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PRINCIPLES OF AERIAL PHOTOGRAPHY AND PHOTOGRAMMETRY

By

F. V. Drobyshev

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PRINCIPLES OF AERIAL PHOTOGRAPHY AND PHOTOGRAMMETRY

By: F. V. Drobyshev

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F. V. Drobyshev

OSNOVY AEROFOTOS"YEMKI I FOTOGRAMMETRII

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ABSTRACT: A text book of 8 chapters, 257 pages whose purpose, according to the author, is to outline the basic processes and methods of operation on the creation of topographic maps by means of aerial photos. In addition, the theory and construction of basic aerial photography and photogrammetric instruments are considered. The book is intended as a text for students of optical and mechanical specialties at institutes of geodesy, aerial photography and cartography. English translation: 257 pages.

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

* ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

FOLLOWING ARE THE CORRESPONDING RUSSIAN AND ENGLISH DESIGNATIONS OF THE TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}
rot	curl
lg	log

ANNOTATION

The book outlines the basic processes and methods of operations on the creation of topographic maps by means of aerial photos. Furthermore, the theory and construction of basic aerial photography and photogrammetric instruments are considered.

The work is intended for students of optical and mechanical specialties at institutes for engineers of geodesy, aerial photography, and cartography.

INTRODUCTION

The course "Principles of aerial photography and photogrammetry" contains descriptions of photographic and photogrammetric processes and instruments applied during the creation of maps of terrain, plans of engineering constructions, and other objects.

Photography is the initial process from which photographs of the assigned section of terrain are obtained. Photographing can be carried out both from the ground and from aircraft. Accordingly there exist ground photography and aerial photography.

Although the goal of photogrammetric operations on the creation of maps is common to the purpose of geodesic operations, the first differs from the second by the fact that in them processes of measurements are executed from photographs and in laboratory conditions. This circumstance makes it possible to apply highly productive instruments of stationary type for the creation of maps.

The widest application is found by aerial photography and the photogrammetric operations connected with it; therefore we will begin our course with them.

Vertical aerial photography for cartography is executed by aerial cameras with focal lengths $f_k = 36$ to 500 mm. The height of photographing can be 400 to 6000 m. Table 1 shows the scales of the obtained aerial photos, depending on the values of f_k and H.

On aerial photos of scale 1:2000 used in large-scale cartography terrain is depicted in detail with its small elements (shrubs, paths, grassy vegetation, etc.).

Table 1

H, м	f_k, мм		
	70	100	200
400	1:5714	1:4000	1:2000
500	1:7143	1:5000	1:2500
750	1:10714	1:7500	1:3750
1000	1:14286	1:10000	1:5000
2500	1:35714	1:25000	1:12500
4000	1:57143	1:40000	1:20000
6000	1:85714	1:60000	1:30000

On aerial photos of scales 1:10,000-1:50,000 topographic objects are depicted very completely; therefore, at present aerial photos on these scales serve as the basic material for detailed cartography of a country.

Aerial photos of scales 1:60,000 used for small-scale cartography give a generalized picture of the terrain. Small structures on such aerial photos will not be identified exactly, and sometimes are not at all conspicuous; certain country roads and small inhabited localities are not seen. In such cases additional survey by an aerial camera with a longer focus objective is done in addition to basic aerial photography. Aerial photos obtained by the former method serve only for identification of details on aerial photos taken by the basic aerial camera.

Aerial photos of the indicated scales are used to create topographic maps and photoplans on scales of 1:1000, 1:2000, 1:5000, 1:10,000, 1:25,000, 1:50,000, 1:100,000.

Map making on the scale 1/M from aerial photos of scale 1/m requires both a general change of scale, and conversion of the central projection, which an aerial photo represents as the plane of intersection by the pencil of rays, into an orthogonal projection. The instructions in effect specify accuracy of horizontal position of points on a map of 0.4 mm, and in height – 1/3 of the section of horizontals. Accepted cross sections of horizontals (in m) for maps of different scales are given in Table 2.

Table 2

Scale of map						
1:1000	1:2000	1:5000	1:10000	1:25000	1:50000	1:100000
0,5 1,0	0,5 1,0 2,0	1,0 2,0 2,5 5,0	1,0 2,5 5,0	2,5 5,0 10,0	10,0 20,0	20,0 40,0

Several sections for a map of one scale are established depending upon the drop of heights of the terrain. The ratio of scales of aeria photos and the map $1/m:1/M = n$ can be within limits of 0.5-6.

Most frequently photogrammetric methods are used for creation of medium-scale and large-scale topographic maps.

Accordingly, maps of the scale 1:25,000 are used for needs of large economic construction, cartography of industrial regions, and other purposes.

Maps of scale 1:10,000 serve as a basis for special surveys; they are used in exploring communication routes, construction of engineering projects, in geological surveys, land management, forest management, and for making maps on smaller scales.

Maps on scales 1:2000 – 1:5000 are used in planning cities, industrial regions, railroad junctions, and other objectives.

Another type of map in the form of photomaps, created from aerial photos, is widely applied in land management, designing inhabited localities, exploratory work, and in other branches of the Soviet economy.

Photogrammetric methods are applied also for solution of scientific problems not directly connected with topographic survey, for instance, in the study of the surface form of the agitated sea, determining deformation of the terrestrial surface, in constructions, and in other problems.

Also known is the use of aerial photos for the study of the history of changes of the earth's surface – for instance, shift of river channel, for assessment, study of places of meteorite fall, etc. By means of aerial photography of water surface, the position of reefs and shallows, the presence of schools of fish, etc., are established.

Photogrammetric methods are applicable also during treatment of photographs of microsurvey of mechanical, biological, or physical objects. Thus it is possible to determine their dimensions or, for instance, the path of movement of elemental particles in a cloud chamber, and so on.

In photogrammetric work various instruments are used based on the use of precision mechanics and optics, and more recently on electrical computers and electronics.

Construction and application of these instruments requires knowledge of the principles of aerial photography and photogrammetry. Therefore the content of the course includes a description of flight-survey work the theory of the connection of points of terrain and aerial photos, consideration of map making methods, and description of photogrammetric instruments, both Soviet and foreign. Furthermore the theory and practice of application of ground stereophotogrammetric survey is expounde

CHAPTER I

THE NATURE OF AERIAL PHOTOS AND THEIR USE

§ 1. General Information

An aerial photograph constitutes a photographic image of the earth's surface, obtained with the help of an aerial camera. At angles of tilt within limits of 0-5° aerial photos are designated vertical; at angles of more than 5° they are designated oblique. Geometrically both the one and the other type of aerial photos constitute a central projection of terrain; front and rear nodal points of the objective serve as summits of pencils of rays.

Fig. 1 shows a diagram of projecting rays proceeding from points O, N, and M. The ray going from point M of the terrain passes through combined (front and rear) nodes of the objective – point S, and fixes it at point m of the aerial photo.

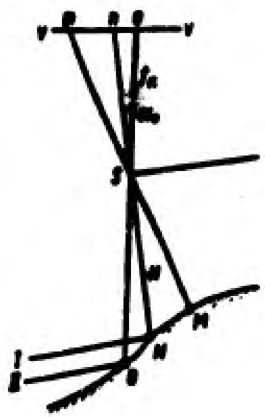

Fig. 1.

Principal point o of the aerial photo is the name given the base of the perpendicular dropped from the rear nodal point of the objective to the plane of the aerial photo. The distance from the rear nodal point of the objective to the plane of the auxiliary frame of the aerial camera is called the principal distance, or focal length of the chamber of the aerial camera f_k. Plumb line NSn is called the nadir line, and the point of its intersection with the plane of the aerial photo is called the point of nadir n of the aerial photo.

The angle between the principal ray So and nadir line Sn is called the angle of tilt α_0 of the aerial photo; segment SN is the height H of the station, or height of photographing. Points N and O are traces on the terrain corresponding to the nadir line and principal axis. One of the planes (I or II) passing through points N and O is taken as the initial plane of the map.

In Fig. 2 is depicted an aerial photo, whose principal point is determined by intersection of lines passing through coordinate marks of the auxiliary frame of the aerial camera. Line vv, which is the trace of the plane passing through principal ray So and the perpendicular to the plane of the aerial photo, is called the principal vertical; in the plane of the principal vertical is the angle of tilt α_0 of the aerial photo. Line hh, perpendicular to line vv, is called the principal horizontal. During treatment of isolated aerial photos lines vv and hh are taken correspondingly as axes x and y. With respect to the axes of flight marked by the arrow, the line of marks composes angle $\kappa = 0° \pm 5°$, and the line of the principal vertical vv - angle κ_0 within limits of 0-360°. Thus, orientation of the aerial photo in space is determined by angles κ_0, α_0 of the vectorial angle system.

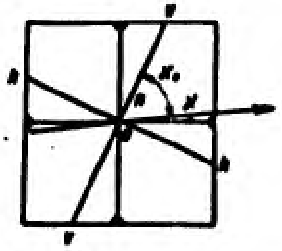

Fig. 2.

The position of the aerial photo can be determined also by angles of tilt relative to selected coordinate axes X, Y.

For creation of maps aerial photos of size 18 × 18; 18 × 24; 30 × 30 cm are used; in aerogeodesic production 18 × 18 cm is the basic size.

During use of an aerial photo it is necessary to know the position of its principal point and the origin of coordinates with respect to straight lines passing through marks of the aerial photos. Coordinates Δ_{x_0} and Δ_{y_0} of the principal point and also the value of f_k are called elements of internal orientation of aerial photos.

In the process of manufacture of the aerial camera and its adjustment we strive to bring magnitudes Δ_{x_0} and Δ_{y_0} to zero; therefore the point of intersection of straight lines passing through the marks is the origin of coordinates of points of the aerial photo. On edges of the aerial photo, besides marks there are photographed data specifying the readings of the level, time, focal length, etc. These data can be obtained also from the data sheet of the aerial camera.

Given determinations are true both for vertical and oblique aerial photos, i.e., aerial photos with large angles of tilt of the optical axes of the aerial camera (over 5°) with respect to the vertical.

Oblique aerial photos have a peculiarly sharp distinction of scales of image in various parts, considerably complicating the process of map making. Therefore in the Soviet Union such photos almost never are used in cartography. Oblique aerial photography at angles of tilt of aerial photos of 20-30° has certain value for determination of the state of ice in the artic and antarctic.

§ 2. The Connection of Coordinates of Points of the Aerial Photo with Points of Terrain

It is accepted that the positive direction of axes X toward the terrain will be the direction proceeding to the right from the origin of coordinates, and that of axis Y — forward from it. Correspondingly, are considered the angles between the vertical line and right or front position of the optical axis are considered to be positive angles of tilt of aerial photos.

Origin of coordinates on the aerial photo at point n. Let us consider that axes x of the aerial photo coincides with the direction of the principal vertical vv, and axis y with the line perpendicular to it and passing through nadir point n. Tracing through point e (Fig. 3) line en' parallel to axes X on the terrain, we obtain

$$\frac{en'}{X_n} = \frac{nS - nn'}{H}; \quad en' = x_n \cos \alpha_0; \quad nn' = x_n \sin \alpha_0; \quad nS = \frac{f_k}{\cos \alpha_0}.$$

After substitution we have

$$\frac{x_n \cos \alpha_0}{X_n} = \frac{\frac{f_k}{\cos \alpha_0} - x_n \sin \alpha_0}{H}. \qquad (1)$$

Erecting perpendiculars to the principal vertical from points e and E we find for corresponding segments

$$e_1 e = y_n \text{ and } EE_1 = Y_n$$

$$\frac{y_n}{Y_n} = \frac{\frac{f_k}{\cos \alpha_0} - x_n \sin \alpha_0}{H}. \qquad (2)$$

By formulas (1) and (2) in the presence of known values of α_0; f_k; H; x_n and y_n it is possible to calculate coordinates X_n, Y_n of points of terrain. With different signs of x_n and α_0 it is necessary to replace the minus sign in the formula with a plus. The scale of the aerial photo at point n is equal to

$$\frac{1}{m} = \frac{f_k}{H \cos \alpha_0}. \qquad (3)$$

Origin of coordinates at point c (at the point of intersection of the principal vertical with the bisector of angle α_0 and called the point of zero distortions of the aerial photo). Tracing the line ec' parallel to EC, we obtain ec' = ec = x_c (Fig. 4). Triangle ecc' is isosceles; Δ ecc' = Δ ec'c = $90° - \frac{\alpha_0}{2}$; $So_1 = So = f_k$.

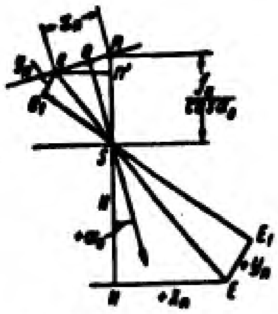

Fig. 3.

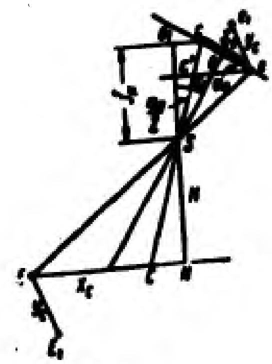

Fig. 4.

From similarity of triangles Sc'e and SCE we obtain

$$\frac{x_c}{X_c} = \frac{f_k - x\sin\alpha_0}{H}. \tag{4}$$

Tracing from points e and E the lines perpendicular to the principal vertical, as before we find the relation of ordinates of points of the aerial photo and terrain:

$$\frac{y_c}{Y_c} = \frac{f_k - x_c\sin\alpha_0}{H}. \tag{5}$$

The scale of the aerial photo at point c is equal to

$$\frac{Sc}{SC} = \frac{\frac{f_k}{\cos\alpha_0}}{\frac{H}{\cos\alpha_0}} = \frac{f_k}{H}. \tag{6}$$

i.e., it is equal to the scale of the horizontal aerial photo. It remains the same also along the line passing through the point of zero distortions and parallel to the principal horizontal. This line is called the line of the undistorted scale.

Ratio $\frac{f_k}{H}$ or 1/m is called the scale of aerial photography.

The given formulas for the connection of coordinates of points of the aerial photo and terrain serve as initial for the derivation of certain formulas of vertical aerial photography and also are used during investigations of photogrammetric instruments.

If the direction of tilts of a pair of aerial photos does not coincide with coordinate axes of the instrument, the formulas for connection of coordinates of points will have a more complicated expression [1], [3] and they will serve for creation of models of aerial photos on glass containing a number of conditional points of terrain expressed in the system of spatial coordinates. Such models are used during investigation of accuracy of work of stereophotogrammetric instruments.

§ 3. Distortion of the Aerial Photo Due to Relief of the Terrain

If the angle of tilt of the aerial photo is equal to zero, flat terrain is depicted in the form of an exact map: if there is relief, points of an aerial photo are displaced relative to their positions on a map. In Fig. 5 it is shown that orthogonal projection of point E_0 on the plane of the map passing through point O is displaced with respect to the ray of projection eE. Therefore on the aerial photo there will be formed distortion $ee_0 = \delta$, the value of which can be determined by formula

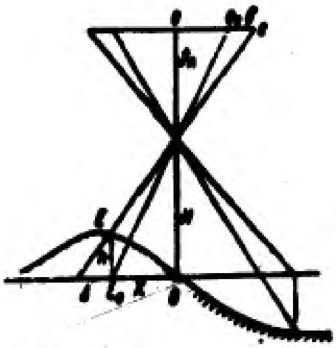

Fig. 5.

$$\frac{\delta}{\Delta} = \frac{f_k}{H}.$$

Furthermore

$$\frac{\Delta}{h} = \frac{X}{H-h}.$$

Hence we will find the value of correction on axis x

$$\delta = \frac{f_k h X}{H(H-h)} \qquad (7)$$

for points located above the initial plane.

The correction with $H = f_k$ for arbitrary radius R (accepted instead of Δ) and points located above or below the initial plane is calculated by the formula

$$\Delta = \frac{Rh}{H-h}. \qquad (8)$$

Example. If $R = 80$ mm (in scale of map); $h = -50$ mm; $H = 1000$ m, value $\Delta = 4.2$ mm. For derivation of a horizontal position of the point it is necessary to defer the correction in the direction of the center of the aerial photo.

With small relief the value of δ can attain several millimeters; on aerial photos of mountain terrain $S_\delta = 20$ mm and more; therefore the transition from an aerial photo to a map of terrain requires a corresponding conversion of projections.

At tilts of aerial photos from 0 to 3° the order of magnitudes of distortions remains the same on horizontal aerial photos.

§ 4. The System of Processes of Map Making by Aerial Photos

As was already noted, the location of points of terrain depicted on the aerial photo does not correspond to their position on the map, due to the difference of the scale of the image in various parts of the aerial photo under the influence of its tilt and of the relief of terrain. Therefore to make a map of terrain from aerial photos it is necessary to eliminate the variety of scale of the image of the aerial photo. Such a conversion of the varied scale of an image to a single scale is called rectification. It is carried out on photorectifiers (projectors). The rectified image obtained on the screen of the instrument, is exposed on photographic paper. After photographic treatment we obtain a rectified aerial photo. By assembling rectified aerial photos on a plane table with reference points we obtain a photomap of the terrain.

For rectifying (and assembling) every aerial photo it is necessary to have on the plane table the geodesic coordinates of three, and in certain cases four, points not located on one straight line.

Coordinates of these points can be obtained either from geodesic field work or by photogrammetric methods, by means of thickening the net of geodesic reference points.

Photogrammetric thickening of the vertical basis is executed by methods of vertical triangulation, photo surveying or by measuring a three-dimensional model of the terrain created by pairs of aerial photos.

Relief of the terrain of the photo map is depicted by contours which are drawn in the field or, which is more widespread, by methods of stereotopographic survey. These methods also are based on measurement of the terrain model created by pairs of aerial photos.

For instance, adjacent aerial negatives are fixed in the instrument which consists of two projecting chambers identical in elements of internal orientation of the surveying camera. Establishing them in the position which they occupied at the time of photographing, we obtain an intersection of analogous projecting rays, as a result of which the terrain model will be formed.

By measuring spatial coordinates of a model by attachments fixed on the screen of the instrument it is possible to create a topographic map in outlines and contours.

With the help of such instruments as the multiplex, it is possible not only to make maps but also to thicken the geodesic basis. Therefore it is accepted to call such instruments and corresponding methods of work <u>universal</u>.

Another solution for the problem of depiction of relief and thickening of elevations is based on use of the dependence between the elevation of points of terrain and the difference of their longitudinal parallaxes.

Basic instruments for this purpose are stereometers, with the help of which contours are traced directly on aerial photos themselves.

From the aerial photos the contours are transfered to the corresponding photo map. If however, the terrain displays considerable relief, then from aerial photos there are made 3-4 times smaller copies which are projected onto the topographic plane table by means of special projectors. By means of circumscription of outlines and contours by pencil there is obtained a map of one or another scale.

Due to differentiation of the entire complex of work into a series of operations executed on different instruments, this method of map making has acquired the name <u>differentiated</u>.

CHAPTER II

AERIAL PHOTOGRAPHY AND AERIAL CAMERAS

§ 5. Aerial Photography of Terrain

Aerial photography of terrain for cartography is carried out by an aerial camera fixed in an aircraft. The latter shifts horizontally and rectilinearly during the time of the entire surveying of the terrain section. After completion of one flight line, the aerial photographer lays a second route in the opposite direction, parallel to the first, and so on.

Film is exposed at intervals of time such that part of the site taken on the preceding aerial photo is retaken on each subsequent aerial photo.

Longitudinal overlap of aerial photos in line of flight survey should be more than 50%. Fig. 6 shows the position of aerial photos of two routes. In this the overlap of flight lines is taken as less than 50%.

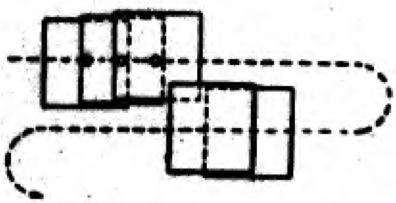

Fig. 6.

When an assignment for aerial photography on the scale $\frac{1}{m} = \frac{f_k}{H}$, is received, the first preliminary requirement is to make the necessary calculations of height of photographing, bases of photographing, intervals between exposures, etc.

Height of photographing H relative to the initial (average) plane of terrain is calculated by the formula

$$H = mf_k; \tag{9}$$

bases of photographing are determined by the formulas

$$b_x = \frac{l_x(100 - P_x)}{100} \tag{10}$$

and

$$b_y = \frac{l_y(100 - P_y)}{100}. \tag{11}$$

where l_x and l_y are the lengths of sides of the aerial photo lengthwise and across the line of flight; P_x and P_y are the longitudinal and transverse overlap.

With $l_x = l_y = 180$ mm, $P_x = 60\%$, and $P_y = 30\%$, correspondingly we obtain

$$b_x = \frac{180(100 - 60)}{100} = 72 \text{ mm},$$
$$b_y = \frac{180(100 - 30)}{100} = 126 \text{ mm}.$$

Knowing the scale of survey $\frac{f_k}{H}$ one can determine segments B_x and B_y on the terrain. Thus at $\frac{f_k}{H} = 1:10{,}000$ segments b_x and b_y of 72 and 126 mm length correspond to 720 and 1260 m on the terrain.

After these calculations one can determine the quantity of aerial photos appearing on the surveying section.

For determination of intervals between exposures, calculate the time during which the aircraft will fly the distance equal to $B_x = b_x \cdot m$ – the base of photographing. If the aircraft flies with speed W km/hr or $\frac{W}{3.6}$ m/sec, the interval between exposures is

$$\tau = \frac{B_x}{W} 3.6. \tag{12}$$

For instance during a speed of flight of 300 km/hour and $B_x = 720$ m the value of $\tau = 8.6$ sec.

Before the beginning of aerial photography the light filter is installed and duration of exposure is determined depending on illumination and color of the terrain, relative aperture of the objective and sensitivity of unexposed aerial film, focal length, and the multiplicity factor of the light filter [14].

Illumination of comparatively flat terrain at solar altitudes of 15° and above is considered sufficient for aerial photography. The predominant color of the terrain, for instance green or yellowish, requires correspondingly larger or smaller exposure time or use of one or another type of film. Presence of haze or upper overcast increases exposure time.

For determination of exposure[1], taking into account all the enumerated factors, exposure meters are used. In spite of the fact that exposure is determined before takeoff for aerial surveying, in flight there is used an exposure meter wedge, or several test aerial photos are made, on which subsequently the degree of accuracy of the exposure meter data is checked and time necessary for high-quality development of a large quantity of aerial photos is determined.

When all calculations are completed lines of flight [runs] of aerial photography are planned on the flight map on a scale of 1:50,000-1:200,000.

Besides the runs, on the map are drawn the turns for the approach of the aircraft to neighboring runs and the individual reference points and boundaries of the surveyed section. The aerial photographer during the flight draws on the map new reference points, useful for laying out runs. By means of the intercom the aerial photographer transmits to the pilot all necessary information about reference points and the direction of movement of the aircraft according to the flight map.

Before the aerial survey the course of travel of the aircraft is determined on the operating altitude on outlined directions of aerial photography runs, taking into account drift angle and groundspeed of the aircraft for more precise definition of the value of intervals between exposures.

The direction and speed of the flight of the aircraft depend on the direction and speed of the wind. The deviation of the aircraft from the assigned direction caused by the influence of the wind is called drift angle, and the speed of shift of the aircraft with respect to terrain is called groundspeed.

So that the aircraft flies in the assigned direction, it is necessary to turn it by a lead angle, depending on the drift angle.

Interval between exposures is established on the command instrument of the aerial camera.

Aerial photography is commenced after leveling the aerial camera and turning it by the lead angle. The rectilinearity of runs, the designated altitude of photographing, overlap, and assigned position of the principal axis of the aerial camera must all be maintained.

Permissible lengths of runs, depending upon scales and focal lengths, are given

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[1]Exposure is the name applied to the quantity of light equal to the product of illumination of the photographic film and the duration of exposure time (time during which the objective of the camera is open).

in Table 3.

Table 3

Scale of map	Scale of aerial photo	Focal length of the camera mm	Length of run, km
1: 2000	1: 2000 — 1: 4000	70, 100, 200	5
1: 5000	1: 5000 — 1:10000	70, 100, 200	15
1:10000	1:10000 — 1:20000	70, 100, 200	25
1:25000	1:25000 — 1:35000	70, 100, 200	45

The process of aerial photography can continue for 4-6 hours. At the same time, the number of clear cloudless days in a season, can be small – for instance, in central and southern latitudes of the Soviet Union excluding winter months, there will be 40-50 days. This explains the responsibility of the aerial photographer for satisfactory aerial photographing. Table 4 gives the average number of surveying days in different regions of the country.

Table 4

Region of country	Surveying period	Winter
Center	40	8
Urals	30	10
South	70	14
Caucasus	55	11

During large-scale cartography high overcast does not prevent aerial photography; therefore the number of surveying days is doubled.

Basic photogrammetric requirements for aerial photography in reference to topographic cartography include:

1. Axes of runs must be parallel to assigned lines (frames of trapezoids, lines of orienting points, valleys, etc.) with precision of +0.015 M, corresponding to 15 mm in the scale of the aerial survey (M – denominator of the scale of photographing).

2. Runs must be rectilinear. Nonrectilinearity causes change of transverse overlap.

Displacement of principal points of aerial photos from the line connecting the principal points of the first and the last aerial photos of the run should not exceed 3 cm, i.e., 3% for the case when the length of the line on the rough mounting of aerial photos is equal to 100 cm.

3. Longitudinal overlap of aerial photos in % is established in accordance with requirements of completed work and can have the value:

$62+38\frac{h}{H}$	with minimum permissible	56,
$80+20\frac{h}{H}$	" " "	78,
$90+10\frac{h}{H}$	" " "	89,

where h is the greatest elevation of points of terrain above the average plane and H is the height of photographing.

Hence it is clear that during survey of mountain terrain longitudinal overlap of aerial photos should be near 90%.

4. Transverse overlap of aerial photos, in %, for the various scales of aerial photography is:

1:10000—1:24000	equal to	$34+66\frac{h}{H}$;
1:25000—1:34000	"	$32+68\frac{h}{H}$;
1:35000 smaller	"	$30+70\frac{h}{H}$.

5. Edges of aerial photos must be parallel to the axis of the run. The nonparallelism of sides of the aerial photo to the axis of the run should not exceed 5°, since with a greater angle the processing of aerial photos is complicated.

6. The height of photographing above the average plane of the surveyed section should not deviate from the assigned value by more than 3% over plains and by 5% in mountain regions, and heights of photographing to 1000 m should deviate not more than 50 m.

7. Elevation of the station of photographing should be fixed by a statoscope (at a height of 2000 m elevations are obtained with an accuracy of 0.7-2 m).

8. Angles of tilt of aerial photos (f_k = 140 mm and less) should not exceed 1°30' and, as an exception, 2°. The number of aerial photos with angles of tilt more than 2° should not exceed 10% of their total number on the surveyed section; in accordance with this, aerial photos with angles of tilt over 3° (at f_k = 200 mm) are not accepted.

Sometimes angles of tilt of aerial photos are obtained with great accuracy.

For instance, with leveling of the aerial camera with the help of a level they can be on the order 1°30'-0°40'.

The fulfillment of point 8 should be not only the aerial photographer must work, but also the flight engineer [?], since during a survey flight the aircraft changes its position in longitudinal direction up to 5° due to expenditure of fuel.

With gyroscopic setting of the aerial camera the mean value of longitudinal and transverse angles of tilt of aerial photos is 10-12'; when their limiting value is allowed equal to 1°; 80-85% of the aerial photos have angles of tilt on the order of 20'.

9. The photographic quality of aerial negatives should satisfy the following of requirements:

a) Fog (D_0) should not be greater than indicated by the factory:

b) minimum density (D_{min}) on the edge of the aerial photo in a ⌀ 2 mm circle is within the limits

$$D_{min} = D_0 + (0.45 \pm 0.15);$$

c) contrast ratio (γ) lies within these limits: for mountain terrain $\gamma = 1.0 \pm 0.25$, for remaining regions $\gamma = 1.5 \pm 0.25$.

d) cloud formations or shadows from them, scratches, spots, streaks, or spots of light should not prevent completion of photogrammetric work with the necessary accuracy.

e) fixation and washing unexposed aerial film should ensure its long-lasting storage.

For manifestation of the quality of aerial photography the aerial photos are assembled on a plywood board of large dimensions (2 × 1 m) in such a way that analogous contours of aerial photos coincide. Then the percent of longitudinal and transverse overlapping of aerial photos is checked, the angles of relative and external tilt of aerial photos are determined by means of a field stereometer on the basis of measurements of transverse parallaxes and readings of the statoscope, and then measures for elimination of deficiencies of aerial photography are outlined.

Conditions of aerial photography. For production of aerial photos useful for cartography, stability of the course the aircraft is following and horizontal position of aerial photos at the time of photographing are necessary. In the meantime a number of factors of physical and technical order disturb the mentioned conditions. These include the wind, (its direction and intensity), the speed of the aircraft, and the displacement of its center of gravity after expenditure of fuel.

Wind velocity at the height of photographing can reach angle as far as the path of flight to the course of the aircraft, cause change in aircraft position and the course of its path. Therefore during prolonged flights one must repeat measurements of drift angle and groundspeed. With a change in the magnitude of drift of an aircraft the aerial camera is turned correspondingly.

Gustiness of the wind appears in irregularity of lift strength of wings of the aircraft. If one were to look from rectilinearly travelling aircraft through the perpendicularly located viewfinder to the earth, one can be certain that the ray of sighting would trace a zigzag line. The aerial camera is inclined together with the aircraft and only the brevity of exposure (1/50-1/50 sec) ensures necessary sharpness of image of the site.

The execution of high quality aerial photography requires the application of special aeronautical instruments [6].

§ 6. Aircraft

The speed of the aerial survey aircraft should be from 100-350 km/hr, and the greatest altitude of its flight should be 6-8 km (during aerial photography of mountain terrain).

The cabin of the aerial photographer should be adjusted for viewing the terrain in all directions.

For training purposes and surveys in scales of 1:2000 to 1:17,000 single-engine aircraft are used (AN-2); for surveys in scales of 1:17,000 to 1:60,000 twin-engine passenger aircraft (LI-2, IL-12 monoplanes) are appropriately equipped.

In Table 5 characteristics of aircraft used during aerial photography are given.

In aircraft LI-2 and IL-12 the aerial photographer is in the navigator's section in direct proximity to the pilot and flight mechanic. The radio operator and flight technician with the aerial camera are in the central cabin. In the rear part of the aircraft there is a dark cabin for reloading cassettes of the aerial camera with unexposed aerial film. For the purpose of creating the best surveying temperature conditions, during survey at high altitudes the walls of the cabins of some hatches are sealed with plane-parallel glass, thickness to 20-25 mm (taper up to ~20"). Thus in the aircraft IL-12 there are three glass hatches, of which the main one has dimensions of about 500 × 500 mm. Therefore photographing can be carried out simultaneously by two or three aerial cameras.

Table 5

Characteristic	Type of aircraft		
	AN-2	LI-2	IL-12
Power of motor, horsepower	820	2 × 820	2 × 1530
Surveying speed, km/hr	160	220	300
Rate of climb, in minutes			
to 1000 m	6	4	9
2000 m	13	10	17
3000 m	24	16	26
Ceiling, m	5200	5750	6000
Takeoff run, m	200	430	450
Landing run, m	120	430	780
Range, km	2000	2000	2300
Crew: first pilot	1	1	1
second pilot	–	1	1
flight mechanic	–	1	1
radio operator	–	1	1
aerial photographer	1	1	1
flight technician	1	1	1

In the equipment of an aerial photography expedition there are 1 to 3 aircraft. An airport is selected on the territory of the surveyed region not further than 20-40 km from the base of the expedition. It is preferable to have the airport in meadows with low grass; sandy or dusty airports are not permissible, since dust lifted by the propellers harmfully affects mechanisms of the motor and the aerial camera.

§ 7. Aerial Cameras

During aerial photography for cartography it is possible to use single-lens and multilens aerial cameras. From 1924-1930 single-lens aerial cameras with angle of field of view of 62° were used (for creation of maps in scales 1:50,000-1:100,000). On aerial photos there were depicted small sections of the terrain. Therefore, to increase the area photographed during one exposure, there were created aerial cameras with 2, 3, 4, 5, and 9 objectives. Of small application in the USSR were the four-objective aerial cameras by which it was possible to photograph the terrain forward, downwards, to the right, and to the left from the flight line, and also aerial cameras with 9 objectives with a total angle of field of view equal to 140°. The central aerial photo was obtained as vertical, and the lateral ones as oblique photos.

As investigations showed, the angles of tilt of the principal axes of chambers of the aerial camera with respect to the principal axis of central chamber, as a result of inaccuracy of assembly, could differ from assigned tilts (for instance, 45°) by 5-8'. To eliminate their influence on map accuracy was required additional work during laboratory processes of treating aerial photos. This explains why certain designers strove to construct multilens serial cameras, in which aerial photos were obtained with the help of prisms or mirrors on a single film. Another advantage of such a solution consisted in the fact that only one cassette was required for the entire assembly. However, multiobjective or multiple-chamber cameras were used for a short period (1928-1935), since the obtained oblique aerial photos, having considerable variety of scale of image over the field, complicated the processes of map creation. The angle of field of view of the assembly on the whole and, consequently, the reduction of flight-surveying hours, are very significant in aerial photography in northern, poorly accessible regions.

With the appearance of aerial cameras with wide-angle and super wide-angle objectives, multiobjective aerial cameras lost their value, expecially for large-scale cartography.

The great variety of existing aerial cameras leads to the necessity of classifying them.

It is possible to establish the following criteria of classification: purpose, angular field of view, magnitude of focal length, by type of film used, by frame size, by principle of operation, method of film levelling, and other criteria.

According to purpose aerial cameras are divided into topographic and nontopographic classifications.

Topographic aerial cameras have great angle of field of view and give high accuracy of image on an aerial photo.

In construction they are complicated since they have automatic-action exchangeable cassettes and must ensure constancy of elements of internal chamber orientation. On the photography there are fixed coordinate marks, the value of focal length, etc.

The aerial camera is fastened in the aircraft above an opening in the floor (hatch), covered by shutters. The dimensions of the hatch depend on how wide-angle is the objective, the thickness of the floor, and the allowance for oscillations of the aerial camera ±5°.

The aerial camera with the photographic set-up occupies an area in the cabin of approximately 500 × 700 mm and up to 500 mm in height. In the aircraft LI-2 the apparatus is located near the entrance door of the passenger cabin, but in small aircraft it is in the central part of the cabin of the aerial photographer.

The aerial camera should ensure the obtaining of clear aerial photos over the entire field through equal intervals of time and should operate reliably in conditions of vibration and large temperature oscillations.

Work with the aerial camera should be simple. Therefore automatic mechanisms are used for frame replacement, to shutter, winding of the aerial film, its levelling, etc, are used.

At present the camera most perfected is the topographic camera AFA-TE (АФА-ТЭ) (Fig. 7). The AFA-TE unit consists of the following parts: frame cassette, and command instrument with electric drives. The camera is located on a camera mount [16].

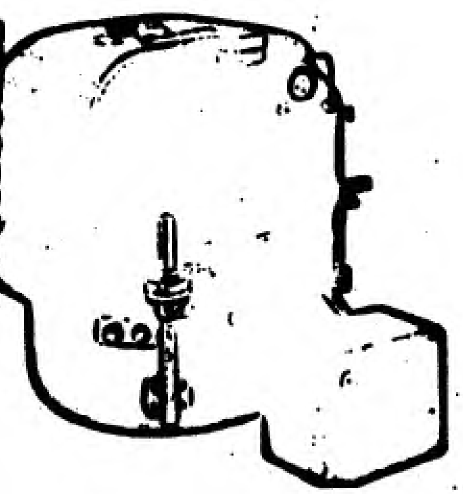


Fig. 7.

The frame (Fig. 8). The frame has the form of a cylinder of one or another height, depending upon the magnitude of focal length (36-500 mm). It endoses cone the lower part of which contains the objective, shelter, and auxiliary frame of size 180 × 180 mm.

The auxiliary frame has coordinate marks in the form of triangular flanges. Lines drawn through the summits of the marks intersect at the principal point of the aerial photo. In the corners of the frame auxiliary objectives are located recording the readings of the level, the time, and sequence number of the aerial photo. Circular graduation lines of the level are marked with the interval of 1° and present the possibility of estimating angles of tilt of aerial photos with an accuracy up to ±5'.

By the photographic image of the clock it is possible to count off the extent of exposure with accuracy up to one second.

The frame contains the shutter-drive assembly in the form of an MS-160 electric motor (26 v, 15 wt, 160 rpm).

Aerial survey objectives. The front and rear nodes serve as centers of projection of the objective. Between the optical components of the objective there is an air gap unilized for accommodating the blades of the shutter and sometimes a light filter.

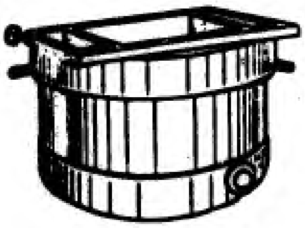


Fig. 8.

The aerial survey objective is characterized by the angular field of view, the magnitude of focal length f', the degree of accuracy of the image (distortion), resolving power, and distribution of illumination over the field of the image.

The characteristics of objectives applied in the AFA-TE (Fig. 9) are given in Table 6.

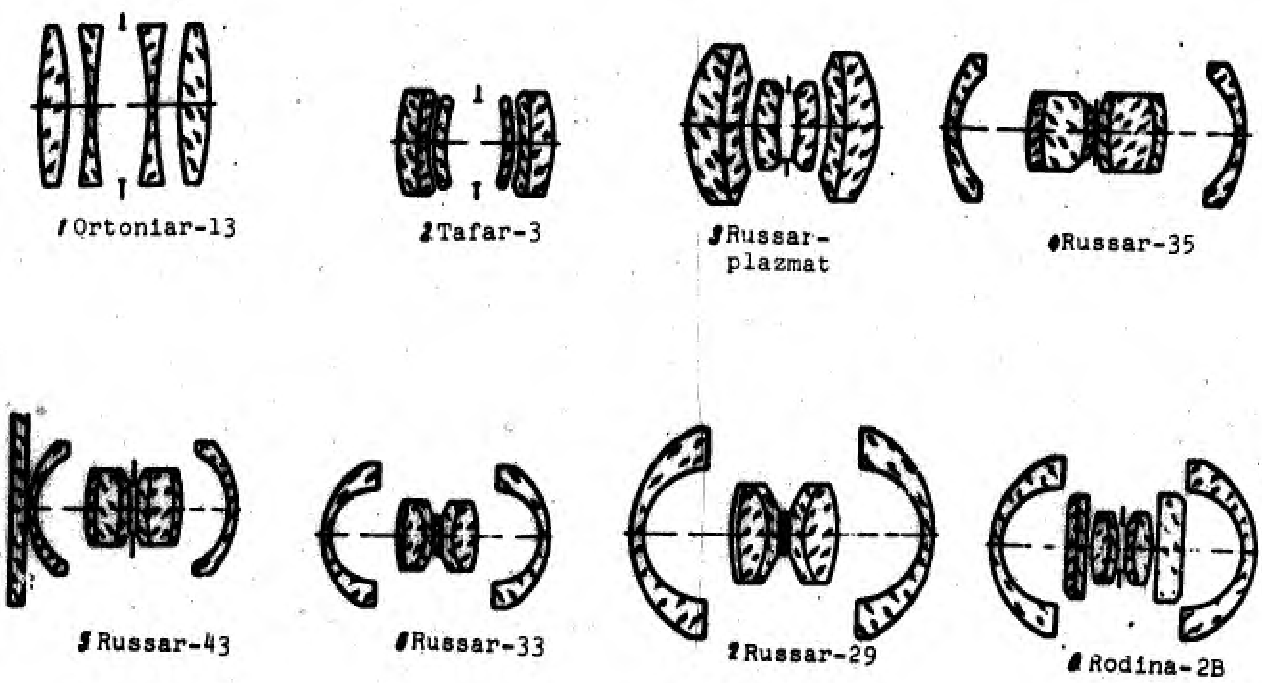


Fig. 9.

Table 6

Fig. No.	Type of objective	Focal length mm	Field of view in degrees	Relative aperture	Resolving power, lines/mm	
					in the center	on the edge
1	Ortoniar-13	500	29	1:7	30	18
2	Tafar-3	350	40	1:6	30	20
3	Russar-plazmat	200	65	1:6,3	45	24
4	Russar-35	200	65	1:9	35	23
5	Russar-43	140	85	1:6,8	35	15
6	Russar-33	100	104	1:6,8	35	19
7	Russar-29	70	122	1:6,8	25	12
8	Rodina-28	55	133	1:8,2	35	12
9	Russar-38	36	148	1:7,7		

Objectives with f' of 500 to 200 mm at present are used in aerial photography for purposes of interpretation of aerial photographs [3], [4], and aerial photography of cities. Objectives with f' of 140 to 100 mm can be used during surveys of mountain terrain. Objectives with the largest application in the USSR are those with f' = 70 mm, useful for surveys of terrain with various forms of relief (hilly, plains, and others).

Objectives with f' = 55 mm are used for surveys of plains; objectives with f' = 36 mm [16] are intended for geophysical aerial surveys and for mine and geological surveys; furthermore they are used during surveys from low heights; at the present time they are not used for cartography.

In Fig. 9 shows diagrams of certain of these objectives. The calculated distortions of the image given by them does not exceed 0.01 mm. In practice on edges of the field this distortion can sometimes be 0.03-0.04 mm.

Distribution of illumination over the image field of objectives 1-3 occurs according to formula

$$E_1 = E_0 \cos^4 \beta,$$

where E_0 and E_1 are the illuminations in the center and on the edges of the field, respectively β is half the angle of field of view. Irregularity of distribution of densities over the field of the negative can be corrected during the printing of positives from it.

Objectives 4-8 ensure light distribution over the field of the image according to the formula

$$E_1 = E_0 \cos^3 \beta. \tag{13}$$

An objective of type Russar-38 gives still better light distribution.

An intrinsic property of Russar-38 aerial photography objectives is the connection of angles of entrance and output of pencils according to the law of tangents; according to this $\frac{tg\ \beta}{tg\ \beta'} = k$, where $k = 1.5 - 2$*. Therefore the angle of incidence of a ray with the surface of the film on the edge of the image does not exceed 50-60° which facilitates the conditions of obtaining an exact image and improves distribution of illumination of the field.

Decrease of distortion can be ensured by application of precalculated clamp glass*. On clamp glass there sometimes is plotted a centimeter grid in the form of crosses for purposes of calculation of the influence of distortion of unexposed aerial film (for instance, as is done in one of the foreign aerial cameras – the Williamson).

Shutter. Of the numerous types of shutters available, the louver and intralens types are used at present in Soviet topographic aerial cameras. In Fig. 10 shows a diagram of a louver shutter. Its blades at the time of exposure take a position parallel to the principal ray and upright; after further revolution the blades, overlapping one another, close the objective.

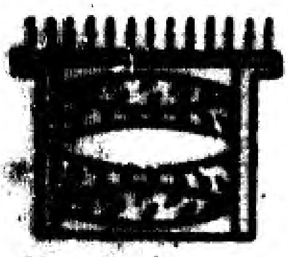


Fig. 10.

The AFA-TE with cones with f_k = 350 and 500 mm, louver shutters with optical efficiency of 62% and exposure to 1/300 sec are used.

In remaining cones intra lens shutters of system N. P. Vertiporokh (ZV-1 and [illegible]V-1) and intralens high speed shutters developed by S. P. Shokin and G. G. Gordon are used.

In Fig. 11 shows the diagram of the four-blade shutter ZV-1 of reversible movement.

Movement of parts of the shutter is carried out by oscillation of flat spring 1, one end of which is secured and the other enters a recess in ring gear 2. The spring has a moveable support in case 14, with the help of which the length of exposure is changed.

The shutter is cocked by cam 3, having clockwise rotation; along with this the guide of the ring turns it, bends spring 1 and turns four gears with blades 13 to a position at which the aperture of the objective still remains covered.

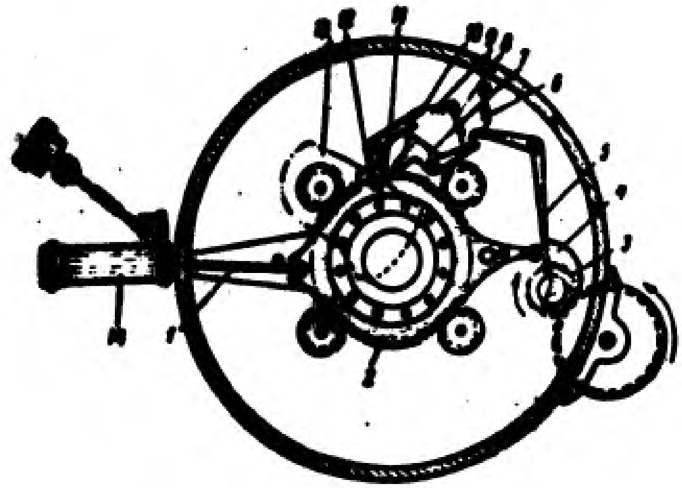


Fig. 11.

Finger 4 of cam 3 deflects stopper pawl 7 fro ring gear 2 by means of lever 5. Its flange goes behind the stop of lever 10 during this.

When cam 3 frees the guide of the ring, it rotates under the action of the cocked spring. The ring revolves the gear with blades up to the moment at which the aperture of the lock is opened. During backward motion of the ring and blades shutting of the shutter aperture occurs.

Further movements of the ring is limited by pawl 7, in which rests flange 11 of the ring. The turn of pawl 7 to the necessary position is carried out by spring 8 when finger 12 of the ring strikes on lever 10.

Springs 8, 9, and 6 return pawl 7 and levers 10 and 5 to the initial position.

Exposure time is changed from 1/45 to 1/120 sec by shifting the nut for reduction of length of the working part of spring. Efficiency of the shutter at light diameter 13 mm approaches 90%. An experiment showed reliability of construction even after 20,000 exposures, which is explained by the excellent damping of oscillatory movement of the spring.

Aerophotosurvey from heights of 500-1500 m and aircraft speeds of 200-300 km/hr requires exposure time of 1/100-1/500 sec, which the above-described shutters are not able to ensure. High-speed intralens shutters (Fig. 12) [ZBS] (ЗБС) solve this problem. In shutter ZBS the aperture of the objective is covered by three disks A, B, and C with lateral slots, and blade D.

Electric motor M, obtaining an impulse from the command instrument, continuously revolves annular gear 1 and cam 2, through a system of gears periodically striking against lever 3.

Disks are set on gears with the ratio of angular velocities 6:4:3. Through 5.5 turns of disk A, blade D, which is attained by the support of cam 2 on the arm of lever 3 of the blade, will start to be opened. After 0.5 turn of disk A it will open aperture 4 of the objective completely.

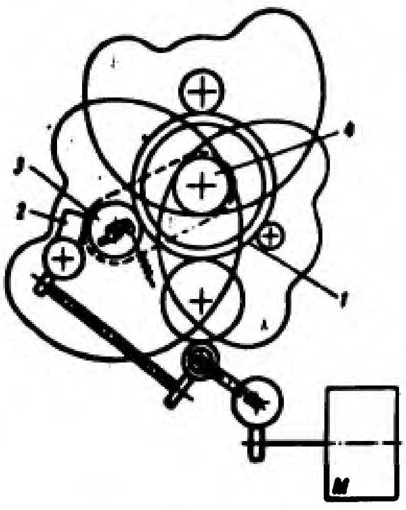


Fig. 12.

On the sixth turn of disk A the slots of all disks will coincide and exposure will occur. At the diameter of aperture of the objective of 30 mm, optical efficiency of the shutter is equal to 0.755. Exposure can be established from 1/30 to 1/1000 sec by changing the rpm of the electric motor. It is necessary to note that during one cycle of work of the shutter disk will make 60 turns.

The extent of exposure time is set by hand by changing the voltage fed to the rotor of the motor as a function of illuminance of the terrain. However automatic regulation of exposure time is possible, using special light-sensitive devices.

The cassette is a mechanism which winds, levels, and stops the aerial film. Depending upon the type of construction the full cycle is completed in 1.4-4 sec.

Winding of aerial film is done by an electric motor and is regulated in some constructions by lever attachments, and in others by measuring shafts. Since the length of the aerial film is about 60 m, the diameters of the supply and takeup spools with aerial film changes within a large range, which complicated the cassettes mechanisms with lever attachments. Cassette mechanisms with measuring shafts are simpler; the length of the circumference of the shaft is equal to the length of one aerial photo, taking into account the free space. Fig. 13 shows the diagram of such a cassette used in the topographic aerial camera (AFA-TE). Measuring shafts in it have electrical contacts for timely completion of one or another process of work of the cassette machanism.

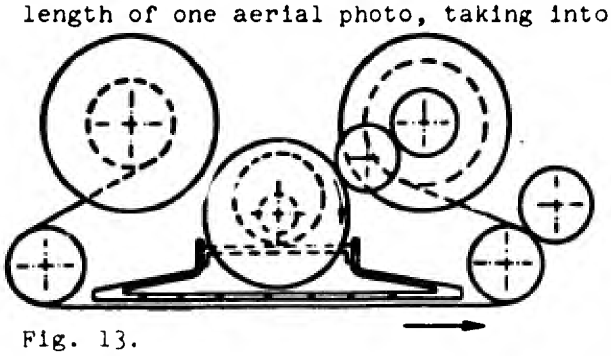


Fig. 13.

Flattening of aerial film into a plane can be executed by several methods:

a) clamping the aerial film by means of a plate to a plane-parallel glass if the angle of field of the aerial camera does not exceed 60°, or to a plane-concave lens with angles of field up to 120°. Such flattening is used in experimental types of aerial cameras of 1960 issue;

b) air pressure on film, pressed into the space between the objective and the film (pressure of 20-25 cm on a water column);

c) suction of film to the plane of the clamp "table" by means of a Venturi tube fastened on the outside of the aircraft.

The first method gives very good results, but only with complete removal of dust, which causes fast abrasion of the glass surface; with plane-concave lenses the difficulties arise in centering them, and there occurs a loss of light up to 12-19% (at f_k = 100 ... 70 mm).

The air pressure method, used in certain Zeiss aerial cameras, requires the presence of a very powerful pressure table, since the total pressure reaches up to 6.5 kilograms on it, and on walls of the chamber is still more, which causes distortion.

The suction method does not have such disadvantages. Flattening aerial film into a plane starts from certain tension and pressure of its spring (up to 5 kilograms) to the auxilliary frame. Then starts the suction of unexposed aerial film to the surface of the table, having a series of perforations (∠ 0.8 mm) and channels, located through 15 mm. The obtained accuracy of film flattening is equal to 0.01-0.02 mm. The table surface and auxiliary frame should be prepared with the same accuracy.

Rotation of spools of the cassette is produced by an electric motor, analogous to the motor of the frame.

Camera mount. The purpose of the camera mount is to give the aerial camera a horizontal position, to turn it by the lead angle and to decrease the influence of vibration of the aircraft on the sharpness of the image.

The camera mount consists of a ring and Cardan frame, provided with lifting screws, and a base on shock absorbers (mainly, springs). The latter are fastened to the floor of the aircraft. The aerial camera, provided with rollers, is located on the inside ring, enabling a change of angles of rotation. There are camera mounts which consist of a ring on three lifting screws and repetition ring for the aerial camera.

The camera mount should ensure levelling of the aerial camera within limits of ±7° in any direction and its rotation in a horizontal plane at angle $\kappa = \pm 30°$.

It is possible to fasten the AFA-TE either to the three-point type camera mount or to the gyrostabilizing installation H-55.

Continuous adjustment of the camera mount is executed by the flight technician. With the help of a level located above the cassette it is possible to establish the aerial camera horizontally. Application of a gyrostabilizing installation sharply improves results of a survey.

Command instrument. For establishing intervals between cycles of work of the cassette mechanisms and the body of the aerial camera a command instrument containing a mechanical or electrical interval meter is used.

The topographic command instrument KPT-3 (Fig. 14), produced by TsNIIGAiK (Central Scientific Research Institute of Geodesy, Herophotogeodesy and Cartography) is intended for pulse circuit aerial cameras; it contains an electric motor (MN-145) with 10-watt power and 111 rpm on output axis and working at a voltage of 25.5-28.5 v. The centrifugal regulator ensures constancy of speed of motor rotation and, consequently, the correct establishment of intervals between exposures.

The axle of the electric motor sets two cams in rotation. If the first cam is engaged in work then the ratchet wheel it sets into work gives a range of intervals from 5 to 100 sec with intervals of 1 sec. If we include the second cam, the range of intervals will be 1-20 sec with 0.2 sec intervals.

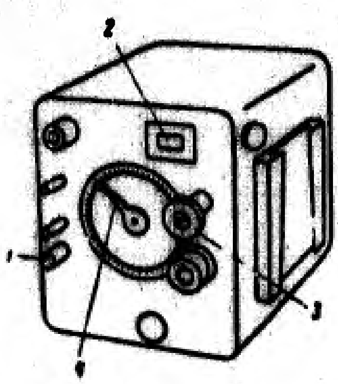


Fig. 14.

The length of the period of start-stop operation of the output shaft of the command instrument depends on the angle of movement of pointer 4, established by the limiter the control knob for the instrument dial. As a result of this set-up the ratchet wheel mechanism presses on the starting lever in good time and locks the microswitch, sending an impulse to the aerial camera.

Due to this there is set the distributive mechanism of the aerial camera, connected in turn with the cassette mechanism is set in motion shutter, exposure counter, and with contacts of the electric motor of the cassette and small electric lamps. From the control instrument impulses also go to lamps of the photorecording system of the statoscope and radio altimeter. The control instrument should execute exactly the work cycle of the aerial photography assembly. Investigations of existing types of control instruments, for instance, the KPT of the plant of aerogeodesic instruments, indicated accuracy of cycles on the order of 0.5 sec. With transition to large-scale cartography from low altitudes it is necessary to increase the accuracy of time intervals between cycles of intervals to 0.1 sec. Such accuracy is ensured by the above described control instrument KPT-3. The instrument has small dimensions and weight (2.7 kg) knob 1 engages a shutter for separate exposures; in window 2 there is observed the exposure counter; shaft 3 establishes time intervals, counted off by pointer 4.

Among foreign designs the Fairchild topographic aerial camera T-11 with the Planigon objective f' = 152 mm is well-known. The cassette contains a spool with film for 450 aerial photos. The intralens shutter gives a small exposure time.

The automatic aerial camera Wild RS7-a, which photographs on plates is well-known. The absence of distortions on glass negatives presents the possibility of applying them for spatial phototriangulating and other especially exact work. The infragon objective is set in the aerial camera f' = 100 mm and relative aperture 1:5.6, and size of plates 150 × 150 mm. They are in two cassette boxes, 40 plates each. The viewfinder is the central part of the aerial camera.

For execution of very exact photogrammetric work film aerial cameras are used which have glasses a grid through 10 mm. Here there is the possibility of calculating the influence of distortion of the objective, and the errors due to lack of flattening of film and its accidental distortion.

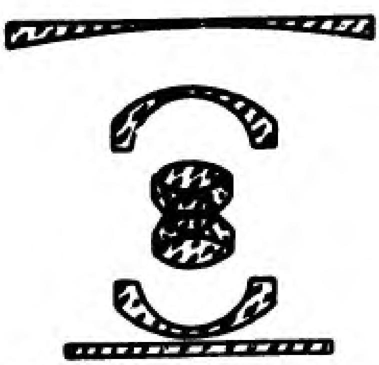


Fig. 15.

In Fig. 15 gives the diagram of the objective Russar-54c in the Soviet aerial camera AFA-TES-7 with f_k = 70 mm.

§ 8. Aeronautical Instruments

Aeronautical instruments used in aerial photography ensure rectilinearity of runs, assigned overlaps assigned heights of photographing and horizontal position of aerial photos. Certain instruments (radio altimeters, statoscope, gyroscope) permit determining the elements of external orientation of aerial photos in the process of aerial photography.

1. Shadow Course Indicator

The shadow course indicator serves for laying out rectilinear runs.

This instrument is based on use of the solar shadow incident from a vertical rod on an evenly revolving travel limb. The main parts of the instrument (Fig. 16) are the base, electromagnetic drive travel limb, azimuthal limb, mirror, and rod.

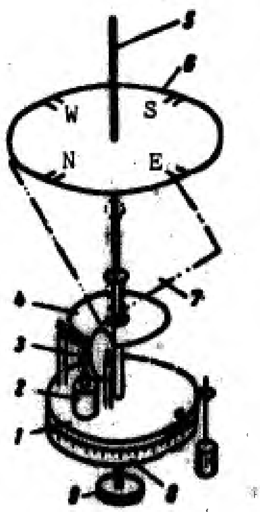


Fig. 16.

On basis 1 is located solenoid 2, ratcher 3, revolving through screw worm gear 4. The latter rotates vertical axis 5, which arries frictionally set limb 6 of frosted glass with designation of compass points (N, S, W, E) and 12 auxiliary courses. Shadow from pin (gnomon) of axis 5 drops to the limb; it is visible through mirror 7 from below. From current pulse proceeding from the control instrument the solenoid sets in motion the travel limb and azimuthal limb 8. Head 9 serves for establishment of initial position of limbs.

The travel limb revolves with speed of 15° per hour, which is equal to the speed of the apparent motion of the sun. If the aircraft flies in a straight direction, the shadow does not depart from the established course. With a shift of the shadow from the mark of the travel limb, corresponding changes are introduced in the course of the aircraft. The shadow course indicator is situated behind the pilot in the center of a transparent sphere of the ceiling and is in a horizontal position. Before the pilot there is an easily oriented mirror, in which the shadow from the gnomon, superimposed on the travel limb, is observed.

Of wider application is the potentiometric distant-reading compass, located in the tail of the aircraft, where there are fewer magnetic masses. An instrument of this type, for instance the PDK-45, looks like a vessel on Cardan suspension. In the vessel is a compass rose with three contact brushes, joined through the potentiometer by wires with an indicator located in the cockpit. For correct operation of the compass it is first of all necessary to eliminate its deviation.

2. Deck Optical Viewfinder

The most convenient instrument for determining the drift angle and speed is the deck optical viewfinder, fastened in the floor of the aircraft (Fig. 17).

In the instrument complex are the telescope, universal pivot, stop watch, and electric cable.

Rays from points of terrain proceed through prisms 1 and 2 to objective 3, focusing the image of the terrain on the glass surface of level 5. The level tube bubble with diameter up to 5 mm jointly with the image of the terrain is projected by system 6-7 onto grid 8 with circular and rectilinear scales in direct image, observed through eyepiece 9. The field of view of the system is equal to 30°, magnification is 1.2^{x}; length of telescope is 1070 mm.

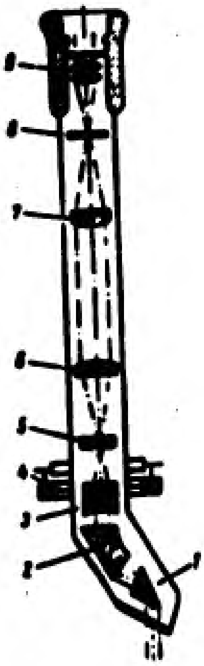


Fig. 17.

For observation of reference points located on the basic flight line, it is possible by turning prism 1 to deflect the ray of sighting from -15° to +75°.

The telescope is fixed to Cardan 4, fastened to the upper part of the universal pivot, built into the floor of the aircraft. The upper part of the pivot can revolve by angles from 0 to 360° and secured by a foot pedal which permits establishing the line of the focal grid of the viewfinder in the direction of the line of flight and determining drift angle of the aircraft. Angles are counted off on a scale plotted on the heel pivot with an accuracy up to 1°, and reported to the flight technician for correction of the aerial camera position.

For establishment of the assigned longitudinal overlap the ray of sighting is inclined forward at angle according to the formula

$$\operatorname{tg} A = \frac{B}{H}, \qquad (14)$$

where B_x is the base on the terrain between projections of centers of aerial photos, and H is the height of photographing.

According to the time of passage in telescopic sight of the shown base the interval between exposures is established and, consequently, the percent of longitudinal overlap of the aerial photos. Time reading is taken on the stop watch, fastened to the instrument.

From the movement of the outline of the terrain along the focal grid of the scope the parallelism of edges of the cassette of the aerial camera to the axis of the run ($\kappa = 0$) is simultaneously established.

3. Sighting Pendant

The sighting pendant, used for determining the value of transverse overlap of aerial photos, is fastened on a bracket in the center of the transparent spherical windows of the navigator's section. In Fig. 18 is depicted the usual type of sighting pendant to B-3. The instrument consists of a frame and inclined ranging rod; it has small dimensions on the order 100 × 40 mm. Establishing the plane of sighting for the angle corresponding to the position of the center of the neighboring run, the aerial photographer intersects the necessary reference points and marks them on the map. These reference points are used when laying out the neighboring run on the aircraft's reverse course.

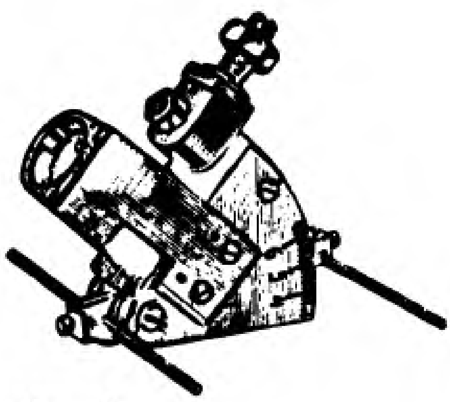


Fig. 18.

4. Turn-setting Device

To line upon the next neighboring run route the aircraft sometimes must make complicated turns, especially during large scale aerial photography when runs are not a great distance from each other. Since it is expedient to establish the path of the aircraft in this case from arcs of the circumferences, we use methods of approach according to a turn-setting device. Instrument KZL-3 represents a frame-limb (180°) with two rods revolving near the center, with length up to 300 mm, fastened at a 90° angle. The rods are established on one or another precalculated bank angle and the pilot sights through a rod to the visible skyline and the aircraft is held in a slanted position until the rod covers the line of the horizon. As a result of these actions the aircraft completes a path along the arc of the circumference of a determined radius. In Fig. 19 shows lines of approach of the aircraft from one run to another. These runs,[1] consisting of straight lines and arcs of circumferences, are selected depending upon the distance between them, speed of the aircraft, and speed and direction of the wind. With simultaneous use of a st watch the turn-setting makes it possible for the pilot to correctly construct the elements of approach to the following run.

Fig. 19.

5. Altimeter

For production of aerial photos of a certain scale the aircraft rises to the corresponding altitude, determined by the altimeter. For fixing of the flight profile an altitude recorder is used (Fig. 20). It consists of drum 1, steadily revolving by means of clockwork; block of aneroid boxes 2; and mechanism of transmission to pointer 3 with a pen filled with non-freezing and non-drying ink. The pen continuously traces the curve of altitude fluctuation of the aircraft (barogram). The barogram depicts the degree of adherence to the assigned flight altitude during aerial photography with a precision of ±50 m.

Fig. 20.

Radio altimeter. For determining altitudes of stations of photographing with respect to the earth's surface an impulse radar installation – radio altimeter – is used. It consists of a transreceiver working on short wave (68 cm), an indicator, the main part of which is an electron-beam tube with annular scan, a small electric motor, two antennas, and a cable. Radio altimeter readings are fixed

[1] A. P. Lyubimov. Approaches to the subsequent run for large-scale aerial photography. Moscow., Geodezizdat, 1952 (Transactions of TsNIIGAiK (Central Scientific Research Institute of Geodesy, Aerial Photography, and Cartography), Issue 89).

on a narrow film by a photorecorder synchronously with statoscope readings. Radio waves are emitted by an altimeter as separate impulses. Being reflected from earth's surface (vegitation is not an obstacle), the radio waves enter the receiver. At the time of transmission of radio wave impulses a secondary voltage appears on the electrode located in center of the screen of the electron-beam tube, as a result of which the electron beam is deflected from the luminescent circular scan circumscribed by it and gives a "blip". With the reception of reflected radio waves on the central electrode of the tube there also appears an additional voltage provoking a subsequent "blip". Its distance initial blip corresponds to distance from the aircraft to the nearest point of terrain. The screen of the indicator has a scale of distances with the value of the interval of 20-10 m. The type reading of the radio altimeter is depicted in Fig. 21. The blip near the zero reading, corresponding to the distances with a multiple of 1500 m, is not taken[1] into account, but the one adjacent to it, which moves in the clockwise direction, corresponds to distance D from the aircraft to the closest point of the earth's surface. Distance D is determined by photo recording tape with the help of a magnifier or projection lantern.

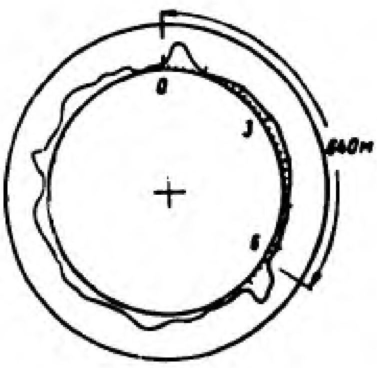


Fig. 21.

In order to obtain H, the height of stations of photographing, i.e., the distance from the aircraft to the earth's surface along the direction of the optical axis of the aerial camera, it is necessary to determine the correction ΔD and to place it in the formula

$$H = D + \Delta D, \qquad (15)$$

where D is the radius of the sphere touching the earth's surface at the point nearest the aircraft, counted off by radio altimeter readings.

Correction ΔD is determined as the result of corresponding measurements of aerial photos on the stereometer.

Radio altimeter readings are very valuable in small-scale and medium-scale cartography, during construction of photopolygonometric nets, and in carrying out other forms of work.

At present the topographic radio altimeter RVTD of decimetric range of waves is used; it has a scale with units from 0 to 500 m. The smallest division

[1] In altimeter RV-10 and in RVTD the initial blip is located at zero of the scale.

corresponds to 5 m, which permits calculating altitude with precision of 0.5-1.0 m.

The emissive power of the RVTD is 20 times more than that of the RV-10 (it is equal to 450 watts) and the scale is increased three times.

The blue glow of an electron-beam tube is used, improving the conditions of photographing of the scale. The electrical part of the circular scan is done so that the trailing edge of the initial blip is eliminated and the steepness of subsequent blips is increased.

The photorecorder (ARFA-7) consists of a chamber, illuminator, and two cassettes fastened on a common chassis (Fig. 22).

On the drawing: 1 – RVTD; 2 – illuminator of the electron-beam tube scale; 3 – chamber, the objective of which has f' = 50 mm, and relative aperture 1:2; 4 – block of two cassettes; 5 – chassis; 6 – electric cord for power supply of the mechanisms.

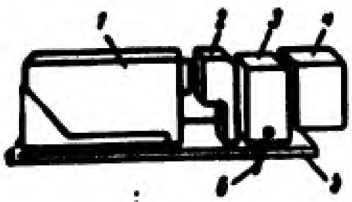


Fig. 22.

Film, 61.5 mm wide and 20 m long, ensures exposure of 300 frames synchronously with the exposures of the basic aerial camera AFA-TE. On the film in the corner of every frame are recorded the readings of a clock with second hand, strictly coordinated with the clock readings of the AFA-TE.

6. Statoscope

For certain photogrammetric processes it is necessary to determine the altitude differences of air stations with great accuracy. Statoscopes working synchronously with the aerial camera solve this problem very successfully.

The statoscope, based on the principle of measuring the difference of air pressures, was first developed by the great Russian scientist D. I. Mendeleyev. The instrument (Fig. 23) consists of a v-shaped manometer tube (4), containing isoamyl alcohol; leg (3); triple-vent tap (2); and bottle (1) placed in a thermostat (vessel with double walls, between which there is a vacuum). One end of the tube is always in contact with the atmosphere, and the other – with the air mass of the bottle.

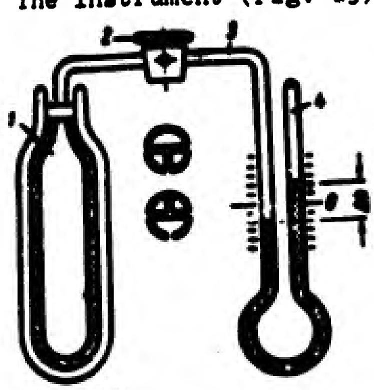


Fig. 23.

By means of the triple-vent tap it is always possible to connect the second end of the tube with the outside air and thereby to bring the liquid in both legs of the manometric tube to the manometric tube to the same level. If the tap is then closed, with a change in aircraft altitude the difference Δh will appear between the levels of columns of the liquid. This difference is registered by the recorder during its exposure. Since the altitudes of aerial photography stations usually vary within limits of 10-20 m, the length of the manometer tube will be selected appropriately, depending on pressure drops and specific gravity of the liquid. During takeoff and landing of the aircraft the statoscope must be turned off so that the liquid does not flow out.

The most modern is the statoscope-automaton of the K. P. Bychkovskiy system [16] working during all regimes of flight without attention by the operator. The instrument consists of two coupled statoscopes, an automatic switching tap, a servosystem, and a photorecording camera (Fig. 24).

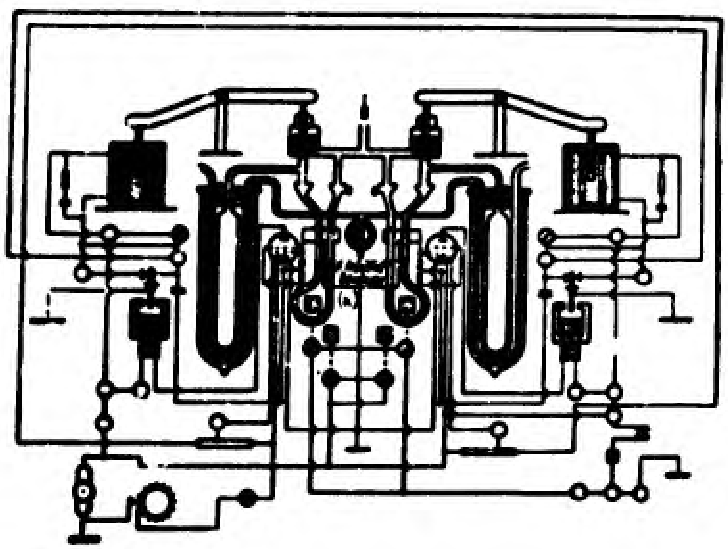


Fig. 24.
KEY: (a) to the Verturi tube.

In the initial moment of work of the instrument the liquid levels in the statoscope manometer tubes are identical.

Above the liquid levels are electrodes connected in the circuit of a highly sensitive relay connected in turn to an intermediate electromagnetic relay which governs the work of automatic taps. After power is supplied through the electron relay one of the parts of the statoscope, for instance the left, is switched on. During this time the solenoid closes the automatic tap.

After operation of the left relay the other electron tube, for instance the right one can no longer switch on the right relay, since the circuit of the right potentiometer is broken and the potential difference between anode and cathode of the right tube disappears. Therefore during a change of altitude of photographing the liquid in the right statoscope remains unmoving on the former level, when the liquid is the left statoscope reaches the contacts and turns off the anode current of the left line, the left relay returns to the initial position and the solenoid, deprived of power, opens the tap. With this the level of the liquid in the left statoscope assumes its initial position. Simultaneously the tube, relay, and solenoid of the right statoscope, are switched on, starting to react to the continuing pressure drop.

Fig. 25 shows a diagram of a photorecorder and statogram. Light from lamp 1 through condenser 2 and mirror 3 continuously illuminates meniscus 4 of the liquid from below. The derived luminescent point is projected through objective 7 and mirror 6 onto moving photographic film 5 in the form of a curve dependent on the pressure drop. The other meniscus is projected onto the same film in the form of a point at the time of photographing.

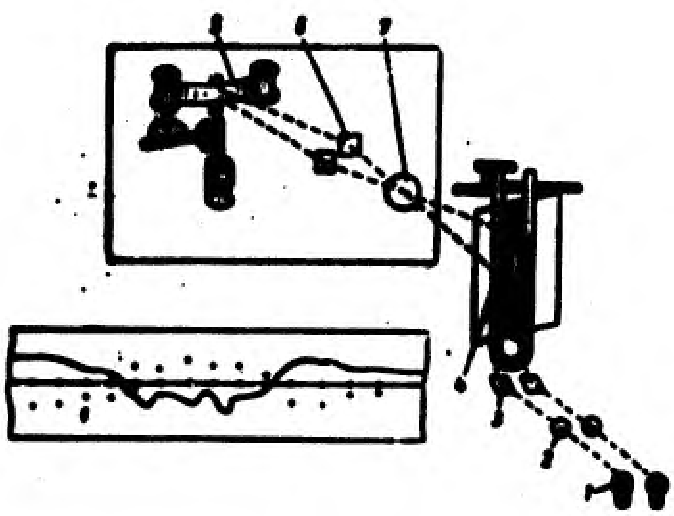


Fig. 25.

Simultaneously the menisci of the second statoscope are projected continuously and as pulses. Since at the moment of work of the first statoscope the second statoscope does not show elevations, the continuous illumination of the meniscus gives a horizontal line of the film, while intermittent light during exposure gives points on the line. The difference of the distances between the external point and curve on neighboring sections, expressed in millimeters and multiplied by the conversion factor q_H, gives the value of elevation of one station in comparison with the other in meters, i.e.,

$$\Delta H = l q_H, \qquad (16)$$

where l is the distance on the film.

Conversion factor q_H for the statoscope is calculated by the formula [16]

$$q_H = Q[k_0(1 - \beta t_H^\circ) + 0{,}5\, w P_H], \qquad (17)$$

where

$$Q = \frac{7991}{v_\phi P_H}(1 + \alpha t^\circ). \qquad (18)$$

Here the number 7991 corresponds to the height of the air column with density corresponding to normal pressure and a temperature of 0° at sea level.

Table 7 gives the values of the components of formula (17).

Table 7

Designation of Components of the Formula	Designation	Standard Digital Qualities
Magnification of the recording system of the statoscope	v_ϕ	1
Barometric pressure of the flight altitude in mm Hg	P_H	453.7
External air temperature	t°	-15°
Air temperature in the cabin	t°_H	+3°, 5
Temperature coefficient of air expansion	α	0.00367
Temperature coefficient of isoamyl alcohol expansion	β	0,00117
Density ratio of isoamyl alcohol and mercury at 0°	k_0	0,06125
Constant of the statoscope	w	0,000048

At heights of photographing on the order of 2000 m the error of determination of elevations by the statoscope attains 0.5-1.0 m. This magnitude depends on a number of factors, including the location of isobars. If isobars (lines of identical pressure) in a given region have a slope, then during the period of the aerial photography route systematic errors in determined elevations of altitudes of photographing will appear. Random errors in determined elevations depend on local heterogenities of atmosphere; they appear at low altitudes of flight (400-1000 m) which in a number of cases, for instance during large-scale cartography, can prevent use of statoscope readings.

7. About Instruments for Horizontalizing [Rectification] of Aerial Photos

Knowing values of angles of tilt of aerial photos, it is easier to execute their subsequent treatment; therefore instruments for determination of these angles are very necessary. Still a larger value has the direct obtaining of aerial photo horizontals.

It is possible to record angles of tilt of the aerial camera by various methods.

Let us note the method of photographing the horizon by special camera, joined with a surveying aerial camera. Shutters of both cameras work synchronously; therefore, measuring the deflection of the line of the horizon from the line of marks, it is possible by the formulas for tying elements of survey and the parameters of the camera to obtain angles of tilt of the aerial camera. Accuracy of their determination can be up to 10-15'.

This method has disadvantages, caused by sometimes illegible image of the line of the horizon and the necessity of considering elevation of points of terrain in the zone of the visible horizon.

More convenient is the method of determining angles of tilt of the aerial camera by means of an installation joined with it and consisting of two gyroscopes with a photorecorder the so-called gyrovertical). It also gives accuracy of determination of angles up to 10-15'.

The most interesting are methods of automatic leveling of the aerial camera. In the 1930's camera mounts with Cardan suspension of the aerial camera were developed. In this case the axis of the freely suspended aerial camera was established vertically under the influence of its own weight. It was necessary to apply movable weight compensators, preserving the position of the center of gravity of the aerial camera during rewinding of film in the process of aerial photography. The least amount of deviation of the aerial camera from its horizontal position attains was ±30'. The disadvantage of this method is the great duration of damping of oscillations of the camera, requiring application of shock absorbers.

The best solution of the problem is attained by application of automatic leveling of the entire aerial camera by means of gyroscopic attachments. This is carried out in the gyroframe (power) gyroscopic system N-55 for the AFA-TE [9], [17].

At its basis lies the use of the principle of the symmetrical gyroscope with three degrees of freedom. For this purpose electric motors of gyroscopes are suspended inside two mutually perpendicular frames in such a way that the center of gravity of the system is on the intersection of three coordinate axes (astatic system).

During fast rotation of the rotor of the electric motor (first degree of freedom along κ) the assigned direction of its axis in space (second and third degrees of freedom) is retained independently of the position of support of Cardan joints, in the given case of the floor of the aircraft. However, a certain amount of friction in axes of suspension of Cardan joints causes precessional movement of the external ring of the Cardan joint.

Therefore in the instrument is applied a correctional device either in the form of short pendulums influencing rheostats of the two electric motors which stabilize the platform of the gyroframe, or in the form of a four-contact round electrolytic level (Fig. 26).

In the horizontal position of the gyroframe 1 (Fig. 27) the electrical resistances between the contacts and the body are identical. At slopes of gyroframe with the level and AFA 5 the resistances change and actuation of the correctional motors occurs through an attachment - correction block 2. This block contains a relay only to the sign of the signal given by the level. Due to this sensitivity of correctional unit of 45" to 2'.5 is attained. Inverter 3 and control panel 4 with the assembly are connected with installation by electric aries.

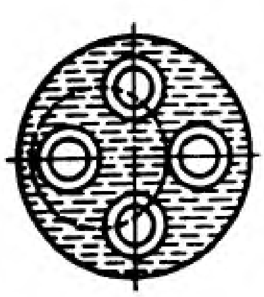


Fig. 26.

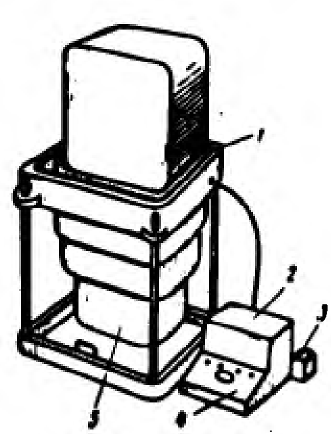


Fig. 27.

The N-55 installation is designed for use with AFA-TE whose objectives have focal lengths of 55, 70, 100, 140, and 200 mm. In the installation there is an autobalancer, consisting of a movable load, preserving the position of the center of gravity of the camera during film rewinding. The magnitude of the load is selected depending upon the weight of the charged spool.

The accuracy of gyrostabilization reaches up to 10-15', which significantly improves the conditions of laboratory treatment of aerial photos.

§ 9. Factors Influencing the Quality of Aerial Photos

1. Aircraft Speed

At high speeds of flight of aircraft and application of slow of wide-angle survey objectives, requiring lengthy exposure, noticeable distortions appear. Let us assume that (Fig. 28) during the time τ the aircraft shifts by the magnitude $v\tau$ (v is the aircraft speed in m/sec), in consequence of which some point of terrain is depicted on the aerial pho) by the segment $mm_1 = \delta_1$.

Fig. 28.

Since

$$\frac{\delta_1}{v\tau} = \frac{f_k}{H},$$

then

$$\delta_1 = \frac{f_k}{H} v\tau. \tag{19}$$

At $f_k = 100$ mm, $H = 3000$ m, $\tau = 1/100$ sec, and $\delta_1 = 0.02$ mm (least measured magnitude on the aerial photo) the allowed speed of the aircraft will be 60 m/sec. or 216 km/hr.

Experiments showed that a certain blurring of the image occurring with an increased aircraft speed does not prevent formulation of a chart from aerial photos. At considerably high speeds of flight blurryness of the image noticeably affects the exactness of the composed chart; this can be eliminated by reduction of exposure time.

2. Oscillations of the Aerial Camera

Linear shifts of the camera mount do not show up on blurryness of the image, but its oscillations on angle i (during exposure) cause blurryness, the magnitude of which is determined by the dependence (Fig. 29)

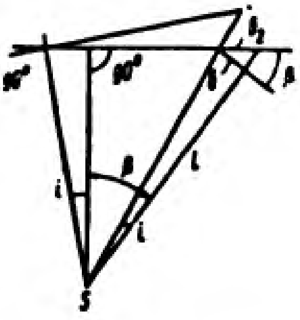


Fig. 29.

$$i = \frac{\delta}{l}.$$

where

$$\delta = \delta_2 \cos\beta;$$

therefore the permissible value of tilt of the chamber is equal to

$$i = \frac{\delta_2 \cos^2\beta}{f_k}. \quad (20)$$

For instance, at $\delta_2 = 0.04$ mm, $\beta = 35°$, and $f_k = 100$ mm, the value of $i = 1'$.

3. Rotation of the Aerial Camera in Its Own Plane

Rotation of the aerial camera during exposure taken place due to vibration of the camera mount in the horizontal plane or due to shocks. Considering the error from rotation separately from the movement of the aircraft, we find the value of bluryness (Fig. 30).

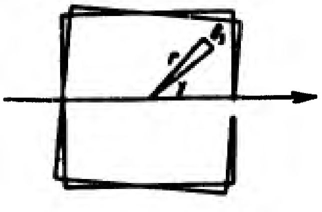


Fig. 30.

$$\delta_3 = r\gamma = \gamma\sqrt{x^2+y^2}, \quad (21)$$

from which the permissible magnitude of rotation will be obtained as

$$\gamma = \frac{\delta_3}{\sqrt{x^2+y^2}} \quad (22)$$

For instance, at $\delta_3 = 0.04$ mm and $r = 100$ mm the value of $\gamma = 1'20''$.

4. Flattening Unexposed Aerial Film

Independent of the accepted method of flattening aerial film in the foral plane (by counterpressure or suction), the film is strained at first and is tightened on the edges of the flattening plane; with this the tension should not cause any appreciable distortion of the aerial film.

Another source of errors in aerial photos is waviness of aerial film, caused by its not adjoining the flattening plane of the cassette. This leads not only to certain blurryness of the image, but also to distortion. Let us suppose that as a result of poor flattening circle δ of scattering was obtained (Fig. 31), depending on the diameter d of the effective aperture of the objective.

Then

$$\frac{d}{f_k} = \frac{\delta_4}{\Delta}. \qquad (23)$$

from which

$$\Delta = \frac{\delta_4 f_k}{d}.$$

At $\frac{d}{f_k}$ = 1:5, and δ_4 = 0.02 mm the value of Δ will be equal to 0.1 mm: it will not affect sharpness of the image, but due to unevenness of the film surface distortion of the image occurs, calculated by the formula

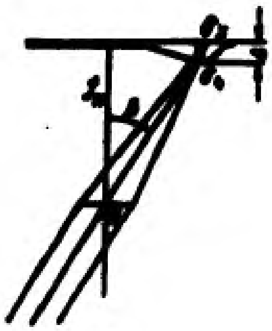


Fig. 31.

$$\delta_5 = \Delta \operatorname{tg} \beta \qquad (24)$$

The value of δ_5 will be appreciable under conditions when the objective is a very wide angle. This error is revealed on the aerial photography base by means of stereoscopic measurement on aerial photos in zones of triple overlap of dislocation of analogous points relative to a straight line located perpendicularly to the line of flight. Another method, interpolation of transverse parallaxes as a function of y, namely $\frac{q}{y} = \phi$, is used (see below).

For general investigation of the influence of cassette mechanism operation on film flattening it is possible to apply the method of Docent Ye. P. Arzhanov. The method consists in applying a point-source-type lamp in the aerial camera chamber. Beams from the lamp, located near the auxilliary frame, cast shadows on the film, depending on the waviness of the film and unevenness of the table. Shadows appear even from dust particles on the latter.

5. Distortion

The objective should produce an image corresponding geometrically to the right central projection. With its distortion the entrance and exit beams are tilted to the principal optical axis by angles A ≠ A; when they are known it is possible to calculate the true value of r and to compare it with measured r' (Fig. 32).

By the formula

$$dr = r' - f_k \operatorname{tg} A'$$

it is possible to calculate photogrammetric distortion error in various parts of the aerial photo. To exclude the error due to distortion in photogrammetric equipment is complicated; therefore it is perferable to use objectives whose distortion is less than 0.02 mm.

Since image accuracy has decisive value in photogrammetric measurements, after manufacture of the objective its distortion along two mutually perpendicular directions is investigated. By means of change of value of focal length it is possible to improve the graph of distortion.

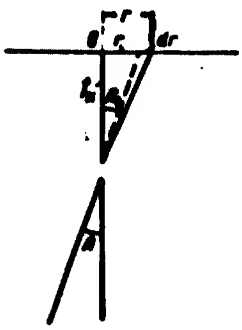


Fig. 32.

Fig. 33 gives calculated distortions of certain of the enumerated objectives. They are very small.

During the investigation of such objectives, sometimes deviation of curves of distortions of the theoretical are revealed. Investigation of this question showed that disturbances of theoretical distortion depend on accuracy of lens centering, on the intervals between them, and also on the influence of mountings on the lenses (due to temperature changes in the survey process or deviations from the most favorable focal length).

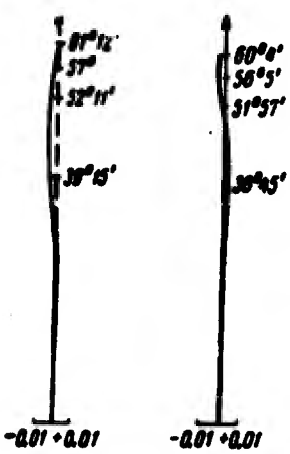


Fig. 33.

Determination of objective distortion can be done photographically, using light filters. More exact results are obtained than during visual observations

If the magnitude of objective distortion is larger than 0.03 mm it must be then eliminated in the process of measurements. One method consists of reverse projection of aerial photos through the objective, similar to surveying. In the case of orthogonal observation of aerial photos there are used (for instance, in wild instruments) compensational glass plates (Fig. 34).

Compensational plates of thickness up to 5 mm have surface curves only on the one side. The negatives are applied to the flat side.

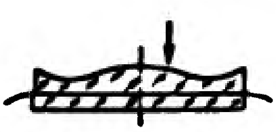


Fig. 34.

6. Distortion of Aerial Photos Caused by Curvature of the Earth and Refraction of Rays into the Atmosphere

In Fig. 35 shows the movement of a tilted ray during aerial photography of the earth's surface from a high altitude.

The direction of ray SM_1 deviates additionally due to refraction in the atmosphere. In conditions of orthogonal aerial photography this deflection is measured in seconds and consequently it is possible to ignore it.

If the height of photographing is H, the point of terrain M, the point of the aerial photo m, and the points of geometric conformities M_1 and m_0, then in the system of axes of one another photogrammetric instrument we will obtain

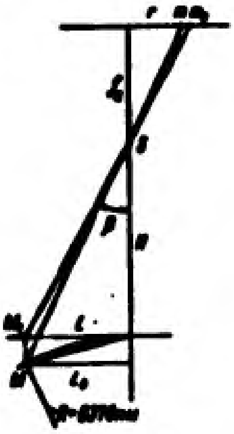


Fig. 35.

$$\frac{MM_1}{L_0}=\frac{L_0}{2R}.$$

From which

$$MM_1=\frac{L_0^2}{2R}.$$

Further from Fig. 35 it follows that

$$\frac{r}{f_k}=\frac{L_0}{H+M_1M}.$$

and

$$\frac{r+mm_0}{f_k}=\frac{L_0}{H}.$$

From which

$$mm_0=\frac{rL_0^2}{2RH}.$$

But since

$$L_0=L=H\,tg\,\beta=H\frac{r}{f_k},$$

the extent of the influence of the Earth's curvature is

$$mm_0=\frac{r^3H}{2Rf_k^3}. \qquad (25)$$

In connection with the fact that at present aerial photography can be produced at any height above the Earth's surface, the question arises concerning altitude limits for execution of topographic survey.

Example. At r = 100 mm, H = 7 km, R = 6371 km and f_k = 70 mm, the value mm_0 = 11 mm.

The value MM_1 in the same conditions attains ≈5.8 m.

Fig. 36 shows the graph of distortions mm_0 as a function of different altitudes of aerial photography. Great values of distortions of aerial photos due to the Earth's curvature make it necessary to face the question of an expedient method of using aerial photos for triangulating and treatment of aerial photos for creation of maps under conditions of speically high altitudes of survey.

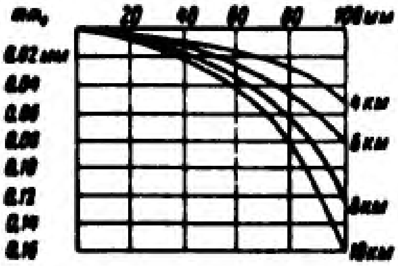


Fig. 36.

In all given photogrammetric instruments and methods it is accepted that the level surface of the Earth, within limits of the processed aerial photo, or section of small extent and heights to 3-4 km, for practical purposes is flat.

7. Resolving Power of the Objective

The resolving power of the objective should correspond to the resolving power of the emulsion (0.01 mm). However, it is difficult to obtain identical sharpness of the image in the middle and on the edges of the aerial photo, since one of the aberrations (curvature of field) during calculation of optics sometimes will be disregarded for the sake of image accuracy.

Objectives with an angle of field of view 60-70° sometimes have very large resolving power reaching, during visual study of the center of the focal field in the microscope, up to 275 lines per mm, which exceeds data of aerial photography photomaterials. The latter circumstance makes it necessary to examine the sharpness of the image of the hatchured target by photographing on slide plates, distinguished by their large resolving power. The number of strokes per mm of image, clearly visible on the photograph, corresponds to the magnitude of the resolving power of the lens, determined by the formula

$$R = \frac{1470}{k}, \qquad (26)$$

where k is the reciprocal of the relative aperture of the objective, by which it is possible to present the expected theoretical quantity of lines per mm.

According to tests of a series of wide-angle objectives conducted in laboratory conditions, it has been established that in the center of an aerial photo 1 mm the magnitude of the resolving power does not exceed 40 lines mm and on its, 20 lines mm. During aerial photography, in view of engine vibration and other factors affecting the aerial camera, the number of lines resolved per mm sometimes decreases even more and approaches 10-5 on the edges of the aerial photo.

The degree of accuracy of combination of the plane of the best image with plane of the light-sensitive layer can also affect the sharpness of the image. Permissible deflection Δf depends on the diameter d of the objective aperture and the permissible diameter δ of the circle of scattering. For instance, at d = 20 mm, f' = 100 mm and δ = 0.02 mm, from the relationship

$$\frac{d}{f'}=\frac{\delta}{\Delta f}$$

we will obtain

$$\Delta f=\frac{f'\delta}{d}=0{,}1\ \text{мм}.$$

Consequently the focal plane should coincide with the plane of the light-sensitive layer with a precision of $\Delta f = \pm 0.1$ mm. On this basis it is sometimes possible to adjust the objective in the aerial camera with the visual appraisal of sharpness on frosted glass. Investigations show that the value of f_k' for objectives of one and the same issue differ within limits of $\pm 0.02\ f_k'$.

8. Dependence of Overlaps and Magnitude of Exposure Time on Height and Speed of the Flight

Example 1. Let us suppose that aerial photography is executed on the scale 1:40,000; focal length of the camera is 100 mm; height of photographing is 4000 m and speed of the aircraft is 120 km/hr. In this case the longitudinal overlap is equal to 60%. Then according to formula (10) the base on the aerial photo will be equal to

$$b_x=\frac{180(100-60)}{100}=72\ \text{мм},$$

and on the terrain

$$B_x=40\cdot 72=2880\ \text{м}.$$

If the speed of the aircraft is

$$v=\frac{120\cdot 1000}{3600}=33{,}3\ \text{m/sec},$$

the interval between exposures will be correspondingly equal to

$$t=\frac{2880}{33{,}3}=86{,}4\ \text{sec}.$$

In this time interval the cycle of work of the cassette should be completely accomplished. When the permissible magnitude of bluring of the image is 0.05 mm the exposure time, according to the formula (19), can have the value

or

$$0{,}05 = \frac{100}{4000} 33{,}3\tau$$

$$\tau = \frac{1}{16{,}7} \text{ sec}$$

If the control instrument works with a precision of 0.5 sec the error in overlap will be determined from the equation

$$\frac{86{,}4}{0{,}5} = \frac{60\%}{x}.$$

from which x = 0.35%. This is 2% less than the permissible magnitude according to the instruction, which is fully acceptable.

Example 2. Let us suppose that aerial photography is executed on the scale 1:40,000 at aircraft speed 360 km/hr. Then in conditions of example 1 the work cycle of the AFA will be equal to 28.8 sec and exposure time will equal 1/120 sec. An error in the work of the control instrument of 0.5 sec results in an error in overlap of 1.04%, which is considered acceptable.

Example 3. Let us suppose that aerial photography is executed on the scale 1:3000 at aircraft speed 120 km/hr. Then on the base corresponding to longitudinal overlap of 60%, the work cycle of the AFA will be 6.4 sec. Exposure for blurring of the image 0.05 mm the exposure time should be 1/222 sec. The control instrument should not give errors in overlap of more than 2%, therefore accuracy of its work should be on the order of 0.2 sec.

Example 4. Let us suppose aerial photography is executed on the scale 1:3000 and speed of flight is 360 km/hr. Then for conditions of example 3 the work cycle of the AFA will be 2.16 sec. Exposure time should be 1/666 sec. The control instrument should not give error in overlap of more than 2%; therefore the accuracy of its work should be 0.07-0.1 sec.

Thus, during small-scale aerial photography (1:40,000-1:60,000) exposure time should be 1:50-1:120 sec and the accuracy of work of the control instrument should be 0.5 sec. Existing equipment satisfies these conditions.

During aerial photography on the scale 1:3000 exposure time should be on the order of 1/222 sec if the aircraft speed is decreased to 120 km/hr. The work cycle of the AFA should be not more than 2 sec and accuracy of work of the control instrument, on the order of 0.1 sec.

9. Appraisal of Quality of Flight

It is important to control the measuring qualities of aerial photos, in particular to determine angles of tilt of aerial photos of a given flight. This control is carried out at the aerial photography base of the expedition.

Angles of tilt of aerial photos are established most quickly according to readings of the circular level. However, due to horizontal accelerations of the aircraft the errors of readings can reach 100% of the magnitude of the actual angle of tilt of aerial photos.

At angles of tilt equal to 1°, errors of determination of angles of tilt should not exceed 10'. Such accuracy can be obtained on the M. D. Konshin and V. B. Orlov field stereometer. In Fig. 37 is given the diagram of the instrument, on which 1 is the base; 2 and 4 are aerial photos; 6 is the basic support; 10 and 11 are micrometer screws shifting the left aerial photo by x and q; 3 and 8 are threads drawn tight above the aerial photos; 7 is a micrometer screw turning the right thread holder; 5 and 12 are screws κ_1 and κ_2 of the aerial photos.

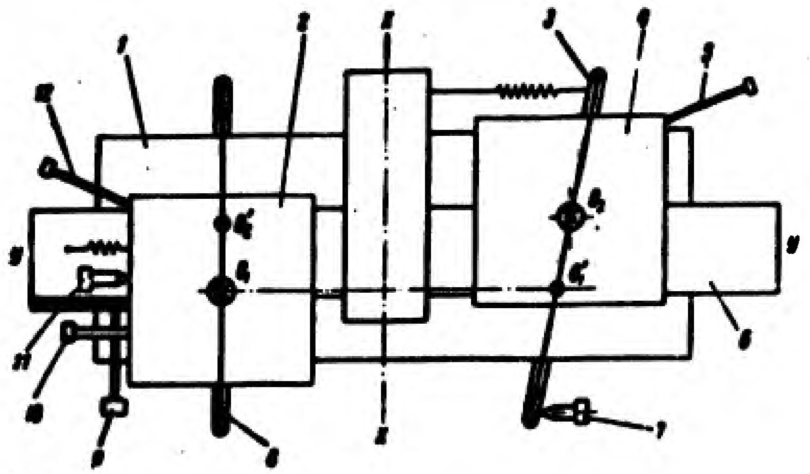


Fig. 37.

Axes X and Y of the instrument are turned by 90° from the usual location.

Above the aerial photos is located a stereoscope. For stereoscopic identification of points it shifts across the instrument. Aerial photos are shifted to the left and to the right by rack and pin 9. The photos are arranged stepwise in such a manner that their centers O_1 and O_1' are approximately on one line. The left aerial photo is oriented under the left thread according to the identical points of the right aerial photo. When the right thread is rotated until it coincides with the left points the angle of descent between threads is found; this is functionally connected with the elements $\Delta\alpha_x$ and ε (see below). In an hour one can determine with good accuracy the angles for approximately 15 pairs of aerial photos.

§ 10. Investigation of the Aerial Camera.

All instruments entering into the composition of flight survey equipment will be subjected to investigation both in the process of their manufacture in the factory, and before their use.

1. Determination of Elements of Internal Orientation of the Aerial Camera

The elements of internal orientation include the coordinates (Δx, Δy) of the principal point of the aerial photo (with respect to lines connecting the marks of the auxiliary frame), and the value of focal length f_k of the aerial camera.

For determination of elements of internal orientation a goniometer is used. Fig. 38 shows the construction of one of the precision goniometers. In this instrument the tubes have identical focal length f' = 270 mm with relative aperture of the objective of 1:6 and angle of field of view of approximately 1°. The front tube contains an autocollimating eyepiece. Between the telescopes there is a round plate on which a large limb is established on three lifting screws. To the upper part the limb is divided into 30-minute intervals, and in the lower part into 10-minute intervals, observed through the microscopes with a micrometer. One turn of the drum of the micrometer is equal to 5'. The value of a division of one interval is equal to 5", which permits measuring the turn of the limb with the chamber fixed on it (without cassette) with the same accuracy. The instrument comes with a number of components for assembling the chamber, and also counterweights for locating the center of gravity of the chamber above the center of the limb.

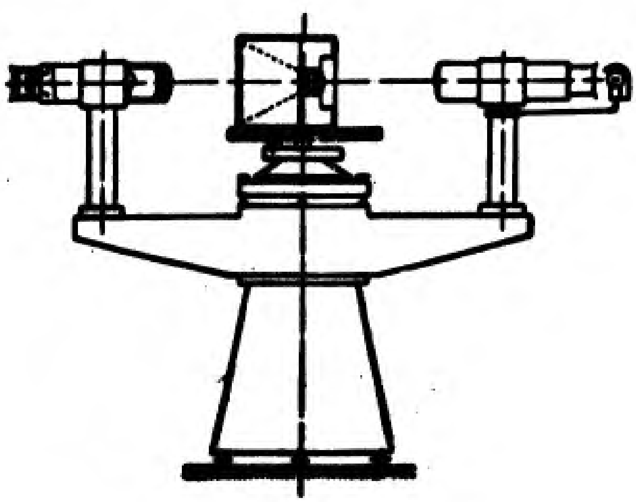


Fig. 38.

Before the investigation of the aerial camera the goniometer is adjusted: for observing parallelism, first, of the sighting axes of the telescopes, second, of vertical and horizontal threads of telescopes, and the perpendicularity of the axis of rotation of the limb to the sighting axes.

Parallelism of sighting axes of telescopes is attained by lining up the crossing points of the threads of grids, for which one of the telescopes is given adjustment movement.

Parallelism of the vertical and horizontal threads of telescopes is established with the help of the corrective screws of the grid.

Perpendicularity of the axis of rotation of the limb to the sighting axes is verified with the help of a plane-parallel glass, utilized for autocollimation, and fastened on the base with three lifting screws; the glass is mounted on the limb between the tubes. Illuminating one of the eyepieces of the telescope, the observer seeks exact coincidence of threads with its reflection, for which are used the lifting screws of the limb and glass. Then the limb with the glass is turned 180°; in case of nonperpendicularity of the axis of the limb to one of the combined sighting axes of the scopes the intersection of the threads does not coincide with its reflection. Errors are eliminated by lifting screws of the base and limb.

After this the body of the aerial camera, free from the cassette, is mounted on the limb in such a way that the objective of the aerial camera is above the axis of rotation of the limb and simultaneously on the line of sighting axes. To the auxiliary frame of the body of the aerial camera is fastened a measuring grid made on plane-parallel glass. The lines of the grid should pass through the coordinate marks of the body of the aerial camera.

Balancing the frame with counterloads with respect to the center of the limb, the focal plane of the chamber is mounted perpendicularly to the sighting axes of the scopes. With this the center of the grid is observed in one of the tubes and in the other the autocollimation of the point of intersection of threads over the same grid.

After preparation for observations horizontal angles are measured by matching the point of intersection of the threads of the telescopic sight into hatchures of the measuring grid through every 10 mm. For derivation of formulas utilized during calculations we will turn to Fig. 39, on which m and n are symmetric strokes of the grid located at distance d from its center; o is the principal point of the aerial photo o_1 is the center of the measuring grid and α and β are the measured angles.

Then

$$\frac{d}{\sin\alpha}=\frac{So_1}{\cos(\alpha-\theta)}.$$
$$\frac{d}{\sin\beta}=\frac{So_1}{\cos(\beta+\theta)};$$

from which

$$\frac{\sin\alpha}{\cos(\alpha-\theta)}=\frac{\sin\beta}{\cos(\beta+\theta)}$$

or, after conversion,

$$\mathrm{tg}\,\theta=0{,}5(\mathrm{ctg}\,\beta-\mathrm{ctg}\,\alpha). \quad (27)$$

For calculating angle θ angles α and β are measured repeatedly and from obtained values of θ take the average; thus, since the difference between angles α and β is small and does not exceed 2-3', the influence of distortion of the objective of the aerial camera on the results of calculation does not show. The value of the focal length is determined from the equations

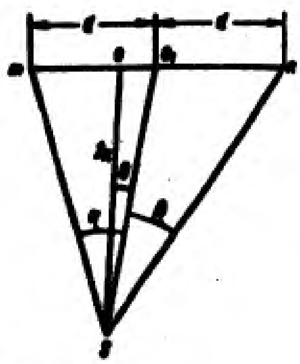


Fig. 39.

$$d-oo_1=f_k\,\mathrm{tg}(\alpha-\theta),$$
$$d+oo_1=f_k\,\mathrm{tg}(\beta+\theta);$$

from which

$$f_k=\frac{2d\cos(\alpha-\theta)\cos(\beta+\theta)}{\sin(\alpha+\beta)}. \quad (28)$$

Knowing angles $\phi_1 = \alpha - \theta$ and $\phi_1' = \beta + \theta$ and taking f_{cp} from the series of determinations of f_k, it is possible to calculate distortion of the objective by the formulas

$$\Delta_1=f_{cp}\,\mathrm{tg}\,\varphi_1-(d_1-oo_1);$$
$$\Delta_2=f_{cp}\,\mathrm{tg}\,\varphi_2-(d_2-oo_1);$$
$$\cdots\cdots\cdots\cdots\cdots$$
$$\Delta_n=f_{cp}\,\mathrm{tg}\,\varphi_n-(d_n-oo_1).$$

and then to construct a graph of distortion for different values of angle ϕ.

Accuracy of determining f_k (at $f_k \approx 200$ mm) on the goniometer is approximately equal to 0.02 mm, and that for the position of the principal point is approximately 0.03-0.04 mm.

Here

$$\Delta x = oo_1 = f_k \operatorname{tg} \theta.$$

For total determination of the position of the principal point (Δx, Δy), it is necessary to turn the camera 90° and to conduct measurements along the y axis.

It is necessary to note that if the graph of distortion Δ_1, Δ_2, ..., Δ_n has zones going out beyond the limits of accuracy of measurement, then it is possible to improve it at the expense of a change of the value of f_k. For simplifying the processes of calculation under the condition of minimum value of distortion nomographs are offered.

According to certain investigations distortion of objectives, measured in several radial directions, gives asymmetric curves, distinguished from the accepted standard curve. If the scattering of curves does not exceed permissible limits, then such an objective can be used.

To accelerate the process of investigation of objectives at present new instruments and systems of calculations are proposed. Thus, for instance, there exists an autocollimating method developed in enterprise No. 10. The instrument consists of a microscope, mirror, and set of prisms of birefringence with different, very exactly determined constant angles for reverse deflection of rays. Observing in the microscope, located above the principal point of the grid of the auxiliary frame of the camera, it is possible through the camera objective the prism, and again through the objective to observe lateral points of the grid with known coordinates. Magnitudes of shift of lateral points relative to the principal point also appear as distortions of the objective.

2. Determining Shutter Efficiency

Testing the shutter consists in determining the optical efficiency and period of damping of its parts. Photographic and photoelectrical methods exist. Fig. 40 shows a diagram of one of the instruments used for the purpose. The tested shutter is disposed in such a way that the condensed bundle from light source L passes through shutter v_1 and objective o of the instrument to the drum with film F, revolving with great speed. Disk S with holes rotates in front of the film, due to which multiple printing of the image of the opened and closed shutter occurs.

Along side these images is a printed sinusoid - time scale. The sinusoid will be formed by photographing a luminescent point obtained from the light source and passing through the prism and mirror on the chamber. Under the influence of electrical current the chamber oscillates 200 times sec, which permits guaging 1/200-1/400 sec and, by interpolation, still smaller fractions of a second. The sinusoid is photographed in 1/30 sec (time of exposure) by means of shutter v_2 within 2-3 sec before and after testing the aerial photography shutter.

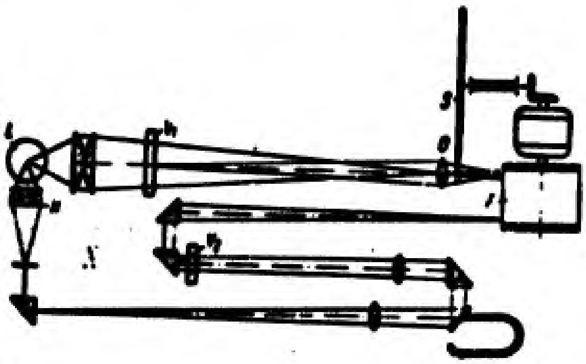


Fig. 40.

Fig. 41 depicts the process of opening the blades of the shutter. The optical efficiency of the shutter is the ratio of the quantity of light energy passing through the shutter to its quantity at the moment of opening and closing of the shutter. The coefficient is determined by the relation of the sun of areas of the open to the sum of the total areas of working apertures of the shutter.

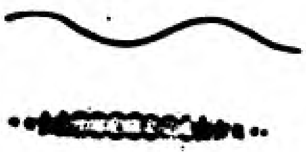


Fig. 41.

Let us note that from the initial moment of work of the shutter to the moment of its full opening there passes very little time - in all 1/600-1/700 sec. The desired efficiency is on the order of 70-90%.

The shutter is checked on various speeds in the beginning, in the middle, and at the end of damping period of work operation.

The service life of the shutter is determined on the basis of the number of exposures obtained up to the moment of its destruction, which is fixed by a counter connected with the working shaft of the shutter. The aerial camera with shutter is tested in a refrigeration chamber under conditions similar to aerial photography, i.e., with temperature down to -30°.

3. Checking the Cassette Operation

The checking of cassette operation consists of establishing the dependability of winding, the correctness of the frame assembly, the supplied quantity of unexposed aerial and film, the degree of its tension and flattening.

A check is conducted on exposed aerial film with the aerial camera motor engaged and the cassette open. Aerial film tension should not be excessive to avoid unnecessary and nonuniform distortion. It is necessary to establish if electric currents are sufficiently well removed from the surface of the aerial film so that during aerial photography discharge traces do not appear on the aerial photo. Constancy of the amount of the segment of re-reeled aerial film is verified over its entire length. The quality of flattening the aerial film in the plane is checked by several methods. One method consists of applying four thin wires, tightened above the aerial film 15-20 mm from its edges. From the amount of curvature of the shadow traces conclusions are made on the degree of flattening of the film.

Suctioning the unexposed aerial film ensures accuracy of flattening up to 0.02-0.03 mm (in approximately 80% of the cases).

As we already noted many factors influence the sharpness of the image on the aerial photo. Some of them (for instance, distortion) can be calculated, but others can be revealed only by special methods.

The joint influence of objective distortion and failure of the film to lie flat in the focal plane of the cassette on the accuracy of the image can be determined by means of stereoscopic measurement of coordinates of points of the aerial photo (for instance, of the rough surface of ice on a lake) and by the study of the degree of curvature of the obtained model (investigation, of Ecklund in Sweden).

§ 11. The Photographic Process

After aerial photography the cassette of the aerial camera is delivered to the photolab. Aerial film is developed as a whole (uncut) in developing instruments.

The most convenient are spiral instruments (Fig. 42) in the form of large metal reels. The coil with aerial film is fastened to the front part of instrument, and the end of the aerial film is attached to the axis of the apiral reel; aerial film is reeled into a spiral.

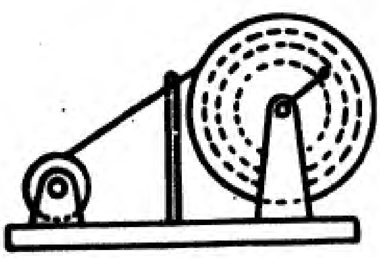


Fig. 42.

The drum with aerial film up to 30 m in length is dipped into a tank with the developer, and within a short time (for instance, 7 minutes) - into a tank with water and then into a fixing bath. Fixing lasts

15 minutes on the average and washing in circulating water (t = +18°C) about 30 minutes. After washing, the aerial film is dried on speical drying drums. The film, fastened with linen clamps, drys in a space with open windows (spring, summer) which are closed by gauze; with closed windows it is necessary to create air movement. The developing process should preserve the resolving power of the emulsion layer of the film, and the process of drying should not cause distortion.

Photo prints are prepared from uncut film on contact machines [14], which have the usual electric lamps and the red lamps used for packing light-sensitive material. For production of identical exposures an exposure meter, automatically turning on the light in the contact machine during photocopying, is installed next to the printer.

Three or four prints are prepared for photogrammetric and other work.

Aerial film has a thickness of about 0.16 mm and consists of nitrocellulose, coating of sodium silicide and gelatinous emulsion in which there are light-sensitive particles of silver halide.

The usual photographic emulsions have a maximum sensitivity to beams of the violet zone of the spectrum, with a wavelength near 400 mμ; for human eyes the brightest are yellow beams with a wavelength of about 550 mμ. In order to obtain good measuring qualities of aerial photos, measures are taken for elimination of nonconformity of color transmission in one-tone gamma of black color by which details of the aerial photo are depicted. Indicated nonconformity is eliminated by introducing dyes into the emulsion and applying light filters. According to the type of emulsions, aerial films are divided into ordinary, isochromatic, panchromatic, and infrachromatic. Fig. 43 gives the curves of sensitivity of the eye and the film panchrome 10, without a light filter [14].

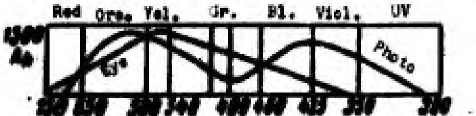


Fig. 43.

For aerial photography major use is made of panchromatic aerial films sensitized to all rays of the spectrum and, depending upon conditions of the landscape, enabling use of filters of any color.

The resolving power of Soviet panchromatic aerial film 11/400 is equal to 140 line/mm; its light sensitivity is about 500° plus (Cin the GOST 2817-45 system), which permits the use during aerial survey of an objective with a small relative aperture 1:6.3-1:9.

For exact photogrammetric measures glass negatives are used which, after development, have extremely minute distortion. Also widely used are diapositive slides.

Slides have a gelatinous layer sensitized only to blue-violet rays which is required during photoimprinting from negatives of one-tone gamma. Their sensitivity is approximately 100 times less than the sensitivity of aerial film, but the resolving power of the emulsion layer is considerably larger than for aerial film. Therefore aerial photos can be reduced two or three times without decreasing the quality of the image.

Photographic paper consists of a base covered by barium sulfate, having a pure white color, and a light-sensitive gelatinous layer. Contrast and normal, dull and glossy sorts of photographic paper of much varied quality exist.

After development and drying photographic paper is considerably distorted. Distortion occurs in all layers and is nonuniform as well (lengthwise and across the paper). This distortion for different sorts of paper fluctuates within limits of 0.423-0.273% along the roll and 0.399-0.155% across it. Therefore, during use of photoprints for measuring, photographic paper is preliminarily glued in the dark with starched glue onto glass. At the same time the photographic paper is oriented in accordance with the direction of the fiber of paper and perpendicularly to the axis of the photorun. Such orientation allows for decreasing the influence of distortion appearing during development. Furthermore, distortion can be decreased by means of preliminary 1.5 hr soaking of the photographic paper in water. After that, as the paper dries, photocopies are made. This process partially lowers the quality of the photographic paper. In a number of cases copies are made on slide plates from negatives to preserve for an unlimited time the measuring qualities of the image.

There are available a great many developing substances which affect the density, contrast, and other qualities of negatives [15].

We give the formula of one of the contrastive methylhydroquinone developers (KTsl) used during processing materials in aerial surveys.

1. Water 1000 cm^3 at a 10° temperature, distilled or boiled.
2. Metol 2 g (developing substance), reducing the silver halide.
3. Hydroquinone 10 g, having the same purpose as the metol.

4. Anhydrous sodium sulfite 52 g, preserving the developer from deterioration for several weeks.

5. Anhydrous [sodium corbonate] 40 g (alkali), neutralizing the acid (hydrobromic) precipitated during the development and accelerating process.

6. Potassium bromide 4 g (delaying the process of development), promoting the obtaining of contrastive aerial photos. It is also an anti-fog substance.

Below is given the formula of a solution used for fixing aerial photos:

1. Boiled water 1000 cm^3 at 18° temperature.

2. Thiosodium sulfate (hyposulphite) 250 g.

3. Sodium sulfite, crystalline, 50 g.

4. Potassium metabisulfate 17 g.

5. Chrome alum 10 g for tanning positives, for the purpose of increasing the mechanical strength of the layer.

Very important criteria of photographic quality of aerial photos are the density and the degree of contrast of the image. According to the character of transmission of the relationship of brightnesses of objects the image is contrastive, normal or dull. The greatest density of the image on the negative should be on the order of 1.6-1.7 arbitrary units.

Photographic paper is selected in conformity with the density and contrast of the negatives. The quality of the image on photographic paper, depending upon its sort, is characterized by the following data; normal 1-1.5; contrastive 1.8; and strongly contrastive 2.65, in the same units.

Heterogeneity of illumination of aerial film in the center and on edges leads to the fact that the density of the photoimprint is also nonuniform. But more essentially is the fact that aerial photos, especially of mountain terrain, are unnecessarily contrastive in detail; therefore to increase the resolving power of the image still in the process of photocopying it is necessary to improve its quality. Such a problem is solved by electron photocopying apparatuses, both Soviet and foreign. We give the diagram of the electronic copying instrument (Fig. 44), developed in TsNIIGAiK [18] (Central Scientific Research Institute of Geodesy, Aerial Photography and Cartography).

The instrument consists of printing table 1, electron-beam tube (CRT) 2, objective 3, photomultiplier 6, and a device for automatic regulation of exposure.

Exposing aerial photography negative 4 on photographic paper 5 is executed by the electron tube through the objective.

The intensity of the electron beam (in the form of a spot, diameter 4-6 mm) scanning the negative changes automatically, depending on the degree of density of the sections. Strengthening the illumintion of darker sections of the aerial photo and decreasing in light places is attained by a photomultiplier with an amplifier, located above the photographic paper.

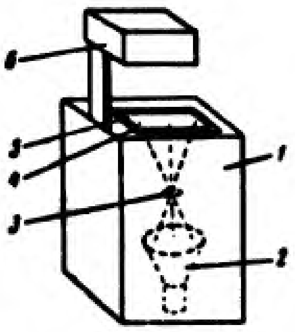


Fig. 44.

Prints obtained on an electronic copying instruments have a considerably better processing of details than prints obtained by the usual printing method. Is noticed also that accuracy of measurements on aerial photos of mountain terrain is increased approximately 1.5 times.

In recent years paper covered with a layer of semiconductor emulsion was used for photocopying. If one were to charge the electropaper with a current of high voltage (to 10,000 v) in darkness, then by the usual method the pro-exposed contact image can be developed in benzene which has been blacked or dyed any other color. Since development and drying (under a draw pipe) continue not more than one minute, distortion of the paper does not occur. Resolving power of electropaper at present is now lower than for ordinary photographic paper. Therefore it is applied most frequently for hachured printing.

CHAPTER III

Construction of a Flat Support Net with the Help of Aerial Photos

§ 12. General Remarks

In map-making by aerial-photos it is necessary to determine the geodesic coordinates of not less than three contour points identified on each aerial-photo and on the terrain. For this reason the volume of field geodesic work on aerial-photo control tie becomes very large. Even in the first years of the development of aerial-photography (1925-1930) remarkable properties of aerial-photos with small angles of tilt were found in that the central directions on them were distorted insignificently, and they permitted securing aerial-photos with the necessary quantity of control points by photogrammetric means. With this three or four times fewer geodesic support points (i.e., 4-5) are required on one topographic plane table than during continuous tying.

At present a number of methods have been developed for thickening the control net, the earliest of which is the method of plane phototriangulation. In connection with this let us consider certain properties of aerial-photos utilized with the given method.

§ 13. Distortion of Central Directions

1. Influence of Tilt of the Aerial Photos on Distortion of the Central Directions

Angles between directions drawn from the principal point to any points of the aerial-photo differ minutely from the identical angles on the terrain if the angle of tilt of the aerial-photo does not exceed 3-4° and the terrain is flat.

Suppose that nh_1 (Fig. 45) is the direction of the principal vertical of the aerial-photo, and point h_1 is the point of descent located on line h_1h_1' of the horizon, obtained by intersecting of the plane in which the aerial-photo is located with the horizontal plane conducted through the center of projection.

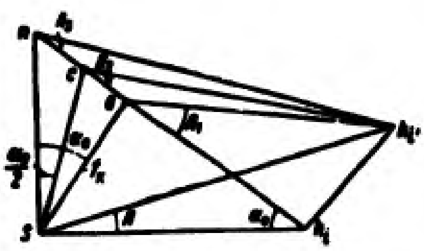


Fig. 45.

Let us establish a connection between angles A on the terrain or, which is the same, on the horizontal plane Sh_1h_1' and angles A_1, A_2, and A_3 on the aerial-photo. Let us determine the value of central angle A_1, the summit of which is at principal point o of the aerial-photo. From triangles h_1oh_1' and h_1Sh_1', having a common side, h_1h_1', we write

$$oh_i \cdot \mathrm{tg}\, A_1 = Sh_i \cdot \mathrm{tg}\, A;$$

$$Sh_i = \frac{oh_i}{\cos a_0}.$$

and therefore

$$\mathrm{tg}\, A_1 = \frac{\mathrm{tg}\, A}{\cos a_0}. \qquad (29)$$

We will determine the value of angle A_3 from triangles h_1nh_1' and h_1Sh_1', the summit of which n at the nadir point of the aerial-photo is

$$nh_i \cdot \mathrm{tg}\, A_3 = Sh_i \cdot \mathrm{tg}\, A;$$

but

$$Sh_i = nh_i \cdot \cos a_0,$$

and therefore

$$\mathrm{tg}\, A_3 = \mathrm{tg}\, A \cos a_0. \qquad (30)$$

Determine the value of angle A_2, having its summit at point c located on the bisector of angle nSo. Triangles h_1ch_1' and h_1Sh_1' are isosceles since sides h_1h_1' are common and $ch_1 = Sh_1$, and therefore

$$\angle A_2 = \angle A. \qquad (31)$$

Thus, central angles of aerial-photo A_1 are greater than the identical angles on the terrain; angles A_3 are smaller, and angles A_2 are equal to the angles on the terrain.

Point c is called the point of zero distortions. Segments located on the horizontal passing through this point are equal to segments of a strictly horizontal aerial-photo. Let us give formulas which allow determining the distortion of an angle measured at the principal point; formula (29) is written in the form

$$\operatorname{tg} A_1 - \operatorname{tg} A_1 \cos \varepsilon_0 = \operatorname{tg} A_1 - \operatorname{tg} A$$

or

$$\operatorname{tg} A_1 2 \sin^2 \frac{\varepsilon_0}{2} = \frac{\sin (A_1 - A)}{\cos A_1 \cos A} .$$

Hence, taking sin $(A_1 - A) = \Delta A$ and $A_1 = A$ we find the value of ΔA in minutes:

$$\Delta A = 3438 \sin 2 A_1 \sin^2 \frac{\varepsilon_0}{2} . \qquad (32)$$

When $A_1 = 0°$ and $A_1 = 90°$ the distortions of the central angles are equal to zero, and with $A_1 = 45°$ the distortions will be maximum. Distortions of angles on an aerial-photo with the summit at the nadir point are determined by the same formula, but with the signs reversed. The distortions of angles at any point of the aerial-photo are expressed by more complicated dependences. Table 8 gives the maximum distortions of central directions depending on the magnitude of the angle of tilt of the aerial-photo.

Table 8

Angle of tilt of aerial photo in degrees	Distortion of central directions in minutes	Note
1	0,26	$A_1 = 45°$
2	1,0	
3	2,3	

Taking into account the small tilt of the aerial camera at the time of photographing (2-3°) in a number of cases, as we will see later it is possible to disregard the error in directions due to tilt of the aerial-photo.

2. Influence of Relief on Directions Traced from the Principal Point

The displacement of points of the aerial-photo which appears due to the influence of forms of relief causes distortion of directions drawn to these points from the principal point of the aerial-photo.

Displacement δ (Fig. 46) of the point of terrain M is determined by the expression

$$\frac{\delta}{h} = \frac{r}{H}$$

or

$$\delta = r \frac{h}{H} . \qquad (33)$$

Expressing h and H in meters in this formula and r in millimeters, we will find the magnitude of displacement δ on the aerial-photo due to the relief. On Fig. 46 is shown this displacement, leading to error $\Delta\kappa$ of a direction drawn from the principal point o on the aerial-photo is shown on the right.

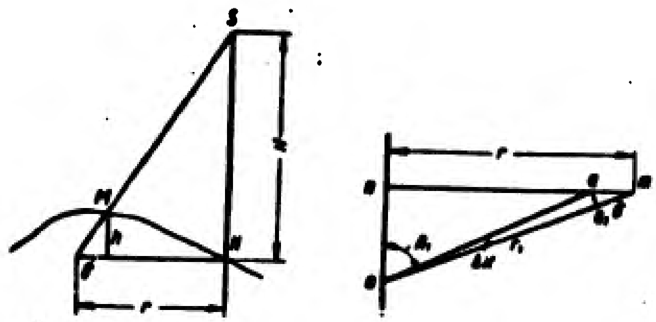


Fig. 46.

The error in direction arising from the relief is equal to

$$\Delta\kappa = \frac{aa_1}{r_1 - \delta}.$$

Dropping δ because of its smallness and replacing aa_1, we find

$$\Delta\kappa = \frac{\delta \sin(nmo)}{r_1}.$$

Placing value δ here we obtain

$$\Delta\kappa = \frac{hr}{Hr_1} \sin(nmo).$$

In addition

$$\frac{\sin(nmo)}{no} = \frac{\sin A_1}{r}$$

or, taking into consideration Fig. 45, on which $no = f_k \operatorname{tg} \alpha_0$, we find

$$\sin m = \frac{f_k \operatorname{tg} \alpha_0 \sin A_1}{r}.$$

Thus, we finally obtain

$$\Delta\kappa = \frac{h f_k \alpha_0}{r_1 H} \sin A_1. \qquad (34)$$

Suppose that distortions are allowed in directions $\Delta\kappa = 5'$, then with $H = 2500$ m, $f_k = 100$ mm, $\alpha = 3°$, $r_1 = 100$ mm, and $A_1 = 90°$, we find that the value of allowance of oscillations of heights of points of terrain above the average plane $h_{max} = \pm 74$ m.

Consequently, distortions due to relief appearing in directions drawn from the principal point of the aerial-photo, at small values of h, will be perceptible magnitudes; at the same time directions drawn from the nadir point do not have these distortions. Therefore the measurement or construction of angles from nadir points of aerial-photos is preferable to that from the central points.

The position of nadir points cannot be determined directly; instead, on the basis of measurements on aerial-photos we find positions of the conditional nadir points.

By applying a stereocomparator in which the cassettes can be established at the angles of tilt of the aerial-photos it is possible to obtain directions free from both forms of errors.

However, in connection with the increase in the quality of aerial-photography at present ensuring the obtaining of almost horizontal aerial-photos (with angles < 1°) it is inexpedient to complicate the stereocomparator.

§ 14. Construction of Plane Phototriangulation

Small distortions of central directions on horizontal aerial-photos permits drawing sufficiently accurate directions to contour points located on overlapping parts of the aerial-photo. Directions to identical points traced on two neighboring, correspondingly oriented aerial-photos give in crossing the vertical position of these points.

The simplest method is graphic construction of rhombic series separately for each run of aerial-photos. The construction of phototriangulation nets by this method consists of the following processes: the selection of points on aerial-photos and their perforating them on the negatives, the manufacture of the stencils of the directions, stacking of one stencil of directions on the other, on which a rhombic series is constructed. Instead of stencils it is possible to apply other thin transparent material (celluloid, astralon, khostafan etc.). Points for the net are selected in such a way that the figure of the bearings will be, as far as possible, of regular form. The points must be clear and easily recognized on mutually overlapped parts of neighboring aerial-photos. Usually at first working centers are selected (contour points located in direct proximity to the center of the aerial-photo) within limits of a circle with the radius $r = 0.02\ f_k$. In the absence of suitable contour points in the center of aerial-photos, the principal points of aerial-photos are taken as working centers. In regions with considerable elevations, instead of central points are pinned on conditional nadir points. Then so-called tie points are selected, located pairwise in the zone of triple overlap of aerial-photos on lines perpendicular to the line of centers; the tie points will be separated from the line of centers by approximately 60 mm. On every aerial-photo four transformation points are selected and superimposed each of them is in the middle of the zone of quadruple overlap of aerial-photos of adjacent runs. Then the image of points of the geodesic control net are identified and superimposed. The common adjacent points are also laid on all aerial-photos (negatives). Then all points superimposed on a given negative are punched in the stencil and the directions from the central point are drawn to them with India ink (Fig. 47).

The net of phototriangulation is constructed on the assembly table. For this purpose one stencil of directions is put on the other in such a way that the lines connecting the central point of a given aerial-photo with the image of the central point of a neighboring aerial-photo (initial directions) will coincide. Selecting a distance between centers of the first two stencils (base of phototriangulation o_1o_2) which is 5-7 mm larger or smaller than the base on the aerial-photo (Fig. 48), combine lines o_1o_2 of the first stencil with line o_2o_1 of the second. As a result of this, rays o_1-1 and o_2-1, o_1-3 and o_2-3, o_2-2 and o_1-2, o_1-4 and o_2-4 will cross among themselves, and points of their intersections will be the orthogonal projection of points 1, 2, 3, and 4 of the terrain. On the first two stencils, fastened with weights is put the third in such a way that beam o_3-o_2 coincides with direction o_2-o_3 on the second stencil, and beams o_3-3 and o_2-4 passes through the already present intersection of beams o_1-3, 0_2-3 and o_1-4, o_2-4. Thus, the position of the central point of the third stencil will be determined by graphical solution of lateral intersection, whereas the position of points 1, 2, 3, and 4 was found by [direct] intersection. As a result of the constructions point o_3 will be located from o_2 at a certain distance, expressed in the scale of the first base, just as the position of new points 5, 6,..., is found by intersection. All the subsequent stencils are joined analogously. Besides phototriangulation tie points, in the intersection of corresponding directions we obtain also the position of other

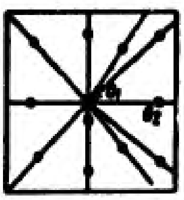


Fig. 47.

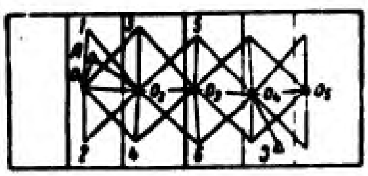


Fig. 48.

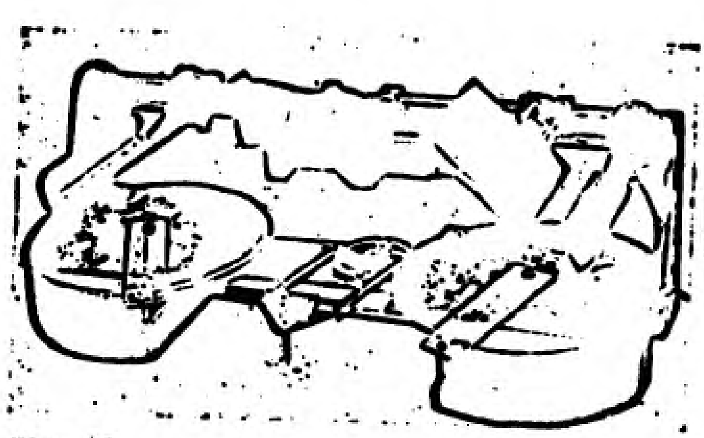
points of aerial-photos, including those for which geodesic coordinates are known. After construction of a phototriangulation net all these points are superimposed on the long stencil.

The rhombic net obtained on the arbitrary scale should have the minimum two geodesic control points A and B, located at the beginning and end of the route. This gives the possibility of its reduction, i.e., reduction to the scale of the topographic plan by points A_0, B_0 and to the geodesic system of coordinates of the state net. Relationship $\frac{AB}{A_0B_0}$ is the coefficient of reduction of the phototriangulation net. In view of complexity of composition and coordination, two-route and solid nets are used less often than rhombic nets.

The graphic method phototriangulating is used most widely in topographic-geodesic work.

The property of small distortions of central angles sometimes is used for construction from aerial-photos of exact plans of small sections of hilly or mountainous site. For this central directions on corresponding points of contours of adjacent aerial-photos are traced on stencils. Then control points on stencils are matched with the corresponding points of the topographic plane table, are oriented and matched to the initial directions, and the sites of crossing of rays are pinned to the plane table.

By a more mechanized method intersections of points are obtained on instruments with radial rulers. One of them (design of the firm of Hilger and Watts, England) is shown in Fig. 49.



GRAPHIC NOT REPRODUCIBLE

Fig. 49.

Semiframes rotate above the aerial-photos, around their centers. The upper part of each of semiframe constitutes a transparent ruler with a hatchure, and the lower part – a metallic ruler, the edge of which is located in the same plane as the hatchure. At the bottom of the instrument is a system of a parallelogram with pins, touching the edges of the lower rulers.

Depending on the displacement of pins established by the operator, i.e., the base of treatment, the cutting of points in that or another scale (from 0.5 to 2^{x}) is possible. With the pencil located on the parallelogram system it is possible to mark the intersected points and in the end to trace the plan.

The disadvantage of the instrument is that in using it, it is impossible to execute intersection of points located on lines of centers and nearby them. It is possible to construct these poi s only by artificial methods, for instance, displacement of working centers of rulers by 10-15 mm. As a result of this, certain errors appear in the position of determined points.

During treatment of aerial-photos of mountainous and even hilly terrain by the shown method, it would be necessary to use the points of true nadirs, the position of which is unknown. This would complicate even more the given method of construction of the plan. When gyrostabilized aerial-photos are used the errors of construction will be small.

About the accuracy of graphic phototriangulation. The accuracy of the position of phototriangulational points is influenced by the errors of directions caused by: 1 – tilt of the aerial-photo (1-2'); 2 – errors of identification and pinning points (0.05-0.08 mm, which at a length of direction of 80 mm amounts to 3-4'); 3 – errors in relief (5-6' for directions from principal points of aerial-photos and at elevations up to 50-70 m); 4 – errors caused by nonuniform distortion of aerial film (2-3'); 5 – errors of drawing (2-3') and connection of stencils (5'). The totality of these errors causes subsequent distortion of phototriangulational series and inconstancy of its scale. After reduction of the series the greatest value of the mean square error of the position of points is obtained in middle of the rhombic series; these errors are calculated by Yu. P. Zhukov's formula

$$\varphi = 0{,}35\, bk\tau \sqrt{n^2 + 2n}, \qquad (35)$$

where b is the length of the base of phototriangulational net; k is the coefficient of reduction; n is the number of bases of the phototriangulational net; τ is the accuracy of the drawn direction, characterized by the mean square error calculated on the basis of calculation of all above-mentioned factors.

Since many points of neighboring rhombic nets are common, it is possible to tie the phototriangulational nets together and, consequently, to refine the position of their points.

Considering the necessity of determining the position of points of thickening with an accuracy to 0.4 mm, dispose the geodesic control points are located with field tying at a distance equal to the length of the frame of the plane table of a given scale, which composes 5-8 bases of the phototriangulation net. Besides graphic and analytic phototriangulation, executed on the basis of measurement of central directions on aerial-photos, in recent years photopolygonometry has been used for production of control points in the plan. The method of photopolygonometry consists of analytic determination of coordinates of summits of directions on the basis of bases of photography measured on aerial-photos and angles between them, and of graphic crossing of rectified points. Measured bases on the aerial-photos are corrected for elements of relative orientation and converted to one scale according to readings of the radio altimeter and statoscope.

Analytic phototriangulation. To increase the accuracy of phototriangulation stereocomparators are used, based on the measurement of coordinates of points of aerial-photos with the precision of 0.02 mm. Central angles of aerial-photos are determined by formulas of the type

$$\operatorname{ctg} A = \frac{x}{y},$$

where x and y are coordinates of points of aerial-photos with respect to the center, measured after orientation of aerial-photos along the initial direction.

Then by calculating the length of sides of triangles and directional angles by the corresponding formulas of geodesy, determine the conditional coordinates of a series of points and reduce the net by means of calculations. The analytic method is very laborious. However, when electronic computers are used analytic phototriangulation becomes more effective method of thickening according the horizontal net.

For this purpose there are also created specialized stereocomparators with increased accuracy of readings (to 2-x μ). They are very massive instruments. During application of stereoscopic marks on points of the aerial-photo a punch card automatically made which contains the values of coordinates of points in the binary system. The punch cards proceed then to an electronic computer where, according to the assigned program, trigonometric equations are automatically solved and coordination of points of the net is executed.

With this method of work a is more expedient to connect blocks of nets, as a result of which considerable areas are secured by control points.

§ 15. Reduction of the Net

It is possible to reduce horizontal nets by graphic, analytic, or optical-mechanical methods. The more widely applied method is optical-mechanical reduction, executed on special projection instruments called projection printers.

1. Projection Printers

The projection printer is intended for optical change of scale of phototriangulation nets, carried out on 600 × 600 mm tracing paper. Figure 50 shows an instrument consisting of bed 1, screen 2, carriage 3 with objective, scale inversor 4, negative carriage 5, and illuminator 6.

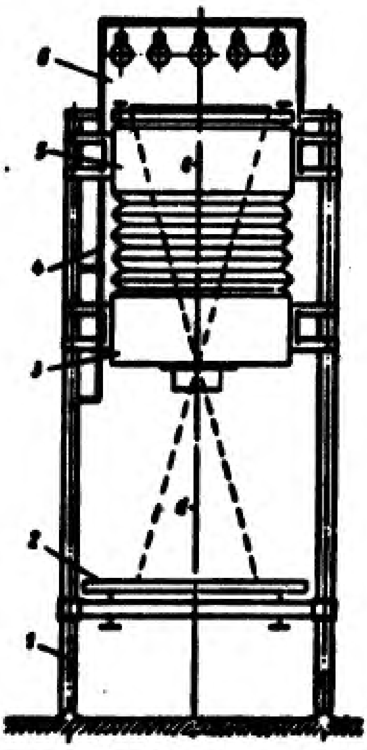


Fig. 50.

Tracing paper with points of the phototriangulation net is laid in the cassette of the negative carriage, then projected onto the plane table placed on the screen. For change of scale both carriages are shifted up and down and disposed at distances c and d, satisfying the condition of optical linkage by the formula

$$\frac{1}{c}+\frac{1}{d}=\frac{1}{f'}. \qquad (36)$$

where f' is the focal length of the projection objective.

Since the carriages are connected by a scale inversor (rhombic), sharpness of the image during the change in the scale of projection is attained automatically. Carriages are set in motion by a hand wheel.

Focal length f' the projecting objective is equal to ~500 mm and the relative aperture is 1:9. An objective with a magnification of 0.6-1.5$^{\times}$ ensures projection of stencils with dimensions up to 600 mm, when outer rays have small tilt. The latter circumstance is very important for decreasing projection errors, since unevenness of the screen on which the rays fall leads to displacement of projected points from the true position.

The bed of the instrument consists of tubes located on lifting screws and step bearings. The total height of the instrument reaches 3 m; therefore it is necessary to use a stepladder during the work.

The screen is made from seasoned wood or from a marble slab. With the help of screws it is possible to set the screen in a horizontal position.

The objective part of the instrument is connected to the negative carriage by blinds for elimination of exposure of the screen. The negative carriage carries the cassette located, like the screen, on lifting screws.

The illuminating device consists of a rectangular housing in which are located 16 lamps with total power up to 800 watts. In certain printer constructions daylight lamps are used, giving more uniform illumination. For stacking the tracing paper in cassette the covering glass, fastened on a hinged frame, is raised.

The rhombic inversor consists of six levers, of which four are of length a and two of length b (Fig. 51). Carriages N and S carry, correspondingly, the negative and objective. Hinge B remains motionless and stands at distance f' from the screen.

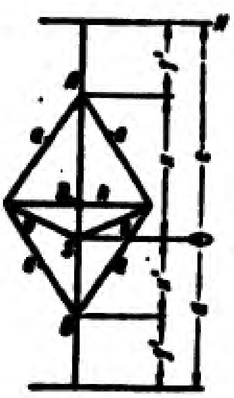


Fig. 51.

During shift of the carriage with the objective, a resultant motion of the carriage with negative occurs; in this

$$a^2 = h^2 + (AD)^2;$$
$$b^2 = h^2 + (DS)^2,$$

but

$$AD = \frac{x + x'}{2},$$

$$DS = x - AD = \frac{x - x'}{2};$$

from which

$$a^2 - b^2 = \frac{(x + x')^2 - (x - x')^2}{4} = xx'.$$

Let us establish the condition that the length of levers satisfy the formula

$$a^2 - b^2 = f'^2, \qquad (37)$$

where f' is the focal length of the objective.

Then we obtain automatic linkage of planes of the negative and the screen. The inversor is calculated on coefficients of magnification of 0.6-1.5^{x} with focal length of the objective f' = 500-600 mm. For exact solution of equation (37), using an objective with a focal length distinguished from the nominal, short levers are made extendable. Simultaneously it is necessary to change segments correspondingly from the negative and the screen to hinges of inversor A and B, equal to f'.

The following requirements are set for the projection printer:

1 - screen should be a plane;

2 - the objective should give an exact image with distortion on the edges of the field of not more than 0.2 mm;

3 - the cassette should have a clean plate glass, without bubbles and cords.

Check of the instrument is carried out with magnification equal to unity.

In order to execute inversor adjustment the latter is established in the initial position, in which the scale of projection is equal to unity and the sharp image is kept.

In the initial moment of screen adjustment, the cassette and plane of mounting of the objective are established in a horizontal position with a precision of 1-2'. Final adjustment of the printer consists of additional turns of the screen and combination of the image of the control net (300 × 300 mm) with the copy, placed on screen, at a scale of projection equal to unity.

Work with reduction consists of combining the images of control points with their position on the plane table, attaining minimum divergences in the position of points common to adjacent runs. After that all phototriangulation points are transposed to the plane table.

Accuracy of reduction can be influenced by noncorrespondence of the surface of the plane table to a plane, insufficient illumination of the screen, error of pinhole points on the plane table in darkened areas. The total error of position of points on the plane table should be less than 0.4 mm.

Other types of photoprinters are based on principles of preliminary photographic or mechanical decrease of nets for the purpose of reduction of dimensions of the instrument. Let us consider one of them.

2. The Drobyshev Optical Reducer

In the instrument are used nets, decreased by 2-2.5 times with the help of a special pantograph, piercing points with a diameter of 0.1-0.2 mm with a needle in the "plane table." The construction of the device may be seen from Fig. 52. On a frame made from tubes there is a screen, covered by massive plate glass and set in a horizontal position. In the rear part of the bed is a vertical directrix and next to it a scale gage inversor in the form of a plate with a curved margin. The gage is touched by a roller, fastened on the flange of the camera carriage. The plane table is illuminated in the carriage by lumin^scent lamps. Further the beams go through an objective (Industar-13, f' = 300 mm) and a mirror to the screen. The chamber with a cassette, mirror, and objective is moved by a reversible electric motor, which is engaged from a foot pedal. To guarantee smooth and easy movement, the carriages are balanced by counterweights.

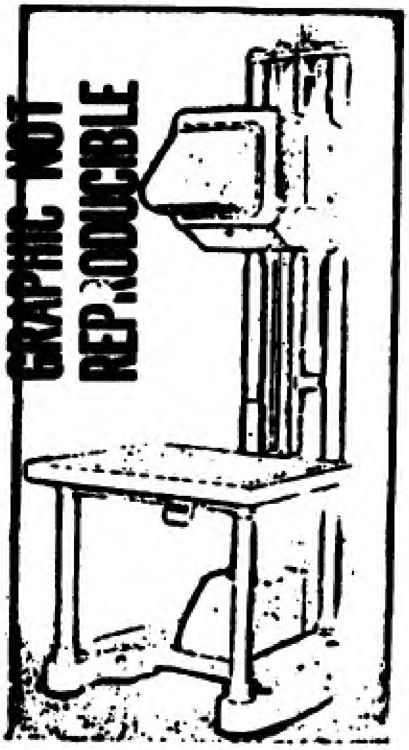


Fig. 52.

Constant sharpness of image is attained by application of a gage-type inversor. It is known that

$$\frac{1}{c}+\frac{1}{d}=\frac{1}{f};$$

since

$$d=ac, \tag{38}$$

then

$$c=f\left(1+\frac{1}{a}\right).$$
$$d=f(1+a). \tag{39}$$

In formulas (38) and (39) d is the length of segments constituting magnitudes of shift of the entire housing upwards and downwards and c is the magnitude of displacement of the cassette in the housing to the left and the right.

Construction of the gage is done after counting the values c and d during magnifications of

$$a=1{,}0;\ 1{,}1;\ 1{,}2\ldots$$

With a change of distance d the carriage is established automatically in a position of sharpness.

Adjustment of the instrument consists of making the basic directrix vertical and the cassette and screen horizontal with an accuracy up to 30". Mirrors must compose angles equal to 45° with the horizontal plane.

After adjustment of the instrument, the control net is laid in the cassette and sharpness of image is obtained; then with a rod-compass segments of the image are measured along the sides of any rectangle of the projecting net. For production of equal sides of a chosen rectangle, change the position of the screen with the lifting screws.

If during shift of the cassette upwards or downwards the sharpness of the image is disturbed, change the position of the inversor. If this causes the image of the principal point to be shifted to one side, then shift the objective in one or two directions.

In order to check the work of the pantograph and optical reductor, check the correctness of combination of luminescent points of the plane table with points of phototriangulation on the tracing paper. After that superimpose on the screen the plane table with control points, which coincide with the image of the control points of phototriangulation. Copy and sign other points. The scale of the net is changes by 0.6-1.6 times

3. Multiprojector Projector Printer

The F. P. Shevchenko projector printer consists of 12 projectors of the multiplex type mounted on a common bed. Separate nets, consisting of four to six aerial-photos each and reduced on a special vertical projector to 6 × 6 cm size, are projected on a common screen with magnification of 5-10 times. With such a method of using phototriangulation nets it is possible to have a very rarefied initial control net and to obtain a well-linked system of points.

This instrument found application during creation of maps on a scale 1:100,000, and also for production of refined aerial mosaics on large scales (1:10,000).

The N. P. Kozhevnikov and F. P. Shevchenko printer consists of four projectors with three degrees of freedom (on axes x, y, and z) each. They permit use of slides of reduced nets of 9 × 12 cm size. The instrument makes it possible to reduce and to tie run and double nets, intended for creation of maps on a scale of 1:25,000.

CHAPTER IV

Rectifying Aerial-Photos and Composition of the Photomap

§ 16. Principles of Rectification

After completion of phototriangulation and obtaining on the plane table a large quantity of control points the problem appears of transfering contours from aerial-photos to the plane table. For this purpose one should use aerial-photos free from distortions due to angles of tilt and elevation differences between centers of projection. If rectified aerial-photographs are glued to plane table with control points, up will obtain a photomap which is a very valuable document for creation of maps of various scales.

Rectification consists of laying the aerial-photo in the projection apparatus (photo rectifier) in a certain position with respect to the screen of the instrument. During projection an image is obtained which corresponds to the plan of the terrain (with a flat terrain) in the given scale. Two cases are possible 1 – a restored cluster of projecting beams similar to the cluster at the time of photographing; 2 – a restored cluster of projected beams not similar to the surveyed cluster.

In the first case the objective of the photo rectifier should be separated from the negative by a distance equal to the focal length of the surveying camera f_k; the tilt of the negative with respect to the screen should be equal to the angle of tilt α_0 of the principal axis of aerial camera when the principal vertical is perpendicular to the axis of tilt of the screen; planes of the screen, negative, and objective have to intersect on one straight line (conditions of perspective linkage)[1]; the focal length of the objective must be selected in accordance with the scale of sharp images obtained on the screen (condition of scale linkage). The last condition, in connection with different coefficients of magnification applied in production, makes it necessary to have a set of objectives, and this prevents wide application of a rectifier with similar bundles or rectifiers of the first kind.

Investigations of properties of the oblique bundle shows that it is possible to execute rectification also with inequality of focal lengths of the surveying and projecting cameras, but then the objective of the latter must be set up on an arc drawn from the point of intersection of the principal vertical with the plane of the horizon, with a radius $S_1 = \frac{f_k}{\sin \alpha_0}$, and the plane of the screen should be tilted at a corresponding angle. It is also necessary to observe the scale and oblique linkage of planes.

Such instruments are called rectifiers of the second type; they have one projecting objective. Correct rectification are requires not three control points, as during rectification of the first kind, but four[2]. One advantage of the second kind is the possibility of projecting wide-angle aerial-photos ($2\beta = 100°-122°$) with objectives with an angle of field of view 60°.

1. Rectification of the First Kind

Rectification of vertical aerial-photographs. During rectification the plane of the cassette should be tilted with respect to screen by the angle α_0 Fig. 53, and the image on the screen should be in the chosen scale. For a horizontal [vertical] aerial-photo obtained from height H the scale of photographing

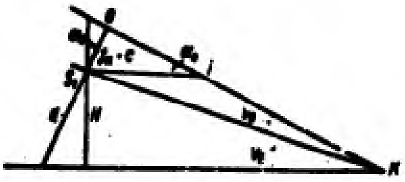


Fig. 53.

$$\frac{f_k}{H} = \frac{1}{N}. \qquad (40)$$

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[1] M. M. Rusinov. Optics of aerial photography instruments. ONTINKTP (Department of Scientific Technical Information) Leningrad, Moscoe, 1936.

[2] Aleksapol'skiy, N. M. Photogrammetry. Moscow, Geodezizdat, 1956.

For an oblique aerial-photo the scale on the principal horizontal is determined by the relationship

$$\frac{\frac{f_k}{H}}{\cos\alpha_0} = \frac{1}{N_1}$$

or

$$\frac{1}{N_1} = \frac{\cos\alpha_0}{N}. \tag{41}$$

The last expression is the conditional measure of scale for both cases of aerial-photography. The scale of rectification can be considered the ratio of distance c (from the negative to the objective) to distance d (from the objective to the screen)

$$\frac{c}{d} = \frac{\cos\alpha_0}{n},$$

where n is the coefficient of increase of the image.

In the first system of rectification

$$c = f_k$$

and

$$d = \frac{cn}{\cos\alpha_0}.$$

If in the formula of optical linkage

$$\frac{1}{c} + \frac{1}{d} = \frac{1}{f}$$

we replace c and d by their values, we can obtain

$$c = f\left(1 + \frac{\cos\alpha_0}{n}\right)$$

and

$$d = f\left(1 + \frac{n}{\cos\alpha_0}\right)$$

Therefore

$$f = \frac{f_k n}{n + \cos\alpha_0}. \tag{42}$$

By this formula it is possible to calculate the sought values of focal lengths of objectives of rectifiers, ensuring conditions of the scale of linkage at various coefficients of enlargement of the image on the screen. With a small α_0 (from 0 to 10 degrees) formula (42) can be presented in the form

$$f = \frac{f_k n}{n + 1}. \tag{43}$$

For observing the condition of oblique linkage it is necessary to turn the objective at angle ε, corresponding to the magnitude of angular eccentricity:

$$\varepsilon = \nu_p$$

Let us define ν_p. We have:

$$e = f_k = aK \operatorname{ctg} \nu_p,$$
$$f_k + d = aK \operatorname{ctg} \alpha_0,$$

therefore

$$\frac{f_k}{f_k + d} = \frac{\operatorname{ctg} \nu_p}{\operatorname{ctg} \alpha_0},$$

from which

$$\operatorname{tg} \nu_p = \frac{f_k + d}{f_k} \operatorname{tg} \alpha_0. \qquad (44)$$

2. Rectification of the Second Kind

Let us assume that aerial-photo aob, objective S_1, and screen AOB are located according to the first principle of rectification. In this case S_1i is the line of the horizon, and point i is the point of descent (Fig. 54).

We place the objective of the rectifier in new point S_2, and the screen is shifted to the position in which point a, o, b of the aerial-photo will be projected on the screen in the form of points A_1, O_1, B_1. Then, so that segments A_1O_1 and O_1B_1 are equal to segments AO and OB, the plane of the screen should be parallel to the line of the modified horizon S_2i. This follows from the theorem of projective geometry, according to which two series of points, projective to a third series, are projective among themselves. In this case series of points A, O, B, and ∞ of the terrain and points A_1, O_1, B_1 and ∞ of the screen are projective to points a, o, b, and i.

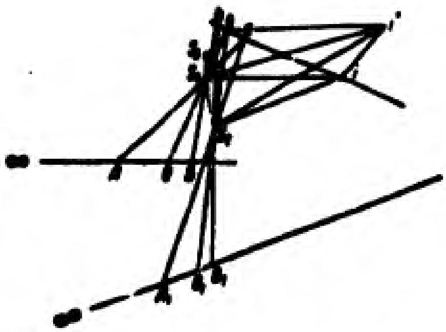


Fig. 54.

It is necessary to be certain that the change of the position of the objective does not cause distortion of the angle along another direction perpendicular to the plane of the figure. Suppose that central angle i'oi on the aerial-photo is the image of angle $i'S_1i$ on the terrain. Then it is necessary that with modified position of objective S_2 the given angle on the screen be kept, i.e., $i'S_2i = i'S_1i$, which is possible only in that case when $S_2i = S_1i$. Consequently, due to the inequality of focal distance of the chamber of the rectifier to the focal length of the aerial camera, the objective must be placed on an arc of circumference whose center is at the point of descent. The distance of this point from the center of the aerial photo and from the objective will be correspondingly

$$ol = f_k \operatorname{ctg} \alpha_0 \tag{45}$$

and

$$S_2 l = S_1 l = \frac{f_k}{\sin \alpha_0} \, . \tag{46}$$

It is possible to place objective S_2 on the arc and above point S_1, for instance, at point S_3.

Basic angles of rectification. Let us define basic angles ν_p and ν_E which is characterized the relative position of the planes of the negative and screen (Fig. 55) with scale and oblique optical linkage of the aerial photo and the screen taking into account the geometric requirement $S_2 l = S_1 l$. During calculations the nodes of the objective, in view of the small distance between them (0.5-2.0 mm), can be replaced by one point. According to the condition of oblique linkage, the planes of the negative, objective, and screen pass through point K. Then, with optical linkage of points m and M we obtain

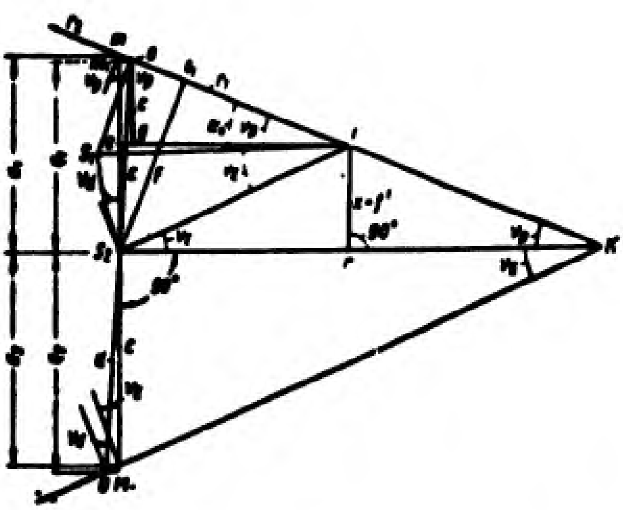


Fig. 55.

$$\frac{1}{a_1} + \frac{1}{a_2} = \frac{1}{f}$$

or

$$\frac{a_1 + a_2}{a_2} = \frac{a_1}{f} \, .$$

At the same time from the similarity of triangles mMK and mS_2l we have the relationship

$$\frac{a_1 + a_2}{a_2} = \frac{mK}{lK} \, ,$$

in which the right side of the equality can be determined from the similar triangles S_2Km and rKl, which gives

$$\frac{mK}{lK}=\frac{a_1}{x}:$$

Consequently,

$$\frac{a_1+a_2}{a_2}=\frac{a_1}{x},$$

i.e.,

$$f=x. \qquad (47)$$

Let us determine angle ν_E. Since

$$f=S_2 l \sin \nu_E$$

and

$$S_2 l=\frac{f_k}{\sin \alpha_0},$$

then

$$f=\frac{f_k \sin \nu_E}{\sin \alpha_0};$$

from which

$$\sin \nu_E=\frac{f \sin \alpha_0}{f_k} \qquad (48)$$

or

$$\sin \alpha_0=\frac{f_k \sin \nu_E}{f}.$$

For determination of angle ν_p from triangles OoK and S_2oi it follows that

$$\frac{c}{d}=\frac{lo}{lK}=\frac{f_k \sin \nu_p}{f \operatorname{tg} \alpha_0}.$$

Replacing the ratio c/d with its expression, we have

$$\frac{\cos \alpha_0}{n}=\frac{f_k \sin \nu_p \cos \alpha_0}{f \sin \alpha_0};$$

from which

$$\sin \nu_p=\frac{f \sin \alpha_0}{f_k n}. \qquad (49)$$

Comparing formulas (48) and (49), we see that between angles ν_p and ν_E there is the simple dependence

$$\sin \nu_E=n \sin \nu_p. \qquad (50)$$

For determining angles of tilt of negative ν_c and screen ν_d we write

$$\frac{c}{\sin(\nu_p+\nu_E)}=\frac{ol}{\cos \nu_d}=\frac{S_2 l}{\cos \nu_c}$$

or

$$\frac{f\left(1+\frac{1}{n}\right)}{\sin(v_p+v_g)}=\frac{f_b}{\operatorname{tg}\alpha_0\cos v_d}=\frac{f_b}{\sin\alpha_0\cos v_c}.$$

Therefore

$$\cos v_d=\frac{f_b\sin(v_p+v_g)}{f\left(1+\frac{1}{n}\right)\operatorname{tg}\alpha_0}. \qquad (51)$$

$$\cos v_c=\frac{f_b\sin(v_p+v_g)}{f\left(1+\frac{1}{n}\right)\sin\alpha_0}. \qquad (52)$$

Dividing equation (51) by equation (52), we obtain the formula for determination of angle α_0:

$$\cos\alpha_0=\frac{\cos v_d}{\cos v_c}. \qquad (53)$$

The value of angular eccentricity ε of the objective is

$$\varepsilon=v_d-v_g.$$

Another expression of eccentricity follows from the formula

$$\operatorname{tg}\varepsilon=\frac{m_1o}{q_1}=\frac{kc-kq}{m_1c+f}=\frac{\frac{f_b}{\sin\alpha_0}\cos v_g-f_b\operatorname{ctg}\alpha_0}{f_b\operatorname{ctg}\alpha_0\sin v_p+\frac{f_b\sin v_g}{\sin\alpha_0}},$$

from which

$$\operatorname{tg}\varepsilon=\frac{\cos v_g-\cos v_p\cos\alpha_0}{\sin v_g+\sin v_p\cos\alpha_0}. \qquad (54)$$

On the basis of given formulas it is possible to calculate the required wide-angleness of projection objective. For this purpose it is necessary to plot segments or_1 and or_2, from center equal to the half diagonal of the rectified aerial-photo. Angles mS_2r_2 and mS_2r_1 or the angles of the light bundle, where their maximum value appears with minimum enlargement. Thus, the working angle of the objective is equal to

$$2\beta=2(\varepsilon+oS_2r_1).$$

Optical scale linkage of planes of the aerial-photo and screen can be executed along different lines. Along the line mM it iwll be characterized by the equation

$$\frac{1}{a_1}+\frac{1}{a_2}=\frac{1}{f}.$$

The formula of linkage of segments along the axis S_2o has the form

$$\frac{1}{q_1}+\frac{1}{q_2}=\frac{1}{f}$$

or

$$\frac{1}{e}+\frac{1}{e'}=\frac{1}{\frac{f}{\cos \theta}}. \tag{55}$$

§ 17. Systems of Photorectifiers

The basic axis of photorectifiers usually is vertical, which predetermines the type of instrument. With this axis it is possible to combine one or another line of the theoretical system of rectification for instance S_2m, S_2o, S_2o_1 (Fig. 55).

Rectifiers corresponding to the given diagram have different operating properties.

1. The Great Rectifier

The great rectifier (ФТБ) [FTB], manufactured in the USSR, is an improved construction of the Zeiss rectifier.

The structural axis of the instrument coincides with the principal optical axis of projection of the objective S_2m. Therefore, for correct rectification it is necessary to displace the principal point of the negative from the structural axis by the magnitude of eccentricity mo and to fulfill conditions of scale and oblique linkage. Scale linkage of planes of an aerial-photo and the screen is executed in accordance with the formula

$$xx'=f^2 \tag{56}$$

with the help of a rectangular inversor (Fig. 56). Around the center C revolves a square, consisting of two rulers, fastened at a 90° angle. With a shift of the carriage with center C upwards or downwards the upper carriage with the negative is set in motion, as a result of which linkage of planes of the negative and screen is attained. Formula (56) is observed during any increase of projection, for the reason that perpendicular CS_2 (console with objective S_2) is the average proportional segment between segments x and x'.

Oblique linkage of planes of the aerial-photo, screen, and objective is attained with the help of the linear oblique inversor (Fig. 57).

To the cassette of the rectifier at point m and to the screen at point M are fastened levers l_1 and l_2 perpendicular to planes of the negative, screen, and axes of their rotation. At distances r guides y_1 and y_2 of the carriages are fastened. At point i is the axis of rotation of inversor ruler NN, joined with levers ℓ_1 and ℓ_2 and carriages. Such a system strictly solves the problem of oblique linkage of segments of the aerial-photo and screen. Actually,

$$\frac{S_1K_1}{f} = \frac{a_1}{y_1}$$

and

$$\frac{S_2K_2}{f} = \frac{a_2}{y_2}.$$

Furthermore,

$$\frac{a_1}{y_1} = \frac{a_2}{y_2},$$

and consequently

$$S_0K_1 = S_0K_2.$$

Thus, planes of the aerial-photo and the screen intersect on the extended principal plane of the objective.

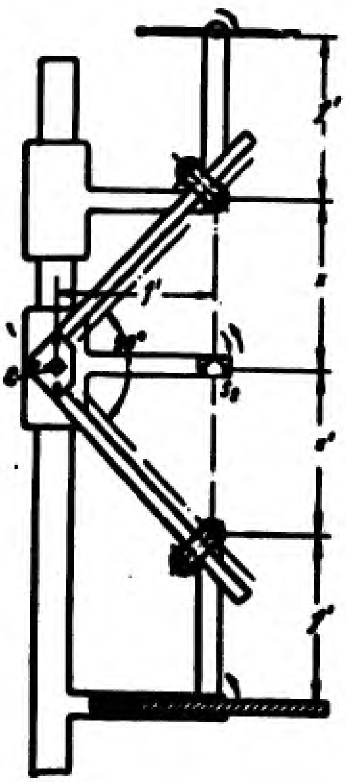


Fig. 56.

We find eccentricity mo of the aerial-photo (Fig. 55) from equation

$$mo = ml - ol,$$

where

$$ml = \frac{ol}{\cos v_p} = \frac{f_k \cos v_E}{\sin \alpha_0 \cos v_p},$$

$$ol = f_k \operatorname{ctg} \alpha_0,$$

from which

$$mo = \frac{f_k}{\sin \alpha_0}\left(\frac{\cos v_E}{\cos v_p} - \cos \alpha_0\right). \qquad (57)$$

Principal parts of the instrument (Fig. 58) are: the base, vertical rods, screen, center carriage with objective and illuminator, negative carriage, scale, and oblique inversors.

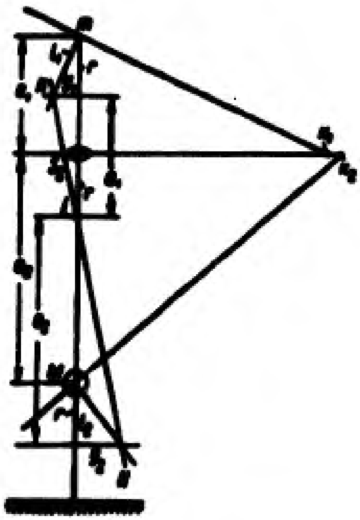


Fig. 57.

The base is a hollow casting with two foot controls of changing the scale of projection and tilts of the screen. On the base are fastened three guide rods, united by a rim. The carriage and counterweights shift along the guide.

The screen (1 × 1 m) is tilted at angle v_E from the rotation of the eight foot control. Angles of tilt are counted off along the limb. Slope of the screen at an angle to 45 degrees is possible; this permits rectification of aerial-photos with angles of tilt approximately 20 degrees.

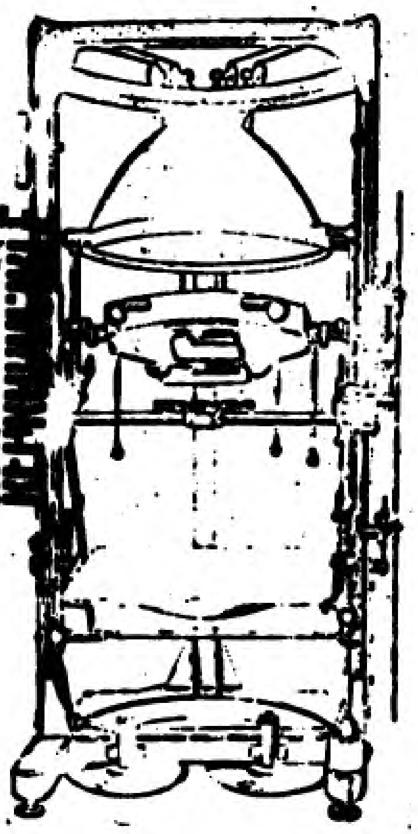


Fig. 58.

On the middle carriage is the objective "Beam" with focal length 180 mm and $2\beta = 76°$. Its working aperture is established with the help of an iris diaphragm within the limits 1:6.8 to 1:36. On the middle carriage there is also an illuminator, which is a reflector of ellipsoidal form. In one of its foci there is a lamp and in the other, the entrance pupil of the objective, which ensures good illumination of the field of the image.

The negative carriage contains a cassette, tilted on angles ν_p around an axis parallel to the axis of rotation of the screen. The cassette with the negative can revolve 360° degrees in its own plane; furthermore, there is the possibility of transferring it by magnitudes of eccentricity Δx, Δy.

Scale linkage of the planes of the screen and the negative is decided by two angular inversors, located on the right and left of the instrument directors. Summits of right angles inversors are set in motion by two coupled screws during shifting of the middle carriage (with rotation of the left foot control); therefore the upper parts of the inversors transfer left and right sides of the negative carriage to identical segments, as a result of which the horizontal position of axis of the negative carriage is ensured.

The oblique inversor consists of a long ruler, revolving around the axis, two levers, and two carriages, shifting along a horizontal guide. The axis of rotation of the inversor ruler stands apart from the center of the objective at a distance $z = 180$ mm, equal to f' (focal length of the objective); at the same distance from the axes of rotation of the cassette and screen are rollers of horizontal carriages, joined with the inversor ruler. When the screen is tilted the lower carriage is displaced and, as a consequence the inversor ruler turns. The latter, in turn, displaces the upper carriage and, consequently, tilts the negative.

Working movements in the instrument:

1 – change of scale of projection with the left foot control;

2 – turn of the aerial-photo in its own plane on angles κ with the help of a crown-shaped cogwheel, by hand;

3 – introduction of angles of tilt of the cassette and screen ν_p and ν_E by the mechanism of oblique linkage with the right foot control;

4 – introduction of eccentricity of the aerial-photo Δx, Δy (manually).

Construction of this instrument is greatly improved.

2. Small Rectifier

The small photorectifier (ФТМ) [FTM] manufactured in the USSR, is the photorectifier Zeiss SEG-IV.

This instrument is intended for rectifying aerial-photos of vertical aerial-photography. Fig. 59 shows the system established in the basis of construction of the instrument. Its structural axis is $S_2 o_1$.

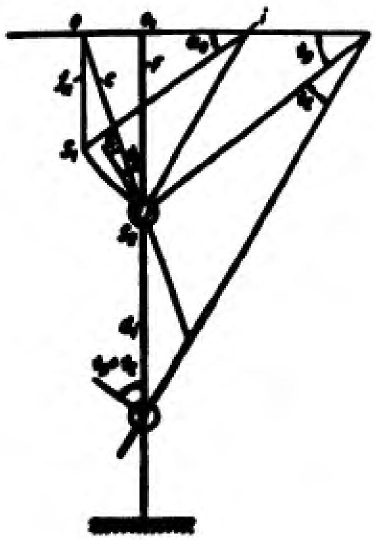


Fig. 59.

The negative remains horizontal during rectification and the screen is inclined on angle $\nu_p + \nu_E$. For optical linkage the principal plane of the objective is inclined on angle ν_p.

Furthermore, the negative must be shifted by the magnitude of eccentricity $o_1 o$, laid out on axes x and y. Scale linkage along the axis $S_2 o_1$ should be executed by the formula

$$\frac{1}{P}+\frac{1}{d_1}=\frac{1}{\frac{f}{\cos\nu_p}}\cdot \qquad (58)$$

In practice it is executed with the help of a ribbon inverter [inversor]. Its arrangement is based on use of symmetric point c.

Fig. 60 shows geometric elements of the ribbon inverter. If interval x + x' is divided in half and triangle cae is constructed with leg ae equal to f', then

$$(ec)^2-(ac)^2=(ae)^2.$$

Furthermore,

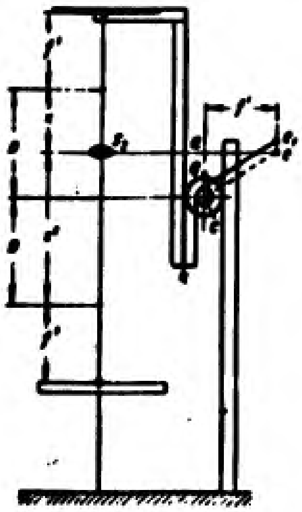


Fig. 60.

$$x' = p - q; \quad x = p + q,$$

where

$$p = \frac{x + x'}{2}.$$

From which

$$xx' = (p - q)(p + q)$$

or

$$xx' = p^2 - q^2.$$

Therefore, if ce = p and ac = q, then a similar system automatically solves the equation of Newton

$$xx' = f'^2$$

and equal to it

$$p^2 - q^2 = f'^2. \quad (59)$$

So that point c would always occupy a center position, during its vertical shift connected with the movement of the objective carriage, the negative carriage should shift two times faster. With fulfillment of this condition the objective and negative are established in a position according to the formula (59) in which a sharp image is obtained.

Construction of the inverter. On bed 1 (Fig. 61) carriage 2 shifts with the negative, and the carriage with objective 3. Beams from the negative pass through the objective to screen 4. Roller of focal lengths 5 is located on objective carriage 3 and is separated from roller 6 of the middle support by the distance $e_1S = p$. The projection of segment e_1S on the horizontal plane is equal to f'. The place of median point e_1 is the location of roller 6, connected with tape roller 5. The end of the tape is connected to a rod whose upper end is connected to negative carriage 2. Another tape, supporting the negative holder, passes through roller 7, fastened on the same support as roller 6.

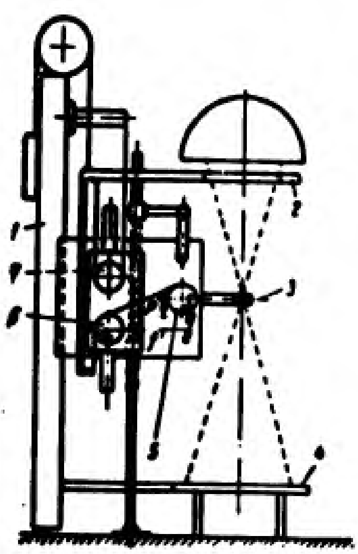


Fig. 61.

Rollers 6 and 7 of the middle support shift with a speed twice smaller the speed of movement of the negative carriage, which ensures the middle position of roller 6.

At a scale of projection of 1:1 rollers 5 and 6 have to be set on one height which is used during adjustment of the inverter. By means of winding the tape on roller 5 it is possible not only to execute adjustment, but also to remove blurryness of the image caused by noncompliance with the formula of linkage.

Oblique linkage of planes of the objective and screen is carried out by a tangentially-oblique inverter (Fig. 62). With slope of the screen at angle $\nu_p + \nu_E$ through the system of gear transmission, flexible transmission, and a tangential screw the support of the movable part and the lever of the objective are set in motion. The latter is tilted on angle ν_p. In this

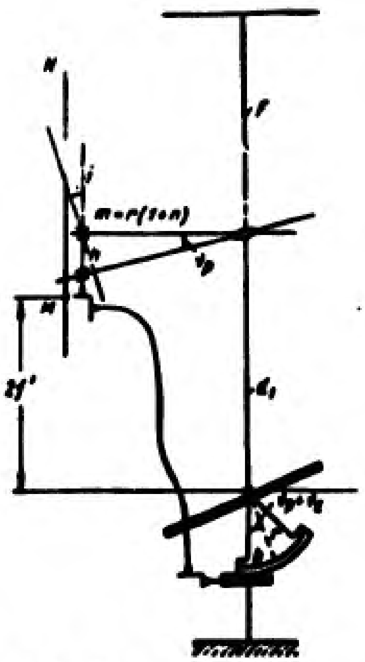


Fig. 62.

$$h = r\,\mathrm{tg}(\nu_p + \nu_E) = kr\,\mathrm{tg}\,\nu_p,$$

where r is the distance of the pusher of the lever of the screen from the axis of its rotation.

Inasmuch as

$$F = f'\left(1 + \frac{1}{n}\right) \text{ and } d_1 = f'(1 + n),$$

then

$$k = \frac{F + d_1}{F} = 1 + n.$$

Therefore it is possible to write the equation in the form

$$r\,\mathrm{tg}(\nu_p + \nu_E) = (1 + n)\,r\,\mathrm{tg}\,\nu_p.$$

From the equation it follows that distance m of the pusher of the lever of the objective from the axis of its rotation is a variable magnitude and depends on the increase of n

$$m = r(1 + n).$$

Since the distance from the objective to the screen is expressed by the formula $d_1 = f'(1 + n)$, the change of length m of the lever is connected with a change in d_1 by the dependence

$$m = \frac{r}{f'} d_1.$$

Therefore the change of length m of the lever can be carried out with the help of a ruler set at an angle i to the structural axis of the instrument and connected with mobile part m. Angle i of tilt of the ruler is determined by the equation

$$\operatorname{tg} i = \frac{f}{f'}. \tag{60}$$

At a coefficient of increase n = 1, the length of lever m = 2r, and the point of tangency by pusher of the ruler is at a distance of 2f' from the screen.

The eccentricity of the negative is shown in Fig. 59. The formula determining the magnitude of eccentricity has the form

$$o_1 o = \frac{f_k}{\operatorname{tg} \alpha_0} = \frac{f}{\operatorname{tg}(\nu_p + \nu_g)}. \tag{61}$$

The base of the photorectifier (Fig. 63) is a metal table to which are fastened the frame with the screen and the vertical bed. Along the bed is located the ruler of oblique inverter, slanted at angle i. The screen is located on Cardan axes; with the help of hand controls it is possible to tilt it in two directions, which leads to the necessity of applying two oblique inverters. One of them revolves holder of the oblique on angle ν_{p_x} and the second [rotates] the semicardan joint with the objective fastened in the holder on angle ν_{p_y}. The semicardan joint with the objective fastened in the holder.

Fig. 63.

The scale inverter is located on the side of the bed and is set in motion by the foot control through telescope shaft.

Before photocopying it is possible to set the diaphragm and to close the objective "Beam" with a red light filter.

On the carriage of the objective is located an illuminator with a high-power lamp located in the focus of an ellipsoid. Therefore the position of the lamp with respect to the objective is fixed.

In the cassette of the carriage it is possible to place aerial photos with the size 300 × 300 mm with continuous film.

With the screw it is possible to introduce eccentricity Δy of the cassette with the aerial photo, and by shifting the actual aerial photo eccentricity Δx (readings of magnitudes of introduced eccentricity are lacking).

The instruments are used for rectifying vertical aerial photographs during preparation of photomaps utilized during the creation of maps of scales 1:10,000-1:25,000. The range of enlargements of projections is within limits of 0.6-2.5.

Table 9 gives data characterizing the photorectifiers FTB and FTM used in the USSR.

Table 9

Data	Type photorectifier	
	FTB	FTM
Size of aerial-photos, cm	18×18...30×30	18×18...30×30
Dimension of the screen, cm	100×100	60×60
Focal length of the objective, mm	180	180
Enlargement	0,7...5,0	0,7...2,5
Tilt of the screen, degrees	45	14,5
Eccentricity, mm		
Δx	± 30	± 50
Δy	±30—70	± 50
Turn of the aerial-photo	360°	0°
Power of light source, watts	150	150
Height of the instrument, m	2,8	2,4

Photorectifier SEG-V (Fig. 64), make in the Federal Republic of Germany, is characterized by application of an illuminator of a new form, consisting of two plates with concentric deepenings (Fresnel lenses) and with a greater range of magnification (0.5-6.4^{x}).

Fig. 64.

In the instrument, designed on the system of rectification of the small photorectifier, mechanisms for tilt of the screen are located lower than the latter. This creates convenience during use of the instrument, since the screen is accessible from three sides.

In the latest models of the SEG-V eccentricities of the negative, depending on angles of tilt of aerial photos in flight, are introduced in the instrument automatically by an electrical computer.

Furthermore, there is an attachment for levelling the total density of the image near the density of the chosen aerial-photo, which ensures the creation of a high quality photoplans. In one of the photorectifiers (Hilger, England) an electron-beam tube is used for the same purpose.

Working adjustments of the instruments. For obtaining rectified aerial-photos of corresponding accuracy and good quality it is necessary to fulfill the condition of geometric and optical linkage of photorectifier parts. Besides this the values of zero readings of scales of the instrument are also determined. After adjustments the sharpness of the image at all magnifications is checked.

In vertical photorectifiers start the adjustment by putting the screen in horizontal position with the help of a level with division values down to 45", fixed in the direction parallel to the axis of rotation of the screen. Using the lifting screws of the photorectifier, bring the bubble to the middle of the level. Then with the help of a level check the horizontal position of the axes of rotation of the cassette and the plane of mounting of the objective.

For adjustment of the scale inverter place the measuring grid in the cassette, project it on the screen and bring the image to the scale 1:1, combining the image with the control grid located on the screen. The distance between the negative and screen should equal to 4f' and the indices of scale c and d should equal reading of 2f' and 4f'. After that sharpness of the image should be retained during any changes of positions of the carriages in height.

The oblique inverter is adjusted by means of correction of the screen and by mounting the objective and negative in a horizontal position and the elements of the inverter in the initial position.

After this the sharpness of the image should be retained at any tilts of the screen and negative cassette.

The illumination system should well concentrate the light bundle on the entrance pupil of the objective and ensure uniform illumination of the field of image.

§ 18. System of Rectification on a Horizontal Screen

The basic axis of the photorectifier coincides with the tangent to arc S_1S_2. On the given diagram (Fig. 65) it is shown that the axis of rotation of the chamber is at point R. Segment oR is the adjusting magnitude f_e in the instrument. Tilt of the aerial-photo on angle $\nu_p + \nu_E$ around point R automatically introduces the necessary eccentricity, i.e., geometric conditions of rectification are executed

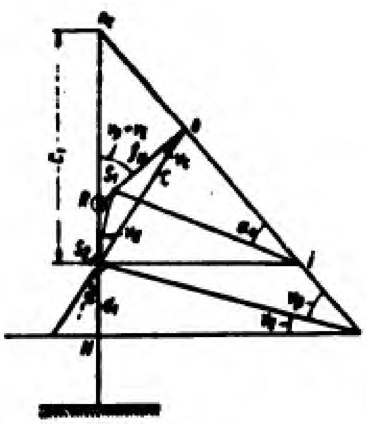


Fig. 65.

very simply. With this the negative should revolve in its plane on angles κ; scale linkage should be carried out along the line Nn_1 by the formula

$$\frac{1}{c_1}+\frac{1}{d_1}=\frac{1}{\frac{f}{\cos(\nu_d-\varepsilon)}};$$

the negative and objective should be inclined correspondingly on angles $\nu_p + \nu_E$ and $(\nu_d-\varepsilon)$.

Rectification is possible by means of turning the negative around the line passing through point n_1 at angle $\nu_p \nu E$ and introduction of eccentricity

$$n_1 o=\frac{c_1}{\sin(\nu_p+\nu_E)}-\frac{f_k}{\operatorname{tg}\alpha_0}$$

This eccentricity must be expanded along axes x and y.

We will find the value of adjusting elements oR and S_2R:

$$\frac{oR}{\sin\nu_d}=\frac{c}{\sin(\nu_p+\nu_E)}=\frac{oi}{\sin(90^\circ-\nu_d)},$$

where

$$oR=f_e \quad \text{and} \quad oi=f_k \operatorname{ctg}\alpha_0.$$

Consequently,

$$f_e=f_k \operatorname{tg}\nu_d \operatorname{ctg}\alpha_0. \tag{62}$$

For composition of a nomograph of values f_e it is necessary to know ν_d. From triangle S_2oi

$$(oi)^2=c^2+(S_2i)^2-2(S_2i)c\cos(90^\circ-\nu_d),$$

from which

$$\cos(90^\circ-\nu_d)=\frac{c^2+\left(\frac{f_k}{\sin\alpha_0}\right)^2-f_k^2\operatorname{ctg}^2\alpha_0}{2c\frac{f_k}{\sin\alpha_0}}$$

and

$$\sin\nu_d=\frac{(c^2+f_k^2)\sin\alpha_0}{2cf_k}. \tag{63}$$

Analogously

$$\sin\nu_e=\frac{(c^2-f_k^2)\operatorname{tg}\alpha_0}{2cf_k}. \tag{64}$$

For determining the value of the setting S_2R of the axis of rotation of the chamber we write

$$\frac{S_2R}{\sin v_e}=\frac{f_e}{\sin v_d},$$

from which

$$S_2R=f_e\frac{\sin v_e}{\sin v_d}. \tag{65}$$

The position of axis R of the rotation of the chamber determines the form of rectification (first or second kind), since with the approach of point S_2 to S_1 the value of f_e will approach the value of f_k.

§ 19. System of a Rectifier with a Vertical Axis Passing Through the Nadir Point

Considering Fig. 66, we establish the connection

$$S_1i=S_2i=\frac{f_k}{\sin\alpha_1}=\frac{F}{\sin\alpha_2},$$

from which for small angles

$$\alpha_2=\frac{F}{f_k}\alpha_1. \tag{66}$$

Eccentricity of the aerial-photo oo_1 will be determined by a simple formula if we use the point of zero distortions c; with this

$$oo_1=co_1-co=F\frac{\alpha_2}{2}-f_k\frac{\alpha_1}{2}$$

From this equation, after substitution of value α_1 and multiplication of the right side by F/F we find the value of eccentricity of the aerial-photo:

$$oo_1=0{,}5\frac{F^2-f_k^2}{F}\alpha_2. \tag{67}$$

Tilt of aerial-photo α_n will be determined from the expression

$$\operatorname{tg}\alpha_n=\frac{no+oo_1}{F}.$$

Putting the values oo_1, $no = f_k\alpha_1$ and replacing α_2, according to the formula (66) we obtain

$$\alpha_n=\frac{F\frac{\alpha_2}{2}+f_k\frac{\alpha_1}{2}}{F}.$$

After replacing α_1 by $\frac{\alpha_2 f_k}{F}$ and multiplying the first member of the numerator of the right side by F/F we obtain

$$\alpha_n=0{,}5\,\alpha_2\frac{F^2+f_k^2}{F^2}. \tag{68}$$

The angle of tilt of the screen is equal to α_e. Inasmuch as according to conditions of rectification $NK||S_2i$, then

$$\alpha_e = 90° - \alpha_n - (90° - \alpha_2) = \alpha_2 - \alpha_n;$$

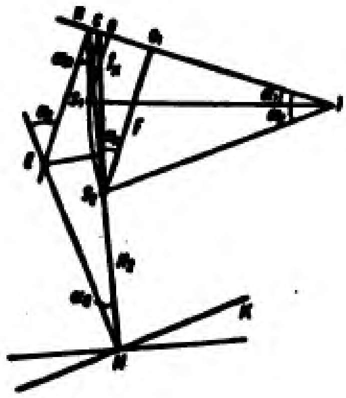


Fig. 66.

replacing α_2 with its expression from equation (68), we obtain

$$\alpha_e = \alpha_n \frac{F^2 - f_k^2}{F^2 + f_k^2}. \qquad (69)$$

For instance, at $f_k = 100$ mm and $F = 250$ mm

$$\alpha_e = 0{,}72\,\alpha_n.$$

Automatic linkage of planes of the aerial-photo and the screen will follow in the case when we use levers nE and NE; for the given example they will have a ratio of arms equal to 0.72.

With this method of projection a large angle of tilt of an aerial-photo, obtained during rectification according to the principle of the second kind, is replaced by smaller angles of tilt of the aerial-photo and the screen.

During transition to another scale of projection in conditions of optical linkage the ratio F/f_k will change and consequently angle α_e will be changed. Therefore it is expedient to apply the given system of rectification for instruments in which sharpness of the image is kept at large changes of H_2 without disturbing the magnitude of focal length of the projecting chamber – for instance, in drawing projectors.

In this case redrawing the image according to zones can be done for relief considerable in height without any shift of the nadir point on the screen.

§ 20. Application of Photorectifiers

1. The Process of Rectification

In production is applied the method of rectification according to control (rectification) points, consisting of the fact that with the working movements are sought the combinations of projections of rectified points with their position on the plane table, placed on the screen. The plane table is a copy from the basic plane table for one or several aerial-photos. Since control points are placed on the plane table in orthogonal projection, in the presence of relief the oblique image of control points cannot be combined with their orthogonal position. As was shown, errors in relief are expressed by the formula

$$\Delta = \frac{rh}{H-h}.\qquad (70)$$

where r is the distance of a point from the center of the aerial-photo on a plane table, expressed in scale of the plan; h is the elevation of a point above the plane of rectification.

Therefore corrections for relief, calculated by formula (70), are introduced in the position of points on the plane table.

If there are four points with heights A_1, A_2, A_3, A_4, then it is convenient to take the plane of rectification at the average height

$$A_0 = \frac{A_1 + A_2 + A_3 + A_4}{4}, \qquad (71)$$

since in this case the corrections for relief will be the smallest. Sometimes the plane of rectification is taken as common for a series of aerial-photos.

Corrections are se on plane tables according to directions drawn from the nadir point or the principal point of the aerial-photo. If neither of these points is on the plane table, then take the tracing paper and superimpose on it from the aerial-photo not less than four control points and the principal point, and then from the latter trace direction to all the remaining points. The obtained bundle of directions is the fitted in the corresponding four points of the plane table by means of graphical solution of the reverse bearing, since the angles between central directions are little distorted. After this superimposed the principal point from the tracing paper onto the plane table.

The preparation of negatives for rectification consists of pinning with a needle those points corresponding to control points of the plane table. For this purpose place the negatives on a mounting table with lower illumination. The diameter of points pinned on should not exceed 0.1-0.2 mm. Then determine the coefficient of shrinkage of the photographic paper, for which a contact print is developed from the measuring grid.

Suppose that a segment of 100 mm grid, after drying, becomes equal to m mm; then the coefficient of shrinkage of photographic paper will be k = m/100. Its calculation is carried out by lowering the screen before photocopying or by removal of the base placed under the plane table during rectification. For determining the height of shift of the screen (or thickness of the base) it is possible to write the proportion

$$k=\frac{d}{d+e},$$

where d is the distance from the objective of the rectifier to the screen at the time of rectification and d + e is the distance from the objective to the photographic paper due to lowering the screen by magnitude e at the time of photocopying.

From the equation we find

$$e=d\left(\frac{1}{k}-1\right). \tag{72}$$

Rectification takes place by means of a consecutive change of scale of the image (along two points found on a line passing through the center) and tilt of the screen for combination of luminescent points of the aerial-photo with the third and fourth points of the plane table. In case of inconsistency of points the cassette (FTB) is revolved on angle κ and the process is repeated in the same sequence until exact combination occurs (0.3-0.4 mm). Moreover in the photorectifier FTB and FTM eccentricities of the aerial-photo are used movements dependent on angles of slope of the cassette.

After that the scale of image is increased in accordance with the distortion ratio of the photographic paper; the objective is closed by a red light filter; and put photographic paper on the screen. By removing the light filter the image is exposed, image, after which the print is developed and, if its quality is satisfactory, the following aerial-photo is rectified.

The rectified image is free from errors caused by elevations of stations and angles of tilt of aerial-photos, but its points are displaced due to the influence of relief. Therefore, during fluctuations of relief which cause displacement of points on the map by more than 0.4 mm, conduct photocopying of the rectified image by zones with an appropriate coefficient of magnification. It is established by means of combination of projections of rectified points of aerial-photos with corresponding points of the plane table, corrected for relief relative to the average plane of the given zone.

2. Assembling of Photomaps

For creation of a photomap of a site place rectified aerial-photos on the plane table in such a manner so that their control points coincide with those of the plane table; for convenience of combination pierce the point with a punch with a strike diameter of 1.5 mm.

In the middle of the overlap of neighboring aerial-photos make a cut with a lencet; the line of the cut should intersect the least [possible] number of contours. To avoid cutting the plane table place cardboard under the aerial. Then superfluous parts of aerial-photos, and give the remaining parts on the plane table for connecting them to the photomap.

The noncoincidence of contours on the line of the cut should not exceed 0.7 mm. Aerial-photos are glued with amylacetate glue, which does not cause distortion; glue can be removed from the surface of the photomap with acetone.

During rectification according to zones cut the corresponding part from every aerial-photo and glue it on the photomap.

After preparation put the photomap under a press or press it with weights for a twenty-four hour period. Then trace the frame on the photomap, mark the ends of the kilometer grid, mark the control points, and note inhabited localities and so on.

A photomap is used:

1) in the national economy (rural [4] and forest economy, during construction, for hydrotechnical purposes, etc.);

2) for creation of topographic maps by the combined method, in which the contours are placed on the photomap with the help of a plane table and telescopic alidade in field conditions;

3) for creation of topographic maps by means of transfering to the photoplan contours plotted on aerial-photos in laboratory conditions, with the help of stereoapparatuses.

3. Special Cases of Rectification

With the theory of rectification is disturbed compression, estension, and misalignment of the image are possible. Let us place an object in the form of a square on the screen of the photorectifier in such a way that two sides of it are parallel to the axis of rotation of the screen. Let us then set up the screen and, consequently, the cassette in a very slanted (up to 20 degrees) position. After this, by illuminating the square we obtain on the cassette an image in the form of a trapezoid. If one were to prepare a negative, place it in the cassette of the photorectifier with those same set ups of screen and cassette, then on screen will again be depicted a square.

In case of introduction of lateral eccentricity of a negative with a slanted screen, i.e., its displacement parallel to the axis of rotation of the screen, we will obtain small displacements of the image in the raised (upper) part of the screen and large displacements in the lower part of it; the dimensions of the sides of the image are not distorted.

Thus, after double rectification the initial square can be obtained in the form of a rhombus inclined to the left or to the right, the more, the greater the eccentricity of the negative.

For compression or extension of the image of the square it is necessary to introduce transverse (to the axis of rotation of the screen) eccentricity and additional tilt. As a result we obtain:

a) small displacements in the direction of slope of the screen of the upper part, and large displacements of the lower part of the image in that same direction; since in this changes of various parts of the image are unequal, we will observe compression of the figure with simultaneous disturbance of the parallelism of its lateral faces;

b) parallelism of sides during additional slope of screen;

c) small change of scale of the image.

Thus, by affecting (after introduction of eccentricity) of slope of the screen with the steering wheel and changing the scale, it is possible to turn the initial square into a compressed or elongated rectangle. By combining both movements of eccentricity on the basis of the initial square, it is possible to obtain the figure of compressed and elongated rhombs.

The given method is used in the USSR during creation of varieties of type, and also during rectification of aerial-photos of evenly tilted sections of the terrain. In last case the result is attained during a single rectification.

4. Courses of Development of Photorectifiers

The use of radio altimeters and statoscopes in the process of aerial-photography presents the possibility of sufficiently exact fixation of the height of an air station above the Earth's surface. Therefore during rectification of a photograph it is completely possible to establish beforehand the scale of the projected image on the phototransformer.

The presence of stabilizing devices in the aircraft and in the camera mount of the aerial camera at present presents the possibility of obtaining aerial-photos with small angles of tilt.

Formulas for tying coordinates of points of an aerial-photo and the terrain show that at angles of tilt of aerial-photos of 10-15', it is possible to use them as horizontal photos for the creation of the contour part of a map. Therefore the photorectifier will serve only for changing the scale of the image; the only working movement will be the linked transposition of the objective and negative along the structural axis. Oblique linkage, eccentricity of negatives, and their rotation in a plane appears unnecessary.

Furthermore, there arises the necessity of creating larger magnifications of projection that is accepted (for instance, instead of 1.5, up to 6-7).

One of the instruments with such magnification, without oblique linkage and without eccentricity, was built by the wild plants in 1960. The development of instruments of optical conversion of mountain aerial-photos into orthogonal is noted. These instruments are based on the principle of slot projection.

CHAPTER V

DIFFERENTIATED METHODS OF CREATION OF TOPOGRAPHIC MAPS

§ 21. General Principles

Differentiated methods of creating topographic maps consist of processes executed on different instruments. Included in these processes are: determination of heights of points of terrain by aerial-photos, independent of their horizontal position; drawing of relief on aerial-photos; map-making with the use of the photomap; and others.

With the help of comparatively simple instruments (stereometers) it is possible to produce a drawing of relief directly on the aerial-photos. This is sufficiently effective and in many cases satisfies accuracy requirements. With the help of photorectifiers or projectors the contour parts of the map are composed.

Thus, the process of creation of a topographic map is divided into a series of stages or operations. This method of map-making is called in the USSR the method of differentiated processes. Differentiated methods were widely developed and at present also include processes of determination of elements of relative and external orientation of aerial-photos, processes of thickening of the horizontal and high-altitude control net, and others.

Investigations of causes of the appearance of a difference of longitudinal parallaxes, their distortions, and structure of transverse parallaxes presented the possibility of analytic solution of the problem of orientation of the stereopair.

At present topographic maps of almost all scales are created by differentiated methods. The effectiveness of the shown methods is a function of the quality of aerial photography and also increases on the requirement for topographic-geodesic substantiation decreases.

Determination in flight of heights of photographing and elevation differences between ends of bases and the fixation of tilts or stabilization of the aerial camera create especially favorable conditions for more precise definition of results of treatment of aerial-photos.

Lessening of topographic-geodesic substantiation obtainable on the basis of application of the methods of extension on the stereometer, an undistorted model, and thickening of the control net on universal instruments all create conditions for increasing the effectiveness of the method.

§ 22. Axes of Coordinates of Aerial-Photos of a Stereopair

The position of points of the aerial-photos is measured in a plane rectangular system of coordinates.

As the direction of axis x of aerial-photos it is convenient to take the trace of the vertical basic plane, passing through nadir points n_1 and n_2 (Fig. 67), and as the direction of axis y, lines perpendicular to the x-axis and passing through principal points o_1 and o_2 of the aerial-photos. Selection of these axes of coordinates also predetermines their origin, being in points e_1 and e_2, called the exnadirs, which are characterized by the fact that right angles $n_1e_1o_1$ and $n_2e_2o_2$ on the aerial-photos correspond to right angles on the terrain. The indicated characteristic follows from the fact that angles measured on the aerial-photo at points o_1 and n_1 are connected with corresponding angles measured on points O_1 and N_1 of the terrain by the dependences:

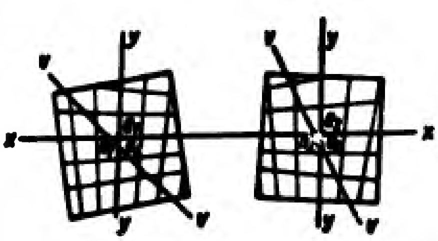


Fig. 67.

$$\operatorname{tg} n_1 = \operatorname{tg} N_1 \cos \alpha_0,$$

$$\operatorname{tg} o_1 = \frac{\operatorname{tg} O_1}{\cos \alpha_0}.$$

On the basis of these dependences it is possible to write

$$\operatorname{tg} n_1 \operatorname{tg} o_1 = \operatorname{tg} N_1 \operatorname{tg} O_1, \qquad (73)$$

from which it follows that angles N_1 and O_1 are angles of a right triangle, i.e., $\Delta E_1 = 90°$.

Thus, in spite of the presence of angles of tilt of aerial-photos it is possible to have the principal coordinate axes both on the terrain, and on the aerial-photo as perpendiculars. The locus of points at which angle e is not distorted will be a circle with the center located on the principal perpendicular and half way between the nadir point and the principal point.

The position of points of the exnadir and nadir with respect to the center of projection is determined by the formulas (Fig. 68)

$$\left.\begin{aligned} Se &= \frac{f_k}{\cos\omega} \\ aS &= \frac{Se}{\cos\alpha} = \frac{f_k}{\cos\omega\cos\alpha} \end{aligned}\right\}. \qquad (74)$$

since angle neS is a right angle.

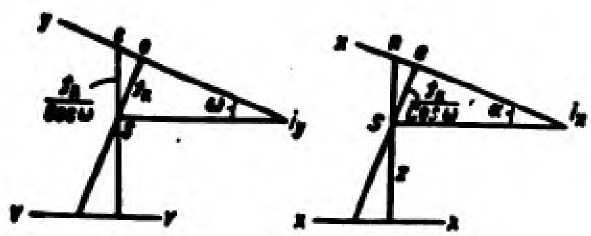


Fig. 68.

Correspondingly the distance from the point of the exnadir to the line of the horizon along axes of coordinates y and x will be

$$el_y = eo + ol_y = f_k(\operatorname{tg}\omega + \operatorname{ctg}\omega), \qquad (75)$$

$$el_x = \frac{f_k}{\cos\omega}\operatorname{ctg}\alpha. \qquad (76)$$

We will derive the formulas of connection of coordinates of points of the terrain and the aerial-photo in the selected system of coordinates.

In agreement with Fig. 69, which shows the section of the stereopiar along the vertical basal plane, for the left aerial-photo $e_1m_1 = x_1$ and segment $S_1e_1 = \frac{f_k}{\cos\omega_1}$.

Tracing through determined point M the line MD parallel to the axis x of the left aerial-photo, we obtain from the similarity of triangles $S_1e_1m_1$ and S_1DM:

$$\frac{x_1}{\frac{f_k}{\cos\omega_1}} = \frac{DM}{D_1S_1} = \frac{X_1\cos\alpha_1}{ES_1 - DE}$$

or

$$\frac{x_1}{\frac{f_k}{\cos\omega_1}} = \frac{X_1\cos\alpha_1}{\frac{H_1}{\cos\alpha_1} - X_1\sin\alpha_1}. \tag{77}$$

This equation permits establishing a mathematical linking of points on the terrain and the aerial-photo located in the vertical basal plane.

Such an equation of linkage can be obtained also for points located in a plane perpendicular to it and passing through axis y, i.e.,

$$\frac{y_1}{\frac{f_k}{\cos\omega_1}} = \frac{Y_1}{H_1 + Y_1\sin\omega_1}. \tag{78}$$

On the basis of formulas (77) and (78), which determine the position of points of descent of lines parallel to the axes of coordinates of the aerial-photo, and the formulas of connection of coordinates of points of the terrain and the aerial-photo, it is possible to construct an image of a rectangular coordinate grid on an aerial-photo. Here the origin of the coordinate system is at the point of the exnadir. When working with stereometers, for practical purposes take the principal point as the origin of coordinates, which somewhat lowers the usual accuracy of determination of heights of points.

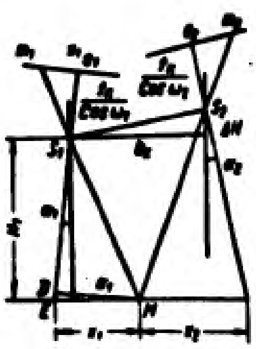


Fig. 69.

§ 23. The Connection of Differences of Longitudinal Parallaxes Δp with Differences of Heights h of Points of Terrain. Base and Height of Photographing

Determination of heights of points of terrain and the image of relief are based on dependences between difference Δp of longitudinal parallaxes of two points on aerial-photos and the difference Δh of heights of these points on the terrain.

Fig. 70 gives the diagram of intersection of points lying in a plane passing through axis X and centers of projection S_1 and S_2, from which it is clear that according to a strictly vertical survey

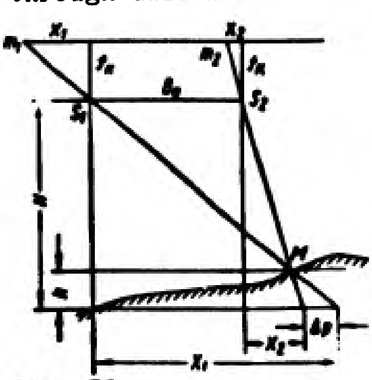


Fig. 70

$$\frac{\Delta P}{h} = \frac{B_0}{H - h}.$$

In the scale of the aerial-photo

$$\Delta p = \frac{b_0 h}{H - h}. \tag{79}$$

In formula (79) the values of Δp and b_0 are expressed in millimeters and those of h and H - h in meters.

It is possible to determine the valve of the difference of longitudinal parallaxes of a point by the equation

$$\Delta P = X_1 - X_2 - B_0 \tag{80}$$

or in the scale of the aerial-photo

$$\Delta p = x_1 - x_2 - b_0. \tag{81}$$

Formulas (79) and (81) are basic for determining heights of points.

Let us find the value of Δp with tilt of aerial-photos of the stereopair. In this case coordinates x_1, x_2 will be distorted and, consequently, the magnitude Δp will also be changed.

Fig. 71 shows the change of coordinate cm of the aerial-photo due to tilt of the photo on the small angle α_1; it is equal to Δ_{x_1}. Let us determine its content for which we will write

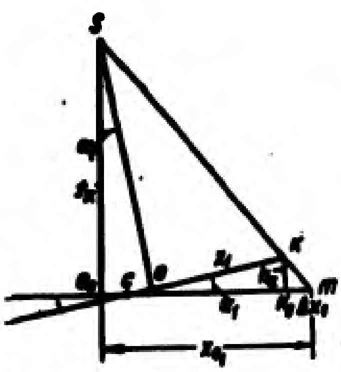


Fig. 71.

$$\frac{\Delta x_1}{h_a} = \frac{ck + \Delta x_1}{f_k}.$$

where

$$h_a = ck\,\alpha_1,$$

and

$$ck = x + \frac{f\alpha_1}{2} = x_c.$$

Consequently,

$$\Delta x_1 = \frac{x_c(x_c + \Delta x_1)}{f_k}\alpha_1,$$

whence

$$\Delta x_1 = \frac{x_c^2}{f_k - x_c\alpha_1}\alpha_1, \tag{82}$$

and for negative x_c

$$\Delta x_1' = \frac{x_c^2}{f_k + x_c\alpha_1}\alpha_1. \tag{83}$$

At $\alpha \leq 3°$ the values of corrections Δ_{x_1} are obtained by these formulas with an accuracy of 0.01 mm.

Such distortion of a section will be apparent also on the right aerial-photo, but in addition to it there is added the distortion due to the excess of point S_2 above point S_1.

In the absence of an excess between points S_1 and S_2 the relation of segments on the right aerial-photo and the terrain has the form

$$\frac{x_2'}{X_2} = \frac{f_k}{H}.$$

and in the presence of excess

$$\frac{x_2}{X_2} = \frac{f_k}{H + \Delta H}.$$

Solving the equations jointly, we obtain the value of the correction in the form

$$\Delta x_2 = x_2' - x_2 = \frac{X_2 \cdot \Delta H_2}{H}, \tag{84}$$

or in the scale of the aerial-photo

$$\Delta x_2 = \frac{x_2}{f_k} \delta H_2. \tag{85}$$

For a tilted aerial-photo instead of x_2, i.e., the distance from the center of an aerial-photo to the image of a point, we must take the distance x_0, where

$$x_0 = f_k \frac{\alpha_2}{2} + \frac{x^2_{c_2} \alpha_2}{f_k - x_{c_2} \alpha_2}. \tag{86}$$

The true difference of longitudinal parallaxes for points on axis x, expressed in the scale of the aerial-photo, has the form

$$\Delta p_0 = \Delta p_i + \delta p_x$$

where Δp_i is the measured difference of parallaxes; δp_x is the sum of corrections for tilt of aerial-photos along axis x and the excess between centers of projections. The sum of corrections for the difference of longitudinal parallaxes relative to the principal point of the right aerial-photo with elimination of constant terms is equal to

$$\delta p_x = \frac{x^2_{c_1}}{f_k - x_{c_1} \alpha_1} \alpha_1 - \frac{x^2_{c_2}}{f_k - x_{c_2} \alpha_2} \alpha_2 - \frac{x_0 \delta H}{f_k}. \tag{87}$$

Fig. 72 shows the distortion of coordinates x as a function of angles ω.

The value of Δx_3 is determined by the relationship

$$\frac{\Delta x_3}{y} = \frac{x}{el_y} = \frac{x}{f_k(\mathrm{ctg}\,\omega + \mathrm{tg}\,\omega)}.$$

Dropping the terrain $\mathrm{tg}\,\omega$ in the denominator because of its smallness, we obtain

$$\Delta x_3 = \frac{xy}{f_k} \omega. \tag{88}$$

The expression of the correction for the difference of parallaxes will have the form

$$\delta p_y = \frac{x_1 y_1}{f_k} \omega_1 - \frac{x_2 y_2}{f_k} \omega_2. \tag{89}$$

Thus, the general formula of correction for the difference of longitudinal parallaxes will be

$$\delta_p = \frac{x^2_{c_1}}{f_k - x_{c_1} \alpha_1} \alpha_1 - \frac{x^2_{c_2}}{f_k - x_{c_2} \alpha_2} \alpha_2 - \frac{x_0 \delta H}{f_k} + \frac{x_1 y_1}{f_k} \omega_1 - \frac{x_2 y_2}{f_k} \omega_2. \tag{90}$$

This formula, we will see below with the same or other simplifications is applied in the theory of stereometers. Furthermore, it can serve for investigation of accuracy of work of calibrating instruments by means of indicators.

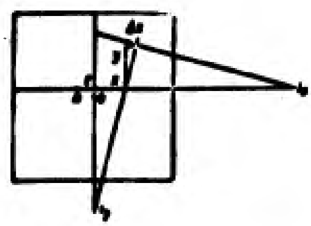


Fig. 72.

Elements of orientation κ_1 and κ_2 are not given in formula (90), inasmuch as they are introduced immediately in the phase of the first stage of orientation.

For calculation of the correction for the difference of longitudinal parallaxes by this formula, it is necessary preliminarily to determine elements of relative and external orientation of aerial-photos. Formulas of connection of coordinates of a photograph and terrain also show that solution of a problem of intersection of points is possible only in the case when angles of tilt α and ω of the principal axis are known. Physical methods of determining these angles at present do not give sufficiently accurate results, and therefore it is necessary to use photogrammetric methods for solution of the problem.

If coordinates of three or four control points are determined on the terrain, by means of one of numerous methods of double-point interpolation in space it is possible to find both the coordinates of the center of projection and angles of tilt α and ω. However, these solutions are very laborious.

Another method consists of preliminarily determining elements of relative orientation of aerial-photos and then elements of external orientation. By calculating the corrected differences of longitudinal parallaxes of points, one can determine the elevations between them by formula (79). For this it is necessary also to know the value of the base and height of photographing.

<u>Determination of the base of photographing</u>. Determination of the base of photographing is possible by means of measurements by aerial-photos. On horizontals of aerial-photos the distance between principal points is equal to the base in the scale of the aerial-photo.

In the presence of angles of tilt of aerial-photos the distance between their centers cannot be taken as the value of the base. Considering Fig. 73, on which aerial-photos are given angles of tilt identical in sign and magnitude H is taken as equal to f_k, it is possible to formulate two expressions of the true base b_0:

$$b_0=(f_k-h_0)\,\mathrm{tg}(A_1-\alpha_1)-(f_k+\delta H-h_0)\,\mathrm{tg}\,\alpha_2$$

and

$$b_0=(f_k+\delta H)\,\mathrm{tg}(A_2-\alpha_2)+f_k\,\mathrm{tg}\,\alpha_1.$$

Considering that tg A_1 and tg A_2 correspondingly are equal to $\frac{b_1}{f_k}$ and $\frac{b_2}{f_k}$ and that $\alpha_1 - \alpha_2 = \Delta\alpha_x$, after a number of conversions and simplifications we obtain

$$b_0=b_1+\frac{b_1^2}{f_k}\alpha_1+f_k\Delta\alpha_x-h_0\Delta\alpha_x-\frac{h_0 b_1}{f_k}$$

and

$$b_0=b_2-\frac{b_2^2}{f_k}\alpha_2+f_k\Delta\alpha_x+\frac{\delta H}{f_k}b_2.$$

Designating

$$\frac{b_1+b_2}{2}=b' \qquad (91)$$

and taking

$$\frac{b_1^2}{f_k}\alpha_1-\frac{b_2^2}{f_k}\alpha_2=\frac{b'^2}{f_k}(\alpha_1-\alpha_2),$$

$$-\frac{h_0 b_1}{f_k}+\frac{\delta H}{f_k}b_2=-\frac{b'}{f_k}(h_0-\delta H),$$

we obtain the value of the base as the average of two determinations

$$b_0=b'+\left(f_k+\frac{b'^2}{2f_k}-\frac{h_0}{2}\right)\Delta\alpha_x-\frac{h_0-\delta H}{2f_k}b'. \qquad (92)$$

The difference between values of the base on every aerial-photo is small even in the presence of angles of tilt; with symmetric tilts of aerial-photos the bases are identical, but essentially differ from b_0. In Table 10 there are given values of b_1 and b_2 calculated exact formulas with $b_0 = 70$ mm.

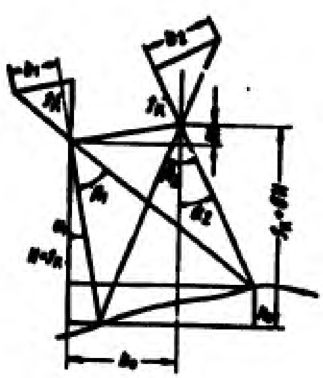


Fig. 73.

Error in determination of base b_0 by formula (92) is equal approximately 0.5-1 mm; without taking into account angle ($\alpha_1 - \alpha_2$) this error could attain 13.8 mm (with $\alpha_1 - \alpha_2 = 4°$). Therefore bases should be determined by formula (92) [11].

Determining the value of the height of photographing. The height of photographing (H) above the earth's surface is determined from geodesic control points, points of phototriangulation, or directly from results of treatment of statoscope and radio altimeter readings. Depending on the relief of the terrain and the scale of the composed map the accuracy of determination of heights of photographing should be within limits of 2-10 m.

Table 10

Angle of tilt of the left aerial-photo α°_1	Angle of tilt of the right aerial-photo α°_2	Base on the left aerial-photo b_1, мм	Base on the right aerial-photo b_2, мм	Difference of bases $b_1 - b_2$, мм	Notes
+2	+2	69.7	70.7	−1.0	$b_0 = 70$ мм
0	−2	63.0	62.2	0.8	$f_k = 200$ мм
0	+2	77.0	78.1	−1.1	$\Delta H = 0$
+2	−2	56.2	56.2	0.0	$h_0 = 0$

§ 24. Dependence of Transverse Parallaxes on Elements of Relative Orientation of Aerial-Photos

Magnitudes determining the position of one aerial-photo of the stereopair relative to another are called elements of relative orientation of aerial-photos. Usually the basic system of elements of relative orientation is accepted, in which the relative position of aerial-photos is determined by angles τ_1 and τ_2 between the directions of principal rays and the perpendicular to the base, and also by angle ε between the principal basic planes containing principal rays. (Figs. 74 and 75). Elements of relative orientation in a given system are also angles κ of aerial-photos between the trace of the principal basic plane and the x-axis. Determination of elements of relative orientation is based on connections of angles with values of transverse parallaxes of points of aerial-photos. The transverse parallax of a point is the difference of its ordinates on the left and right aerial-photos of the stereopair.

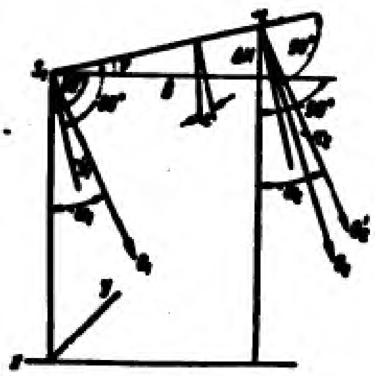


Fig. 74.

For deriving formulas of connection of elements of relative orientation of aerial-photos and transverse parallaxes of points we take the left aerial photos as conditionally horizontal. On the basis of formulas (88) and (86) for differences of longitudinal parallaxes the expressions of distortions of ordinates of points 3 and 5 of the standard system will take the form:

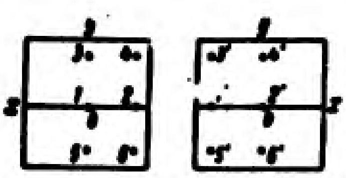


Fig. 75.

$$\Delta y_3 = 0;$$
$$\Delta y_5' = \frac{b y_5'}{f_k}\tau_2 + \frac{y_5'^2}{f_k - y_5'\tau_2}\varepsilon_2,$$
$$\Delta y_1 = 0,$$

$$\Delta y_6' = -\frac{b y_6'}{f_k}\tau_2 + \frac{y_6'^2}{f_k - y_6'\varepsilon_2}\varepsilon_2$$

where

$$\varepsilon_2 = \omega_1 - \omega_2,$$

y – coordinates from the point of zero distortions.

The difference of these dependences gives the values of transverse parallaxes calculated by the formulas

$$q_5 = \Delta y_5 - \Delta y_5' = -\frac{b y_5'}{f_k}\tau_2 - \frac{y_5'^2}{f_k - y_5'\varepsilon_2}\varepsilon_2,$$

$$q_6 = \Delta y_6 - \Delta y_6' = \frac{b y_6'}{f_k}\tau_2 - \frac{y_6'^2}{f_k - y_6'\varepsilon_2}\varepsilon_2,$$

with

$$y_5' = y_6'; \quad y = \frac{y_5' + y_6'}{2}$$

$$q_5 - q_6 = -\frac{2by}{f_k}\tau_2, \tag{93}$$

$$q_5 + q_6 = -\frac{2y^2\varepsilon_2}{f_k - y\varepsilon}. \tag{94}$$

from which

$$\tau_2 = -\frac{f_k(q_5 - q_6)}{2by}, \tag{95}$$

$$\varepsilon_2 = -\frac{f_k(q_5 + q_6)}{2y^2 - y(q_5 + q_6)}. \tag{96}$$

Taking the right aerial-photo as initial, analogously we obtain

$$\tau_1 = -\frac{f_k(q_3 - q_4)}{2by} \tag{97}$$

and

$$\varepsilon_1 = -\frac{f_k(q_3 + q_4)}{2y^2 - y(q_3 + q_4)}. \tag{98}$$

Considering that $(\alpha_1 - \alpha_2) = (\tau_1 - \tau_2)$, we obtain

$$\Delta\alpha_x = \tau_1 - \tau_2 \tag{99}$$

and

$$\varepsilon = \frac{\varepsilon_1 + \varepsilon_2}{2}. \tag{100}$$

These formulas are approximate; the influence of relief (considerable) is not considered in them. At small angles of tilt of aerial-photos (up to 2°) the second members of denominators of formulas (94), (96), (98), are not considered.

From the elements of relative orientation of aerial-photos τ_1 and τ_2 (Fig. 75) elements of external orientation can be calculated: in the first approximation:

$$90° - \tau_1 + \alpha_1 = 90° - \tau_2 + \alpha_2,$$

from which

$$\alpha_1 - \tau_1 = \alpha_2 - \tau_2 = \upsilon$$

and

$$\tau_1 - \tau_2 = \alpha_1 - \alpha_2.$$

In addition,

$$\alpha_1 = \tau_1 + \nu, \qquad (101)$$

$$\alpha_2 = \tau_2 + \nu. \qquad (102)$$

Angles α_1 and α_2 are elements of external orientation, and the value of ν is calculated by the formula

$$\nu = \frac{\Delta H}{B}, \qquad (103)$$

where ΔH is found by statoscope readings and B, from phototriangulation. Knowing angle ν, it is possible to calculate the values of α_1 and α_2 by formulas (101) and (102).

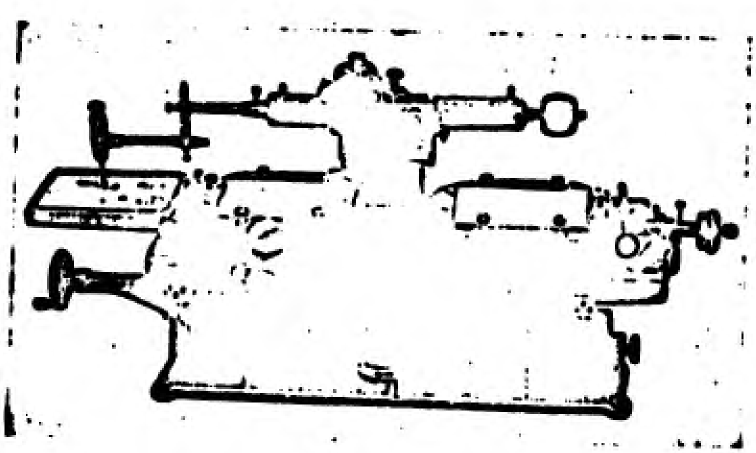
For determination of elements of relative orientation, measure the transverse parallaxes in six standardly located points [9].

§ 25. Instruments for Measuring Coordinates and Parallaxes of Points of Aerial-Photos

1. Stereocomparator

This instrument serves for stereoscopic measurement of coordinates of points of aerial-photos, longitudinal and transverse parallaxes, and also differences of longitudinal parallaxes of points. Points of aerial-photos are observed stereoscopically, in addition to being enlarged, ensuring great accuracy of obtained results. For measurements on aerial-photos stereocomparators with cassette size 18 × 18 cm and 30 × 30 cm are used.

The stereocomparator of the first type (Fig. 76), manufactured in the USSR, consists of a cast iron bed of a box-like section, a base carriage, parallactic supports, and a detachable binocular microscope.



GRAPHIC NOT REPRODUCIBLE

Fig. 76.

The upper part of the bed is polished and leveled off; the sides of the rail-like flanges of the bed are parallel. The base carriage x_1 shifts on the bed, carrying the cassettes and supports. To ensure easy movement the carriage shifts on four ballbearings, two on each side of the carriage; two of them have eccentric axes to ensure exact movement of the carriage along axis x. In the left part of carriage x_1 is a support with the round cassette of the left aerial-photo; in the right part is a support of differences of longitudinal parallaxes $\Delta p = x_1 - x_2 - b$ and on it a support of transverse parallax $q = y_1 - y_2$. The round cassette of the right aerial-photo is on the support of the transverse parallax.

The coordinates of x_1, for various points of the aerial photograph are measured by the shift of the main carriage continuing negatives or aerial-photographs and having a fixed line of sight. The shift of parallactic supports is used to measure the longitudinal parallaxes, their difference Δp, and the transverse parallaxes q. The reading of coordinates is produced with the help of a vernier, divided into 25 parts on a 12 mm length, which gives accuracy of reading of 0.02 mm. The parallactic screws have a step of 1 mm; the drum of the screw is divided into 100 parts; therefore accuracy of readings of Δp and q is equal to 0.01 mm. Measurement of coordinates y_1 of points of the aerial-photo is attained by shifting the binocular system across the bed.

The binocular system (Fig. 77) consists of each branch of the entrance prism, objective I, complicated prism of triple reflection, objective II, and the eyepiece – in the focal plane of which is the mark for fixation of points of the aerial-photo. The slope of the outgoing sighting axes is 30°, which ensures a comfortable position of the head of the observer.

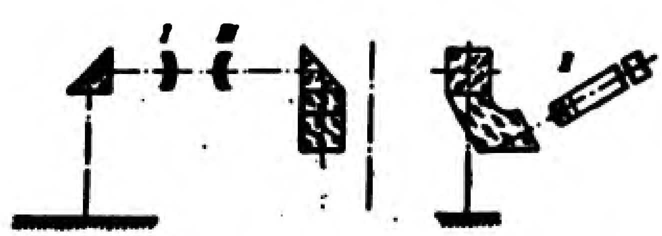


Fig. 77.

Use of two objectives I and II with parallel movement of rays between them instead of using one objective is done for the purpose of simultaneous obtaining of a sharp image of both aerial-photos and identical enlargement of the image, which is attained by changing the position of one of the objectives. Overall enlargement of the system is equal to $4^{\times}$. Instead of the given binocular it is possible to have a binocular with an $8^{\times}$ enlargement, using for this instead of objective II the more long-focus objective III. Replacement of the binocular is executed during transition to the treatment of photos of ground stereophotogrammetric survey.

The glasses with measuring marks are fastened on small supports, allowing their placement on optical axes with high accuracy. The ends of the marks are on parallel optical axes approximately 60 mm apart. This enables an observer even with a small base line of the eyes to make observations with a convergent position of axes of the eyes. By using marks with the upper parts shifted inward by 0.1-0.3 mm, it is possible to obtain a vertical stereomark with the help of which it is convenient to measure parallaxes on aerial-photos. During adjustment of the binocular, ends of marks are placed on sighting axes and diaphragms are shifted toward one other by 0.5 mm. In consequence of this the stereomark seems to be suspended in front of the focal plane in zone of the observed site. The dimensions of marks equal approximately 0.5 × 0.08 mm. The diameter of field of view of the binocular microscope is less than the diameter of the front lens of the eyepiece and consequently the optical base of instrument can be changed by shift of the eyepiece only. The latter are shifted with the help of a screw, having on the edges a right and left thread, or the eyepieces are given eccentric rotation to opposite sides. The optical base enters into limits of 55-72 mm.

For measurement of coordinates x_1, y_1 and differences of longitudinal parallaxes with required accuracy, carriages and supports of the instrument must shift smoothly, rectilinearly, and without vibrating. The directions of movement of carriage x_1 and support y_1 must be perpendicular.

Directions of movements are verified with the help of an 18 × 18 cm measuring grid, which is a glass plate with a grid of squares 5 mm on a side. The lines of the grid, approximately 0.04 mm thick, are located perpendicularly to each other.

By orienting the grid in such a manner that the tips of sighting marks do not descent from the lines during shift of the carriage from left to right and shifting the binocular along axis y, it is possible to check the rectilinearity of guide y and the degree of its perpendicularity to the axis of movement of carriage x_1.

For elimination of revealed nonperpendicularity the guide of the transverse support can be turned to a better position. The same grids are used to investigate degree of perpendicularity of the support of differences of longitudinal parallaxes verses transverse parallaxes, and accuracy of work of parallactic screws.

Errors of screws can be periodic or running. Periodic errors appear on every turn of the screw. Furthermore, the parallactic screw should be checked during the period of all movement 0-60 mm. With the deflection of readings from their true values a graph is composed - a curve, which can be used during treatment of observations. The end of the screw should have a hardened ball. To eliminate play during measurement, the nut of the screw is notched and tightened by a check nut. The parallactic supports should shift under the influence of springs and contact the end of the screw by tempered plates.

Measurement of longitudinal parallaxes. Before beginning work set the eyepiece to obtain clear visibility of marks, then establish the optical base for the merging of focal fields into one whole. After this use by the rack screw to shift the binocular upwards and downwards until a sharp image of the negatives is obtained. Good accuracy of vertical shift of the binocular is attained by use of a trihedral guide locked on all sides.

Aerial-photos are centered and oriented along the initial direction, using screws κ_1, κ_2 and Δp. Then by movements of basic and parallactic supports, and also of the binocular, complete the stereoscopic matching of the mark on points of the model. By carrying out readings on the parallactic screw and x and y scales it is possible to obtain all data for calculations of heights and coordinates of points.

Data of the Instrument

Dimension of cassettes, cm	18 × 18
Base of output beams, mm	300
Shift of carriages x, y, mm	±90
Shift of support p, mm	0-60
Shift of support q, mm	±15
Accuracy of measurement of p and q, mm	0.01
Accuracy of measurement of x, y, mm	0.02

Since the shift of support p can be larger than 60 mm, the instrument is given an additional support on the left cassette.

In 1954 in the K. Zeiss plant (Iona) the stereocomparator 1818 was created for treating 18 × 18 cm, aerial photos, distinguished from former instruments by the system of arrangement of optical components and counting devices (Fig. 78).

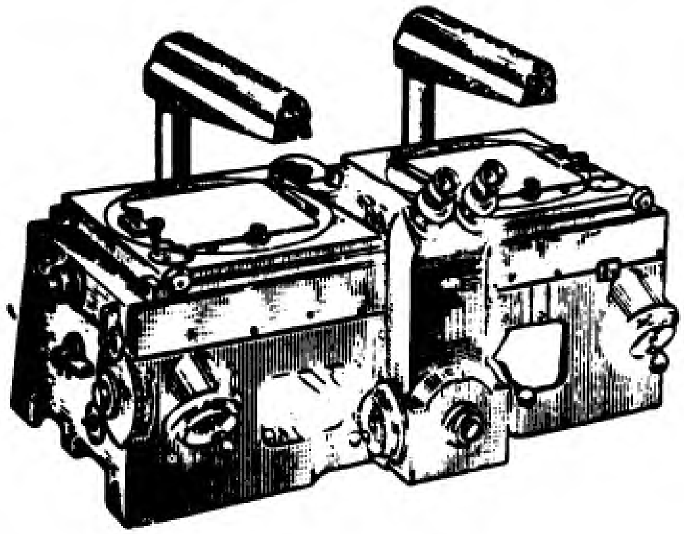


Fig. 78.

During treatment of photographs of ground stereophotogrammetric survey the parallactic screws measure not the difference of longitudinal parallaxes Δp, but parallaxes p.

The binocular part of the optical system is fixed, as was the case at one time in Soviet stereometers. The movable part of the optical system is under the negative. For illumination of negatives luminescent lamps are used. Readings of coordinates and parallaxes with an accuracy up to 5 μ are produced on conveniently located drums.

2. Stereoscopes and Parallactic Plates

In expedition conditions and also during the composition of maps on a small scale it is possible to measure parallaxes of points also with simple instruments, for instance, with four-mirror stereoscopes (Fig. 79) with attachments. These instruments should enable the observer to see the entire area of stereoscopic pairs of aerial photos equal on the average to 130 × 180 mm, or part of it – 60 × 60 mm. The stereoscope consists of four parallel mirrors, in pairs, inclined

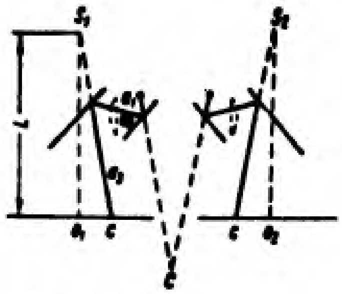


Fig. 79.

on a 45° angle to the horizontal plane.

Let us assume that S_1 and S_2 are imaginary centers of projection found at distance L = 260 - 350 mm from the plane of aerial-photos. Main points of the latter should be on the base of perpendiculars dropped from points S_1 and S_2.

The distance between external mirrors of the stereoscope is usually 150-200 mm; therefore aerial-photos located alongside can be revolved in their planes on angles $\kappa = \pm 15°$. It is necessary to consider that rays of sighting directed towards identical points with the indicated location have convergence close to normal (convergence of 15°). Therefore the stereoeffect is obtained without eyestrain.

During creation of the stereoscope it is necessary to consider the following:

1. Middle points c of the stereopair are approximately 30 mm from main points o_1 and o_2.

2. Value of $L = a_1 + a_2 + a_3$.

3. The base of eyes of the observer is equal to 55-72 mm.

4. The center of rotation of the eye-ball is approximately 12 mm from the pupil. The surface of the eye with respect to the small mirror is located not closer than 15 mm. The diameter of the pupil in conditions of weak illumination reaches up to 6 mm.

5. Thickness of the mirror influences the position of the imaginary center of projection; with the index of refraction of glass, for instance, n = 1.51, turn of beams occurs as if in its center.

6. The intersection of longitudinal planes of mirrors with the plane of aerial-photos should occur on parallel lines. This purely geometric condition is complicated by the additional requirement that the mirror not be wedge-shaped; otherwise the line of centers traced on paper and observed in the stereoscope is often in the form of two parallel lines or a broken line. Permissible break of the line depends on the vertical convergence of eyes of the observer which can be accepted as 10' which with a length of ray of 280 mm and for points located on the edge of the aerial-photo gives a break of 30'. This magnitude can be the initial during counting of the allowance for the total taperdness of

mirrors of the stereoscope.

There is widespread use of the folding lens-mirror stereoscope [LZ] (ЛЗ) (Fig. 80) built according to the given diagram. Mountings with external mirrors and legs, and also internal mirrors (sometimes prisms) are hinge-fastened on the holders. Between the mirrors are located planoconvex lenses in quadrangular mountings. Enlargement of lenses is 1.5$^{\times}$; the field of view of the lenses is approximately 100 × 100 mm. Sometimes lenses are removed; then the stereopair is totally visible.

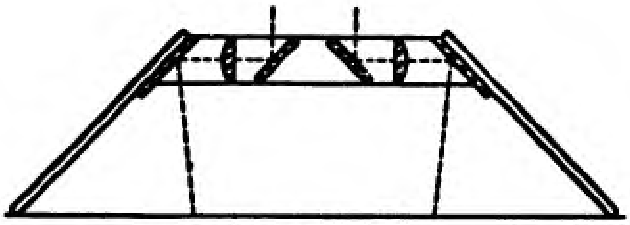


Fig. 80.

In order to obtain a good stereoeffect, lines connecting the principal points of the aerial-photos must be parallel to base line of the eyes and continue each other. Eyes of the observer must be in direct proximity to the small mirrors; then the observed field will be of maximum dimensions. Depending on the properties of eyes of the observer the centers of the stereopair must be on convergent or parallel (but not divergent) sighting axes; the visible transverse parallaxes have to be eliminated. With disturbance of these conditions distortion of the stereomodel is observed. When the aerial-photo are turned in their own plane distortion appears as tilting of the stereomodel across the line of the base, and with inequality of distance of centers of projection from the aerial-photos it appears as tilt of the stereomodel along the line of the base. Compression occurs with divergence of sighting beams of over 6-10°. With decrease and increase of distances from the eyes to the aerial-photos there is observed a corresponding change in the heights of points, which signifies distortion of the stereomodel in the vertical direction.

The indicated phenomena are explained by a change in the magnitudes of visual parallaxes depending on the position of aerial-photos and eyes of the observer relatively to true centers of projection. Furthermore, distortion of the stereomodel depends also on values of angles of tilt of aerial-photos; for vertical aerial photographs it is small.

Parallactic plates, proposed by the author, serve for the measurement of aerial-photos located under stereoscope of differences of longitudinal and transverse parallaxes. In one of the variants the composition of the measuring assembly of the stereoscope includes three glass or plexiglas plates with leveled sides (Fig. 81).

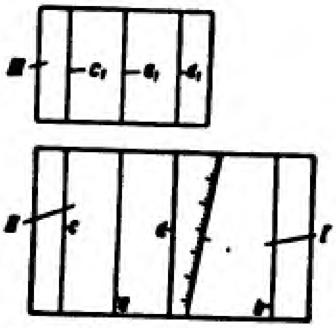


Fig. 81.

On the plates there is a millimeter scale (measuring lines with thickness 0.05 mm and an index). The angle between the tapered side and measuring lines $\gamma = 5°44'20"$. Therefore during a shift of one plate along the other it is possible to bring the lines together or to separate them. Since in this case $\sin \gamma = 0.1$, every millimeter of shift of the plate corresponds to 0.1 mm of drawing together of the lines; this condition is used during measurement of differences of longitudinal parallaxes. The distance between lines a and b of plates II and I is equal to the distance between middle points of the stereopair placed under the stereoscope.

Using plates II and I, measure the differences of longitudinal parallaxes of points after stereoscopic guiding of lines on points of the model. By turning the system of plates II, I, and III clockwise 90° above the fixed aerial-photos, it is possible to measure transverse parallaxes of six standardly located points by lines c and c_1, a and a_1, d and d_1, 60 mm apart from one another.

Plates ensure measurement of parallaxes with accuracy up to 0.05 mm. There are different forms of plates.

§ 26. Measurement of Transverse Parallaxes on the Stereocomparator

Transverse parallaxes of points of aerial-photos are measured for determination of elements of relative orientation by formulas (99) and (100). Combine the principal points of aerial-photos with centers drawn on cassettes and orient these aerial-photos along the initial direction, joining first the left mark of the binocular with the principal point of the left aerial-photo, and transfer the base carriage along axis x of the instrument. If the right mark does not pass through the principal point of the right aerial-photo, then transfer the latter by the necessary magnitude by means of the transverse parallax screw. After that join the left (right) mark with any contour located near the principal point of the left (right) aerial-photo, correcting noncoincidence of the right (left) mark with an identical contour by means of the longitudinal parallax screw and rotation of the right (left) aerial-photo is its own plane. Completing orientation of aerial-photos along the initial direction and producing readings by the transverse parallax screw, shift the binocular along axis y by +60 mm (or +50 mm) to point 3 (Fig. 74), and measure transverse parallax q_3. Then, shifting the binocular to point 5 (y = -60 mm or -50 mm), measure transverse parallax q_5. Analogously repeat the operation by guiding the mark to point 4' (x = +b; y = +60 mm) and to point 6' (x = +b; y = -60 mm), after which the measured values of transverse parallaxes an used for calculation of elements τ and ε.

Table 11 gives a chart of notes of readings and calculations.

Table 11

No. of points	Reading, mm	Transverse parallax	Notes
1	6,61	0	$y=60$ мм
2	6,61	0	$b=77,5$ "
3	7,36	+0,75	$f_k=68,3$ "
4	5,98	−0,63	$\rho'=3438$
5	7,92	+1,31	
6	9,38	+2,77	

If

$$q_3-q_5=-0{,}56 \text{ мм},\quad q_3+q_5=+2{,}06 \text{ мм},$$
$$q_4-q_6=-3{,}40 \text{ "},\quad q_4+q_6=+2{,}14 \text{ "},$$

then after substitution of the values in formulas (95)-(98) we obtain

$$\tau_2=+14',1,\quad \varepsilon_2=-68',3,$$
$$\tau_1=+85',8,\quad \varepsilon_1=-71',1,$$

and also according to formulas (99) and (100),

$$\Delta\alpha_x=71',7$$
$$\varepsilon=-69',7.$$

Calculations are done with the help of a slide rule.

Stereocomparators of Increased Accuracy

In recent years stereocomparators with precision of readings up to two microns have been produced in foreign countries. Fig. 82 shows the stereocomparator TA-3 "Nistri", which allows overlaying three adjacent aerial-photos for the purpose of exact identification of points of triple overlap.

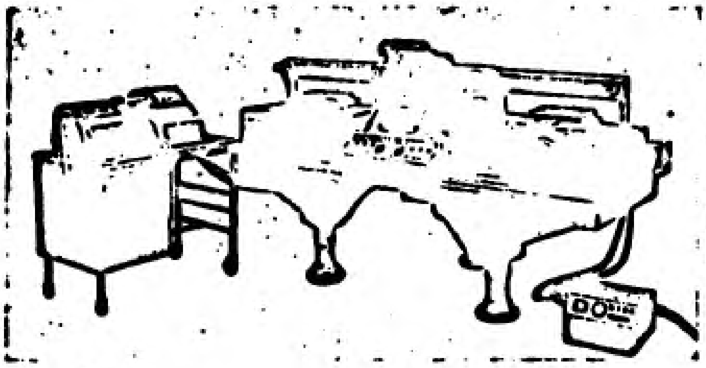


Fig. 82.

Next to the stereocomparator is a table with an electric typewriter for automatic recording of the coordinates of observed points.

Fig. 83 shows the stereocomparator SOM (France), designed for use with aerial-photos of size 300 × 300 mm and also ensuring accuracy of readings of coordinates of 2 μ. A binocular microscope gives an enlargement of the image from 4 to $12^{\times}$.

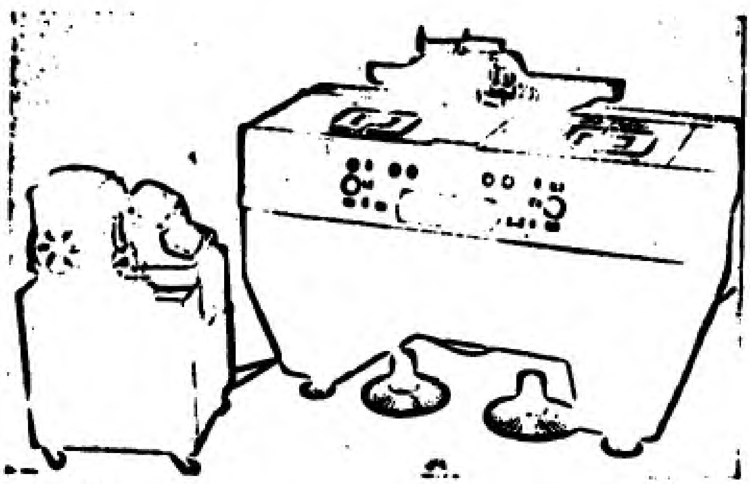


Fig. 83.

Shift of the carriages in the instrument is ensured by electric motors with a speed of $\approx$40 mm per/sec.

It is necessary also to note that in certain new stereocomparators the readings of coordinates are taken from the nearest lines of measuring grids, fastened in carriages, which frees the results of measurement from a number of errors, connected with inaccurate manufacture of carriages or actuating screws.

In certain stereocomparators screens and electronic optical systems are used to increase the accuracy of readings of coordinates.

There are also stereocomparator models arranged with a perforator of coordinates, cards of which enter a computer.

§ 27. Mechanized Solutions of Formulas for Tying Coordinates of Points of Terrain and Aerial-Photos

During aerial photography from the same height and strictly vertical direction of the principal axis of the aerial camera, the difference of longitudinal parallaxes on the aerial-photos is connected with elevation differences of points by the dependence

$$h = \frac{\Delta p}{b + \Delta p} H. \tag{104}$$

It can be solved if the base and height of photographing, and difference of longitudinal parallaxes are known. However, due to tilt of the principal axis and different heights of photographing of the aerial-photos of the stereopair the difference of longitudinal parallaxes of points is distorted. Therefore on the stereocomparator the distortion of magnitudes would be measured but consequently, the elevation of points determined it would be erroneous.

However, there are stereometers which can automatically correct the measured differences of longitudinal parallaxes, after which it is possible to use formula (104). Distortions in differences of longitudinal parallaxes are corrected by correctional attachments, by which are solved equations of connection of coordinates of points of terrain and the aerial-photo. Let us note that the change of abscissas due to the influence of angle α is characterized by equation (82); angle ω by equation (88); and, finally, the influence of elevation δH by formula (85).

Principles of mechanized solution of equations. For eliminating errors in abscissas X_2 caused by longitudinal tilt α of the aerial-photo there must be included in the stereocomparator an additional carriage x_2 (which transforms it to a stereometer), with a shift of which is considered distortion due to tilt and elevation differences of stations for points of aerial-photos located on axis x. Carriage X_2 is connected with the carriage of aerial-photo x_2 by a convergent device consisting of two rulers which mechanically solve the equations of connection of coordinates of points of terrain and the photograph.

For eliminating errors in abscissas caused by transverse tilt of the aerial-photo it is necessary to shift the sighting marks along axis y, not along the perpendicular to the axis of the abscissas, but along the line of descent, i.e., during match on a point with the abscissa x, the mark should move at variable angle ρ_ω to the axis of ordinates. One of the structural solutions is to have the mark move along slanting directions (stereometer SM-2). In other instruments (stereometers SM-1, SM-3) elimination of errors in abscissas caused by transverse tilt of the aerial-photo is attained by rotation of the negative on angles ρ_ω by a special correctional device. This method, like the first one, can be considered exact during possible rotation around the point of intersection of a given line of descent with axis y. Such solution of the problem, however, turned out to be difficult; therefore, rotation of an aerial photo takes place around the center or exnadir; transverse parallaxes appearing in this are removed by displacement of one of the negatives.

<u>Mechanized solution of the equation of the connection along axis X</u>. Let us assume that the base carriage X moves parallel with the aerial-photo along axis X of the instrument. Around the fixed axis of rotation S, perpendicular to axes x of aerial-photos, revolve two rulers; between them there is fixed the angle of tilt α of the aerial-photo. One of these rulers is connected by means of roller a with the carriage of the photo and the second, SC, is connected by means of roller C with carriage X (Fig. 84). A shift to the right (or to the left) of the basic carriage along the axis X causes a turn of the system of rulers to position PSe and thereby a shift of carriage of the aerial-photo to segment x. Suppose that Sa is the line of sight, and point e the point of exnadir shifting to segment x. Then

$$\frac{x}{\frac{f_k}{\cos\omega}}=\operatorname{tg}A,$$

$$X=H[\operatorname{tg}(A-\alpha)+\operatorname{tg}\alpha]=\frac{H\sin A}{\cos A\cos^2\alpha+\sin A\sin\alpha\cos\alpha},$$

from which

$$\operatorname{tg}A=\frac{X}{\frac{H}{\cos^2\alpha}-X\operatorname{tg}\alpha},$$

and consequently,

$$\frac{x}{\frac{f_k}{\cos\omega}}=\frac{X}{\frac{H}{\cos^2\alpha}-X\operatorname{tg}\alpha}. \qquad (105)$$

According to the equation, the point of connection e, joined with ruler Se, should stand apart from point S at distance $\frac{f_k}{\cos\omega}$, and point of connection P – at distance H, equal to the height of photographing.

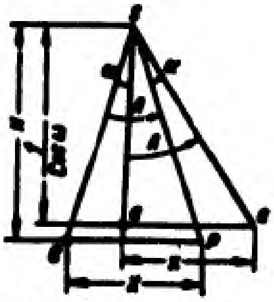


Fig. 84.

Mechanized rotation of the aerial-photo. Earlier it was shown that in the stereometer SM-1 for elimination of error in the abscissas due to transverse angle of tilt of the aerial-photo, the method of rotation of negatives in their own plane is accepted. In this case rotation of negatives occurs near the center or exnadir. The correctional device which solves the problem is depicted in Fig. 85, on which are shown: carriage 1 of the negative, shifting along guide bed 2; circular cassette 3 revolving about its own center; correctional ruler 4, established on the bed at certain angle ρ_0 to the axis X of the direction of movement of carriage 1.

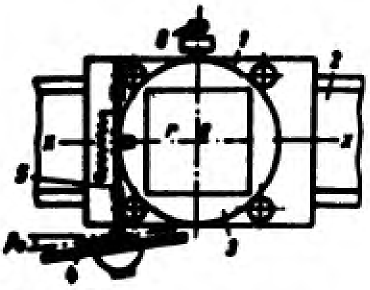


Fig. 85.

Perpendicular to axis X on the carriage there is mounted pusher 5, carrying the roller on the end; the latter under the action of the spring touches with correctional ruler 4. The pusher is joined with the circular cassette by a pin which rotates it. The pin is at distance r from the center of rotation of the cassette. Carriage 1 is shifted with the help of roller 6 of focal lengths.

In the position depicted on the figure the sighting axis of the branch of the binocular is directed towards the point of exnadir e, and the axis of the roller of the pusher is combined with the axis of rotation of the ruler; therefore any of the fixed angles ρ_0 does not cause displacement of pusher 5, or consequently, and rotation of the negative. If, however, the carriage with the pusher and the negative is shifted to coordinate x (Fig. 86), the pusher is shifted to segment a, turning the negative on angle ρ_ω; the line of descent mi will occupy a position parallel to the axis Y. In eyepiece point m will be observed. Then

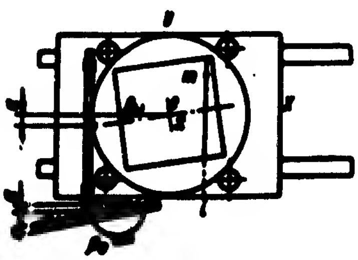


Fig. 86.

$$\operatorname{tg}\rho_\omega=\frac{a}{r}.$$

$$\operatorname{tg}\rho_0=\frac{a}{x}.$$

By solving the equations jointly we obtain

$$\operatorname{tg}\rho_0=r\frac{\operatorname{tg}\rho_\omega}{x}.$$

In agreement with Fig. 87

$$\operatorname{tg}\rho_0 = \frac{a}{d_y} = \frac{a\sin\omega}{\frac{f_k}{\cos\omega}}; \tag{106}$$

therefore

$$\operatorname{tg}\rho_0 = \frac{r}{\frac{f_k}{\cos\omega}}\sin\omega. \tag{107}$$

It is simple to note that ρ_0 will be constant for a given angle ω. With $\omega = 3°$, $r = 100$ mm and $f_k = 200$ mm, the value of $\rho_0 = 1°38'$; therefore expression $\operatorname{tg}\rho_0$ can be presented as

$$\rho_0 = \frac{r}{f_k}\omega, \tag{108}$$

after which formula (88) will take the form

$$\Delta x = \frac{xy}{r}\rho_0. \tag{109}$$

Another solution of correction of errors in differences of longitudinal parallaxes due to the transverse angle of tilt ω is based on automatic establishment of line of descent with the help of one of the filaments. In this case the right filament rotates; it is fastened on revolving filament holder $i_y y'$, which is turned with the help of ruler ki.

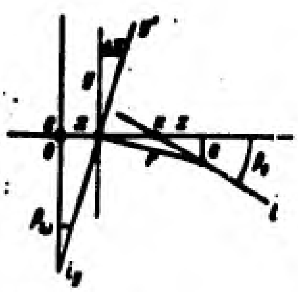


Fig. 87.

Extension of threadholder r, joined with correctional ruler ρ_0 (Fig. 87), establishes the filament in position $y' i_y$.

According to the drawing:

$$a = x\operatorname{tg}\rho_0,$$
$$a = r\sin\rho_\omega,$$

from which

$$\rho_0 = \frac{r\rho_\omega}{x} = \frac{r}{f_k}\omega.$$

This equation for determination of ρ_0 differs little from the more strict equation. Correctional ruler kl is under the cassette and is fixed to axis x at angle ρ_0. In such position the ruler shifts jointly with the differential support and the aerial-photo.

Since the center of the filament holder lies on axis x, at zero value of x the angle of descent of the filament is also equal to zero, which corresponds to the requirement of the theory.

§ 28. Stereometers

Stereometers developed by F. V. Drobyshev, are basic stereophotogrammetric instruments of the differentiated method. In the prewar period they were widely used in the creation of small-scale maps, both of mountainous and flat terrain.

At present they are used for treatment of aerial-photos during creation of large-scale maps of plain and hilly terrain.

The topographic stereometer [STD-1] (СТД-1) with four correctors was in use for a long time. At present it has been replaced by the topographic stereometer STD-2, in which, as compared to STD-1, are added to correctors, (design M. D. Konshin), with which more exact operation of the instrument is possible.

The precision series of instruments was represented stereometers [SM-1] (СМ-1), SM-2, SM-3, and SM-4 differing only in their structural elements.

1. Stereometer SM-4

This instrument (Fig. 88) has the same purpose as the topographic stereometer, i.e., it serves for drawing contours on the aerial-photos, but it is more frequently used for measurement of coordinates, transverse parallaxes, and differences of longitudinal parallaxes.

Fig. 88.

The main parts of the instrument (bed, carriage and binocular) are analogous to the main parts in the stereocomparator. Furthermore, the instrument has convergent and correctional devices.

At the bottom of bed are lamps for illumination of the negatives. On the precision-made edges of the upper part of the bed the basic carriage, constructing coordinate x, shifts on ballbearings. On the right part of it there are located the supports of longitudinal and transverse parallaxes. On the latter is the cassette of the right negative and table for the contact print. Thus, by introducing the difference of longitudinal parallaxes Δp it is possible to measure the magnitude of the coordinate of the point of the right aerial-photo:

$$x_2 = x_1 - b - \Delta p.$$

In the left part of the carriage x is the support, which, with the help of the convergent device accomplishes repeating movement with respect to the carriage. On the repeating support there is a double circular cassette, the lower part of which serves for automatic turning of the aerial-photo at angles ρ_ω and upper for establishing it at angles κ.

The convergent device is located behind the left part of the instrument; it consists of two rulers located in a vertical plane and connected by a micrometer screw. Between the rulers there is established angle $\Delta\alpha_x$ of relative orientation of aerial-photos. Through hinged attachments the rulers are united with rollers of heights H and focal lengths f_k. Their values are established with the help of micrometer screws on the supports of basic and differential carraiges of the left aerial-photo.

The correctional device (see Fig. 85) is made according to the system given earlier.

With rotation of the aerial-photo on angles of descent ρ_ω transverse parallaxes appear (Fig. 89), disturbing the stereoeffect. Simultaneously there appear small distortions of differences of longitudinal parallaxes δp_x, expressed by the dependence

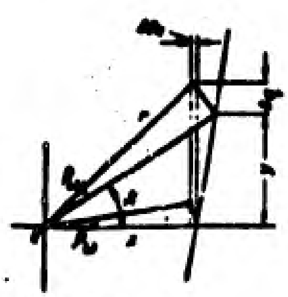


Fig. 89.

$$\delta p_x = x(1 - \cos\rho_\omega) = 2x\sin^2\frac{\rho_\omega}{2}$$

Transverse parallax Δq, caused by rotation of the negative around the center, is determined by the expression

$$y + \Delta q = r\sin(A + \rho_\omega).$$

Expanding sin (A + ρ_ω) in series and being limited by two terms, we find

$$y + \Delta q = r\sin A + r\rho_\omega\cos A,$$

but

$$y = r\sin A,$$

therefore

$$\Delta q = r\rho_\omega \cos A. \qquad (110)$$

For r = 100 mm; A = 45°; and ρ_ω = 2/57 the value Δq = 2.5 mm.

In order to remove the transverse parallax, it is necessary to displace the right aerial-photo by the transverse parallax screw.

For observation of points located outside the axis x of the aerial-photographs an optical system is applied consisting of a fixed binocular and a movable part located on support y. The mobile part of the binocular is shifted by the magnitude of coordinate y within the limits of 180 mm. Since between objectives of movable and fixed parts rays go in a parallel bundle, during shift of the movable optical part sharpness of the image is retained.

On the left on the movable part is fastened a magnifier for taking readings of coordinates y. On the movable part there are also fastened lamps for illumination of aerial-photos from above.

2. Topographic stereometer STD-2

The instrument (Fig. 90) consists of a bed, a base carriage, convergent and correctional devices, and a mirror stereoscope. The base carriage is shifted by means of a rack screw.

Fig. 90.

In left part of basic carriage (x) is located a parallactic support and on it – the cassette of the left aerial-photo. In the right part of the carriage there is a differential support and on it – the cassette of the right aerial-photo.

Above the aerial-photos strained filaments 0.04-0.05 mm thick are drawn taut on turning holders. They are the lines of descent.

Above the aerial-photos on the transverse traverse there is located on the carriage a four-mirror stereoscope with enlargement of about $2^{\times}$. In the right part of the bed there is a supporting surface for the operator's hands.

Differences of parallaxes can be measured by a parallactic screw with an accuracy of 0.02 mm. Formula (90) can be modified in such a way that the variable members will pertain to the right aerial-photo. Considering that small angles of tilt of aerial-photos ($\leq 3°$) magnitudes of the second order of smallness can be disregarded and constants can be excluded by means of replacing x_1 with its value

$$x_1 = b + x_2 + \Delta p,$$

we find the new form of the formula for determining distortion of the difference of parallaxes:

$$\delta p = \frac{x_2^2}{f_k}(\alpha_1 - \alpha_2) + \frac{x_2}{f_k}(2b\alpha_1 - \delta H) + \frac{x_2 2\Delta p_l}{f_k}\alpha_1 + $$

$$+ \frac{x_2 y_2}{f_k}(\omega_1 - \omega_2) + \frac{y_2}{f_k} b\omega_2 + \frac{y_2}{f_k}\Delta p_l \omega_1 + \frac{2b\alpha_1}{f_k}\Delta p_l. \qquad (111)$$

Convergent, correctional, and other devices of the topographic stereometer solve six terms of the formula, besides term $\frac{2b\alpha_1}{f_k}\Delta p_l$, which is established on the parallactic screw as a constant correction (steps as a function of Δp_1). The convergent device (Fig. 91) serves for correcting the influence of the difference of longitudinal angles of tilt of aerial-photos, which is expressed by the quantity $\frac{x_2^2}{f_k}(\alpha_1 - \alpha_2)$. It consists of two rulers with a common center of rotation. Between the rulers there is established angle $\Delta\alpha_x$, equal to $\alpha_1 - \alpha_2$, after which they are secured. The cursor of heights, whose roller is pressed to the right ruler of the convergent device, is set in accordance with the value of $\delta H'$, equal to $2b\alpha_1 - \delta H$.

During shift of the basic support to the left and to the right, the right cassette with the aerial-photo delays or accelerates its own movement. In accordance with the first and second members of the formula, a correction is attained of the difference of longitudinal parallaxes of points located along the line of centers of the aerial-photos. The third term is solved by cursor 5 (see Fig. 93), touching with correctional ruler 6, fixed on angle γ_1 (Fig. 93).

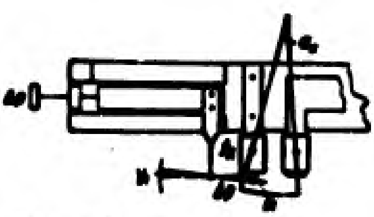


Fig. 91.

Since the correctional ruler stands on the platform, fastened parallactic support, then with introduction of Δp cursor 5 shifts by the magnitude

$$m = \Delta p_l x_2.$$

as a result of which the right carriage will shift additionally by the magnitude of Δx_3 determined from the relationship

$$\frac{\Delta x_3}{m} = \frac{x_3}{f_k}$$

or

$$\frac{\Delta x_3}{\Delta p_1 \gamma_1} = \frac{x_3}{f_k}.$$

Comparing this equation with the value of the third member of the right side of the formula (111)

$$\Delta x_3 = \frac{x_3 \Delta p_1}{f_k} 2e_{\mu}$$

we find the value of the adjusting angle

$$\gamma_1 = \frac{\Delta p_1}{f_k} 2e_{\mu}. \qquad (112)$$

The fourth member of the formula is solved by the filament correctional device with the condition of establishment of the correctional ruler on angle ρ_0. Turn of the filament holder is carried out by extension r.

The fifth member of the formula is solved by the same filament holder, having repetitive setting κ_0 (angle between the filament and axis y with zero value of coordinate x).

The sixth member of the right side of the formula is solved by the attachment which revolves the left filament (Fig. 92). It consists of a ruler fastened to the parallactic support and fixed at angle γ_2 to line κ, and from the T-form pusher attached to support x in a position perpendicular to the axis x. On this pusher there is a roller, contacting the correctional ruler under action of the spring. Therefore upon the introduction of differences of parallaxes the pusher is displaced. With the longitudinal ruler o the pusher contacts the lever roller, rigidly joined with the left filament holder of the topographic stereometer.

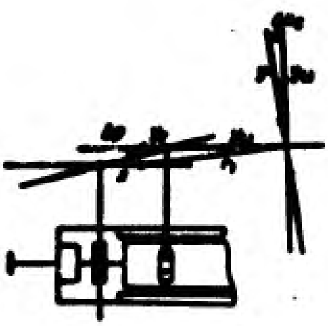


Fig. 92.

With introduction of the value Δp_i the correctional attachment constructs the magnitude

$$\Delta x_6 = y_6 \gamma_m,$$
$$\Delta p_1 \gamma_2 = r_1 \gamma_m,$$

from which

$$\Delta x_6 = \frac{\Delta p_1 y_2}{r_1}\gamma_2.$$

Equating them to the sixth member of right side of the formula (111)

$$\Delta x_6 = \frac{\Delta p_1 y_2}{f_k}\omega_1,$$

we find the value of the adjusting angle γ_2 of the correctional ruler:

$$\gamma_2 = \frac{r_1}{f_k}\omega_1. \qquad (113)$$

The values of α_1 and ω_1 can be found both by means of calculations and in the process of orientation of aerial-photos.

Fig. 93 depicts the system of the topographic stereometer STD-2, the additional mechanisms of which, as compared to topographic stereometer STD-1 with its convergent and correctional devices, are: 1 – the pusher, fastened on the base carriage x, having a T-shaped form and joined with the extension of the left filament holder with the roller touching the ruler 2; 2 – correctional ruler γ_2; 3 – support of installations $\Delta p = 0$, fastened on the parallactic support; 4 – axis of rotation of correctional ruler 2, located on support of installations $\Delta p = 0$, and 6 – ruler γ_1.

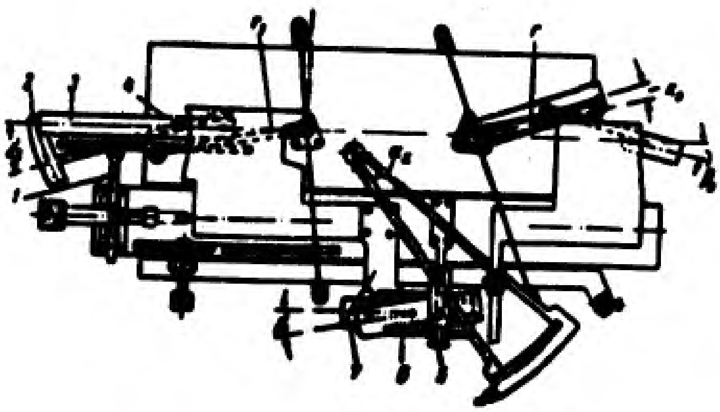


Fig. 93.

During orientation of aerial-photos support 7 is established in the zero position so that during matching of the filament to initial point o_2 corrections are carried out only with a change of longitudinal parallaxes. Pusher 5 during this time is also located in the zero position.

Knowing angles α_1 and ω_1, it is possible to establish the values of γ_1 and γ_2. In Table 12 gives characteristics of stereometers STD-2 and SM-4.

Table 12

Characteristics of the instrument	Stereometer	
	STD-2	SM-4
Dimensions, mm	[illegible]	1070×640×520
Movement of x, mm	− [illegible] + [illegible]	− 80 + 95
" y, mm	± [illegible]	± 100
" f_k and H, mm	[illegible] — 110	92 — 110
Angle [illegible] in degrees	± 5	± 10
" [illegible]	± 5	± 10
" [illegible]	± 5	± 10
Repetitive κ	± 5	± 10
Difference of longitudinal parallaxes Δρ, mm	45 — 90	45 — 90
Transverse parallaxes q, mm	—	± 20
Magnification[x]	2,5×	[illegible]×
Diameter of field, mm	55	[illegible]
Accuracy of readings of screws for measurement of parallaxes, mm.	± 0,01	± 0,01

Since 1958 the instrument has been widely applied in a number of establishments using aerial photography materials. The most effective application is for the creation of maps of plain and hilly terrain.

3. Adjustments of the Topographic Stereometer

1. Adjustments are completed in factory conditions. The base carriage of the instrument and differential support have to shift exactly and smoothly. The basic carriage is adjusted with the help of mobile ballbearings. Differential support is shifted by oscillation on four balls, of which high durability and accuracy are required. Smoothness of movement of carriage and support, and also absence of slack, determined by rocking with the hands or, still better, revealed by an indicator, serve as criteria of quality of adjustment.

2. Guides for setting H and f_k have to be perpendicular to the axis of the base support with an accuracy up to 2'. The cursors on them have to shift smoothly and exactly.

3. The left filament should be situated perpendicularly to axis x and pass from the center of rotation of the left cassette at distance equal to the reading on the parallactic screw; the right filament during this time should pass through the center of rotation of the right cassette.

4. Rulers of the convergent device should be exact and de-centered by the

magnitude of the radius of rollers with an accuracy up to 0.05 mm.

5. With location of correctional ruler parallel to the axis x ($\rho_0 = 0$) the right filament should be perpendicular to the axis x (κ – repetitive also should be equal to zero).

6. Small mirrors of the stereoscope should be fixed at approximately 3° from their normal position for creation of natural convergence of sighting axes of the eyes. It is expedient that turn of the reflectors be carried out by the observers themselves.

7. The lenses of the stereoscope should be fixed in a vertical position.

Below we outline working checks executed for determination of zero points of scales of the instrument.

1. Determine zero point of scale $\Delta\alpha_x$ of the converging device. Cursor f_k is transfered along the ruler and the base carriage is shifted along axis x until the movement of the measuring grid of right cassette under the thread disappears, indicating the establishment of the ruler in a position perpendicular to axis x. For establishment of the second ruler of the convergent device in the initial position transfer cursor δH forward and back and change angle $\Delta\alpha_x$ until movement of the right cassette is eliminated, after which zero point of the scale $\Delta\alpha_x$ is determined as scale reading with an accuracy up to 2'.

Note. In the construction of topographic stereometer (STD-2) this check is conducted by means of observation of three points (two main and one intermediate) located on axis x, measurement of the movement of the differential support by the indicator, and solution of the corresponding equations.

2. The magnitudes of f_k and H and the zero points of their scales are determined as a result of lining up the filament with marks of the grid, located on distances x_1 and x, multiples of 5 mm, under the condition of change of position of the cursor by the quantity d. The value of f_k is determined from the relationship

$$\frac{x}{f_k} = \frac{x_1 - x}{d}.$$

The zero point of the f_k scale is equal to the difference of the obtained value of f_k and the corresponding scale reading. In exactly the same way the value of H and the zero point of the scale d are determined.

3. Zero point of scale ρ_0 is defined as the scale reading with the right

filament lines up (in extreme position) with one of the marks of the measuring grid, oriented along the initial direction. For this first fasten movable small knots on the threads at points above the centers of cassettes; these knots determine the initial direction of the axis of the aerial-photos during movement of the cassette.

4. The check of length of lever r is based on solution of the equation

$$r = \frac{xy}{\Delta x}\rho_0,$$

where Δx is the displacement of a point of the filament with respect to the line of grid during establishment of angle $\rho_0 \approx 3°$; x, y are the coordinates of this point, measured with the help of the grid.

By determining the magnitude of r (≈86 mm), it is possible to establish ρ_0 in the instrument, corresponding to known angle $\omega_1 - \omega_2 = \Delta\omega$, i.e.,

$$\rho_0 = \frac{r}{f_k}\Delta\omega.$$

Additional correctional devices must be reliably established in the zero position, at which in the principal point of the second aerial-photo the value $\Delta p = 0$.

§ 29. Processes of Creation of Topographic Maps

The first and basic process is determination of elements of relative orientation of aerial-photos, corresponding to restoration of the model on instruments of optical projection. From the elements of relative orientation, calculate conditional angles of tilt of aerial-photos used in thickening the vertical [planar][1] and vertical basis and drawing of relief. Thickening of the planar basis is executed according to the method of vertical [planar] phototriangulation, photopolygonometry,

In a number of cases it is expedient to determine the [planar] position of points of the multiplex or stereoplanigraph. Photogrammetric thickening of the field vertical control is executed during creation of maps of scales 1:25,000-1:100,000 by one of the methods considered below, depending on the character of the region of survey.

The following process – stereoscopic depiction of relief – is executed on the topographic stereometer; in certain cases depiction of relief is executed with

[1]According to available references both the Russian words плановая [planovaya] and высотная [vysotnaya] translate as vertical in reference to aerial photography. [Trans. Ed. note].

help of the stereoscope by a considerable quantity of peg points, the heights of which are determined by photogrammetric methods.

Horizontals [contours] are drawn on unrectified aerial-photos; on these same aerial-photos trace contours (according to photointerpretation data).

The final process involves composition of the map according to aerial-photos with image of relief and contours.

1. Method of Treatment of Aerial-photos on the Topographic Stereometer

In order to use the stereometer or topographic stereometer to solve the problem of automatic correction of differences of longitudinal parallaxes in process of their measurement or during tracing of horizontals [contours], it is necessary preliminarily to set the correctional attachments in accordance with elements of orientation of aerial-photos of the given stereopair. This process of establishing correctional adjustments is called orientation of aerial-photos. The quantity $\Delta\alpha_x$ utilized for adjustment, can be obtained from elements of relative orientation of aerial-photos, determined preliminarily. Magnitudes ρ_0, δH, and κ are established by four to six vertical control points located on the aerial-photos as is shown in Fig. 94. Marks of these points can be determined by geodesic methods in the field or by application of photogrammetric methods of thickening of the vertical control (for maps of medium scales). The process of orientation according to vertical control points consists in seeking by a change of adjustment of the correctional attachments to effect the coordination of measured differences of longitudinal parallaxes of these points with their values precalculated by the formula

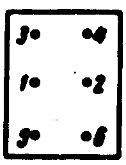


Fig. 94.

$$\Delta p = \frac{b_0 h}{H - h},$$

where h is the height of points of vertical thickening above the initial points.

Preliminarily the aerial-photos are centered in cassettes and oriented along the line connecting their principal points.

If preliminarily we determine the angles of tilt α_1, α_2 and ω_1, ω_2 of aerial-photos, then we orient the latter along the line connecting the nadir points. The distances from the center of projection to points of the exnadir are equal in this case to $\frac{f_k}{\cos \omega_1}$ and $\frac{f_k}{\cos \omega_2}$; with small magnitude of angles ω_1 and ω_2 magnitude f_k

is established on ruler of focal lengths of the convergent device. For calculation of true differences of longitudinal parallaxes of vertical control points it is necessary to determine the value of base b_0 and height of photographing H by formula (92).

The height of photographing above the initial point is determined by comparison of perspectively conjugate segments on the aerial-photo and terrain (layout). For comparison of segments it is necessary preliminarily to introduce correction for relief in one of them (aerial-photo or layout), inasmuch as the distance between orthogonal projections of points is measured on the layout. To decrease the influence of distortions of segments of aerial-photos due to their tilt in order to determine H accurately, select a segment on a straight line passing through the principal point of a given aerial-photo; the ends of the segment must be symmetrical relative to the principal point.

If D is the distance between points 4-6 on the terrain measured by the phototriangulation net and d is the corresponding distance on the aerial-photo, then

$$H_4 = \frac{D}{d} f_k + \frac{\Delta R}{d} f_k, \qquad (114)$$

where ΔR is displacement due to relief of point 6, equal to

$$\Delta R = \frac{R(h_6 - h_4)}{H_4},$$

and R is the distance from point 6 to the point of nadir on the phototriangulation net.

The value of height of photographing H_4 for determination of correction for relief is calculated by the formula

$$H_4 = H_a - h_4,$$

where H_a is the height of photographing relative to sea level, determined by a barogram with an accuracy up to ±50 m; it is possible also to determine H_4 according to the radio altimeter, corrected by statoscope readings; h_4 is the elevation of point 4 above sea level.

The process of work on the instrument consists of the following.

Establish on the parallactic screw of the topographic stereometer the reading corresponding to the calculated difference of longitudinal parallaxes, and observe the spatial position of the filament relative to the corresponding point of the model. If the filament seems located higher or lower than the surface of model, then seek contacts in that point of the model by setting the corresponding corrective or convergent device. Measurements and calculated differences of parallaxes at point 1 are made equal by changing the adjustment of the ruler of heights; in points 4 and 6 by the repetition device, turning the right filament on constant angle κ; in points 3 and 5 by adjustment of angle ρ_0 on the correctional device.

A pair is considered oriented when the difference between precalculated and observed differences of longitudinal parallaxes does not exceed 0.02 mm. Since orientation of aerial-photos on the topographic stereometer is executed on the basis of heights of a series of photographed points, errors in these heights will lead to incorrect orientation, which can be revealed by control points located on the stereopair.

Tables 13 and 14 show an example of orientation of aerial-photos on the topographic stereometer. Values of geodesic heights and precalculated parallaxes are given in Table 13.

Table 13

No. of points	Geodesic height A, м	Elevation h, м	Precalculated Δp, мм	Notes
2	277,0	—	—	$b_0 = 63{,}9$ мм
1	296,8	+ 19,8	+ 0,57	$H_0 = 2223$ м
4	355,5	+ 78,5	+ 2,34	$\Delta\epsilon_x = +0°45'$
3	298,5	+ 21,5	+ 0,62	
6	239,6	− 37,4	− 1,06	
5	255,5	− 21,5	− 0,62	

In Table 14 gives the results of observations of difference of parallaxes [9].

By executing orientation one can determine heights of points by results of measurement of the difference of their longitudinal parallaxes or by horizontals [contours] on the aerial-photo. For drawing horizontals, first calculate the difference of longitudinal parallaxes by elevations of horizontals relative to the initial point.

For instance, if initial height A = 277 m; b = 63.9 mm; H = 2223 m, and the selected horizontal has height of 200 m, we obtain

$$\Delta p = \frac{63{,}9\,(200-277)}{2223-(200-277)} = -2{,}14 \text{ мм.}$$

Changing the parallactic screw reading by the calculated magnitude of the difference of longitudinal parallaxes permits obtaining a reading corresponding to the selected section, which is established on the scale of the screw. Then with a shift of both aerial-photos along axis x of the instrument and of the observation system along axis y, it is possible to note points of the model with which the spatial mark coincides, and to connect them with a smooth curve, called the horizontal [contour] of the given section. Analogously on the scale of the parallactic screw it is possible to establish a reading corresponding to another section, and thereby on the area of the stereopair to dr[illegible] horizontal of any section.

Table 14

No. of points	Reading on the parallactic screw, mm	Reading should be mm	Observed differences of parallaxes, mm	Notes
2	63,94	—	—	
1	64,78	64,51	+ 0,27	Correction of δH
2	66,06	—	—	
1	64,53	64,52	+ 0,01	
4	65,56	65,29	− 0,36	Correction of κ_g
4	65,27	65,29	− 0,02	"repetitive"
3	64,80	64,57	+ 0,23	Correction of ρ_0
3	64,85	64,57	− 0,08	
2	63,91	—	—	
1	64,46	64,48	− 0,02	
4	65,26	65,26	+ 0,06	
6	62,79	62,86	− 0,06	
4	65,24	65,26	− 0,01	Correction κ_g
6	62,83	62,86	− 0,02	
3	64,55	64,53	+ 0,02	
5	63,22	63,29	− 0,07	Correction ρ_0
3	64,51	64,53	− 0,02	
5	63,26	63,29	− 0,03	
			Residual errors	Final result
2	63,92	—	—	$\Delta\kappa_g = +0°.48'$
1	64,47	64,48	− 0,02	$\rho_0 = 360°\ 40'$
4	65,24	65,26	− 0,02	$f_k + \Delta H = 202,42$ mm
3	64,51	64,51	− 0,06	$b = 63,9$ mm
6	62,84	62,86	− 0,02	$H_0 = 2223$ m
5	63,27	63,30	− 0,03	

After drawing the relief, turn the right aerial-photo 180°, but instead of the left photo place the following aerial-photo on the carriage of the instrument and orient it to the right. Therefore the drawing of relief is executed again on the right aerial-photo, i.e., only on even (or only on odd) aerial-photos of the flight line.

The accuracy of determination of heights of points on stereometers depends on errors of magnitude of the difference of longitudinal parallaxes Δp entering the formula of elevations (104). Errors Δp depend on a series of factors: distortion of the objective, film not flat at the time of exposure, nonuniform distortion of negatives and photographic paper, errors of the instrument, and others.

As operational experience shows, on aerial-photos glued on glass, the total error in measured differences of longitudinal parallaxes is approximately equal 0.03-0.05 mm; in a forest region (during aerial photography in spring) it is 0.06-0.07 mm.

Differentiating formula (104) and disregarding the term of the second order of smallness, we obtain

$$dh = \frac{d(\Delta p)}{b} H;$$

with b = 72 mm and d(Δp) = 0.03 mm

$$dh = \frac{H}{2400}.$$

In certain cases d(Δp) = 0.06 mm) dh is lowered to magnitude $\frac{H}{1200}$; this should be considered during appraisal of accuracy of work executed on the instrument.

A list and a short description of certain processes of treatment of separate stereopairs on the topographic stereometer are given below.

1. Drawing of points of conditional nadirs. By the formulas

$$\Delta x_n = -f_k \alpha \quad \text{and} \quad \Delta y_n = -f_k \omega$$

calculate and draw on the negatives the points of conditional nadirs. Calculation of angles α and ω is executed from elements of relative orientation.

2. Determination of heights of photographing H. The value of H is found by formula (114) or by data of the radio altimeter.

3. Determination of base b_0. This is calculated by formula (92).

4. Determination of heights of points of aerial-photos. If in the field only the heights of points 3, 4, 5, and 6 are determined, the heights of the other points are determined by one of the photogrammetric methods of thickening of the vertical control net expedient in this case (see §§ 30, 31, and 32). Orientation can be executed along four points (3, 4, 5, and 6), not determining the heights of points near the principal point.

5. Mounting of aerial-photos along the line connecting nadir points. In the topographic stereometer for this purpose we use small knots on threads located above centers of cassettes; under the small knots we should find the points of nadir of the aerial-photos. By shifting and revolving the aerial-photos we find

the approximate position at which the transverse parallaxes are observed along line x.

6. <u>Setting of angle $\Delta\alpha_x$</u>. By formula (99) calculate angle $\Delta\alpha_x$ and establish it on the convergent device of the topographic stereometer, where, if angle $\Delta\alpha_x$ is positive, the rulers should be extended. If the length of correctional ruler $d \neq f_k$, then the established angle $\beta = \frac{d}{f_k} \Delta\alpha_x$. Angle ρ_0 also is established beforehand.

7. <u>Calculation of true differences of longitudinal parallaxes</u>. Calculations are made for points 1, 3, 4, 5, and 6 with respect to point 2, taken in the topographic stereometer as initial on the basis of the formula

$$\Delta p = \frac{b_0 h}{H_0 - h}.$$

where h represents the elevations of points 1, 3, 4, 5, and 6 above point 2.

8. <u>Orientation</u>. The process of setting the correctional attachments is described above [9]. During orientation along four points relative to point 4 a somewhat different system is applied.

9. <u>Tracing of horizontals [contours]</u>. Horizontals are traced by hand. The most exactly obtained are horizontals of terrain rich in contours, and having no massive woods.

The process of drawing horizontals completes the work on the topographic stereometer.

2. Composition of Topographic Maps

The final stage after drawing of relief is composition of the original of the map, in process of which the central projection (image on the aerial-photo) is reduced to the orthogonal projection of the layout.

In regions with small differences of heights of points of terrain the map is composed from the photomap with horizontals plotted on it.

During transfer of horizontals from aerial-photos to the photomap the Bashtan stereoscope is used (Fig. 95). It consists of a table on which the aerial-photo was horizontals is placed; a mirror, located above the photo at a 45° angle; lenses and an internal mirror. A mirror with lenses and eye apertures shifts with respect to the aerial-photo and the mirror.

Due to this the scale of the visible image of the photo can be changed until

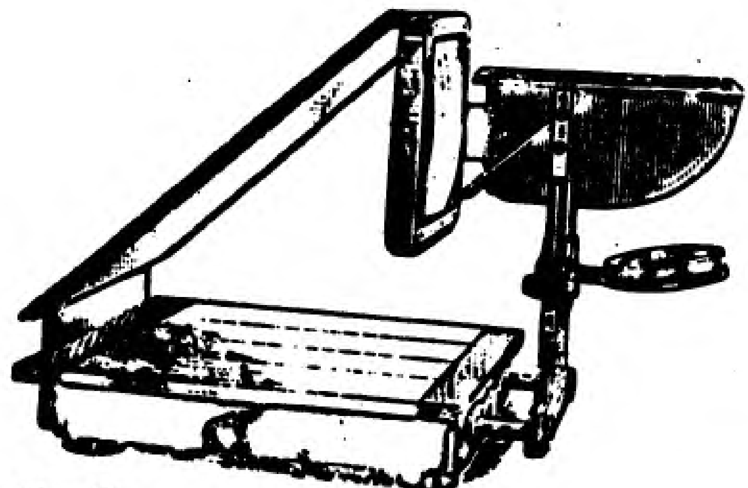


Fig. 95.

it coincides with the image of the corresponding part of the photomap and causes a stereoeffect; after that transfer the horizontals to the photomap. Work is conducted also with different scales of the photo and photomap (up to 30%). Photograph the photomap, prepare a blue impression, and trace the original of the map.

During composition of a map from individual aerial-photos proceed in the following way.

At first trace the horizontals and contours of the photo with India ink, then bleach the photos and prepare negatives from them reduce in size (approximately four times). For this purpose stack the prints in such a manner that the nadir point coincides with the projection of the center of the projector cassette (Fig. 96), given in a horizontal position, after which photographing is accomplished on a slide plate placed on the auxiliary frame of the projector.

The obtained negative is placed in the projector, which for rectification is set at angles α and ω and shifted in height until the coincidence of rectified points of the plane table with images of these points. Introduce preliminarily corrections for relief in the position of control points on the plane table. In the presence of great difference of heights on the area of the aerial-photo, project the latter by zones, lifting or lowering the projector above the screen.

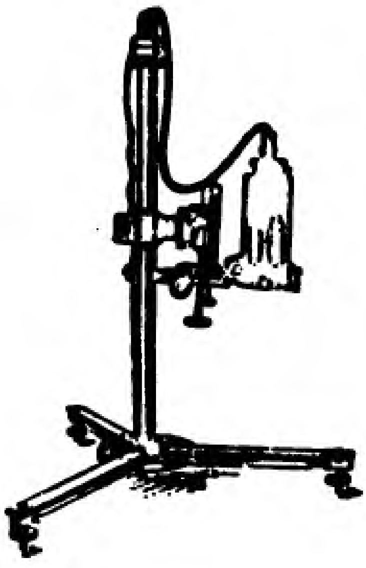


Fig. 96.

In addition bring the image of the zones to the same scale. The magnitude of shift in height of the projector is calculated by the formula

$$\Delta H' = 2 \cdot 0.4 \frac{H'}{d_0},$$

where 0.4 is the accuracy of the horizontal position of points in mm; H' is the distance from the nodal point of the projection objective to the plane table in mm; d_0 is the distance on the screen from the nadir point to the point of projection.

After transfer of horizontals and contours of the first zone to the plane table, set the projector at the height of the second zone without changing its angles of tilt and continue redrawing.

As a result of work on the projector a topographic plan is obtained in pencil; then it is traced in India ink according to the operative instructions.

In the USSR there exists the method of composition of maps with the help of a mountain photorectifier (created by F. P. Shevchenko, S. A. Pylayev, L. V. Pavlov, and P. P. Popov); it consists of tracing and rectifying in horizontals an aerial photo obtained as a result of treatment on the topographic stereometer. Then it is cut along the boundaries, for instance, along every fourth horizontal. After that all the cuts are filled with rubber cement and the aerial-photos are set on the assembly grid. With the help of special type faces of defined height, from the photo (by stretching of the rubber film and lifting of cuttings), leaving the upper zone below, a reverse model of the terrain is created. When it is photographed on a mountain photorectifier a negative is obtained not having distortions for relief. Then from negative a copy is prepared, which is used for composition of a topographic layout.

Using the given method of cartography, attain somewhat greater accuracy than during redrawing of contours and horizontals on the projector, especially in case of a complicated situation.

§ 30. Thickening of the Vertical Net by the Method of the TsNIIGAiK[1]

For creation of maps of the scale 1:25,000 use TsNIIGAiK method is also used. It is an analytic method of phototriangulation [9], [16] and consists of the following processes:

1. Determination of elements τ_1, τ_2 and ε of relative orientation of aerial-photos. Transverse parallaxes are measured in the stereocomparator or on the SM-4; for more precise definition of obtained results, elements τ and ε are calculated by formulas containing correction members.

2. Determination of conditional angles of tilt of all aerial-photos. Taking the first photo as initial with angles of tilt $\alpha = \omega = 0$, by summation of the quantities $\Delta t = \tau_2 - \tau_1$ and ε determine the angles of tilt of other photos with respect to the initial one. This gives the possibility then by the formulas

$$\Delta x = -f_k \alpha,$$
$$\Delta y = -f_k \omega$$

to determine the position of the conditional nadir point of every aerial-photo. For calculation of angles α use also statoscope readings.

3. Vertical phototriangulation. Taking points of conditional nadirs as working centers of directions on contour points, compose a rhombic net; include in it also the points utilized in subsequent processes of map making.

Vertical phototriangulation from points of conditional nadirs gives good results even with considerable relief, but with tilt of the conditional plane of less than 2°. After reduction and coordination of the net on available geodesic points, the plane table will be secured by control points in the layout.

4. Determination of heights of photographing. If there are no radio altimeter readings and elevation of points of the terrain does not exceed 100 m on the stereo-pair, determine height H by the simplified formula:

$$H = \frac{B_0 f_k}{b_0},$$

where B_0 is the distance between points of conditional nadirs on the terrain, measured on the phototriangulation net; b_0 is the distance between points of conditional nadirs on the aerial-photo, corrected for its tilt.

With great differences of heights of points of terrain consider also elevation

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[1]Central Scientific Research Institute of Geodesy, Aerial Photography and Cartography.

ΔH between ends of the base and the elevation between projections of conditional points of nadir to the terrain; calculate their values by the formulas

$$\Delta H = Bv \quad \text{and} \quad h = \frac{H}{b}\Delta p.$$

5. <u>Determination of corrected differences of longitudinal parallaxes Δp</u>. This work can be produced on an exact stereometer when elements of settings of $\Delta\alpha_x$, ρ_0, δH are known. If elevations are larger than 100 m, the corrected differences of longitudinal parallaxes are determined by formula (90), and the value of Δp is measured on the stereocomparator prior to the construction of the vertical net.

6. <u>Calculation of elevations</u>. By the formula of elevations

$$h = \frac{\Delta p}{b + \Delta p}$$

we find the values of conditional elevations of points on the area of the stereopair above the conditional point of nadir of the right photo of the stereopair.

7. <u>Reduction of conditional elevations to one level</u>. This process is executed on the basis of collation of elevations of points in zones of triple overlap. Differences of elevations of the same points obtained for the adjacent stereopair give a difference of initial levels of these stereopair; the mean value of levels is subtracted from elevations in subsequent stereopairs by which their reduction to the level of the preceding [pair] is achieved.

8. <u>Geodesic orientation and coordination of the vertical control net</u>. On the basis of three or four heights of geodesic points construct a graph of corrections for calculated values of heights of points of the net. This procedure is analogous to the process of geodesic orientation and coordination of the net during triangulation on the multiplex (see § 37).

The obtained vertical control points are necessary during work on topographic stereometers.

§ 31. Method of the Undistorted Model [9], [11]

Dependences between longitudinal parallaxes and heights of points of the model can be considerably simplified if we use values of transverse parallaxes of the same points (see formula (132)).

In the scale of the aerial-photo under the condition of orientation along the initial direction and exclusion of a constant number $f_k(\omega_1 - \omega_2)$ during orientation along the line passing through points of zero distortions, the value of transverse parallax q will be

$$q = -\frac{x_1 y_1}{f_k}\tau_1 + \frac{x_2 y_2}{f_k}\tau_2 - \frac{y_2^2}{f_k}(\omega_1 - \omega_2). \tag{115}$$

Replacing x_1 with $x_2 + b$ and taking $y_1 = y_2$, we obtain

$$q = -\frac{x_2 y_2}{f_k}\tau_1 - \frac{b y_2}{f_k}\tau_1 + \frac{x_2 y_2}{f_k}\tau_2 - \frac{y_2^2}{f_k}(\omega_1 - \omega_2).$$

After multiplication of both parts of the equation by x_2/y_2,

$$q\frac{x_2}{y_2} = -\frac{x_2^2}{f_k}\tau_1 - \frac{b x_2}{f_k}\tau_1 + \frac{x_2^2}{f_k}\tau_2 - \frac{y_2 x_2}{f_k}(\omega_1 - \omega_2).$$

If in a later conversion according to equations (99), (101), and (103) we replace the value of $\tau_1 - \tau_2$ with $\alpha_1 - \alpha_2$ and the value of τ_1 with $\alpha_1 - \frac{\delta H}{b}$, we obtain

$$q\frac{x_2}{y_2} = -\frac{x_2^2}{f_k}(\alpha_1 - \alpha_2) + \frac{\delta H}{f_k}x_2 - \frac{b x_2}{f_k}\alpha_1 - \frac{y_2 x_2}{f_k}(\omega_1 - \omega_2). \tag{116}$$

Furthermore, the value of the difference of longitudinal parallaxes according to formula (111) (without taking into account the third and sixth members, which are functions of elevations of points) will be equal to

$$\Delta p = \Delta p_i + \frac{x_2^2}{f_k}(\alpha_1 - \alpha_2) - \frac{\delta H}{f_k}x_2 + \frac{2 b x_2}{f_k}\alpha_1 + \frac{y_2 x_2}{f_k}(\omega_1 - \omega_2) + \frac{b y_2}{f_k}\omega_1. \tag{117}$$

Summing up the equation, we obtain

$$\Delta p + q\frac{x_2}{y_2} = \Delta p_i + \frac{b}{f_k}(x_2\alpha_1 + y_2\omega_1),$$

from which

$$\Delta p = \Delta p_i - q\frac{x_2}{y_2} + \frac{b}{f_k}(x_2\alpha_1 + y_2\omega_1). \tag{118}$$

Taking into account members containing Δp_1, we find

$$\Delta p = \Delta p_i - q\frac{x_2}{y_2} + \frac{x_2}{f_k}(2\Delta p_i + b)\alpha_1 + \frac{y_2}{f_k}(b + \Delta p_i)\omega + \frac{2b\Delta p}{f_k}\alpha_1. \tag{119}$$

From formula (119) it follows that the first two members solve the problem of determining the difference of parallaxes of points of relative orientation of aerial-photos; the remaining members determine the dependence of the difference of parallaxes on elements of relative orientation and elevation of points. Those and other members can be determined graphically, since all members of the equation have a linear character.

In Fig. 97 shows how the expression $q\frac{x_2}{y_2}$

$$\frac{\delta q}{q}=\frac{x_2}{y_2}.$$

Values of $\frac{q}{y_2}$ for different points of the model can be obtained from a graph constructed on the stencil by values $\frac{q}{y_2}$, measured for three points located on edges of the model. Values of $\frac{q}{v_2}$ for these points are interpolated linearly along the sides connecting the points. After that points with identical values of $\frac{q}{y_2}$ are connected by lines. Using the graph one can determine the value of $\frac{q}{y_2}$ for points located in any part of the pair, and then calculate by the formula

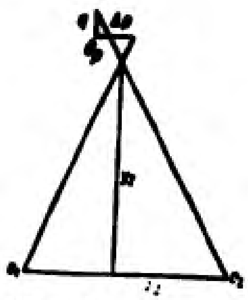

Fig. 97.

$$h=\frac{\left(\Delta p_1-q\frac{x_2}{y_2}\right)H}{b+\left(\Delta p_1-q\frac{x_2}{y_2}\right)} \tag{120}$$

the values of elevations of points of the model.

Then find for geodesic points the difference between photogrammetric and geodesic heights $A_\Phi - A_\Gamma$ and construct a second graph analogous to the first. From this graph of geodesic orientation we find corrections for photogrammetric heights of all points of thickening. The presented method is called the method of the undistorted model.

The value of $q\frac{x}{y}$ can be determined with the help of an automatic attachment to the stereocomparator (Fig. 98) developed by G. V. Romanovskiy. The attachment consists of vertical carriage 1; Cardan joint 4 connected to binocular carriage y; Cardan joint 3 belonging to carriage 1; external Cardan joint 2; holder 6, located on the repetition carriage (Δx, Δy) 7, and precision rod 6. The distance from the center of oscillation of Cardan joint 4 to the center of oscillation of Cardan joint 3 is changed by the movement of vertical support 1 until the transverse parallaxes of the point is eliminated (by displacements of carriage 7 with the aerial-photo). After this measure the longitudinal parallax of the point.

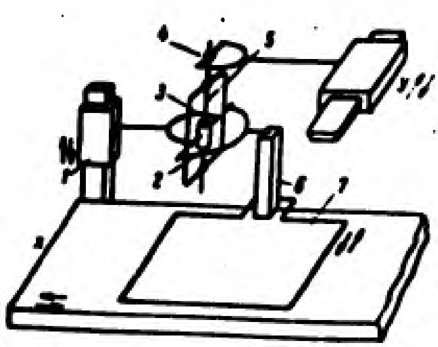

Fig. 98.

The method of the undistorted model gives high accuracy of determination of elevations of points. It is used for treatment of photo runs and for obtaining not only the elevations, but also the vertical position of points.

The quantity of calculations characterizing spatial triangulation considerably decreases with application of the correction mechanism of G. V. Romanovskiy, and the productivity of labor is increased.

§ 32. Spatial Triangulation with Help of the Electronic Computer

In connection with the appearance in USSR of different types of electronic computers there appeared the possibility of their use in solving equations of tying of coordinates of points of terrain and aerial-photos. The process of phototriangulation [13] involves these stages: measurement on a high-precision stereocomparator of the coordinates of points, then transferring their values from the decimal to the binary system. After preparation of punched tape according to a given program the latter is fed into an electronic computer (for instance, the "Ural"). Results of the machines work are fixed on forms.

The high speed and accuracy of work of the machine opens the way for mass application of equalizing calculations in the region of phototriangulation from aerial-photos and obtaining of vertical control points for work on simplified stereophotogrammetric instruments.

CHAPTER VI

UNIVERSAL STEREOINSTRUMENTS OF OPTICAL PROJECTION OF AERIAL-PHOTOS FOR CREATION OF TOPOGRAPHIC MAPS

§ 33. Principal of Spatial Resection

During treatment of a pair of aerial-photos it is possible to obtain topographic layout of the terrain with image of relief in contours, for which it is first necessary to create a 3-D model of the terrain (optical, mechanical or by analytical method), and then to produce its measurement.

Let us consider the solution of this problem by the method of projection of neighboring aerial-photos (Fig. 99) on a common screen and stereoscopic measurement of the model. Double projectors and multiplexes are based on this method.

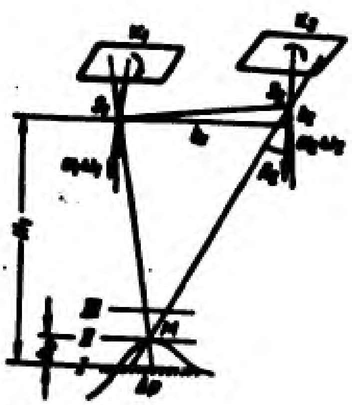

Fig. 99.

In chambers of the projector, located relative to one another at the distance of the chosen base, place the slides of aerial-photos in their actual form or decreased in size. Projecting chambers have the same relative position that they occupied at the time of aerial photography.

The slides are projected on the common screen.

If the distance from rear nodal point S of the projecting objective to the layer of emulsion of the aerial-photo is equal to the focal length of the surveying camera, then projecting rays are similar to surveying rays, as during rectification of the first kind. The projecting rays of both aerial-photos will then form at the intersection a model similar to the terrain. The plane of the screen can occupy any position along the height (for instance, I, II), making it possible to produce a section of the spatial model with different horizontal planes for determination of elevations and the image of relief by horizontals. For exact spatial resection of a point it is necessary that projecting cameras be established in accordance with angles of tilt of the surveying camera, that aerial-photos be correctly oriented in the projection cameras, and that the location of both centers of projection be similar to their position during aerial photography with respect to both the base and the elevations of points of photographing.

Fig. 100 shows the position of principal axis So of the aerial camera in the system of coordinates XYZ. This position is characterized by angles α_0, κ or α, ω. The first system (α_0 is the angle of tilt of the aerial-photo in the plane of the principal vertical, and κ is the angle between the line of coordinate marks and the principal vertical) was used during the study of properties of a single aerial-photo. When using pairs of aerial-photos for the creation of space resection, it is more convenient to express their tilts by angles α and ω lying in coordinate planes XSn and YSo', passing through points of projection. Therefore, every projection camera should be provided with the means for tilting it at angles α and ω of from 0 to ±5° and rotation of the aerial-photo in its own plane from 0 to 360°.

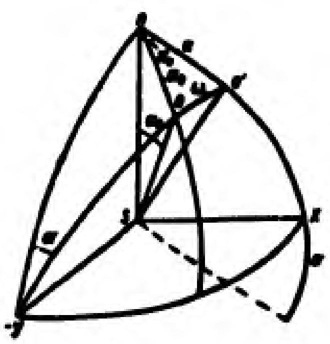
Fig. 100.

Furthermore, it is possible to establish one projector with respect to another at magnitudes b_x and b_z, determining the base and elevation of one station relative to the other.

For tracing contours on the plane table match the mark with points of the contour, if necessary shifting the screen along the height so that in its plane one image is obtained.

If the intersection of analogous rays is not located in the plane of the screen, then on the latter the two images of similar points are obtained; the distance between them corresponds to the difference Δp of longitudinal parallaxes on the aerial-photos.

The connection between the difference of longitudinal parallaxes of a point on the screen and the elevation of a point is determined by the formula

$$\frac{b_x - \Delta b_x}{\Delta p} = \frac{H-h}{h},$$

where $b_x - \Delta b_x$, Δp are expressed in millimeters.

Then the value

$$\Delta p = \frac{(b_x - \Delta b_x)h}{H-h},$$

where h and H are expressed in meters.

From this equation after conversion it is possible also to write

$$h = \frac{\Delta p H}{(b_x - \Delta b_x) + \Delta p}. \tag{121}$$

Thus, by measuring the magnitude b_x as the distance between the projectors, correcting it for the magnitude

$$\Delta b_x = b_x \operatorname{tg} A_2 \tag{122}$$

(A_2 is the angle formed by projected ray S_2M with the principal ray of right cluster), and then placing in formula (79) the value of H expressed in meters, it is possible to find the value of h also in meters according to the difference of longitudinal parallaxes. It is practical to calculate h not by the formula, but by the known scale of the model. For instance, if the scale of the model is equal to 1:10,000, then an outcrop of one millimeter on the scale of the model corresponds to 10 m on the terrain. Determination elevations of points, consequently, consists of raising or lowering the screen from its initial position until the mark is matched with the identical images of a determined point and measuring the magnitude of the shift of the screen in height.

Exact fixing of the points giving a single image in the plane of the screen, is possible in the case when two images are projected alternately from two to five times per second with the help of shutters revolving in front of the objectives. With this method of flickering the illumination of the screen is sufficiently great, and rectification of the image (in 0.2-0.3 mm) is sensed quite well by the observer in the form of apparent movement of the projected points. Fixed points located in the plane of the screen upon their connection by a pencil give the horizontal of an assigned section. The speed of shutter rotation should not exceed five blinks per second, since the eyes of observer, preserving the impression of the preceding projection, will perceive no shift of the image, even with considerable divergence of points on the screen. The disadvantage of the method of blinking should be considered to be the great eye fatigue of the observer. Therefore the given method of observation of the model is rarely used.

In double and multiple projection stereoscopic observation of aerial-photo images most frequently is used; in this connection we will consider the structure of the eye and the process of formation of a visual stereomodel.

§ 34. Monocular and Binocular Observation

1. Structure of the Eye

The eye is similar to a photographic camera (Fig. 101). In the front part of the eye is the cornea 1; pupil 2 with a diameter 3-7 mm, executing the function of a diaphragm; and crystalline lens 3, constituting the objective. The crystalline lens of the eye has the ability to change its focal length. Therefore the image on the retina always remains sharp, regardless of the distance to the observed subject. The focal length of the crystalline lens is changed by means of expansion or contraction of its siliar muscles 4, which receive appropriate impulses from visual cells. The adaptability (accomodation) of the eye for examining of objects depends on the remoteness of the eye of the observer. For instance, focal length of the eye at rest is equal to 22.79 mm, while under stress it is 18.93 mm.

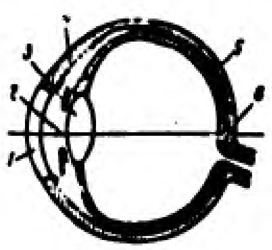

Fig. 101.

The capacity for accommodation of eyes of observers changes with age (Table 15).

Table 15

Age in years	Distance to a near point mm	Capacity for accomodation in diopters	Note
20	80	11,7	Distant point of view is at infinity
30	130	7,8	
40	210	4,8	Diopter $D = \frac{1000}{f\,mm}$
50	[illegible]	2,6	

For eyes aided by spectacles the values of the distances to the nearest points change; therefore increases for various observers are different. The distance of best sight for the normal eye, accepted for optical calculations, is equal to 250 mm.

In the rear part of the eye is the retina 5, on which there is constructed the inverted image of objects seen. The retina consists of visual elements, rods and cones with dimensions of about 0.004 mm; the visual cells in every eye number about 140 million. Simultaneously the field visible to the eye is characterized by these angles; upwards +40°, downwards -60°, into the nose +50°, out to the temples -75°[1]. However, visual acuity is preserved only in the middle of the retina on the yellow spot 6, with a diameter of 2 mm (in angular measure ~40'). In the middle of the yellow spot is its most sensitive part – around recess with a diameter of approximately 0.3 mm. Visual acuity in the recess exceeds by 20 times the acuity of marginal vision. The line passing through the nodal point of the crystalline lens and recess is called <u>the optical axis of the eye</u>.

At a certain distance from the yellow spot on the retina is the exit point of the ophthalmic nerves, the so-called blind spot, with a diameter of 2 mm. Rays falling on the blind spot do not give a visual sensation. To prove this, place two points on a paper 65 mm apart and parallel to the base of the eyes. When you close the left eye and look with the right (located opposite the right point) at the left point, it is evident that at a distance of 15 cm from the eye the right point disappears. The angle between rays proceeding to the fovea centralis and the blind spot is approximately 12°. The distance between nodal points of crystalline lenses of eyes is called the <u>ocular base</u>. The ocular base is different for different people, varying within limits of 55-72 mm.

<u>Resolving power of the eye</u>. The resolving power of monocular vision, as established by experiment, is equal to ~55", which agrees with the precalculated value. Separate objects are represented only when there is no less than one unstimulated cell between the edges of the image. Actually, since the dimension of a visual cell is 0.004 mm and the focal length of the crystalline lens is approximately 15 mm, then

$$\varepsilon = \frac{d}{f} = 0{,}000267,$$

which corresponds to 55" in angular measure.

The resolving power of monocular vision is increased to 20" if the observed object is not a point, but closely located lines; this is explained by the fact that a number of visual participate cell in the observation.

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[1]S. V. Kravkov. The eye and its work. Moscow-Leningrad, Publication of Academy of Sciences of the USSR, 1950.

The resolving power of the eye depends also on illumination of the image on contrast. With an increase in illumination, the diameters of pupils decrease, which leads to an increase in their resolving power (Table 16).

Table 16

Illumination lux	Resolving power of the eye in minutes (of arc)
1	1,21
10	0,74
50	0,62
100	0,51
200	0,38

The greatest visual acuity during observation of two series of set marks is attained with monochromatic illumination (3-5 times higher than with normal illumination).

Stereoscopic vision. During observation of an object with both eyes the latter automatically turn in such a way that the highly sensitive pits of the retinas turn are located on the sighting axes of the eyes. Objects located on the terrain outside points of fixation, are projected to the side of the recesses; if the distance to these objects is not the same as the distance to points of fixation, the magnitude of shift of their images in the left and right eyes is not the same, and differences of their shifts, or parallaxes as they are called cause the representation of a 3-D arrangement of the objects. The least difference of angles $d\delta$, formed by intersection of directions to the point of fixation and other observed points at which the difference of the distances to points of space is sensed, is called the resolving power of stereoscopic vision; it is equal to 30" for separately taken points. Let us determine the shift dL of points in depth (Fig. 102) which is not perceptible with stereovision. Differentiating the equation

$$\delta = \frac{b}{L},$$

we obtain

$$d\delta = -\frac{b \cdot dL}{L^2},$$

from which

$$dL = -\frac{d\theta L^2}{b}. \qquad (123)$$

In Table 17 there are given the values dL.

Table 17

Distance L m	Depth dL mm	Note
0,250 1,0 10,0	0,14 2,2 220,0	$b = 65$ mm $d\theta = 30''$

<u>Stereoscopic effect in examining a pair of neighboring aerial-photos or their projections on the screen</u>. As is known, every aerial-photo overlaps the neighboring one along the photo run by 60-70%. Depending on relief of the terrain, identical points of neighboring photos or their projections occupy a different position relative to the principal points, as also on the retinas of eyes of the observer (the only distinction lies in the fact that on aerial-photos parallaxes are constructed not on a spherical surface but on a plane parallel to the base). Therefore during simultaneous observation of the left aerial-photo by the left eye, and the right photo by the right eye, the observer perceives then a model of terrain, which is called a <u>stereoscopic model</u>. When the relative positions of the aerial-photos are renewed there is obtained a reverse stereoscopic effect, and when they are turned in their own planes by 90° a zero stereoeffect is achieved. Since in double projectors the images of two aerial-photos are projected on a common screen, for stereoscopic observation it is necessary to use a means whereby it is possible for the left eye to scan only the left image and the right eye only the right image. Such means are anaglyphs and polaroids.

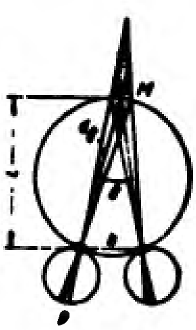


Fig. 102.

2. Anaglyphs

<u>The anaglyphic method</u> of observation consists of examining through ocular light filters images which are "dyed" same color with the help of light filters fixed in the projecting cameras. Anaglyphic light filters are celluloid (or glass) films, one red and the other green. These colors are basic, mutually complementing one another, since with the imposition of one on the other a white color is obtained. Through the red filter of the spectacles one can see a red mase and through the green filter a green image. Therefore during examination through

anaglyphic light filters of the mixed colors of the projections obtained on the screen from left and right aerial-photos, each eye sees only one corresponding image, due to which the stereoeffect is created.

Other combinations of complementary hues are also possible. The best result are obtained when one of the filters passes yellow rays and the second passes dark-bluish. The ratio of the luminous fluxes with this is equal to approximately 2.28 and corresponds to the known position that the red part is darker. According to Kravkov ("The Eye and Its Work") the rays of complementary hues (pairs with wave lengths 616 and 487 mμ in joint projection only give a white color when the ratio of their intensities is approximately equal to 2.722.

In the projectors the light filters are located between the source of light and the negative. For correct selection of filters in projectors it is necessary also to consider the spectral composition of the rays of an electric lamp. Therefore anaglyphic light filters for examining printed stereophotos or geometric drawings by the typographical method are necessarily different from light filters for examining anaglyphic images.

Illumination of the anaglyphic image composes approximately 30% of the illuminance without filters; therefore the need appears of increasing the power of the illuminator.

3. Polaroids

To increase the brightness the image is projected and examined through polaroids, consisting of aerial film in which microscopic crystals of an appropriate substance, (for instance herapathite – artificial "tourmaline," consisting of quinine sulfate periodide) are oriented in one direction.

Light rays passed through the polaroids are polarized in a determined plane; they are not depolarized during reflection from an aluminized screen. Therefore, if in the path of light rays passing from projectors there are set two polaroids polarizing the light in two mutually perpendicular directions, and the images projected on the screen are viewed through spectacles with polaroid lenses instead of glasses, then the polaroid of the spectacles whose plane of polarization coincides with the polaroid of one of the projectors will pass the image projected by this projector, but will not pass the other image. Analogously the second polaroid of the spectacles will pass to the eye of the observer the image projected by the second projector, i.e., each eye of the observer will perceive the image of only one aerial-photo, but in observation with both eyes the observer will see a 3-D model of the terrain.

Polarization is carried out within the wavelength limits 630-670 mμ, allowing application of filters even when they are heated by the illuminating installation (120° during brief work and 70° during prolonged work).

As compared to Nicol's prism passing 50% of white light, polaroids pass in all 40% of the light falling on them. But then the angle of observation through them can deviate by 35° from the vertical, which allows the use of polaroids also for double projection with the angle of the field of vision of objectives being on the order of 62-70°.

§ 35. Regarding Sharpness of the Image on the Screen

Sharpness of the image on the screen is preserved within small limits (in depth), since the focal length of the chamber of the projector remains constant. Fig. 103 shows rays passing through the edges of the diaphragm $d = 2a$ of the objective. The depth of sharpness is determined by the equation

$$\frac{\delta}{d} = \frac{h}{H},$$

from which

$$h = \pm \delta \frac{H}{d}, \qquad (124)$$

where δ is the permissible diameter of the circle of scattering, which can be taken as equal to 0.2 mm.

Fig. 103.

At $H = 300$ mm, $d = 2$ mm we obtain $h = \pm 30$ mm, and with the scale of the image equal to 1:5000, $n = \pm 150$ m.

For practical purposes the sharpness of the image remains acceptable in large limits, equal approximately to ±3h. In M. D. Konshin's work[1] results are given concerning the investigation of the magnitude of the error of measurement of parallaxes caused by blurryness of the image. This error does not change its value even with a diameter of the circle of scattering equal to 0.5 mm.

Parallaxes are measured two to four times more exactly than this corresponds to the resolving power of the aerial-photo. With a decrease in the effective

[1]Collection of TsNIIGAiK, Issue 10. "Research in Photogrammetry," Moscow, Geodezizdat, 1941.

aperture of the objective of the projector the depth of sharpness is increased, but the resolving power of the image becomes worse, since diffraction of light begins to influence it.

Two objects are depicted separately by the objective if the angle between the objects is larger than ρ'', determined by the expression

$$\frac{\rho''}{206265} = 1{,}22\,\frac{\lambda}{d},$$

where λ is the wavelength, equal to 0.00055 mm; d is the diameter of effective aperture of the objective, in accordance with which

$$\rho'' = \frac{138}{d}. \qquad (125)$$

Table 18 shows comparative data on the resolving power of the objectives and the eyes, as calculated by formula (125).

Table 18

Type of objective	Focal length f, mm	Effective aperture of the objective f, mm	Resolving power ρ'' Theoretical	ρ'' Practical	Note
Tessar	200	44	3	20	Diameter of a grain of emulsion 0.02 mm
Russar	100	18,5	8	41	
The eye	16	3	46	45	Diameter of visual cells 0.004 mm.
Ortoniar of the double projector	100	6	23	23	
Re2beIM to the multiplex	22,5	2	60	60	

From this it is clear that to preserve the resolving power of the image at 20" (in the aerial camera) it is necessary that the diameter of effective aperture d of the projecting objective be equal to 6 mm.

§ 36. Double Projector[1]

In the given instrument (Fig. 104), intended for creation of a large scale map from aerial-photos, projectors are used with focal lengths equal to the surveying f (on the order of 150 mm); therefore the slides serve as copies of

[1] The firm Bausch (United States).

the aerial-photos.

The illuminating system consists of two condensers united to a telescopic lever, connected with the plotting table. In the path of the light bundles there are placed anaglyphic filters. The image of aerial-photos is projected onto a small screen of the plotting table; it is observed through anaglyphic spectacles.

A section of the model is introduced by means of a change in the height of the screen of the surrounding table. With a shift of the table contours and outlines are traced with a pencil on the plane table, located on the screen of the instrument.

In another construction of a double projector there is used an ellipsoidal illuminator manufactured by means of deposition of a metal on the surface of a glass ellipsoid (Fig. 105). In one focus there is the source of light and in the other, the entrance pupil of the objective. The system ensures good brightness of the image.

Fig. 104.

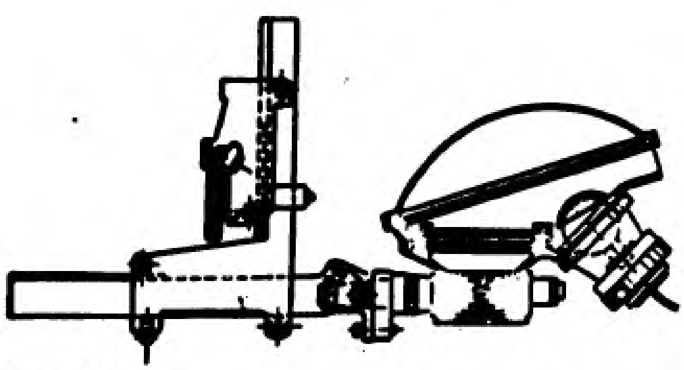


Fig. 105.

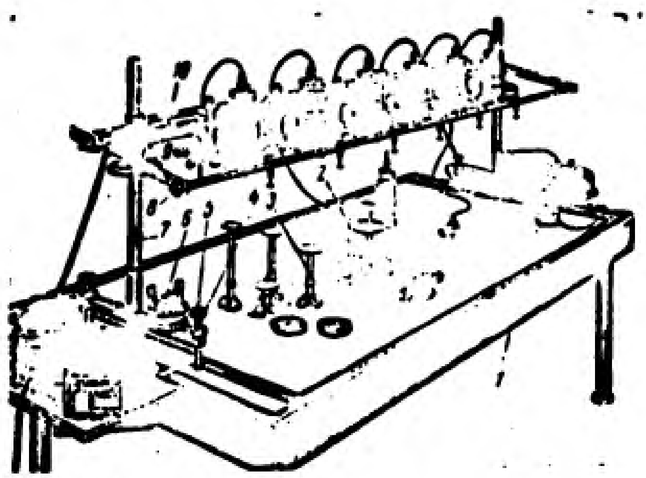
§ 37. Multiplex

1. Description of the Instrument

The instrument is intended for the construction of spatial [3-D] photo-triangulation nets and the creation of topographic map.

The composition of the instrument includes six to nine (and more) projectors producing an image on a common screen. With longitudinal overlap of aerial-photos of more than 50%, it is possible to construct a model of terrain over a considerable extent. Since the line of shift of the aircraft constitutes a curve in space of three dimensions, the projection of points of stations onto coordinate planes of a single system of coordinates gives values of components of base b_x, b_y, b_z, the possibility whose establishment is provided by the construction of the instrument. Considering that at the time of photography aerial-photos have tilts and turns in their own plane, projectors are assigned tilts along the flight line by angles α, across the flight line by angles ω, and rotation in the plane of aerial-photos is carried out through angle κ.

Fig. 106 shows an exterior view of the K. Zeiss wideangle multiplex with accessories.



GRAPHIC NOT REPRODUCIBLE

Fig. 106.

On table 1, covered by plate glass 5, is a bed with T-form screw supports 7 and a transverse crossbar with guiding arm 10. The crossarm with projectors can be lifted or lowered by means of wheel 9, by which movement one or another scale of the model is set up. It is possible to tilt the bed along axes X and Y by means of screws 4. On the crossbar arm there are suspended six sections of projectors, having the base settings b_x (to left – right), b_y (forward – backward) and b_z (upwards – downwards). Furthermore, each projector has three angular installations α, ω, κ. The projectors are protected from shocks by a safety rod with clamping screw 8.

On the table is a small plotting table board 2; it serves for circumscription by a luminescent mark of outlines and contours of the image of the terrain. For accelerating the process of orientation four height [vertical] tables 3 are used.

For measurement of angles of tilt of aerial-photos there is used two-level holder 6. It is installed on the projector Cardan ring after the illuminator is removed.

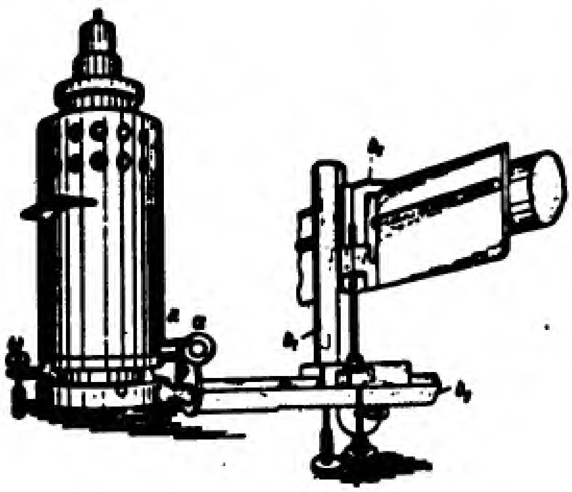


Fig. 107.

We show a multiplex projector (Fig. 107), which has linear installations b_x, b_y, b_z and angular α, ω, κ. These installations are secured by the projector's holder device. On the internal Cardan ring of the holder there is located a projector of small weight and dimensions, since the natural actual aerial-photos are not used but copies reduced in size (by 3.3 or 4.5 times). The diagram of the location of the optical elements of the multiplex projector depicted in Fig. 108 shows that the ray from the point source lamp drops on the first lens of the aspherical condenser, passes through the green (or red) light filter, through the second lens of the aspherical condenser, through the slide, and spherical segment with the mark. The rays are collected on the entrance pupil of the objective and thus the image of the slide is projected onto the screen.

The image of aerial-photos on the screen is enlarged as compared to the original (aerial negative) by approximately 3-4 times which makes it possible to observe the image with the naked eye with sufficient accuracy.

Images of neighboring aerial-photos are colored in complementary hues.

During examination of images through anaglyphic spectacles the model of terrain is observed, if slides in projectors are given a relative position similar to the position of aerial-photos at the time of the survey.

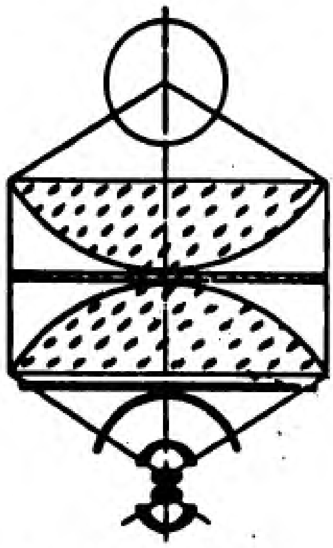


Fig. 108.

Fig. 109 shows the external view of the Soviet wideangle multiplex, 1960 model. On a table with thick plate glass, which is the screen, there are two vertical columns along which the girder with projectors is moved, allowing construction of the model in a zone of good sharpness of the image. The girder with projectors can also be tilted lengthwise or turned across the flight path of aerial-photos, which is used in geodesic orientation of the flight path model.

The most important attachment of the multiplex is the small plotting table (Fig. 110). Along vertical stand 3 with the help of a screw one can shift the small screen with point aperture 1, illuminated from below by the lamp; the vertical position of the screen is counted off on a scale. To the base of the table there is fastened attachment 4 with a pencil whose point is located on the plumb line passing through the point aperture. On this screen are sections of the model examined stereoscopically but its measurement is derived by a mark; the pencil traces both the outlines and contours of the terrain.

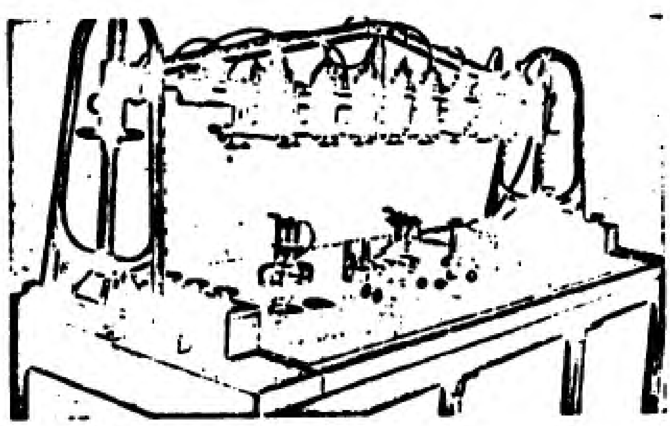


Fig. 109.

Slides for the multiplex are prepared on the reducer, an exact projector - a vertical type construction (Fig. 111). The negative is placed on glass 1 with coordinate marks and pressed to it by a second glass. The negative is illuminated from below by the lamp. On the folding body there is fastened frame 2 for slides and objective 3 with a focal length f' = 70 mm.

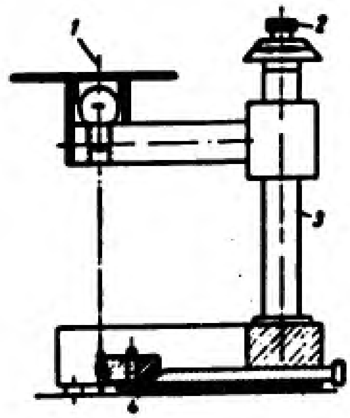


Fig. 110.

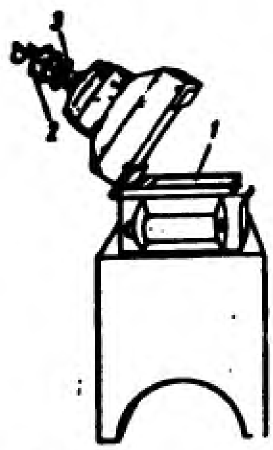


Fig. 111.

With the micrometer screw it is possible to set the position of the objective in height, and to read on the scale the position of the auxiliary frame which corresponds to the required reduction factor while retaining the sharpness of the image.

Table 19

Multiplex data	Multiplex type			
	Normal $2\beta = 62°$, $f_k = 210$ мм	Wide-angle $2\beta = 106°$ $f_k = 100$ мм	M-9	Super wide angle $2\beta = 122°$, $f_k = 70$ мм
Number of projectors	6—9	6—9	9	9
Degree of reduction of aerial-photos	4	4,5	3,3	3,3
Focal lengths of projectors f_p, мм	45	22	31,2	21
Dimension of slides мм	40×60	40×60	40×60	58×58
Focal length of objectives f, мм	40,0	20,7	28,6	19,6
Zone of sharpness мм	250—380	250—380	250—380	250—360
Value b_x	80—100	80—120	80—120	100—140
" b_y	±65	±60	±55	±60
" b_z	±40	±40	±35	±40
Angles of tilt in degrees α, ω, κ.	±8	±8	±7	±8
Angles of rotation in degrees	360	360	360	360

During calculation of objective of the reducer residual distortions of the surveying and projecting objectives are taken into account. For exact work of the reducer it is necessary that the surface of the slide be strictly parallel to the surface of the negative and the reduction factor be determined correctly. Correctness of the setting of the coefficient can be checked by comparing the facsimile of the control grid. Table 19 gives certain data on various multiplex types; of these the sideangle and super-side angle types are manufactured in the USSR and in the German Democratic Republic.

2. Checks of the Instrument

1. The screen should be a flat surface. With large dimensions of the screen (glass or marble) depressions can be observed exceeding the allowance of accuracy of measurement of heights (0.1 mm).

Check of the screen is conducted with the help of a level of 45-second accuracy, and correction of sag is done with screws of the screen.

2. The directrices of the table should be perpendicular. The plotting table mark and tip of the pencil pin also have to be on one perpendicular line. Observing on the mark the projection of the same point with two positions of the table, differing by 180°, it is possible by divergence of tilts to judge the necessity of resharpening of the pencil point or correction of position of the pencil holder.

3. The principal point on the spherical segment of the projector should be on the perpendicular to the plane of the auxiliary frame, passing through the nodal point of the objective. The plane of the auxiliary frame of the projector is brought to a horizontal position with the help of a striding level and then under the image of the principal point bring the mark of the plotting table. If after piercing the point with the pencil the table is removed, then, in the presence of error δ in the position of the principal point of a spherical segment, the image deviates from the marked position of the point by the quantity Δ.

The magnitude of the error will be determined by the relationship

$$\frac{\delta}{f_{\text{п}}}=\frac{\Delta}{h}.$$

With focal length of the projector $f_{\text{п}}$ = 22.93 mm, the permissible error of sighting is Δ = 0.3 mm and the height of the small screen of the plotting table above the screen is h = 100 mm the value of tolerance $\delta = \pm 0.06$ mm.

4. Determination of focal lengths of projectors is possible by means of projecting the measuring grid onto the plane of the small screen of the plotting table and onto the plane of the screen (Fig. 112). As a result of measurements of lengths of projections of segment, f_{π} can be determined from the equation

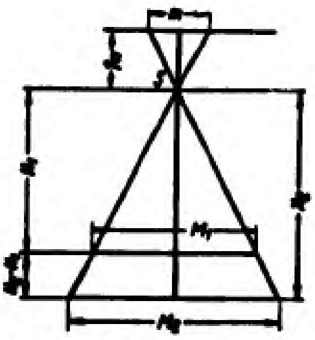


Fig. 112.

$$\frac{m}{M_2-M_1}=\frac{f_{\pi}}{H_2-H_1},$$

where m is the segment of the measuring grid; for instance to 40 mm and passing through the center of the grid; M_1 is the segment of image m in the plane of the small screen of the plotting table; M_2 is the segment of image m in the plane of the screen; H_2-H_1 is the distance from the multiplex screen to the small screen of the table, equal approximately to 100 m.

With accuracy of measurement H_2-H_1 up to 0.05 mm and of M_2 and M_1 up to 0.1 mm the mean square error of determining $m_{f_{\pi}} = \pm 0.03$ mm.

The inequality of focal lengths of projectors of a given multiplex should not exceed 0.02 mm, and their deviation from the assigned value should not be more than 0.5 mm. Then the disturbance of similarity of clusters of projected rays will not lead to perceptible distortions of the model.

5. Determination of the reduction factor. For manufacture of slides with reduction factor $\frac{f_k}{f_{\pi}}$ it is necessary to establish the corresponding distances c and d between the negative, objective, and frame for the slide.

Knowing focal length f' of the objective of the reducer, it is possible to calculate values of segments c and d established in the reducer by the formulas

$$c=f\left(1+\frac{1}{n}\right) \text{ and } d=f(1+n).$$

After their installation and the photographing of the measuring grid, measure the segments of the obtained slide on the stereocomparator. If the value of n, calculated as the ratio of segments of grid and slide, differs from the assigned value by more than 0.005, then it is necessary to change the length of segments c and d to repeat the photographing.

6. After the checks indicated above, make a general instrument check for which in projectors place slides of measuring grids, establish them in a horizontal position and on it conduct spatial phototriangulating along the X-axis of the instrument. In case of divergence of more than 0.1 mm in the heights of points common to neighboring pairs, replace or readjust the projectors. During this check the values of zero points of scales of basic components are also manifested by means

of readings on the corresponding scales.

For eliminating the influence of sag of the bed on the determination, the zero points of b_z, measure the position of chambers of projectors with the help of a support with an indicator adjoining at the measuring tip to the summit points of the objectives.

§ 38. Orientation of Aerial-Photos

For creating a topographic layout on the multiplex, it is necessary first to restore a model which is similar to the terrain. The process of orientation of one projector relative to another is called relative orientation of aerial-photos.

Reduction of the obtained model to the assigned scale and its orientation relative to the geodesic system of coordinates is called external or geodesic orientation.

1. Relative Orientation

The relative position of aerial-photos of the stereopair is determined by five quantities, called the elements of relative orientation.

With correctly fixed aerial-photos analogous rays intersect, forming a model of the terrain. Intersection of rays occurs independently of the dimension of the chosen base and relief; consequently, the presence of longitudinal parallaxes on screen does not prevent intersection of rays when they are extended.

With incorrectly fixed aerial-photos analogous rays do not intersect; besides longitudinal parallaxes there appear also transverse (noncoincidence of two images of the same point on lines perpendicular to the base).

Thus, the process of relative orientation of aerial-photos in the instrument consists of elimination of transverse parallaxes of points on the screen by appropriate settings of projectors.

The effects of the individual movements of the projector on the magnitude of change of transverse parallaxes of different points of the stereopair are unequal. Therefore, for relative orientation of aerial-photo select those points in which the corresponding movement causes the most effective elimination of transverse parallaxes. Furthermore, select an order of movements in which each subsequent movement does not cause the appearance of transverse parallaxes on the preceding points.

Two methods of relative orientation of aerial-photos exist: relative orientation with a horizontally fixed base and relative orientation with a nonhorizontal base.

Relative orientation with a horizontally fixed base. Let us establish two neighboring multiplex projectors on one height by means of shifting one of the projectors to a height at which the transverse parallax along axis X will not be observed. Then between neighboring projectors we establish one or other base b_x arbitrary in magnitude. Transverse parallax in five points of the stereopair are eliminated by rotation of screws in the order given in Table 20.

Table 20

Point	Screw	Note
2	κ_1	Diagram of location of points is shown in Fig. 94.
1	κ_2	
4	$\Delta\alpha_1$	
3	$\Delta\alpha_2$	
5	$\Delta\omega$	
6	Control	

Rotation of the left projector through angle κ_1 causes a change in the transverse parallax at point 2 on the screen according to the expression[1]

$$Q_1 = -X_1 \kappa_1. \tag{126}$$

Analogous rotation of the right projector through [this?] angle changes the transverse parallax at point 1:

$$Q_2 = X_2 \kappa_2. \tag{127}$$

Elimination of transverse parallaxes in points 4 and 3 is attained by tilt $\Delta\alpha$ of projectors in direction of the axis X. In agreement with Fig. 113, presenting the image of an aerial-photo tilted at angle $\Delta\alpha$, the distortion of the position of point m_0 is $m_2m_0 = q$; therefore

---

[1]Here and subsequently plus signs are used for coordinates and elements of orientation of aerial-photos as follows: for X and Y – to the right and forward; for $\Delta\alpha(\tau)$ and $\Delta\omega(\varepsilon)$ – during deflection of the principal axis from the perpendicular line (normal to the base) in the direction of positive coordinates; for κ – during count from the trace of the basic plane in a counterclockwise direction.

$$\frac{q}{x} = \frac{y}{oi},$$

where

$$oi = f_k \operatorname{ctg} \Delta\omega.$$

Hence we find that in scale of the model for points 4 and 3 the values of transverse parallaxes caused by tilt are determined by the expressions

$$Q_4 = -\frac{X_1 Y_1}{H} \Delta\omega_1 \qquad (128)$$

and

$$Q_3 = \frac{X_2 Y_2}{H} \Delta\omega_2, \qquad (129)$$

where X, Y are coordinates of points of image on the screen and H is the distance from the screen to the nodal point of the projector objective.

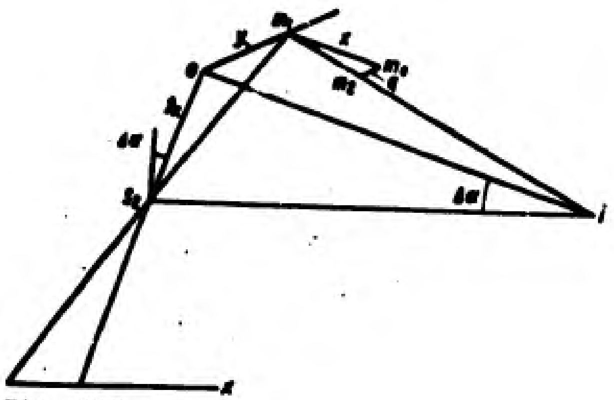


Fig. 113.

At point 5 the transverse parallax is eliminated by tilting the right aerial-photo at angle $\Delta\omega_2$ (or the left through angle $\Delta\omega_1$) in direction of axis Y.

Fig. 114 shows that in the absence of relative transverse tilt $\Delta\omega$ for both images segments o_1M_1 would be identical. With rotation of one aerial-photo at angle $\Delta\omega$ it is possible to obtain projection of segments o_2M_1' (equal o_1M_1) on the screen in the form $o_2'M_2$. Consequently, the transverse parallax

$$Q_5 = o_1o_2' + M_0M_2.$$

We will define the terms of the right sides of the equation:

$$o_1o_2' = H \cdot \Delta\omega,$$

from which

$$\frac{M_0M_2}{Y} = \frac{Y \cdot \Delta\omega}{H}.$$

$$Q_5 = H\cdot\Delta\omega + \frac{y^2\cdot\Delta\omega}{H}. \qquad (130)$$

Thus, transverse parallax Q_5 appears both in the central zone of the stereopair and on its edge. Therefore during its elimination by movement $\Delta\omega$ at point 5 transverse parallaxes in other points will remain which will make several approximations necessary.

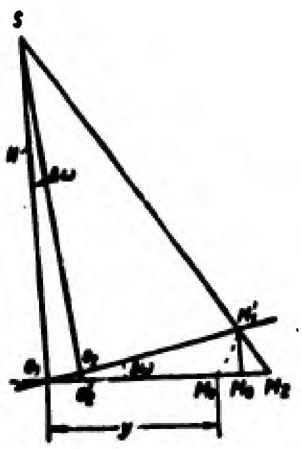


Fig. 114.

For manifestation of the conditions for complete elimination of transverse parallaxes on the basis of observation of it at point 5 we will turn to equation

$$H\cdot\Delta\omega + \frac{y^2}{H}\Delta\omega = \frac{y^2}{H}\Delta\omega\cdot k,$$

from which

$$k = \frac{y^2 + H^2}{y^2} = 1 + \frac{H^2}{y^2}$$

or

$$k = 1 + \frac{f_k^2}{y^2}. \qquad (131)$$

Here k is the coefficient of necessary increase in the transverse parallax in the visible on the screen at point 5.

Table 21 gives coefficients k for different focal lengths of the aerial camera and ordinates of points.

Table 21

Focal length f_k, mm	y, mm	
	60	70
70	2,3	2
100	3,8	3
200	12,0	9

Thus, at point 5 the transverse parallax is not eliminated, but introduced and enlarged k times taken with the opposite sign. After this the process of eliminating transverse parallaxes is repeated in all points.

At point 6 control of orientation is carried out.

The general type of formula of transverse parallax of a point on the screen will be obtained during summation of all its components:

$$Q = -X_1\kappa_1 + X_2\kappa_2 - \frac{X_1Y_1}{H}\Delta\alpha_1 + \frac{X_2Y_2}{H}\Delta\alpha_2 + \left(H + \frac{Y_2^2}{H}\right)\Delta\omega_2, \qquad (132)$$

where

$$\Delta\omega_2 = \omega_2 - \omega_1.$$

Formula (132) for any point of the stereopair can be used for analysis of transverse parallaxes. However, during relative orientation the calculations are not carried out; and the settings of projectors α, ω, κ are found by stepwise approximation. Transverse parallaxes are eliminated with an accuracy of 0.2 mm in the scale of the model.

The shown method of relative orientation is applied to separate pairs of aerial-photos. The process of orientation lasts 15-20 minutes.

Relative orientation on a nonhorizontal base. The left aerial-photo occupies an arbitrary position in the instrument and during orientation remains motionless.

The right aerial-photo is displaced and tilted in such a way that it is possible to establish elements of relative orientation.

The sequence of elimination of transverse parallaxes is shown in Table 22.

Table 22

Point	Screw	Note
2	b_{y_2}	Diagram of location is shown in Fig. 94.
1	κ_2	
4	b_{z_2}	
6	$\Delta\omega_2$	$Q_5 = (129)$
5	$\Delta\alpha_2$	
3	Control 1	

The elimination of transverse parallaxes at point 2 is carried out in accordance with the condition

$$Q_2 = -b_{y_2}, \qquad (133)$$

i.e., it is produced by shifting the right aerial-photo along the X-axis, which replaces the setting of κ_1 of the preceding method.

The elimination of transverse parallax at point 1 is subject to the condition

$$Q_1 = X_2\kappa_2, \qquad (134)$$

i.e., it is produced by rotation of the right aerial-photo through angle κ_2.

Eliminating the transverse parallax at point 4 is attained by introducing elevation of the right end of the base by the magnitude b_{z_2}. In agreement with Fig. 115,

$$-\frac{Q_4}{b_{z_2}}=\frac{Y_4}{H}.$$

from which

$$Q_4=-b_{z_2}\frac{Y_4}{H}. \qquad (135)$$

The transverse parallax at point 6 is eliminated by means of establishment of the relative transverse angle $\Delta\omega_2$, according to equation (130), but with the sign reversed:

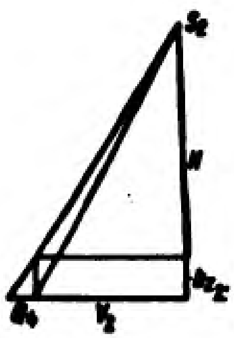


Fig. 115.

$$Q_6=-\left(H\Delta\omega_2+\frac{Y_6^2}{H}\Delta\omega_2\right). \qquad (136)$$

The transverse parallax at point 5 is eliminated by movement of $\Delta\alpha_2$. At point 3 the check of orientation is produced.

Let us give the general form of the formula of the transverse parallax for this method of relative orientation:

$$Q=-b_{y_2}+X_2\kappa_2-\frac{Y}{H}b_{z_2}+\frac{X_2Y_2}{H}\Delta\alpha_2-\left(H+\frac{Y^2}{H}\right)\Delta\omega_2 \qquad (137)$$

or in the system of the origin of coordinates of the left aerial-photo

$$Q=-b_{y_2}+(X_1-B)\kappa_2-\frac{Y}{H}b_{z_2}+\frac{Y_2(X_1-B)\Delta\alpha_2}{H}-$$
$$-\left(1-\frac{Y_2^2}{H^2}\right)H\Delta\omega_2. \qquad (138)$$

The method of suborientation of subsequent aerial-photos to the preceding ones is used to create a model of an entire flight photo run during spatial phototriangulation.

If the elevation between ends of the base (stations) is known, for instance from statoscope readings, then magnitude b_z, equal to this elevation in the scale of the model, is established on the corresponding scale of instrument immediately.

2. External Orientation

The model of terrain obtained as a result of relative orientation has a scale which does not correspond to the scale of the map; also is arbitrarily oriented relative to the geodesic system of coordinates. Therefore it is necessary to bring the model to the determined scale of the plan and to orient it relative to the geodesic system of coordinates, i.e., to carry out external orientation of the mode. For external orientation of the model three geodesic control support points are used; for two of them all three geodesic coordinates have to be known, but for the third -- only its height.

The problem of external orientation of the model is solved by the optical-mechanical method, by changing the basic components b_x, b_y, b_z and the tilt of the entire model with respect to the plane of the screen, which one should consider to be combined with plane XY of the geodesic system of coordinates. As a result, the three control points of the model will be combined with corresponding points of the base, placed on the screen, both in layout and in height.

Usually the model has angles of tilt equal to tilt of the first aerial-photo, taken as the initial and fixed in the instrument in a horizontal position; consequently, the angles of tilt of the model can be on the order of 1-2°. This permits determining the coefficient of reduction of the model from the ratio on the basis of simple comparison of distances between two points of model A and C and points of the plan A_0 and C_0.

$$\frac{A_0C_0}{AC}=k. \tag{139}$$

Then, by establishing in the instrument the new basic components Kb_x, Kb_y, and Kb_z we will obtain a model in the assigned scale.

For tilt of the model use the values of the heights of points A, B, C (Fig. 116).

Let us take point A as the origin of coordinates of the model, i.e., $h_A = 0$. Then, setting at point B of the screen a plotting table with geodesic elevation of point B, expressed in the scale of the model, fixed on it we see noncoincidence of the point of the model with the space mark; by shifting the latter in height, it is possible to measure the quantity h_B — the difference between geodesic and photogrammetric heights. Analogously we find the difference h_C between the geodesic and photogrammetric heights of point C of the model.

In order to determine the angles of tilt of the model it is possible to use an auxiliary construction on the plane table. For this purpose we draw from points C and B segments of axes X and Y to intersection with sides AB and AC or with their extensions at points D and E. After that we determine the difference h of the geodesic and photogrammetric heights of points D and E from the equations

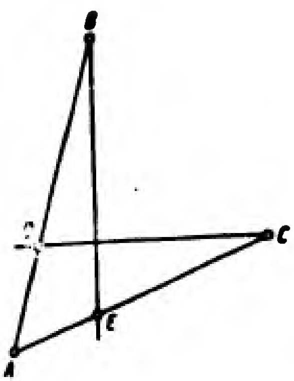


Fig. 116.

$$\frac{AB}{AD}=\frac{h_B}{h_D}$$

and

$$\frac{AC}{AE}=\frac{h_C}{h_E}.$$

The unknown angles of tilt of the model can be determined by the formulas

$$\alpha_i'=3438\frac{h_D-h_C}{DC}, \tag{140}$$

$$\omega_i'=3438\frac{h_E-h_B}{BE}. \tag{141}$$

The values of h_D, h_C, h_E, and h_B are taken with their signs; the obtained signs of angles of tilt of the model indicate the direction of its tilt; with positive α_i', ω_i' it is necessary to lower clockwise the right part of the model remote from the observer.

Tilting of the model is usually carried out by consecutive turns of the bed, with checking of the results of orientation by measurement of elevations between geodesic points of the model. For the horizontal model the measured photogrammetric elevations of points have to be equal to their geodesic values.

The multiplex is used in many countries which employ aerial photography. Its wide application is known for purposes of spatial triangulation, as well as for the creation of plane nets in scales of 1:25,000-1:10,000.

With good selection of light filters and lamps the multiplex can be used for map making on larger scales.

For the purpose of creating photomaps with contours by the slot method, the multiplex is equipped with additional attachments (carriages, photocassettes, and electric drives).

§ 39. Construction of the Spatial Phototriangulation Net and Plan

Measurement of the model obtained as the result of orientation of aerial-photos on the multiplex makes it possible to conduct spatial phototriangulation for the production of new vertical-horizontal control points over a considerable extent, expecially with the use of radio-altimeter and statoscope readings. After thickening the vertical-horizontal control and its coordination, it is possible to process separate models for composing a topographic layout. The process of spatial phototriangulation consists of the construction of a model (relative orientation of aerial-photos), measurement of coordinates of its points, and geodesic orientation of the spatial net.

1. Spatial Phototriangulation with Graphic Turn of the Levelling Net

Setting up the first and second aerial-photos, orient them relatively according to one of the considered methods. The length of the base is taken such that it is possible to construct the model in the zone of best sharpness of the image. Then with an arbitrary base to the second aerial-photo, subvarient the third aerial-photo by the second method (with a nonhorizontal base). After that bring the scale of the second model to the scale of the first. This is achieved by changing the valve of the base S_2S_3' (Fig. 117) to the value S_2S_3, at which common points (for instance, M_2 and M_1) will have identical heights in the two models.

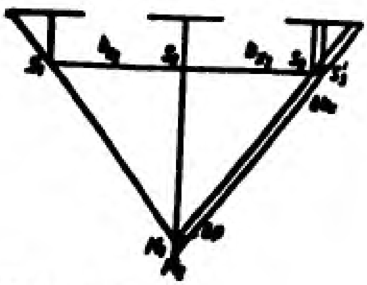


Fig. 117.

It is necessary also to establish new values of the base components b_y, b_z, changing them according to formulas of reduction to valves

$$\Delta b_{y_3} = \frac{\Delta b_{x_3}}{b_{x_3}} b_{y_3}; \quad \Delta b_{z_3} = \frac{\Delta b_{x_3}}{b_{x_3}} b_{z_3}. \qquad (142)$$

After checking the coincidence of heights on other common points found in the zone of triple overlap of aerial-photos and the improvement resulting from additional observations of the position of points S_3, proceed to the suborientation of the fourth and subsequent aerial-photos. Heights of points of the model are measured with the help of plotting tables.

When the length of the net exceeds five to seven bases, noticeable errors appear in the position of points, in view of tilt of the initial aerial-photo, flexure of the net, and its torsion. Let us suppose that on the area of the model (Fig. 118) there are five control points $M_1 \ldots M_5$, located on its edges and in the middle of it. After relative orientation of aerial-photos and determination of the scale of the net, combine one of the points of the model with the mark of the plotting table. Then in other points M_2, M_3, M_4, after establishing geodesic elevations on the measuring table, coincidence of the mark with points of the model will not be observed due to the tilt of the model and errors in its construction.

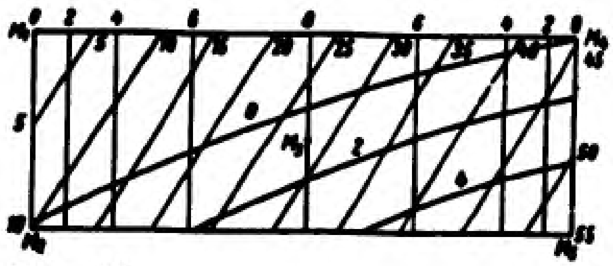


Fig. 118.

First we will coordinate the net on points M_1, M_2 and M_4. Let us assume that the divergence of heights at point M_4 is 45 m, and that at point M_2 is 10 m. Considering the significant length of the photo run in the scale of construction (800-1200 mm and more) and the small tilt of the model (1-2°), we may conclude that with small relief the points are not displaced in the horizontal relation. Therefore, for thickening the horizontal net there is no need to turn the model. For horizontalizing and correlating heights, trace on the base the graph of vertical corrections with an interval of, for instance, 5 m. Determining by graph the correction of heights of points M_3 and M_5 (in the given example the measured height of point M_3 must be corrected by 27.5 m and the height of point M_5 by 54.0 m), compare the corrected height of point M_3 with its geodesic height. A difference in them equal, for instance, to $\Delta h_3 = -8$ m indicates that there is a sag of the model. This sag occurs according to the law of the parabola, $\Delta h = \Delta X^2$ [16]; therefore for the introduction of corrections in points of the net we construct on the same base a second graph of lines of equal corrections. Values of corrections for a series of points of the net, utilized during construction of a graph, are calculated by the formula

$$\Delta h_i = \Delta h_3 - \Delta h_3 \frac{(X_i - X)^2}{X^2}, \quad (143)$$

where Δh_i is the correction for the height of a point, X is the distance from point M_3 to line M_1M_2, and X_i is the distance from the point with correction Δh_i to line M_1M_2.

For instance, with $\Delta h_3 = -8$ m, $X_i = 200$ mm, and $X = 400$ mm, the value $\Delta h_i = -6$ m, etc. Lines of identical corrections should be parallel, but due to errors can approach the edges of the model.

Let us suppose that after construction of both graphs the sum of corrections for point M_5 turns out to be still insufficient and $\Delta h_5 = 4$ m. In this case twisting of model occurs according to the law of the hyperbola, $\delta h = BXY$. To eliminate the influence of twisting of the model on the values of the determined heights of points, we will construct a third graph on the same base. For this one should execute linear interpolation of corrections between points M_4 and M_5, M_2 and M_3, and so on. By connecting the points with identical values of corrections we obtain curves of corrections for twisting of the model.

With the summation of values of corrections with measured height obtained from the graph it is possible to obtain final heights of the control and other points on the area of the model of the photo run.

The quality of completed spatial phototriangulation is estimated prior to its coordination. The mean square error in the determination of heights in middle of the net in the absence of statoscope readings is equal to [11]

$$m_h = \pm 0.15 \frac{H}{b} m_q \sqrt{n^3 + 19n + 48}, \quad (144)$$

where H is the height of photographing in m; b is the base, equal in the scale of the aerial-photo to ≈70 mm; m_q is the accuracy of measurement of transverse parallaxes; n is the number of bases.

Furthermore, by this formula one can determine the maximum permissible distance between control points. For instance, with $m_h = 10$ m and H = 4200 m, the value of n = 7, i.e., with $f_k = 100$ mm the control points should be at a distance to 20.5 km from each other (small-scale aerial photography).

By using statoscope readings (the instrument working with an accuracy of 0.7-2.0 m), it is possible to establish directly on the instrument the value of b_z, but only in that case when if on the first stereopair is determined the scale of the model. Then errors of spatial phototriangulation are essentially decreased.

2. Composition of a Topographic Plan

After obtaining vertical control points and the values of the elements of orientation of aerial-photos by the method of spatial triangulating, we come to treatment of separate stereopairs.

Establishing aerial-photos by obtained data, definitize their orientation according to control points, then establish the table at the height of the chosen section. Observing the model through the spectacles, shift the table without detaching the mark from the surface of the model. Sometimes, to simplify the work of tracing of contours , workers first draw on the plane table the hydrographic net, ravines, etc. After drawing the contours proceed to tracing roads, blocks of inhabited localities, and outlines of woods, then shape the topographic plan according to the requirements of the corresponding instructions.

The scale of the original which is directly obtained on the multiplex, is larger than the map scale; therefore for reducing it to the scale of the created map, use an attachment in the form of a pantograph, fastened with the encircling needle of the plotting table. In other cases photoreductions of the drawn plan are made.

The multiplex is used during composition of maps on scales of 1:100,000, 1:50,000 and sometimes 1:25,000, with the section of relief at each 20-10 m.

CHAPTER VII

Creation of Topographic Maps on Universal Stereophotogrammetric Instruments of the Optical-Mechanical and Mechanical Types

§ 40. Special Features of Universal Instruments

In these universal instruments intersection of points is executed by mobile optical or mechanical elements.

The special features of constructions of universal instruments as compared to the multiplex include: the possibility of carrying out intersection according to the principle of the triangle plus parallelogram, the application of aerial-photos 18 × 18 cm and more in size, the presence of coordinate carriages X, Y, and Z of the space, and the possibility of drawing the map in orthogonal projection according to a stereomodel magnified 6-12 times (for instance, a model of terrain created in the instrument is magnified by 1.5 times, and the image is enlarged by 4-8 times. A map is created in different scales directly on the instrument or on coordinatograph. On certain universal instruments it is possible also to carry out treatment of aerial-photos obtained with very large slopes of the principal axis of the aerial camera at the time of the survey.

Processes of orientation of aerial-photos on these instruments are carried out according to a procedural outline analogous to that of orientation on the multiplex. The same methods are used for spatial triangulation and coordination of the net. Triangulation is produced with switching of the sighting axes and the introduction of a negative base.

Universal instruments of the optical-mechanical type can ensure high accuracy of determination of the planar position and heights of terrain points, which makes them convenient for large-scale cartography (1:10,000 to 1:5000).

At present in aerial photography varied wide-angle objectives are used; they have focal lengths of 210 to 55 mm, which corresponds to angles of the field of view of 62° to 133° for the 18 × 18 cm size.

In connection with this there exist designs of universal instruments based on the use of similar bundles with angles of the field of view up to 105° and [device using] converted bundles of projecting rays with angles of more than 105°. The last instruments are more complicated in the theoretical sense, since the orientation of the aerial-photos requires the installation of additional elements. In spite of these difficulties, the instruments of this group have obtained the widest application in the USSR inasmuch as aerial photography is executed basically with side-angle aerial cameras.

§ 41. Intersection in Space in the Stereoplanigraph

Fig. 119 shows the form of intersection by similar bundles of rays in the stereoplanigraph, it has the form of a triangle united with a parallelogram. The form of intersection is caused by the application of two marks (instead of the one mark in instruments of double projection). The projecting cameras[1] of the stereoplanigraph are located on fixed points S_1 and S_2 instead of points S_1 and S_2' to rays S_1M_1 and $S_2'M_1$ determining by intersection the position of point M_1, there corresponds rays S_2M_0 and S_2M_2, parallel to them. With such location of projectors it is possible to establish bases b_x of any dimensions (equal to zero, positive, or negative). The base components b_y and b_z are established in the space of intersection by movements of the carriages on which marks M_1 and M_2 are fastened (component b_y is not shown on the figure).

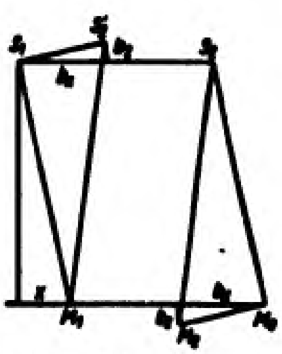


Fig. 119.

The projectors of the instrument construct the images of the aerial-photos on mirrors with marks which are examined jointly with the images in the observed system. With this the observer sees one space mark and a stereoscopic model

[1]These may be photocarriers (Schwidefsky, An Outline of Photogrammetry, 1959) or simply projectors (The Focal Encyclopedia of Photography, Focal Press Limited, 1960). [Tr. Ed. note].

of the terrain. Matching of the mark with an observed point of the model corresponds to measurement of the position of this point in the plan [horizontally] and in height.

Method of Projection. The projectors have objectives similar to the objectives of aerial cameras. Therefore the rays projecting a point of the negative reach the optical system along the same directions in which they entered the aerial camera, thanks to which the influence of distortion of the surveying objective is removed. A small disturbance of the focal length of the objective of the camera (~2% from the f_k of surveying) does not affect the sharpness of the image; this fact is used during the setting of the value of focal length. In this case rays proceed between the objective of the projector and the viewing system as a parallel bundle, ensuring image sharpness even during observation of extreme points of the aerial-photos.

Such are the advantages of the collimational principle of measurement of angles; a diagram of this system is depicted in Fig. 120.

For preservation of sharpness of the image in the plane of the mark during changes of the lengths of the projecting rays there is used, besides an objective identical to the surveying lens, a telephoto lens of variable focal length (Fig. 121); it consists of positive lens 2 and negative lens 1. The distance between the lenses depends on the change $f_э$ in the length of the projecting beam.

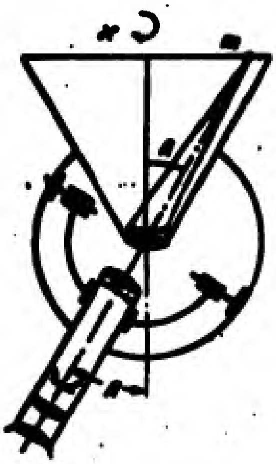


Fig. 120.

If the lenses have the same focal length, then the value $f_э$ of the equivalent focal length of the system is determined by the formula

$$f_э = \frac{f^2}{l}. \qquad (145)$$

where l is the distance between the principal planes of the lenses.

Table 23 gives values of $f_э$ at $f' = 40$ mm

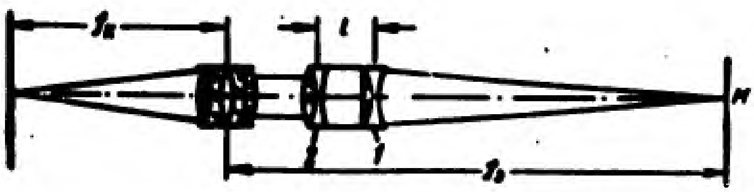


Fig. 121.

Table 23

l, мм	$f_э$, мм
2,5	640
3,2	500
4,0	400
8,0	200
16,0	100
20,0	80

These data show considerable change in the magnitude of $f_э$ corresponds to a small change in the distance between lenses. The change in the distance between lenses occurs by means of an automatic shift of the positive lenses of the system with help of an invertor calculated by formula (145). The invertor constitutes a tube with a steel flange — a cam which is connected to mechanism of shift of positive lens by means of a parallelogram device.

Zoomar revolves around the axes of a Cardan joint which have their point of intersection in the rear modal point of the projection objective.

§ 42. Structure of the Stereoplanigraph

Fig. 122 depicts the diagram of a stereoplaingraph. The base of the instrument consists of solid guides X and Y, which are rigidly connected. Vertical column Y shifts along guide; column Z has a guide for the carriage of heights, carrying the projections. Each projector has three angular movements, utilized for relative orientation of aerial-photos. Furthermore, the projectors jointly revolve around two mutually perpendicular axes, which is necessary for horizontalizing the model.

Telephoto lenses with variable focal length, along with the invertors and parallelograms, ensure sharp images of aerial-photos in the plane of brands M_1 and M_2 of the mirrors. The images, together with the marks, are seen through the observation system of the instrument. Stereoscopic matching on points of the model is executed by shifts of chambers on coordinates Y and Z and of the carriages with the base supports on coordinates X. These movements cause corresponding shifts of the carriages of a coordinatograph located next to the stereo planigraph. A plane table, on which occurs tracing of the map, is set on the table of the coordinatograph.

Carriage Z with the projectors, travelling along the vertical guide, has a horizontally located cylindrical beam whose axis of rotation is parallel to axis X.

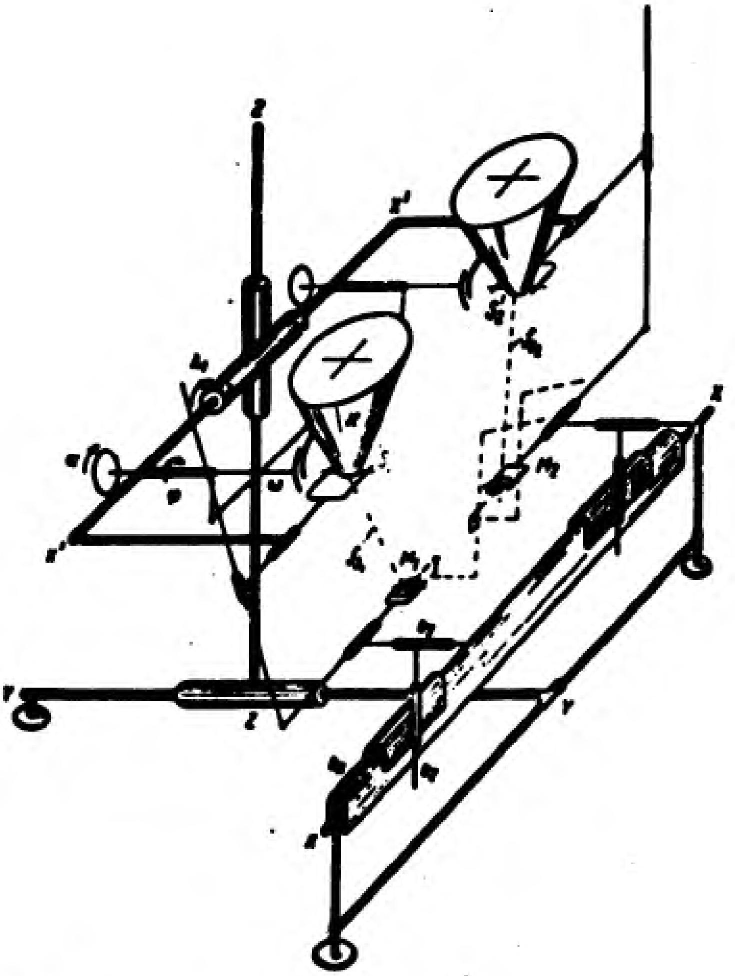


Fig. 122.

By rotating the beam around its own axis it is possible to turn the model by angle λ of the transverse tilt. Perpendicular to the beam there are fixed cylindrical axes – bushings connected by a parallelogram for joint rotation of the projectors by angle ϕ, which corresponds to the longitudinal tilt of the model. In the bushings there are located the first axes $\Delta\alpha_1$ and $\Delta\alpha_2$ of relative rotation of the projectors around axis Y, and on the axes themselves, are sectors $\Delta\omega$ of the relative transverse tilt of the projectors. The negative cassettes possess rotation in own plane at angles κ.

On the brackets of the transverse beam there are also holders (with Cardan joints) for telephoto lenses of variable focal length.

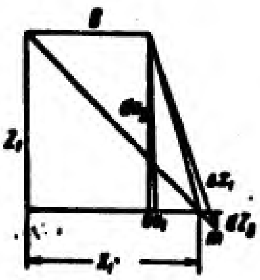


Fig. 129.

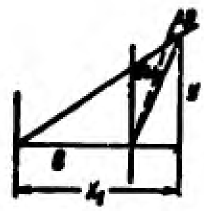


Fig. 130.

§ 43. Stereophotogrammetric Instruments Based on Optical-Mechanical and Mechanical Diagrams

These instruments are intended for the creation of maps from aerial-photos. The bearing [intersection] of points is produced in them by precision rods or bars. As in the stereo planigraphs, they have coordinate carriages X, Y, and Z and settings of base components b_x, b_y, and b_z.

A special feature of such stereophotogrammetric instruments is principle of intersection by spatial rods with tilted projectors or photoholders and shifts of the viewing system. With this there are used methods of observation of aerial-photos by collimating or comparator principles, when the aerial-photos are observed at beams of sight orthogonal to them.

In this case movement of the projectors is produced most frequently in one direction and that of the sighting system in another.

<u>Drawing instrument of Thomson</u> (England). The instrument (Fig. 131) is based on use of intersection by three bars, of which two revolve in plane ZX (behind the instrument), and one in the plane ZY (on the left side of the instrument).

During turns of the bar of transverse directions, located on the left side of the instrument, both projectors rotate around the line of the base by angle β. Rays of sight are directed into the objectives of the projectors vertically. During turning of the rear bars, located in plane ZX of the instrument, there occur corresponding tilts of the optical elements of the viewing system; consequently intersection is carried out and the possibility of observation of identical points of the stereomodel is attained.

Horizontalizing of the model with respect to the axis Y is attained by transverse tilt of chambers, and that with respect to axis X with the help of correction arm. The instrument is designed for treatment of aerial-photos of the

Fig. 131.

size 23 × 23 cm and with a focal length of 152 mm. According to source material, the instrument, which has binocular magnification of 5-15$^{\times}$, gives exact results.

<u>The Nistri photostereograph</u> (Italy). On this instrument it is possible to produce spatial triangulation and automatic tracing of maps from photographs obtained with an arbitrary location of the principal axis of the camera. Intersection of points of the model is produced exact rods, [telescoping] tubes, connecting the optical parts of the projecting system with supports b_z of the coordinate system XYZ.

Depending upon the type of survey (ground or air) the rods are disposed horizontally or vertically; here additional consoles to supports b_z are used.

The instrument (Fig. 132) consists of a massive bed, on which shift carriages X, Y, and Z move; on carriage Z there are supports for setting the basic components b_x, b_y, and b_z. Carriages X and Y have electric motors, synchronously connected with the motors of an electrocoordinatograph. The electrocoordinatograph is equipped with a gearbox for the construction of maps in different scales.

In the front part of the instrument there are photocarriers with objectives

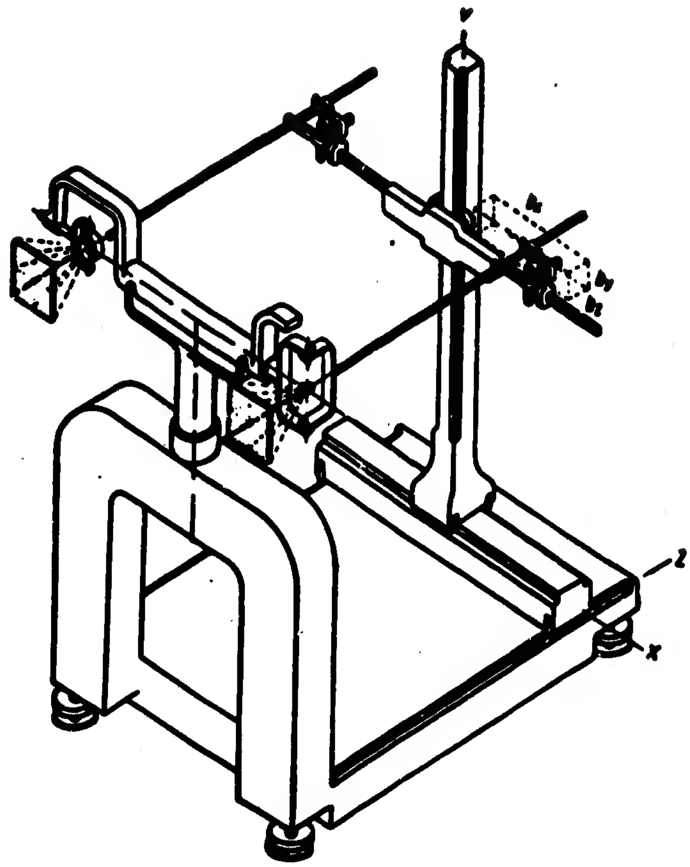


Fig. 132.

identical to the taking objectives and mobile Cardan devices carrying the optical elements of the viewing system.

The mobile Cardan devices are actuated by rods in such a way that in the binocular system a stereomodel of the terrain is visible. The diagram of the front part is shown in Fig. 133.

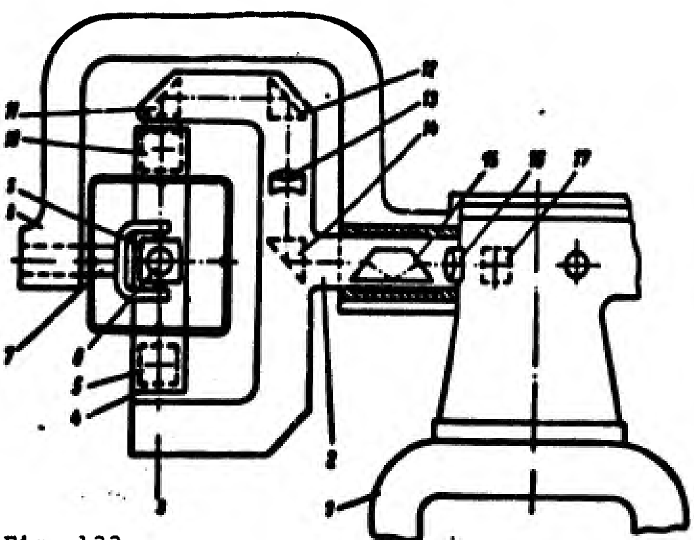


Fig. 133.

On bed 1 there is attached massive holder of the first axis of rotation of semicardan joint 2. On the second axis of rotation 3, containing part of the optical system, there is semicardan joint 4 of intersection rod 6 with mark reflector 5. Cardan 7 bears the photocarrier in the form of a frame with marks for centering of aerial-photos. Cardan 7 is attached to console 8 for common rotation around the X-axis of relatively oriented ground photographs or aerial-photos at angles of from +10 to -110°. The stereomodel is viewed through optical system 9-17.

The image of the luminescent mark is obtained by means of illumination of a point hole in the diaphragm; the latter is in the rod (tube) and is illuminated by a lamp; certain components (prisms 10 and 15) have rotary movement and therefore require very precise manufacture and adjustment.

In the instrument there are applied photocarriers (projectors) with a focal length of 152 mm (objective Rigel, 1:6.3) 23 × 23 cm for aerial-photos and photocarriers with a focal length of 210 mm (objective Aldebaran, 1:4.5) for 18 × 18 cm aerial-photos. Data characterizing the instrument are given in Table 25.

Table 25

Elements of instrument	Numerical data
X, mm	± 300
Y(Z) "	± 300
Z(Y) "	− 210 + 720
b_x "	± 240
b_y "	± 80
b_z "	± 80

<u>The Wild Autographs</u> (Switzerland). Over a number of years there have been released several models of autographs based on mechanical intersection and intended both for treatment of an isolated stereopair and also for spatial triangulation. Models A-5 and A-8 are intended for treatment of isolated pairs of aerial-photos 18 × 18 cm in size, while the model A-7 is designed for treatment of aerial photos by the method of continuation, and also of photographs of ground stereophotogrammetric surveys.

Figure 134 shows an overall view of the Autograph A-7. The lower part of instrument consists of carriages X, Y, and Z with supports b_{x_1}, b_{x_2}, b_{z_1}, b_{z_2}, b_{y_1}, and b_{y_2}; on the latter there are located Cardan sleeves. Marking rods pass through

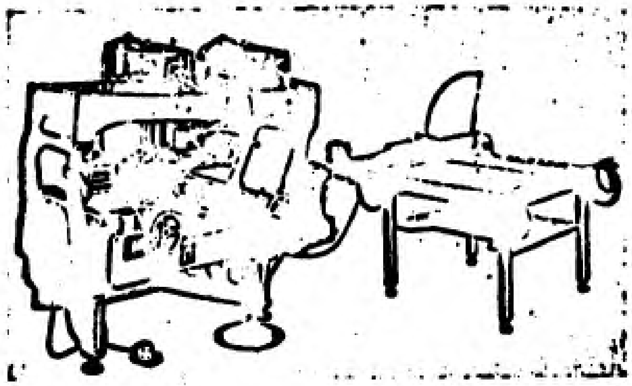


Fig. 134.

them. In the upper part of the instrument there are two massive photocarriers, to which it is possible to impart angles of tilt α and ω, and to the cassettes with photographs, furthermore, also angles κ. The cassette can be set with respect to the center of rotation of the photocarriers at values of focal lengths equal to 98-210 mm.

Under the cassettes there are located radial carriages with prisms, which are activated by the telescopic ends of precision rods, carrying out intersection of determined points. Rays of sight emerge from the prisms on to the aerial-photos orthogonally, as in the stereocomparator. The presence of tilted photocarriers demanded the creation of a complicated optical system, executing a number of functions (rotation of image, introduction of positive and negative stereoeffect). Magnification of the field of the aerial-photo visible in the ocular is equal to $6^{\times}$.

Shifting of carriages X, Y, Z in the system of the model is accompanied by readings of coordinates with a precision of 0.01 mm.

The high accuracy of the instrument is attained by the application in the X, Y, and Z carriages and Cardan sleeves of precision ballbearings and the [illegible] in the mechanisms of the adjusting elements of polished axles and bushings. The guides along which the carriages shift are cylindrical in form. The coordinatograph, located on the right of the instrument, ensures the obtaining of maps of all scales.

At present the Wild plants are creating the Autographs A-9 and B-9. The first of them is adjusted for treatment of aerial-photos reduced by two times and is useful for triangulation (Fig. 135). The tilts of the rods ensure treatment of aerial photos with $f_k = 70$ mm. The diagram of the instrument is similar to that

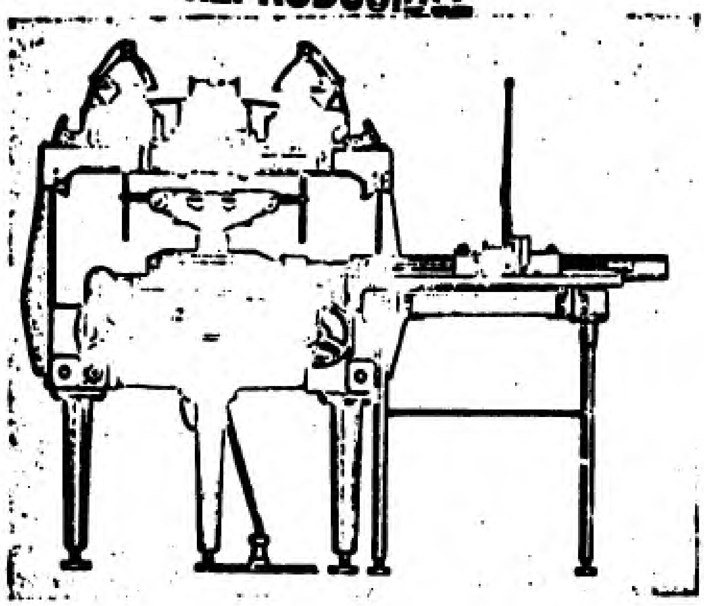


Fig. 135.

of the A-7.

The Autograph B-9 differs from A-9 by the fact that it takes aerial-photos of normal size. The precision rods intersect at one point, and consequently the possibility appears of guiding the height table with one hand. The map is traced next to the instrument with the help of a pantograph connected to the height table.

The Zeiss stereometrograph (German Demorcratic Republic). This stereophotogrammetric instrument (Fig. 136) is based on mechanical intersection and is intended for treatment of aerial-photos of isolated stereopairs of orthogonal aerial photography.

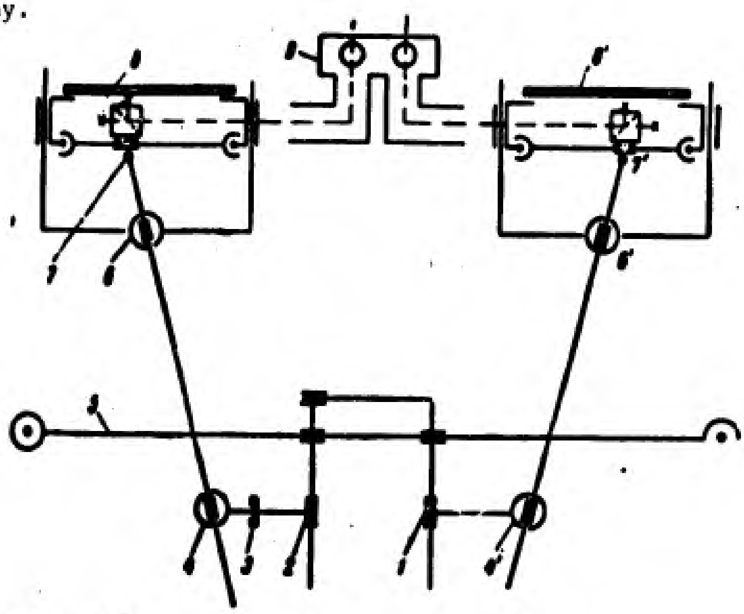


Fig. 136.

Intersection is carried out by spatial precision rods on the principle of a triangle with a parallelogram. The lower ends of the rods are actuated by Cardan sleeves 4 and 4', located on vertical carriages 1 and 2 (Z_1 and Z_2). The upper ends of the rods shift carriages 7 and 7' with prisms for observation of points of aerial-photos 8 and 8'. The projectors with aerial-photos possess adjusting movements b_{y_1} and b_{y_2}.

As compared to that in the stereoplanigraph, the coordinate system has a smaller number of guides, since carriages b_{x_1} and b_{x_2}, joined by a screw, are located on a common guide 5. During their joint movement one can determine the abscissa X of the model, and during transverse movement of guide 5 the ordinate Y can be found.

Determination of coordinate Z is attained during joint shifting of carriages b_{z_1} and b_{z_2} (Y and Y'), of which b_{z_1} carries additional support b_{y_1} (3).

Synchronous rotation of screws b_{z_1} and b_{z_2} is achieved by electrical devices of the selsyn type. Such devices are also provided and for screws X and Y of the instrument.

Replacement of mechanical connections by electrical simplifies control of both the instrument and the coordinatograph and also gives the possibility of sending elements of the movements to the height counter and to the printing machine for fixation of coordinates.

It is assumed that electrical connections preserve the accuracy of the transmitted coordinates even during prolonged operation of instrument.

Settings b_y, α, ω, and κ are carried out by mechanical drives of the usual type.

The projectors are mounted on Cardan joints and can be set on angles of inclination around the points of projection of rods 6 and 6'; the aerial-photos, besides turning by angles κ, can also be decentered along axes X and Y. The mechanism of decentering permits carrying out intersection by converted bundles at angles of tilt of the aerial-photos of not over 30'. Carriages with viewing prisms are shifted in inclined planes by means of the action of rods. To eliminate shifts of images of aerial-photos with luminescent marks superimposed on them during tilting of projectors, automatically turning prisms are used in the optical system.

By changing the height of the position of the cassette in the instrument with the help of micrometers it is possible to establish different values of focal lengths of the projectors while retaining image sharpness and the possibility of observation of aerial-photos with orthogonal position of the visual rays.

The height counter, actuated from the carriages through an electrical connection, is in front of the observer, near binocular 9. The instrument is encased in a light-tight housing (Fig. 137).

GRAPHIC NOT REPRODUCIBLE

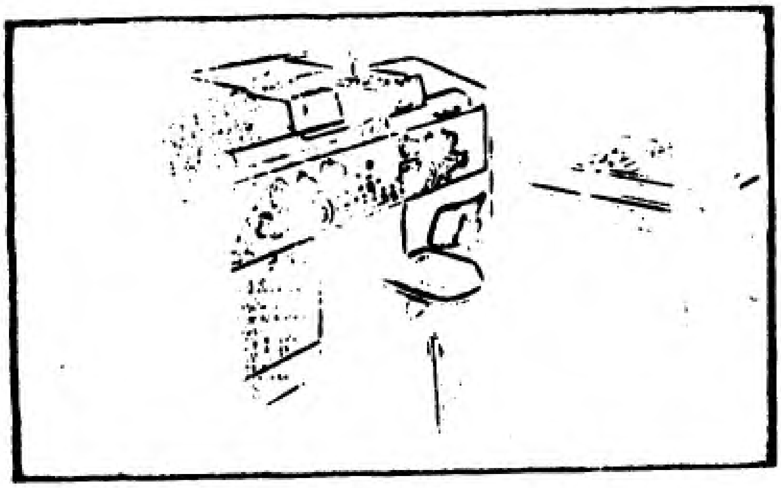


Fig. 137.

Characteristics of the instrument are given in Table 26.

Table 26

Elements of the instrument	Numerical data
Size of aerial-photos, cm	18×18; 23×23 an 14×14
Magnification	8,7×
Focal length, mm	98 — 215
Tilts of aerial-photos, in degrees	± 5
Angle of rotation of aerial-photos, in degrees;	± 30
Bases: b_x, mm	0 — 280
b_y :	− 10 + 10
b_z :	− 15 + 15
Coordinates: X, :	± 200
Y :	− 250 + 250
Z :	+ 125 + 310
Dimensions of the instrument, cm	210×150×130
Weight kg	1200
Area occupied by coordinatograph, cm	110×120

The scale of the map is established by means of a set of gears with a gear ratio of 0.25 to 5.

§ 44. Stereofot.[1]

The instrument is intended for small-scale and medium-scale cartography from aerial-photos of orthogonal surveys. It is based on the principle of optical-mechanical intersection. In Fig. 138 are shown rods S_1M and S_2M, intersecting at point M. By means of parallelogram thrusts collimators C_1 and C_2, are actuated; their axes are always parallel to the marking rods. Above collimators are projected on their surface, and simultaneously field of observation is illuminated. So that slides will always be located horizontally, on the way path of the rays of the projection two mirrors are installed. Of these the lower is large and immobile and the upper is a small mirror, fixed at an angle equal to the angle of tilt of the aerial-photo.

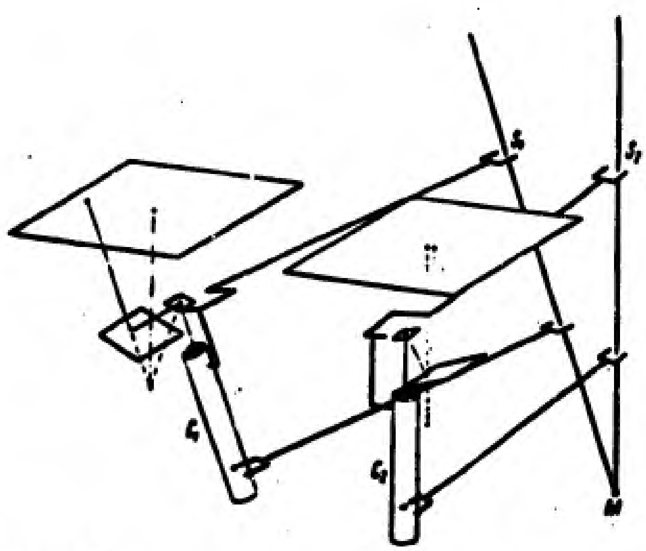


Fig. 138.

Fig. 139 shows an external view of the instrument (France), which consists of an upper part set at a height and equal to section h and containing a binocular system for observation of aerial-photos, and a lower part consisting of a table with mechanisms on its surface (mechanism of circumscription and a pantograph, connected at one end with the point of intersection of the rods). Stereoscopic observation of points of aerial-photos is carried out by means of the mobile

[1]No expansion found for this term. [Trans. Ed. note].

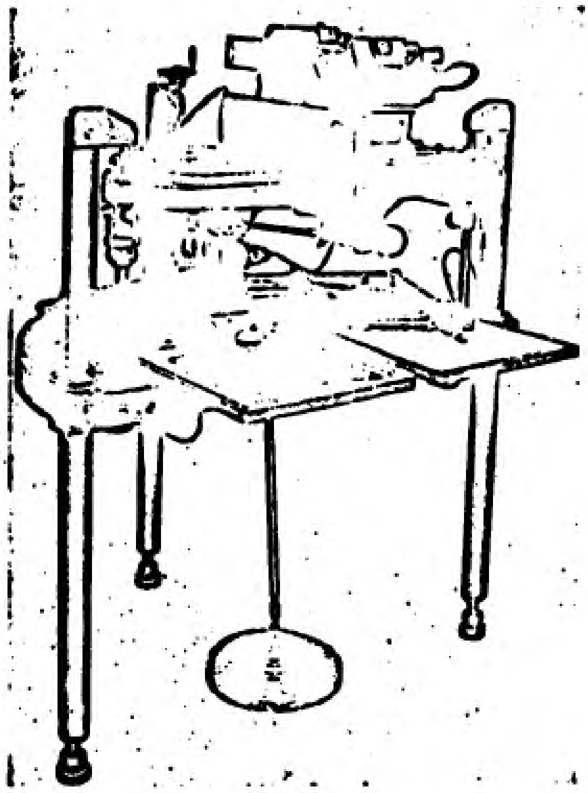


Fig. 139.

sleeves of the optical system, set by hand. With a magnification of $2^{\times}$ the field of view equals 90 mm. The brightness and sharpness of a mark on the edges of the fields of the photos drop as compared to that in the central parts; to improve the visibility of the mark regulators of illuminance (rheostats) and other means are applied. On the instrument it is possible to process only isolated stereopairs on paper sized up to 400 × 400 mm, with an increase in the scale of the map up to two as opposed to the surveying scale.

Stereoinstrument of Poivilliers (SOM;[1] France). The instrument is intended for treatment of aerial-photos from orthogonal aerial photography and is among the instruments of class I accuracy (Fig. 140). It contains in the lower part a coordinate-base system and in the upper part a travelling carriage with the aerial-photos; they shift left or right and up or down, which ensures simplicity of the optical system. Calculation of the influence of angles is produced by correctors. Precision rods connect the base system with the photo carriages and

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[1]Société d'Optique at de Mécanique, Paris. [Tr. Ed. note].

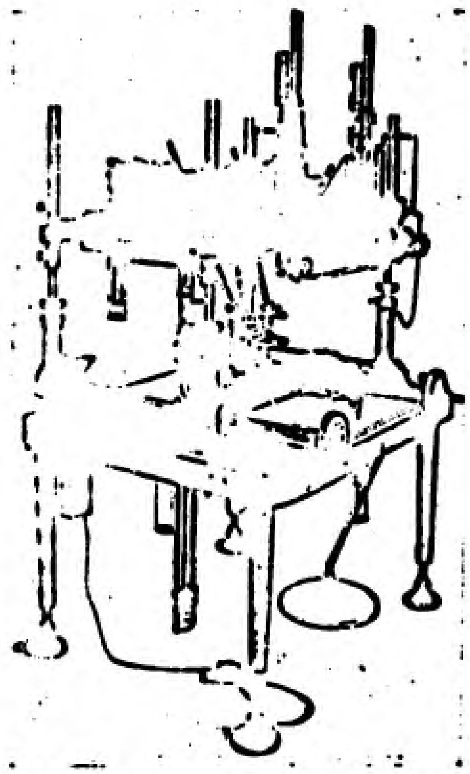


Fig. 140.

ensure constant intersection of determined points. The instrument is used for treatment of aerial-photo with f_k = 120 to 130 mm. The map is traced on a table located in front of the observer.

§ 45. Experimental Designs

Since the process of intersection in any stereophotogrammetric instruments reduces to shifting of carriages of the aerial-photos or optics and the carriages of spatial coordinates, the solution of the problem can be attained by different methods, with the use of:

a) mechanical connections, for instance, toothed transmissions, telescopic shafts, precision rods;

b) electrical drives based on the Wheatstone bridge;

c) selsyn connections, and

d) photoelectric and electron coupling.

The first type of connection has been applied for a long time and at present is of basic value in photogrammetric-instrument-making. The other methods as yet are applied during experimental works. Thus, in the Canadian instrument stereomat (Hobrow [?] Fig. 141), based on the scheme of a double projector, use is made of carriages X and Y carrying a television tube with the mark in the middle [20]. The motors of carriages X and Y transfer the carriage with the tube until the beams proceeding from identical points of the aerial-photos intersect on the mark. At this instant the pencil is switched on and automatic tracing of contours begins. As compared to hand tracing it doubles the speed of the work and gives good accuracy of contours in height.

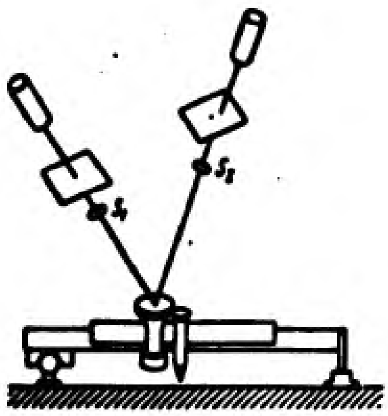


Fig. 141.

However, on aerial-photos with wooded and populated areas the contour trace comes out confused and must be deciphered.

Worthy of attention is the process of automatic orientation of aerial-photos; it occurs more clearly than drawing of relief and contours.

First the X and Y wheels are used to guide the television tube with the mark consecutively into each of five zones of the stereopair with clear points, where some settings (κ, α, ω, b_z) act on the process of orientation.

Corresponding screws are connected with electric motor drives, which automatically remove the transverse parallaxes in the given point.

Another instrument, the "Planitop" (Fig. 142), produced in the released FRG for small-scale cartography, is based on the application of a stereoscope with marks, mobile photocarriages, and potentiometric devices. They solve the relationships which in preceding instruments of the same type were solved by the mechanical method [19].

The size of the processed aerial-photos is 23 × 23 cm and that of the parallactic range is +25 mm. Treatment in various scales is carried out on a pantograph with magnification of $0.5-2^{x}$. Height values are obtained in meters. Posing of relief is attained by direct shift of the carriages by hand. The represented model is designed for treatment of single pairs, composed from photo-prints.

It is necessary to note a stereophotogrammetric instrument of a new form (Aeromet, Fig. 143), based on the use of photoelectric intersection and selsyn transmissions; this instrument falls into the I-II accuracy class. In it all

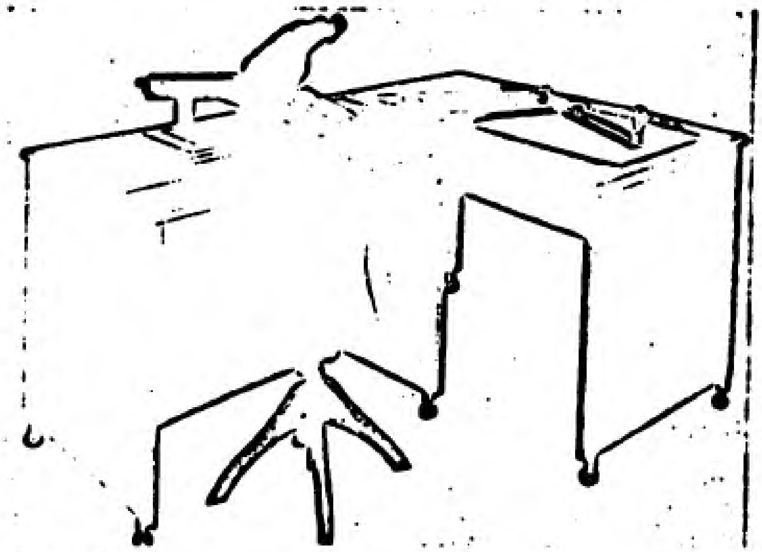


Fig. 142.

mechanisms are enclosed in a housing; tracing of the map is produced on the upper part of the instrument. Control of the instrument is by handles located on the front part of the instrument housing.

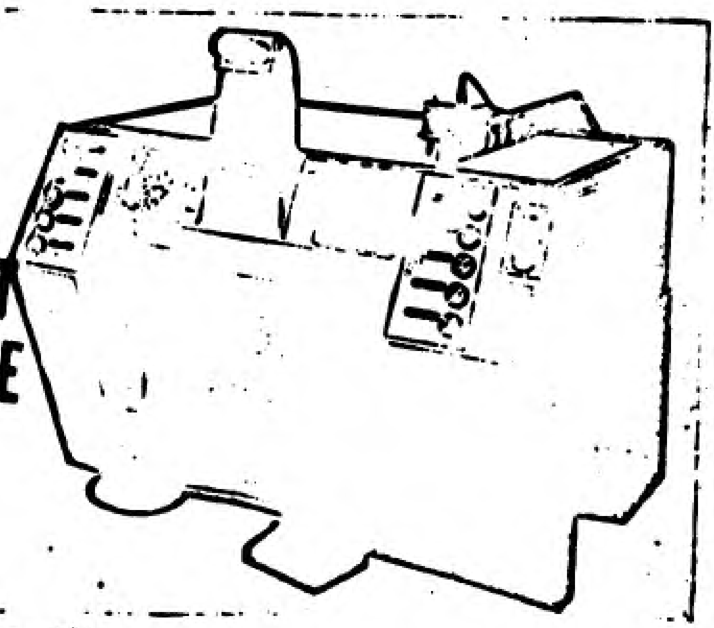


Fig. 143.

§ 46. Projection by Converted Bundles of Projected Rays

The idea of using converted bundles was first developed by M. D. Konshin.[1]

Aerial-photos obtained by wide-angle objectives with f_k = 70, ..., 50 mm, cannot be treated on the stereoplanigraph and other instruments designed on the principle of preservation of the similarity of bundles of projected rays to the surveying bundles. Therefore there appears the problem of using affine conversions, with which the treatment of super-wide-angle aerial-photos becomes possible. To develop the properties of these conversions we will consider certain relationships for linear and angular elements of rectified [transformed] bundles.

Rectified aerial photographs. Fig. 144 depicts intersection of points by similar and converted bundles of rays for rectified aerial-photos. Considering the corresponding triangles, it is possible to write:

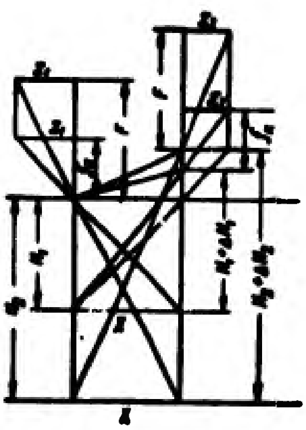


Fig. 144.

$\frac{x_1}{f_k}=\frac{X}{H_1}$ — for the left photo, $\frac{x_2}{f_k}=\frac{X}{H_1+\Delta H_1}$ — for the right,

and for the case of converted bundles:

$\frac{x_1}{F}=\frac{X}{H_2}$ — for the left photo and $\frac{x_2}{F}=\frac{X}{H_2+\Delta H_2}$ — for the right

(where F is the focal length of the projectors).

Comparing the equations, we obtain

$$\frac{F}{f_k}=\frac{H_2}{H_1},$$

$$\frac{F}{f_k}=\frac{H_2+\Delta H_2}{H_1+\Delta H_1}.$$

The ratio of the focal length of the projectors to the focal length of the aerial camera is called the coefficient of affinity, i.e.,

$$k=\frac{F}{f_k}, \qquad (151)$$

or the coefficient of change of coordinates Z of points of the model. The latter is easy to prove.

We have

$$d(H_2+\Delta H_2)=kd(H_1+\Delta H_1)$$

or, designating d(H + ΔH) through h, we will obtain

[1]M. D. Konshin. Photogrammetric treatment of photographs with converted bundles. Tr. TsNIIGAiK, Issue 44, M., Geodezizdat, 1944.

$$h_2 = kh_1 = \frac{H_2}{H_1} h_1,$$

where h is the change of coordinate Z of points of the model. Analogously,

$$\Delta H_2 = k \Delta H_1. \qquad (152)$$

Thus, the projection of rectified aerial-photos with coefficient of affinity k causes a change in coordinates Z of points of the model proportional to k.

Coordinates X and Y of points of the model are not changed, which is clear from the relationships:

$\frac{1}{M} = \frac{f_k}{H_1}$ — scale of rectified aerial-photo,

$\frac{1}{M}\frac{H_1}{f_k} = \frac{1}{M_0}$ — scale of image on plan in the presence of similar bundles of projected rays (here and below H_1 and H_2 are magnitudes in the instrument).

The scale of the plan $1/M_0$ with converted bundles is

$$\frac{1}{M_0} = \frac{1}{M}\frac{H_2}{F}.$$

From the ratio of scales of plan we obtain

$$\frac{1}{M}\frac{H_2}{F} : \frac{1}{M}\frac{H_1}{f_k} = \frac{H_2 f_k}{H_1 F} = 1,$$

i.e., coordinates of points of the plan are not changed.

<u>Tilted aerial-photos</u>. In the presence of angles of tilt of aerial-photos which constitute a stereopair, changes in the heights of points of the model occur also in accordance with the dependence $h_2 = kh_1$. Angular settings of aerial-photo are also changed proportionally to k.

According to the diagram of projection by converted bundles (Fig. 145), it is possible to establish that the tilt α_1 of the chamber [projector] is not equal to the angle of inclination of the aerial-photo in the instrument and that the necessity arises to introduce decentering o_1o of the aerial-photo. For determination of the value of α_2 we will compose the equation

$$S_1 l = S_2 l = \frac{f_k}{\sin \alpha_1} = \frac{F}{\sin \alpha_2} \qquad (153)$$

or for small angles (from 0 to 4°)

$$\frac{F}{f_k} = \frac{\alpha_2}{\alpha_1}. \qquad (154)$$

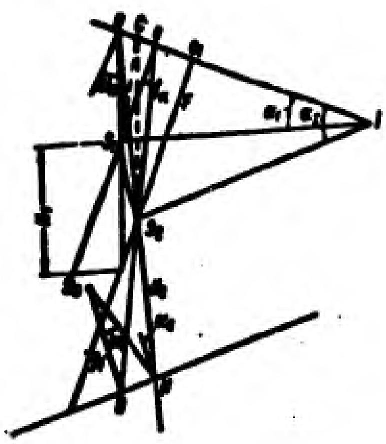


Fig. 145.

Thus, new angle of setting of the projector in the plane of the drawing is

$$\alpha_2 = k\alpha_1. \qquad (155)$$

The magnitude of decentering oo_1 of the aerial-photo is determined by the equality

$$o_1o = co_1 - co,$$

in which segments co_1 and co are taken from the point of zero distortions co

Therefore

$$o_1o = F\frac{\alpha_2}{2} - f_1\frac{\alpha_1}{2}.$$

Placing the value α_1 from equation (155) and multiplying the second member of the right side by F/F, we will find the value of decentering in the form

$$o_1o = 0{,}5F\left(1 - \frac{1}{k^2}\right)\alpha_2. \qquad (156)$$

Nadir point n of the aerial-photo is projected on the screen at point N by a beam inclined at angle α_e. We will determine angle α_e of the slope of the nadir line. Since

$$\alpha_2 = \alpha_n + \alpha_e, \qquad (157)$$

then, substituting in it the value of α_n from equation (68), we will obtain

$$\alpha_e = 0{,}5\alpha_2\left(1 - \frac{1}{k^2}\right). \qquad (158)$$

The image has its conditional center of projection S_3, seperated from the principal point of the image o by distance f_1.

The distance of point O from nadir point N is

$$ON = f_1 \operatorname{tg}\alpha_1 \approx f_1\alpha_1.$$

After substitution of the value of α_1 from equation (155) we will obtain

$$ON = \frac{1}{k} f_1\alpha_2.$$

Knowing the location of points o and n on the aerial-photo, it is possible to measure the image corresponding to segment ON and to calculate the value of f_1 by the formula

$$f_1 = \frac{ONk}{\alpha_2}$$

or

$$f_1 = \frac{H_1}{H} f_r \qquad (159)$$

The value of f_1 for a rectified aerial-photo can be determined also on the basis of measurements of the distance d (on the aerial-photo) and d_0 (on the rectified image) between points located on a line, perpendicular to the principal vertical and passing through the point of zero distortions.

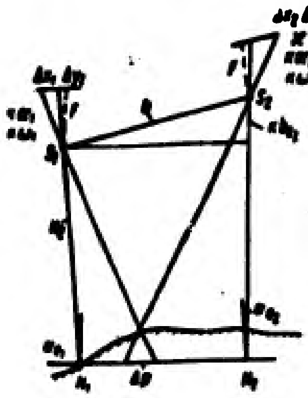


Fig. 146.

Thus

$$\frac{d}{f_r} = \frac{d_0}{f_1},$$

whence

$$f_1 = \frac{d_0}{d} f_r \qquad (160)$$

Formula (160) permits revealing, when necessary, the scale of a rectified aerial-photo with great accuracy.

Thus, for production of an undistorted model during affine projection of slanted air-photos it is necessary (Fig. 146):

1 – to establish the projectors at on angles $\alpha_2 = k\alpha_1$, $\omega_2 = k\omega_1$, where α_1 and ω_1 are angles of tilt of the aerial-photos at the time of exposure;

2 – to establish focal length F of the projectors in conformity with the coefficient of affinity and the angle of tilt of the aerial camera.

3 – to decenter the aerial-photo by the magnitudes

$$\Delta x = 0{,}5F\left(1 - \frac{1}{k^2}\right)\alpha_2, \qquad (161)$$

$$\Delta y = 0{,}5F\left(1 - \frac{1}{k^2}\right)\omega_2; \qquad (162)$$

Here point n of the nadir of the aerial-photo is projected on the screen by a slant beam (slopes α_e and ω_e).

§ 47. Photocartograph

Fig. 147 depicts the diagram of the photocartograph (author's design) for treatment of aerial-photos in conditions of converted bundles of rays. The instrument constitutes a unique double projector on which it is possible to trace topographic maps and to prepare prints for the composition of a photomap. It is possible also to produce phototriangulation on aerial-photos.

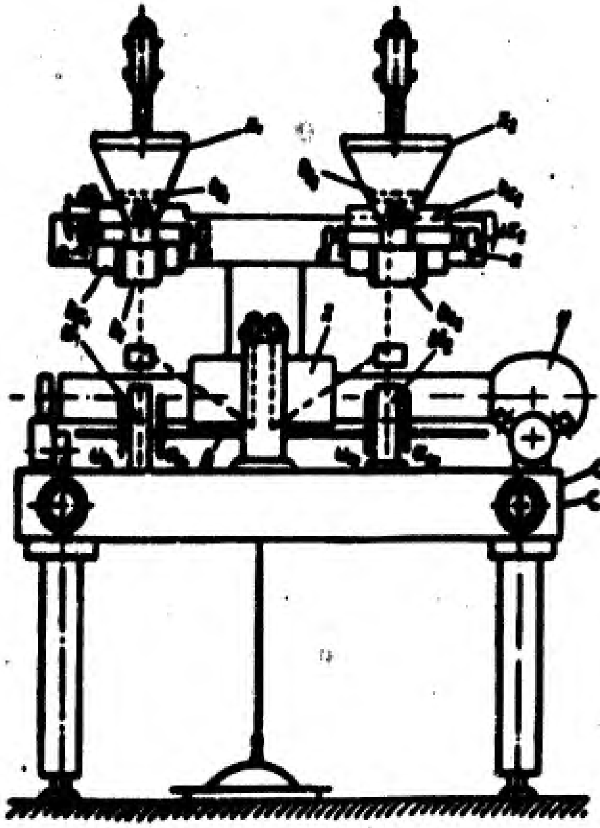


Fig. 147.

On bed there are carriages Y and X, shifted by manual wheels. Carriage X has a massive holder with base supports b_x, b_{y_1}, b_{z_1}, b_{y_2}. and b_{z_2}. The projectors are attached to the supports.

Under the projectors there is a large screen with sites, in the center of which marks are plotted; images of aerial-photos (negatives) are projected onto these sites. The screen can be shifted up or down, as a result of which one can determine the value of coordinates Z.

Air-photos are illuminated by lever condensers located on supports b_{y_1} and b_{y_2}. The lower hinged ends of the levers of the supplementary vertical support are located on the same horizon with brands M_1 and M_2, thanks to which only the necessary places on the negatives are illuminated. For illumination of the entire field of aerial-photos detachable reflectors are used. The projected images, jointly with the marks, are viewed from above through a fixed binocular system located on the front part of the bed. The total magnification of the optical system is 43X.

With a shift of the projectors by magnitudes X, and Y and of the screen by the magnitude Z it is possible to observe points of the model located in any part of the stereopair.

With a shift of carriages X and Y there is a simultaneous shift, with the help of Cardan transmissions, of the carriage of the coordinatograph, on which there is traced a map in the corresponding scale.

Each projector contains in its lower part four removable objectives of different focal lengths, and in the upper part supports of decentering Δx, Δy with cassettes revolving on angles κ. With the help of micrometers a cassette can be set according to height at the value of focal length F.

In the given form the instrument is useful for processing aerial-photos with small angles of tilt; spatial intersection is carried out in it in the same way as in stereoplanigraph equipped with attachments for decentering movements of the cassettes.

For production of an exact map from aerial-photos with converted bundles of projected rays it is necessary, according to the investigations of Prof. A. N. Lobanov, that the angle of tilt

$$\alpha_0 < \sqrt{\frac{m_q}{2x(k^2-1)}}. \qquad (163)$$

where

$$m_q = \frac{\Delta H h}{(H-h)(H+\Delta H-h)}$$

is the error in removal of the transverse parallax.

For instance, when $m_q = 0.02$ mm, $x = 70$ mm, and $k = 2$ the angle α_0 should be less than 30', which undoubtedly requires exact fulfillment of process of making aerial photographs to be utilized for large-scale cartography. Such conditions arise also during work on the Zeiss stereometrograph.

<u>Correctional attachments</u>. For treatment of aerial-photos with large angles of tilt there are introduced correctors of space directions, displacing the measuring marks in the plane of screen in conformity with the angles of displacement of nadir directions from the plumb line. The values of these angles are determined by relationship (158) and also by

$$\alpha_y = 0{,}5\,\alpha_1\left(1-\frac{1}{k^2}\right). \qquad (164)$$

The correctional attachments are attached to the screen of the instrument and to the bed. Figure. 148 shows screen 1, on which are located two mutually perpendicular supports. The lower support carries pusher 2 and the upper, pusher 3 and ring 4 with the measuring mark. On fixed stand 5, attached to the column of the instrument, there are located correction bars N and N_1 with micrometers of angles α_e and ω_e. During a vertical shift of the screen vertical pushers 2 and 3 displace the ring with mark 4 in two directions, by which there is carried out the necessary correction of space directions of rays proceeding from objective S. An analogous attachment exists for the second mark.

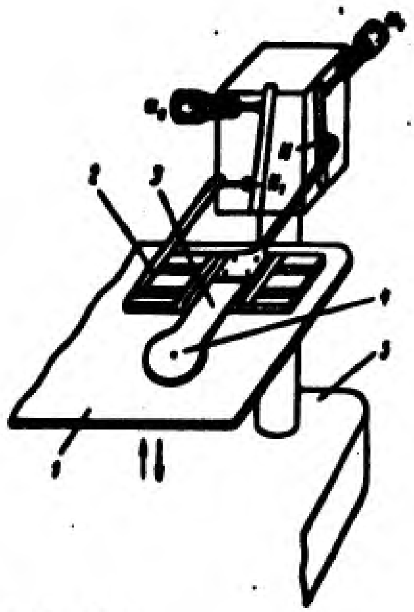


Fig. 148.

Displacements of the mark in the field of view, although small, may cause a visual parallax; it is removed by one of the ocular wedges.

According to the presented theory of intersection for creation of a map it is possible to use both the stereoplanigraph and also other instruments of this type, equipped only with supports of decentering of cassettes, if the correction Δb_x is introduced in the base of the model by the formula

$$\Delta b_x = h_2(\alpha_{e_1} - \alpha_{e_2}). \qquad (165)$$

Taking $\Delta b_x \leq 0.05$ mm, one can find the magnitude of steps h_2, within the limits of which errors in determination of heights are insignificant. However, in this case there remains an error in the position of contours of the map, equal during outlining of contours by the left mark to the quantity

$$\Delta m = h_2 \alpha_{e_1},$$

and during their circumscription by the right mark to

$$\Delta m' = h_2 \alpha_{e_2}. \qquad (166)$$

The magnitudes of Δm and $\Delta m'$ should be less than 0.4 mm – the error of the graphic position of the point in the plan. The inconveniences connected with the introduction of corrections disappear in the presence of correctional devices.

§ 48. Stereoprojector SPR-2 (Romanovskiy)

This instrument is intended for the creation of topographic maps from aerial-photo taken by aerial cameras with various focal lengths (55-210 mm). Intersection of determined points of the model is produced mechanically.

On the lower part of bed of instrument there is a plane table and under it carriages X, Y, Z (Fig. 149). Supports of base components b_x and b_z are installed on vertical carriage (Z). Precision rods revolve around the centers of projection; at the lower end they are connected with the Cardan devices of the base components and at the upper end with the Cardan devices of the photocarriers. The ratio of the distances from the points of centers to the Cardan joints of the aerial-photos and to the base components is determined by the scale of model, which can be varied from 0.5 to 2 as compared to the scale of the aerial-photos.

Fig. 149.

This installation also predetermines the coefficient of affinity of the model. A vertical shift of carriage Z gives a section of the model.

A peculiarity of the instrument is that in it the aerial-photos occupy a horizontal position independently of the angles of tilt of the aerial camera during the aerial photography. Such location of aerial-photos is attained through the application of correctional mechanisms, automatically displacing the sighting axes of the binocular system by the magnitudes of the corrections. The aerial-photos must have been decentered beforehand by $\Delta x = f_k \operatorname{tg} \alpha$; $\Delta y = f_k \operatorname{tg} \omega$ (first setting).

Let us assume that the precision rod (Fig. 150) has obtained a slope around point S and instead of position 1 occupies position 2. Then the carriage of the aerial-photo will obtain a shift by the magnitude x_0 and under objective 3' of the viewing system point a_0 of the aerial photo will be established. Correction bar 5 with correction site 4, under the influence of movement of the photocassette by the magnitude x_0, will obtain a slope on angle β. Due to this, cam [finger] 6, which is joined with the site and was displaced during orientation by the magnitude d_α, will descend by the quantity

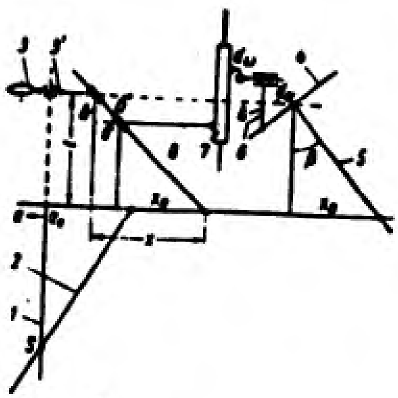


Fig. 150.

$$\Delta = d_\alpha \cdot \operatorname{tg} \beta',$$

which will cause a shift of support 7 by the same magnitude Δ and will force the second bar 8 to tilt at angle β; therefore

$$\delta = \Delta \operatorname{tg} \beta.$$

Substituting the expression for Δ, we obtain

$$\delta = d_\alpha \operatorname{tg} \beta \operatorname{tg} \beta'$$

or

$$\delta = d_\alpha \frac{x_0}{l} \cdot \frac{x}{l} = d_\alpha \cdot \frac{x x_0}{l^2}.$$

Application of values $\operatorname{tg} \beta'$ requires the introduction of decentering on cassettes of lower parts of the correction bars by magnitudes $\Delta x' = f_k \frac{\alpha}{2}$; $\Delta y' = f_k \frac{\omega}{2}$ (second setting).

It is also known that

$$\delta = \frac{x x_0}{f_k} \alpha, \qquad (167)$$

therefore

$$d_\alpha = \frac{l^2}{f_k} \alpha. \qquad (168)$$

Thus, by setting cam 6 on values d_α, d_w (third setting) it is possible to introduce automatically the correction in radial directions on the aerial-photos, inasmuch as bars 5 and 8 are inclined in two directions.

By means of the hinged joint of bar 8 the objective is displaced exactly by the quantity δ and consequently point a of the aerial-photo is observed.

The instrument is equipped with counters, rheostats for adjustment of illumination of aerial-photos, and switches of directions of movements; Cardan joints are led off from the instrument for connection with a coordinatograph. The instrument has shown excellent qualities and high accuracy of determination of points in height (1:4000H) and in the plan [2].

On the basis of the [SPR-2] (СПР-2) there has been created the experimental instrument [SPS] (СПС) – a stereoprojector for triangulation on aerial-photos. In this instrument there is conducted full automation of settings of dependent correction mechanisms, ensuring high speed of orientation of aerial-photos and a certain increase in the accuracy of their treatment.

§ 49. Mechanical Intersection Based on Change of Focal Length

Strict intersection with converted bundles can be realized during mechanical projection by applying directly correction of the magnitude of focal length of the aerial-photo [6]. A diagram of the solution of the problem is shown in Fig. 151.

A ray proceeding from point M_1 of the terrain through objective S_1 of the aerial camera gives images m_0 and m_1, respectively, on a horizontal aerial-photo and on one tilted at angle α_1. Let us draw a ray from point m_0 to a new center of projection, S_2, and continue it to point M_2 located on segment X. Segment X is separated from point S_2 by distance H_2.

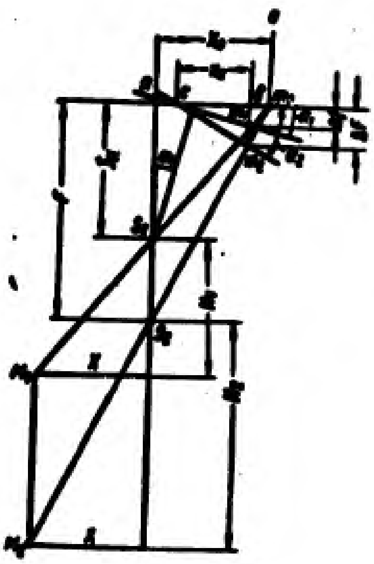


Fig. 151.

Drawing through point m_1 a line parallel to S_1n, we will obtain point m_2 at the intersection with direction $m_0S_2M_2$. The plane passing through points c and m_2 can be called the correction plane. It is inclined at angle α_2 and possesses the ability to convert a tilted aerial-photo into a level photo.

Designating as f_k and F, respectively, the focal lengths of the taking and projecting cameras, we will write the relationships

$$\frac{x_1}{f_k} = \frac{X}{H_1}$$

and

$$\frac{x_2}{F} = \frac{X}{H_2}.$$

whence

$$H_2 = \frac{F}{f_k} H_1. \qquad (169)$$

For tilted aerial-photo cm_1 we have correction δ to the position of a point determined from the equation

$$\frac{\delta}{\Delta f} = \frac{x_1}{f_k};$$

and for the correctional plane we have

$$\frac{\delta}{\Delta F} = \frac{x_2}{F}.$$

Here

$$\Delta f = x_c \alpha_1,$$
$$\Delta F = x_c \alpha_2,$$
$$\alpha_2 = \frac{F}{f_k} \alpha_1. \qquad (170)$$

According to the diagram, similarity of the model is retained if each aerial-photo of the stereopair is inclined at its angle of tilt α_1 and the correction plane is displaced by the magnitude

$$\Delta = f_k \operatorname{tg} \frac{\alpha_1}{2} \qquad (171)$$

and to inclined at angle α_2. Certain movements can be united. Intersection can be produced by the spatial rod $M_2S_2m_2$, joined with the correction plane at point m_2.

Sighting ray m_1m_2 should be vertical. With tilt of rod $M_2S_2m_2$ and shift of the sighting ray the observed point is determined.

During observation of points located outside axis xx, rod $M_2S_2m_2$ is tilted and constructs a direction taking into account the slope of the line of the aerial-photo. Intersection is carried out without any limitations with respect to focal lengths and tilts of the aerial-photos.

§ 50. Stereographs

On the basis of the theory of mechanical intersection by converted directions presented in § 49 the author has created a number of stereographs. Fig. 152

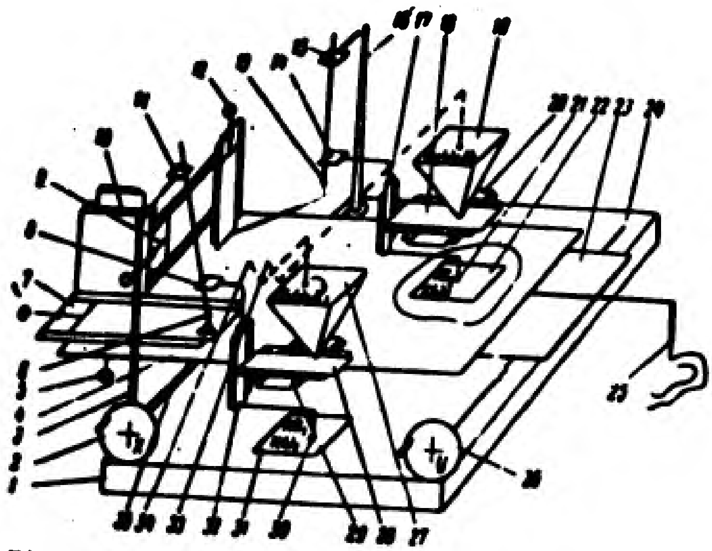


Fig. 152.

shows the diagram of the location of structural elements of the [SD-1] (СД-1). On bed 1 are carriages 23 and 4, shifted along directions X and Y by means of hand wheels 26 and 2. On carriage 4 is column on which vertical carriage 10 moves; it is shifted up or down by means of screw 5, connected to the foot control (pedal). Furthermore, on carriage 4 there are also differential supports 28 and 29 for the left aerial-photo and supports 18 and 20 for the right photo 19. Photos 19 and 27, located in the instrument along axis Y, are set at angles α_1, ω_1, κ_1 and α_2, ω_2, κ_2 and at small angles α, ω are placed in a horizontal position, but with the introduction of decenterings Δx, Δy. The differential supports move the aerial-photos in two directions. To supports 28 and 18 there are attached supports 32 and 17 of focal increments, carrying correspondingly spherical spindles 30 and 21 and also Cardan joints 8 and 14.

Spherical spindles 30 and 21 rest on correction planes 31 and 22, set at angles $k\alpha_1$; $k\omega_1$; $k\alpha_2$; $k\omega_2$. The Cardan joints are connected with the intersecting rods, rocking in the upper Cardan joints 11 and 15 (centers of projection). The lower parts of the rods are joined with spherical spindles 6 and 13, shifted by the screws of the base supports 7 (b_x), 9 (b_y); and 12 (b_z).

Since the support of heights with the base installations is located above carriage 4, having movement X, Y (hand wheels and screws 2, 35 and 26, 24), then the intersecting rods will move through the Cardan joints of the carriage with aerial-photos in position of intersection. The photos are observed in fixed optical system 33, 34, located on stands 3 and 16.

For measurement of heights of points of a terrain model or the setting of a section its base system is shifted up or down. Pole 25 is connected with a pantograph (SD-1), with which the map is traced.

Stereograph SD1-M [7], [9] and [10]

The main parts of instrument (Fig. 153) are: table on extending legs, bed with correction planes, coordinate carriages X and Y base device with height counter, Cardan mechanisms, optical system, pantograph (in the most recent model a coordinatograph).

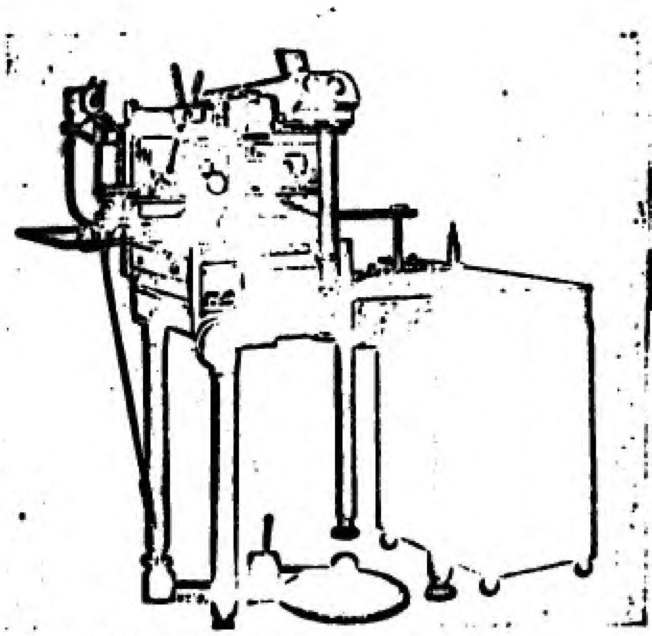


Fig. 153.

Coordinate carriages X and Y are assembled on precision ballbearings and are shifted by hand wheels within limits of the field of a single stereopair. Two speeds of importing are motions are available: 2.5 and 8 mm per turn of the wheels. In the right part of carriage X there are located repetition carriages X and Y with photocarriers and in left part, the base device.

Mechanism of correction planes. This mechanism is disposed on a separate plate, fixed in the lower part of the bed, and consists of four spindles shifted by balance beams (Fig. 154). Since the arms of the beams are identical in length, the readings on the micrometers characterize the magnitude of rise of the spindles. On them are located correction planes, consisting from steel base and a glass plate, polished

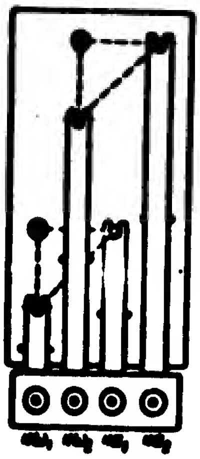


Fig. 154.

with great accuracy. By means of counterweights the planes are always drawn to the spindles. The angles of inclination of the planes are determined by the formula

$$k\alpha = k\omega = \frac{h_{\alpha,\omega}}{r}. \qquad (172)$$

Since r = 90 mm and h_{max} = ±8 mm, then, considering the conversion factors (k = 1.3; 1.9; 2.4), we will find it possible to incline the plane from 0 to 6°, which corresponds to a change of α and ω of 0 to 3°.

From the theory of the stereograph it follows that the point of rotation must be higher than the point of tangency of the spindle. With a lower location of the point errors in the magnitude of F appear, in consequency of which the need for its [their] removal ar!ses. Let us determine the value of the error ΔF_1.

In Fig. 155 we can see the location of the correction plane; here I is the correct position of the plane and II is its actual position.

We have

$$m_1 = f_k \frac{\alpha}{2} k\alpha,$$

$$m_2 = r_1 \frac{k\alpha}{2} k\alpha.$$

Therefore the error in F is equal to

$$m_1 + m_2 = 0.5 k\alpha^2 (f_k + r_1 k). \qquad (173)$$

At r_1 = 31 mm and angle α = 3° the value of the error can be 0.25 mm. In the presence of tilts of aerial-photos not exceeding 1° to 30', the error in F does not effect the final result of orientation of the photos. With larger values of angles of tilts the values F it can be co:rected by meanc of the corresponding supports ɔf focal lengths.

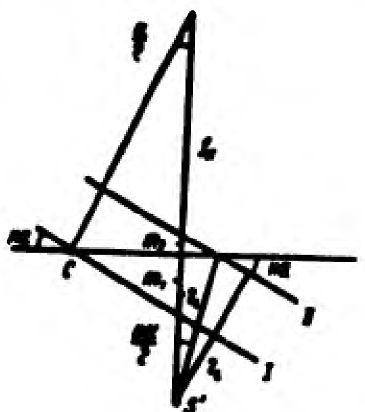


Fig. 155.

<u>Mech nism of the base system</u>. The mechar!sm is located on carriage X, in its left part, and consists of supports and two spindles. It permits introducing the values of b_{x_1}, b_{y_2}, and b_{z_2}, and also the value h of the section of the terradn. Rods carrying out intersection of determined points rest on spherical spindles of the mechanism. A shift of the

vertical [height] carriage, executed by the pedal, is accompanied by rotation of the disks of a mechanical counter (Fig. 156). Holder 3 of the counter is located on the upperpart of base device 8. On the holder there is a support with a worm wheel having 100 teeth. The wheel [together] with disk 4 engages the worm, which carries change wheel 2 at the bottom and drum 6 at the top.

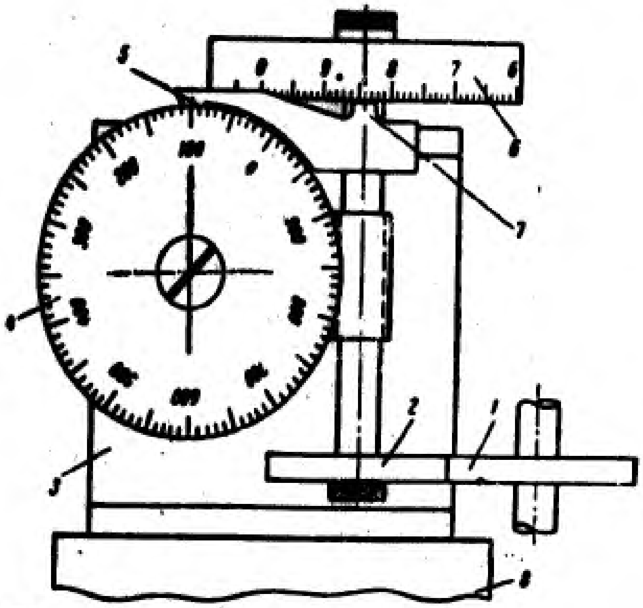


Fig. 156.

With rotation of the vertical screw (with step of 1 mm) of gears 1 and 2, the worm transmission, and disk 4 and drum 6 connected with it are set in motion with respect to index marks 5 and 7.

Let us assume that the scale of heights of the model is equal to 1/10,000; then by selecting gear 2, equal in the number of teeth to gear 1, after one turn of the vertical screw we will obtain on drum 6 (divided into 100 parts) a reading of 10 m with the value of the least scale division 0.1 m. On disk 4, also divided into 100 parts, at this time there will be fixed a displacement of one division.

Consequently, one full turn of disk 4 will signify a section of the model of 1000 m. For the same numbering of disks, but at a model scale of 1:25,000 it is necessary to install a gear 2 of a smaller diameter, namely by 2.5 times. For cases of other vertical scales of the model gears 2 and 1 must have corresponding gear ratios.

Designating the number of teeth of gears 1 and 2 as a and b, we can write the formula for selection of gears:

$$\frac{a}{b}=\frac{M_B}{M}. \qquad (174)$$

where $1/M = 1/10{,}000$ and $1/M_B$ is the sought scale.

A peculiarity of the counter is the presence of smooth feed up to contact of its gear with the pinion gear of the vertical screw. This makes it possible to use all possible combinations of gears. With a set of 30 gears the number of combinations of scales reaches 700, which ensures the use of any scales of the processed aerial-photos.

Cardan mechanisms. They consist of Cardan joints with two axes of rotation and precision rods. The values of the focal length of the height of the section are established from one side of the center of projection, as a result of which the size of the instrument is kept small.

Cardan sleeves each carry six ballbearings. The first and second axes of each Cardan joint must intersect with the axis of the precision rod with an accuracy of 7-10 μ. Therefore for the axes of the Cardan joints there are provided adjusting movements: along the axis Y – movement of the support screws; along the axis X – eccentric movements of the ballbearings; and along the axis Z – shifts of the first axis of rotation of the Cardan joint up or down. Adjustment is executed with the help of indicator in conditions of a turn of the Cardan joints and their sleeves by 180 degrees in both the vertical and a slanted position of the rods.

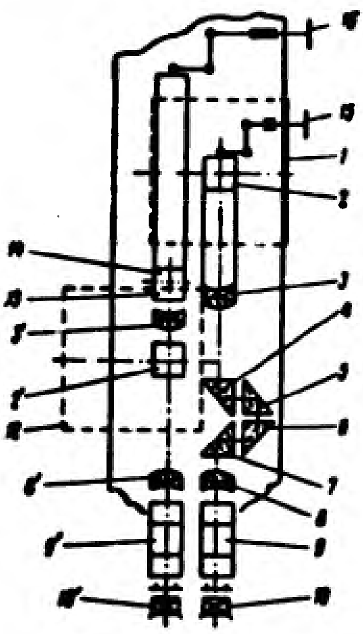


Fig. 157.

Optical system. Figure 157 gives a diagram of the optical system of the SDI-M. In the right branch rays from aerial-photo 1 pass through prism 2, objective 3, then as a parallel bundle to prisms 4, 5, 6, and 7, into objective 8, and then through trapezoidal prism 9 to the focal field with the mark; the image is viewed through eyepiece 10.

In the left branch rays from photo 12 pass through prism 2' of mirror 13, 14, of objective 3', then as a parallel bundle into objective 8' and through trapezoidal prism 9' into the focal field of eyepiece 10'. The total magnification of the optical system ≈4ˣ.

The optical is introduced by shifting branch 10-9-8-7. By movement of block 3-2 decnetering Δy_2 is introduced, movement of block 2'-3'-13-14 introduced decentering

Δy_1. For this purpose micrometers 15 and 16 are applied. Decenterings Δx_1 and Δx_2 are introduced by displacement of the aerial-photos.

Pantograph. Instrument consists of four hinge-connected arms, cursors on the arms, fixed axis of pole, and a bar utilized for smooth change of the scale of the map (Fig. 158).

Attaching one of the cursors in outlined position, move it until needle 1 contacts line of polar ruler. Than shift the second cursor until pencil 2 contacts the same line; this completes the setting of a new scale of drawing. In order to determine the scale of the model, the positions of points of the model and the plan are superimposed with the needle and pencil. Multiplying the obtained plan scale of the model by a conversion factor, we obtain the vertical scale of the model. Since 1963 pantographs have been replaced by coordinatographs, designated as SD-2.

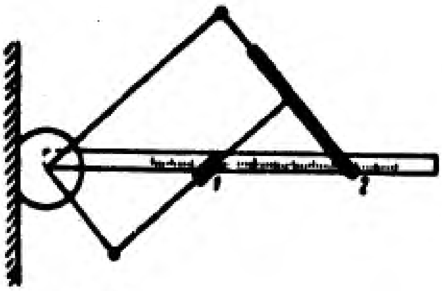


Fig. 158.

Orientation of aerial-photos and their treatment. For treatment of aerial-photos on the instrument, first carry out preparatory works: calculate conversion factors k and stack the negatives on the panel with the emulsion down, as in their surveying location. The scale of the model is taken as 1.1 to 1.2 of the scale of the photo (setting b_x). Then the transverse parallax observed in five standardly located points is removed by adjusting movements of the instrument, according to the diagram (Fig. 159).

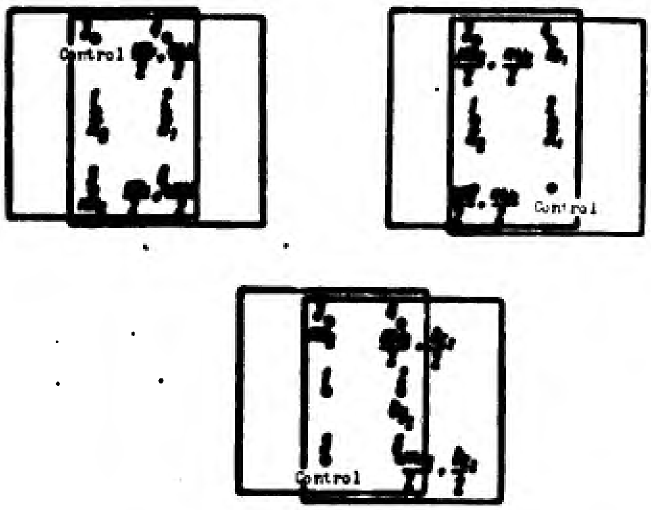


Fig. 159.

After removal of transverse parallaxes decenterings of sighting rays are introduced

$$\Delta y_1 = f_k \omega_1;\quad \Delta y_2 = f_k \omega_2. \qquad (175)$$

and also decentering of the aerial-photos:

$$\Delta x_1 = f_k \alpha_1;\quad \Delta x_2 = f_k \alpha_2. \qquad (176)$$

Subsequent processes of scaling and turning of the model are based on the use of horizontal and vertical points of the plane table and subsequent setting of the gears of the counter.

After final orientation of the aerial-photos tracing of the map is carried out just as on the stereoplanigraph and on other instruments.

Stereograph USD-1

The instrument constitutes a development of preceding designs of stereographs. In it the theories of correction mechanisms are more strictly carried out and a number of improvements are made. Attachments for calculation of azimuthal distortions of aerial photos are introduced, allowing the use of horizontal location of aerial-photos. Installations for decentering images in the instrument are connected with mechanisms of the correction planes. Settings of all basic components are realized. In the optical system there is applied switching of sighting rays, allowing use of the instrument not only for treatment of isolated stereopairs but also for spatial triangulation on aerial-photos. Tracing of the map is produced on a coordinatograph, located on the table of the instrument on the right.

From Fig. 160 changes in the theory of converted bearings are evident. Here c is the point of zero distortions of aerial-photos; m_0 is a point on an orthogonal aerial-photo; m is a point on a tilted photo; m_1 is is the projection of point m on the horizontal plane, m_2 is a point of the correction plane located on plumb line m_1m_2, I is an aerial-photo tilted at angle $\alpha_1(\omega_1)$; and II is a correction plane tilted at angle $\alpha_2(\omega_2)$.

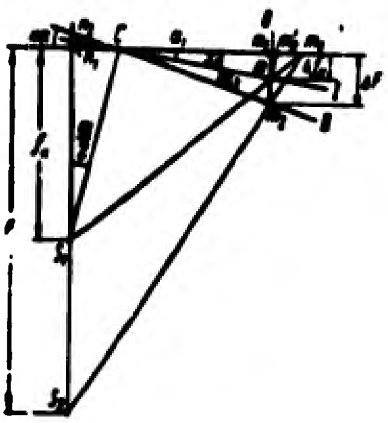


Fig. 160.

With horizontal location of the aerial-photo m will appear at point m_0'.

Therefore the sighting ray, directed on point m_1, must be shifted from the center by the magnitude

$$\delta = 2y_n \sin^2 \frac{\omega}{2}$$

or

$$\delta = 0{,}5 y_n \omega^2. \qquad (177)$$

For instance, when y_n = 80 mm and ω = 3° the value of $\delta \approx$ 0.1 mm.

Such displacement of points on the aerial-photo is connected with azimuthal distortions of nadir directions in the tilted photo. In fact, from Fig. 161 it is clear that

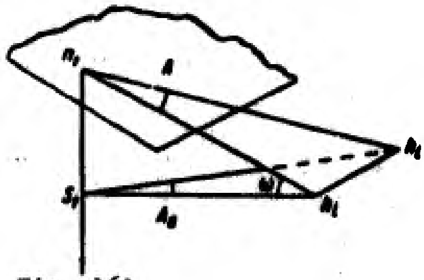


Fig. 161.

$$hh_1 = n_1 h_1 \operatorname{tg} A = S_1 h_1 \operatorname{tg} A_0,$$

whence

$$\operatorname{tg} A = \operatorname{tg} A_0 \cos \omega.$$

When A_0 = 45° we will obtain

$$A_0 - A = \Delta\gamma = \frac{\omega^2}{4}. \qquad (178)$$

Distortion $\Delta\gamma$ will be considered in the case when we construct displacements δ by the ray of sighting in a direction from the center of the aerial-photo. Let us note that these displacements do not depend on the sign standing before the value of the angle of tilt of the aerial-photos; they, as it were, stretch the image forward or back or to the left or the right in the presence of angle α. Let us assume that we have simultaneously slopes α and ω of the photo. Then the principal vertical will pass between axes x and y (Fig. 162). Displacement $m_0 m_1$ can be expressed, by analogy with (177), by the formula

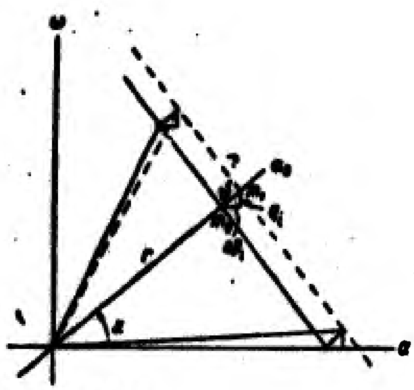


Fig. 162.

$$\Delta r = 0{,}5 r \alpha_0^2. \qquad (179)$$

For small slopes of the aerial-photo the value of α_0^2 can be determined from the equation

$$\alpha_0^2 = \alpha^2 + \omega^2.$$

Correction Δr can be expanded along axes x and y:

$$\Delta p_i = \Delta r \cos \varkappa;$$
$$q_i = \Delta r \sin \varkappa,$$

where angle κ of the position of the principal vertical can be determined by the formula

$$\operatorname{tg} \varkappa = \frac{\omega}{\alpha}$$

or from relationship of segments $k\omega$, $k\alpha$ of the correction micrometers.

By means of stencils, on which the main verticals of aerial photographs are plotted and perpendiculars are dropped to them from points, one can determine value of r and then calculate the corrections Δp_1 and q_1 [sic].

If angles of tilt α and ω are larger than 1°30' corrections Δp_1 and q_1 [sic] will exceed 0.01 mm [7] and then they must be introduced by the decentering screws. It is possible to install a special correction device (Fig. 163), so which will solve the problem of the introduction of corrections automatically.

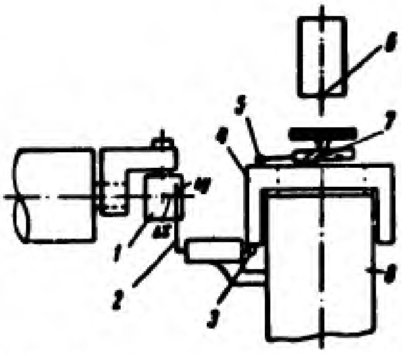


Fig. 163.

To sleeve of the upper Cardan joint there is fastened a parallelogram for actuating small site 1. Its slopes are obtained as equal to the slopes of the precision rod. A bearing with axis and mutually perpendicular levers 2 and 3 is attached to decentering carriage 8. The end of bar 2, displaced by the magnitudes

$$\Delta x = f_k \alpha$$

and

$$\Delta y = f_k \omega,$$

will be shifted by the action of the site by the magnitudes:

$$h_{t_x} = \Delta x \operatorname{tg} A = \Delta x \frac{x_n}{k f_k} = \frac{x_n f_k}{f} \alpha,$$

$$h_{t_y} = \frac{y_n f_k}{f} \omega$$

or – in the direction of the main vertical – by

$$h_{t_r} = \frac{r_n f_k}{f} \alpha_0,$$

where k is a conversion factor. Bar 3 will shift support 4, and the latter through bar 5 will turn around axis 7 a special glass plate, through which luminescent mark 6 is projected. Cardan system 7-5 can be aligned in azimuth, with which axis 7 of bar 5 will be disposed perpendicular to the main vertical. As a result there will occur displacement of the image of the mark as a function of the slope of the plate and its thickness.

It is known that

$$\frac{\sin i}{\sin i'} = n$$

or for small angles (from 0 to 6°)

$$i' = \frac{i}{n}.$$

where n $\approx$ 1.51 and is the index of refraction of the glass. Displacement of a ray passing through the slanted plate is equal to

$$\delta \approx (mi - mi')\cos i \approx m(i - i')$$

or

$$\delta = m\left(\frac{ni - i}{n}\right) \approx \frac{m}{3} i = \Delta r_1$$

where i is in radians. At a plate thickness of m = 3 mm the value is δmm = i. Thus, in the presence of r_1 equal to the length of bars 2, 3, and 5, the magnitude of correction

$$\varepsilon_0 = \frac{\Delta r_1}{r_1} = \frac{r_0 f_1 e_0}{k \cdot r_1} \qquad (180)$$

is considered by the special attachment.

For transition to a new focal length the plate of thickness m is replaced by a plate with a different thickness.

<u>Structure of the instrument</u>. Figures 164 and 165 show diagrams of the instrument. On bed 1 are located base carriages X and Y of the instrument, i.e., 5 and 6. In the left part of carriage X there is located base device 28, and in the right part, pencil mechanism 43 or coordinatograph manual controls 38, 42 connected to it.

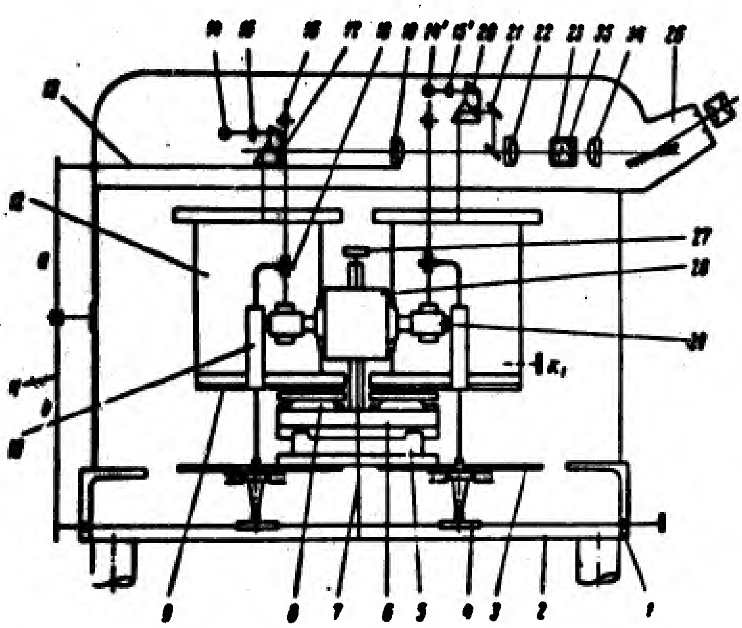


Fig. 164.

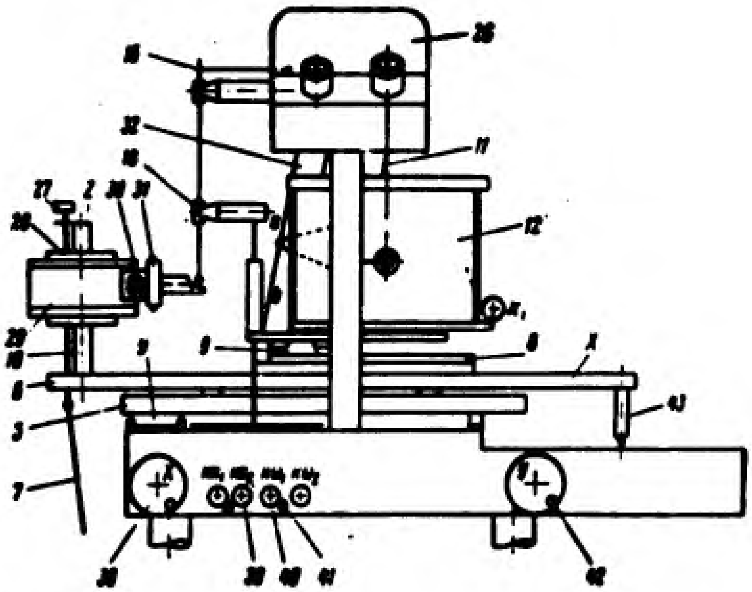


Fig. 165.

In the middle part of the carriage X are differential carriages x and y (8 and 9), and on them – photocarriers 12 with the aerial-photos. Besides a photocarrier upper differential carriage Y carries vertical carriage 10 (and also 29) of focal changes. The carriage is equipped with Cardan 18 connected to precision rod 16. The position of the Cardan joint in terms of height is determined by the position of the spindle of the carriage on the correction plane.

The figure also shows front plane 3, set in a slanted position by a micrometer connected with support 4.

The pole of the plane, in the form of a spindle with a ball on the end, can be inclined both in the plane of drawing (on angle k_{ω}) and also perpendicular to it (on angle k_{α}), for which the corresponding micrometers 39, 40 are applied. Here similar micrometers can be connected by satellite gear 41 for joint turning of the planes, which becomes necessary during turning of the model. During the tilting of the planes there occurs a certain lowering of them, namely by the magnitude $2f\sin^2\frac{k_{\omega}}{2}$, with which the planes pass through the points of zero distortions of the aerial-photos.

Fig. 166 shows the arrangement of the correction plane in section. The length of the shank is determined by comparison of correction dF according to formula (173) and the magnitude of drop dF of the plane. Table 27 gives data on the

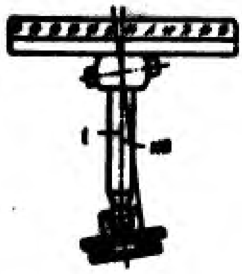


Fig. 166.

setting of planes.

Thus, the error ΔF of setting of the planes with maximum tilts of the aerial-photos is within acceptable limits.

Pusher 2 (see Fig. 164), which by means of the supports sets the slope of the plane by the formula $h' = r_0 k\alpha$ (also $h_1' = r_0 k\omega$) extends beyond the limits of the instrument, where through balance beam 11 it adjusts carriage 13 (with prism 17, luminescent mark 14, and objective 19 by a magnitude equal in this case to

$$\Delta y = f_k \omega.$$

The ratio of the arms of the beam should be equal to

$$\frac{a}{b} = \frac{f_k \omega}{r_0 k \omega} = \frac{f_k}{r_0 k}. \qquad (181)$$

The base device with components of settings b_x29, b_y30, and b_z31 can be shifted by the magnitudes X, Y, and Z. Thanks to this precision rod 16, joined at the bottom with the Cardan joint of the base device, shifts Cardan joint 18 with photocarrier 12 by magnitudes x and y. The photocarrier has a detachable cassette, adjusted in the process of orientation at angle κ.

Table 27

f_k, мм	$k\alpha$, в мм	$k\alpha$, In radial measure	k	$\alpha°$	dF_{Theor}, мм	$dF_{Pract.}$, мм	ΔF, мм.	Note
55	6,5	5°	2,5	2°	0,18	0,18	+0,06	$F = 140$ мм
70	7,5	6	2,0	3	0,34	0,34	+0,02	$l = 75$ "
100	5,3	4°12'	1,4	3	0,25	0,20	−0,05	$r_1 =$ [illegible]0 "
140	2,3	2	1,0	2	0,04	0,04	−0,05	
200	1,5	1 3	0,7	1°30'	0,02	0,02	0,00	

Thus, by rotating hand wheels 38, 7, and 42 and observing the aerial-photos through fixed binocular 26, it is possible to measure the height of points stereoscopically and to trace contours on the table of the instrument.

Let us consider the diagram of the optical part of the instrument (Fig. 167). Here the right branch is represented with block of two prisms 25 raised up, optical elements 14, 15, 17, 19, 24. Left branch has elements 14, 15, 20, 21, 22, 23, 35, 34.

The block of two prisms (25) when lowered changes the direction of rays,

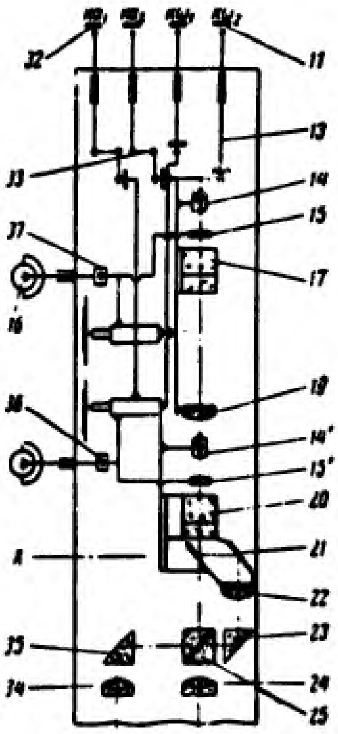


Fig. 167.

which is required during the treatment of aerial-photos by the method of continuation. Correspondingly the base components b_x are set both as positive and negative direction.

When balance beam 11 is rocked by carriage 13 decentering Δy is established, and during rocking of balance beam 32 through angle [corner] 33 decentering Δx of branch 14, 17, and 19 is set. Between objectives 19-24 there is parallel movement of rays.

The carriage of the second branch are arranged analogously; the output rays of this branch are on common line Y with first branch.

During settings of carriages of decenterings Δx, Δy there is also a shift of the bars of the mechanism for azimuthal corrections of Δx and Δy (36, 37). By means of parallelograms of upper Cardan joints plane rear [sites] tilted during work actuate (through bars) plane-parallel plates 15 and 15', located ahead of optical blocks 17 and 20. Thanks to this the images of the sighting marks are displaced with respect to the aerial-photos.

Coordinatograph. Its composition includes carriages X' and Y', a pencil mechanism with a solenoid, and two reductors for changing the coefficients of magnification n of the model. Figure 168 shows the diagram of a coordinatograph.

Let us assume that the scale of plan is equal to

$$\frac{1}{M_0}=\frac{1}{M_r}n, \quad (182)$$

where $1/M_r$ is the scale of the model.

Let us select gears c and d at which

$$n'=\frac{M_r'}{M_0}.$$

Coincidence of points of the model and the plan is attained by changing the base b_x of the model.

For vertical scale of the model we have the relationship

$$\frac{1}{M_n}=\frac{k}{M_r'}.$$

Since at the value the gears cannot be used to set the scale $1/M_B$, the spindles sliding along the correction planes are given vertical shifts within limits up to ±5 mm. Calculating the value of $1/M_B$ by the formula, we will find on special tables the nearest scale $1/M_B'$, and the pair of gears a and b corresponding to it, which gives

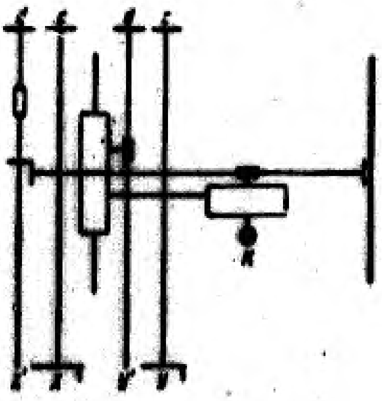


Fig. 168.

$$\frac{1}{M_B'} = \frac{f'}{M_f'} . \qquad (183)$$

From this it is possible to determine the new value

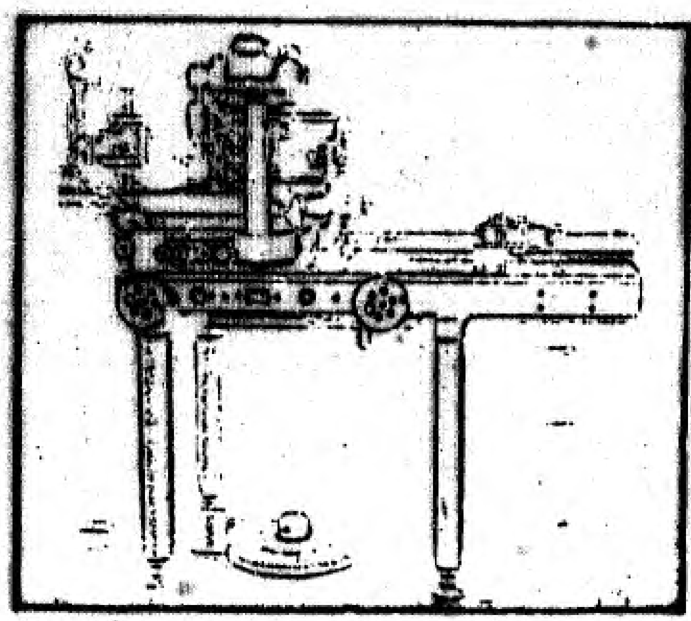
$$f'' = kf_k \qquad (184)$$

which is set in the instrument.

After that full coordination of the plane and vertical scales of the model is obtained.

Checks and Adjustments of the Instrument

The stereograph [USD-1 (УСД-1) (Fig. 169) consists of a number of units and attachments mounted on common bed. Each of the units is adjusted after assembly in accordance with technical requirements for accordance with technical requirements for accuracy of operation and on smoothness of movements of the component elements.



GRAPHIC NOT REPRODUCIBLE

Fig. 169.

The instrument is installed in a location with a strong foundation on cement footings.

The left V-shaped rail bed is considered a base and is set in a horizontal position with a level with 10-second accuracy. Then the rails of carriages X, x, y, and F are set also in horizontal and mutually perpendicular position; great sensitivity and accuracy in movements of the carriages is achieved.

The base device is assembled separately and on the control plate carriages b_x, b_y and b_z are set in a mutually perpendicular position by means of squares and indicators, under the condition that the base guide Z is fixed in a vertical position with the help of a level on the square. Correction mechanisms contain glass plates with tangs (l = 75 mm) for realization of tilt by means of the tilt support. Supports x, y are set in a mutually perpendicular position by means of squares and an indicator; equality of lengths of tangs, which is necessary for further turning of the model be equal increments $k\omega$ or $k\alpha$ is established on the basis of calculations.

Fig. 170 represents plane 1 inclined by $k\omega$ (approximately 5 degrees). After the indicator attached to carriage y of the instrument is shifted by segment b

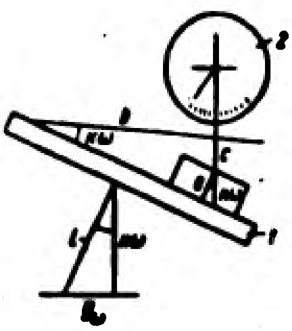


Fig. 170.

of the instrument is shifted by segment b (on the order of 140 mm), it becomes necessary to place a measuring plate with thickness a under the measuring rod of indicator 2; segment a is measured by the indicator. After this we will obtain the equation

$$b\frac{\sin k\omega}{\cos k\omega}=c+\frac{a}{\cos k\omega},$$

whence

$$b\sin k\omega-a=c\cos k\omega. \qquad (185)$$

Selecting from tables the best value of $k\omega$, we will find the length of the tang by the formula

$$l=\frac{y_\omega}{\sin k\omega}, \qquad (186)$$

where y_ω is the difference of the readings of $k\omega$ indications of the micrometer.

The length of the tang of the second correction plane is determined analogously. Then the length of one of the tangs is fitted to the length of the other. The arms of the Γ-shaped pushers of the correction planes are made equal.

The given investigation makes possible the simultaneous obtaining of scale division values of micrometers $k\alpha_1$, $k\alpha_2$, $k\omega_1$, and $k\omega_2$.

The Cardans of the instrument are investigated for intersection of their axes X, Y at the point through which the axis of the precision rod passes. This check is conducted on a special Cardan joint holder by means of an indicator during turning of Cardan joints and sleeves by 180 degrees.

Cassette glasses are adjusted under the microscope for centering of the position of the central small cross located the intersection of the coordinate marks, above the axis of rotation of the corresponding photocarriers.

The binocular system with the mechanisms of the carriages is adjusted on a separate support. In the vertical position the balance beam of decentering Δx and Δy of the outlets visual axes have to be coordinated with the data of the drawing. Balance beams of decentering must work in accordance with formula (186); it is necessary to align the arms of the balance beam in such a way that the introduction of decenterings for identical values of $k\alpha_1$ and $k\alpha_2$, and also $k\omega_1$ and $k\omega_2$ will also be identical.

After complete assembly of the instrument it is necessary to carry out a check by the indicator of the accuracy of operation of the nut of the vertical screw of carriage Z by means of direct and reverse matching with screw z on the same reading of the height drum; the permissible deviation should not be greater than 0.010 mm; check of allowances Δp by means of the indicator attached to the carriage Y_2 and resting in carriage Y_1 during matching in height by hand wheel Z downward from above and vice versa on the same reading on the height drum; the permissible magnitude $\Delta p \leq 0.015$ mm.

The working checks of the instrument are outlined below:

setting of bars in the vertical position by means of up or down shift of carriage Z and observation in the binocular of the central part of the grid located in the cassette, which gives the value of the zero points of X, Y, b_{y_1}, b_{y_2}, b_{x_1} b_{x_2};

finding the values $H_1 = F_1$ and $H_2 = F_2$ by means of the indicator attached to carriage X of the instrument. By solving the equation

$$\frac{l_1}{l_2}=\frac{F}{F+\Delta Z}, \qquad (187)$$

in which l_1 and l_2 are segments of grid and ΔZ is the difference of readings of the position of carriage Z in height (on the order of 60 mm), one can find the value of F and the zero point of Z with an accuracy of a hundredth of a millimeter. This operation makes it possible to establish the zero points of b_{z_1} and b_{z_2}.

The overall check of the instrument is done on a grid model, by means of observation in 10-15 grid points of heights at zero settings of the cassettes, correction planes, and base components [2] and [10].

The expected accuracy of the model in height should be within limits of 1/12,000 H to 1/15,000 H.

Orientation of aerial photos on instrument differs in the fact that thanks to simultaneous setting of the values $k\alpha$ and $k\omega$ and decenterings Δx and Δy the transverse parallax in 4 point is removed by the same method as during work on the stereoplanigraph, when account is taken of the coefficient of orientation

$$k_0 = \frac{f_k^2}{2y^2}.$$

In other respects the process of treatment of aerial-photos is identical to that in methods presented earlier.

Data on the instrument USD-1 are given in Table 28.

Table 28

Elements of the instrument	Numerical data
Size of aerial-photos, mm	180×180
Focal lengths, mm	55 . . . 200
Magnification of optics, x	4,5 and 5,6
Focal length of instrument, mm	140 ± 5
Angles of tilt inclination of aerial-photos α, ω in degrees	± 3
Angles of tilt $k\alpha$, $k\omega$ degrees	± 6
Settings of b_x, mm	± 100
Settings of b_y, b_z, mm	± 12
Settings of $\varkappa$, in degrees	± 10
Coordinates X, Y, mm	± 150
Coordinates Z, mm	135 — 225
Magnification of coordinatograph, x	0,5 — 3
Dimensions of working part of table, mm	740×744

§ 51. Universal Instruments with Photocopying Devices

During the creation of maps on universal stereophotogrammetric instruments, after tracing of contours there is executed the process of transferring details of the terrain from the aerial-photos to the plane table. When the quantity of contours is large the use of instrument is not always rational. Therefore recently inserts (patches) of photorectified complicated zones have been used.

However, the presence of relief complicated this process of creation of a map.

Hence there appears a requirement to obtain for this purpose a photomap only in orthogonal projection, making it possible to trace contours directly on it.

The problem of the creation of an orthogonal photomap is solved by many methods, of which we will note:

a) rectification of aerial-photos in terms of vertical [height] zones (2-7) on photorectifiers;

b) optical rectification of 3-8 zones on a common sheet of photographic paper;

c) survey rectification by means of the creation of a spatial model on a survey photorectifier and photographing it on a common sheet of photographic paper;

d) slot rectification executed on the multiplex (United States);

e) slot projection by means of an aggregate consisting of a projector with $f_{\Pi} = 200$ mm and a 3-section multiplex (USSR); the aggregate was designed by Ye. I. Kalantarov in 1958;

f) slot projection by a stereoprojector (G. V. Romanovskiy) with a photoattachment containing a slot device (1961).

Instruments of the last two types possess the property that orthogonal photocopying is executed on the basis of mutual orientation of aerial-photos during their stereoscopic observation, that ensures high accuracy of rectification of the image.

The photostereograph FSD (author's design) described below constitutes a special instrument, consisting of a stereograph and a slot photocopying device.

Intersection by converted rays is based on a change in the focal length of photocarriers, just as in the stereograph. Points of intersection and joints of the precision rods with the Cardans of the photocarriers are located on different sides of the centers of rotation of the rods.

In this case the possibility of inclining the plane around a point located higher than the plane at distance l_1, which extends focal length F in accordance with requirements of the theory of intersection. From Fig. 151 (see above) it is clear that

$$\frac{om_2}{om_1}=\frac{\Delta F}{\Delta f_k}=k,$$

whence the lengthening of focal length is

$$oa_2 = k \cdot oa_1$$

or

$$oa_2' = kf_k \frac{a}{2} a = 0{,}5 F a^2. \quad (188)$$

Actual lengthening of focal length is

$$oa_2' = l_1 \frac{ka}{2} ka = 0{,}5\, l_1 k^2 a^2.$$

Therefore

$$\Delta F = 0{,}5 a^2 (F - l_1 k^2). \quad (189)$$

At $l_1 \simeq 50$ mm, $\alpha = 3°$, $f_k = 70, \ldots, 100$ mm and $F = 135$ mm the value $\Delta F \leq$ ≤ 0.08 mm. The mechanism of tilting of planes obtained with this solution is very simple.

In Fig. 171 shows a front view of the instrument. On three stands there is upper level 1 with centers of rotation of precision rods and mobile photocarriers with aerial-photos, established at angles κ_1, κ_2, and decenterings Δx, Δy. On the right on the plate there is a mechanical height counter. Screen 3 of the instrument is shifted in the vertical direction by means of a foot control and transmissions. On the screen is located coordinatograph 6 with carriages X and Y and base installations b_{x_1}, b_{x_2}, b_{y_1}, b_{y_2}, b_{z_1}, and b_{z_2}. To carriage X (upper) there is connected plane table 2, and to the screen, pencil [stylus?] device K on console 4. On the left detachable photocassette 5 can be connected [illegible] X. Above it on console 7 there is located slot Щ for photocopying.

Fig. 172 (side view) shows the location of precision rods with focal carriage 20 and correction planes 9, set at angles $k\alpha$, $k\omega$ for the purpose of ensuring the changes required by theory at the time ofintersection of focal lengths of the photocarriers of the instrument. The aerial-photos are placed in the cassettes of the instrument with the emulsion down, as in the stereographs SD-1 and USD-1.

With displacement of the aerial photos and the plane table by the precision rod on opposite sides there is attained:

a) the possibility of stereoscopic observation of points of the aerial-photo,

b) possibility photocopying the photos during a movement of the photocassette under the motionless slot.

<u>Observation and photocopying</u>. For observation and photocopying it is necessary

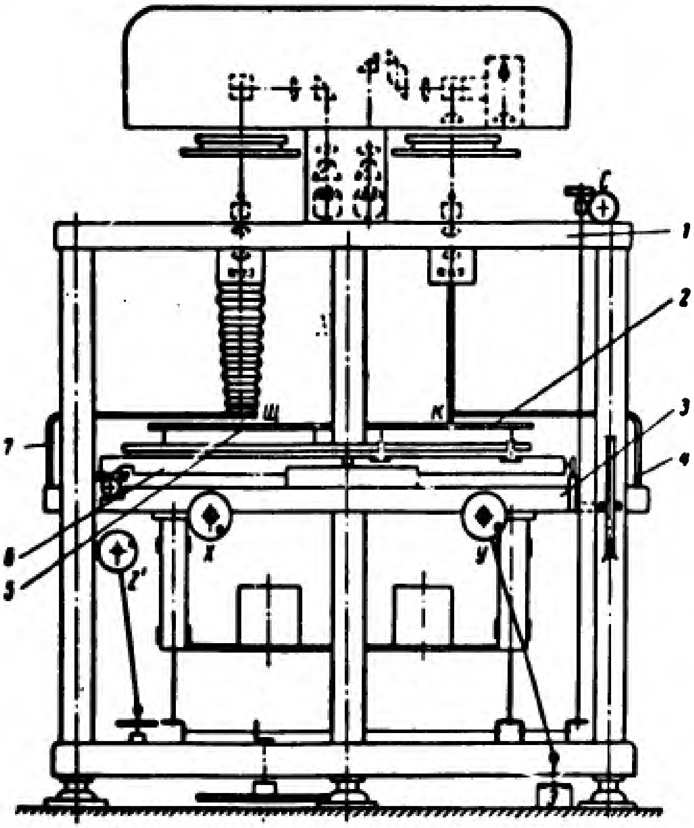


Fig. 171.

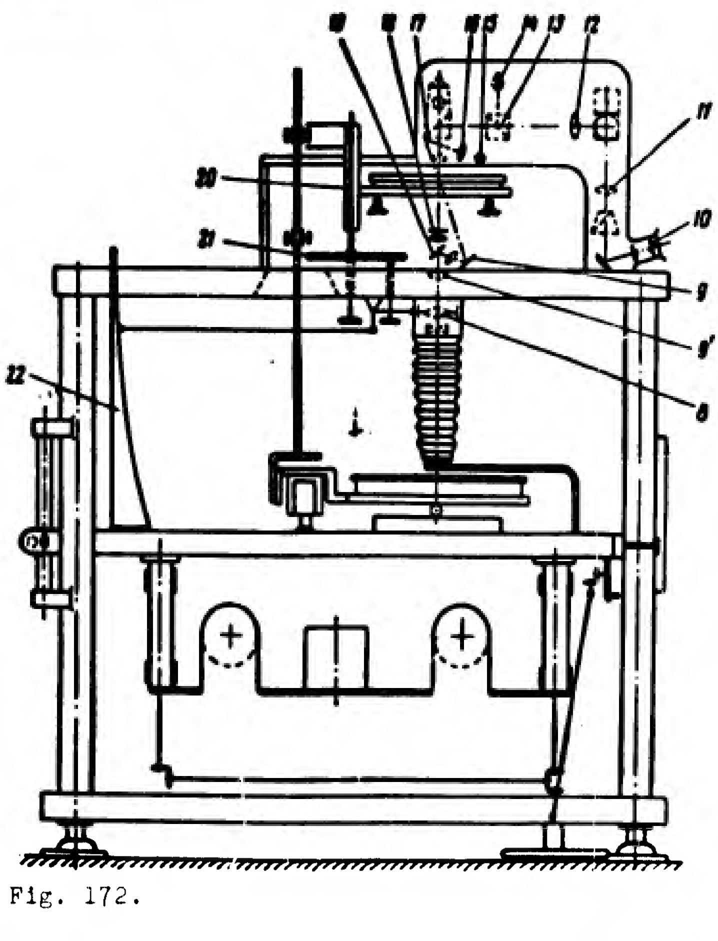


Fig. 172.

to illuminate an aerial-photo both from below and from above. Observation is produced through eyepiece 10, a prism, and objective 11. Then the rays, as a parallel bundle, enter objective 12 and through semisilvered cube 13 and the prism they reach the aerial-photo. Luminescent mark 14 is superimposed on the image field of the photo, which is illuminated by lamp 18 through frosted glass. Simultaneously lamp 15, through small objective 16 and mirror 17, illuminates the observed part of the negative (slide) from above. The light then enters objective 9, separated from the illuminated place by a distance equal to the f' of the objective (in this case f' = 135 mm) and then reaches mirror 19. Another beam, proceeding through the same point of the photograph, reaches a photocell for adjustment of illuminance of the image.

Projection of the image of the aerial-photo is produced by objective 8 of variable focal length through the slot onto photographic paper located in the photocassette.

Focusing of the image is carried out by templet 22 and a system of rods and pushers acting on the positive lens of the objective with variable focal length.

The right branch of the optical system of the instrument has an analogous arrangement with projecting objective 9 and condensor illuminator, allowing the obtaining on the plane table of the image of the right aerial-photo for marking certain of its points whenever the operator wishes. Figure 173 shows an experimental model of the instrument without a tent.

Before the beginning of operations, a base with control points is fastened on the plane table; it is used to carry out normal stereophoto grammetric and geodesic orientation of the model with the introduction of decentering of aerial-photos and tilts of the correction planes at angles $k\alpha$ and $k\omega$.

Then the electric motor system is switched on; it shifts the photocassette forward and back at a definite speed, each time shifting it along the axis X by a magnitude of 2.5-7.5 mm.

During movement of the photocassette operator looks in the binocular and, using the vertical bond wheel, holds the mark on the surface of the model.

After termination of scanning the photocassette is transferred to the photolab for development of the photographs and drying. The obtained orthophotographs can be attached to the plane table of the instrument, and tracing of contours directly

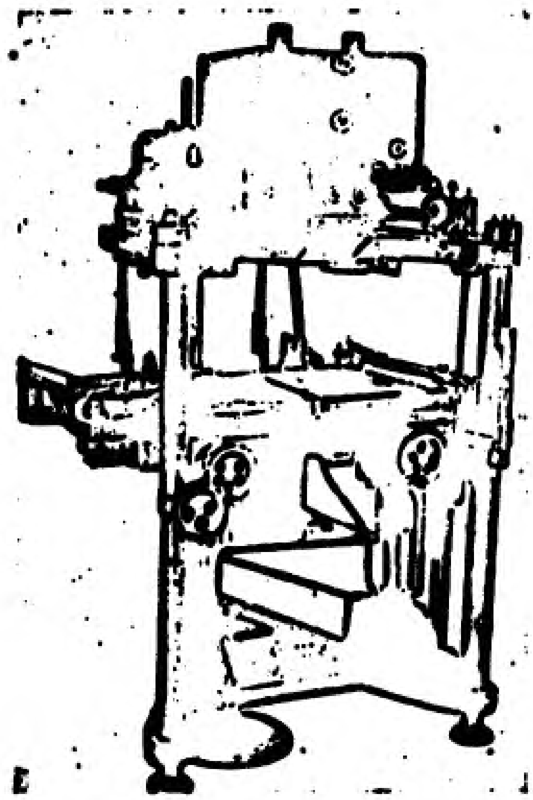


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Fig. 173.

on it can be undertaken. Other forms of work are possible on the instrument. The treatment of aerial-photos and other photoscanning can be carried out by the method of continuation. A MIIGAiK[1] electrocoordinatograph is attached to the instrument located on a table to the right of the operator. Transmission of movements from carriages X and Y of the photostereograph to carriages X' and Y' of the electrocoordinatograph is based on use of synchronously working selsyn motors, connected in pairs by electric drives. Changes in the scale of the traced map are attained by means of the reductors of the instrument.

[1]Moscow Institute of Engineers of Geodesy, Aerial-Photography, and Cartography. [Trans. Ed. note].

CHAPTER VIII

TERESTRIAL STEREOPHOTOGRAMMETRIC SURVEY AND ITS APPLICATION

§ 52. Purpose and Principles of Terrestrial [ground] Stereophotogrammetric Survey

Terrestrial [ground] stereophotogrammetric survey is used for the creation of maps of small sections of terrain, plans of hydrotechnical and architectural constructions, during the investigation of deformation of buildings, bridges, and railroad ways [12], during the determination of the volume of worked-out mines, and so on. Most frequently this form of survey is used for the creation of maps on scales of 1:5000-1:2000 under conditions of full visibility of the object and its sufficient illumination. Also the application of ground stereophotogrammetric surveying as a method of horizontal and vertical thickening in mountain regions is known.

Field works are carried out with phototheodolites, representing precise cameras with attachments for setting the elements of external orientation.

The main beam of phototheodolite, also called the principal axis of photography, is set most frequently in horizontal plane.

Photography is produced from the two ends of a base; the phototheodolite is arranged in a determined position with respect to the base. The theodolite, part of a set of equipment, is used to measure the bearing from one end of the base to geodesic points and the second end of the base, and also determined the length of the base according to a horizontal stadia rod.

The widest use is made of stereophotogrammetric surveying with the principal axes of the phototheodolite perpendicularly and evenly deflected with respect to the base.

Measurements on the photographs are produced on stereocomparators, stereo-planigraphs, or stereoautographs, making it possible to construct the plan and profile of terrain along chosen directions. Processing of photos is based on tying of coordinates of points of the terrain and the photograph.

Formulas for tying of coordinates of points of the terrain and the photograph constitute particular cases of the formulas of aerial-photogrammetry. The only distinction is in the fact that the axis Z of aerial-photography corresponds the axis Y of ground stereophotography.

§ 53. Tying of Coordinates of Points of Terrain and Photograph

Principal axes of the phototheodolite located perpendicularly to the base. Figure 174 depicts a diagram of surveying at which the principal axes are horizontal and are perpendicular to the projection of the base. Terrain point M was depicted on positives in corresponding points. By tracing the direction $S_1M_1 \parallel S_2M$, we will obtain

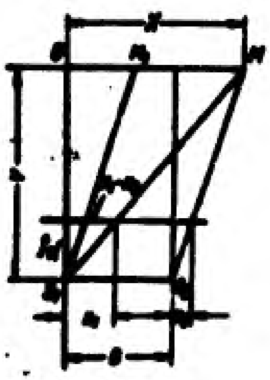

Fig. 174.

$$M_1M = B.$$

It follows from this that

$$\frac{f_k}{Y} = \frac{x_1 - x_2}{B},$$

$$\frac{x_1}{X} = \frac{f_k}{Y} = \frac{x_1 - x_2}{B}.$$

Analogously, for determination of third coordinates we have

$$\frac{z_1}{Z} = \frac{f_k}{Y} = \frac{x_1 - x_2}{B}.$$

The quantity $x_1 - x_2 = p$ is called the horizontal parallax. With the given equations one can determine all three spatial coordinates of a determined point by the formulas:

$$Y = \frac{Bf_k}{p}, \qquad (190)$$

$$X = \frac{Yx_1}{f_k} = \frac{Bx_1}{p}, \qquad (191)$$

$$Z = \frac{Yz_1}{f_k} = \frac{Bz_1}{p}. \qquad (192)$$

From formula (190) it follows that identical parallax characterized the position of a plane parallel to base S_1S_2. If one of stations, for instance S_2, is located at a different height, coordinates z_1 and z_2 of a certain point will not be equal on both photographs. The quantity $q = z_1 - z_2$ is called the vertical parallax. For its determination we will write two equations:

$$Z = Y\frac{z_1}{f_k}$$

and

$$Z - B\,\mathrm{tg}\,\varepsilon = Y\frac{z_2}{f_k},$$

where ε is the angle of inclination of the base, and B tg ε is the elevation of the second end of the base above the first. Solving the equation, we obtain

$$B\,\mathrm{tg}\,\varepsilon = Y\frac{(z_1 - z_2)}{f_k}$$

or

$$q = \frac{Bf_k\,\mathrm{tg}\,\varepsilon}{Y} = p\,\mathrm{tg}\,\varepsilon. \tag{193}$$

In this case the vertical parallax does not affect the value of the determined coordinates.

Evenly deflected axes of the phototheodolite. Axis Y is also in the plane containing the bearing of the principal axis of the phototheodolite for the left photograph. For derivation of formulas for tying coordinates of points of terrain and photographs we will draw through point S_1 perpendicular S_1S_3 to axis Y (Fig. 175) and also line S_1M_1 parallel to line S_2M. Placing photograph p_2 in position p_3, referred to center S_3, we will obtain an arrangement analogous to that shown in Fig. 174. Therefore relationship

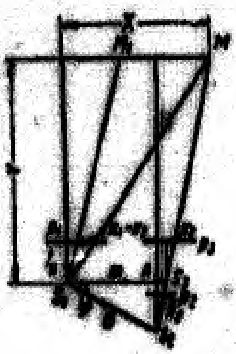

Fig. 175.

$$\frac{M_1M}{x_1 - x_2} = \frac{Y}{f_k} = \frac{S_1S_3}{x_1 - x_3}.$$

is valid, whence

$$Y = \frac{S_1S_3 \cdot f_k}{p},$$

where

$$S_1S_3 = m + n = B\cos\varphi + B\sin\varphi\,\mathrm{tg}\,A_2 = B\left(\cos\varphi + \frac{x_2}{f_k}\sin\varphi\right).$$

Thus, the value of coordinate Y will be determined by the formula

$$Y = \frac{B}{p}(f_k\cos\varphi + x_2\sin\varphi), \tag{194}$$

where

$$p = x_1 - x_2 \tag{195}$$

is the measured horizontal parallax.

With deflection of principal axes of phototheodolite clockwise to the right the equation will take the form

$$Y = \frac{B}{p}(f_k \cos\varphi - x_2 \sin\varphi). \tag{196}$$

With the use of formula (194) or (196) the quantity x_2 is calculated preliminarily by formula (195). Coordinates X and Z of point M are determined through elements of left photograph; therefore the corresponding formulas have the form (according to formulas (191)-(192):

$$X = \frac{Y x_1}{f_k}, \quad Z = \frac{Y z_1}{f_k}. \tag{197}$$

In this case the vertical parallax has a complicated expression, although as before it does not affect coordinates X, Y, and Z.

<u>Convergent and divergent location of principal axes</u>. With convergence (convergent) or divergence (divergent) of the principal axes (Fig. 176) of the phototheodolite at the time of photographing the formulas for tying of coordinates take the form

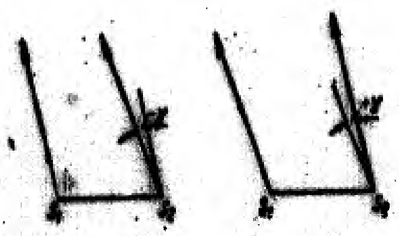

Fig. 176.

$$Y = \frac{B}{p}(f_k \cos\varphi + x_2 \sin\varphi),$$

where

$$x_2 = f_k \operatorname{tg}(A_2 - \gamma), \quad p = x_1 - x_2. \tag{198}$$

Coordinates X and Z of point M are determined by formulas (191) and (192). These forms of surveying are rarely applied.

Using the formulas for tying of coordinates, it is possible to study the character of distortions of the stereomodel for different cases of location of the principal axis of the phototheodolite.

Analysis of the formula for determination of Y shows that with perpendicularly located principal axes the value of Y does not depend on x_1 and z_1; therefore equation (190) determines the plane parallel to the base and separated from it by distance Y. Consequently, the lines of equal parallaxes are straight lines, parallel to the base.

With evenly deflected axes the value of Y is determined by equation (194) of paraboloid surfaces passing through the ends of the base (Fig. 177). With divergent location of axes hyperboloid surfaces correspond to equation Y, and with convergent location of axes, ellipsoid surfaces. When p = 0 the surface turns into a cylinder passing through the ends of the base.

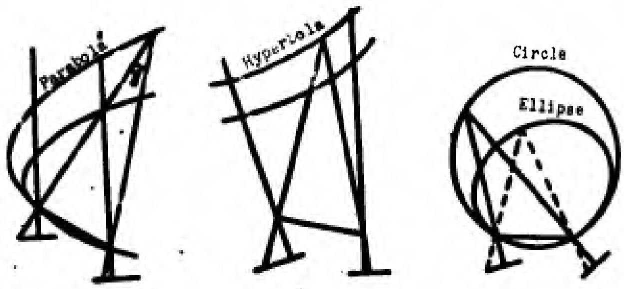

Fig. 177.

§ 54. Means for Ground Photography

1. Phototheodolite

For ground photography, determination of bases, and tying of surveying stations to the geodesic net use is made of phototheodolites, i.e., instruments made up of cameras and theodolites of thirty-second accuracy.

There exist phototheodolites in which the camera and theodolite are separate but have identical fittings for their installation in turn on the same support.

Theodolites applied for the creation of maps must have:

1 – a camera with exact reference frame and with marks determining the position of the principal point of the photograph; 2 – a pattern for fixation of the form of the survey, the number of the photograph, and the value of camera focal length with an accuracy of 0.01 mm; 3 – an objective ensuring exact image (distortion of image must not exceed 0.01 mm); 4 – an orienting attachment for installation of the camera in the required position with respect to the base with an accuracy of 20".

The phototheodolites [SZ/V, TAL] (C3/B, ТАЛ) and 19/1318 of the plant "Geodesy" are used. The phototheodolite TAL is applied in field conditions; the dimensions of its reference frame are 65 × 90 mm. The cassette part is adapted also for survey on film. The focal length of the camera is 55 mm; the relative aperture of the objective is 1:6.6 to 21.5. The magnification of the geodesic telescope is 12^{x}; the limbs ensure accuracy of measured angles of up to 1'.

There are also phototheodolites on a rod with a base of 1 m for photographing formations, open pits, etc. For surveys of ocean shores of seas ship phototheodolites are used; they are fastened pedestals in the bow and stern parts of the vessel and have electrical synchronous shutters.

The most exact is the phototheodolite 19/1318 (Fig. 178), constituting a modified SZ/V phototheodolite.

<u>Phototheodolite 19/1318</u>. The set of the phototheodolite includes Camera, theodolite with tangential screw, three tripods, three holders [?], a base rod, sighting marks, 48 one-way cassettes, 13 × 18 cm, in two boxes, an umbrella, a field bag, and a positioning attachment.

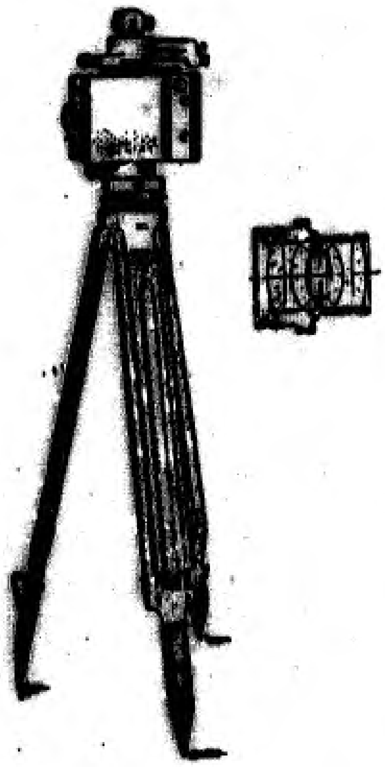

Fig. 178.

The camera is made of an aluminum alloy. In the lower part there is a hollow axis for joining with the holder, which has a micrometer shift which makes it possible to turn the camera at various angles with respect to the base. In the front part of the camera body there is a support with an Ortoprotar objective (f' = 190 mm; rel. ap. 1:24). The angle of the field of view of the objective in the horizontal plane is 45° and that in the vertical is up to 57° with the objective in the extreme position. The objective gives a very exact image, the distortion of which does not exceed 0.01 mm.

The support of the surveying objective carries inside the camera: mirror, index and small objective (with f' = 40 mm). The index, having the form of an acute triangle, is illuminated by reflected natural light of the sky and at the time of exposure is projected onto the left edge of the photograph. The image of the index on the photograph determines the position of the main horizontal of the photograph.

In the rear part of the body there is a reference frame in the form of an outcropping rectangular part; in it there are set two marks which determine the position of the main vertical and are depicted on the negatives. On photographic plate there are also printed a number of stations from 1 to 99, for which two drums with numbers are used.

To the left of the reference frame there is a drum for printing the codes of the forms of survey (A and B – left and right station of normal survey; AL and BL – left and right station for principal camera axes deflected to the left and AR and BR -- for axes (deflected to the right).

In the upper part of the camera there is an orienting device (Fig. 179) with the help of which the camera is set in the required position with respect to the base. The orienting device consists of visual and adjusting telescopes, joined together, and also of glass limb 1 with divisions set immovably at the bottom of the attachment. Above the limb housing 2 rotates; it has sighting and adjusting telescopes 8 and 9, joined together, and also a series of prisms. Telescope 8 20X magnification and contains an eyepiece, crosshairs with a bisector, focusing lens, prism 5, objective 6, and inclined prism 7.

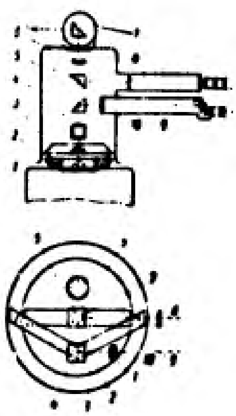

Fig. 179.

The ocular part of the telescope contains a complex prism consisting of a rhomb, a trapezoidal component, and a small cube, intended for uniting strokes displaced by 180°. Further the rays go through objective 10, prism 4, and rhombic prisms 3. Since prism 4 is displaced from the center of the limb, the entire telescope 9 is displaced to the side from telescope 8.

During orientation of the phototheodolite telescope 9 is set in such a manner that the reading on limb 1 will be equal to 0 and 180° or 30° and 210°, and is then locked; the camera is then turned around the vertical axis until the bisector of telescope 8 coincides with the image of the sighting mark or the range rod fixed on the opposite end of the base.

To facilitate setting of the optical axis of the camera in a symmetrical position with respect to perpendiculars to the line of the base, indices A, B, AL, AR, BL, BR. are marked on upper part of the camera.

<u>Theodolite with tangential screw</u>. This theodolite is intended for tying the ends of the base to geodesic points according to method of resection and for extending the theodolite traverse. It differs from the usual theodolite in the

fact that it is easily removed from the support of the tripod and attached to that of the camera; therefore the vertical axis of the theodolite is automatically established on the place of the vertical axis the camera telescope. The theodolite has reiterative [?] horizontal and vertical limbs, separated by 360°. With the help of scale microscopes, whose magnification is $36^{\times}$, corresponding angles are read off with an accuracy of 30". The magnification of the theodolite telescope is $30^{\times}$; sharpness of the image is attained by movement of theinternal lens, and sharpness of the grid of lines by shifting the eyepiece. Range marks make it possible to determine from the rod the length of the base under the condition of perpendicularity to it of the average [middle] beam of sighting, by the formula

$$B_s = 0{,}1 + 100\,m, \tag{199}$$

where m is the difference of readings along the rod.

Furthermore, the range finder is used during laying of theodolite traverses; during measurement of bases the tangential screw of the theodolite is used, and gives high accuracy.

Essentially the tangential screw is a micrometer screw for turning the telescope along the horizon within limits of ±3°.

To determine the length of the base set the theodolite on one end and the range rod on the other; the rod, 1 m in length is set horizontally and perpendicular to the base. The telescope of the theodolite is matched to the middle of the range rod and the tangential screw is set in the middle position (reading 10.00); with this its axis is perpendicular to the rod. Then the tangential screw is used to match consecutively the intersection of the lines the telescope to the left and right mark of the rod (Fig. 180) and [they are] read off on the drum of the screw.

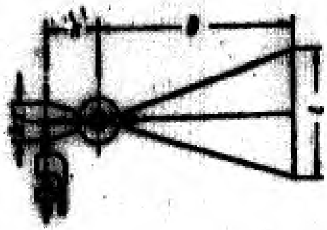

Fig. 180.

Independently of the difference in elevation between the ends of the base, the difference n of readings permits determining the projection of base B to the horizontal plane. Let us determine the base as a function of readings on the drum of the screw. The step of the screw is usually equal to 0.5 mm and the drum of the screw has 100 divisions; consequently the linear shift of the screw during its rotation is equal to 1/200 mm per division. The magnitude of the radius k is established as equal to 100 mm or, in divisions of the drum, k = 20, 000τ. According to Fig. 180, with n divisions

$$\frac{B}{30000} = \frac{l}{n\tau},$$

whence the value of the base

$$B = \frac{Nl}{n\tau} = k\frac{30000}{n}l \qquad (200)$$

This expression is a working formula for determination of the projection of the base to the horizontal plane. The value of coefficient kl is verified on a 20-30 m segment of the terrain, thoroughly measured with a tape.

2. Tripods with Supports

Tripods consist of six legs of round section, joined in pairs and fixed into metallic parts. The support of the tripod, having three base screws, services in turn the camera, the theodolite, and the range rod; it has locking attachments, which when fastened ensure complete immobility of the instrument. On the support there is a small round 5-minute level and lifting screws for bringing the phototheodolite or range rod to the horizontal position.

3. Range Rod

Range rod 1-3 m in length is applied as horizontal rod. It consists of a middle metallic part with a threaded channel and two wooden tubes 90 mm in diameter with metallic tips. Inside the tubes there are rods of steel, with having small temperature coefficient of expansion. When the tubes are screwed in the rods touch the precision flanges of the middle part. With such a device constancy of the length of the range rod is ensured. The middle part of the rod there is attached a telescopic sight with crosshairs in the focal plane, rotating in the vertical plane and perpendicular to the axis of the rod. A telescope with a focal length of approximately 60 mm serves for setting the rod perpendicularly to the middle beam of sight of the theodolite.

§ 55. Checks of Phototheodolites

Checks are made before beginning of work and during operations. Some checks require appropriate attachments.

The objective of the camera should give sharp images, since measurements on the negatives are produced with an accuracy of 0.02 mm. On the edges of field at an angle of 25° it should have a resolving power of 70-80 lines/mm. During investigation of the objective on optical benches elements of internal orientation and distortion of image are also determined.

The vertical axis of rotation of the phototheodolite should be vertical. The check is produced according to the level of the instrument during a turn around the vertical axis by 180° and it is corrected by screws of the level and the instrument with an accuracy of 20". The plane of the reference frame of the camera should be parallel to the vertical axis of rotation of the camera. The observance of this condition is checked by photographing two plumb lines 7-8 m in length at a distance from the instrument of approximately in 20 m; the distance between the plumb lines is 10-15 m. After development of the negatives the distance between the images of the plumb lines is measured on the stereocomparator. The images of the two plumb lines should be parallel.

Let us assume that (Fig. 181) S is the center of projection; o is the principal point of the photograph; oi is the image of the first plumb line, located approximately in the central part of the photograph; mn is the image of the second plumb line; i is the point of convergence of the lines depicting the two plumb lines. Then, taking segments z symetrically with respect to axis x, we will obtain

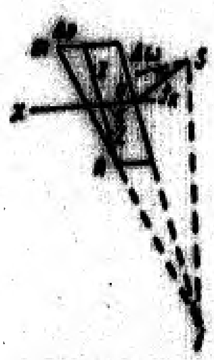

Fig. 181.

$$\frac{\Delta p}{2z}=\frac{x}{oi}$$

or

$$\frac{\Delta p}{2z}=\frac{x}{f_k}\Delta\omega,$$

whence

$$\Delta\omega=\frac{\Delta p\cdot f_k}{2xz}. \qquad (201)$$

Here $\Delta\omega$ is the angle composed by the plane of the negative with plumb line. With displacement of the image of plumb line Δp = 0.02 mm, f_k = 190 mm, 2z = 80 mm and x = 70 mm, this angle is approximately equal to 2'.

The main horizontal of the photograph, passing through the principal point, should be in the horizontal plane passing through the center of the objective. The check is carried out with a level and rod fixed 10-15 m from the phototheodolite, and also a millimeter ruler set on the phototheodolite.

If the readings of the rod and ruler are equal to N and h, respectively, and the distance from the base of the ruler fixed on the phototheodolite to the center of the objective is d, then the line of the horizontal on the photographic plate should be at a height corresponding to a reading on the rod of N - (h + d). The rod is photographed by the phototheodolite; when there is any noncoincidence of the image of the rod reading on the photographic plates with the line of the coordinate marks in coordinates z of the center of the objective one should introduce a correction. With a levelling accuracy of 2 mm per 10 m the accuracy obtained in the determination of the horizon is 2/10,000 = 40", which is fully sufficient for manifestation of the error in the element of internal orientation, Δz, of the camera.

The main vertical of the photograph should pass through the marks of the reference frame and the principal point. To check this condition in the absence of an adjusting attachment, photograph the plumb line depicted in the zone of location of marks. After development and drying of the negative, compare the images. When there is an adjusting and drying of the negative, compare the images. When there is an adjusting attachment photographing is not required. The attachment consists of a frame in which there is placed a glass with a double line in the middle, about 130 mm in length. Above the line stroke there is an easily shifted magnifier, intended for observation of the exact matching of the line with the marks of the reference frame and the image of the plumb line in the bisector of the lines; if it is necessary, the position of the frame marks are corrected.

The optical axes of the visual and adjusting telescope must lie in the same vertical plane with the optical axis of the camera. The check is produced with the help of an adjusting attachment. When the telescopes are turned so that the image of the figures 0-180° of the limb coincide, of camera the line of the adjusting is matched through the camera objective to point of site.

If the thread of the telescope is displaced from the object, then the position of the grid of telescope threads is corredted by a shift of the ocular bisector [SZ/B] (C3/B) or by turning the sighting system (19/1318).

Negatives should lie tightly against the reference frame, in order to obtain correct values of focal length and coordinates of points. A corresponding error (for instance, 0.1 mm) is caused by poor work of the springs and by curvature of the negative glass. To eliminate this error it is recommended to use plates or negatives made from plate glass. The error is revealed by comparison of the distances ℓ between marks in the camera and ℓ_1 on the negative.

If from relationship

$$\frac{l_1}{l} = \frac{f_k + \Delta f_k}{f_k}$$

we will subtract by unity, then we will obtain

$$\Delta f_k = \frac{f_k(l_1 - l)}{l}. \qquad (202)$$

When f_k = 180 mm and l = 180 mm we will find that $\Delta f_k = l_1 - l$. Hence we conclude that the distance betv :n images of marks should be sustained with an accuracy of 0.02-0.03 mm – the necessary accuracy of the value of focal length.

The chamber of the phototheodolite and the cassette must be light-tight. To check the observance of this condition place phototheodolite with the cassette filled with photographic film, under sunlight for 10 minutes and then develop the plates [sic]. The manifestation of traces of coordinate marks should not be considered, since their appearance depends on the influence of the metal on the emul.ion.

After these checks, determine the focal length and the position of the principal point of the photograph, which is the origin of reading of coordinates. These determinations are carried out on special optical benches.

§ 56. Phototheodolite Survey

Surveying is preceded by reconnaissance of the terrain for selection of the stations of the phototheodolite survey, i.e., bases of photographing, the establishment of the sequence of operations on tying the stations to the geodesic control net, the distribution of control points, and the manifestation of dead zones.

The lengths of the bases of photographing depend on the scale of the map to be composed. During selection of the base it is necessary to be sure that it provides a good view of the terrain to be photographed and that from each end of the base it will be possible, under given circumstances of illumination, to produce a survey in the three positions of the principal axis of the camera (normal, deflected to the left, and deflected to the right).

Location of the principal axis of the phototheodolite at survey is shown in Fig. 182.

Tying of stations to the geodesic control net can be accomplished by laying out the theodolite traverses (by the theodolite of the set), the solution of intersections

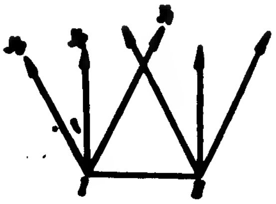

Fig. 182.

and resections, etc.

On the control points tripods are set up with shields or sighting marks, which will be depicted on the photographs; tying of control points to bases permits using them for control of the operations.

With a standard deviation of axes by 31°30' the angle of the field of view from each station should not exceed 120°. At the same time a section of terrain located between the stations and also located in front of the base at distance Y = 4B is not processed, since in these zones the stereoeffect is poor. When it is necessary to process deed zones it is necessary to produce a survey from other bases.

The survey of the terrain is executed in the sequence presented below. On one end of the base establish a phototheodolite on a tripod and on the second end a tripod with a signting mark. Then carryout horizontalizing and orientation of the principal axis of the phototheodolite on the mark. After photographing in three positions of the principal axis on station I, move the phototheodolite manually to station II, orient the camera, and carry out photographing also in three positions of the principal axis of camera. When photographing of the site is completed on station II, remove the camera and set in its place a theodolite with a tangential screw. Measure the direction to the brand fixed on the end of the base, to separate objects, and to visible geodesic points. Carry out the measurements at two positions of the telescope. After measurement of directions measure the length of the base on a range rod, set in place of a mark on station I. Using the tangential screw, match the crosshair consecutively on the left and right ends of the rod and find the magnitude of the difference in readings, and consequently also [that of] base B, where for more precise definition of the obtained result the quantity B is measured 4-6 times. A diagram of recording of the measurements of the base is shown in Table 29.

Bases with lengths up to 100 m are determined with an accuracy of 1:1000 (with length of rod l = 1 m). If because of the conditions of survey the base should be greater than 100 m, then use an auxiliary base with length $b=\sqrt{Bl}$, measured by the tangential device. For this select point III on the terrain, located approximately at an angle of 70-80° to the line of the base and separated from

Table 29

Readings on the drum of the tangential screw		Differences in readings $n_1 - n$	Note
n_1	n_2		
1272	800	472	Average $n = 472$
1283	810	473	$B = \frac{20\,000}{472} = 42.4$[1] m
1297	815	482	
1261	790	471	

station I by a distance of 10-50 m (Fig. 183). At point III set a range rod perpendicular to the auxiliary base I-III, which permits a theodolite set up at station I to measure the angle γ and allows the tangential screw to measure the length of base I-III. Then transfer the theodolite to station II and there measure the angle $\alpha_1 + \alpha_2$ (on the order of 3-6°) with help of a tangential screw, giving higher accuracy than direct measurement of the angle on the limb. In this case the tangential screw is set in the middle position, at which its axis is perpendicular to the rod, and the telescopic sight is directed approximately on the middle of the auxiliary base; then the angles α_1 and α_2, the sum of which is equal to the unknown angle, are measured. Angles α_1 and α_2 are calculated by the formulas

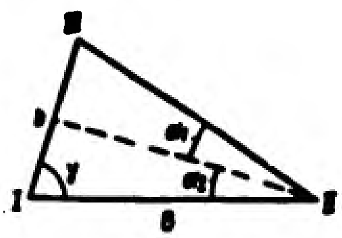

Fig. 183.

$$\left.\begin{aligned} \mathrm{tg}\,\alpha_1 &= \frac{n_0 - n_1}{20\,000} \\ \mathrm{tg}\,\alpha_2 &= \frac{n_2 - n_0}{20\,000} \end{aligned}\right\}, \qquad (203)$$

where n_0, n_1, and n_2 are readings on the down of the screw at the middle and extreme positions of the tube, respectively.

The unknown base is equal to

$$B = \frac{b \sin(\alpha + \gamma)}{\sin \alpha}. \qquad (204)$$

§ 57. Processing of Photographs of Ground Survey

Independently of the purpose of ground stereophotogrammetric survey (composition of map of terrain or determination of the spatial coordinates of its

[1]Here 20,000 is the "constant" of the range attachment.

individual points), measurements are made on a stereocomparator. First measure the coordinates x_1, z_1 and parallaxes p of points is of the photographs and then by formulas (190)-(193) calculate the coordinates of these points relative to the left end of the base of photographing. Using the geodesic coordinates of station S_1 and the azimuth of the principal axis of the camera, calculate the coordinates of the determined points.

Use of the stereocomparator. The stereocomparator intended for the treatment of negatives of ground stereophotography differs from those described earlier only in the size of the cassette (13 × 18 cm) and magnification (6^x). Checks of the instrument remain the same. With this instrument it is possible to measure coordinates and parallaxes with the high accuracy necessary during the creation of large-scale maps and profiles.

During measurements the negatives are set on cassettes with the emulsion side down and are oriented, i.e., parallelism of the axes of coordinates of photograph to the corresponding axes of instrument is sought. For this, transfering the binocular to the distant position along axis z, shift carriage x_1 until the left mark appears exactly on the mark of the main vertical of the left photograph. Then, with carriage x_1 motionless shift the binocular to the near position, after which half of displacement of the mark from the mark of the main vertical is removed by screw κ for rotation of the photograph in its plane, and other half is eliminated by hand wheel X_1. After repetition of the process, seek the accurate adjustment of negative at which the mark coincides with the upper and lower marks. At this instant, with the help of mobile vernier, fix the zero point of scale x_1. Then match the mark onto the mark of the main horizontal and record the zero point of scale z_1. These observations can be produced monocularly. The setting of the right negative is done analogously, though by using parallactic screws p and q and screw κ_2 for rotation of the right photograph. Mobile verniers establish the values of zero points of x, p, and z.

Measurement of coordinates x_1, z_1 and parallaxes p is begun from some characteristic point and is then conducted consecutively over the entire area of the stereopair. Bringing in under the left mark the image of sought contour points (by movements of x and z), rotate the parallactic screw until stereoscopic matching of the mark with the terrain point is achieved.

In the process of fulfillment of works it is very frequently necessary to remove the vertical parallax, which although it does not affect the accuracy of measurement does disturb the stereoeffect.

Besides readings of coordinates and parallaxes, on the fields of journals hand sketches of the observed area are sometimes made.

Drafting board. To accelerate the process of treatment of materials of stereophotogrammetric survey graphical solution of the equations connecting the coordinates of points of the photograph and the terrain is applied. Figure 184 depicts a diagram of the accessory intended for this (drafting board). The board has axis S on which there rotates ruler [bar] SL with a leveled edge. At five times the focal length from axis S and perpendicular to the conditional principal axis So there is plotted scale x_1 with 5-mm divisions, corresponding to the millimeter divisions of the stereocomparator.

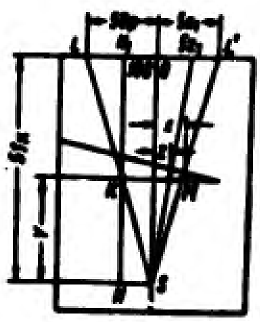

Fig. 184.

To the left of principal axis So at distance 10B (expressed in the scale of the plan) there is constructed line NN_1, parallel to So. Magnified parallax 50p is plotted to the left of point o on the earlier plotted scale.

If the bar is set in position SL, its intersection with line NN_1 will determine point K, located at distance Y from point N_1. Actually, according to Fig. 184

$$\frac{50p}{10B} = \frac{5f}{Y},$$

whence

$$Y = \frac{Bf}{p}.$$

For determination of coordinate X at point K set the side of triangle, perpendicular to So. The bar is turned to position SL, where $oL' = 5x_1$. At the intersection of the edge of the triangle with fixed direction SL' point M with coordinate X is determined. On the same fivefold-enlarged scale there is established reading z_1, which makes it possible to determine the elevation Z of point M above the left end of the base. For convenience of work the values of divisions of scales of the stereocomparator are written on the scale of parallaxes and the scale x_1.

Treatment of photographs with deflected axes can be carried out on the same instrument, but by a more complicated construction. From the results of these determinations it is possible to compose a map of a section directly on the drafting board. The given method of processing photographs is recommended only in the absence of automatic stereoinstruments.

Magnitudes of parallaxes and base. For construction of a map with a precision of 0.001 with accuracy of measurements of parallaxes of 0.01 mm it is necessary before the survey correctly to establish the length of the base.

Differentiating formula $Y = \frac{Bf_k}{p}$ in terms of Y and p, we obtain

$$dY = -\frac{Bf_k}{p^2} dp,$$

whence

$$\frac{dY}{Y} = -\frac{dp}{p}.$$

With dp = 0.01 mm and dY/Y = 0.001 we obtain $|p_{min}|$ = 10 mm.

The maximum value Y at f_k = 200 mm corresponds to the given parallax:

$$Y_{max} = \frac{B200}{10} = 20B. \quad (205)$$

Formula (205) is applied during the selection of the length of the base at a given Y_{max} (depending on the scale of the proposed map). The angle of intersection on distant objects in this case is equal to 3°.

The minimum distance is determined by the maximum value of the parallactic angle γ, with [its] apex at the observed point; here

$$B = Y_{min} \operatorname{tg} \gamma.$$

During ground stereophotography the angle should not exceed 15°, since large changes of parallaxes of neighboring points appearing [at greater angles] cause eye fatigue. It follows from this that

$$Y_{min} = 4B. \quad (206)$$

Thus, the maximum parallax has the value (at f_k = 200 mm)

$$p_{max} = \frac{Bf_k}{Y} = 50 \text{ мм}. \quad (207)$$

It follows from this that the parallax screw of the stereocomparator should ensure measurements within the limits 0-50 mm.

§ 58. Stereoautographs

Diagram of intersection. Stereoautographs are intended for automatic tracing of a topographic map from photographs of a ground stereophotogrammetric survey with horizontal location of the principal axis of the phototheodolite.

The instruments are based on the use of the properties of mechanical intersection. Fig. 185 depicts a diagram of intersection at the time of survey. Terrain point M is depicted on the negatives at points m_1 and m_2. Negatives are placed in the instrument in such a manner that the observer sees their direct image; consequently, the negatives are turned at 180° relative to their true position in the phototheodolite. Angles A_1 and A_2 between directions to the object and the main rays will remain the same, but their signs will change, if directions S_1M and S_2M pass through the principal point of the photographs. At a diatance from the photographs equal to the focal length f_k of the camera we will dispose axes S_1 and S_2 of rotation of the straightedges carrying out intersection; the short arms of these bars, S_1o_1 and S_2o_2. will be hinge-connected with the negative carriages. Then with intersection of the straightedges at point M the regular angles A_1 and A_2 will be constructed and the images m_1 and m_2 of determined point M will appear on the sighting axes K_1 and K_2 of the binocular. Consequently, the observer will see a single stereoscopic image of the point, combined with the floating spot of the binocular. The base of the survey. expressed in the scale of the map, is usually small (10-15 mm). Therefore in the instrument point S_2, corresponding to the right end of the base (survey station), is displaced along line S_1S_2; instead of stations S_1 and S_2, stations S_1 and S_3 are used. Carriage MM_0, travelling along axes X and Y, is located parallel to line X. Point M_0 is the initial point, relative to which base b is set. With this the intersection of point M has the form of quadrangle, consisting of triangle S_1S_2M and parallelogram $MS_2S_3M_1$.

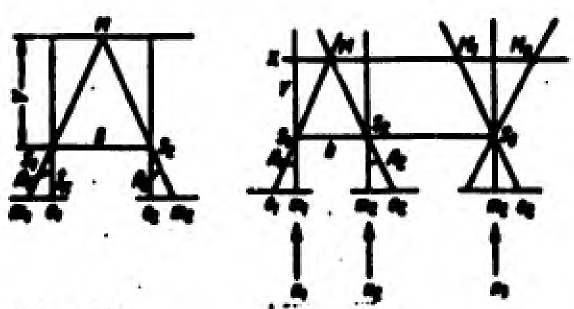

Fig. 185.

The advantage of intersection if the form of "triangle plus parallelogram" as compared to intersection from base S_1S_2 lies in the fact that stations S_1 and S_3

remain motionless with different bases. Consequently, the sighting axes K_1 and K_3 of the binocular microscope are also stationary.

During convergent survey, when the principal axes compose one or another angle, short arm S_3o_2 of the straightedge is set at this angle with respect to the long arm S_3M_1. Thanks to this the negatives remain in a single plane and the sighting axes of the binocular, as before, are stationary.

By this principle it is also simple to automate intersection with equally deflected axes (Fig. 186). With respect to line S_1S_3 to the instrument base b is set at angle ϕ. To obtain the direction S_3M_1, parallel to direction S_2M, it is necessary to establish point M_1 with respect to the initial point M_0 by the magnitudes:

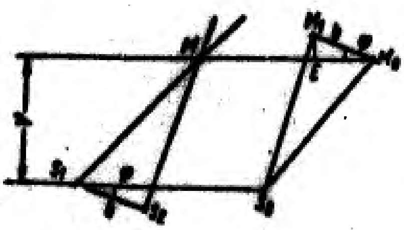

Fig. 186.

$$EM_0 = b\cos\varphi, \quad (208)$$

$$EM_1 = b\sin\varphi, \quad (209)$$

which is attained by the application of base supports which are mutually perpendicular. Thus, at any position of the principal axes in the horizontal plane automatic intersection requires settings of three elements: the angle of convergence and the two base components.

Automatic determination of coordinate Z is carried out (Fig. 187) with help of a third straightedge, S_1C, called the height arm. With this the elevation h of a point of the terrain above point S_1 is obtained at point M'. The short arm, the height arm, composes a 90° angle with its long arm; it is connected with the binocular microscope, thanks to which the latter shifts. With a turn of arm S_1C by angle β_0 the binocular is displaced by the magnitude of coordinate Z. Consequently, arm S_1C solves the equation

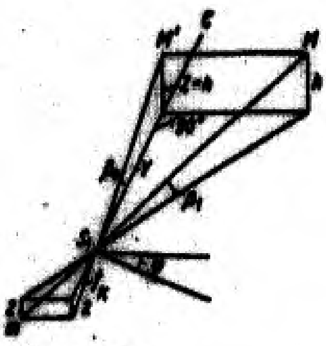

Fig. 187.

$$\frac{z}{Y_b} = \frac{Z}{Y}.$$

The first stereoautographs (designed by the engineer Orel) were built by Zeiss in 1910-1912. They remained unchanged up to 1940 and were successfully applied for the creation of maps on scales of 1:2000 to 1:25,000. Soviet stereoautographs [6], distinguished by compact arrangement of parts, were produced from 1932 through 1939.

At present we have the Zeiss stereoautograph (German Democratic Republic), produced since 1953. The instrument is applied in a number of establishments using ground stereophotogrammetric survey.

<u>Zeiss Stereoautograph</u>. The set of the instrument (Fig. 188) includes an optical-mechanical system, realizing intersection of determined points in conditions of stereoscopic observation, and a coordinatograph with a reductor, which allows tracing maps in several scales.

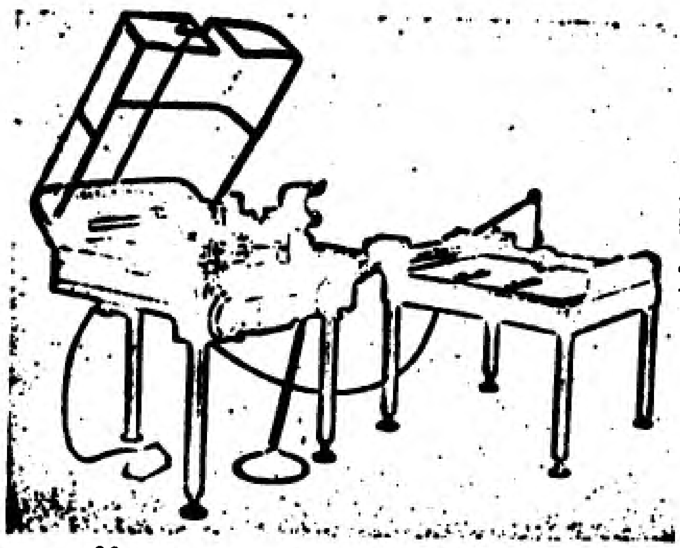

Fig. 188.

While in the models of stereoautographs mentioned above three intersection arms were used, in this design there are four of these arms, thanks to which transverse parallaxes are not observed in the model during work on the instrument.

We present the diagram of the arrangement of the instrument (Fig. 189). In the foreground there are carriages with negatives from terrestrial photosurvey.

Direction arms L_{x_1} and L_{x_2} rotate around centers I and II, representing photographing stations. They are joined by a short arm at distance f_k and at distances Y_1 and Y_2 by a long arm with the negative carriages. Carriages X and the base components b_x and b_y are correspondingly hinge-connected. Carriage X_1 with distance piece can be shifted forward and backward by a hand wheel and to the left and the right by hand wheel X. As a result the position of points will be determined by two coordinates. The rotation of the wheel screws is transmitted to the screws of the coordinatograph, where the map is traced.

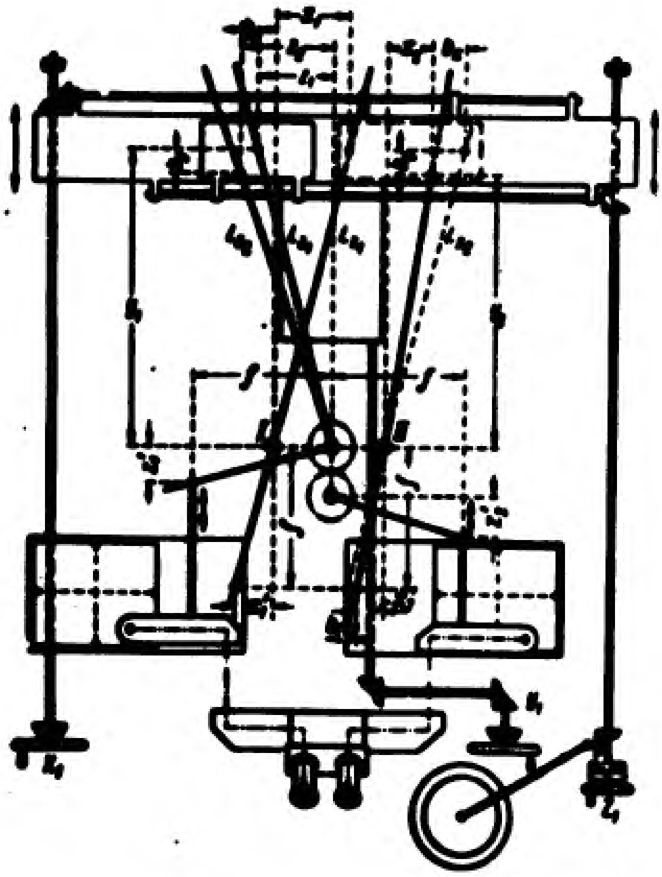

Fig. 189.

Bridge clarified-I carries besides guide X with a carriage, the distance piece carries guide Z, also with a carriage, located higher than the carriage X.

Two L-shaped levers are joined by their short arms to the left and right parts of the optical system, and by the long [arms] L_{z_1} and L_{z_2} to the joint of carriage Z_1 and that of supports b_y, b_z.

Hand wheel Z can be used to introduce section [profile?] of the terrain and thereby to determine the third coordinates of points.

The optical system (Fig. 190) consists of two symmetrical branches. The left branch contains movable part 3, 4, 5, picking up rays from the photograph illuminated by lamp 1 through reflector 2, and fixed part 6, 7, 8, 9, 10, 11; in the focal field of the eyepiece the image of the photograph is sketched.

Through semisilvered pentaprism 8 the additional part 16, 15 gives an image of mark 14, illuminated by lamp 12 with condenser 13. Construction of the luminescent mark by parallel movement of beams 4-7 requires exact parallelism of rays 5-6 of axis Y of the instrument.

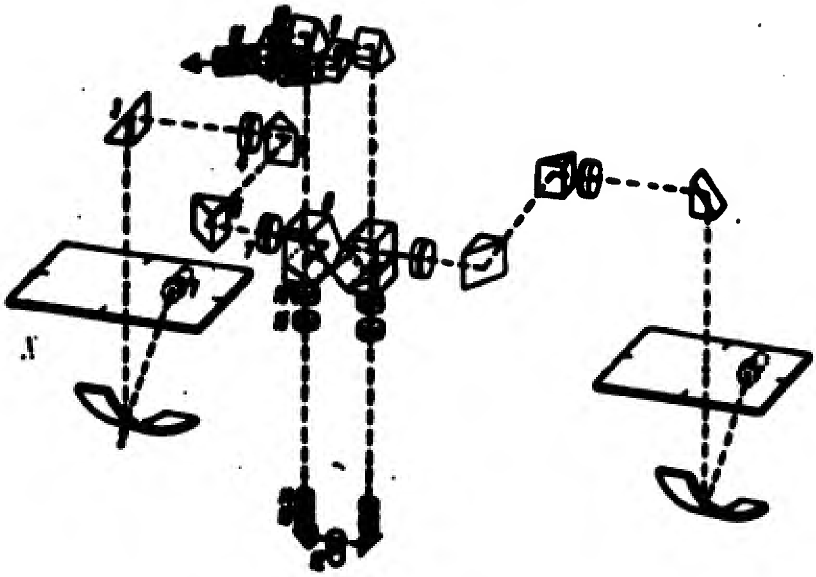

Fig. 190.

The coordinatograph permits carrying out approximate manual setting of pencil above a point of the plane table after removal of the guide nuts of screws X and Y; exact setting is done by means of two screws located close to the solenoid of the pencil hoist [lifter].

During adjustment of the instrument set all parts in the horizontal position (with the help a level with one-minute accuracy). Check also that all movements of the parts of the instrument (X, b_x, Y, b_y, Z, x_1, z_1, f_k) are carried out along directions parallel or perpendicular to guide Y, with an accuracy of 0.02 mm. This is attained with the help of a measuring grid and a telescope with a magnification of 6×. Then determine the zero points of scales X, Z, Y, b_x, b_y, and b_z in approximately the same way as on the stereoplanigraph.

For processing photographs on the stereoautograph levers are set in a position in which readings on the X, b_x, b_y, and b_z scales are equal to zero. After that the negatives are oriented on the carriages in such a way that the main verticals of the negatives pass through the line of sight of the binocular; at the zero position of the arm the line of sight of the left branch of the binocular should pass through the main horizontal of the left photograph. Then b_x and b_y are set in the scale of the map, the plane table is laid on the drafting board of the instrument, and processing of the stereopair is begun.

For drawing horizontals, set the appropriate readings on the scale of heights and, using movements of the hand wheels of the X and Y carriages, move the mark over the stereoscopic model in such a manner that it continuously touches its surface. The pencil will draw on the plane table the trace of the total movement of the carriages.

Shifting the mark by movements of X, Y, Z, trace the contours of roads, woods, precipices, etc. Table 30 presents data characterizing the stereoautograph.

Table 30

Elements of instrument	Numerical data
Size of photograph, cm	[illegible] × [illegible]
Focal length, mm	[illegible] . . . [illegible]
Horizon line marks on the negative, mm	± [illegible] . . . − [illegible]
Convergence γ_0, in degrees	± [illegible] . . . − [illegible]
Base: b_x, mm	[illegible] . . . [illegible]
b_y, mm	± [illegible] . . . − [illegible]
b_z, mm	± [illegible] . . . − [illegible]
Coordinates X, mm	− [illegible] . . . + [illegible]
Y, mm	− [illegible] . . . + [illegible]
Z, mm	− [illegible] . . . + [illegible]
Magnification, X	[illegible]
Transmissions to coordinatograph, magnification	[illegible]
Dimension of plane table of coordinatograph, mm	[illegible] × [illegible]
Area occupied by stereoautograph, cm	[illegible] × [illegible]
Height of instrument, cm	[illegible]
Area occupied by coordinatograph, cm	[illegible] × [illegible]
Weight of instrument, kg	[illegible]
Weight of coordinatograph, kg	[illegible]

The instrument is used for tracing topographic maps, and also for the creation of mining survey plans, for the treatment hydrotechnical photographs, etc. [12].

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