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FLUERIC PRESSURE- AND TEMPERATURE-INSENSITIVE OSCILLATOR FOR TIMER APPLICATION

by

Carl J. Campagnuolo
Stacy E. Gehman

February 1968

U.S. ARMY MATERIEL COMMAND
HARRY DIAMOND LABORATORIES
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FOREWORD

This report summarizes the work performed under order No. PRON-GG-7-80614-01-GG-A9 for Picatinny Arsenal, Dover, N. J., under the technical cognizance of George R. Taylor.

CONTENTS

SYMBOLS

A	$1 + \frac{\Delta Q_s(R_1 + R_2)}{P_0}$
a	constant
a_0	free speed of sound
a_1	constant
a_2	constant
B	$\frac{R_1(1-m) + R_2(1-n)}{R_1(n-a) + R_2(m-a)}$
b	constant
C	capacitance
C_0	constant
$\mathbf{c}$	complex speed of wave propagation
D	$\frac{R_1(1-\beta) + R_2(1-\alpha)}{R_1(\alpha-b) + R_2(\beta-b)}$
f	frequency
K_1	constant
L	inertance per unit length
m	constant
n	constant
P_0	total pressure in feedback loop
$\bar{P}_0$	normalized pressure in feedback loop
P_{10}	constant
P_{11}	constant
Q	flow
$\dot{Q}$	flow acceleration or $\frac{dQ}{dt}$

Q_b	bias flow in feedback network
Q_s	flow through R_2 required to switch
Q_1	flow into capacitor
Q_2	flow through R_2 less Q_b
Q_2^+	Q_2 during charging cycle
Q_2^-	Q_2 during decay cycle
$\dot{Q}_2^+$	acceleration of Q_2^+
$\dot{Q}_2^-$	acceleration of Q_2^-
ΔQ_s	flow difference required to switch
$\overline{\Delta Q_s}$	normalized flow difference required to switch
R	$\frac{R_1 R_2}{R_1 + R_2}$
R_g	gas constant
R_0	resistance per unit length of duct
R_1	feedback resistor
R_2	feedback resistor
R_{10}	constant
R_{20}	constant
R_{11}	constant
R_{21}	constant
S	area of duct
T	temperature
t	time
ΔT	temperature change
$\overline{\Delta T}$	temperature change normalized to midrange temperature

W	angular frequency
X	τ_s/RC
α	constant
β	constant
μ	fluid viscosity
ϵ	density of fluid
τ_s	halfperiod
τ_{so}	halfperiod at ambient temperature

ABSTRACT

Three fluid relaxation oscillators using R-C-R feedback loops were tested to establish the feedback resistances required to make the oscillator frequency insensitive to temperature and pressure. The frequency of the three oscillators, geometrically similar but with different feedback resistances, was measured as a function of stagnation pressure and temperature. With modifications guided by these data, one of the oscillators showed frequency variations of less than 2 percent with changes in input pressure of 6 to 30 psig and changes in temperature of 75° to 200°F. A theoretical analysis indicates that an oscillator frequency simultaneously insensitive to temperature and pressure can be achieved using a lumped R-C-R network in the feedback loop.

1. INTRODUCTION

An oscillator whose frequency is not affected by changes in stagnation pressure and temperature is basic to the development of pneumatic timers and logic circuits that must operate under severe environmental conditions. HDL (Harry Diamond Laboratories) has developed such an oscillator consisting basically of a high-gain bistable amplifier with a feedback network (fig. 1). The feedback is a lumped R-C-R (resistance-capacitance-resistance) network. Some of the fluid from the power jet is returned to the control port through the feedback causing the unit to oscillate. The amount of fluid entering the capacitance is determined by resistance R_1; the fluid leaving it, by resistance R_2. Thus R_1, R_2, and the capacitor volume determine the frequency of the oscillator. In the experiment, the oscillator exhausted into a binary device that amplified the signal output and maintained a fixed load. Frequency changes with variation of stagnation pressure and temperature were investigated for different resistances in the feedback loop.

Theoretical considerations are presented in appendix A to show that insensitivity to temperature and pressure can be obtained simultaneously by using a lumped R-C-R network in the feedback loop.

2. THEORETICAL CONSIDERATIONS

2.1 R-L Feedback Network

Previous studies by J. M. Kirshner (ref 1) showed how the transmission line equations can be applied to the feedback loop in designing an oscillator whose frequency is insensitive to temperature. Theory states that the magnitude of the complex speed of propagation in a duct of constant cross section is given approximately (ref 2) by:

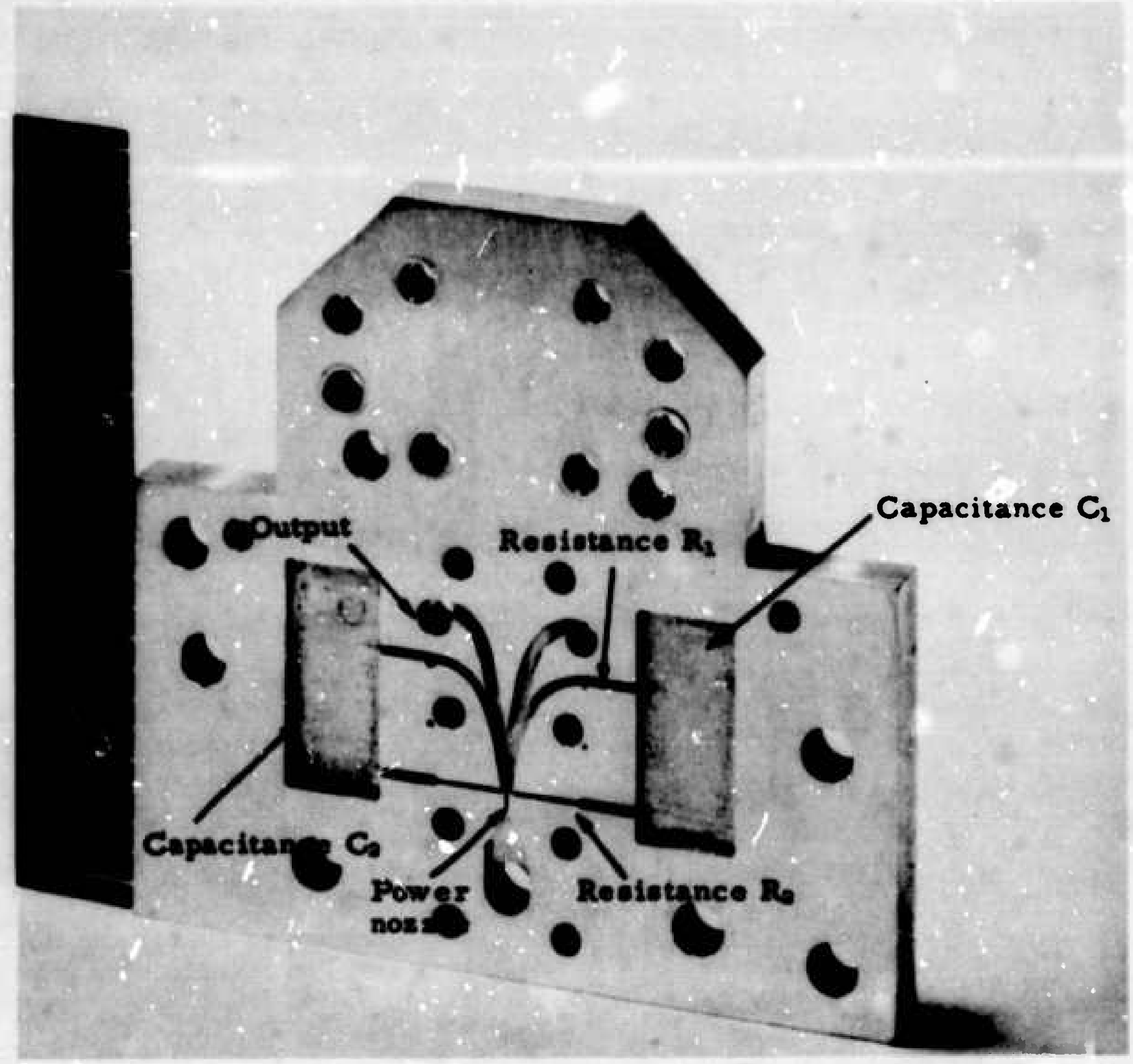

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Figure 1. Relaxation oscillator.

$$|c|^4 = \frac{a_0^4}{1 + \frac{R_0^2}{W^2 L^2}} \tag{1}$$

where

$\lvert c \rvert$	complex speed of wave propagation
a_0	free speed of sound
R_0	resistance per unit length of duct
W	angular frequency
L	inertance per unit length

R_0 and L for a circular duct are given by:

$$R_0 = \frac{8\pi\mu}{S^2} \qquad L = \frac{\rho}{S}$$

where μ = viscosity of fluid
S = area of duct
ρ = density of fluid used

From the ideal gas law,

$$\rho = \frac{P_0}{R_g T}$$

where R_g = the gas constant
T = absolute temperature
P_0 = pressure in feedback loop

Substituting R_0, L, and ρ in equation (1), $|c|$ is given in terms of the temperature and pressure as:

$$|c|^4 = \frac{a_1}{\frac{1}{T^2} + \frac{a_2 T^{3/2}}{W^2 S^2 P_0^2}} \tag{2}$$

where a_1 and a_2 are constant.

From equation (2) it is apparent that for $T = 0$ and $T \to \infty$, $|c| = 0$; hence, $|c|$ has a maximum at some temperature, and it is least sensitive to temperature in the vicinity of the maximum; consequently, the frequency W is least sensitive to temperature in the region of the maximum. Hence, if a uniform duct is used in the oscillator feedback path, so that the curve of $|c|$ is flattened and the maximum occurs over a wide temperature range, then the oscillator frequency will be temperature insensitive. Further inspection of equation (2) shows that for large S, the speed of propagation is pressure insensitive; for small S, the propagation speed $|c|$ and, consequently, the frequency becomes pressure sensitive. Hence in Kirshner's analysis, temperature insensitivity is achieved by continuously compensating the speed of propagation uniformly along L-R (inductance-resistance) feedback path, even though pressure insensitivity is sacrificed.

2.2 R-C-R Network

An oscillator insensitive to pressure and temperature requires a lumped R-C-R feedback network. The compensation process can be explained using the diagrams in figure 2. Figure 2a shows the operating path for each cycle in a plane composed by the phase difference between the flow acceleration $\dot{Q}$ through the network and the flow rate Q entering the circuit. Figure 2b represents the sawtooth response of the R-C-R network; the experimental response is shown in figure 2c. In figure 2a, the charging of the network is represented by the points 1-2, the switching of the amplifier by 2-3, and the decay portion of the cycle by 3-4; the amplifier switches back at 4-1, and a new cycle begins. From the diagram, the period of oscillation τ is the line integral around the phase plane limit cycle, so τ is given by

$$\tau = \int_1^2 \frac{dQ}{\dot{Q}} + \int_2^3 \frac{dQ}{\dot{Q}} + \int_3^4 \frac{dQ}{\dot{Q}} + \int_4^1 \frac{dQ}{\dot{Q}} \tag{3}$$

at 1, in the phase plane, $\dot{Q} = P_0/R_1^2C - 2Q_b/R_1C$ (4)

at 3, in the phase plane, $\dot{Q} = 2Q_s/R_1C$ (5)

(see appendix A).

The second and last integral in equation (3) can be neglected, because the switching time of the amplifier is much smaller than the period of oscillation. For the period to remain unchanged, the sum of the integrals in equation (3) must be a constant. The denominator (R_1^2C) in equation (4) becomes temperature insensitive if the term $1/R_1C$ in equation (5) increases with temperature. This condition is necessary because the acceleration $\dot{Q}$ is a function of temperature. For a temperature increase, the charging and decay paths shift as shown in figure 2d. However, the time integrals along these paths remain unchanged, so that the charge time and the decay time sum to the same period as required by equation (3).

The compensation in the network takes place as follows: As the temperature rises, the resistance of the network increases, causing the bias flow to diminish. If this increase in resistance were the only change in the network, the frequency of oscillation would drop. However, the tank capacitance decreases with higher temperature with a consequent rise in frequency. Hence by adjusting the size of

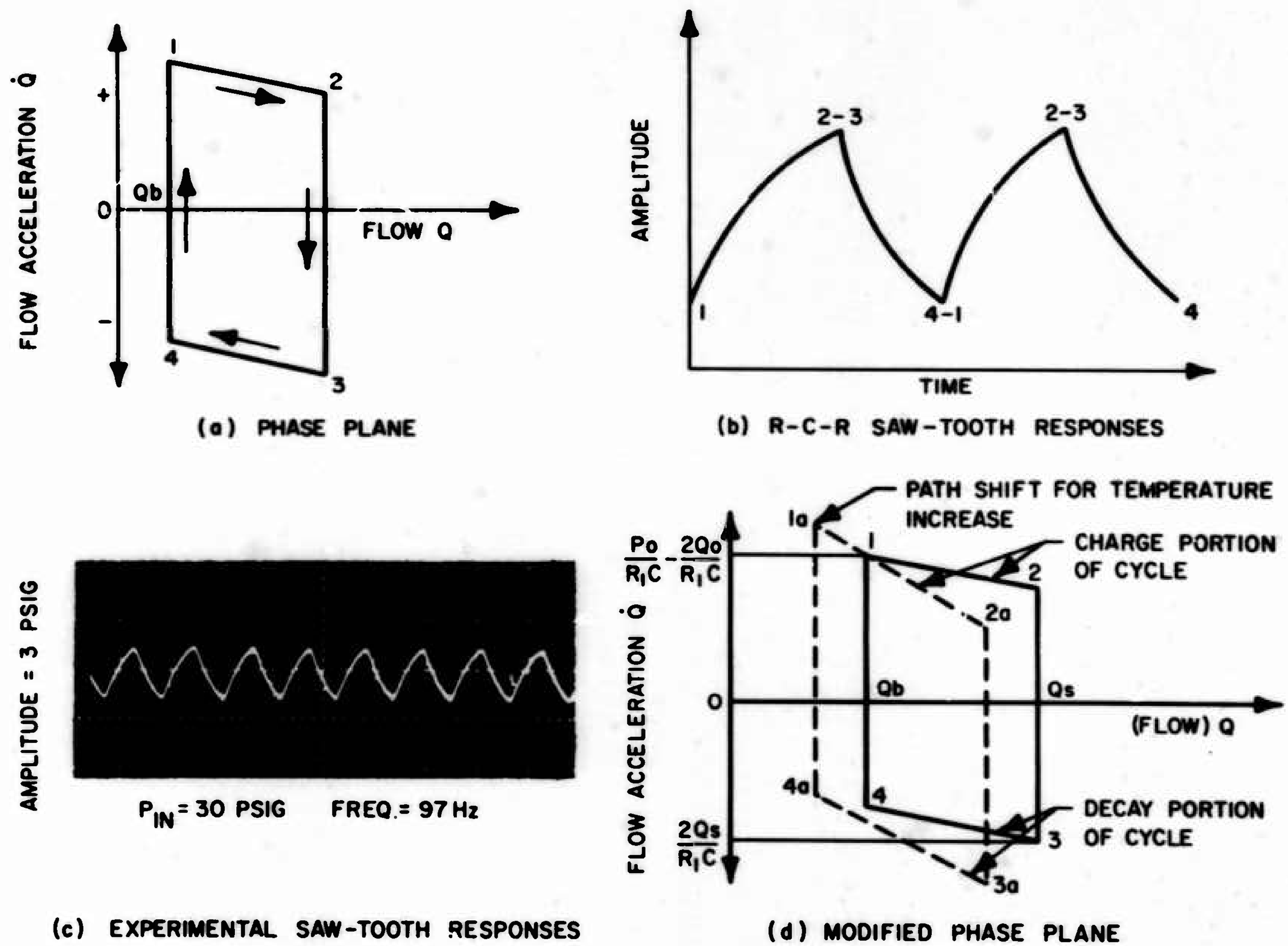

Figure 2. Phase plane and saw-tooth responses.

the resistances and volume of the capacitance so that one compensates the other, temperature insensitivity can be achieved. A similar argument is proposed for pressure independence. Appendix A demonstrates how temperature and pressure insensitivity can be achieved simultaneously.

2.3 Temperature Insensitivity Criteria

To establish the criteria for temperature and pressure insensitivity, the flow conditions in the network must be considered. The flow through R_2 during the charge portion of the cycle is given by

$$Q_2^+ = \frac{P_0}{R_1 + R_2}\left(1 - e^{-t/RC}\right) + Q_b e^{-t/RC} - Q_b \qquad (6)$$

where

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

P_0 = total pressure at entrance of the network

Q_b = network bias flow

The flow through R_2 during the decay portion of the cycle is expressed as

$$Q_2^- = Q_s e^{-t/RC} - Q_b \qquad (7)$$

where

Q_s = flow required to switch

If it is assumed that the switching mechanism is the flow difference ΔQ_s between the two control ports and that this flow difference is temperature independent, then the switching condition can be expressed as the difference between equation (6) and (7), or

$$\Delta Q_s = \frac{P_0}{R_1 + R_2} + \left(- \frac{P_0}{R_1 + R_2} + \Delta Q_s\right) e^{-\tau_s/RC} \qquad (8)$$

where

$$\Delta Q_s = Q_s - Q_b$$

as defined in the figure, and

τ_s = the time at which switching occurs, or the half cycle.

The half cycle τ_s can be expanded as a Taylor series,

$$\tau_s(T + \Delta T) = \tau_{s0} + \overline{\Delta T}\frac{d\tau_s}{dT} + \frac{\overline{\Delta T}^2}{2!}\frac{d\tau_s^2}{dT^2} + \cdots \qquad (9)$$

where

$$\overline{\Delta T} = \frac{\Delta T}{T}$$

For small temperature variations, the second order term and all higher order terms can be neglected since $\overline{\Delta T}$ is small. Thus for the half cycle to remain constant over a small temperature range requires that

$$\frac{d\tau_s}{dT} = 0 \tag{10}$$

By taking the total derivative with respect to temperature of the flow difference ΔQ_s in equation (8) and by using the condition imposed by equation (10), we obtain

$$e^X = 1 + A \cdot BX \tag{11}$$

which is the criterion for temperature insensitivity. (See appendix A for complete mathematical analysis.) In equation (11),

$$X = \frac{\tau_s}{RC}$$

$$A = \left(1 + \frac{\Delta Q_s(R_1 + R_2)}{P_0}\right)$$

$$B = \frac{R_1(1-m) + R_2(1-n)}{R_1(n-a) + R_2(m-a)}$$

and

$$R_1 = R_{10}T^n, \quad R_2 = R_{20}T^m, \quad P_0 = P_1T^a$$

where

a, n, m, R_{10}, R_{20}, and P_1 are constants.

Graphically equation (11) shows the locus of points along which $d\tau_s/dT = 0$ or $df/dT = 0$. Hence for temperature insensitivity, it is required to operate along this curve (See fig. A3 in appendix A).

2.4 Pressure Insensitivity

Pressure insensitivity can be obtained by the use of the previous equations and assumptions. In fact if the total derivative of equation (8) is taken with respect to pressure and $d\tau_s/dP = 0$, then the equation for pressure insensitivity can be written as

$$e^X = 1 + A \cdot DX \tag{12}$$

where

$$D = \frac{R_1(1-\beta) + R_2(1-\alpha)}{R_1(\alpha-b) + R_2(\beta-b)}, \quad X = \frac{\tau_s}{RC}$$

and

$$R_1 = R_{11}P^{\alpha}, \quad R_2 = R_{21}P^{\beta}, \text{ and } P_0 = P_{11}P^{\alpha}$$

where α, β, b, R_{11}, R_{21}, and P_{11} are constant.

By close inspection of equations (11) and (12), if $B = D$, it is apparent that, for $df/dT = 0$ and $df/dP = 0$, the equations represent the same curve, and therefore temperature and pressure insensitivity can be obtained simultaneously.

This paper shows that by properly choosing the size of R_1, C, and R_2 experimentally, pressure and temperature insensitivity can be achieved simultaneously. The theoretical analysis demonstrates that the simultaneous attainment of pressure and temperature insensitivity is physically possible, and the results shown in the various experimental graphs substantiate the predictions. However, in the analysis it was assumed that the network is strictly an R-C-R feedback. For the oscillator design it was necessary to include some inductance; hence, the function for ΔQ_s becomes very complex, but for the frequency range of interest, the device is still described by equations (6) and (7).

3. RELAXATION OSCILLATOR (R-C-R)

3.1 Physical Characteristics

A relaxation oscillator (fig. 1) is basically an R-C-R feedback type; some of the fluid from the power jet is returned to the control port through the feedback network causing the unit to oscillate. The amount of fluid entering the capacitance is determined by the resistance R_1, placed in the upper portion of the capacitance, and the fluid leaving it by the resistance R_2, located at the bottom of the capacitance. Hence, R_1, R_2, and the capacitor volume determine the filling time of the capacitance, which will in turn determine the frequency of the oscillator.

The oscillatory mode is excited only for pressure ratios for which the jet spreads to occupy the full width of the output channel. This is necessary to achieve a feedback process that will induce oscillation. The spreading of the power jet is a function of the input pressure and the pressure of the field into which it is operating. If the pressure at the output of the oscillator is atmospheric, a high pressure at the input is required to achieve the pressure ratios necessary for oscillation. Such behavior is normal and is characteristic of jet flow.

In this particular case, the oscillator exhausts into a binary device (fig. 3), which has a pressure below ambient in its interaction region. The amplifier control area sets a fixed load on the oscillator output which causes a back pressure. The back pressure induces the oscillator power jet to spread and forces a portion to feed back into the R-C-R network initiating oscillations.

The binary amplifier and the oscillator have a common supply, so that a change of input pressure in one is accompanied by a change in the other. This action is needed because some of the increase in flow through the oscillator nozzle is conveyed to the lower pressure region in the binary amplifier control ports.

In addition, the binary amplifier is provided with a set of bleeds, located in the separation region. The function of the bleeds is to exhaust any increase in back pressure that arises when the amplifier

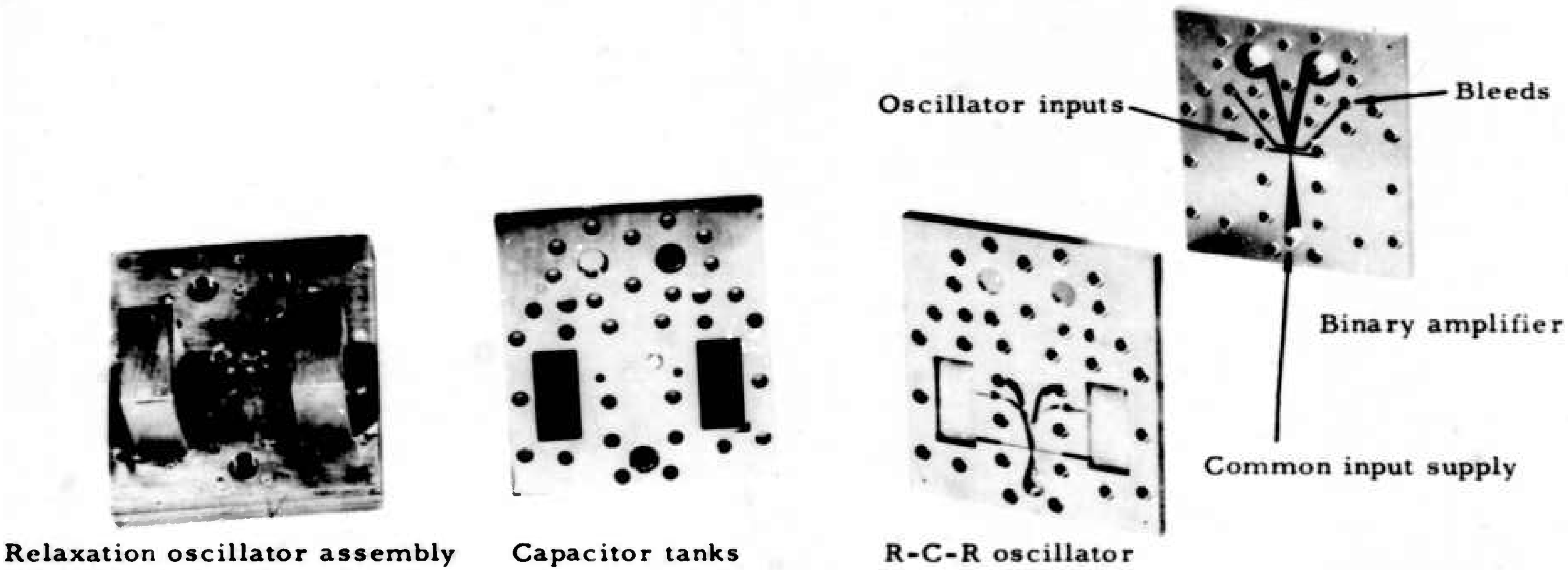

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Figure 3. Relaxation oscillator and digital amplifier.

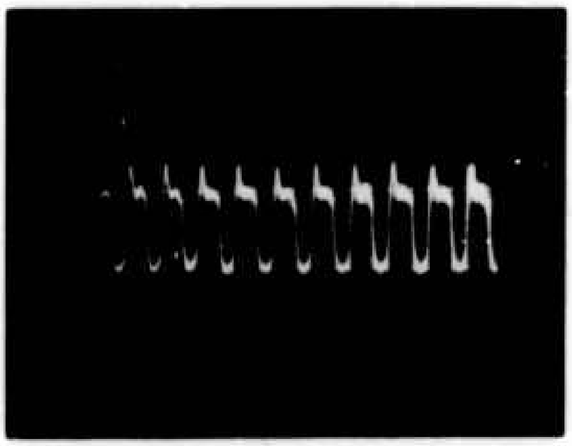
is loaded. The binary amplifier is used as a buffer because an appreciable gain is needed to amplify the oscillator output. Even though a binary amplifier was used, a quasi-sinusoidal output was obtained (fig. 4).

3.2 Experimental Results

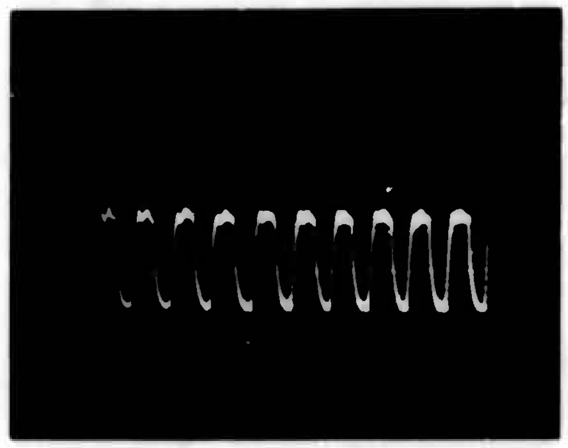
For the experimental analysis three geometrically similar oscillators were constructed, each having a different resistance R_2 (fig. 1) in its feedback loop. The capacitance volume (0.366 in.^{3}) and the size of the resistance R_1 were the same for all three.

The resistances R_2 were similar in depth and length, but their width was 0.010, 0.015, and 0.018 in. The frequency was measured by a piezoelectric crystal installed at one of the two outputs of the element, fully loading this channel. This generated a distortion in the output signal, so a matching load was provided for the opposite port. A plot of the frequency as a function of input pressure is shown in figure 5 for all three oscillators. From the graph it appears that the element with a resistance of 0.018-in. width (top curve) was least sensitive to pressure in the range from 6 to 30 psig. The change in frequency in the above range was at most 0.06 percent. The remaining two elements had a more limited pressure range with greater frequency variation (1.4 percent for the 0.015-in. width and 4.3 percent for the 0.010-in. width resistances). In figure 5 are included the size of the resistive loads used with each oscillator.

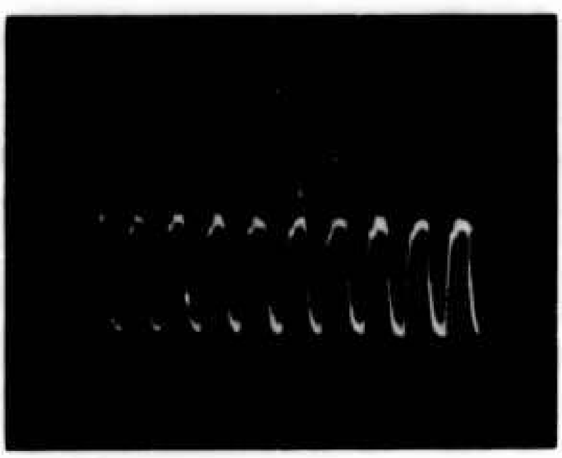
The three oscillators were tested for temperature insensitivity by supplying them with heated air. A copper coil approximately 10 ft long was connected to a brass tank and placed in an oven. The tank was used to measure input stagnation pressure. The oscillators were placed outside the oven and were carefully wrapped in asbestos to minimize heat loss to the surroundings. A copper constantan thermocouple was placed between the stagnation tanks and the oscillator input. The frequency output was monitored by the piezoelectric transducer as in the previous test. The oven temperature was allowed to rise to its maximum and when equilibrium was reached between the oven temperature and the air stagnation temperature, the frequency output of the oscillator was measured. As the oven was cooling the frequency was monitored. Figure 6 and 7 summarize the results obtained using two input pressures (10 and 3 psig). From figure 6, it is noted that the best results are obtained with the 0.010-in. width resistance. The maximum change of frequency for this case is 4 percent over a temperature range from 70° to 200°F. For the other two oscillators, the maximum frequency change was 9.45 and 12.80 percent for the 0.015- and 0.018-in. resistances. For pressure input of 3 psig (fig. 7), the trend is the same as in figure 6. The 0.010-in. resistance seems to result in the least change in frequency over the temperature range used.



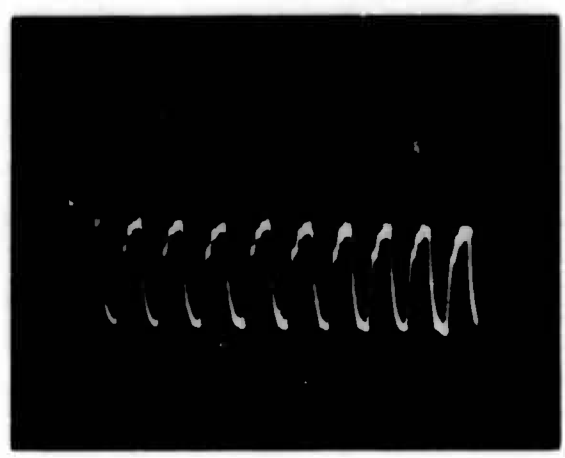
P_{IN} = 3 PSIG FREQ = 96 HZ



P_{IN} = 10 PSIG FREQ = 97 HZ



P_{IN} = 15 PSIG FREQ = 96 HZ



P_{IN} = 30 PSIG FREQ = 94 HZ

Figure 4. Relaxation oscillator output waveform.

The curves described thus far have shown that the oscillator with the 0.018-in. resistance yielded the best pressure insensitivity and the one with a 0.010-in. resistance produced the optimum temperature invariance. Thus, the data demonstrate that none of the three oscillators was simultaneously pressure and temperature insensitive. Hence, a minor change of resistance in the feedback network was required to flatten the temperature curves, so the frequency would become less temperature sensitive. For this purpose two adjusting screws were placed in feedback resistance R_2 to control the amount of fluid entering the feedback loop. The screws were placed in the oscillator having an R_2 0.010 in. wide. This oscillator was chosen because it displayed the least temperature sensitivity. The screws were adjusted until the oscillator frequency was invariant with changes of input pressure. Figure 8 shows that the frequency output of the oscillator was flat from 8 to 30 psig. The unit was also found to display a high degree of temperature insensitivity. Figures 9 and 10 show the effect of temperature variation on frequency. For pressure inputs of 3 and 10 psig, the maximum change in frequency was 2.2 percent when the temperature was varied from 80° to 200°F. For an input pressure of 20 psig, a 3-percent maximum frequency change was observed for the same temperature range (fig. 11).

The restriction caused by the screws was measured and another unit was built having a similar geometrical configuration. Figure 12 indicates that for this new unit the frequency varied less than ±1 percent for a pressure input from 6 to 30 psig. The test for temperature insensitivity showed that the frequency varied less than 1 percent from 77° to 175°F, for an input pressure of 10 psig (fig. 13). It has been observed that the oscillators tested are pressure sensitive in the range 2 to 6 psig. This discrepancy occurs because of the volume of the capacitor tanks (0.366 in.^{3}). At these low pressures the flow entering the feedback loop encounters less resistance, so the discharging time of the capacitors decreases, thereby increasing the frequency. This condition can be corrected by slightly increasing the capacitor size or by increasing resistor R_2. However, a change in R_2 would sacrifice the temperature insensitivity of the element.

4. CONCLUSIONS

Experimental studies using three geometrically similar relaxation oscillators with different resistances in the feedback network have shown that a change in frequency of less than ±2 percent is achieved over a pressure range from 6 to 30 psig. A frequency variation less than 1 percent can be obtained over a temperature range from 77° to 175°F.

The theoretical analysis presented in appendix A has shown that temperature and pressure insensitivity can be obtained simultaneously when using a lumped R-C-R network. Further, the theory provides a firm basis for further understanding, analyzing, and refining the timer oscillator.

5. REFERENCES

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2. Kirshner, J. M., "Fluid Amplifiers," McGraw-Hill Book Company, New York, pp 175-176.

6. ACKNOWLEDGEMENT

The authors thank John Delawter for his craftsmanship in the building of experimental units, Henry Lee for his patience in gathering and reducing the necessary data, and F. M. Manion for his contribution to the theoretical analysis of the R-C-R network to obtain $d\tau_s/dT = 0$.

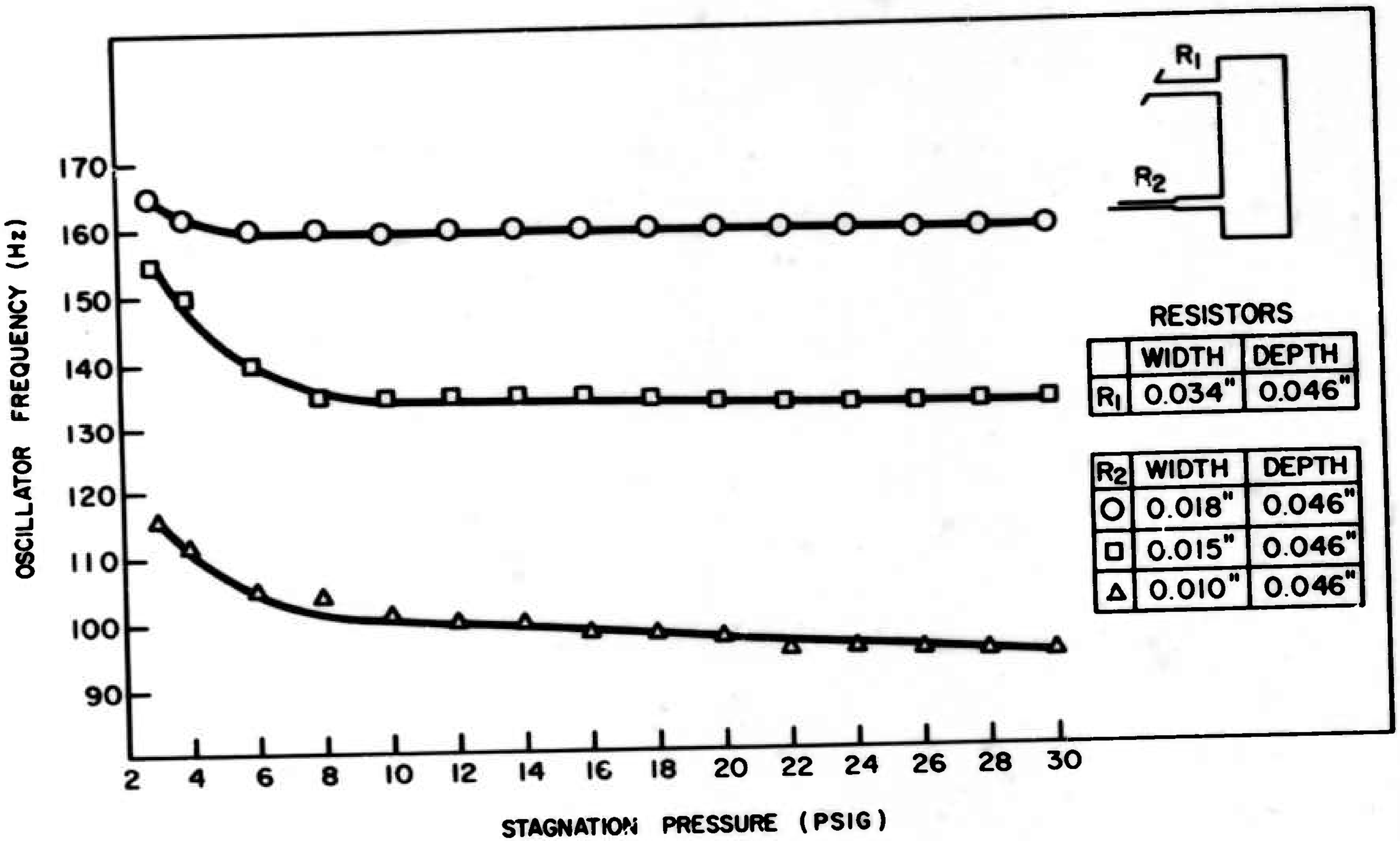

Figure 5. Oscillator frequency versus stagnation pressure.

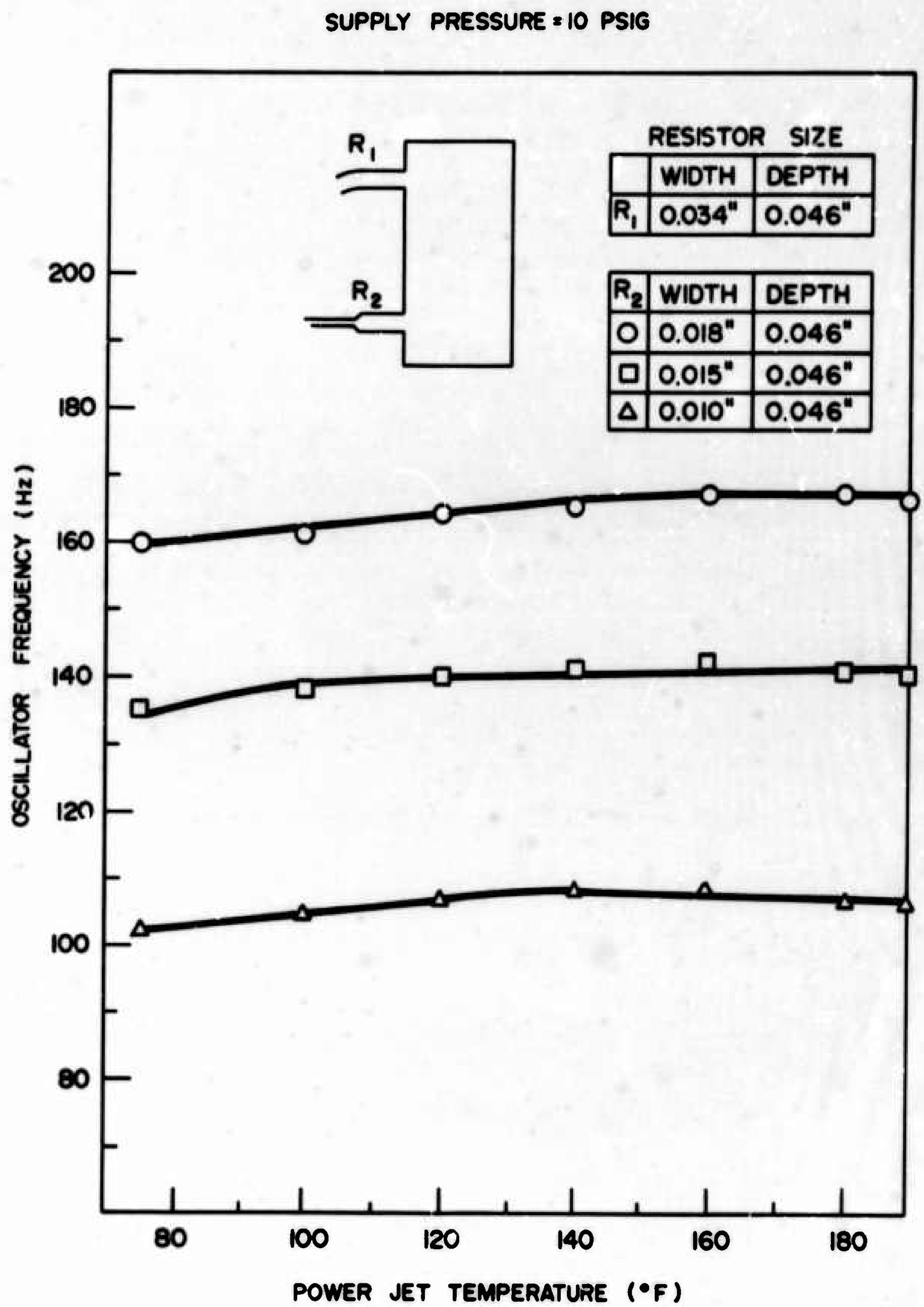

Figure 6. Oscillator frequency versus stagnation temperature.

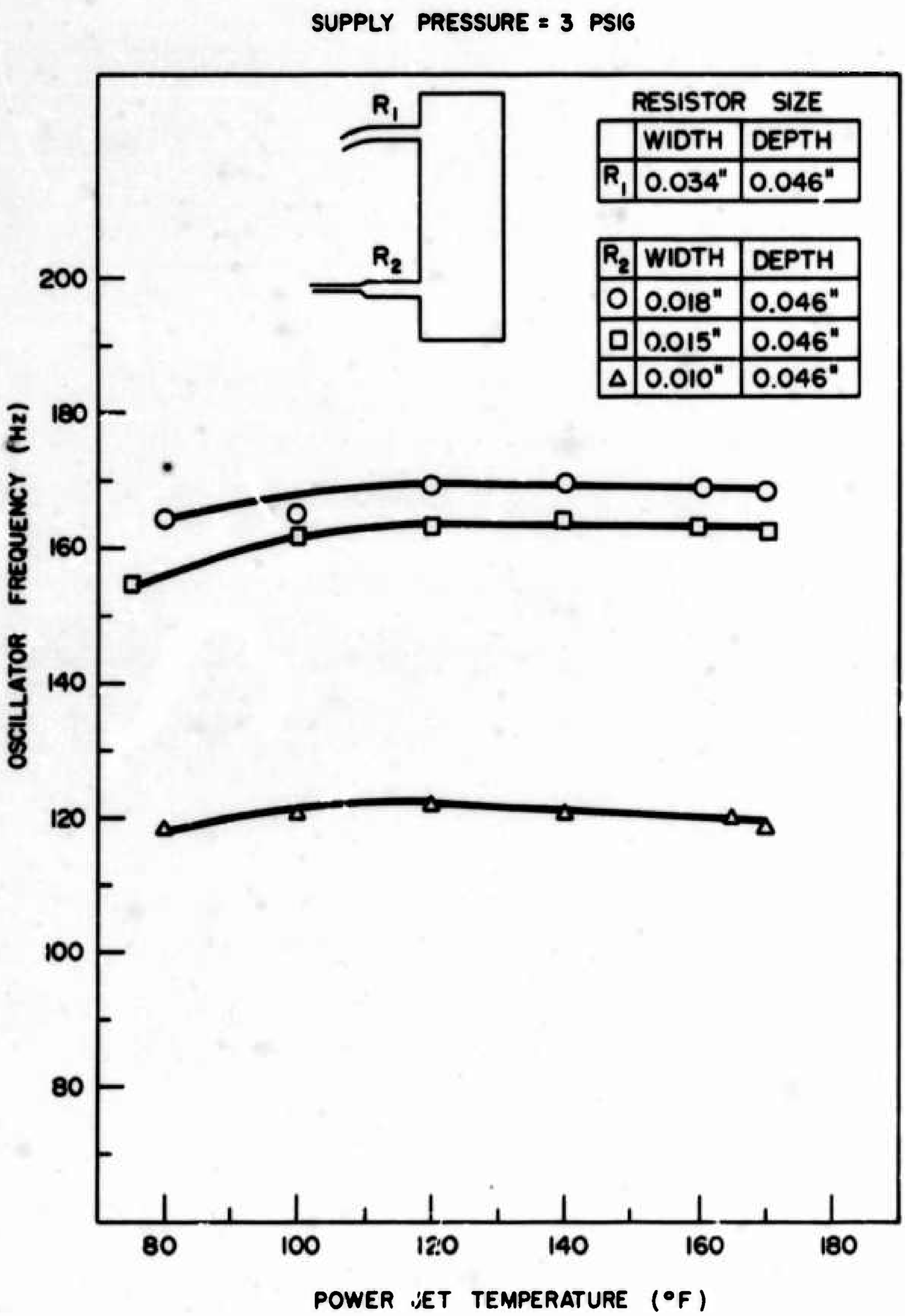

Figure 7. Oscillator frequency versus stagnation temperature.

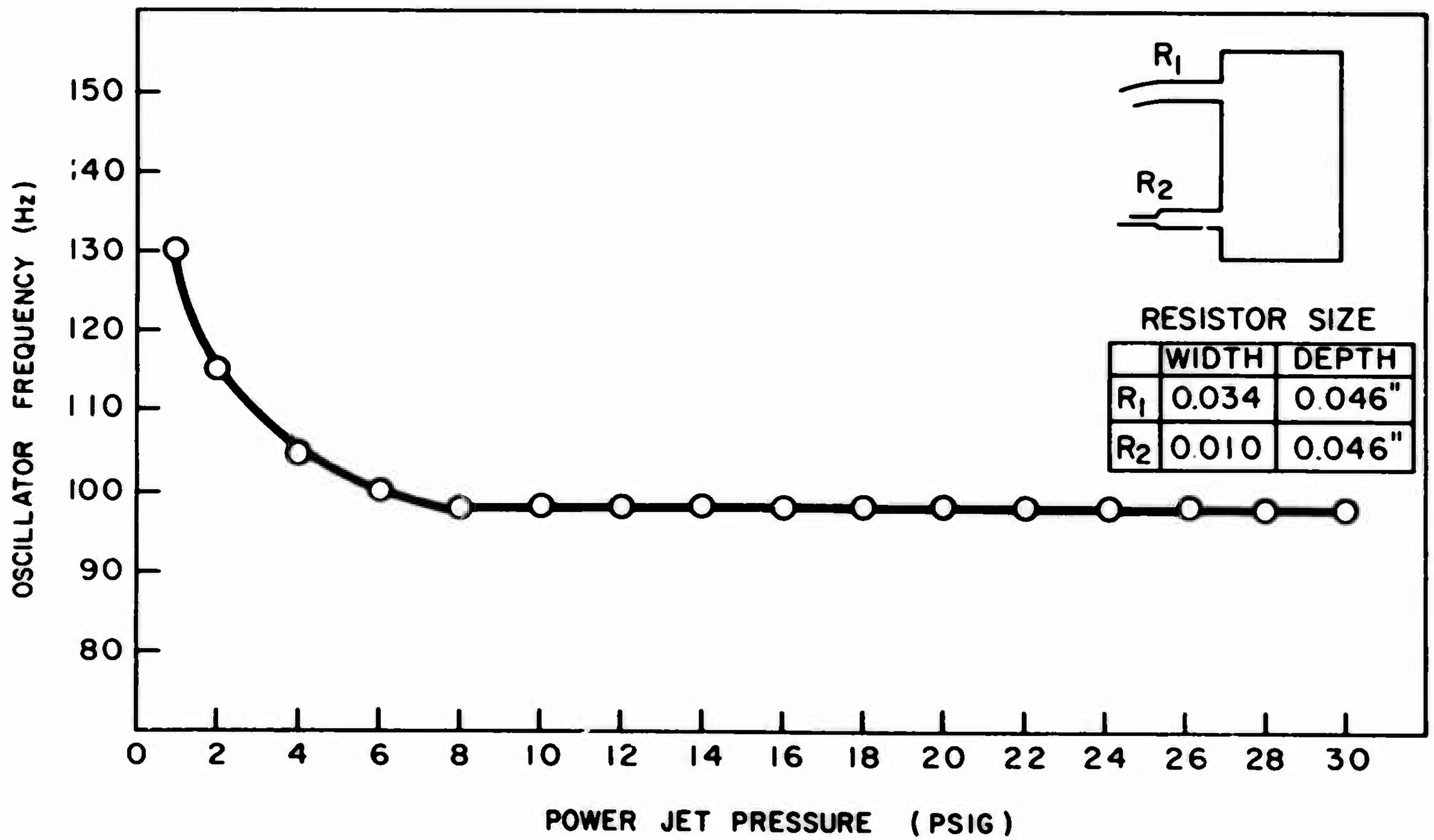

Figure 8. Oscillator frequency versus stagnation pressure.

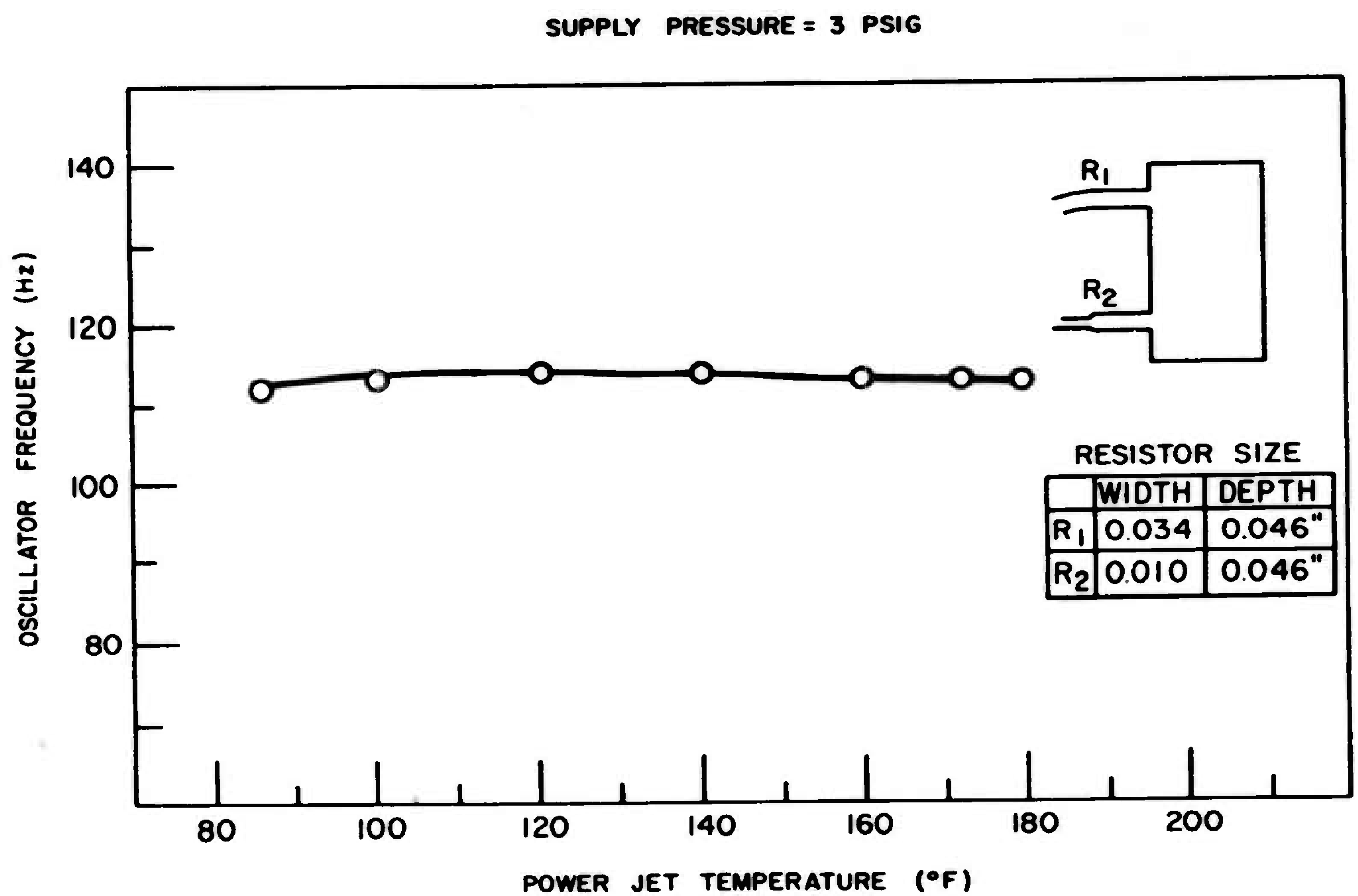

Figure 9. Oscillator frequency versus stagnation temperature.

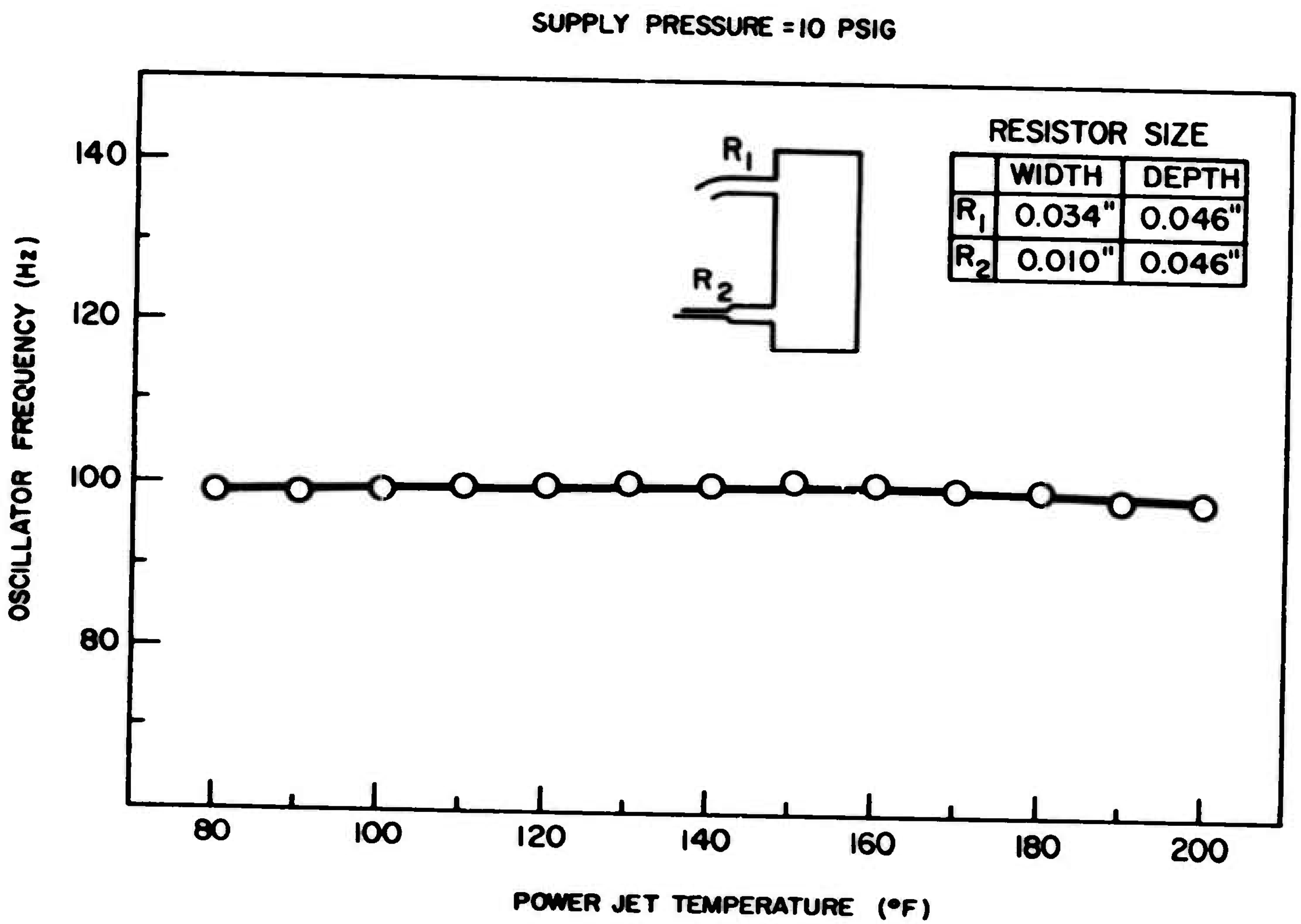

Figure 10. Oscillator frequency versus stagnation temperature.

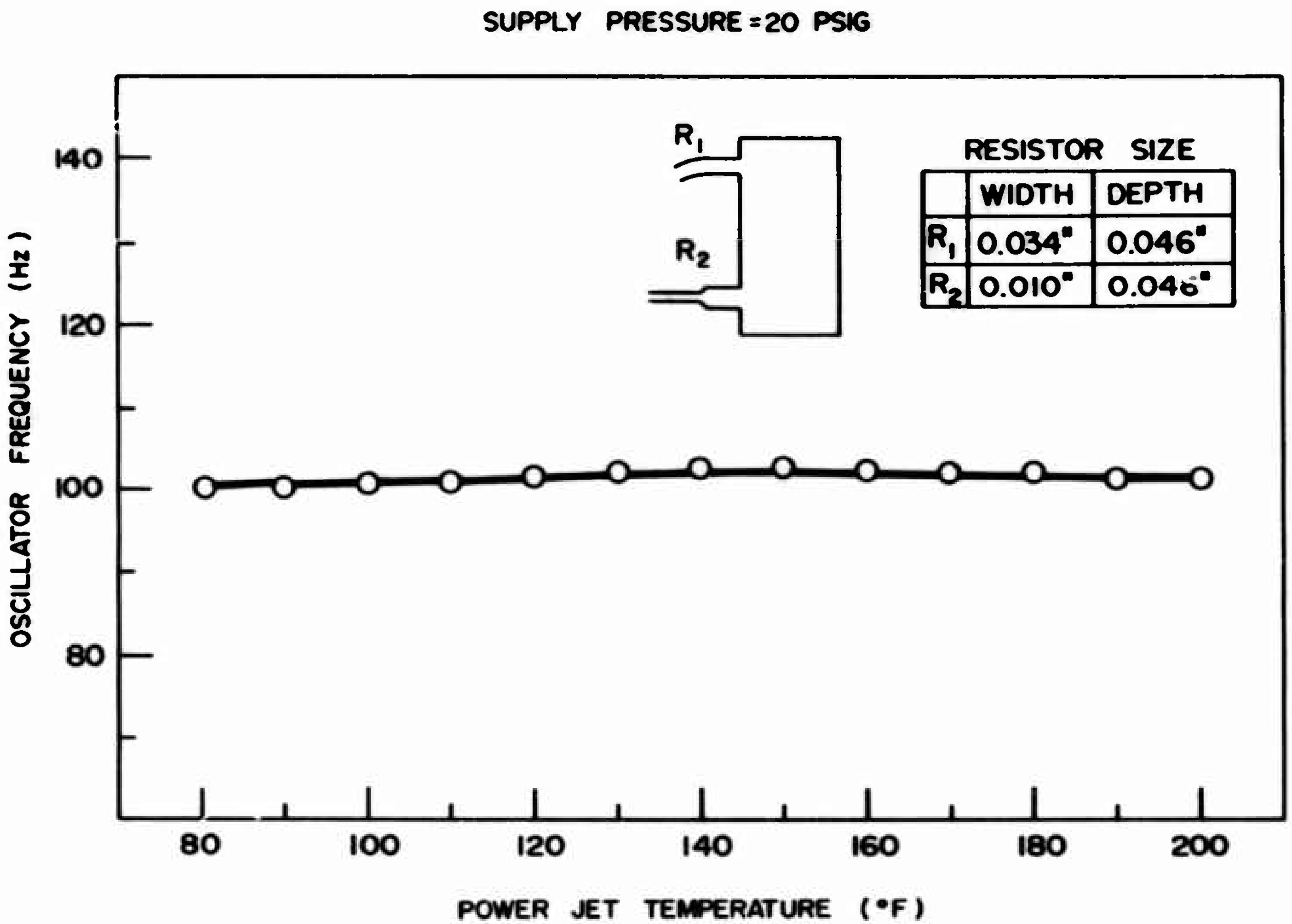

Figure 11. Oscillator frequency versus stagnation temperature.

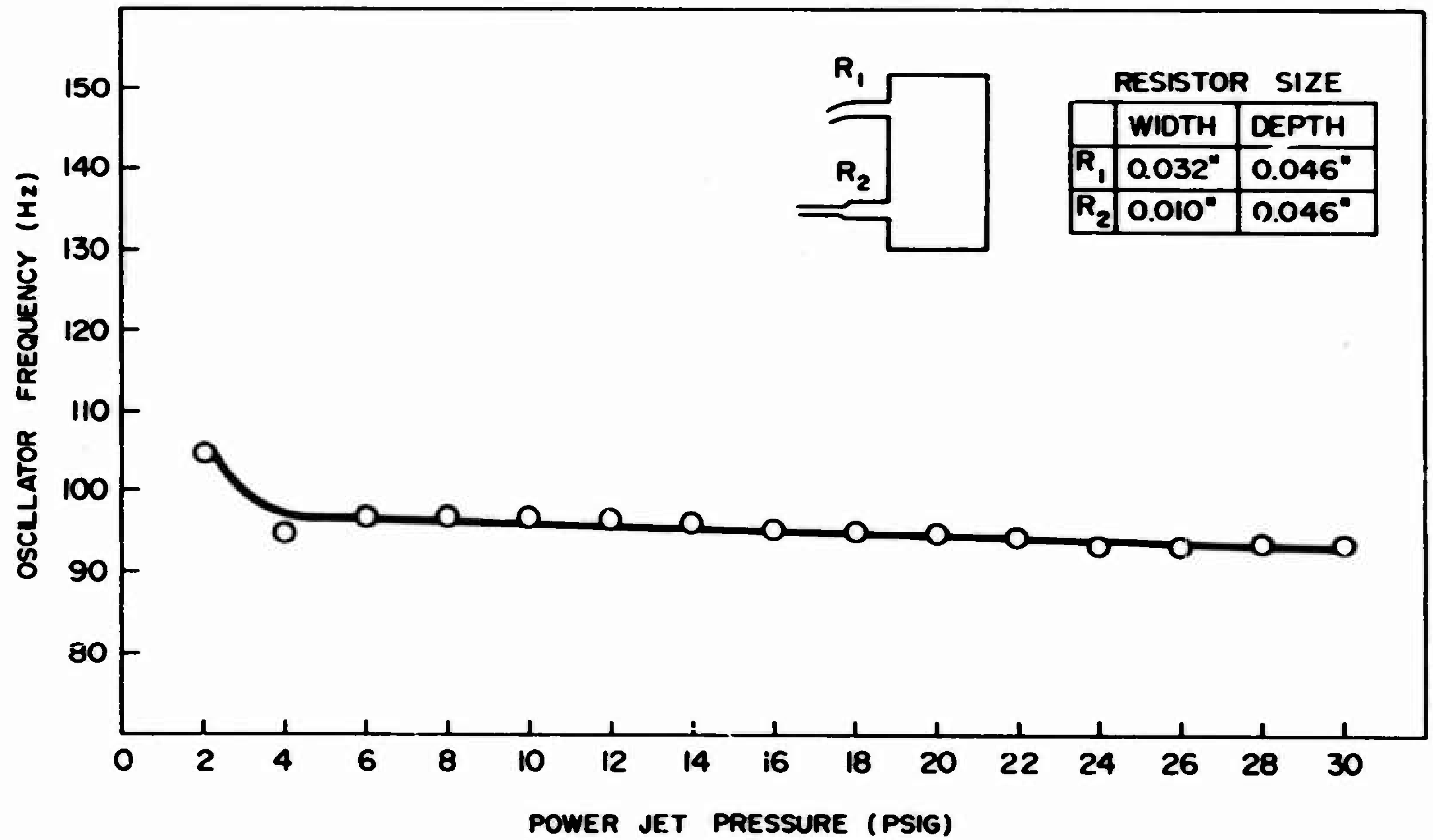

Figure 12. Oscillator frequency versus stagnation pressure.

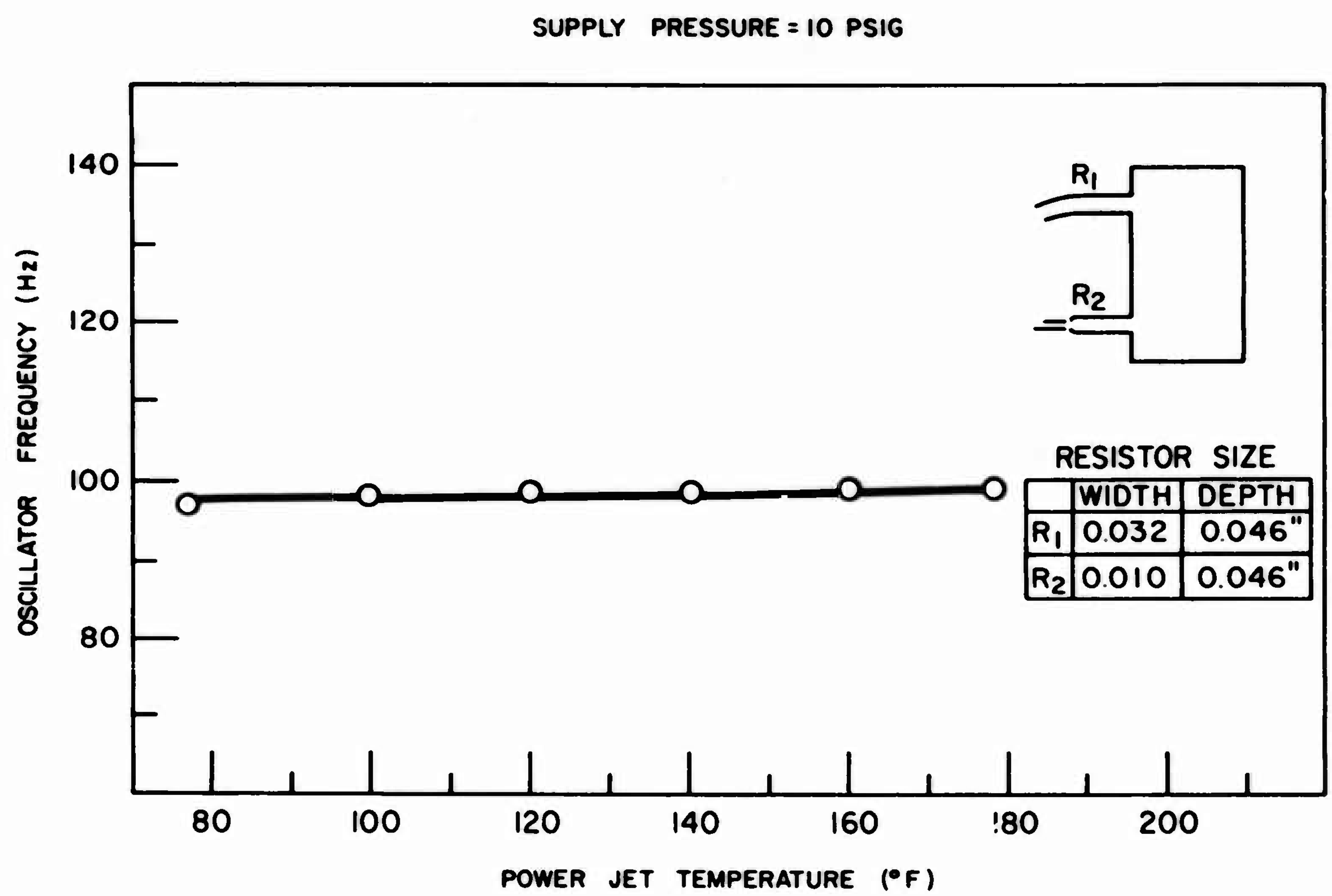

Figure 13. Oscillator frequency versus stagnation temperature.

APPENDIX A. — OSCILLATOR THEORY

This analysis establishes the conditions required for a constant oscillator frequency with variations in stagnation temperature and pressure. The oscillator selected consists of a bistable amplifier with a lumped R-C-R feedback network (fig. A1). To establish the criteria for pressure and temperature insensitivity, it is necessary to consider the flow condition through each branch of the network.

The total flow into the network (through R_1) is $Q_1+Q_2+Q_b$ (fig. A2). The flow into the capacitor is Q_1, and Q_2+Q_b is the flow through R_2. Q_b is a bias flow always present in the network; hence the total pressure P_0 can be written as:

$$P_0 = R_1(Q_1+Q_2+Q_b) + \frac{1}{C}\int Q_1\,dt \tag{A1}$$

where

$$\frac{1}{C}\int Q_1\,dt = R_2(Q_2+Q_b) \tag{A2}$$

The pressure drops across R_1 and R_2 are $R_1(Q_1+Q_2+Q_b)$ and $R_2(Q_2+Q_b)$. Differentiating equation (A2) with respect to time and substituting the results in equation (A1), the control flow Q_2+Q_b is obtained in differential form, so

$$P_0 = \left(R_1R_2C\frac{d}{dt}(Q_2+Q_b)\right) + (R_1+R_2)(Q_2+Q_b) \tag{A3}$$

To solve this equation, it is necessary to consider the initial conditions in the charging portion of the cycle, or

$$P_0(0) = P_0 \quad \text{and} \quad Q_2(0) = 0 \tag{A4}$$

With this initial condition, the solution of equation (A3) can be expressed as:

$$Q_2^{+}(t) = \frac{P_0}{R_1+R_2}\left(1-e^{-t/RC}\right) + Q_b e^{-t/RC} - Q_b \tag{A5}$$

where

$$R = \frac{R_1R_2}{R_1+R_2}$$ and Q_2^{+} is the flow during the charging cycle which increases until switching occurs. At switching the capacitor discharges. The initial conditions for the decay cycles are,

$$P_0 = 0 \quad \text{and} \quad Q_2(0) + Q_b = Q_s \tag{A6}$$

where Q_s is the total flow through R_2 required to switch.

Applying the initial conditions (A6) in equation (A3), an expression for the flow during the discharge cycle is obtained, so

$$Q_2^- = Q_s e^{-t/RC} - Q_b \tag{A7}$$

If it is assumed that the switching mechanism can be expressed by the difference in flow ΔQ_s between the two control ports and that this difference is independent of temperature, then the criterion for switching is written:

$$\Delta Q_s = \left(Q_2^+(\tau_s) + Q_b\right) - \left(Q_2^-(\tau_s) + Q_b\right) \tag{A8}$$

Note that $Q_s = \Delta Q_s + Q_b$

By substituting the expressions for Q_2^+ and Q_2^-, equation (A8) becomes

$$\Delta Q_s = \frac{P_o}{R_1 + R_2} - \left(\frac{P_o}{R_1 + R_2} + \Delta Q_s\right) e^{-\tau_s/RC} \tag{A9}$$

where τ_s is the half cycle.
If the flow and pressure terms in equation (A9) are normalized with respect to the input flow and pressure, (A9) can be rewritten as

$$\overline{\Delta Q_s} = \frac{\overline{P_o}}{R_1 + R_2} - \left(\frac{\overline{P_o}}{R_1 + R_2} + \overline{\Delta Q_s}\right) e^{-\tau_s/RC} \tag{A10}$$

where the terms with the bar are normalized.
In equation (A10) it is assumed that $\overline{\Delta Q_s}$ is constant with changes of input flow and pressure. The temperature dependence of the half cycle is determined by expanding τ_s in the Taylor series

$$\tau_s(T+\Delta T) = \tau_{s_0} + \overline{\Delta T}\,\frac{d\tau_s}{dT} + \frac{(\overline{\Delta T})^2}{2!}\,\frac{d^2\tau_s}{dT^2} + \ldots$$

Temperature insensitivity requires that $d\tau_s/dT = 0$

$$\frac{d\tau_s}{dT} = 0 \tag{A11}$$

Note that for a small temperature range(± 10 percent), the second and higher order terms in the series can be neglected and $\overline{\Delta T} = \Delta T/T$.

By assuming that the switching flow $\overline{\Delta Q_s}$ is temperature independent, its derivative with respect to temperature equals zero or

$$\frac{d\overline{\Delta Q_s}}{dT} = 0 \tag{A12}$$

Carrying out the above derivative and using $\overline{\Delta Q_s}$ from equation (A10) and dropping the bar since it is understood that the terms are normalized, it follows that

$$0 = \left(\frac{P_o}{(R_1+R_2)^2}\left(\frac{dR_1}{dT} + \frac{dR_2}{dT}\right) - \frac{1}{(R_1+R_2)}\frac{dP_o}{dT}\right)\left(e^{-\tau_s/RC} - 1\right) -$$

$$\left(\left(\frac{P_o}{R_1+R_2} + \Delta Q_s\right) e^{-\tau_s/RC}\,\frac{d}{dT}\left(-\frac{\tau_s}{RC}\right)\right) \tag{A13}$$

The derivative of ${}^{-}s/RC$ can be expressed as

$$\frac{d}{dT}\left(-\frac{{}^{-}s}{RC}\right) = -\left(\frac{1}{RC}\frac{d\,{}^{-}s}{dT} - \frac{{}^{-}s}{R^{2}C}\frac{dR}{dT} - \frac{{}^{-}s}{RC^{2}}\frac{dC}{dT}\right) \qquad (A14)$$

A1. TEMPERATURE INSENSITIVITY

The criterion for temperature insensitivity requires that $\frac{d\,{}^{-}s}{dT}$ be equal to zero, so equation (A14) reduces to

$$\frac{d}{dT}\left(-\frac{{}^{-}s}{RC}\right) = \frac{{}^{-}s}{RC}\left(\frac{1}{R}\frac{dR}{dT} + \frac{1}{C}\frac{dC}{dT}\right) \qquad (A15)$$

Now dR/dT can be written as

$$\frac{dR}{dT} = \frac{d}{dT}\left(\frac{R_1R_2}{R_1+R_2}\right) = R\left(\frac{1}{R_2}\frac{dR_2}{dT} + \frac{1}{R_1}\frac{dR_1}{dT}\right) - \left(\frac{R}{R_1+R_2}\right)\left(\frac{dR_1}{dT} + \frac{dR_2}{dT}\right) \qquad (A16)$$

Substituting equations (A15) and (A16) in equation (A13)

$$\left(\frac{P_0}{(R_1+R_2)^2}\left(\frac{dR_1}{dT} + \frac{dR_2}{dT}\right) - \frac{1}{(R_1+R_2)}\frac{dP_0}{dT}\right)\left(e^{-{}^{-}s/RC} - 1\right) =$$

$$\left(\left(\frac{P_0}{R_1+R_2} + \Delta Q_s\right)e^{-{}^{-}s/RC}\right)\left(\frac{{}^{-}s}{RC}\left(\frac{1}{R}\frac{dR}{dT} + \frac{1}{C}\frac{dC}{dT}\right)\right) \qquad (A17)$$

Rearranging terms and dividing both sides by

$$\left(\frac{P_0}{(R_1+R_2)^2}\left(\frac{dR_1}{dT} + \frac{dR_2}{dT}\right) - \left(\frac{1}{R_1+R_2}\right)\frac{dP_0}{dT}\right)$$

yields

$$e^{-{}^{-}s/RC} - 1 = \frac{\left(\left(\frac{P_0}{R_1+R_2} + \Delta Q_s\right)e^{-{}^{-}s/RC}\right)\left(\frac{{}^{-}s}{RC}\left(\frac{1}{R}\frac{dR}{dT} + \frac{1}{C}\frac{dC}{dT}\right)\right)}{\left(\frac{P_0}{(R_1+R_2)^2}\left(\frac{dR_1}{dT} + \frac{dR_2}{dT}\right) - \left(\frac{1}{R_1+R_2}\right)\frac{dP_0}{dT}\right)} \qquad (A18)$$

Adding +1 to both sides and multiplying by $e^{{}^{-}s/RC}$, equation(A18) becomes

$$e^{{}^{-}s/RC} = 1 - \left(1 + \frac{\Delta Q_s(R_1+R_2)}{P_0}\right)\frac{{}^{-}s}{RC}\left(\frac{\left(\frac{1}{R}\frac{dR}{dT} + \frac{1}{C}\frac{dC}{dT}\right)}{\left(\frac{1}{R_1+R_2}\right)\left(\frac{dR_1}{dT} + \frac{dR_2}{dT}\right) - \frac{1}{P_0}\frac{dP_0}{dT}}\right) \qquad (A19)$$

Using the expression for dR/dT obtained from equation (A16), $\frac{1}{R}\frac{dR}{dT}$ can be written as

$$\frac{1}{R}\frac{dR}{dT} = \frac{1}{R_2}\frac{dR_2}{dT} + \frac{1}{R_1}\frac{dR_1}{dT} - \left(\frac{1}{R_1+R_2}\right)\left(\frac{dR_1}{dT} + \frac{dR_2}{dT}\right) \qquad (A20)$$

Substituting the above equation into (A19) and dividing the numerator and denominator of the last bracket by

$$\left(\frac{1}{R_1 + R_2}\right)\left(\frac{dR_1}{dT} + \frac{dR_2}{dT}\right)$$

the numerator becomes

$$\frac{(R_1 + R_2)\left(\frac{1}{R_2}\frac{dR_2}{dT} + \frac{1}{R_1}\frac{dR_1}{dT}\right)}{\frac{dR_1}{dT} + \frac{dR_2}{dT}} - 1 + \frac{\left(\frac{R_1 + R_2}{C}\right)\frac{dC}{dT}}{\frac{dR_1}{dT} + \frac{dR_2}{dT}}$$

and the denominator is

$$1 - \frac{\frac{R_1 + R_2}{P_0}\frac{dP_0}{dT}}{\frac{dR_1}{dT} + \frac{dR_2}{dT}}$$

At this point it is necessary to introduce functional relationships for R_1, R_2, P_0, and C which are

$$R_1 = R_{10}T^n \qquad P_0 = P_{00}T^a$$

$$R_2 = R_{20}T^m \qquad C = C_0 T^{-1}$$

The derivatives of the above terms are

$$\frac{dR_1}{dT} = \frac{nR_1}{T}, \quad \frac{dP_0}{dT} = \frac{aP_0}{T}, \quad \frac{dR_2}{dT} = \frac{mR_2}{T}, \quad \frac{dC}{dT} = -\frac{C}{T}$$

where a characterizes the dependency of P_0 on temperature due to output resistances. Typically $0 \leq a \leq 1/2$. Substituting the above relations in the numerator and denominator of the last bracketed term of equation (A19) makes that term

$$\frac{R_1(m-1) + R_2(n-1)}{R_1(n-a) + R_2(m-a)} = B$$

For convenience, the remaining terms are defined

$$\frac{\tau_s}{RC} = x \qquad A = \left(1 + \frac{\Delta Q_s(R_1 + R_2)}{P_0}\right)$$

and equation (A19) becomes

$$e^x = 1 + (A \cdot B)x \tag{A21}$$

The above equation is the condition that must be satisfied so that $\frac{d\tau_s}{dT} = 0$, or for the frequency to be independent of changes of stagnation temperature. Equation (A21) is the locus of points at which $d\tau_s/dT = 0$, and since $f = 1/2\tau_s$, then $df/dT = 0$. The equation is shown graphically in figure A3. From figure A3, if x is too large, $df/dT < 0$, since $dR/dT > 0$; if $df/dT > 0$, the frequency of oscillation increases with temperature.

In order to attain a solution for $df/dT = 0$, equation (A21) must be satisfied and it can only be satisfied if $A \cdot B > 0$, which implies that the exponents m and n cannot be greater than one simultaneously. Experiments have shown that m and n lie between 0.5 and 1.75; 0.5 is characteristic of an orifice-type resistor and 1.75 is typical of a laminar capillary tube.

To illustrate that τ_s can be made constant, assume that $a = 0$ (output load is temperature independent), $m = n = \frac{1}{2}$ (orifice), and $R_1 = R_2$; then $R = \frac{1}{2}R_1$ and $B = 1$. Thus the differential equation for the charge portion of the cycle becomes

$$\frac{dQ_2^+}{dt} = \frac{P_0}{R_1^2 C} - \frac{2(Q_2^+ + Q_b)}{R_1 C} \tag{A22}$$

and for the decay portion

$$\frac{dQ^-}{dt} = - \frac{2(Q_2^- + Q_b)}{R_1 C} \tag{A23}$$

The initial condition for the decay cycle is $Q_2^- = \Delta Q_s$. Equation (A22) shows that $\dot{Q}_2^+$ decreases with increasing temperature. This is indicated by the second term of the equation, where R_1C is proportional to $T^{-1/2}$; however, the first term P_0/R_1^2C is independent of temperature. In equation (A23) $\dot{Q}_2^-$ increases negatively since R_1C is proportional to $T^{-1/2}$. The above analysis is diagrammed in the phase plane (fig. A4). The diagram shows that for a temperature increase, the charging and decay paths shift (dashed curves); however, the time integral along these paths remain unchanged. Therefore the charge time and decay time sum to the same period; hence the frequency is temperature insensitive.

A2. PRESSURE INSENSITIVITY

The condition for pressure insensitivity can be obtained by using the assumptions of the previous section.

By defining the functional relationships

$$R_1 = R_{11} P^{\alpha}$$

$$R_2 = R_{21} P^{\beta}$$

$$P_0 = P_{11} P^b \quad \text{and} \quad C = \frac{k_1}{P_0}$$

equation (A19) can be written as

$$e^{\tau_s/RC} = 1 + \left(1 + \frac{\Delta Q_s(R_1 + R_2)}{P_0}\right) \frac{\tau_s}{RC} \left(\frac{R_1(1 - \beta) + R_2(1 - \alpha)}{R_1(\alpha - b) + R_2(\beta - b)} \right) \tag{A24}$$

where derivatives in (A19) have been taken with respect to P instead of T.

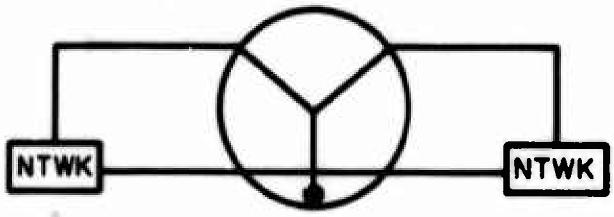

Figure A1. R-C-R feedback oscillator.

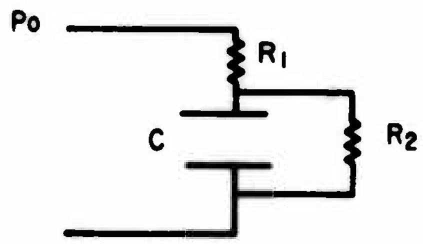

Figure A2. Feedback network.

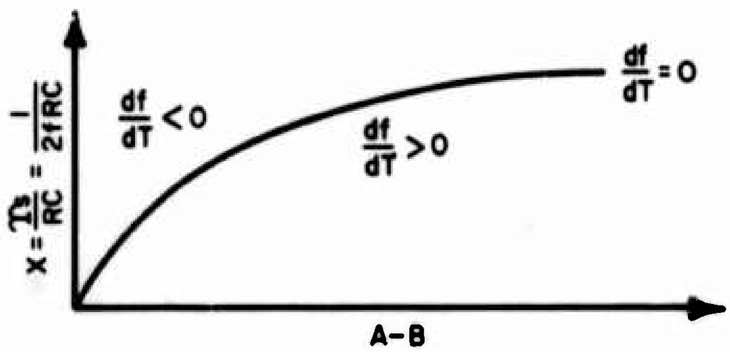

Figure A3. Locus of points for temperature insensitivity.

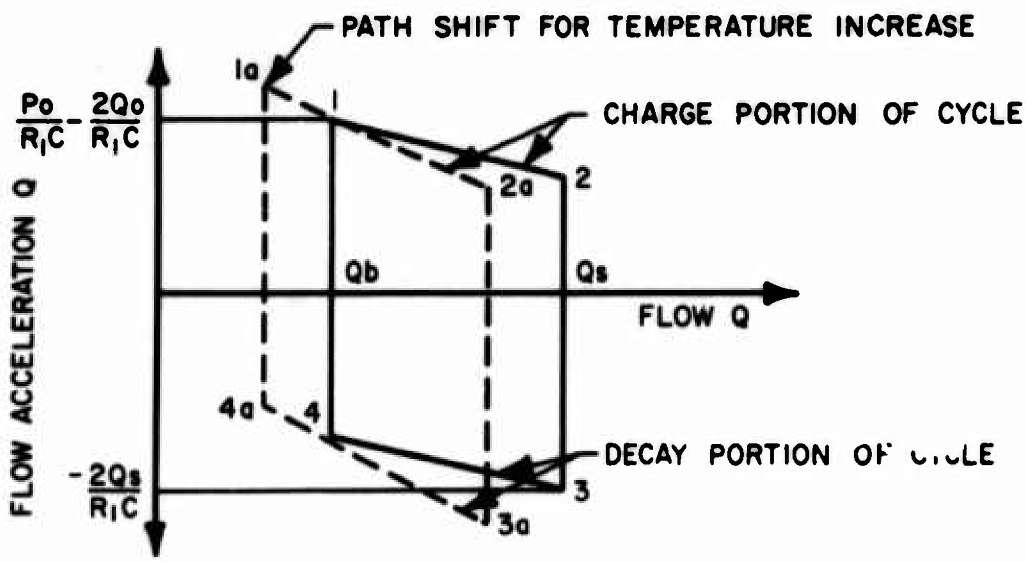

Figure A4. Modified phase plane.

and letting

$$D = \frac{R_1(1-\beta)+R_2(1-\alpha)}{R_1(\alpha-b)+R_2(\beta-b)} \qquad x = \frac{\tau_s}{RC}$$

$$A = \frac{1+\Delta Q_s(R_1+R_2)}{P_0}$$

equation (A24) is written as

$$e^x = 1 + (A \cdot D)x \tag{A25}$$

If D = B, where B was defined in the previous section, then the locus of points defining $d\tau_s/dP = 0$ falls on the same line for temperature insensitivity. If D = B, pressure and temperature insensitivity will not occur simultaneously (see fig. A5). From figure A5.

For $D > B$, if $\frac{df}{dP} = 0$, then $\frac{df}{dT} > 0$

$D = B$ if $\frac{df}{dP} = 0$, then $\frac{df}{dT} = 0$

$D < B$ if $\frac{df}{dP} = 0$, then $\frac{df}{dT} < 0$

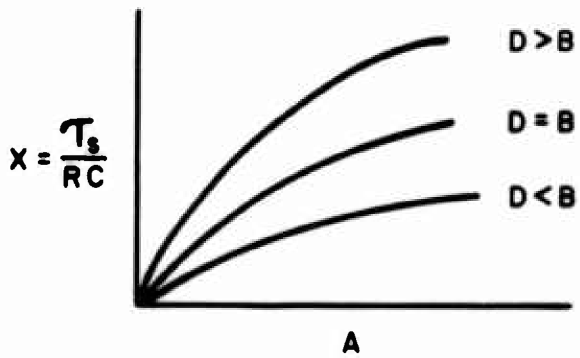

Figure A5.

A3. DISCUSSION

The equations developed thus far serve to indicate that pressure and temperature insensitivity can be obtained simultaneously by careful design. The assumptions used to derive the equations may not be true for all cases. For instance, the assumption that ΔQ_s is independent of pressure and temperature may not hold. Also it should be noticed that the exponents m, n, a, α, β, and b are functions of Reynold number; hence their values will change during a pressure or temperature test. However, equations (A21) and (A24) do indicate that the development of a pressure and temperature insensitive oscillator can be achieved.

It is important to note that the criterion for temperature and pressure insensitivity was that df/dP or df/dT equal zero. This is true only if a small pressure and temperature interval is used (80° F above or below room temperature). If a larger temperature or pressure range is required, the second derivative $d^2f/dT^2 = 0$ from the Taylor series must be included. In addition for the case of B = D, it is required that $n = \alpha$, $m = \beta$, and $a = b$. In the analysis it was assumed that the feedback is strictly an R-C-R network. However, in the design of an oscillator it is necessary to include some inductance. This causes the function for ΔQ_s to become very complex, and the solution of equations is not easily obtained.

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13. ABSTRACT

Three fluid relaxation oscillators using R-C-R feedback loops were tested to establish the feedback resistances required to make the oscillator frequency insensitive to temperature and pressure. The frequency of the three oscillators, geometrically similar but with different feedback resistances, was measured as a function of stagnation pressure and temperature. With modifications guided by these data, one of the oscillators showed frequency variations of less than 2 percent with changes in input pressure of 6 to 30 psig and changes in temperature of 75° to 200°F. A theoretical analysis indicates that an oscillator frequency simultaneously insensitive to temperature and pressure can be achieved using a lumped R-C-R network in the feedback loop.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Fluericsl						
Fluidics						
Oscillator R-C-R						
Temperature-Pressure Insensitive Oscillatcr						
Timer (Pneumatic)						