop to 6. If no high priority demand had been received meanwhile, stock on hand would still be 9 and 3 units could be applied against low priority backorders.

A penalty is affixed for every day a unit demanded is backordered. The penalty is 1 if the unit was demanded in a low priority demand, C_h if it was a high priority demand. C_h is larger than 1.

THE OPTIMUM SCHEDULED RESERVE LEVELS

Assumptions

1. High priority demand in any period is independent of demand in any other period, and whether that demand was satisfied. The time between successive opportunities to change the scheduled reserve level is considered a period.

2. No penalty costs are incurred on a low priority unit backordered from the time it is received until the beginning of the next period. Equivalently, the assumption can be stated: low priority demand always occurs at the beginning of a review period. The beginning period time sequence is: receive low priority demand, recompute scheduled reserve level, issue if warranted against any outstanding low priority demands.

3. Assumption 3a is that the probability distribution of demand for a period, about the forecast of demand for that period, is known. Assumption 3b is that the distribution and forecasted demand are the same for each period.

4. High priority demands always occur at the midpoint of a review period.

Assumptions 3b and 4 are made for ease of exposition only.

Outline of Existence Proof

The difference in the expected penalty cost if a reserve of x or of x + 1 is kept in period i is defined recursively (equation 1). This difference is independent of stock on hand and net stock* over a range of x determined

*net stock, as used here, does not include the replenishment stock on order. It equals on hand minus back orders.

by these quantities (lemma 1). Over this range the difference is monotonic increasing (lemma 2). Either there is an optimum x in this range or at one of the boundary points (proposition 1). By making the range large we see that there is an optimum scheduled reserve level which is independent of net stock and stock on hand (proposition 2).

Basic Recursive Equation

1a) For $k \leq x$

$$E_{i+1,k,\theta}(x+1) - E_{i+1,k,\theta}(x) = A(x) + B(x)$$

$$A(x) = 1 - (C_h/2) \cdot p(d \geq x + 1)$$

$$B(x) = \sum_{d=x+1-s_i}^{x} p(d) \cdot \left[E_{i,k',\theta'}(x + 1 - d) - E_{i,k',\theta'-1}(x-d) \right] + \sum_{d=x+1}^{\infty} p(d) \cdot i \cdot (1-C_h)$$

1b) For $k > x$: difference is zero

1c) For $x \geq \theta$: difference is undefined

$E_{i+1,k,\theta}(x)$ is: the total expected penalty costs to the system during periods $i + 1$ to 1. (There are i periods after period $i + 1$ before stock replenishment). This expectation is conditional upon some set $\{S_i\}$ of scheduled reserve levels being used for periods 1 to i.

x is stock to be kept as a reserve for period $i + 1$.

θ is stock on hand (before any issuing) at the beginning of the period and θ' at the beginning of the next period.

$p(d)$ is probability of high priority demands totaling d units in the period.

$p(d \geq x)$ is probability high priority demands total at least x units.

k is net stock, or on hand minus backorders outstanding. k' is net stock at beginning of next period.

C_h is penalty per period per high priority unit backordered. For low priority, penalty is 1 .

For $k > x$ (condition 1b) all outstanding backorders are satisfied whether the reserve kept is x or $x + 1$. Under assumption 2, this determines all action until the beginning of the next period. $x \geq \theta$ (condition 1c) implies that the reserve level x cannot be increased by 1 because it already equals the stock on hand.

Otherwise, the difference in expected penalty costs may be broken down into the difference for period $i + 1$, $A(x)$, and the difference thereafter, $B(x)$. With a reserve of $x + 1$, and $k \leq x$, one more low priority unit must be backordered during period $i + 1$ than with a reserve of x , at a penalty cost of 1 . On the other hand, one less high priority unit will be backordered if high priority demand totals at least $x + 1$: the saving is then $C_h/2$ using assumption 3b .

The difference in expected penalty for periods 1 to i depends upon what happens in period $i + 1$. If high priority demand in period $i + 1$ exceeds the reserve stock level $(d \geq x + 1)$, then there will be no stock on hand at the beginning of period i. The system is frozen in a sense with 1 more low priority and 1 less high priority unit backordered if the initial reserve were $x + 1$ than if it were x . If high priority demand in period $i + 1$ is low $(d \leq x - S_i)$, on hand at the end of period $i + 1$ will be $\geq S_i$, the reserve level for period i , regardless of whether the initial reserve was x or $x + 1$; therefore, there is no difference in expected penalty from the beginning of period i . Between the extreme cases $(x + 1 - S_i \leq d \leq x)$, the reserve kept for period i will be the reserve kept for period $i + 1$ minus "d" , for this quantity will be smaller than S_i .

Lemma 1: The difference of equation 1 does not depend on k or θ so long as $k \leq x \leq \theta - 1$.

We use an induction argument. For period 1 equation 1 reduces to

$$(1') \quad E_{1,k,\theta}(x+1) - E_{1,k,\theta}(x) = 1 - (C_h/2) \cdot p(d \geq x + 1)$$

in the range for x being considered. In this range the value clearly does not depend on k or θ. Using the induction premise, equation 1 indicates the only way the difference for period $i + 1$ might depend on k or θ is if the probability that $K' \leq x - d \leq \theta' - 1$ depended on k or θ, where k' and θ' are net stock and on hand for period i . x - d will always fall in this range, however. $k \leq x$ and $k' \leq k - d$ implies $k' \leq x - d$. If a reserve of x + 1 is kept for period i + 1, on hand at the end of i + 1 is x + 1 - d. Hence $\theta' - 1 = x - d$.

Lemma 2: The difference of equation 1 - $\Delta E_{i+1}(x)$ - is monotonic increasing as a function of x for the range of x such that the difference is not trivially zero, and for any $\{S_i\}$ which is not demonstrably non-optimal.

We again use an induction argument. For period 1 the statement is true as $p(d \geq x + 1)$ is monotonically decreasing.[2] For period i + 1

$$\Delta E_{i+1}(x+1) \text{ minus } \Delta E_{i+1}(x) = T_0 + T_1 + T_2 + T_3 + T_4,$$

which after doing the subtraction reduces to:

$$T_0 = p(d \leq x - S_i) \cdot 0$$

$$T_1 = p(d = x + 1 - S_i) \cdot [-\Delta E_i (S_i - 1)]$$

$$T_2 = \sum_{d = x+2-S_i}^{x} p(d) \cdot [\Delta E_i (x+1-d) - \Delta E_i (x-d)] \text{ (term vanishes for } S_i < 2)$$

$$T_3 = p(d = x+1) \cdot [\Delta E_i (0) + C_h/2 + i \cdot (C_h - 1)]$$

$$T_4 = \sum_{d=x+2}^{\infty} p(d) \cdot 0$$

That T_1 is ≥ 0 will be shown in the proof of proposition 1. The use of the non-optimal clause is clarified there. Each term of T_2 is ≥ 0 by the induction assumption. T_3 is ≥ 0 so long as ΔE_i (0) is

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[2] Without assumption 4 we would have to put things a little differently. Let z be the time remaining in period i + 1 after the x + 1[st] high priority unit is demanded, z being defined as 0 if this event does not occur in the period. Then E(z) is monotonic decreasing with x and $C_h E(z)$ should be put in place of $(C_h/2) \cdot p(d \quad x + 1)$ in equation 1. T_4 is no longer exactly zero.

$\geq -C_h/2 - i(C_h - 1)$. Actually, $\Delta E_i(0)$ must be $\geq 1 - C_h/2 - i(C_h - 1)$, the last two terms representing the maximum benefit which could be realized by reserving an extra unit of stock.

Proposition 1: Let net stock in beginning of period i be k , on hand be θ Let S_i^* be an optimum value for the scheduled reserve level, possibly dependent on k and θ . The set of optimum S_i^* for period i , $\{S^*\}_i$, is non-empty.

If we look at all values of x where it is not trivially zero $\Delta E(x)$ must be always non-negative, always non-positive, or there can be at most one change of sign, the change being from negative to positive as x increases. This is a restatement of lemma 2. If $\Delta E(x)$ is always non-negative, a subset of $\{S^*\}_i$ is all $S_i \leq k$, and if it is always non-positive all $S_i \geq \theta$ form a subset. If there is a change of sign, let x' be the maximum x for which the $\Delta E(x)$ is negative. $S_i = x' + 1$ is in $\{S^*\}_i$.

In the proof of lemma 2, we had to show $\Delta E_i(S_i - 1)$ was non-positive. We now see that if it were positive S_i would be demonstrably non-optimal (for period i monotonicity was assumed by induction).

Proposition 2: There exists at least one optimum scheduled value for the reserve level for period 1 + 1 which is independent of k, θ .

Choose k small, θ large. By proposition 1, there is at least one optimum value, which may depend on k, θ. We show that any optimum value for k, θ is also optimum for net stock = k' , and on hand = θ' when $k \leq k' \leq \theta' \leq \theta$. Suppose the value is in the interval (k' + 1, θ' - 1) . By lemma 1, $\Delta E(x)$ in this interval is the same for the two cases. Therefore, the same reasoning which proves the value optimum in one case, proves it optimum for the other. If there is a value of $S_i^*(k, \theta)$ - notation has obvious interpretations - which is $\leq$ k' , then $\Delta E(x)$ is non-negative for $x \geq k'$, for both cases, and all values of $S_i \leq k'$ are optimum for the (k', θ ') case. Similar reasoning takes care of optimum values of $S_i^*(k, \theta) \geq \theta'$

The Algorithm

The optimum schedule is calculated recursively using equation 1 . For each period $\Delta E(x)$ is calculated, beginning with x = 0 and incrementing x by 1 until a positive value for the difference is obtained. (θ is taken implicitly to be larger than x, k negative.) This provides the optimum value for that period. At the same time the values for the differences are saved for use in the next period (begin with period 1, then period 2 and so on.)

In doing the calculations, assumption 4 should be relaxed. Hence, an expression is needed for the expected time remaining in a period after the $x+1$st high priority unit is demanded, given that at least $x+1$ high priority units are demanded in one period. For the Stuttering Poisson distribution the desired expression is

$$1-(x+s)/\lambda s - \left[\sum_{d=0}^{x} d \cdot p(d)/\lambda s - p(d \le x)\right] / p\,(d \ge x+1)$$

where λ is the frequency of demand per review period and s is the average order size.[3] For the Stuttering Poisson distribution there is a nice algorithm available for calculating the necessary probabilities.[4]

SOME EXPERIMENTAL RESULTS

Schedule Quantities

Figure 1 gives some of the optimum scheduled reserve levels calculated, assuming the demand distribution was stationary and Stuttering Poisson with known parameters. Note that as the variance to mean ratio (VMR) gets small, the values approach those which could be obtained by a deterministic model:

$$2) \qquad S_i^* = \left[(C_h-1)/C_h\right] \cdot \lambda s \cdot i$$

Note also that a higher VMR does not necessarily result in a higher reserve stock.

Controlled Experiment

The results obtained from the algorithm discussed above were compared with those obtained under three alternative policies. Alternative 1 was to set no reserve. Alternatives 2 and 3 were heuristics: alternative 2 was to set the reserve equal to expected high priority demand before stock replenishment; alternative 3 was to set the reserve to what would be the

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[3] This result is based on work done by Edward Bruckner, University of Pennsylvania, and Karl Kruse, U.S. Army.

[4] See Feeney, G. J., and Sherbrooke, C. C., "(S - 1, S) Inventory Policy under Compound Poisson Demand," Management Science, January 1966, pp. 409-411.

optimum value for a deterministic model (equation 2 above). For C between 2 and 10, alternative 3 gives a simple way of reflecting the importance attached to high priority demands in the size of the reserve.

A simulation program was written which embodied the model of section 1. In the controlled experiment we used artifically generated demand history. Demand was given a stationary Stuttering Poisson distribution. The forecasted parameters for the distribution of demand were set equal to the actual parameters. There were 2 cases as depicted in Figure 2, with 324 trials per case. Stock on hand at the beginning of a trial was set equal to high priority demand forecasted to occur during the trial. There were 56 days in each trial, comprising four 2-week periods.

Figure 3 gives the results on relative penalty sizes. Three values of the penalty cost C_h were used; 2, 4, and 10. As a "quick and dirty" indication of the significance of the improvement of using the algorithm vs. the other three alternatives, t statistics were calculated where the % improvement was small. In these calculations, trials giving o differences in penalty costs were ignored.

"Real World" Experiment

Another set of simulations was performed on actual demand data. Historical data was collected on 235 items, with 336 days of history used in the actual simulation per item. Trial periods of 56, 84 and 112 days were tested. For a 56 day trial period, there would be 6 (i.e. 336/56) trials per item. As in the controlled experiment, on hand inventory at the beginning of a trial was set equal to forecasted high priority demand for the trial period. One demand forecast, and VMR estimate, was made at the beginning of the simulation, and a new one after 1/2 year.[5] Reserve levels were changed bi-weekly (a weekly period was tried with little difference in results). There was some problem in aggregating penalty costs incurred by item into total penalty costs, because with a straight summing undue weight would be given to an item for which demands were typically for large number of units.

The results are given in Figure 4. A special run with the artificial data exhibited the same tendency for the no-reserve policy to do worse for a longer trial period. It is, of course, true that improvement which can be expected from a reserve policy will increase with the ability to accurately forecast high priority demand, which will vary in different applications.

[5] The reader may notice that this meant the demand forecast actually changed during one trial for each item.

We might add that the forecasts used in the simulation were simple projections of demand experience which tended to be poorer than the Army would experience in practice. There was no reasonably simple way for us to produce simulation forecast errors which we could be sure neither understated nor overstated typical Army forecast error; hence, we tried to be conservative by inclining toward the latter. Finally, we used the Stuttering Poisson distribution in compiling the tables for rule 4 knowing we might possibly have been able to find a more appropriate distribution, hence get somewhat better tables, if it turned out the additional work could be justified.

CONCLUSIONS

The results of the "controlled" experiment (figure 3) indicated that the improvement to be realized by use of the algorithm was not substantial. Particularly striking was the result for case 2 for a weight of 10. Despite marked differences in the reserve levels calculated by the simple rules and the algorithm, the simple rules captured most of the improvement potential rationing could offer (92.5% of the potential was captured by rule 2). It is possible that this conclusion is sensitive to the distribution of demand.

The results of the "real world" experiment (figure 4) encouraged us in our conviction that rationing did have a place in the Army Materiel Command, as well as helping to confirm our decision to abandon consideration of sophisticated rules.

Figure 1 - Some Optimum Levels

Mean/√VMR per period		Periods To Go Till Arrival of Stock							
		1	2	3	4	5	6	7	8
1/1	Ch = 2	0	1	1	2	2	3	3	4
	Ch = 4	1	2	3	3	4	5	6	7
	Ch = 10	2	3	4	5	6	7	8	9
1/10	Ch = 2	0	0	0	0	0	1	1	2
	Ch = 4	0	0	2	4	5	7	8	9
	Ch = 10	0	5	8	10	12	14	15	17
1/100	Ch = 2	0	0	0	0	0	0	0	0
	Ch = 4	0	0	0	0	0	0	0	0
	Ch = 10	0	0	0	0	0	0	8	15
8/1	Ch = 2	4	8	12	16	20	24	28	32
	Ch = 4	6	12	18	24	30	36	42	48
	Ch = 10	8	16	23	31	38	45	53	60
8/10	Ch = 2	0	6	10	14	18	22	26	30
	Ch = 4	6	14	21	28	35	41	47	54
	Ch = 10	13	24	34	43	51	59	68	76
8/100	Ch = 2	0	0	0	0	0	0	2	9
	Ch = 4	0	0	9	27	40	52	62	72
	Ch = 10	0	40	67	88	105	120	134	147
8/250	Ch = 2	0	0	0	0	0	0	0	0
	Ch = 4	0	0	0	0	0	7	28	46
	Ch = 10	0	0	46	89	122	149	150*	150*
8/500	Ch = 4	0	0	0	0	0	0	0	0
	Ch = 10	0	0	0	2	65	116	150*	150*

*Computer Program did not compute values larger than 150

Figure 2 - Two Artificial Cases

	Case I	Case II
mean demand/pd-hi prior.	2.1	7.0
VMR hi prior.	5.0	19.0
mean demand/pd - lo prior.	4.0	21.0
VMR - lo prior.	11.0	30 (constant order size of 30)

	Reserve Levels for Period							
Ch = 2	4	3	2	1	4	3	2	1
rule 1	0	0	0	0	0	0	0	0
rule 2	8	6	4	2	28	21	14	7
rule 3	4	3	2	1	14	11	7	4
rule 4	3	2	1	0	11	6	1	0
Ch = 4								
rule 1	0	0	0	0	0	0	0	0
rule 2	8	6	4	2	28	21	14	7
rule 3	6	5	3	2	21	16	11	5
rule 4	8	6	4	1	27	21	13	2
Ch = 10								
rule 1	0	0	0	0	0	0	0	0
rule 2	8	6	4	2	28	21	14	7
rule 3	8	6	4	2	25	19	13	6
rule 4	13	11	8	4	47	37	27	13

Figure 3 - Results from artificial trials: relative penalty costs
(Penalty costs as a % of those using algorithm)

	Case I	Case II
Alt. 1 vs. algorithm		
Ch = 2	101.1%	101.0%
Ch = 4	113.2%	112.7%
Ch = 10	145.7%	144.3%
Alt. 2 vs algorithm		
Ch = 2	105.1%	105.0%
Ch = 4	100.2% ("t" = 2.42 n = 125)	100.3% ("t" = 2.25 n = 227)
Ch = 10	103.2%	103.3%
Alt. 3 vs algorithm		
Ch = 2	100.7%	101.2%
Ch = 4	100.9%	100.8%
Ch = 10	103.2%	104.8%

Figure 4: Results using actual demands

(Penalty costs as a % of those using alternative 1, i.e. no reserve)

	Ch = 2	Ch = 4	Ch = 10
56 days per trial			
alt. 2 vs no reserve	103.0%	100.4%	97.7%
alt. 3 vs no reserve	100.9%	99.9%	97.7%
algorithm vs no reserve	100.0%	99.7%	98.1%
84 days per trial			
alt. 2 vs no reserve	103.1%	99.1%	94.6%
alt. 3 vs no reserve	100.7%	99.0%	94.7%
algorithm vs no reserve	100.1%	99.0%	93.4%
112 days per trial			
alt. 2 vs no reserve	102.4%	98.4%	94.1%
alt. 3 vs no reserve	100.6%	98.2%	94.4%
algorithm vs no reserve	99.9%	98.4%	93.4%

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13. ABSTRACT Stock rationing is the attempt to insure that scarce material goes where it can do the most good. Two forms of stock rationing are investigated for application at the NICP level.

A reserve for high priority requisitions is designed to channel scarce stock away from low priority users to high priority users. The value of maintaining a reserve is empirically shown. A mathematically optimum method for setting the size of the reserve level is developed. This and other methods are empirically compared. A simple rule for setting reserve levels is recommended which bases the level at any given time on the high priority demand expected during the time remaining before stock replenishment. It is shown how to implement this rule in a multi-priority system.

Reduced shipments can be made to economize on scarce stock. The use of such a procedure is investigated in the case where the requisitioner is ordering in economic quantities to replenish his own supplies rather than for immediate use. It is shown that proper application of the procedure provides a way of improving supply performance which is inexpensive relative to the costs of raising the safety level. ( )

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